NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

*Technical Report 32-1240*

# *ELAS—A General-Purpose Computer Program for the Equilibrium Problems of Linear Structures*

## *Volume I. User's Manual*

*Senol Utku*

*Fevzican A. Akyuz*



JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

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Subject: Errata

Gentlemen:

The following paragraph and Table IV-1a should replace the description of Input Item 13 on page 19 of Technical Report 32-1240, Vol. I, entitled "ELAS - A General-Purpose Computer Program for the Equilibrium Problems of Linear Structures, Vol. I, User's Manual," by Senol Utku and Fevzican A. Akyuz, dated Feb. 1, 1968:

Input Item 13 (Angle Types - Fixing Local y and z Axes). No input card is required for this input item if the content of the IMFI field of the control card (Input Item 2) is zero; otherwise, one or more cards are prepared, as shown in the first line of Input Item 13 in Table IV-1, to list the different angles $\phi$ in degree units. Angle $\phi$ is a signed quantity, the absolute value of which is not greater than 90. Let l, m, n denote the direction cosines of the local x axis, $\alpha$, $\beta$, $\gamma$ the direction cosines of the local y axis, and p, q, r the direction cosines of the local z axis. One does not need to compute these quantities; however, the signs of l, q and $\alpha$ are used in determining the sign of $\phi$. When the local x axis is not parallel to the overall Y axis, i.e., $|l|$ and $|n|$ are not simultaneously less than or equal to 0.0001, $\phi$ is the angle between the local y and the overall Y axes. In this case, the local y axis should be directed such that $\beta = \cos\phi$, and angle $\phi$ should carry the negative of the sign of the (ql) product (when ql is zero, its sign may be assumed negative) if $|l| > 0.0001$, and $\phi$ should carry the sign of $\alpha$ (if $\alpha$ is zero, its sign may be assumed positive) if $|l| \leq 0.0001$ but $|n| > 0.0001$. When the local x axis is parallel to the overall Y axis, i.e., $|l|$ and $|n|$ are simultaneously less than or equal to 0.0001, $\phi$ is the angle between the local y and the overall Z axes. In this case, the local y axis should be directed such that $\gamma = \cos\phi$, and $\phi$ should carry the sign of $\alpha$ (if $\alpha$ is zero, its sign may be assumed positive). These statements are summarized in Table IV-1a, below. For the local x, y, z axes and the overall

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X, Y, Z axes, see Fig. VI-1. In Table III-4, column 16 shows which element type requires the angle specification. The contents of input cards of this input item appear in Output Item 2 (Section VI-B).

Table IV-1a. Table for determining the direction of the local y axis and the sign of angle $\phi$

| Parameter | $\lvert l \rvert > 0.0001$ | $\lvert l \rvert \leq 0.0001$<br>$\lvert n \rvert > 0.0001$ | $\lvert l \rvert \leq 0.0001$<br>$\lvert n \rvert \leq 0.0001$ |
|---|---|---|---|
| Positive direction for local y axis | Such that $\beta = \cos \phi$ | Such that $\beta = \cos \phi$ | Such that $\gamma = \cos \phi$ |
| Sign of $\phi$ | Negative of the sign of (ql)[a] | Sign of $\alpha$[b] | Sign of $\alpha$[b] |

[a]If (ql) is zero, its sign may be assumed negative.

[b]If $\alpha$ is zero, its sign may be assumed positive.

Very truly yours,

John Kempton, Manager
Publications Section

JK:cs

## Foreword

This work is dedicated to the memory of Professor M. Inan, whose recent untimely death was a loss to the academic and scientific world and a personal loss to the authors. We both had the privilege of studying under Prof. Inan's tutelage at the Technical University of Istanbul, and remember him as a brilliant teacher and a great humanist as well. His guidance and teachings were a major shaping influence in our lives, having first inspired our interests in directions that have led to our present field of work.

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# Contents

## Contents (contd)



## Tables

## Contents (contd)

### Tables (contd)



### Figures

# Contents (contd)

## Figures (contd)

## Abstract

A general-purpose digital computer program (named ELAS) for the in-core solution of linear equilibrium problems of structural mechanics is described for potential and actual users in Volume I of this report and documented in Volume II. The program requires minimum amount of input for the description of the problem. The solution is obtained by means of the displacement method and the finite element technique. Almost any geometry and structure may be handled because of the availability of lineal, triangular, quadrilateral, tetrahedral, hexahedral, conical, and triangular and quadrilateral torus elements. The piecewise linear deflection distribution assumption insures monotonic convergence of the deflections from the stiffer side with decreasing mesh size. The stresses are provided by the best-fit strain tensors in the least-squares sense at the mesh points where the deflections are given. The selection of local coordinate systems whenever necessary is automatic. The core memory is efficiently used by means of dynamic memory allocation, an optional mesh-point relabelling scheme, imposition of the boundary conditions during the assembly time, and the straight-line storage of the rows of the stiffness matrix within variable bandwidth and the main diagonal. The number of unsuppressed degrees of freedom that can be handled in a given problem is 500 to 600 for a typical structure, but might far exceed these average values for special types of problems; the execution time of such problems is about four minutes in 32K IBM 7094 Model I machines. The program is written in FORTRAN II language. The source deck consists of about 8000 cards and the object deck contains about 1400 binary cards. The physical program (standard ELAS) is available from COSMIC, the agency for the distribution of NASA computer programs.

# I. Introduction

ELAS, a general-purpose digital computer program for the in-core solution of linear equilibrium problems of structural mechanics, is described in two volumes. Volume I, the *User's Manual*, contains the information necessary for the use of ELAS. Volume II, *Documentation of the Program*, contains flow charts, block diagrams, source program listings, and other pertinent information related with the released program (standard ELAS). The physical program is available from the NASA agency COSMIC.*

In this volume, a general description of the program is given in Section II. In Section III, certain fundamental concepts necessary for input preparation and output interpretation are explained. Input preparation is described in detail in Section IV; Section V deals with the arrangement of the physical program; Section VI describes the output. A complete list of error messages and their explanations are contained in Section VII, and suggestions are given for the diagnosis of the related errors. Section VIII presents two sample problems, together with related input information and examples of computer printouts of standard ELAS output.

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# II. General Description of the Program

## A. Purpose

ELAS† is a general-purpose digital computer program that handles the equilibrium problems of linear structures of one-, two-, or three-dimensional continuum. The program requires as input (1) the coordinates, in an overall coordinate system, of the mesh points of a random one-, two-, or three-dimensional mesh established in the material volume of the structure of one-, two-, or three-dimensional continuum, respectively; (2) the geometrical, topological, material, and loading characteristics of the mesh elements; (3) the list of prescribed deflections and forces at the mesh points; and (4) a few program control parameters. As output it provides (1) the deflections at the mesh points, (2) the stresses at the mesh points, and (3) the listings of the input data. The different types of structures and their combinations that ELAS can handle are given in Table II-1.

## B. Method of Solution

ELAS generates the governing equations for the unknown deflections of the mesh points that define the stationary point of the total potential energy functional associated with the given loading and unknown deflections. If the distribution of the deflections in a mesh element is not known, it is assumed to be linear. Thus the coefficient matrix of the unknown deflections is always positive definite, symmetric, and usually bandwidth limited and sparse. Upon request, ELAS relabels the mesh points internally to decrease the bandwidth of the coefficient matrix. Those coefficients that are in the upper half of the variable band are generated and stored. The system of equations is solved with a special Cholesky algorithm. The computed deflections are then augmented by the prescribed ones, rearranged in the user's labels, and printed out. The stresses are computed upon request. In structures of two- or three-dimensional continuum, the best-fit strain tensors at the mesh points are used in the stress computations. The stresses and the deflections are expressed in the local coordinate systems, and printed out together with the direction cosines of the local axes with respect to the overall axes. The local coordinate systems at the mesh points are different than the overall coordinate system in the case of general shells and shells of revolution, and at the boundary points. When appropriate, the stresses in the overall coordinate system are also provided. The selection of the local coordinate systems is automatic unless otherwise specified.

†First two syllables of the word ELASTICITY.

**Table II-1. Types of structures that ELAS can handle (shaded squares indicate compatible combinations for ELAS)**

| Case number | Description of case | 1 Planar truss | 2 Space truss | 3 Planar frame | 4 Space frame | 5 Gridwork frame | 6 Plane stress[a] | 7 Plane strain[a] ($\varepsilon_z = 0$) | 8 Plate bending | 9 General solid | 10 General shell; bending, membrane | 11 General shell, membrane | 12 Solid of revolution | 13 Shell of revolution, membrane | 14 Shell of revolution; bending, membrane |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Planar truss | ■ | | | | | ■ | ■ | | | | | | | |
| 2 | Space truss | | ■ | | | | | | | ■ | | ■ | | | |
| 3 | Planar frame | | | ■ | | | | | | | | | | | |
| 4 | Space frame | | | | ■ | | | | | | ■ | | | | |
| 5 | Gridwork frame | | | | | ■ | | | ■ | | | | | | |
| 6 | Plane stress[a] | ■ | | | | | ■ | | | | | | | | |
| 7 | Plane strain[a] ($\varepsilon_z = 0$) | ■ | | | | | | ■ | | | | | | | |
| 8 | Plate bending | | | | | ■ | | | ■ | | | | | | |
| 9 | General solid | | ■ | | | | | | | ■ | | ■ | | | |
| 10 | General shell; bending, membrane | | | | ■ | | | | | | ■ | | | | |
| 11 | General shell, membrane | | ■ | | | | | | | ■ | | ■ | | | |
| 12 | Solid of revolution | | | | | | | | | | | | ■ | ■ | |
| 13 | Shell of revolution, membrane | | | | | | | | | | | | ■ | ■ | |
| 14 | Shell of revolution; bending, membrane | | | | | | | | | | | | | | ■ |

[a]Cases 6 and 7 may not exist simultaneously.

## C. Limitations

In a given problem, ELAS can handle up to 99 different types of each of the following: materials, temperature changes, temperature gradients, cross-sectional areas, moments of inertia, angles defining principal axes of cross-sections, torsional constants, thicknesses, and pressures. The number of mesh points and the number of mesh elements may be as high as 9999; however, the shaded area in the schematic description of the governing equations shown in Fig. II-1 is the limiting factor in most of the problems for a given computer. For the shaded area in Fig. II-1, as much as 75% of a 32K core memory is allocated (roughly 19,000 36-bit words). The limited experience with the use of this program indicates that one may solve problems up to about 600 unsuppressed degrees of freedom in 32K machines. In a 64K machine, ELAS may handle up to about 1500 unsuppressed degrees of freedom. Since the storage allocations are done dynamically at the execution time, ELAS may be used in machines of different core capacity without change. The average run time of a 600-unsuppressed-degree-of-freedom problem in IBM 7094 Model I machines is of the order of four minutes.

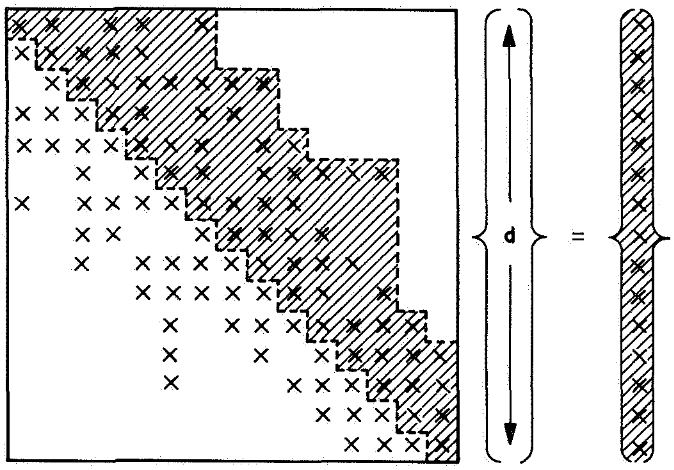



**Fig. II-1. Sketch of governing equations in deflections (only the shaded area need be stored)**

### D. Programming Language

ELAS is a system of programs and subprograms, all of which are written in FORTRAN II language, with the exception of the three subprograms named TICK, SEBIN, and LEBIN, which are in FAP language (they constitute less than 0.3% of the whole program). FORTRAN II was selected because it uses less core area (and consequently allows more area for the user) for the system programs during the execution time than the other languages.

### E. Computer Hardware and Operational System

The program has been developed for the 32K IBM 7094-7044 direct-coupled system; however, it may be used in other systems that have the FORTRAN II compiler and the FAP assembler. The operational system should be one that is compatible with the FORTRAN II compiler and the FAP assembler used.

### F. Brief Description of the Physical Program

The program consists of four chain links. The deck arrangement is shown in Fig. II-2. The user may arrange

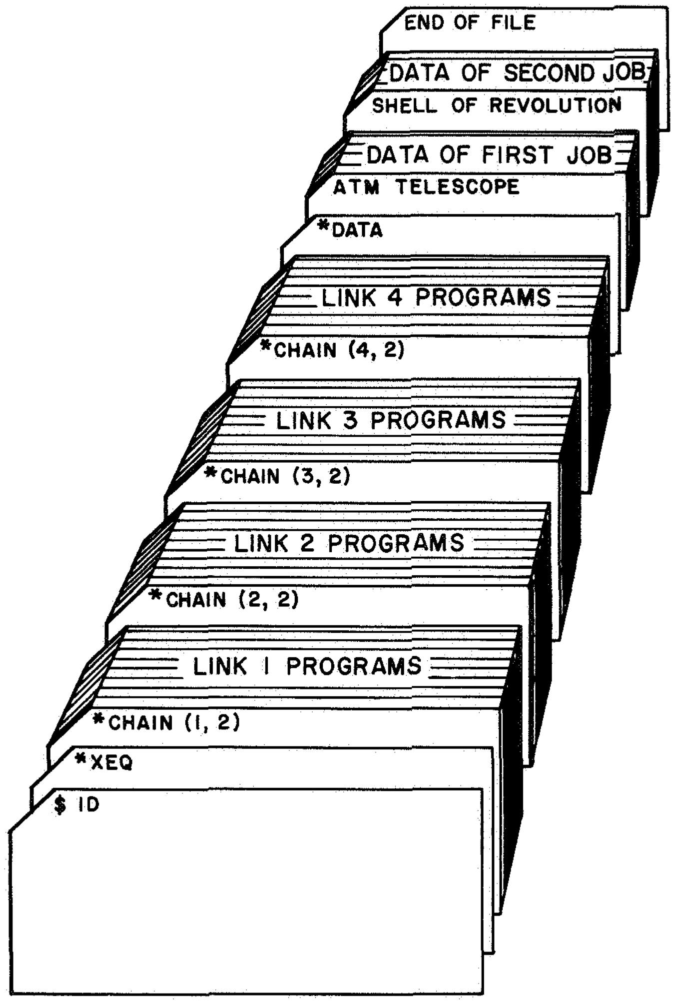



**Fig. II-2. Physical arrangement of ELAS program**

successively the data decks of an indefinite number of different jobs that are solved successively one after the other. In executing a given job, the user may employ any one of the following options: Link 1 only; Links 1 and 2; Links 1, 2, and 3; or Links 1, 2, 3, and 4. The source deck consists of about 8000 cards and the object deck contains about 1400 binary cards.

# III. Definition of an Equilibrium Problem for a Digital Computer

## A. Degrees of Freedom at a Point

The number of pieces of independent scalar information at a point, necessary to define the state related with the primary unknown system variables, is an important parameter in determining the storage area. In the ELAS program, the primary unknown system variables are the deflections. The number of pieces of independent scalar information needed to determine the deflection state at a point is the same as the number of deflection degrees of freedom. Given a problem, ELAS assumes that all points have the same number of degrees of freedom. Of course, any number of these may be prescribed. Degrees-of-freedom directions are those implied by the overall coordinate system, namely, the displacements along, and the rotations about, the $X$, $Y$, and $Z$ axes of the overall coordinate system. In Table III-1, the deflection degrees of freedom at a point in the structures listed in Table II-1 are given. The structures that have the same deflection degrees of freedom at a point are the compatible structures indicated in Table II-1 by shaded squares. The last column in Table III-1 contains the number of degrees of freedom at a point of the structure. The necessary storage area is roughly proportional to the square of this number; therefore, when options are available in the structural idealization, the structure with fewer degrees of freedom at a point is preferable.

## B. Definition of the Geometry of the Structure

In order to solve an equilibrium problem, the geometry of the structure should be defined. This can be done by constructing a mesh in the material volume of the structure by means of the coordinates of the mesh points in the overall coordinate system. Depending upon the type of the structure, the mesh is one-, two-, or three-dimensional, as shown in Fig. III-1. In the structures of two- and three-dimensional continua, the more refined the mesh, the better the approximation. One can select the mesh points at will, provided that they include the points of the structure where the deflections and stresses are requested, and also those points where deflections and/or concentrated loads are prescribed.

The mesh points may be thought of as joined by straight lines or planes to define subdomains that are not overlapping, and as covering the material volume of the structure completely. These subdomains are called "finite elements." In ELAS, line segment, triangle, quadrilateral, conical segment, tetrahedron, hexahedron, triangular torus, and quadrilateral torus options are available for elements. Actually, there are 18 different types of elements for the types of structures listed in Table II-1. Each element has an identification number called "element type number." This should not be confused

**Table III-1. Deflection degrees of freedom at a point for different cases of structures**

| Case number | Case description \ Degree-of-freedom direction[a] | 1 Displacement along X | 2 Displacement along Y | 3 Displacement along Z | 4 Rotation about X | 5 Rotation about Y | 6 Rotation about Z | 7 Number of degrees of freedom |
|---|---|---|---|---|---|---|---|---|
| 1 | Planar truss | ■ | ■ | | | | | 2 |
| 2 | Space truss | ■ | ■ | ■ | | | | 3 |
| 3 | Planar frame | ■ | ■ | | | | ■ | 3 |
| 4 | Space frame | ■ | ■ | ■ | ■ | ■ | ■ | 6 |
| 5 | Gridwork frame | | | ■ | ■ | ■ | | 3 |
| 6 | Plane stress | ■ | ■ | | | | | 2 |
| 7 | Plane strain | ■ | ■ | | | | | 2 |
| 8 | Plate bending | | | ■ | ■ | ■ | | 3 |
| 9 | General solid | ■ | ■ | ■ | | | | 3 |
| 10 | General shell; bend., memb. | ■ | ■ | ■ | ■ | ■ | ■ | 6 |
| 11 | General shell, membrane | ■ | ■ | ■ | | | | 3 |
| 12 | Solid of revolution | ■ | ■ | | | | | 2 |
| 13 | Shell of revolution, membrane | ■ | ■ | | | | | 2 |
| 14 | Shell of rev.; bend., memb. | ■ | ■ | | | | ■ | 3 |

[a]X, Y, Z refer to the axes of the overall coordinate system.

with the element labels, which are sequential integer numbers assigned on a one-to-one basis to the elements of a given structure. In Table III-2, the relationship of the elements to the types of structures is shown. The user may refer to Table III-2 to find the element type number of the suitable element(s) for his structure. If two or more options are available, the element with the larger number of vertices may be preferred, since this will minimize the necessary storage and minimize the input.

The necessary storage area is also roughly proportional to the number of mesh points, but not to the total number of elements (the necessary storage area increases with the number of elements at a slower rate). Once the mesh is established, every mesh point should be labelled sequentially with integer numbers starting from one. If there are $n$ mesh points, there are $n!$ different types of possible labelling. A system with the least difference between labels for the neighboring mesh points (points connected to each other with elements) is preferable, since the storage area is also proportional to the largest difference in the labels of neighboring mesh points. The user may request ELAS to do the internal computations with a better labelling system than his own. If such a request is made, the program internally finds a better labelling system, and performs internal operations using these new labels; however, the output is always in the user's labelling system. The extra machine time necessary for relabelling is a function of how well the user's original labelling system meets the above criterion.

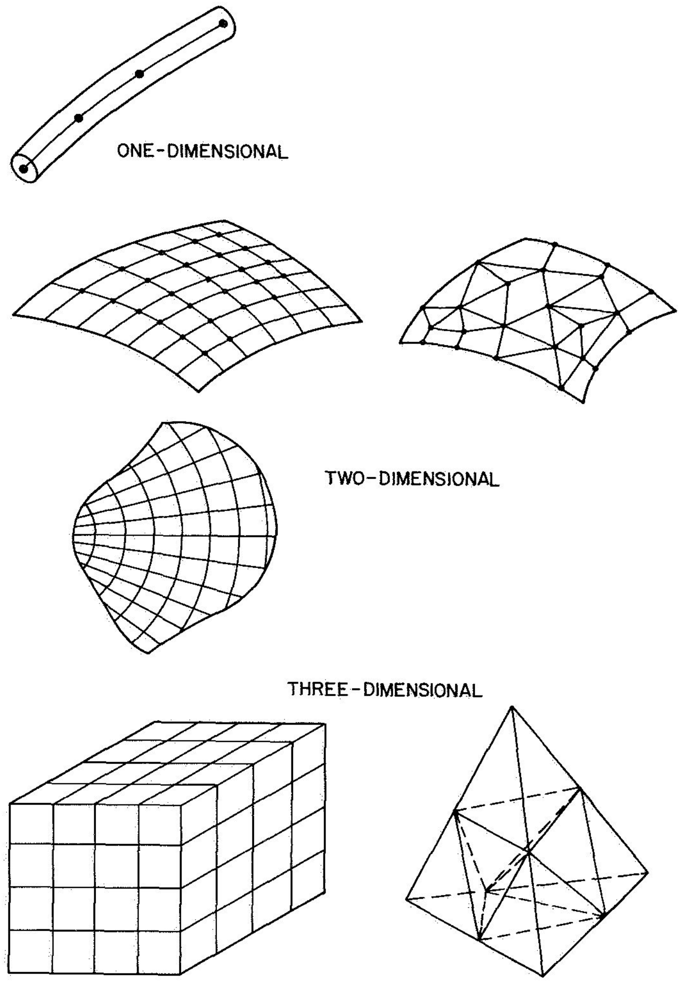



**Fig. III-1. One-, two-, and three-dimensional meshes**

Like mesh points, all the elements of a given structure must be labelled sequentially with integers, starting from

**Table III-2. Types of elements available for different cases of structures (element type numbers are shown in the shaded squares)**

| Case number | Case description \ Element type — Column number | 1 Line segment | 2 Triangle | 3 Quadrilateral | 4 Conical segment | 5 Tetrahedron | 6 Hexahedron | 7 Triangular torus | 8 Quadrilateral torus |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Planar truss | 1 | | | | | | | |
| 2 | Space truss | 1 | | | | | | | |
| 3 | Planar frame | 2 | | | | | | | |
| 4 | Space frame | 4 | | | | | | | |
| 5 | Gridwork frame | 3 | | | | | | | |
| 6 | Plane stress | | 5 | 6 | | | | | |
| 7 | Plane strain | | 5 | 6 | | | | | |
| 8 | Plate bending | | 7 | 8 | | | | | |
| 9 | General solid | | | | | 9 | 10 | | |
| 10 | General shell; bending, membrane | | 11 | 12 | | | | | |
| 11 | General shell, membrane | | 13 | 14 | | | | | |
| 12 | Solid of revolution | | | | | | | 15 | 16 |
| 13 | Shell of revolution, membrane | | | | 17 | | | | |
| 14 | Shell of revolution; bending, membrane | | | | 18 | | | | |

one. If there are $m$ elements, there are $m!$ possible labelling systems, and every one of these is equally feasible for ELAS.

In Table III-3, certain important properties of each of the available 18 elements of ELAS are listed against the element type number. In Table III-4, the necessary information for the definition of an element is indicated by shaded squares. The cross-hatched squares in the table indicate the optional information, such as temperature change, temperature gradient, and pressure. In the absence of these, the information related with the cross-hatched squares may be omitted (the cross-hatched squares under column 11 of Table III-4 should be interpreted as mass-density-type numbers that may be used in the computation of steady-state stress and deflection computation in the rotating solids of revolution about their axes of revolution). Note that in Table III-4 the definition of an element requires the type number of certain properties. The different properties for the structures listed in Table II-1 are indicated as the column headings in columns 10–20 of Table III-4. In defining an element, the type number of the necessary property is given, rather than its numerical value. The user should make a list of different values used in the whole structure for each of the properties required by the elements, and assign successive integer numbers to each entry in the list. As an example, if the cross-sectional area type of a bar element (element type 1) is given as 5, this means that the value of the cross-sectional area is the fifth entry in the list of different cross-sectional areas. The node labels indicated in columns 2–9 of Table III-4 are the mesh-point labels shared by the element. The convention of which vertex of the element is the first, and which is the second, etc., is described in Table III-5 for each of the available 18 elements.

## Table III-3. Element properties

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| Element type number | Element geometry | Number of nodes (vertices), IMS | Degrees of freedom per node, IDEG | Number of words for element description, I8 | Case No. of structure (Table III-2) for which this element may be used | Nodal line on the first material axis direction | Nodal line or nodal plane on which the pressure may exist | Pressure direction | Orientation of overall coordinate system with respect to structure | Local coordinate system of element |
| 1 | Line segment | 2 | 2 | 5 | 1 | 1-2 | 1-2 | $ | ○ | ■ |
| 1 | Line segment | 2 | 3 | 5 | 2 | 1-2 | 1-2 | $ | Any | ■ |
| 2 | Line segment | 2 | 3 | 6 | 3 | 1-2 | 1-2 | $ | ○ | □ |
| 3 | Line segment | 2 | 3 | 5 | 5 | 1-2 | 1-2 | † | ○ | □ |
| 4 | Line segment | 2 | 6 | 8 | 4 | 1-2 | 1-2 | $ | Any | □ |
| 5 | Triangle | 3 | 2 | 6 | 6, 7 | ● | 1-2 | * | ○ | ■ |
| 6 | Quadrilateral | 4 | 2 | 7 | 6, 7 | ● | 1-2 | * | ○ | ■ |
| 7 | Triangle | 3 | 3 | 6 | 8 | ● | 1-2-3 | ** | ○ | ■ |
| 8 | Quadrilateral | 4 | 3 | 7 | 8 | ● | 1-2-3-4 | ** | ○ | ■ |
| 9 | Tetrahedron | 4 | 3 | 6 | 9 | ● | 1-2-3 | ** | Any | ■ |
| 10 | Hexahedron | 8 | 3 | 10 | 9 | ● | 1-2-3-4 | ** | Any | ■ |
| 11 | Triangle | 3 | 6 | 6 | 10 | 1-2 | 1-2-3 | ** | Any | △ |
| 12 | Quadrilateral | 4 | 6 | 7 | 10 | 1-2 | 1-2-3-4 | ** | Any | △ |
| 13 | Triangle | 3 | 3 | 6 | 11 | 1-2 | 1-2-3 | ** | Any | △ |
| 14 | Quadrilateral | 4 | 3 | 7 | 11 | 1-2 | 1-2-3-4 | ** | Any | △ |
| 15 | Triangular torus | 3 | 2 | 6 | 12 | ● | 1-2 | * | ◐ | ■ |
| 16 | Quadrilateral torus | 4 | 2 | 7 | 12 | ● | 1-2 | * | ◐ | ■ |
| 17 | Conical segment | 2 | 2 | 5 | 13 | 1-2 | 1-2 | (*) | ◐ | ▲ |
| 18 | Conical segment | 2 | 3 | 5 | 14 | 1-2 | 1-2 | (*) | ◐ | ▲ |

**Legend**

○ structure is in the overall (X-Y) plane

◐ the mesh is in the overall (X-Y) plane and overall Y axis is the axis of revolution

● first material axis is the overall X axis

* normal to nodal line 1-2 and the overall Z axis, and away from element

** normal to surface shown in column 8 and in direction of local normal

(*) local z-axis direction

$ Perpendicular to the element in the plane established by the element and the overall X axis. The direction is such that the angle between the perpendicular and the X axis is less than 90 deg

† in the direction of overall Z axis

■ parallel to overall axes (for element type 1, for stresses, local system as in □)

△ x axis: nodal line 1-2; z axis: normal to middle surface, which sees labels counterclockwise

▲ x axis: nodal line 1-2; y axis parallel and opposite to Z axis

□ x axis: nodal line 1-2; y axis: one of the principal axes of the cross sections

## Table III-4. Necessary and optional information for element definition

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Element type number | Node label of first vertex | Node label of second vertex | Node label of third vertex | Node label of fourth vertex | Node label of fifth vertex | Node label of sixth vertex | Node label of seventh vertex | Node label of eighth vertex | Material type number | Thickness type number | Cross-sectional area type number | y moment of inertia type number | z moment of inertia type number | Torsional constant type number | Angle fixing principal directions type number | Pressure type number | Uniform temperature increase type number | Temperature gradient along y type number | Temperature gradient along z type number |
| 1 | N | N | | | | | | | N | | N | | | | | O | O | | |
| 2 | N | N | | | | | | | N | | N | | N | | | O | O | O | |
| 3 | N | N | | | | | | | N | | | N | | N | | O | | | O |
| 4 | N | N | | | | | | | N | | N | N | N | N | N | O | O | O | O |
| 5 | N | N | N | | | | | | N | N | | | | | | O | O | | O |
| 6 | N | N | N | N | | | | | N | N | | | | | | O | O | | O |
| 7 | N | N | N | | | | | | N | N | | | | | | O | | | O |
| 8 | N | N | N | N | | | | | N | N | | | | | | O | | | O |
| 9 | N | N | N | N | | | | | N | | | | | | | O | O | | |
| 10 | N | N | N | N | N | N | N | N | N | | | | | | | O | O | | |
| 11 | N | N | N | | | | | | N | N | | | | | | O | O | | O |
| 12 | N | N | N | N | | | | | N | N | | | | | | O | O | | O |
| 13 | N | N | N | | | | | | N | N | | | | | | O | O | | |
| 14 | N | N | N | N | | | | | N | N | | | | | | O | O | | |
| 15 | N | N | N | | | | | | N | O | | | | | | O | O | | |
| 16 | N | N | N | N | | | | | N | O | | | | | | O | O | | |
| 17 | N | N | | | | | | | N | N | | | | | | O | O | | |
| 18 | N | N | | | | | | | N | N | | | | | | O | O | | O |

N = Necessary information    O = Optional information

Table III-5. Convention for ordering the vertices of elements

| Element type number | First vertex | Other vertices |
|---|---|---|
| 1 | Any | The remaining |
| 2 | Any | The remaining |
| 3 | Any | The remaining |
| 4 | Any | The remaining |
| 5 | Any | Counterclockwise sequence about overall Z axis |
| 6 | Any | Counterclockwise sequence about overall Z axis |
| 7 | Any | Counterclockwise sequence about overall Z axis |
| 8 | Any | Counterclockwise sequence about overall Z axis |
| 9 | Any | Counterclockwise sequence for the first three vertices about the normal of their plane, heading towards the fourth vertex |
| 10 | Any | * |
| 11 | Any | Counterclockwise sequence about local normal** |
| 12 | Any | Counterclockwise sequence about local normal** |
| 13 | Any | Counterclockwise sequence about local normal** |
| 14 | Any | Counterclockwise sequence about local normal** |
| 15 | Any | Counterclockwise sequence about overall Z axis |
| 16 | Any | Counterclockwise sequence about overall Z axis |
| 17 | *** | The remaining |
| 18 | *** | The remaining |

*Counterclockwise sequence for the first four vertices on the same face about the normal heading towards the other four vertices. The fifth vertex lies diagonally across the first vertex. The last four vertices also establish a counterclockwise sequence about the normal of their face, heading towards the first four vertices.

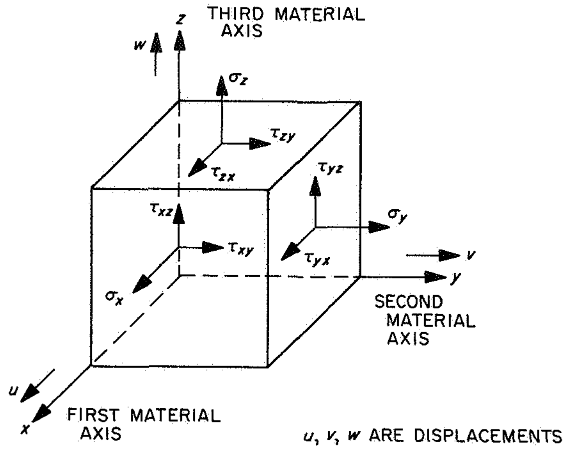
**Local normals head always to the same side of the space divided by the middle surface.

***The one with smaller meridional arc length (the meridional curve should have a direction).

## C. Definition of the Material

As far as ELAS is concerned, the material at a point is defined by the material matrix and the vector of thermal expansion coefficients (not necessary if there is no temperature loading) referred to a right-handed orthogonal axes system, that is, the material axes, at the point. In an element, the material is assumed to be homogeneous; that is, the orientation of the material axes and the related material matrix and the vector of thermal expansion coefficients do not change in a given element. However, they may change from one element to another. The orientation of the first material axis in an element is given in column 7 of Table III-3. In structures of one-dimensional continuum (element types 1–4, inclusive), the orientations of the second and third material axes are not important. In the structures of two-dimensional continuum (element types 5–8, 11–14, 17–18, inclusive), the third material axis is in the thickness direction, and the second axis is normal to both the first and the third axes. In general solids (element types 9 and 10), the material axes are assumed to be parallel to the overall coordinate system. In solids of revolution (element types 15 and 16), the second material axis is assumed to be in the direction of the axis of revolution, and the third axis is normal to both the first and the second axes.

The definitions for stresses, strains, and displacements are given in Fig. III-2a. The corresponding material matrix and the vector of thermal expansion coefficients are as shown in Fig. III-2b. In general, $21+3=24$ pieces of scalar information are necessary to define the material matrix and the vector of thermal expansion coefficients of an element. For isotropic, orthotropic, and general material cases, the number is $2+1=3$, $9+2=11$, and $21+3=24$, respectively. If the whole structure falls into one of the latter three categories, the amount of input to define one type of material is as shown in Figs. III-2c, 2d, and 2e. If for the line elements 1–4, inclusive, an orthotropic or general material is assigned, the values $D_{11}$, $D_{12}$, and $\alpha_1$ are assumed as $E$, $G$, and $\alpha$. If for two-dimensional elements 5–8, 11–14, and 17–18, inclusive, a general material is assigned, regardless of the input, $D_{15}$, $D_{16}$, $D_{25}$, $D_{26}$, $D_{35}$, $D_{36}$, $D_{45}$, $D_{46}$, and $D_{34}$ are automatically assumed to be zero; for elements 17 and 18, $D_{13}$ and $D_{23}$ are also assumed to be zero. For three-dimensional elements 9, 10, 15, and 16, orthotropic materials should not be assigned. For torus elements 15 and 16, when general material is assigned, regardless of the input, $D_{15}$, $D_{16}$, $D_{25}$, $D_{26}$, $D_{35}$, $D_{36}$, $D_{45}$, and $D_{46}$ are automatically assumed to be zero. In the early stages of the execution, there is no internal checking in ELAS to determine whether or not the material matrix used is positive definite. The user should make sure that all of his material matrices are positive definite. A non-positive definite material matrix may cause non-positive definiteness in the overall stiffness matrix. Such a situation is detected by ELAS at Link 3 during the inversion of the equations.




$$\varepsilon_x = \frac{\partial u}{\partial x},\ \varepsilon_y = \frac{\partial v}{\partial y},\ \varepsilon_z = \frac{\partial w}{\partial z},\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z},\ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

(a)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ & & D_{33} & D_{34} & D_{35} & D_{36} \\ & & & D_{44} & D_{45} & D_{46} \\ & \text{SYM} & & & D_{55} & D_{56} \\ & & & & & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

$\{\sigma\} = [D]\{\varepsilon\}$, $[D]$: MATERIAL MATRIX

$\{\alpha\} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix}$, $\{\alpha\}$: VECTOR OF THERMAL EXPANSION COEFFICIENTS

(b)

**ISOTROPIC CASE**

$$[D_0] = \begin{bmatrix} aE & bE & bE & 0 & 0 & 0 \\ & aE & bE & 0 & 0 & 0 \\ & & aE & 0 & 0 & 0 \\ & & & G & 0 & 0 \\ & \text{SYM} & & & G & 0 \\ & & & & & G \end{bmatrix}$$

$$a = \frac{G\,(4G-E)}{E\,(3G-E)},\quad b = \frac{G\,(2G)}{E\,(3G-E)}$$

$[\alpha] = [\alpha, \alpha, \alpha]$

INPUT: $E, G, \alpha$

(c)

**ORTHOTROPIC CASE**

$$[D_1] = \begin{bmatrix} D'_{11} & D'_{12} & * & D'_{14} & 0 & 0 \\ & D'_{22} & * & D'_{24} & 0 & 0 \\ & & \dagger & 0 & 0 & 0 \\ & & & D'_{44} & 0 & 0 \\ & \text{SYM} & & & D'_{55} & D'_{56} \\ & & & & & D'_{66} \end{bmatrix}$$

* IN PLANE-STRAIN CASE $D_{12}$, OTHERWISE 0

† IN PLANE-STRAIN CASE NOT NECESSARY, OTHERWISE 0

$[\alpha] = [\alpha'_1, \alpha'_2, 0]$

INPUT: $D'_{11}, D'_{12}, D'_{14}, D'_{22}, D'_{24}, D'_{44}, D'_{55}, D'_{56}, D'_{66}$
$\alpha'_1, \alpha'_2$

(d)

**GENERAL CASE**

$$[D_2] = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ & & D_{33} & D_{34} & D_{35} & D_{36} \\ & & & D_{44} & D_{45} & D_{46} \\ & \text{SYM} & & & D_{55} & D_{56} \\ & & & & & D_{66} \end{bmatrix}$$

$[\alpha] = [\alpha_1, \alpha_2, \alpha_3]$

INPUT: $D_{11}, D_{12}, D_{13}, D_{14}, D_{15}, D_{16}, D_{22},$
$D_{23}, D_{24}, D_{25}, D_{26}, D_{33}, D_{34}, D_{35},$
$D_{36}, D_{44}, D_{45}, D_{46}, D_{55}, D_{56}, D_{66}$
$\alpha_1, \alpha_2, \alpha_3$

(e)

**Fig. III-2. Description of the material: (a) definition figure; (b) stress and strain relations and definition of strains; (c) material matrix for isotropic case; (d) material matrix for orthotropic case; (e) material matrix for general case**

## D. Deflection Boundary Conditions

The deflections of the mesh points are referred to the degree-of-freedom directions. One scalar deflection component is associated with each degree-of-freedom direction. The degree-of-freedom directions are those defined by the unit vectors of the overall coordinate system. At a mesh point, the degrees of freedom are ordered in the sequence (from left to right) of the shaded squares in the rows of Table III-1. A deflection component is completely determined by (1) its magnitude, (2) the mesh-point number, and (3) the sequence number of the related degree of freedom at that mesh point (to be obtained from Table III-1). Let $u$ denote deflections. Then, as a general rule, $u_{i,j}$ means the deflection component in the $j$th degree-of-freedom direction of the $i$th mesh point. If the total number of mesh points is $n$, and the number of degrees of freedom per mesh point is $m$, the following should be observed: $i \leq n$ and $j \leq m$. As an example, $u_{25,4}$ means the deflection component in the fourth-degree-of-freedom direction of mesh point 25.

Because of supports, some of the deflection components may be prescribed, or may be linearly dependent on one or more other deflection components. These constitute the deflection boundary conditions. For example, in an unyielding fixed support, the related deflection components are prescribed as zero. If the support is a yielding one, the support settlements in the related degree-of-freedom directions are the prescribed values of the deflections. A hinged support means that only the lineal deflection component(s) at the related mesh point is prescribed as zero. If there is a roller support at a mesh point, this means that there is a linear dependence among the lineal deflection components of that mesh point. This linear dependence may be obtained by writing down the condition that no lineal deflection may exist in the normal direction of the plane on which the roller support is allowed to move. The linear-dependence equations should be expressed in the overall coordinate system. A general linear-dependence equation that is solved for the deflection component with the largest coefficient in magnitude may be formally written as

$$u_{i,j} = a_0 + a_1 u_{i',j'} + a_2 u_{i'',j''} + \cdots \quad (1)$$

This equation is general enough to cover the deflection boundary conditions related with fixed-unyielding, fixed-yielding, hinged, or roller-type supports and many others. For example, if the right-hand side of Eq. (1) contains only the $a_0$ term, it corresponds to a fixed-support condition, which may be unyielding or yielding type, depending upon whether $a_0$ is zero or nonzero, respectively. The coefficients $a_0$, $a_1$, $a_2$, $\cdots$ , and the index pairs $(i, j)$, $(i', j')$, $(i'', j'')$, $\cdots$ , are the inputs of deflection boundary conditions. Each term of the right-hand side of Eq. (1) corresponds to one deflection boundary condition (dbc) input unit. The dbc input units of Eq. (1) may be written as follows:

| | | |
|---|---|---|
| $i, j$ | $i\ , j$ | $a_0$ |
| $i, j$ | $i', j'$ | $a_1$ |
| $i, j$ | $i'', j''$ | $a_2$ |
| $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ | $\cdot$ |

where, in any row, the first index pair defines the degree-of-freedom direction under question, the second index pair defines the related degree-of-freedom direction, and the third quantity is the scalar relating the deflection components in these directions. Note that for the scalar $a_0$, the direction under question and the related direction are the same. The dbc input units of a given problem may be arranged in any order; however, the total number of dbc input units should be known. If any one index pair of a dbc input unit appears more than twice in the whole set of input units of the problem, the other may not appear more than twice. Other than this there is no restriction on the dbc input units, provided that $i$, $i'$, $i''$, $\cdots$ , are legitimate mesh-point numbers, $j$, $j'$, $j''$, $\cdots$ , are legitimate degree-of-freedom sequence numbers, and $a_0$, $a_1$, $a_2$, $\cdots$ , are meaningful scalars.

### E. Prescribed Force Boundary Conditions

The concept of prescribed force boundary conditions is generalized to include all material points of a given structure. They are classified under two categories: (1) prescribed distributed loads at material points, and (2) prescribed concentrated loads at the mesh points. The first category includes thermal loads, pressure loads on the boundary surface, and the gravity loads. Thermal loads and pressure loads are indicated in the definition of the elements. For gravity loads, the direction cosines of the acceleration vector and the unit weight of the structure should be given. Only in the case of solids of revolution can the unit weight vary from element to element.

The loads in the second category are defined by the components along the degree-of-freedom directions. Let $P$ denote the concentrated loads. In general, $P_{i,j}$ means the concentrated load in the $j$th degree-of-freedom direction of the $i$th mesh point. For example, $P_{25,4}$ means the concentrated load in the fourth-degree-of-freedom direction of mesh point 25. The indices and the magnitudes of the prescribed concentrated loads are the inputs of the second-category force boundary conditions. Let $P_{i,j}$, $P'_{i',j'}$, $P''_{i'',j''}$, $\cdots$, denote the prescribed concentrated loads at the mesh points. These are input as

$$\begin{array}{ll} i\,,j & P \\ i'\,,j' & P' \\ i''\,,j'' & P'' \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array}$$

where each row is one concentrated load input unit. Any index pair in the list may appear only once. The total number of concentrated load input units should be known. The concentrated load input units may be arranged in any order, provided that $i$, $i'$, $i''$, $\cdots$, are legitimate mesh-point numbers, $j$, $j'$, $j''$, $\cdots$, are legitimate degree-of-freedom sequence numbers, and $P$, $P'$, $P''$, $\cdots$, are meaningful scalars. In axisymmetrical structures undergoing axisymmetrical deformation, if element types 15, 16, 17, and 18 are used, the scalar $P$ in the concentrated load input unit is the product of the component of the line load acting on the parallel times the radius of the parallel.

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# IV. Preparation of Input

## A. Preparation of Input Cards

The physical deck arrangement of data cards for different jobs is shown in Fig. II-2. The arrangement of the input cards of a given job is shown in Fig. IV-1; the names and numbers appearing on the cards are for identification only, and do not appear on the cards. Every card or card group is an "input item" and the accompanying encircled numbers shown in Fig. IV-1 are the input item numbers. The contents and the formats of input items are given in Table IV-1. Depending upon the condition indicated in the second column of Table IV-1, many of the input items may not be required in a job. The program learns these conditions from Input Item 2, which is the "control card" of input data. This card is explicitly described in Table IV-2. Table IV-3 gives the meanings of the variables listed in Input Item 16 of Table IV-1. The information in Fig. IV-1 and Tables IV-1 through IV-3 may be sufficient for the preparation of input data. However, when the need arises, the user may refer to Section IV-B, in which every input item is explained individually.

## B. Description of Input Items (as Listed in Table IV-1)

*Input Item 1 (Title Card).* This is the very first card of the input data of any job. It may contain any alphanumeric message up to 80 characters. The message appears in Output Item 1 (see Section VI-B).

*Input Item 2 (Control Card).* The information on this card enables the program to determine which of the following input items are necessary; how the storage allocations are to be made; which are the two chain tapes of the program; up to and including which chain link of the program is to be executed; which level of output is to be produced; whether the coordinates of the mesh points, the element descriptions, and the deflection boundary conditions are to be read from cards or generated by the user's version of subroutines CORG, MESG, and BUNG (see Section V); how the local coordinates at the mesh points of shells are to be selected; and, if the structure is subjected to a constant force field, what is the direction and the magnitude of the force per unit volume. In Table IV-2, which gives a summary of the control card, the names and the card column numbers of the fields, their formats and the ranges of their contents, and the location where the additional information may be found are given with a short explanation. The contents of this card appear in Output Item 1 (Section VI-B).

*Input Item 3 (Material Types).* Depending upon the content of the ITYPE field of the control card (Input Item 2), the input card (or cards) of this input item is

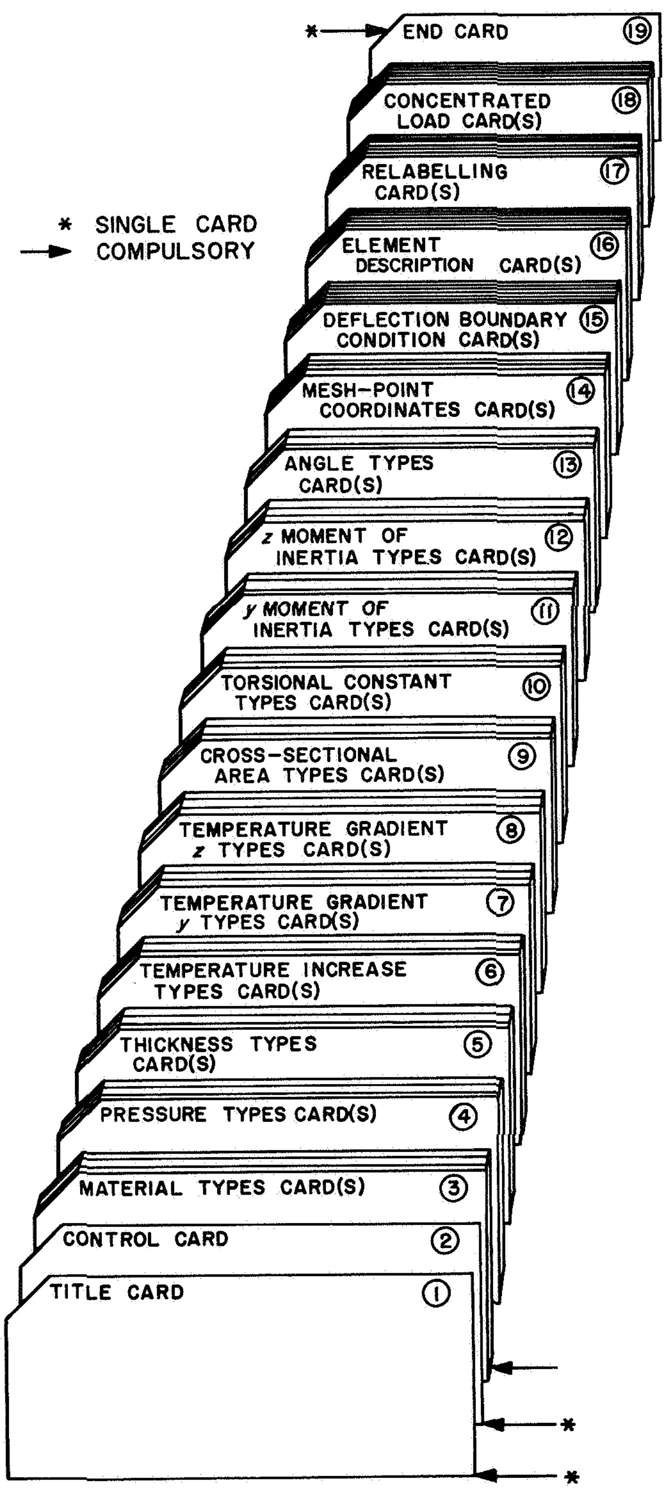


**Fig. IV-1. The data cards of a job**

prepared according to the first, the second, or the third line of Input Item 3 in Table IV-1. (Further information for Input Item 3 may be found in the last paragraph of Sections III-B and III-C.) The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 4 (Pressure Types).* No input card is required for this input item if the content of the IPRS field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 4 in Table IV-1 to list the different pressures. For one-dimensional structural elements, pressure represents "wind pressure*effective beamwidth" and the wind is assumed blowing towards the positive $X$ direction. The pressure direction in various elements is indicated in column 9 of Table III-3. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 5 (Thickness Types).* No input card is required for this input item if the content of the NTIC field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 5 of Table IV-1 to list the different thicknesses. In Table III-4, column 11 shows which element types may require thickness specification. (Note that in Table III-4, element types 15 and 16 are indicated as if they may require thickness specification. These elements are those of solids of revolution. For solids of revolution, the thickness types may be used to indicate the variations in the centrifugal force field. The content of the ACEL field of the control card may be assumed as nominal centrifugal force per unit volume, and the thickness type may be used to denote the multiples of the nominal force; for example, when the thickness is 1, the centrifugal force is the nominal force, and when the thickness is 1.5, the centrifugal force is the 1.5 multiple of the nominal.) The contents of the input card(s) of Input Item 5 appear in Output Item 2 (Section VI-B).

*Input Item 6 (Temperature Increase Types).* No input card is required for this input item if the content of the ISDT field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 6 in Table IV-1 to list the different temperature increases. In Table III-4, column 18 shows which element types may require temperature increase specification. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 7 (Temperature Gradient Types — Local y-Axis Direction).* No input card is required for this input item if the content of the ISDY field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 7 in

Table IV-1 to list the different temperature gradients in the local $y$-axis direction. The direction of the local $y$ axis in an element may be obtained from Table III-3, column 11, using the type number of the element. In Table III-4, column 19 shows which element types may require temperature gradient specification in the local $y$-axis direction. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 8 (Temperature Gradient Types — Local $z$-Axis Direction).* No input card is required for this input item if the content of the ISDZ field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 8 in Table IV-1 to list the different temperature gradients in the local $z$-axis direction. The direction of the local $z$ axis in an element may be obtained from Table III-3, column 11, using the type number of the element. In Table III-4, column 20 shows which element types may require temperature gradient specification in the local $z$-axis direction. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 9 (Cross-Sectional Area Types).* No input card is required for this input item if the content of the IARE field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 9 in Table IV-1 to list the different cross-sectional areas. In Table III-4, column 12 shows which element types require cross-sectional area specification. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 10 (Torsional Constant Types).* No input card is required for this input item if the content of the IMMX field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 10 in Table IV-1 to list the different torsion constants. A torsion constant when multiplied by the shear modulus and divided by the length gives the torsional rigidity of the one-dimensional element. In Table III-4, column 15 shows which element types require torsional constant specification. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 11 ($y$ Moment of Inertia Types).* No input card is required for this input item if the content of IMMY field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 11 in Table IV-1 to list the different moments of inertia about the local $y$ axis. The direction of the local $y$ axis in an element may be obtained from Table III-3, column 11. In Table III-4, column 13 shows which element types require the moment of inertia specification about the local $y$ axis. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 12 ($z$ Moment of Inertia Types).* No input card is required for this input item if the content of the IMMZ field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 12 in Table IV-1 to list the different moments of inertia about the local $z$ axis. The direction of the local $z$ axis in an element may be obtained from Table III-3, column 11. In Table III-4, column 14 shows which element types require the moment of inertia specification about the local $z$ axis. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 13 (Angle Types — Fixing Local $y$ and $z$ Axes).* No input card is required for this input item if the content of the IMFI field of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 13 in Table IV-1 to list the different angles $\phi$ in degree units. Angle $\phi$ is the angle between the local $y$ axis (see Table III-3) and the overall $Y$ axis; $\phi$ is always less than 90 deg. It is positive if the absolute value of angle $\alpha$ between the local $y$ axis and the overall $X$ axis is less than 90 deg. If $|\alpha|$ is less than 0.0081 deg, then $\phi$ is the angle between the overall Z axis and the local $y$ axis (see Section VI, Fig. VI-1). In Table III-4, column 16 shows which element type requires the angle specification. The contents of the input card(s) of this input item appear in Output Item 2 (Section VI-B).

*Input Item 14 (Mesh-Point Coordinates).* If the content of the ICOR field of the control card (Input Item 2) is zero, one or more cards are prepared as shown in the first line of Input Item 14 in Table IV-1 to list the $X$, $Y$, $Z$ coordinates of the mesh points in the overall coordinate system. In the preparation of these cards, any one of the formats shown in the first line of Input Item 14 in Table IV-1 may be used, or the user may employ any combination of these formats. If the content of the ICOR field of the control card (Input Item 2) is not zero, the user should provide his version of subroutine CORG, which should be prepared in accordance with Section V-B and should be included in the physical deck as described

## Table IV-1. Input items (summary of options, contents, and formats)

| Input item No. | Conditions determining options | List of input statements that read the associated input item card(s)[a] | Format (outside parentheses indicate the possibility of multiple cards) |
|---|---|---|---|
| 1 | | ($B_i$, $i = 1, 14$) The card may contain any alphanumeric message | 14A6 |
| 2 | | IN, IT, IDEG, ITYPE, IGEM, ISTR, IH, I8, IBN, IP, IPRS, IMAT, NTIC, ISDT, ISDY, ISDZ, IARE, IMMX, IMMY, IMMZ, IMFI, INX, INP, ISHUF, ICOR, IBUN, IMES, IPIR, ITAP, ITAS, G1, G2, G3, ACEL | 2I4, 6I1, 3I4, 10I2, 9I1, 3F5.4, E10.5 (see Table IV-2 for details) |
| 3[b] | ITYPE = 0 | ($i$, $E_i$, $G_i$, $\alpha_i$, $i = 1$, IMAT) | (3(I2, 3E8.5)) |
| | ITYPE = 1 | ($i$, $D'_{11_i}$, $D'_{12_i}$, $D'_{14_i}$, $D'_{22_i}$, $D'_{24_i}$, $D'_{44_i}$, $D'_{55_i}$ $D'_{56_i}$, $D'_{66_i}$, $i$, $\alpha'_{1_i}$, $\alpha'_{2_i}$, $i = 1$, IMAT) | (I2, 9E8.5/I2, 2E8.5) |
| | ITYPE = 2 | ($i$, $D_{11_i}$, $D_{12_i}$, $D_{13_i}$, $D_{14_i}$, $D_{15_i}$, $D_{16_i}$, $D_{22_i}$, $D_{23_i}$, $D_{24_i}$, $i$, $D_{25_i}$, $D_{26_i}$, $D_{33_i}$, $D_{34_i}$, $D_{35_i}$, $D_{36_i}$, $D_{44_i}$, $D_{45_i}$, $D_{46_i}$, $i$, $D_{55_i}$, $D_{56_i}$, $D_{66_i}$, $\alpha_{1_i}$, $\alpha_{2_i}$, $\alpha_{3_i}$, $i = 1$, IMAT) | (I2, 9E8.5/I2, 9E8.5/I2, 6E8.5) |
| 4 | $1 \leq$ IPRS $\leq 99$ | ($i$, $p_i$, $i = 1$, IPRS) | (8(I2, E8.5)) |
| | IPRS = 0 | No input card | |
| 5 | $1 \leq$ NTIC $\leq 99$ | ($i$, $h_i$, $i = 1$, NTIC) | (8(I2, E8.5)) |
| | NTIC = 0 | No input card | |
| 6 | $1 \leq$ ISDT $\leq 99$ | ($i$, $\Delta t_i$, $i = 1$, ISDT) | (8(I2, E8.5)) |
| | ISDT = 0 | No input card | |
| 7 | $1 \leq$ ISDY $\leq 99$ | ($i$, $(\partial t/\partial y)_i$, $i = 1$, ISDY) | (8(I2, E8.5)) |
| | ISDY = 0 | No input card | |
| 8 | $1 \leq$ ISDZ $\leq 99$ | ($i$, $(\partial t/\partial z)_i$, $i = 1$, ISDZ) | (8(I2, E8.5)) |
| | ISDZ = 0 | No input card | |
| 9 | $1 \leq$ IARE $\leq 99$ | ($i$, $A_i$, $i = 1$, IARE) | (8(I2, E8.5)) |
| | IARE = 0 | No input card | |
| 10 | $1 \leq$ IMMX $\leq 99$ | ($i$, $C_i$, $i = 1$, IMMX) | (8(I2, E8.5)) |
| | IMMX = 0 | No input card | |
| 11 | $1 \leq$ IMMY $\leq 99$ | ($i$, $I_{y_i}$, $i = 1$, IMMY) | (8(I2, E8.5)) |
| | IMMY = 0 | No input card | |
| 12 | $1 \leq$ IMMZ $\leq 99$ | ($i$, $I_{z_i}$ $i = 1$, IMMZ) | (8(I2, E8.5)) |
| | IMMZ = 0 | No input card | |
| 13 | $1 \leq$ IMFI $\leq 99$ | ($i$, $\phi_i$, $i = 1$, IMFI) | (8(I2, E8.5)) |
| | IMFI = 0 | No input card | |
| 14 | ICOR = 0<br>$2 \leq$ IN $\leq 9999$ | ($j$, $X_j$, $Y_j$, $Z_j$, $j = 1$, IN) | (I4, 3E12.4, 40X) or (40X, I4, 3E12.4) or (2(I4, 3E12.4)) |
| | ICOR = 1 | Input card(s) should be as required by the user's version of subroutine CORG (see Section V-B) | |

## Table IV-1 (contd)

| Input item No. | Conditions determining options | List of input statements that read the associated input item card(s)[a] | Format (outside parentheses indicate the possibility of multiple cards) |
|---|---|---|---|
| 15 | IBUN = 0<br>$1 \leq$ IBN $\leq 9999$ | $(i_k, j_k, i'_k, j'_k, a_k, k = 1, \text{IBN})$ | (5(I4,I1, I4, I1, F6.0)) |
| | IBUN = 1 | Input card(s) should be as required by the user's version of subroutine BUNG (see Section V-D) | |
| 16 | IMES = 0<br>$1 \leq$ IT $\leq 9999$ | $(\text{MM}_m, \text{J1W}_m, \text{J2W}_m, \text{J3W}_m, \text{J4W}_m, \text{J5W}_m, \ldots, m = 1, \text{IT})$ | (20I4) (see Table IV-3 for variables of the list) |
| | IMES = 1 | Input card(s) should be as required by the user's version of subroutine MESG (see Section V-C) | |
| 17 | ISHUF = 0 or 1 | No input card | |
| | ISHUF = 2 | $(\text{N}_i, i = 1, \text{IN})$ | (20I4) |
| | ISHUF = 3 | $(\text{N}_i, \text{IMAX}_i, i = 1, \text{IN})$ | (20I4) |
| 18 | $1 \leq$ IP $\leq 9999$ | $(i_l, j_l, \text{P}_l, l = 1, \text{IP})$ | (5(I4, I1, E11.4)) |
| | IP = 0 | No input card | |
| 19 | | No list (the card is punched END in the last three columns) | 77X, 3HEND |
| 20 | | No input card for Input Item 20 if standard ELAS is used; otherwise the input cards of this item are as required by the user's subroutines CAS2, PUNC, (CAS4, AGEL) (see Sections V-E through V-H) | |

[a]Nomenclature

- $p$ pressure
- $h$ thickness
- $\Delta t$ temperature increase
- $\partial t/\partial y$ temperature gradient in local $y$-axis direction
- $\partial t/\partial z$ temperature gradient in local $z$-axis direction
- $A$ cross-sectional area
- $C$ torsional constant
- $I_y$ moment of inertia about local $y$ axis
- $I_z$ moment of inertia about local $z$ axis
- $\phi$ angle determining the orientation of principal axes of cross section in overall coordinate system
- $X, Y, Z$ overall coordinates of mesh points
- $x, y, z$ local coordinates
- $(i_k, j_k), (i'_k, j'_k), a_k$ index pairs and the constant of the $k$th dbc input unit (see Section III-D)
- $i_l, j_l, P_l$ index pair and constant of the $l$th concentrated load input unit (see Section III-E)

[b]The symbols shown in Input Item 3 are defined in Figs. III-2c, 2d, and 2e.

in Section V-A. In this case, the input cards of subroutine CORG are considered as the input cards of Input Item 14.

*Input Item 15 (Deflection Boundary Conditions).* If the content of the IBUN field of the control card (Input Item 2) is zero, one or more cards are prepared as shown in the first line of Input Item 15 in Table IV-1 to list the dbc input units (see Section III-D for dbc input units). If the content of the IBUN field of the control card (Input Item 2) is not zero, the user should provide his version of subroutine BUNG, which should be prepared in accordance with Section V-D and should be included in the physical deck as described in Section V-A. In this case, the input cards of subroutine BUNG are considered as the input cards of Input Item 15.

*Input Item 16 (Element Descriptions).* If the content of the IMES field of the control card (Input Item 2) is zero, one or more cards are prepared as shown in the first line of Input Item 16 in Table IV-1 to list the descriptions of each element. Depending upon the type of element, five to ten words may be used to describe an element, and an additional word to contain a negative integer related with the element label. As shown in Table IV-1, all these words are punched on cards with (20I4) format. The meanings of these words are given in Table IV-3. Note that the description of an element on the cards consists of integer numbers separated by minus signs. The integers are the mesh-point labels of the element and the type numbers of properties describing its material, geometry, and thermal and pressure-loading

**Table IV-2. Summary of the control card (Input Item 2) of input data**

| Name of field | Card columns of field | Format | Range | Description |
|---|---|---|---|---|
| IN | 1–4 | I4 | 2–9999 | Total number of mesh points (see Section III-B) |
| IT | 5–8 | I4 | 1–9999 | Total number of elements (see Section III-B) |
| IDEG | 9 | I1 | 2–6 | Number of degrees at a mesh point (see Table III-1, column 7) |
| ITYPE | 10 | I1 | 0–2 | Material indicator: 0—isotropic, 1—orthotropic, 2—general (see Fig. III-2) |
| IGEM | 11 | I1 | 0–1 | Geometry indicator: IGEM = 0 all Z coordinates are zero[a]; IGEM = 1 not all Z coordinates are zero[a] (see Table III-3, column 10) |
| ISTR | 12 | I1 | 0–1 | Plane strain case indicator: ISTR = 1 plane strain; ISTR = 0 not plane strain |
| IH | 13 | I1 | 2–8 | Maximum number of vertices in elements used (see Table III-3, column 3) |
| I8 | 14 | I1 | 5–10[b] | Maximum number of words for element description (see Table III-3, column 5) |
| IBN | 15–18 | I4 | 1–9999 | Total number of dbc input units (see Section III-D) |
| IP | 19–22 | I4 | 0–9999 | Total number of concentrated load input units (see Section III-E) |
| IPRS | 23–26 | I4 | 0–99 | Total number of different pressures |
| IMAT | 27–28 | I2 | 1–99 | Total number of different materials |
| NTIC | 29–30 | I2 | 0–99 | Total number of different thicknesses |
| ISDT | 31–32 | I2 | 0–99 | Total number of different temperature increases |
| ISDY | 33–34 | I2 | 0–99 | Total number of different temperature gradients $(\partial t/\partial y)$[c] |
| ISDZ | 35–36 | I2 | 0–99 | Total number of different temperature gradients $(\partial t/\partial z)$[c] (see Section III-B, last paragraph) |
| IARE | 37–38 | I2 | 0–99 | Total number of different cross-sectional areas |
| IMMX | 39–40 | I2 | 0–99 | Total number of different torsional constants |
| IMMY | 41–42 | I2 | 0–99 | Total number of different moments of inertia (about *y* axis)[c] |
| IMMZ | 43–44 | I2 | 0–99 | Total number of different moments of inertia (about *z* axis)[c] |
| IMFI | 45–46 | I2 | 0–99 | Total number of angles fixing local *y* and *z* axes[c] |
| INX | 47 | I1 | 1–4 | Number of link after which return-to-beginning-for-next-job is done (see Section II-F and Table V-1) |
| INP | 48 | I1 | 0–2 | Printout indicator: 0—minimum; 1—intermediate; 2—detailed output (see Section VI-A) |
| ISHUF | 49 | I1 | 0–3 | Relabelling indicator: 0—no relabelling; 1—iterate to relabel without reading cards; 2—read cards and iterate to relabel; 3—relabel as shown on cards (see Section IV-B) |
| ICOR | 50 | I1 | 0–1 | Indicator for coordinate generation: 0—read coordinates from cards; 1—generate coordinates via subroutine CORG (user's version) |
| IBUN | 51 | I1 | 0–1 | Indicator for displacement boundary conditions: 0—read from cards; 1—generate with user's version of subroutine BUNG |
| IMES | 52 | I1 | 0–1 | Indicator for element descriptions: 0—read from cards; 1—generate with user's version of subroutine MESG |
| IPIR | 53 | I1 | 0–2 | Local coordinate selection indicator for shells: 0—assume local *x* as 1-2 line of lowest numbered element; 1—assume as principal; 2—read by subroutine AGEL |

Table IV-2 (contd)

| Name of field | Card columns of field | Range | Format | Description |
|---|---|---|---|---|
| ITAP | 54 | I1 | 0–9 | Chain tape number for program (if zero, program assumes 2) |
| ITAS | 55 | I1 | 0–9 | Chain tape number for intermediate storage |
| G1 | 56–60 | F5.4 | (−1.)–(+1.) | Cosine of the angle of acceleration vector with X axis[a] |
| G2 | 61–65 | F5.4 | (−1.)–(+1.) | Cosine of the angle of acceleration vector with Y axis[a] |
| G3 | 66–70 | F5.4 | (−1.)–(+1.) | Cosine of the angle of acceleration vector with Z axis[a] |
| ACEL[d] | 71–80 | E10.3 | Any | Magnitude of acceleration vector times unit mass (unit weight) |

[a]X, Y, Z refer to overall coordinate system.

[b]When I8 = 10, zero should be punched in column 14.

[c]x, y, z refer to the local coordinate system of the element.

[d]In element type 3, ACEL means weight per unit length.

conditions (see last paragraph of Section III-B). The first integer becomes a negative integer because of the minus sign. It is obtained by the lowest three significant digits of the actual element label. For example, if the element label is 7, this integer is −7; if the element label is 1000, this integer is −000; if the element label is 1245, this integer is −245, etc. The program requires that the element descriptions be sequenced with the element labels; that is, the description of the first element be given first, the description of the second be given second, etc. If for the description of an element only $n$ number of I4 fields are necessary, and if the user assigns $n + m$ I4 fields, the last $m$ number of I4 fields is automatically ignored. This feature is especially useful in arranging input cards for easy checking, and making corrections after the cards are punched. The meanings of the symbols used in Table IV-3 are given at the bottom of the table. For the sequence of the mesh-point labels in the element, refer to Table III-5.

If the content of the IMES field of the control card (Input Item 2) is not zero, the user should provide his version of subroutine MESG, which should be prepared in accordance with Section V-C and should be included in the physical deck as described in Section V-A. In this case, the input cards of subroutine MESG are considered as the input cards of Input Item 16.

*Input Item 17 (Relabelling Information).* No input card is required for this input item if the content of the ISHUF field of the control card (Input Item 2) is zero or one. If the content of the ISHUF field is 2, one or more cards should be prepared with (20I4) format to list the labels of the mesh points in the order of another labelling system. ELAS will relabel the mesh points in an optimum way, which is obtained by iteration starting from the labelling system given by these cards. [Similar cards are provided by ELAS as $C_p$ cards of Output Item 5 (see Section VI-F) for every 100 seconds of relabelling time. If a requested relabelling is not accomplished in one run, it may be continued in another run by ISHUF=2 and using the last punched cards of Output Item 5 as the input cards of Input Item 17.] If the content of the ISHUF field is 3, one or more cards should be prepared with (20I4) format to list the user's labels and IMAX values of the mesh points in the order of the labelling system that is desired for internal computations. The IMAX value of a mesh point is the highest-valued new label in the mesh points that are connected with the current mesh point with finite elements. If the former IMAX (in the sense of the new labels) is larger than the IMAX of the current mesh point, the current IMAX is taken as the IMAX of the former mesh point. [Similar cards are provided by ELAS as $C_f$ cards of Output Item 5 (see Section VI-F) at the end of relabelling. If a job is to be executed with relabelling more than once, after the first execution the user may request executions with ISHUF=3 and use the final punched $C_f$ cards of Output Item 5 of the first run as the input cards of Input Item 17 of the succeeding runs.]

*Input Item 18 (Concentrated Loads).* No input card is required for this input item if the content of the IP field

## Table IV-3. Description of element data for different element types

| Element type No. | Meanings of variables appearing in the list of Input Item 16 of Table IV-1[a] | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | MM | J1W | J2W | J3W | J4W | J5W | J6W | J7W | J8W | J9W | J10W |
| 1 | M | 100+IMET | 100(JARE)+ITEM | JPRS | $N_1$ | $N_2$ | | | | | |
| 2 | M | 200+IMET | 100(JARE)+ITEM | JPRS | 100(JMMZ)+JSDY | $N_1$ | $N_2$ | | | | |
| 3 | M | 300+IMET | 100(JPRS)+JSDZ | 100(JMMX)+JMMY | $N_1$ | $N_2$ | | | | | |
| 4 | M | 400+IMET | 100(JARE)+ITEM | 100(JMMX)+JMMY | 100(JMMZ)+JSDY | 100(JSDZ)+JMFI | JPRS | $N_1$ | $N_2$ | | |
| 5 | M | 500+IMET | 100(ITIC)+ITEM | JPRS | $N_1$ | $N_2$ | $N_3$ | | | | |
| 6 | M | 600+IMET | 100(ITIC)+ITEM | JPRS | $N_1$ | $N_2$ | $N_3$ | $N_4$ | | | |
| 7 | M | 700+IMET | 100(ITIC)+ITEM | 100(JSDZ)+JPRS | $N_1$ | $N_2$ | $N_3$ | | | | |
| 8 | M | 800+IMET | 100(ITIC)+ITEM | 100(JSDZ)+JPRS | $N_1$ | $N_2$ | $N_3$ | $N_4$ | | | |
| 9 | M | 900+IMET | 100(JPRS)+ITEM | $N_1$ | $N_2$ | $N_3$ | $N_4$ | | | | |
| 10 | M | 1000+IMET | 100(JPRS)+ITEM | $N_1$ | $N_2$ | $N_3$ | $N_4$ | $N_5$ | $N_6$ | $N_7$ | $N_8$ |
| 11 | M | 1100+IMET | 100(ITIC)+ITEM | 100(JSDZ)+JPRS | $N_1$ | $N_2$ | $N_3$ | | | | |
| 12 | M | 1200+IMET | 100(ITIC)+ITEM | 100(JSDZ)+JPRS | $N_1$ | $N_2$ | $N_3$ | $N_4$ | | | |
| 13 | M | 1300+IMET | 100(ITIC)+ITEM | JPRS | $N_1$ | $N_2$ | $N_3$ | | | | |
| 14 | M | 1400+IMET | 100(ITIC)+ITEM | JPRS | $N_1$ | $N_2$ | $N_3$ | $N_4$ | | | |
| 15 | M | 1500+IMET | 100(ITIC)+ITEM | JPRS | $N_1$ | $N_2$ | $N_3$ | | | | |
| 16 | M | 1600+IMET | 100(ITIC)+ITEM | JPRS | $N_1$ | $N_2$ | $N_3$ | $N_4$ | | | |
| 17 | M | 1700+IMET | 100(ITIC)+ITEM | JPRS | $N_1$ | $N_2$ | | | | | |
| 18 | M | 1800+IMET | 100(ITIC)+ITEM | 100(JSDZ)+JPRS | $N_1$ | $N_2$ | | | | | |

[a]Definitions:

- IMET material type number
- ITIC thickness type number
- JPRS pressure type number
- JARE cross-sectional area type number
- ITEM temperature increase type number
- JSDY temperature gradient along $y$ type number
- JSDZ temperature gradient along $z$ type number
- JMMX torsion constant type number
- JMMY moment of inertia (about $y$) type number
- JMMZ moment of inertia (about $z$) type number
- JMFI angle fixing local $y$, $z$ axes type number
- $N_1$ mesh-point label of the first vertex
- $N_2$ mesh-point label of the second vertex, . . . ,
- $N_8$ mesh-point label of the eighth vertex
- $m$ element label

$M = -\left(m - \left(\frac{m}{1000}\right) * 1000\right)$, which is to be interpreted in FORTRAN integer arithmetic sense

Note: On the cards of Input Item 16 (first option), zero field(s) (of four columns) after the last nonzero field of element description is ignored if provided.

of the control card (Input Item 2) is zero; otherwise one or more cards are prepared as shown in the first line of Input Item 18 in Table IV-1 to list the concentrated load input units (see Section III-E for concentrated load input units). The contents of the input card(s) of this input item appear in Output Item 10 (see Section VI-B).

***Input Item 19 (End Card).*** This is the last card of the data cards of a job to be run with standard ELAS. This card contains the word "END" in columns 78–80; it indicates the end of input cards related with Link 1 of the program.

***Input Item 20.*** No card is required for this input item if the standard ELAS (see Section V-A) is used or if the user's version of subroutines CAS2, PUNC, (CAS4, AGEL) do not require any input cards; otherwise the input cards of these subroutines should be provided in the order of CAS2, PUNC, (CAS4, AGEL). Subroutine CAS2 is called once for every element in the order of the increasing labels of the elements; subroutine PUNC is called only once, and subroutines CAS4 and AGEL are called once for every mesh point in the order of the increasing labels (those of the user) of the mesh points. For a mesh point, subroutine CAS4 is called before subroutine AGEL.

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# V. Preparation of Program Deck

## A. Arrangement of the Physical Program

The program available to the user is called the "standard ELAS." The deck arrangement of the standard ELAS is shown in Fig. II-2. The program consists of four chain links and data cards of one or more jobs. Each link has a number and a name describing its function. The correspondence between link numbers and the names, and the functions of the links are given in Table V-1. The programs in each link of the standard ELAS are listed in Tables V-2 through V-5 with the information about their names, lengths, labels, functions, and the case number of the structure (see Table II-1) that requires them; in these tables, the subprograms necessary for each case are indicated by shaded squares.

**Table V-1. Link numbers, names, and functions**

| Link No. | Link name | Function |
|---|---|---|
| 1 | INPUT | Input of the problem is obtained, ordered, and printed out if requested. Relabelling of mesh points is performed upon request. |
| 2 | GENERATION | Governing equations of the problem are generated by generating and assembling element stiffness and load matrices. Element stiffnesses are saved on tape ITAS. |
| 3 | DEFLECTION | Governing equations are solved for deflections and the deflections are printed out in the overall coordinate system; the forces acting on the structure at the nodes are computed and nodal sets are obtained and saved on tape ITAS. |
| 4 | STRESS | Stresses at the nodes are obtained by using the information on tape ITAS and are printed out. |

In the physical program, the binary cards of each link are preceded by chain control cards as shown in Fig. II-2. The first number shown on a chain control card is the link number, and the second number is the number of the tape that is loaded with the program. The second number should be identical with the number appearing in the ITAP field of the control card of input data. As shown in Fig. II-2, the program tape number of the standard ELAS is 2. On the control card of input data, if the IBUN, ICOR, IMES fields contain 0, and the IPIR field contains 0 or 1, the standard ELAS may be used as it is, provided that the chain program tape number is 2; if a different chain program tape number is to be used, the standard ELAS may be used only after changing the tape number in the chain control cards.

## Table V-2. Programs in Link 1 of ELAS (input link)

| Program name | Length in 36-bit words | Label | Function | Case No. of structure that requires this program | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| | | | **Programs provided by ELAS**[a] | | | | | | | | | | | | | | |
| MAIN | 3291 | ELAS1 | Reads, stores, and checks input data | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| ARAN | 1295 | E1ARN | Relabels mesh points | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| COOR | 152 | E1COR | Reads and stores coordinates | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| EXCH | 223 | E1EXC | Interchanges consecutive rows and columns of connectivity matrix | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| LEBIN[b] | 12 | E1LED | Checks whether a binary bit is 0 or 1 | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| MEST | 415 | E1MES | Reads and stores mesh topology data | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| OUTPT | 104 | E1OPT | Writes information related with relabelling | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| SEBIN[b] | 52 | E1LED | Stores 1 or 0 to prescribed binary bit | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| SRAT | 713 | E1SRT | Generates connectivity matrix and describes topology of stiffness matrix | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| TABL | 424 | E1TBL | Prints first output item | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| TOPO | 977 | E1TOP | Prepares element properties and checks | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| TICK[b] | 15 | TICK | Measures time | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ | ▩ |
| | | | **Programs that may be provided by the user** | | | | | | | | | | | | | | |
| BUNG | 19 | E1BUNG | Dummy subroutine for boundary condition generator | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ |
| CORG | 19 | E1CORG | Dummy subroutine for coordinate generator | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ |
| MESG | 19 | E1MESG | Dummy subroutine for mesh generator | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ | ▧ |

[a]Programs provided by FORTRAN II system: CHAIN, XLOC, (FIL), (FPT), (RTN), (RWT), (STH), (TSH), (SCH).

[b]In FAP language. ▩ Necessary. ▧ Optional (see Section V-A).

## Table V-3. Programs in Link 2 of ELAS (generation link)

| Program name | Length in 36-bit words | Label | Function | Case No. of structure that requires this program | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| | | | Programs provided by ELAS[a] | | | | | | | | | | | | | | |
| MAIN | 1492 | ELAS2 | Generates governing equations | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| ADM | 138 | E2ADM | Adds submatrices to form element stiffness matrix | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| BEAM | 159 | E2BEA | Generates submatrices for element types 2 and 4 | | | ■ | ■ | | | | | | | | | | |
| CODI | 255 | E2COD | Obtains transformation matrix of local/overall coordinates for line element | | | ■ | ■ | ■ | | | | | | | | | |
| CORT | 166 | E2CRT | Obtains coordinates of triangular shell element in local coordinates | | | | | | | | | | ■ | ■ | | | |
| CUTE | 216 | E2CUT | Cuts quadrilaterals and hexahedrons into triangles and tetrahedrons | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| DARN | 155 | E2DRN | Prepares information related with constraints for assembly | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| DMM | 68 | E2DMM | Obtains the product of element stiffness matrix times a vector | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| ELDI | 155 | E2ELD | Obtains unit vector of pressure for line elements | ■ | ■ | ■ | ■ | | | | | | | | | | |
| PLBE | 109 | E2PLB | Generates submatrices for element types 3 and 4 | | | | ■ | ■ | | | | | | | | | |
| RLOC | 99 | E2RLC | Adds submatrices to form element stiffness matrices of line elements | | | ■ | ■ | ■ | | | | | | | | | |
| STFS | 93 | E2STF | Selects proper subroutine for generation of element matrices | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| STRA | 114 | E2STR | Transforms descriptions of element matrices from local to overall coordinates | | | ■ | ■ | ■ | | | | | ■ | ■ | | | |
| S01 | 121 | E2S01 | Generates stiffness and load matrices for element type 1 | ■ | ■ | | | | | | | | | | | | |
| S02 | 137 | E2S02 | Generates stiffness and load matrices for element type 2 | | | ■ | | | | | | | | | | | |
| S03 | 127 | E2S03 | Generates stiffness and load matrices for element type 3 | | | | | ■ | | | | | | | | | |
| S04 | 222 | E2S04 | Generates stiffness and load matrices for element type 4 | | | | ■ | | | | | | | | | | |
| S05 | 147 | E2S05 | Generates stiffness and load matrices for element type 5 | | | | | | ■ | ■ | | | ■ | ■ | ■ | | |
| S07 | 448 | E2S07 | Generates stiffness and load matrices for element type 7 | | | | | | | | ■ | | ■ | | | | |
| S09 | 567 | E2S09 | Generates stiffness and load matrices for element type 9 | | | | | | | | | ■ | | | | | |
| S11 | 148 | E2S11 | Generates stiffness and load matrices for element type 11 | | | | | | | | | | ■ | | | | |
| S13 | 55 | E2S13 | Generates stiffness and load matrices for element type 13 | | | | | | | | | | | ■ | | | |
| S15 | 937 | E2S15 | Generates stiffness and load matrices for element type 15 | | | | | | | | | | | | ■ | | |
| S17 | 311 | E2S17 | Generates stiffness and load matrices for element type 17 | | | | | | | | | | | | | ■ | ■ |

### Table V-3 (contd)

| Program name | Length in 36-bit words | Label | Function | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Case No. of structure that requires this program | | | | | | | | | | | | | |
| S18 | 221 | E2S18 | Generates stiffness and load matrices for element type 18 | | | | | | | | | | | | | | ■ |
| TOPO | 387 | E2TOP | Prepares element properties | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| TRAN | 139 | E2TRN | Transforms the description of a vector from local to overall coordinates | | | ■ | ■ | ■ | | | | | ■ | ■ | | | |
| TRIM | 57 | E2TRI | Generates *M*, *N*, and *L* matrices of thin triangular elements | | | | | | ■ | ■ | ■ | | ■ | ■ | ■ | | |
| TRM | 191 | E2TRM | Performs triple matrix product of order 3 | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| TICK[b] | 15 | TICK | Measures time | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| **Program that may be provided by the user** | | | | | | | | | | | | | | | | | |
| CAS2 | 19 | E2CAS2 | Dummy subroutine | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ |

[a]Programs provided by FORTRAN II system: ATAN, CHAIN, COS, LOG, SQRT, (FIL), (FPT), (RWT), (STB), (STH), (WLR).

[b]In FAP language. ■ Necessary. ▨ Optional (see Section V).

### Table V-4. Programs in Link 3 of ELAS (deflection link)

| Program name | Length in 36-bit words | Label | Function | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Case No. of structure that requires this program | | | | | | | | | | | | | |
| **Programs provided by ELAS[a]** | | | | | | | | | | | | | | | | | |
| MAIN | 548 | ELAS3 | Obtains deflection components in overall coordinates | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| ELST | 340 | E3ELT | Writes on tape ITAS element set information | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| RESI | 251 | E3RES | Computes residual forces at the nodes | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| RESW | 164 | E3REW | Writes residual forces at the nodes | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| TICK[b] | 15 | TICK | Measures time | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| VELAS | 594 | E3VEL | Solves the governing equations by variable-bandwidth Cholesky method | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| **Program that may be provided by the user** | | | | | | | | | | | | | | | | | |
| PUNC | 19 | E3PUNC | Dummy subroutine | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ |

[a]Programs provided by FORTRAN II system: CHAIN, SQRT, (FIL), (FPT), (STH), (RLR), (TSB), (BST), (STB), (WLR).

[b]In FAP language. ■ Necessary. ▨ Optional (see Section V-A).

## Table V-5. Programs in Link 4 of ELAS (stress link)

| Program name | Length in 36-bit words | Label | Function | Case No. of structure that requires this program | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| | | | **Programs provided by ELAS[a]** | | | | | | | | | | | | | | |
| MAIN | 592 | ELAS4 | Obtains stresses at mesh points | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| ABEQ | 478 | E4ABQ | Generates equations for stress boundary conditions at a node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| BEST | 276 | E4BST | Obtains best-fit plane segment at a boundary node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| BOFI | 855 | E4BOF | Finds if a node is on boundary | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| CODI | 255 | E4COD | Obtains transformation matrix of local/overall coordinates for line element | ■ | ■ | ■ | ■ | ■ | | | | | | | | | |
| DIMI | 547 | E4DIM | Generates stresses for line elements | ■ | ■ | ■ | ■ | ■ | | | | | | | | | |
| DINA | 86 | E4DIN | Obtains local coordinate axes at a node in shells | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| EPAN | 314 | E4EPN | Increases node set beyond 9 in shells at a node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| FINDQ | 49 | E4FNQ | Obtains deflections of a node in overall coordinates | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| FINDX | 54 | E4FNX | Obtains overall coordinates of a node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| GENE | 563 | E4GEN | Generates NEL and MAC matrices of a node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| INER | 69 | E4INR | Obtains a vector heading towards the structure at a boundary node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| INLZ | 328 | E4INZ | Initializes scalars, vectors, and matrices at a node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| INV | 517 | E4INV | Inverts matrices up to order 8 by Gauss elimination | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| LEST | 404 | E4LST | Obtains strain components by least squares at a node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| MDIN | 120 | E4MDN | Orients local axes properly at a boundary node in shells | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| META | 412 | E4MET | Generates material matrix in local coordinates in the 1,2,12,3,13,23 order at a node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| QUAD | 719 | E4QAD | Finds local coordinate axes by best-fit quadratic surface at a shell node | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| REVO | 731 | E4REV | Finds local axes by best-fit fourth-order polynomial at a node of shells of revolution | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| ROTA | 357 | E4ROT | Expresses material matrix in local axes | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| SAME | 216 | E4SAM | Expresses stress tensor in overall coordinate system | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| SCAL | 48 | E4SCL | Performs scalar vector product | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| SETA | 489 | E4SET | Generates strain-deflection relationship at a nodal line | | | | | | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |
| STRA | 114 | E4STA | Transforms descriptions of element matrices from local to overall coordinates | ■ | ■ | ■ | ■ | ■ | | | | | | | | | |

## Table V-5 (contd)

| Program name | Length in 36-bit words | Label | Function | Case No. of structure that requires this program | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| STRS | 223 | E4STR | Computes stresses from strains | | | | | | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ |
| TEMP | 68 | E4TMP | Computes nodal line vector in original material axes | | | | | | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ |
| TICK[b] | 15 | TICK | Measures time | | | | | | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ |
| TOPO | 387 | E4TOP | Prepares element properties | | | | | | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ |
| TRAN | 139 | E4TRN | Transforms the description of a vector from local to overall coordinates | ▓ | ▓ | ▓ | ▓ | ▓ | | | | | | | | | |
| UNIT | 98 | E4UNT | Obtains a unit vector along a line segment | | | | | | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ |
| VECT | 76 | E4VCT | Performs vectorial vector product | | | | | | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ | ▓ |
| **Programs that may be provided by the user** | | | | | | | | | | | | | | | | | |
| AGEL | 19 | E4AGEL | Dummy subroutine | | | | | | | | | | ▨ | ▨ | | ▨ | ▨ |
| CAS4 | 19 | E4CAS4 | Dummy subroutine | | | | | | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ |

[a]Programs provided by FORTRAN II system: ATAN, CHAIN, EXP(2), SIN, SQRT, (BST), (FIL), (FPT), (RLR), (STH), (TSB).

[b]In FAP language. ▓ Necessary. ▨ Optional (see Section V-A).

If the number in the IBUN, ICOR, or IMES fields of the control card of input data is larger than 0, the user should provide his own version of subroutines BUNG, CORG, and MESG, respectively, for Link 1. If the number in the IPIR field of the control card of input data is larger than 1, the user should prepare his own version of subroutine AGEL for Link 4. The rules for preparing these subroutines are explained in the following sections. If the user desires to punch or to plot the deflections in overall coordinates, he should provide his own version of subroutine PUNC for Link 3, as explained in Section V-F. If the INP field of the control card of input data contains a number larger than 1, the user may choose to replace subroutines CAS2 and CAS4 of Links 2 and 4, respectively, with his own version of the subroutines explained in Sections V-G and V-H. Once the binary cards of the subroutines developed by the user are obtained, they should replace the corresponding ones in the standard ELAS.

If the user desires to run problems having as many unsuppressed degrees of freedom as possible, in addition to using the efficient relabelling scheme, he may eliminate the subroutines that are not needed for his case (see Tables V-2 through V-5). These may be replaced by dummy subroutines, which may be written as shown in Fig. V-1. It should be remembered that any gain in the program length, especially in Link 2, directly adds to the available storage area for the problem itself.

When using the standard ELAS, if an error is detected in a given job, this job is ignored and the program automatically skips all the input cards related with that job until it encounters a new job. In the standard ELAS, all input data is read in Link 1, and automatic skip procedure is at the end of Link 1. In the user-provided subroutines, if information is being read from the data cards, these data cards should be placed according to Fig. IV-1. If some of these cards are being read in the second, the third, and the fourth links, in case of error, the automatic skipping to the next job will not work.

### B. Rules for Preparing Subroutine CORG

When the number in the ICOR field of the control card of input data is larger than 0, the user should provide his own version of subroutine CORG for Link 1. The objective of this subroutine is to generate the coordinates of the mesh points in the overall coordinates and store them in the allocated area in the memory. The FORTRAN II statements that should be included in the source program of this subroutine are given in Fig. V-2, where the statements indicated by dots are to be provided by the user. The meanings of the variables that appear in Fig. V-2 are given in the comment statements.

### C. Rules for Preparing Subroutine MESG

When the number in the IMES field of the control card of input data is larger than 0, the user should provide his own version of subroutine MESG for Link 1. The objective of this subroutine is to generate the information given in Table III-4 for each element in the structure and store them in the allocated area in the memory. The FORTRAN II statements that should be included in the source program of this subroutine are given in Fig. V-3, where the statements indicated by dots are to be provided by the user. The meanings of the variables that appear in Fig. V-3 are given in the comment statements.

### D. Rules for Preparing Subroutine BUNG

When the number in the IBUN field of the control card of input data is larger than 0, the user should provide his own version of subroutine BUNG for Link 1. The objective of this subroutine is to generate the dbc input items (see Section III-D) and store them in the allocated area in the memory. The FORTRAN II statements that should be included in the source program of this subroutine are given in Fig. V-4, where the statements indicated by dots are to be provided by the user.

```
CNAME..
      SUBROUTINE NAME..
C           NAME.. SHOULD BE REPLACED BY THE NAME OF THE SUBROUTINE WHICH
C           IS TO BE REPLACED.
      RETURN
      END
```

**Fig. V-1. FORTRAN II statements of dummy subroutines**

```
CCORG..
      SUBROUTINE CORG
C           TO GENERATE THE OVERALL COORDINATES OF MESH POINTS OF........
C           SUBROUTINE CORG IS CALLED ONCE.
C           THE MEANINGS OF THE SYMBOLS USED IN THE PROGRAM FOLLOW
C           A     THE NAME OF THE COMMON. THE USER IS NOT ALLOWED TO USE
C                 THE COMMON, EXCEPT FOR B.
C           B     PORTION OF COMMON AVAILABLE FOR THE USERS USE
C                 B IS NOT DESTROYED THROUGHOUT THE FIRST LINK
C           IGEM  INDICATOR WHICH IS ALREADY DEFINED. IGEM=0 MEANS NO Z
C                 COORDINATE IS NECESSARY.
C           IN    NUMBER OF MESH POINTS. ALREADY DEFINED
C           IXX   COUNT OF THE WORD PRECEDING X COORDNTS IN COMMON, DFND.
C           IYY   COUNT OF THE WORD PRECEDING Y COORDNTS IN COMMON, DFND.
C           IZZ   COUNT OF THE WORD PRECEDING Z COORDNTS IN COMMON, DFND.
C           M     USERS LABEL FOR MESH POINTS
      DIMENSION A(1),B(50)
      COMMON A
      EQUIVALENCE (A(1),IN),(A(71),IXX),(A(72),IYY),(A(73),IZZ),
     1(A(78),IGEM),(A(200),B)
      DO 10 M=1,IN
C           COMPUTE OR READ IN THE OVERALL COORDINATES OF THE M TH MESH
C           POINT IN LOCATIONS XX,YY AND ZZ
C     .................
C     .................
C     .................
      IXXM=IXX+M
      IYYM=IYY+M
      IZZM=IZZ+M
      A(IXXM)=XX
      A(IYYM)=YY
      IF (IGEM) 10,10,9
    9 A(IZZM)=ZZ
   10 CONTINUE
      RETURN
      END
```

Fig. V-2. FORTRAN II statements of subroutine CORG

```
CMESG..
      SUBROUTINE MESG
C           TO GENERATE INFORMATION RELATED WITH THE MESH OF ...........
C           SUBROUTINE MESG IS CALLED ONCE
C           THE MEANINGS OF THE SYMBOLS USED IN THE PROGRAM FOLLOW
C           A,IA NAMES OF THE COMMON. THE USER IS NOT ALLOWED TO USE
C                COMMON  EXCEPT FOR B.
C           B    PORTION OF COMMON AVAILABLE FOR THE USERS USE
C                B IS NOT DESTROYED THROUGHOUT THE FIRST LINK
C           IT   NUMBER OF ELEMENTS. ALREADY DEFINED.
C           I8   NUMBER OF WORDS NECESSARY FOR AN ELEMENT. ALREADY DEFND.
C           M    ELEMENT COUNT
C           J1W,J2W,J3W,J4W,J5W,J6W,J7W,J8W,J9W,J10W  ARE THE INFORMATION
C           SHOWN IN TABLE 3.2 FOR THE M TH ELEMENT. USER DEFINES THESE
      DIMENSION A(1),IA(1),B(50)
      COMMON A,IA
      EQUIVALENCE (A,IA),(A(50),J1),(A(51),J2),(A(52),J3),(A(53),J4),
     1(A(54),J5),(A(55),J6),(A(56),J7),(A(57),J8),(A(345),J9),(A(344),
     2J10),(A(3),IT),(A(11),I8)
      DO 10 M=1,IT
C           COMPUTE OR READ IN INFORMATION J1W,J2W,J3W,J4W,J5W,J6W,J7W,
C           J8W,J9W,J10W FOR THE M TH ELEMENT
C     .................
C     .................
C     .................
      J1M=J1+M
      J2M=J2+M
      J3M=J3+M
      J4M=J4+M
      J5M=J5+M
      J6M=J6+M
      J7M=J7+M
      J8M=J8+M
      J9M=J9+M
      J10M=J10+M
      IA(J1M)=J1W
      IA(J2M)=J2W
      IA(J3M)=J3W
      IA(J4M)=J4W
      IA(J5M)=J5W
      IF(I8-6) 9,9,10
    9 IA(J6M)=J6W
      IF (I8-7)8,8,10
    8 IA(J7M)=J7W
      IF (I8-8) 7,7,10
    7 IA(J8M)=J8W
      IF (I8-9) 6,6,10
    6 IA(J9M)=J9W
      IF (I8-10)5,5,10
    5 IA(J10M)=J10W
   10 CONTINUE
      END
```

**Fig. V-3. FORTRAN II statements of subroutine MESG**

```
CBUNG..
      SUBROUTINE BUNG
C          TO GENERATE DBC INPUT UNITS OF .................
C          THE MEANINGS OF THE SYMBOLS USED IN THE PROGRAM FOLLOW
C          A,IA   NAMES OF THE COMMON. THE USER IS NOT ALLOWED TO USE
C                 COMMON EXCEPT FOR B.
C          B      PORTION OF COMMON AVAILABLE FOR THE USERS USE
C                 B IS NOT DESTROYED THROUGHOUT THE FIRST LINK
C          IBN    NUMBER OF DBC INPUT UNITS WHICH IS ALREADY DEFINED BY
C                 THE MAIN PROGRAM. IF THE ACTUAL COUNT OF DBC INPUT
C                 UNITS IS LARGER THAN 1000, THE MAIN PROGRAM DEFINES IT
C                 AS 1000. SUBROUTINE BUNG IS CALLED ONCE FOR EVERY 1000
C                 AND FRACTION THEREFROM.
C          I      DBC INPUT UNIT COUNT
      DIMENSION A(1),IA(1),IDUM(1),DUMMY(1)
      COMMON IA,A
      EQUIVALENCE (A,IA),(A( 9000),DUMMY,IDUM),(A(2),IBN),(A(200),B)
      DO 10 I=1,IBN
C          COMPUTE THE FIRST INDICE PAIR (IQ,JQ),THE SECOND INDICE
C          PAIR (IR,JR),AND THE RELATED CONSTANT (C) OF THE I TH
C          DBC INPUT UNIT (SEE SECTION III.D OF ELAS MANUAL). DATA
C          MAY BE READ IN.
C     .................
C     .................
C     .................
      IDUM(I)=IQ
      IDUM(I+1000)=JQ
      IDUM(I+2000)=IR
      IDUM(I+3000)=JR
      DUMMY(I+4000)=C
   10 CONTINUE
      RETURN
      END
```

**Fig. V-4. FORTRAN II statements of subroutine BUNG**

The meanings of the variables that appear in Fig. V-4 are given in the comment statements.

### E. Rules for Preparing Subroutine AGEL

When the number in the IPIR field of the control card of input data is larger than 1, the user should provide his own version of subroutine AGEL for Link 4. The objective of this subroutine is to generate the direction cosines of the local coordinate axes at the mesh points of shell structures. ELAS refers to the local coordinate systems for the descriptions of deflections, stress resultants, and stress couples. The FORTRAN II statements that should be included in the source program of this subroutine are given in Fig. V-5, where the statements indicated by dots are to be provided by the user. The meanings of the variables that appear in Fig. V-5 are given in the comment statements.

### F. Rules for Preparing Subroutine PUNC

If the user desires to plot or to punch the deflections in the overall coordinate system, he can do so by providing his own version of subroutine PUNC for Link 3. The FORTRAN II statements that should be included in the source program of this subroutine are given in Fig. V-6, where the statements indicated by dots are to be provided by the user. The meanings of the variables that appear in Fig. V-6 are given in the comment statements.

```
CAGEL..
      SUBROUTINE AGEL
C           TO GENERATE THE DIRECTION COSINES MATRIX DIN
C           SUBROUTINE AGEL IS CALLED AT LEAST ONCE FOR EVERY MESH
C           POINT OF TWO AND THREE DIMENSIONAL CONTINUA
C           THE MEANINGS OF THE SYMBOLS USED IN THE PROGRAM FOLLOW
C           A      THE NAME OF THE COMMON. THE USER IS NOT ALLOWED TO USE
C                  THE COMMON EXCEPT FOR C
C           C      PORTION OF COMMON AVAILABLE FOR THE USERS USE.
C                  C IS DESTROYED BEFORE THE NEXT CALL
C           ICN    USERS LABEL FOR THE MESH POINT. ALREADY DEFINED.
C                  SUBROUTINE AGEL IS CALLED ONCE FOR EVERY MESH POINT
C           XII    DIRECTION COSINES OF THE FIRST LOCAL AXIS
C           ETA    DIRECTION COSINES OF THE SECOND LOCAL AXIS
C           ZTA    DIRECTION COSINES OF THE THIRD  LOCAL AXIS
C           DIN    THE MATRIX WHOSE COLUMNS ARE XII,ETA AND ZTA
      DIMENSION A(1),XII(3),ETA(3),ZTA(3),DIN(3,3),C(50)
      COMMON A
      EQUIVALENCE (A(201),ICN),(A(226),DIN),(DIN(1),XII),(DIN(4),ETA),
     1(DIN(7),ZTA),(A(15420),C)
C           COMPUTE OR READ IN THE DIRECTION COSINES OF THE FIRST LOCAL
C           AXIS IN C1,C2,C3 ,THE DIRECTION COSINES OF THE SECOND LOCAL
C           AXIS IN D1,D2,D3,AND THOSE OF THE THIRD LOCAL AXIS IN
C           E1,E2,E3, FOR THE (ICN)TH MESH POINT, IN THE OVERALL
C           COORDINATE SYSTEM
C     .................
C     .................
C     .................
      XII(1)=C1
      XII(2)=C2
      XII(3)=C3
      ETA(1)=D1
      ETA(2)=D2
      ETA(3)=D3
      ZTA(1)=E1
      ZTA(2)=E2
      ZTA(3)=E3
      RETURN
      END
```

**Fig. V-5. FORTRAN II statements of subroutine AGEL**

```
CPUNC..
      SUBROUTINE PUNC
C             TO OBTAIN COORDINATES AND DEFLECTIONS IN OVERALL FOR.........
C             FOR PUNCHING OR PLOTTING
C             SUBROUTINE PUNCH IS CALLED ONCE
C             THE MEANINGS OF THE SYMBOLS USED IN THE PROGRAM FOLLOW
C             A       THE NAME OF THE COMMON. THE USER IS NOT ALLOWED TO
C                     USE THE COMMON, EXCEPT FOR B.
C             B       PORTION OF COMMON AVAILABLE FOR THE USERS USE
C                     B IS DESTROYED AFTER THIRD LINK.
C             ZGEM    INDICATOR WHICH IS ALREADY DEFINED. ZGEM=0 MEANS ALL
C                     Z COORDINATES ARE ZERO.
C             IN      NUMBER OF MESH POINTS. ALREADY DEFINED
C             IXX   COUNT OF THE WORD PRECEDING X COORDNTS IN COMMON, DFND.
C             IYY   COUNT OF THE WORD PRECEDING Y COORDNTS IN COMMON, DFND.
L             IZZ   COUNT OF THE WORD PRECEDING Z COORDNTS IN COMMON, DFND.
C             M       USERS LABEL FOR MESH POINTS
C             BB      ARRAY FOR DEFLECTIONS IN OVERALL COORDINATES. DEFINED.
C             X       X COORDINATE OF THE M TH MESH POINT, DEFINED
C             Y       Y COORDINATE OF THE M TH MESH POINT, DEFINED
C             Z       Z COORDINATE OF THE M TH MESH POINT, DEFINED
C             U       DISPLACEMENT ALONG X-AXIS FOR THE M TH MESH POINT,DFND.
C             V       DISPLACEMENT ALONG Y-AXIS FOR THE M TH MESH POINT,DFND.
C             W       DISPLACEMENT ALONG Z-AXIS FOR THE M TH MESH POINT,DFND.
C             TX      ROTATION ABOUT X-AXIS FOR THE M TH MESH POINT, DEFINED.
C             TY      ROTATION ABOUT Y-AXIS FOR THE M TH MESH POINT, DEFINED.
C             TZ      ROTATION ABOUT Z-AXIS FOR THE M TH MESH POINT, DEFINED
      DIMENSION A(1),BB(999,6),B(50)
      COMMON A
      EQUIVALENCE (A(1),IN),(A(40),ZGEM),(A(71),IXX),(A(72),IYY),
     1(A(73),IZZ),(A( 9000),BB),(A(200),B)
      DO 10 M=1,IN
      IXXM=IXX+M
      IYYM=IYY+M
      IZZM=IZZ+M
      X=A(IXXM)
      Y=A(IYYM)
      Z=A(IZZM)*ZGEM
      U=BB(M,1)
      V=BB(M,2)
      W=BB(M,3)
      TX=BB(M,4)
      TY=BB(M,5)
      TZ=BB(M,6)
C     THE COORDINATES OF THE M TH MESH POINT ARE IN X,Y AND Z
C     THE DEFLECTIONS OF THE M TH MESH POINT ARE IN U,V,W,TX,TY,TZ
C     THE USER MAY USE THEM IN HIS STATEMENTS
      ..................
      ..................
      ..................
   10 CONTINUE
      RETURN
      END
```

**Fig. V-6. FORTRAN II statements of subroutine PUNC**

## G. Rules for Preparing Subroutine CAS2

ELAS determines the amount of output requested by the contents of the INP field of the control card of input data. Output is generated according to the following: minimum output if the INP field contains 0; intermediate output if it contains 1; and detailed output if it contains 2. The meanings of these output levels are explained in Section VI-A. The detailed output includes the element stiffness and load matrices of every element, the overall stiffness matrix, and the loading vector. This abundance of output data may not be desirable. If the user desires to output only a limited number of element matrices, he may do so by prescribing the contents of INP less than 2, and providing his own version of subroutine CAS2 for Link 2. The FORTRAN II statements that should be included in the source program of this subroutine are given in Fig. V-7, where the statements indicated by dots are to be provided by the user. The meanings of the variables that appear in Fig. V-7 are given in the comment statements.

## H. Rules for Preparing Subroutine CAS4

If the content of the INP field of the control card of input data is 2, ELAS provides the details of the best-fit stress computation at every mesh point of two- and three-dimensional continua. INP=2 generates detailed output, which may not be desirable. If the user desires to output only a limited number of best-fit stress computations at mesh points of two- and three-dimensional continua, he may do so by prescribing the contents of INP less than 2, and providing his own version of subroutine CAS4 for Link 4. The FORTRAN II statements that should be included in the source program of this

```
CCAS2..
      SUBROUTINE CAS2
C            TO PRINT OUT SELECTED ELEMENT AND OVERALL SYSTEM MATRICES....
C            SUBROUTINE CAS2 IS CALLED ONCE FOR EVERY ELEMENT
C            THE MEANINGS OF THE SYMBOLS USED IN THE PROGRAM FOLLOW
C            A       THE NAME OF THE COMMON. THE USER IS NOT ALLOWED TO
C                    USE THE COMMON
C            M       THE COUNT OF THE CURRENT ELEMENT, DEFINED.
C            INP     INDICATOR. THE USER MAY REDEFINE IT FOR CURRENT ELEMENT
C                    2 MEANS DETAILED, 1 MEANS NORMAL, 0 MEANS MINIMUM
C                    PRINT OUT.
C            IT      TOTAL NUMBER OF ELEMENTS, DEFINED
C            NN      THE COUNT OF ELEMENT FOR WHICH INP INDICATOR IS TO
C                    BE CHANGED. IF NN=IT+1,THE REFERENCE IS UNDERSTOOD
C                    TO THE GOVERNING LINEAR EQUATIONS OF THE SYSTEM
C                    NN IS DEFINED BY USER.
      DIMENSION A(1)
      COMMON A
      EQUIVALENCE (A(3),IT),(A(25),M),(A(42),INP)
C             DEFINE ONE OR MORE NN, KEEPING IN MIND THAT FOR
C            EVERY ELEMENT, SUBROUTINE CAS2 IS CALLED ONCE.
C            CURRENT ELEMENT NUMBER IS AVAILABLE IN M.
C            SUPPOSE ONE NN IS DEFINED FOR WHICH INP IS TO
C            BE MADE 2. THE STATEMENTS ARE AS FOLLOWS
C     ..................
C     ..................
C     ..................
      IF (NN-M) 9,10,9
   10 INP=2
    9 RETURN
C            THE CALLING PROGRAM RESTORES INP FOR NEXT ELEMENT
      END
```

**Fig. V-7. FORTRAN II statements of subroutine CAS2**

subroutine are given in Fig. V-8, where the statements indicated by dots are to be provided by the user. The meanings of the variables that appear in Fig. V-8 are given in the comment statements.

```
CCAS4..
      SUBROUTINE CAS4
C            TO PRINT OUT DETAILS OF STRESS COMPUTATION  AT MESH POINT....
C            SUBROUTINE CAS4 IS CALLED ONCE FOR EVERY MESH POINT OF TWO
C            AND THREE DIMENSIONAL CONTINUA
             THE MEANINGS OF THE SYMBOLS USED IN THE PROGRAM FOLLOW
             A       THE NAME OF THE COMMON. THE USER IS NOT ALLOWED TO
                     USE THE COMMON
             ICN     THE MESH POINT WHICH IS BEING PROCESSED
             NN      THE MESH POINT FOR WHICH INP INDICATOR IS TO BE CHANGED
C                    NN IS DEFINED BY USER
             INP     INDICATOR.THE USER MAY REDEFINE IT FOR CURRENT NODE
                     2 MEANS DETAILED,1 MEANS NORMAL,0 MEANS MINIMUM
      DIMENSION A(1)
      COMMON A
      EQUIVALENCE (A(201),ICN),(A(42),INP)
             DEFINE ONE OR MORE NN, KEEPING IN MIND THAT FOR EVERY MESH
             POINT OF TWO- OR THREE DIMENSIONAL CONTINUA, SUBROUTINE CAS4
             IS CALLED ONCE. CURRENT MESH POINT NUMBER IS AVAILABLE IN ICN
             SUPPOSE ONE NN IS DEFINED FOR WHICH INP IS TO BE MADE 2.
             THE STATEMENTS ARE AS FOLLOWS
C     ..................
C     ..................
C     ..................
      IF (NN-ICN) 9,10,9
   10 INP=2
    9 RETURN
C            THE CALLING PROGRAM RESTORES INP FOR NEXT MESH POINT
      END
```

**Fig. V-8. FORTRAN II statements of subroutine CAS4**

# VI. Description of Output

## A. Control of Output

In Table VI-1, all possible output items (except error messages) are listed chronologically with respect to the execution time. The output items are listed separately for each link, and a number is assigned to each output item for identification. The last three columns in the table show the levels of output that may be produced for each output item: According to whether the number 0, 1, or 2 is assigned to the INP field of the control card (Fig. IV-1), the produced output is *minimum, intermediate,* or *detailed,* respectively. The output items included in these three modes are shown by the shaded squares in the respective columns of Table VI-1. Thus the user has control of the amount of output by assigning 0, 1, or 2 to the INP field of the control card of input data. Note that for Output Items 14, 15, and 16, when the number assigned to INP is not 2, the user may control the production of output on a selective basis by substituting his version of subroutine CAS2, which is explained in Section V-G. He can similarly control the production of Output Item 22 by means of his version of subroutine CAS4, which is explained in Section V-H. The production of Output Item 5 is also dependent upon the number used in the ISHUF field of the control card of input data. Various levels of Output Item 5 are separately explained in Section VI-F.

Most of the output items are self-explanatory; however, some of them are not provided with enough explanatory messages. In the following sections, every output item is explained individually to furnish the user with sufficient information about the output data.

## B. Description of Output Items of Link 1 (as Listed in Table VI-1)

*Output Item 1.* This is a standard part of output data. The second line and the central column of numbers of this item may change from one job to another. The content of the comment card of the input data (Input Item 1) appears as it is on the second line of this table, and it is left justified. The numbers in the central columns are the parameters appearing on the control card of input data. Output Item 1 always appears on the first page of the ELAS-produced output, and it is self-explanatory.

*Output Item 2.* This consists of one or more tables separated by four blank lines. It always starts on a new page. The number of tables depends on the contents of IMAT, IPRS, ITIC, ISDT, ISDY, ISDZ, IARE, IMMX, IMMY, IMMZ, and IMFI fields of the control card of input data, since each one of these controls one table. If the content of any one of these is zero, the corresponding table does not appear. The tables appear in the

**Table VI-1. List of output items**

| Output item No. | Output item | Minimum INP = 0 | Intermediate INP = 1 | Detailed INP = 2 |
|---|---|---|---|---|
| | **Link 1 (input link)** | | | |
| 1 | Table for title and important constants | ■ | ■ | ■ |
| 2 | Tables for material, loading, and geometry | | ■ | ■ |
| 3 | Table for coordinates of mesh points | | ■ | ■ |
| 4 | Table for element properties | | ■ | ■ |
| 5 | Message and/or tables and punched cards for relabelling | @ | @ | @ |
| 6 | Table for reduced stiffness matrix | | ■ | ■ |
| 7 | Message about necessary storage for reduced stiffness matrix | ■ | ■ | ■ |
| 8 | Message about total common length | ■ | ■ | ■ |
| 9 | Table for diagonal elements of reduced stiffness matrix | | ■ | ■ |
| 10 | Tables for force and deflection boundary conditions | | ■ | ■ |
| 11 | Table for common, integer (IA block) | | | ■ |
| 12 | Table for common, floating (AA block) | | | ■ |
| 13 | Message for the execution time of Link 1 | ■ | ■ | ■ |
| | **Link 2 (generation link)** | | | |
| 14 | Tables for element stiffness matrices | * | * | ■ |
| 15 | Table for upper half of reduced stiffness matrix | * | * | ■ |
| 16 | Table for reduced load vector | * | * | ■ |
| 17 | Message for the execution time of Link 2 | ■ | ■ | ■ |
| | **Link 3 (deflection link)** | | | |
| 18 | Table for reduced solution vector | | | ■ |
| 19 | Table for deflections | ■ | ■ | ■ |
| 20 | Table for forces acting on the nodes | | ■ | ■ |
| 21 | Message for the execution time of Link 3 | ■ | ■ | ■ |
| | **Link 4 (stress link)** | | | |
| 22 | Table for stresses at the nodes | ■ | ■ | ■ |
| 23 | Details of the best-fit stress computation | † | † | ■ |
| 24 | Table for stresses of one-dimensional elements | ■ | ■ | ■ |
| 25 | Message for the execution time of Link 4 | ■ | ■ | ■ |

■ Output produced (blank means no output produced).
@ See Section VI-F for different levels of output.
* May be produced selectively by subroutine CAS2 (see Section V-G).
† May be produced selectively by subroutine CAS4 (see Section V-H).

following order: material properties, pressure types, thickness types, temperature increase types, temperature gradient types along $y$ axis, temperature gradient types along $z$ axis, cross-sectional area types, torsion constant types, moment of inertia types about the first principal axis, moment of inertia types about the second principal axis, and angle types defining principal axes. This order also corresponds to the order of the related fields of the control card as listed above. Each one of these tables is self-explanatory. In the program, and also in the related output items, the type numbers are used instead of the actual numerical values. The contents of these tables are provided by the user in the input data cards.

*Output Item 3.* This table starts on a new page. It contains the coordinates of the mesh points in the overall coordinate system as referred to the user's labels of the mesh points. The contents of Output Item 3 are provided by the user either by means of input data cards or by means of subroutine CORG. The table is self-explanatory.

*Output Item 4.* This table starts on a new page. It contains the information shown in Table III-4, as provided by the user either by the input data cards or by means of the subroutine MESG. The type numbers are those appearing in Output Item 2, and the labels are those of the user. The table is self-explanatory.

*Output Item 5.* This is explained separately in Section VI-F.

*Output Item 6.* This table starts on a new page. It gives the number of elements in the shaded zone of the reduced stiffness matrix of the structure given in Fig. II-1, for each row of the matrix. In each row of the table, there are 10 pairs of numbers. In each pair, the first number is the row label of the reduced stiffness matrix and the second number is the number of matrix elements in the shaded portion of the matrix as sketched in Fig. II-1.

*Output Item 7.* This is a message indicating the total number of matrix elements in the shaded portion of the reduced stiffness matrix shown in Fig. II-1. In the common memory, the number of words allocated for the overall stiffness matrix of the whole structure is equal to the number indicated by this message. This message follows immediately the preceding printed output item.

*Output Item 8.* The ELAS program keeps all input data and all generated information in the common area of the memory. Output Item 8 indicates the length of the total common area for the job being run. The message immediately follows Output Item 7.

*Output Item 9.* This table starts on a new page. Each row of the table contains 10 pairs of numbers. The first number in a pair is the row number of the reduced stiffness matrix, and the second number is the sequence number of the main diagonal matrix element of this row when the matrix elements of the shaded portion of the reduced stiffness matrix (see Fig. II-1) are counted rowwise. The last pair in the table should be ignored.

*Output Item 10.* In this item, there are as many tables as the number of degrees of freedom at a mesh point (that is, as many as the content of the IDEG field of the control card of input data), and each one of these tables starts on a new page. The tables are labelled in the first line with the sequence number of the degree of freedom at a mesh point (see Section III-D, first paragraph). In these tables, every mesh point (called node) is identified with the user's label, and associated with four quantities listed under the column headings P, IBO, IBB, and C. The number under P is the magnitude of the concentrated load acting at this mesh point in the degree-of-freedom direction appearing in the table heading. This quantity, if it is not zero, is the same as the quantity provided by the user in the input cards for the prescribed concentrated loads. The numbers appearing in the remaining three columns are related with the deflection boundary conditions provided by the user either in the input data cards or by the subroutine BUNG. In general, the numbers in the IBB column are the sequence numbers of the primary deflection components in the vector of unknown deflections (see Fig. II-1). The numbers in the IBO column are used in finding the index pairs of the dbc input units, and the numbers in the C column include the constants of the dbc input units. In Table VI-2, a detailed explanation is given for the meanings of the numbers in IBB, IBO, and C columns.

*Output Item 11.* This table starts on a new page under the title IA BLOCK. Each row of the table contains 10 pairs of integer numbers. The first number in a pair is the word count in the common, and the second is the fixed-point-mode interpretation of the contents of the related word. The table gives the contents of the common area in the fixed-point mode at the end of Link 1, or when an error is encountered in Link 1 and the job is terminated. Approximately the first 200 words of the common contain the important parameters and the pointers of various one-dimensional arrays related with the geometry, material, and boundary conditions of the structure. A pointer is a word that contains the address (in this case the word count in the common) of the first word before the related array. In Table VI-3, the common map is given in terms of pointers (whenever necessary). Output Item 11 is especially useful in finding the error related with input data.

*Output Item 12.* This table starts on a new page under the title AA BLOCK. Each row of the table contains 5 pairs of numbers. The first number in a pair is the word count in common, and the second is the interpretation of the contents of the related word in floating-point mode. Therefore, this table is the floating-point interpretation of the information given in Output Item 11. Output Items 11 and 12 may be used together with Table VI-3 in locating any possible input error.

**Table VI-2. Explanation of numbers in IBO, IBB, and C columns of Output Item 10**[a]

| Node | IBO | IBB | C | Explanation |
|---|---|---|---|---|
| $i$ | $-1$ | $k$ | 1. | Deflection component at node $i$ and in direction $j$ ($X_p$) is the $k$th unknown ($x_k$): $X_p = x_k$ |
| $i$ | 0 | $m$ | a | Deflection component at node $i$ and in direction $j$ ($X_p$) is prescribed as a: $X_p = a$ |
| $i$ | $p$ | $k$ | $b$ | Deflection component at node $i$ and in direction $j$ ($X_p$) is dependent on only the $k$th unknown ($x_k$): $X_p = bx_k$ |
| $i$ | 10,000 | $-m$ | 0 | Deflection component at node $i$ and in direction $j$ ($X_p$) is dependent on some unknowns without constant term (the related unknowns may be obtained from negative 5-digit IBO numbers as explained in the last case below) |
| $i$ | $10{,}000+p$ | $-m$ | a | Deflection component at node $i$ and in direction $j$ ($X_p$) is dependent on some unknowns with constant term $X_p = a + \ldots$ (the related unknowns may be obtained from negative 5-digit IBO numbers as explained below) |
| $i$ | $-10{,}000-n$ | $-k$ | $b$ | Deflection component at node $i$ and in direction $j$ ($X_p$) is the $k$th unknown $X_p = x_k$; it is also used in defining the dependent deflection $X_n$: $X_n = \cdots + bx_k$ |

[a]Nomenclature:

- $i$ user's label of the mesh point, $1 \leq i \leq$ IN
- $j$ degree-of-freedom direction appearing in the heading of the table, $1 \leq j \leq$ IDEG
- $p$ $(i-1)$ * IDEG + $j$
- $m$ IN * IDEG + 1
- $k$ entry number in the vector of unknowns (see Fig. II-1)
- $n$ integer, similar to $p$, $1 \leq n < m$
- $a$, $b$ constants appearing in the dbc input units
- $X$ a general deflection component
- $x$ an element of the vector of unknowns (see Fig. II-1)

***Output Item 13.*** This is a message stating the total execution time of Link 1. It follows immediately the preceding printed output item.

### C. Description of Output Items of Link 2 (as Listed in Table VI-1)

***Output Item 14.*** This item includes tables for the element stiffness matrices and load vectors, and other pertinent information for their generation, for every element of the structure (by using INP<2 and subroutine CAS2, these tables may be output for only those elements desired by the user). If the element is quadrilateral in shape, it is automatically divided into two triangles in two different ways. If the element is hexahedral in shape, it is automatically divided into 5 tetrahedrons in two different ways. The program uses the average of the element matrices and load vectors associated with the two different ways of subdivision, and, in the tables of Output Item 14, the matrices and vectors of each subelement are given as if they were independent elements. The tables of an element (or a subelement) in Output Item 14 are separated by four blank lines from those of the next element (or subelement). The printed information for an element (or subelement) consists of three blocks. The first block consists of three lines, the second block contains the element stiffness matrix, and the last block contains the element load vector. The first line of the first block contains 20 integers, the meanings of which are given in Table VI-4. The second and third lines of the first block contain coordinates of the mesh points of the element (or the subelement) in the order of $x_1, y_1, z_1, x_2, y_2, z_2, \cdots$, listed row-wise with 12 pieces of information per line. The coordinates are in the overall coordinate system with the exception of element types 11, 12, 13, and 14 (Table III-3), for which the coordinates are expressed in local coordinates. For these 5 elements, the local coordinate system is located at the first mesh point, and the first axis is on mesh line 1-2. In element type 4, the other two local axes are the principal axes of moment of inertia. In element types 11, 12, 13, and 14, the second axis is in the plane of mesh lines 1-2 and 1-last of the element (or subelement). The third axis is obtained from the condition that the local coordinate system is right-handed and orthogonal. If an element (or subelement) has less than 8 mesh points, the missing points will appear in the three lines of the first block with numbers that should be ignored. Each line of the second block contains 5 pairs of numbers. The first number in a pair is the row-wise count of the matrix element of the element stiffness matrix and the second is its numerical value. Likewise, each line of the third block contains 5 pairs of numbers, the first number in a pair being the element count in the element load vector and the second number its numerical value. Excluding quadrilaterals

**Table VI-3. Common map and meanings of the general constants and arrays in the common block**

| Location in common | Symbol | Brief description |
|---|---|---|
| 1 | IN | Total number of nodal points |
| 2 | IBN | Total number of dbc input units |
| 3 | IT | Total number of elements |
| 4 | IP | Total number of nonzero concentrated load components |
| 5 | IPRS | Number of pressure types |
| 6 | ITYPE | Indicator for material type |
| 7 | IMAT | Number of material types |
| 8 | IDEG | Degree of freedom at a node |
| 9 | INX | Number of last link to be executed |
| 10 | IH | Maximum number of vertices |
| 11 | I8 | Maximum number of words to describe an element |
| 12 | IMMX | Number of torsion constant types |
| 13 | IMMY | Number of *y* moment of inertia types |
| 14 | IMMZ | Number of *z* moment of inertia types |
| 15 | IMFI | Number of angle types |
| 16 | IARE | Number of cross-sectional area types |
| 17–24 | $N_i$ | Labels of the vertices of an element |
| 25 | M | Label of the current element |
| 26 | | (Not used) |
| 27 | ISTR | Indicator for plane strain case |
| 28 | IELT | Element type number |
| 29 | ITEM | Temperature change type number |
| 30 | ITIC | Thickness type number |
| 31 | IMET | Material type number |
| 32 | ISUM | Number of equations |
| 33 | IND | IND = IDEG * IN |
| 34 | IMS | Number of vertices of current element |
| 35 | | (Not used) |
| 36 | IDS | Order of the element stiffness matrix |
| 37 | IORD | Number of words allocated for the reduced stiffness matrix |
| 38 | IORD1 | IORD1 = IORD + 1 |
| 39 | ACEL | Body force per unit volume |
| 40 | ZGEM | Floating-point equivalent of IGEM |
| 41 | ITAP | Chain program tape number |
| 42 | INP | Indicator for output level |
| 43 | IPBG | Integer constant for element load vector |
| 44 | IPEN | Integer constant for element load vector |
| 45 | CONS | Constant for element load vector |
| 46 | IU | Pointer for diagonal element count vector |
| 47 | G1 | First direction cosine of acceleration vector |
| 48 | G2 | Second direction cosine of acceleration vector |
| 49 | G3 | Third direction cosine of acceleration vector |
| 50 | J1 | Pointer for J1W array |
| 51 | J2 | Pointer for J2W array |
| 52 | J3 | Pointer for J3W array |
| 53 | J4 | Pointer for J4W array |
| 54 | J5 | Pointer for J5W array |
| 55 | J6 | Pointer for J6W array |
| 56 | J7 | Pointer for J7W array |
| 57 | J8 | Pointer for J8W array |
| 58 | | (Not used) |
| 59 | IBB | Pointer for IBB array |
| 60 | IBO | Pointer for IBO array |
| 61 | IID | Pointer for material constants array |
| 62 | IIA | Pointer for thermal expansion coefficients array |
| 63 | IDT | Pointer for temperature changes array |
| 64 | IDY | Pointer for temperature gradients array (*y* direction) |
| 65 | ITE | Pointer for thicknesses array |
| 66 | ICAR | Pointer for cross-sectional areas array |
| 67 | ICIX | Pointer for torsional constants array |
| 68 | ICIY | Pointer for *y* moments of inertia array |
| 69 | ICIZ | Pointer for *z* moments of inertia array |
| 70 | ICFI | Pointer for angles array |
| 71 | IXX | Pointer for X coordinates array |
| 72 | IYY | Pointer for Y coordinates array |
| 73 | IZZ | Pointer for Z coordinates array |
| 74 | IIC | Pointer for dbc unit constants array |
| 75 | IDEF | Pointer for unknown deflections array (initially loads array) |
| 76 | IST | Pointer for reduced stiffness matrix of the whole structure |
| 77 | IIS | Pointer for element stiffness matrix |
| 78 | IGEM | Indicator for structures inscribed in Z = 0 plane |
| 79 | IERR | Error indicator |
| 80 | TE | Value of thickness for an element |
| 81 | DT | Value of temperature change for an element |
| 82 | DG | Temperature gradient for an element in direction *y* |
| 83 | AL1 | Thermal expansion coefficient of an element in first material axis direction |

## Table VI-3 (contd)

| Location in common | Symbol | Brief description | Location in common | Symbol | Brief description |
|---|---|---|---|---|---|
| 84 | AL2 | Thermal expansion coefficient of an element in second material axis direction | 334 | IDZ | Pointer for temperature gradients array (z direction) |
| 85 | AL3 | Thermal expansion coefficient of an element in third material axis direction | 335 | ITAS | Scratch tape number |
| 86–106 | D21 | Material constants for an element | 336 | JMFI | Type number of angle defining local coordinate of a frame member |
| 107–130 | P | Loading vector for an element | 337 | JMMZ | Type number of the sectional moment of inertia about local z axis |
| 131–154 | UV | Deflection due to temperature changes for an element | 338 | JMMY | Type number of the sectional moment of inertia about local y axis |
| 155–162 | X | Overall X coordinates of vertices of an element | 339 | JMMX | Type number of the torsional constant about local x axis |
| 163–170 | Y | Overall Y coordinates of vertices of an element | 340 | JARE | Type number of cross-sectional area |
| 171–178 | Z | Overall Z coordinates of vertices of an element | 341 | JSDZ | Type number of temperature gradient along local z axis |
| 179–185<br>186–192<br>193–199 | XD<br>YD<br>ZD | Coordinates of the vertices with respect to the coordinate system parallel to overall and with origin on the first vertex | 342 | JSDY | Type number of temperature gradient along local y axis |
| 200–325 | | General usage area | 343 | JPRS | Type number of pressure |
| 326 | IMES | Indicator for mesh topology input | 344 | J10 | Pointer for J10W array |
| 327 | IBUN | Indicator for boundary conditions input | 345 | J9 | Pointer for J9W array |
| 328 | ICOR | Indicator for coordinates input | 346 | ISDZ | Number of temperature gradients along local z axis |
| 329 | IPIR | Indicator for local coordinate axes selection | 347 | ISDY | Number of temperature gradients along local y axis |
| 330 | PRES | Pressure value for an element | 348 | ISDT | Number of temperature change types |
| 331 | DGZ | Temperature gradient along local z axis for an element | 349 | NTIC | Number of thickness types |
| 332 | DGY | Temperature gradient along local y axis for an element | | | |
| 333 | IPR | Pointer for pressures array | | | |

## Table VI-4. Meanings of the 20 integers in the first line of first block of Output Item 14

| Sequence No. | Symbolic name | Meaning | Sequence No. | Symbolic name | Meaning |
|---|---|---|---|---|---|
| 1 | M | Element label | 12 | ITIC | Thickness type number |
| 2 | N1 | Label of the first vertex | 13 | ITEM | Temperature increase type number |
| 3 | N2 | Label of the second vertex | 14 | JPRS | Pressure type number |
| 4 | N3 | Label of the third vertex | 15 | JARE | Cross-sectional area type number |
| 5 | N4 | Label of the fourth vertex | 16 | JSDY | Temperature gradient (y direction) type number |
| 6 | N5 | Label of the fifth vertex | 17 | JSDZ | Temperature gradient (z direction) type number |
| 7 | N6 | Label of the sixth vertex | 18 | JMMX | Torsion constant type number |
| 8 | N7 | Label of the seventh vertex | 19 | JMFI | Angle (fixing the principal axes) type number |
| 9 | N8 | Label of the eighth vertex | 20 | IDS | Order of the free-free element stiffness matrix |
| 10 | IELT | Element type number | | | |
| 11 | IMET | Material type number | | | |

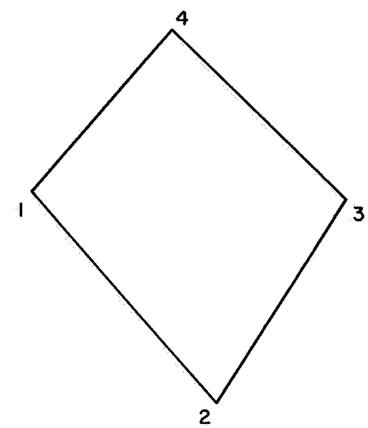
and hexahedrals, the parent element is the same as the subelement. In Table VI-5, the mesh points of subelements associated with quadrilateral and hexahedral parent elements are shown.

*Output Item 15.* This table lists the shaded portion of the upper half of the reduced stiffness matrix (see Fig. II-1) in row order. After two blank lines, it follows the last printed output item. Each line of Output Item 15

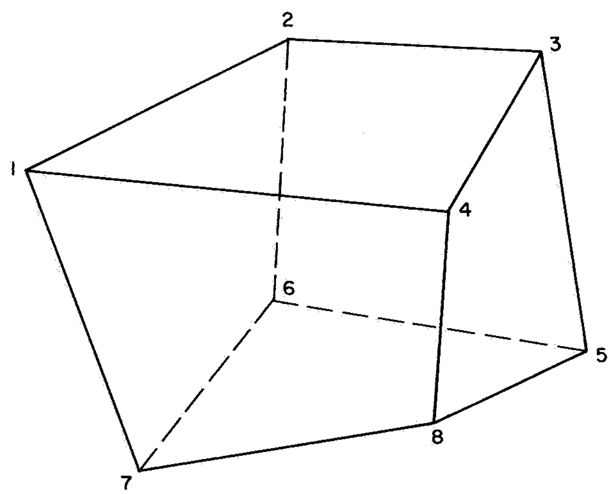
**Table VI-5. Subdivisions of the quadrilateral and hexahedral elements**

A. QUADRILATERAL

| Subdivision | Sequence of vertices: For the 1st triangle | | | Sequence of vertices: For the 2nd triangle | | |
|---|---|---|---|---|---|---|
| Subdivision by diagonal 1–3 | 1 | 2 | 3 | 3 | 4 | 1 |
| Subdivision by diagonal 2–4 | 2 | 3 | 4 | 4 | 1 | 2 |




B. HEXAHEDRON




| Subdivision | Sequence of vertices: For the 1st tetrahedron | | | | For the 2nd tetrahedron | | | | For the 3rd tetrahedron | | | | For the 4th tetrahedron | | | | For the 5th tetrahedron | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Subdivision by cutting corners 1, 3, 6, 8 | 1 | 4 | 2 | 7 | 3 | 2 | 4 | 5 | 4 | 8 | 5 | 7 | 2 | 5 | 6 | 7 | 2 | 4 | 5 | 7 |
| Subdivision by cutting corners 2, 4, 5, 7 | 5 | 8 | 6 | 3 | 7 | 6 | 8 | 1 | 8 | 4 | 1 | 3 | 6 | 1 | 2 | 3 | 6 | 8 | 1 | 3 |

consists of 5 pairs of numbers. The first number in a pair is the word count in common and the second is the numerical value of the stiffness matrix element. The first element of the stiffness matrix is stored in the common word, the count of which is IST+1 (see Table VI-3). Any particular element of the reduced stiffness matrix (which is in the shaded area of Fig. II-1) may be located in the following way: Let $k_{i,j}$ denote the element of the reduced stiffness matrix on the $i$th row and the $j$th column for which $i \leq j$ holds. From Output Item 9 one obtains the sequence number of $k_{i,i}$; let this number be $n_i$. The word count of $k_{i,j}$ in the common is $m_{ij} = \mathrm{IST} + n_i + (j-i)$. The second number in the pair whose first number is $m_{ij}$ is $k_{i,j}$.

*Output Item 16.* This table starts on a new page and lists the right-hand side of the governing equations shown in Fig. II-1. Each line in this output item consists of 5 pairs of numbers. The first number in a pair is the word count in the common and the second is the numerical value of the corresponding element of the right-hand-side vector (namely, reduced load vector). The first word of the reduced load vector is stored at the common word, the count of which is IDEF+1 (see Table VI-3). Let $p_i$ denote the $i$th element of the reduced load vector. The numerical value of this element may be obtained as the second word of the pair whose first number is IDEF+$i$.

*Output Item 17.* This is a message stating the total execution time of Link 2. It follows immediately the preceding printed output item.

### D. Description of Output Items of Link 3 (as Listed in Table VI-1

*Output Item 18.* This table starts on a new page and lists the solution vector of the governing equations shown in Fig. II-1. Each line of this output item consists of 5 pairs of numbers. The first number in a pair is the word count in the common and the second is the numerical value of the element of the solution vector (namely, reduced solution vector). The first word of the reduced solution vector is stored at the common word, the count of which is IDEF+1. Let $v_i$ denote the $i$th element of the reduced solution vector. The numerical value of this element may be obtained as the second number of the pair whose first number is IDEF+$i$.

*Output Item 19.* This table, which starts on a new page, lists the components of the deflection vector at the mesh points as referred to the overall coordinate system. The mesh-point numbers are listed under the column heading NODES. These are user's labels for the mesh points. The table is self-explanatory.

*Output Item 20.* This table starts on a new page and lists the components of the force vector acting at the mesh points of the structure in the free-free state, as referred to the overall coordinate system. If these forces are applied to the structure, the deflections listed in Output Item 19 are induced. There are two exceptions: (1) The thermal loads are not included in Output Item 20, and (2) the roundoff errors are included in Output Item 20. The elements of Output Item 20 may be used (1) in getting an idea about the order of magnitude of the roundoff errors involved in the solution of deflections (the forces listed at an unloaded mesh point in Output Item 20 are the residual forces of the solution), and (2) in obtaining the reactions at the restrained degree-of-freedom directions. The mesh-point numbers under the column heading NODES are the user's. The table is self-explanatory.

*Output Item 21.* This is a message stating the total execution time of Link 3. It follows immediately the preceding printed output item. The number that appears at the extreme right-hand side of the line is the time in seconds spent for the inversion of the simultaneous equations set shown in Fig. II-1.

### E. Description of Output Items of Link 4 (as Listed in Table VI-1)

*Output Item 22.* This table lists the stresses at the mesh points of structures of two- and three-dimensional continua. (Stresses of structures composed of one-dimensional elements, or stresses in the one-dimensional elements of composite structures, are listed in Output Item 24.) Output Item 22 starts on a new page. The heading of the table is self-explanatory. Mesh points on the boundary are indicated with an * sign in the printout. If the quantities related with a mesh point are not in the overall coordinate system, this is indicated by a ** symbol, in which case the direction cosines of the local axes KSI, ETA, and ZTA are also given.

The selection of a local coordinate system at every mesh point for expressing the stress components is automatically accomplished in the following way. At an internal mesh point, the local axes are taken as the material axes at that mesh point, unless the material is isotropic, in which case the local system is such that

(1) it is parallel to the overall system in structures related with element types 5, 6, 7, 8, 9, 10, 15, and 16 (Table III-2), or

(2) in structures related with element types 11, 12, 13, 14, 17, and 18, the third axis ZTA is the middle surface normal (the sense of which is consistent with the node labelling of the elements). The other two axes are determined by

 (a) assuming that the middle surface tangent direction defined by the 1-2 line of the lowest labelled element coming to that mesh point is the first axis KSI,

 (b) assuming that the first axis KSI is in the smaller principal curvature direction (not in shells of revolution), or

 (c) by the user's version of subroutine AGEL.

 (Options a, b, and c are controlled by assigning 0, 1, or 2, respectively, to the IPIR field of the control card of input data.)

At a boundary mesh point, the local first axis KSI is coincident with the outer normal of the boundary surface, and the other two axes are such that

(1) in shells, the local third axis ZTA is the middle surface normal, and

(2) in structures other than shells, the direction defined by the cross-product of the outer normal with the unit vector of the overall axis that makes the largest angle (less than 90 deg) with the outer normal becomes the second axis ETA.

The stress computation is repeated at every mesh point for every material type of every class of elements. In Table VI-6, the class numbers of the 18 elements of the ELAS program are shown. The meanings of the six components of the stresses at a mesh point for a given class are also given in the table. The mesh-point numbers are those of the user. When feasible, the stresses in the overall coordinate system are also computed and printed out. The stress state at point supports is given by distributing the reaction forces to the average boundary area corresponding to the mesh point of the point support. The concentrated reaction forces may be obtained from Output Item 20. The outer normal of the structure at sharp corners is computed as the average of the normals to the faces defining the sharp corner.

***Output Item* 23.** This item details the way that the best-fit stress computation is performed for every mesh point. By using INP<2 and subroutine CAS4, Output Item 23 may be printed out only for desired mesh points. This output item is listed together with Output Item 22. Some of the subitems of Output Item 23 are as follows:

NEL *matrix.* This is a matrix of integer elements. For every finite element related with the current mesh point, there is assigned one row in the NEL matrix. A row contains 17 integer elements. The meanings of these integer elements are given in Table VI-7a.

MAC *matrix.* For every material type at a mesh point, one MAC matrix is produced. For every element class of a given material type, there is one row in the MAC matrix. The meanings of the entries in a row are given in Table VI-7b.

DIN *matrix.* This is a $3 \times 3$ orthogonal matrix, the columns of which are the direction cosines of the unit vectors of KSI, ETA, and ZTA axes of the local coordinate system at the current mesh point.

*Material matrix.* This matrix is the same as shown in Fig. III-2b, with the exception that the third and fourth rows and columns are interchanged.

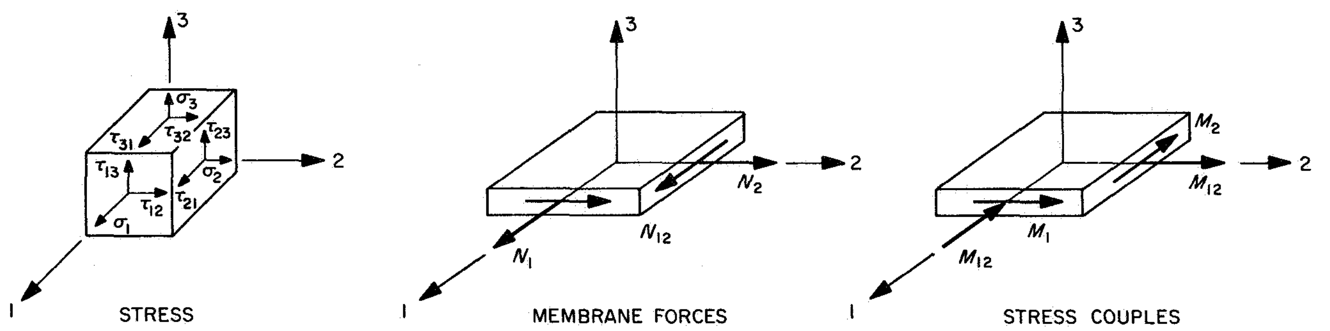
NSET *array.* This is the list of the mesh points of the elements related with the current mesh point. The list excludes the current mesh point.

*Equations for stress boundary conditions.* These equations are obtained by equating the boundary tractions in terms of the stress tensor to the tractions obtained from the forces acting on the structure at the current mesh point (these forces are listed in Output Item 20). The equations are scaled with the first element of the material matrix. In the printout, in the title of this subitem, the number of unknown components of the stress tensor and the number of right-hand sides are indicated as $i \times j$ ($i$ is the number of independent unknown components of the stress tensor, $j$ is the number of right-hand sides in the equations). Note that in shell structures, $i \times j$ may come out as $3 \times 2$, which means that the first right-hand side is for the independent unknown components of the stress tensor for the middle surface stretching, and the second for the curvature changes. After the title, the augmented matrix related with these equations is listed.

Table VI-6. Meanings of the components of stresses at mesh points of two- and three-dimensional continua[a]

| Class No. | Element type No. | Structure type | First component | Second component | Third component | Fourth component | Fifth component | Sixth component |
|---|---|---|---|---|---|---|---|---|
| 1 | 5, 6 | 2-D elasticity | $\sigma_1$ | $\sigma_2$ | $\tau_{12}$* | 0† | 0 | 0 |
| 2 | 7, 8 | Plate, bending | 0 | 0 | 0 | $M_1$ | $M_2$ | $M_{12}$ |
| 3 | 15, 16 | Solid of revolution | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\tau_{12}$ | 0 | 0 |
| 4 | 9, 10 | General solid | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\tau_{12}$ | $\tau_{13}$ | $\tau_{23}$ |
| 5 | 17 | Shell of revolution, membrane | $N_1$ | $N_2$ | 0 | 0 | 0 | 0 |
| 6 | 18 | Shell of revolution, membrane and bending | $N_1$ | $N_2$ | 0 | $M_1$ | $M_2$ | 0 |
| 7 | 13, 14 | Shell, membrane | $N_1$ | $N_2$ | $N_{12}$ | 0 | 0 | 0 |
| 8 | 11, 12 | Shell, membrane and bending | $N_1$ | $N_2$ | $N_{12}$ | $M_1$ | $M_2$ | $M_{12}$ |

[a]Nomenclature:
$\sigma_1, \sigma_2, \sigma_3$ normal stresses; $\tau_{12}, \tau_{13}, \tau_{23}$ shear stresses
$N_1, N_2$ membrane normal forces; $N_{12}$ membrane shear
$M_1, M_2$ bending stress couples; $M_{12}$ twist stress couple
*$\sigma_3$ for plane-strain case; †$\tau_{12}$ for plane-strain case




The axis labels 1, 2, and 3 stand for KSI, ETA and ZTA, respectively

*Prescribed stresses.* The right-hand sides of the preceding subitem are listed as scaled with the number appearing at the extreme right-hand end of the line. These stresses are obtained from the description of the forces (given in Output Item 20) in the local coordinate system by dividing them with the average boundary surface area given as AREA in Output Item 23.

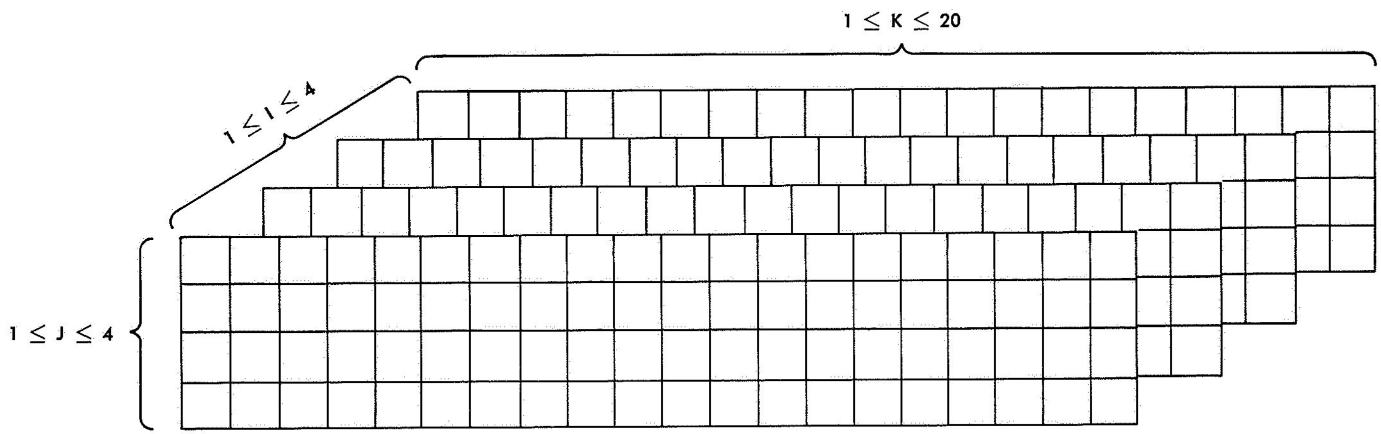
*Strain equations along nodal lines.* These are the equations for the first-order approximation of the strains along the mesh lines defined by the current mesh point and the other mesh points of the element related with the current mesh point. The equations are given as separate blocks for every mesh element related with the current mesh point. A strain equation along a given mesh line is established by equating the strain expressed in terms of the unknown independent components of the strain tensor at the current mesh point to the approximation computed from the deflections of the end points of the mesh line. The title of a block of these equations contains a number of the type $i \times j$, where $i$ denotes the number of independent unknown components of the strain tensor, and $j$ denotes the number of the right-hand sides in an equation ($j=2$ has the same connotation as in equations for stress boundary conditions explained above).

*Best-fit strain tensor components.* The equations for stress boundary conditions (if there are any) and the strain equations along nodal lines are solved for the independent unknown components of the strain tensor by least squares. The results are printed as a single line under the title of BEST-FIT STRAINS. This is the last line of Output Item 23.

## Table VI-7. Meanings of the entries of NEL and MAC matrices

| Label of the element | Element type number | Material type number | Class number (see Table VI-6) | Number of vertices | Thickness type number | Temperature gradient (along z) type number | Pressure type number | Temperature increase type number | Label of first vertex | Label of second vertex | Label of third vertex | Label of fourth vertex | Label of fifth vertex | Label of sixth vertex | Label of seventh vertex | Label of eighth vertex |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| M | IELT | IMET | IC | IMS | ITIC | JSDZ | JPRS | ITEM | N(1) | N(2) | N(3) | N(4) | N(5) | N(6) | N(7) | N(8) |
| | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |

(a) NEL matrix (Output Item 23 contains only the shaded area, where each row corresponds to one element attached to the mesh point under question)




(b) MAC matrix is of MAC (I, J, K) type where I controls material type, J controls the class type. MAC (I, J, 1) is the number of elements of material and class corresponding to I and J. MAC (I, J, K), $K \geq 2$ is the sequence number of the element in the NEL table. Note that at a mesh point there may be at the most four different materials; for each material there may be at the most four class types; and the number of elements of the same class and material may not be greater than 19.

*Output Item 24.* This output item is produced if one-dimensional elements (element types 1, 2, 3, and 4) are used in the structure. This table starts on a new page and lists the forces and the moments acting on the element for each one-dimensional element and for each one of its two end points. These quantities are referred to the local coordinate systems that are parallel to the local coordinate system of the element (see Table III-3, column 11, for the local coordinate system of the element) such that the one on the negative face end (the end where the outer normal is in the opposite direction from the local $x$ axis of the element) is in the opposite sense, and the one on the positive face end (the end where the outer normal is in the same direction as the local $x$ axis of the element) is in the same sense as the local coordinate system of the element (see Fig. VI-1). Let **i**, **j**, and **k** denote the unit vectors of the local coordinate system at an end point. Let **N** and **M** denote the force and the moment vectors acting on the element at this point. These quantities may be expressed in the local coordinate system at the end point as

$$\mathbf{N} = N_x\mathbf{i} + Q_y\mathbf{j} + Q_z\mathbf{k}$$

$$\mathbf{M} = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

where $N_x$ is the normal force, $Q_y$ and $Q_z$ are the shear forces, $M_x$ is the torsional moment, and $M_y$ and $M_z$ are the bending moments. In the table of Output Item 24 these components are listed as $N_x$, $Q_y$, $Q_z$, $M_x$, $M_y$, $M_z$. In any one row of the table, these six components are preceded by three integers that stand for the element label, the mesh-point label of the end point, and the element type number. The labels are those of the user.

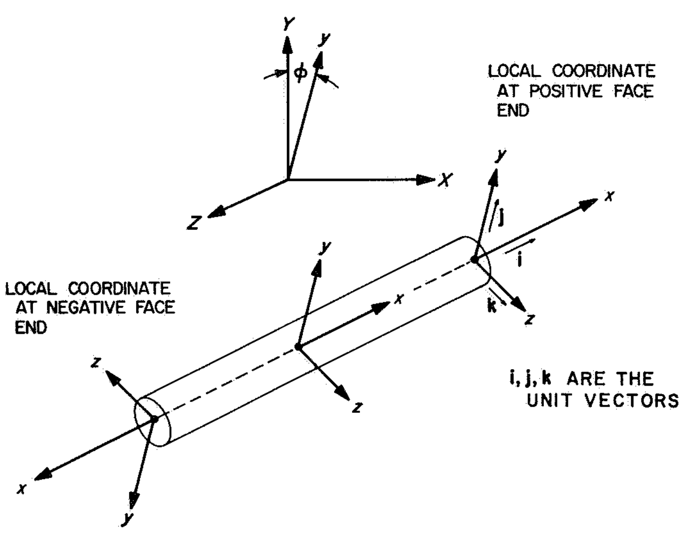



**Fig. VI-1. Local coordinate systems of a line element**

*Output Item 25.* This is a message stating the total execution time of Link 4. It follows immediately the preceding printed output item.

## F. Output Items Related With Relabelling

Depending upon the contents of the INP and ISHUF fields of the control card (Input Item 2), outputs consisting of printout of mesh topology (P), periodical message (M), periodical punched cards ($C_p$), and final punched cards ($C_f$) are produced. Table VI-8 shows the order and the types of outputs produced for different values of INP and ISHUF. These outputs are described in the following paragraphs.

*1. Printout of Mesh Topology Matrix* (**P**). This table starts on a new page, and gives the following information for each mesh point: the new label obtained by ELAS (under the heading NO.), the user's label (under the heading ISIR), the IMIN value (under the heading IMIN), the IMAX value (under the heading IMAX), and the connectivity information (under the heading LABEL MATRIX IN OCTAL). The IMIN value of a mesh point is the lowest-valued new label in the mesh points that are connected by the current mesh point with finite elements. The IMAX value of a mesh point is the highest-valued new label in the mesh points that are connected by the current mesh point with finite elements. The connectivity information of a mesh point consists of a binary row that contains as many binary digits as the total number of mesh points. The count of a digit from left is the new label of the corresponding mesh point. A *zero* in this

**Table VI-8. Outputs for relabelling**

| ISHUF \ INP | 0 | 1 | 2 |
|---|---|---|---|
| 0 | — | — | — |
| 1 | (M, $C_p$), M, $C_f$ | (M, $C_p$), M, $C_f$ | P, (M, $C_p$), P, M, $C_f$ |
| 2 | (M, $C_p$), M, $C_f$ | (M, $C_p$), M, $C_f$ | P, (M, $C_p$), P, M, $C_f$ |
| 3 | — | — | — |

Definitions:
- P printout of mesh topology matrix
- M message
- $C_p$ periodic punched cards (only at the end of every 100 seconds)
- $C_f$ final punched cards
- ( ) used to indicate repetitions every 100 seconds of relabelling time

binary row indicates that the mesh point corresponding to this binary digit is not connected with the current mesh point by means of a finite element. A *one* in the binary row indicates that the mesh point corresponding to this binary digit is connected with the current mesh point by means of a finite element. (The IMIN value of a mesh point is the count of the first nonzero binary digit of the binary row of the mesh point; the IMAX value of a mesh point is the count of the last nonzero binary digit of the binary row of the mesh point.) The connectivity information is printed out in octal format (7O13). If there are more than 252 mesh points in the structure (that is, if the content of the IN field of the control card of input data is larger than 252), the connectivity information is continued on another table (separated by three blank lines from the preceding table) that contains the new label, the user's label, and the connectivity information in excess of the first (in the sense of new labels) 252 mesh points for each mesh point. In relabelling, the maximum number of mesh points is assumed to be 540. If relabelling is requested for cases where IN $>$ 540, subroutine ARAN should be modified suitably. The matrix established by the binary rows of the mesh points is called the "mesh topology matrix." The rows of the mesh topology matrix are sequenced with the new labels of the corresponding mesh points; that is, the first row corresponds to the mesh point whose new label is 1, the second to that of the mesh point whose new label is 2, etc.; therefore, the mesh topology matrix is a symmetric matrix. As far as the distribution of the nonzero elements is concerned, the mesh topology matrix is roughly similar to the coefficient matrix in Fig. II-1. The mesh topology matrix is printed at the beginning and at the end of the relabelling procedure if INP=2.

*2. **Periodic Message*** (M). This message provides information related with relabelling that is an iteration procedure. The basic unit of the iteration is called "sweep." The information provided in this message consists of the total number of zero binary digits – called the "upper off-band element count" – at the right of the IMAXth binary digit at every row of the mesh topology matrix described in the preceding paragraph, the number of sweeps performed, and the iteration time elapsed to attain this state. As the iteration progresses, the upper off-band element count increases. When the upper off-band element count is multiplied by the square of the content of the IDEG field of the control card of input data, one obtains a number that is roughly equal to the upper-right-corner area of the stiffness matrix (the unshaded portion) shown in Fig. II-1; therefore it represents the savings in the storage area. The iteration procedure used in relabelling is based on maximizing the upper off-band element count. The periodic message appears at the start of relabelling, at the end of every 100 seconds of relabelling time, and at the successful termination of the relabelling procedure.

*3. **Periodic Punched Cards*** ($C_p$). These cards are produced at the beginning of every 100 seconds of relabelling time after the first 100 seconds. The cards list in (20I4) format the user's labels of the mesh points in the order attained by iteration at the time of punching. If somehow the execution of ELAS is terminated before the relabelling iteration is completed, the user may rerun this job by setting ISHUF=2 and by using the last produced $C_p$ cards as the input cards of Input Item 17 (see Section IV-B). Care should be taken to separate the last punched $C_p$ cards from the other punched cards.

*4. **Final Punched Cards*** ($C_f$). These cards are punched at the successful termination of the iteration of relabelling. They list in (20I4) format the user's labels and the IMAX values of the mesh points in the order attained by ELAS at the end of relabelling [here the meaning attributed to IMAX is modified as follows: if the IMAX value of a mesh point is smaller than the IMAX value of the previous (in the sense of the new labels) mesh point, the IMAX value is taken as that of the previous (in the sense of the new labels) mesh point]. To save relabelling time in a subsequent run related with the same structure, these cards may be used by setting ISHUF=3 and by using the $C_f$ cards as the input cards of Input Item 17 (see Section IV-B). Care should be taken to separate the $C_f$ cards from the other punched cards.

# VII. Error Messages and Diagnostics

## A. Error Messages

The error messages of the ELAS program are listed in Table VII-1. Table VII-2 gives the chain link number for each of the message numbers listed in Table VII-1, the name of the program that produces the message, the approximate place of the message in the output items of Table VI-1, and the consequence of the detected error. The user has no control over the appearance of the error message in the printed output, unless he makes deliberate errors during the input preparation. The error messages are printed out when a condition that may be detrimental to the solution of the problem is encountered. Depending upon how grave the consequences of the detected error condition, the execution of the job is either terminated or continued. The decision for this is made automatically to provide to the user the best return corresponding to his input. The user should evaluate his return in the light of the message itself and the remarks provided in the last column of Table VII-2. If evaluation indicates that the run should be repeated, the error that caused the first error condition (if possible, also the following ones) should be diagnosed and corrected. The systematic diagnosis of the errors related with the error messages of Table VII-1 is explained in the following discussion. In order to simplify the diagnosis in case of errors, it is suggested that when a job is run the first time, it be executed with INP=1 printout option (INP=2 option is generally not suggested because of the excessive amount of output). If a run is not successful and there is no printed error message, the user should suspect that some of the input cards may be missing.

## B. Diagnosis of Errors Related With the Error Messages of Link 1

*Error Message 1.* This message appears whenever an error condition is encountered in the input information. The message means that at least one item in the user's input is incompatible with the rules of Section IV. In order to find the erroneous item, the last successfully produced output item is located. In Table VII-3 the relation between the output items of Link 1 and the input cards (or programs) is shown. Using this table, one may locate the input that contains the erroneous input item. The exact location of the input item may then be found by means of Output Items 11 and 12 (which are automatically provided) and by Table VI-3, since the information related with the mesh elements, the mesh points, and the dbc input units are used one by one (in the sequence in which they are input) in the generation of the related output item. For example, if the last successfully produced output item is Output Item 3, then

**Table VII-1. List of error messages**

| No. | Error message |
|---|---|
| 1. | INPUT ERROR |
| 2. | THE FOLLOWING DISPLACEMENT BOUNDARY CONDITION(S) CAUSE(S) MORE THAN ONE MULTIPLE CONNECTION FOR THE UNKNOWNS. THEY ARE IGNORED |
| 3. | *i*: IN ELEMENT . . . , ERROR IN MESH TOPOLOGY INFORMATION. NO CORRECTION IS MADE. *ii*: IN ELEMENT . . . , . . . PROPERTY TYPE NUMBER(S) IS OUTSIDE THE RANGE. THE TYPE NUMBER(S) IS ASSUMED LARGEST POSSIBLE |
| 4. | ELEMENT . . . IS UNACCEPTABLE. DISREGARDED |
| 5. | WARNING. LESS THAN 12750 DECIMAL LOCATIONS ARE AVAILABLE FOR THE NEXT LINK PROGRAMS. THOUGH IT MAY BE SUICIDAL, EXECUTION CONTINUES |
| 6. | THE POINT . . . DOES NOT APPEAR IN THE MESH TOPOLOGY |
| 7. | DUMMY AREA OVERLAPS COMMON AREA BY . . . DECIMAL LOCATIONS. RECOMPILE BY CHANGING THE EQUIVALENCES OF DUMMY AND BB IN LINKS 1 AND 3, RESPECTIVELY |
| 8. | ELEMENT . . . , . . . IS UNACCEPTABLE. DISREGARDED . . . |
| 9. | THE VOLUME OF ELEMENT . . . , . . . IS TOO SMALL . . . DISREGARDED |
| 10. | STIFFNESS MATRIX IS NOT POSITIVE DEFINITE . . . |
| 11. | NO SCRATCH TAPE IS GIVEN OR ERROR IN SCRATCH TAPE |
| 12. | MORE THAN 12 NON-ONE-DIMENSIONAL ELEMENTS AT NODE . . . |
| 13. | NODAL STRESS COMPUTATION IS DELETED DUE TO PRECEDING |
| 14. | NO SCRATCH TAPE. STRESS LINK IS NOT EXECUTED |
| 15. | ERROR IN READING ELEMENT SETS FROM TAPE ITAS. STRESS LINK EXECUTION IS DELETED . . . , . . . |
| 16. | NOT ENOUGH INDEPENDENT INFORMATION AVAILABLE |
| 17. | ERROR IN MESH TOPOLOGY. NODE ASSUMED INTERNAL |
| 18. | MORE THAN 4 MATERIALS, FIRST 4 CONSIDERED |
| 19. | MORE THAN 4 CLASSES, FIRST 4 CONSIDERED |
| 20. | MORE THAN 19 ELEMENTS, FIRST 19 CONSIDERED |
| 21. | NOT ENOUGH INFORMATION FOR BEST-FIT QUADRATIC. BEST-FIT PLANE IS USED |
| 22. | NOT ENOUGH INFORMATION FOR MIDDLE SURFACE NORMAL. APPROXIMATE XII AND ZTA VALUES ARE USED |
| 23. | SCRATCH AREA FF OVERLAPS WITH RESIDUAL AREA. PUSH FF FURTHER DOWN BY RECOMPILING LINK 4. |

the error is in the element properties. The number of the element whose properties are prepared incorrectly may be found by obtaining from the IA block the number of the element whose properties are already placed in common; the erroneous element is the one following this.

*Error Message 2.* This message is related with the deflection boundary conditions and provides a list of unacceptable dbc input units. The user should check all of his dbc input units.

*Error Message 3.* There are two submessages in this error message, as shown in Table VII-1. The first submessage (*i*) appears whenever a vertex label is larger than IN (the total number of mesh points) or the number of vertices is larger than IH (maximum number of vertices) in an element. The second submessage (*ii*) indicates errors in element properties. Elemental information should be examined if either or both of these submessages appear.

*Error Message 4.* This indicates that the properties of the element whose number appears in the error message are not stated properly. This information should be examined.

*Error Message 5.* This message indicates that the number of unknown deflection components of this job is too big for the machine used. Sometimes, in spite of this message, the problem may be solved successfully. If the execution is not successful, one should repeat the run by requesting relabelling (ISHUF=1) if it has not already been requested. If the program still fails, the user should check the necessary storage area given in Output Item 8. Usually the program length of Link 2 is more critical than those of the other links. The user may decide to repeat the run by replacing the subroutines of Link 2 that are not required (see Table V-3) by dummy subroutines as described in the fourth paragraph of Section V-A, provided that the number given in Output Item 8, when incremented by the program length of Link 2 in the lower core, is close to the size of the core memory. If such a run also fails, the number of unknowns must be decreased.

*Error Message 6.* The user should check Output Item 4 and a sketch of the mesh to find the reason for the error.

*Error Message 7.* If this message appears, the user is advised to decrease the size of his problem by using a cruder mesh. (In order to recompile as suggested by this

## Table VII-2. Error messages—producing programs and consequences

| Error message No. | Chain link No. | Program | Location of message as referred to output item Nos. | Consequences of the error |
|---|---|---|---|---|
| 1 | 1 | MAIN | 0–14 | Output Items 11 and 12 are automatically produced and this job is skipped uncompleted |
| 2 | 1 | MAIN | 3–4 | Execution is continued with different boundary conditions than intended |
| 3 | 1 | MAIN | 3–4 | Execution is continued with different properties than intended |
| 4 | 1 | SRAT | 4–6 | Execution is continued with lesser material volume than intended |
| 5 | 1 | SRAT | 4–6 | Execution is continued. If overlap occurs, the user's program is skipped completely |
| 6 | 1 | ARAN | 4–6 | No relabelling is performed, and execution is continued |
| 7 | 1 | MAIN | 12–13 | Some of the input information in common may be destroyed; execution continues |
| 8 | 2 | MAIN | 13–15 | Execution is continued with lesser material volume than intended |
| 9 | 2 | S09 | 13–15 | Execution is continued, probably successfully |
| 10 | 3 | MAIN | 17–18 | Job is skipped, uncompleted |
| 11 | 3 | MAIN | 20–21 | Remaining portion of job is skipped |
| 12 | 3 | ELST | 20–21 | Execution is continued to detect all such mesh points |
| 13 | 3 | ELST | 20–21 | Remaining portion of job is skipped |
| 14 | 4 | MAIN | 21–22 | Job is skipped, uncompleted |
| 15 | 4 | MAIN | 22–23 | Job is skipped, uncompleted |
| 16 | 4 | MAIN | 22–23 | Computation of stresses at this mesh point for this case is skipped |
| 17 | 4 | BOFI | 22–23 | Stresses at this mesh point are probably computed wrong. Execution continues |
| 18 | 4 | GENE | 22–23 | Stress computation at this mesh point is not performed for the extra element types |
| 19 | 4 | GENE | 22–23 | Stress computation at this mesh point is not performed for the extra class types |
| 20 | 4 | GENE | 22–23 | Only the first 19 elements are included in the best-fit stress computation |
| 21 | 4 | QUAD | 22–23 | Stresses at this node are approximate. Execution continues |
| 22 | 4 | REVO | 22–23 | Stresses at this node may be very approximate |
| 23 | 4 | MAIN | 21–22 | The job is skipped, uncompleted |

Table VII-3. Correspondence between input and output items of Link 1

| Input item No. | Output item No. |
|---|---|
| 1, 2 | 1 |
| 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 | 2 |
| 14 | 3 |
| 15, 18 | 10 |
| 16 | 4 |
| 2, 17 | 5 |

error message, the user would need Volume II of this report and the program source decks. However, even if the recompilation were carried out properly, the chances are still high that the problem is too big for the core memory.)

### C. Diagnosis of Errors Related With the Error Messages of Link 2

*Error Message 8.* This message indicates that the element properties associated with the mesh element and the subelement indicated by the first two numbers in the message are erroneous. If the last number in the message is 6, this means that the mesh points of the element are involved with improper displacement boundary conditions. If the last number is 2, this is an indication of mesh-point labels being negative. If the last number is 1, it shows that either mesh points are not labelled according to the rules given in Table III-5, or the coordinates of the mesh points are wrong. The user should examine the properties and the coordinates associated with this element (see Output Items 3 and 4).

*Error Message 9.* This is probably not a serious error. It shows that the subelement of the element (indicated by the first two numbers in the message) is a very flat tetrahedron.

### D. Diagnosis of Errors Related With the Error Messages of Link 3

*Error Message 10.* This message indicates that either the structure is not supported properly to eliminate the rigid body motion, or the structure itself is geometrically unstable. The mesh-point number of the trouble spot is indicated by the last number appearing in the message. If this number is a negative number, it shows that there is no unknown displacement and/or rotation in the structure. If this number is positive, from Output Item 9 one finds the number pair whose second number is identical with this number. The first number in this pair is the equation number in the reduced set shown in Fig. II-1 and is called IBB. One searches Output Item 10, column IBB, to find the row that contains the same IBB number in column IBB and $-1$ in column IBO. The mesh-point number in this row is the trouble spot. The defective direction is the one appearing in the heading of the table of Output Item 10. The user should examine element descriptions and material matrices, and a sketch of the mesh at the vicinity of this mesh point for geometric stability.

*Error Message 11.* This message appears when no scratch tape number is given in the ITAS field of the control card of input data and INX=4. If the message appears in spite of the fact that an allowable tape number is indicated for the ITAS field, the tape is a bad one.

*Error Message 12.* In order to accelerate the execution time, it is assumed that not more than 12 elements meet at a mesh point of two- and three-dimensional continua if the stress computation is requested. For every mesh point where more than 12 elements meet, this message is produced with the mesh-point number.

*Error Message 13.* This message always follows Error Message 12. If the stresses are necessary, the user should modify his mesh in two- or three-dimensional continuum at the mesh points indicated by Error Message 12.

### E. Diagnosis of Errors Related With the Error Messages of Link 4

*Error Message 14.* This message has the same meaning as Error Message 11.

*Error Message 15.* The tape prescribed in the ITAS field of the control card of input data is a bad one. The user should change the scratch tape number to another tape number that is allowable.

*Error Message 16.* In the computation of the best-fit strain tensor at a mesh point, there should be at that mesh point at least as many independent mesh-line directions that belong to the same material and the same class type as the number of independent unknown components of the strain tensor (in general shells, this number is always 3). When this message appears, it means that there are not enough independent mesh-line directions at this mesh point. If the stresses of this mesh point are necessary, the run should be repeated by modifying the mesh.

*Error Message 17.* Among the input data describing the mesh (Input Item 16) there are certain topological relationships that are indicative of whether a mesh point is on the boundary or not. The fact that a mesh point is on the boundary is important in the stress computations. If mesh labelling is wrong, the criteria used in finding the mesh points on the boundary may not work. When this happens, Error Message 17 appears. The mistake in labelling the mesh may invalidate the whole solution. Therefore, when this message appears, the user should thoroughly check his mesh topology by using Output Item 4 and a sketch of the mesh.

*Error Message 18.* For the computation of stresses, it is assumed that not more than four different material types may appear at a mesh point. The stress computation for those material types in excess of four is not performed.

*Error Message 19.* For the computation of stresses, it is assumed that not more than four class types may appear for a given material type at a mesh point. The stress computation for those classes in excess of four is not performed.

*Error Message 20.* For the computation of stresses, it is assumed that not more than 19 elements may appear for a given material type and for a given class at a mesh point. The stress computation is performed for the first 19 elements, and those in excess of 19 are not included in the best-fit stress computation.

*Error Message 21.* In the stress computation for general shells, the middle surface normal is obtained by fitting a best-fit quadratic surface to at least the first eight neighboring mesh points at every mesh point. Whenever this is not possible, Error Message 21 appears, and the local coordinate system is taken as the one obtained by a best-fit plane, which may produce unacceptable results. The user should try the IPIR=2 option for such cases by providing his version of subroutine AGEL.

*Error Message 22.* The middle surface normal in shells of revolution is obtained by fitting a fourth-order polynomial to the mesh point and its first four neighbors. If this is not possible, the middle surface normal is taken as the average of the normals of the two neighboring chords meeting at the mesh point. Therefore the local coordinate system is selected rather crudely whenever Error Message 22 appears. However, the stresses may be still acceptable.

*Error Message 23.* When this message appears, all the programs in Link 4 should be recompiled as stated in the message.

# VIII. Sample Problems

## A. Circular Cylinder Subjected to Uniform Circumferential Pressure (Plane Strain)

An infinitely long, circular right cylinder subjected to uniform circumferential pressure of 1 psi is to be analyzed. Because of symmetry, one half of a thin slice of the cylinder may be analyzed as a plane-strain problem. In Fig. VIII-1, the idealized slice as referred to the overall coordinate system ($X$, $Y$, $Z$), the mesh, and the proper boundary conditions are shown. Note that there are 10 mesh elements and 14 mesh points. It is assumed that the thickness of the slice is 1 in., and the material is isotropic with $E = 29.$ psi and $G = 11.$ psi. The program tape number is 2 and the scratch tape number is 3. Relabelling of the nodes is requested. All four links of the program are to be executed for this problem, and the intermediate printout is requested. The list of input cards for this problem is given in Table VIII-1, where the encircled numbers refer to the input item numbers (see Section IV). Table VIII-2 gives the computer printouts, where the encircled numbers are the output item numbers (see Section VI). Table VIII-3 is a list of the punched output cards that are produced as a result of the relabelling request. As indicated in Table VIII-2 by Output Items 13, 17, 21, and 25, the total net execution time is 6.54 seconds. The user may try the same problem with a much more refined grid.

## B. Prism Subjected to Pressure at One End, and Supported Without Friction at the Other

A rectangular right prism subjected to uniform pressure of 1 psi at one end and supported without friction at the other end is to be analyzed. In Fig. VIII-2, the prism as referred to the overall coordinate system ($X$, $Y$, $Z$), the mesh, and the proper boundary conditions are shown. Note that there are 2 mesh elements and 12 mesh points. The material is isotropic and $E = 10.67 \times 10^6$ psi and $G = 4. \times 10^6$ psi. The program tape number is 2 and the scratch tape number is 3. No relabelling is requested. All four links are to be executed. The intermediate printout is to be produced. The list of the input cards is given in Table VIII-4, where the encircled numbers refer to the input item numbers (see Section IV). Table VIII-5 gives the computer printouts, where the encircled numbers are the output item numbers (see Section VI). As indicated in Table VIII-5 by Output Items 13, 17, 21, and 25, the total net execution time is 8.65 seconds. The user may try the same problem with a much more refined grid.

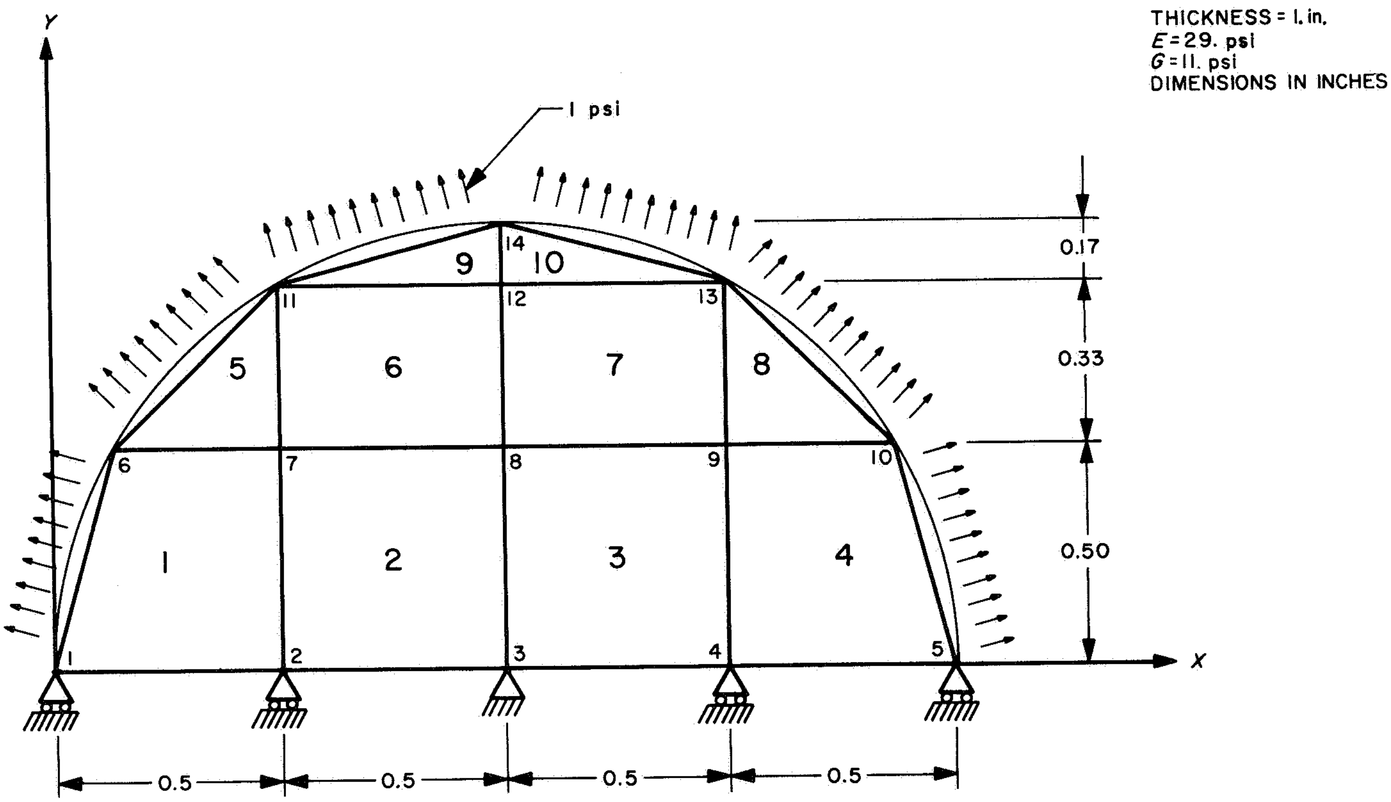



**Fig. VIII-1. Idealization of one half of a thin slice of a long, circular right cylinder**

**Table VIII-1. List of input cards of the circular cylinder problem (encircled numbers on the right are the input item numbers, see Section IV)**

```
CIRCULAR CYLINDER SUBJECTED TO UNIFORM CIRCUMFERENTIAL PRESSURE (PLANE STRAIN)   }(1)
  14  102  147    6       1 1 1                 411     23                       }(2)
 1 29.     11.                                                                   }(3)
 1 1.                                                                            }(4)
 1 1.                                                                            }(5)
   1                                        2  .5                                ]
   3 1.                                     4 1.5                                |
   5 2.                                     6  .13333333   .5                    |
   7  .5            .5                      8 1.           .5                    |(14)
   9 1.5            .5                     10 1.86666667   .5                    |
  11  .5            .86666667              12 1.           .86666667             |
  13 1.5            .86666667              14 1.          1.                     ]
   12   12          22   22         31   31         32   32         42   42      ]
   52   52                                                                       ](15)
-  1 6 1 1    1  6   1   2   7        -  2 6 1 1       7   2   3   8             ]
-  3 6 1 1       8   3   4   9        -  4 6 1 1   1   5  10   9   4             |
-  5 5 1 1    1 11   6   7            -  6 6 1 1      11   7   8  12             |(16)
-  7 6 1 1      12   8   9  13        -  8 5 1 1   1  10  13   9                 |
-  9 5 1 1    1 14  11  12            - 10 5 1 1   1  13  14  12                 ]
                                                                          END    }(19)
```

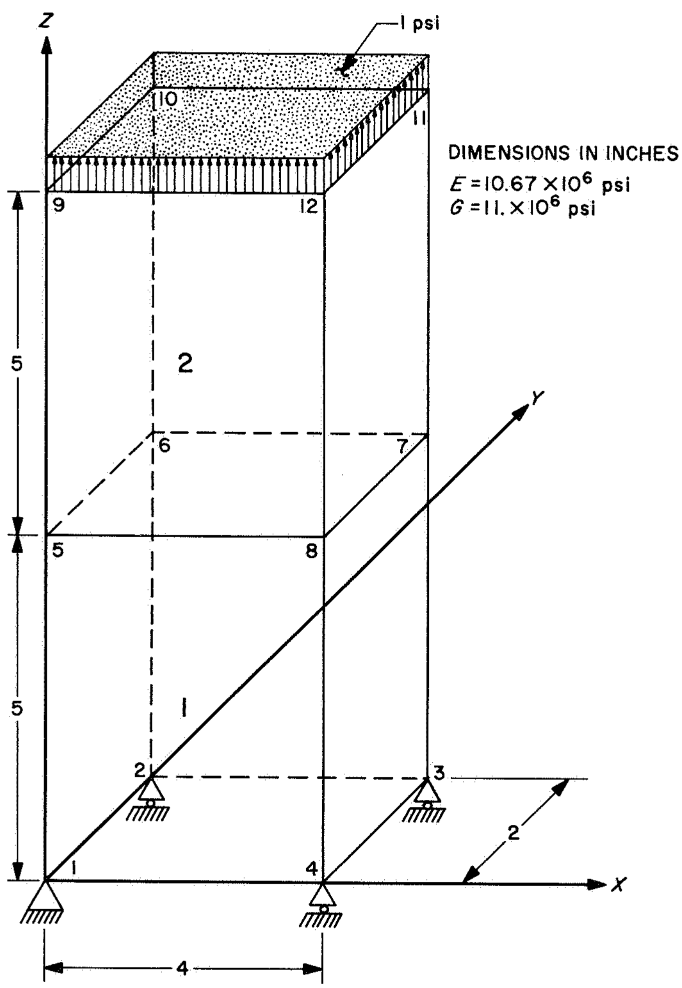



Fig. VIII-2. The prism as referred to the overall coordinate system (X, Y, Z) and the mesh

**Table VIII-2. Computer printouts of the circular cylinder problem (encircled numbers at the left are the output item numbers, see Section VI)**

(1)

```
                              LINEAR ELASTICITY PROBLEM

 CIRCULAR CYLINDER SUBJECTED TO UNIFORM CIRCUMFERENTIAL PRESSURE (PLANE STRAIN)

TOTAL NUMBER OF NODES                      14
TOTAL NUMBER OF FINITE ELEMENTS            10
DEGREES OF FREEDOM AT A NODE                2
ITYPE VALUE                                -0        0 FOR ISOTROPIC, 1 FOR ORTHOTROPIC, 2 FOR GENERAL MATERIAL
IGEM VALUE                                 -0        0 FOR 2-, 1 FOR 3-DIMENSIONAL STRUCTURES
ISTR VALUE                                  1        1 FOR PLANE STRAIN CASE, 0 OTHERWISE
MAXIMUM NUMBER OF CONTACTS IN AN ELEMENT    4
CONTACT NUMBER INCLUDING DUMMIES            7
IBN VALUE                                   6        NUMBER OF SUPRESSED DEGREES OF FREEDOM IF NO MULTIPLE CONNECTIONS
TOTAL NUMBER OF CONCENTRATED LOADS         -0
PRESSURE TYPES                              1
MATERIAL TYPES                              1
THICKNESS TYPES                             1
TEMPERATURE CHANGE TYPES                   -0
TEMPERATURE GRADIENT TYPES ALONG Y         -0
TEMPERATURE GRADIENT TYPES ALONG Z         -0
AREA TYPES                                 -0
TORSION CONSTANT TYPES                     -0
Y MOMENT OF INERTIA TYPES                  -0
Z MOMENT OF INERTIA TYPES                  -0
NUMBER OF ANGLES FIXING PRINCIPAL AXES     -0
INX VALUE                                   4        NUMBER OF THE LINK FROM WHICH RETURN-TO-BEGINNING IS MADE
INP VALUE                                   1        0 MINIMUM PRINT, 1 PARTIAL PRINT, 2 COMPLETE PRINT
ISHUF VALUE                                 1        0 NO RELABELLING, 1 RELABEL, 2 OR 3 READ CARDS FOR RELABELLING
ICOR VALUE                                 -0        0 READ CARDS, 1 CALL SUBROUTINE CORG FOR COORDINATES
IBUN VALUE                                 -0        0 READ CARDS, 1 CALL SUBROUTINE BUNG FOR BOUNDARY CONDITIONS
IMES VALUE                                 -0        0 READ CARDS, 1 CALL SUBROUTINE MESG FOR MESH TOPOLOGY
IPIR VALUE FOR SHELL LOCAL NODAL AXES      -0        0 ASSUME ZERO, 1 COMPUTE AS PRINCIPAL, 2 READ AS INPUT
CHAIN PROGRAM TAPE NUMBER                   2
SCRATCH TAPE NUMBER                         3        0 OR - DO NOT COMPUTE RESIDUALS, OTHERWISE COMPUTE
ACCELERATION*UNIT MASS                    -0.
DIRECTION COSINES OF ACCELERATION         -0.        -0.          -0.
```

(2)

```
 MATERIAL PROPERTIES

TP    E              G              ALPHA                 TP    E              G              ALPHA

 1    0.29000E 02    0.11000E 02   -0.


 PRESSURE TYPES

 1    0.10000E 01


 THICKNESS TYPES

 1    0.10000E 01
```

(3)

```
                                        NODAL COORDINATES

NODE     X              Y              Z               NODE     X              Y              Z

   1   -0.            -0.             -0.                 2    0.50000E 00   -0.             -0.
   3    0.10000E 01   -0.             -0.                 4    0.15000E 01   -0.             -0.
   5    0.20000E 01   -0.             -0.                 6    0.13333E-00    0.50000E 00   -0.
   7    0.50000E 00    0.50000E 00   -0.                 8    0.10000E 01    0.50000E 00   -0.
   9    0.15000E 01    0.50000E 00   -0.                10    0.18667E 01    0.50000E 00   -0.
  11    0.50000E 00    0.86667E 00   -0.                12    0.10000E 01    0.86667E 00   -0.
  13    0.15000E 01    0.86667E 00   -0.                14    0.10000E 01    0.10000E 01   -0.
```

## Table VIII-2 (contd)

(4)

MESH TOPOLOGY / ELEMENT PROPERTY TYPES

| EL NO | ND-1 | ND-2 | ND-3 | ND-4 | ND-5 | ND-6 | ND-7 | ND-8 | ELMT | PRES | MTRL | THCK | DTMP | TGDY | TGDZ | AREA | I-XX | I-YY | I-ZZ | FI-Y |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 1 | 2 | 7 | 0 | 0 | 0 | 0 | 6 | 1 | 1 | 1 | -0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 7 | 2 | 3 | 8 | 0 | 0 | 0 | 0 | 6 | 0 | 1 | 1 | -0 | 0 | -0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 8 | 3 | 4 | 9 | 0 | 0 | 0 | 0 | 6 | 0 | 1 | 1 | -0 | 0 | -0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 5 | 10 | 9 | 4 | 0 | 0 | 0 | 0 | 6 | 1 | 1 | 1 | -0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 11 | 6 | 7 | 0 | 0 | 0 | 0 | 0 | 5 | 1 | 1 | 1 | -0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 11 | 7 | 8 | 12 | 0 | 0 | 0 | 0 | 6 | 0 | 1 | 1 | -0 | 0 | -0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 12 | 8 | 9 | 13 | 0 | 0 | 0 | 0 | 6 | 0 | 1 | 1 | -0 | 0 | -0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 10 | 13 | 9 | 0 | 0 | 0 | 0 | 0 | 5 | 1 | 1 | 1 | -0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 14 | 11 | 12 | 0 | 0 | 0 | 0 | 0 | 5 | 1 | 1 | 1 | -0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 13 | 14 | 12 | 0 | 0 | 0 | 0 | 0 | 5 | 1 | 1 | 1 | -0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(5)

AT 0. SEC. OF RELABELLING -0 SWEEPS PERFORMED. UPPER OFF-BAND ELEMENT COUNT OF MESH TOPOLOGY MATRIX IS 35.
AT 0.28 SEC. OF RELABELLING 44 SWEEPS PERFORMED. UPPER OFF-BAND ELEMENT COUNT OF MESH TOPOLOGY MATRIX IS 55.

(6)

TOPOLOGY OF THE REDUCED STIFFNESS MATRIX

NUMBER OF ELEMENTS RETAINED AT EACH ROW OF UPPER STIFFNESS MATRIX (DIAGONAL INCLUDED)

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 2 | 7 | 3 | 6 | 4 | 9 | 5 | 10 | 6 | 9 | 7 | 8 | 8 | 7 | 9 | 8 | 10 | 7 |
| 11 | 9 | 12 | 8 | 13 | 7 | 14 | 6 | 15 | 7 | 16 | 6 | 17 | 6 | 18 | 5 | 19 | 4 | 20 | 3 |
| 21 | 2 | 22 | 1 | | | | | | | | | | | | | | | | |

(7)

STIFFNESS MATRIX REQUIRES (DECIMAL) 141. STORAGE LOCATIONS

(8)

TOTAL COMMON LENGTH IS (DECIMAL) 817. STORAGE LOCATIONS

(9)

COUNT OF MAIN DIAGONAL ELEMENTS OF ROW LISTED STIFFNESS MATRIX

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 7 | 3 | 14 | 4 | 20 | 5 | 29 | 6 | 39 | 7 | 48 | 8 | 56 | 9 | 63 | 10 | 71 |
| 11 | 78 | 12 | 87 | 13 | 95 | 14 | 102 | 15 | 108 | 16 | 115 | 17 | 121 | 18 | 127 | 19 | 132 | 20 | 136 |
| 21 | 139 | 22 | 141 | 23 | 142 | | | | | | | | | | | | | | |

(10)

FORCE AND DISPLACEMENT BOUNDARY CONDITIONS IN DIRECTION 1

| NODE | P | IBC | IBB | C | NODE | P | IBC | IBB | C |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0. | -1 | 1 | 0.1000E 01 | 2 | 0. | -1 | 4 | 0.1000E 01 |
| 3 | 0. | 0 | 29 | -0. | 4 | 0. | -1 | 19 | 0.1000E 01 |
| 5 | 0. | -1 | 22 | 0.1000E 01 | 6 | 0. | -1 | 2 | 0.1000E 01 |
| 7 | 0. | -1 | 5 | 0.1000E 01 | 8 | 0. | -1 | 11 | 0.1000E 01 |
| 9 | 0. | -1 | 17 | 0.1000E 01 | 10 | 0. | -1 | 20 | 0.1000E 01 |
| 11 | 0. | -1 | 7 | 0.1000E 01 | 12 | 0. | -1 | 13 | 0.1000E 01 |
| 13 | 0. | -1 | 15 | 0.1000E 01 | 14 | 0. | -1 | 9 | 0.1000E 01 |

## Table VIII-2 (contd)

(10)

FORCE AND DISPLACEMENT BOUNDARY CONDITIONS IN DIRECTION 2

| NODE | P | IBC | IBB | C | NODE | P | IBO | IBB | C |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0. | C | 29 | -0. | 2 | 0. | 0 | 29 | -0. |
| 3 | 0. | 0 | 29 | -0. | 4 | 0. | 0 | 29 | -0. |
| 5 | 0. | C | 29 | -0. | 6 | 0. | -1 | 3 | 0.1000E 01 |
| 7 | 0. | -1 | 6 | 0.1000E 01 | 8 | 0. | -1 | 12 | 0.1000E 01 |
| 9 | 0. | -1 | 18 | 0.1000E 01 | 10 | 0. | -1 | 21 | 0.1000E 01 |
| 11 | 0. | -1 | 8 | 0.1000E 01 | 12 | 0. | -1 | 14 | 0.1000E 01 |
| 13 | 0. | -1 | 16 | 0.1000E 01 | 14 | 0. | -1 | 10 | 0.1000E 01 |

(13)

INPUT LINK TOOK 1.73 SECONDS.

(17)

GENERATION LINK TOOK 1.33 SECONDS.

(19)

NODAL DEFLECTICNS

| NODE | CISP. ALONG X | DISP. ALONG Y | DISP. ALONG Z | ROTA. ABOUT X | ROTA. ABOUT Y | ROTA. ABOUT Z |
|---|---|---|---|---|---|---|
| 1 | -C.1652892E-01 | -0. | 0. | 0. | 0. | 0. |
| 2 | -C.8264462E-02 | -0. | 0. | 0. | 0. | 0. |
| 3 | -0. | -0. | 0. | 0. | 0. | 0. |
| 4 | C.8264464E-02 | -0. | 0. | 0. | 0. | 0. |
| 5 | C.1652893E-01 | -0. | 0. | 0. | 0. | 0. |
| 6 | -0.1432507E-01 | 0.8264463E-02 | 0. | 0. | 0. | 0. |
| 7 | -0.8264462E-02 | 0.8264463E-02 | 0. | 0. | 0. | 0. |
| 8 | 0.1040721E-08 | 0.8264463E-02 | 0. | 0. | 0. | 0. |
| 9 | 0.8264464E-02 | 0.8264463E-02 | 0. | 0. | 0. | 0. |
| 10 | 0.1432507E-01 | 0.8264463E-02 | 0. | 0. | 0. | 0. |
| 11 | -0.8264462E-02 | 0.1432507E-01 | 0. | 0. | 0. | 0. |
| 12 | 0.1178010E-08 | 0.1432507E-01 | 0. | 0. | 0. | 0. |
| 13 | 0.8264465E-02 | 0.1432507E-01 | 0. | 0. | 0. | 0. |
| 14 | C.1028673E-08 | 0.1652893E-01 | 0. | 0. | 0. | 0. |

(20)

FCRCES ACTING AT THE NODES

| NODE | FORCE ALONG X | FORCE ALONG Y | FORCE ALONG Z | MOMENT ABOUT X | MOMENT ABOUT Y | MOMENT ABOUT Z |
|---|---|---|---|---|---|---|
| 1 | -0.2500000E-00 | -0.1833333E-00 | 0. | 0. | 0. | 0. |
| 2 | 0.6984919E-09 | -0.5000000E-00 | 0. | 0. | 0. | 0. |
| 3 | -0.4167669E-07 | -0.5000000E-00 | 0. | 0. | 0. | 0. |
| 4 | -0.4656613E-08 | -0.5000000E 00 | 0. | 0. | 0. | 0. |
| 5 | 0.2500000E-00 | -0.1833333E-00 | 0. | 0. | 0. | 0. |
| 6 | -0.4333333E-00 | 0.2500000E-00 | 0. | 0. | 0. | 0. |
| 7 | -0.2002344E-07 | -0.3818423E-07 | 0. | 0. | 0. | 0. |
| 8 | -0.6053597E-08 | -0.3166497E-07 | 0. | 0. | 0. | 0. |
| 9 | 0.1490116E-07 | -0.1862645E-08 | 0. | 0. | 0. | 0. |
| 10 | 0.4333333E-00 | 0.2500000E-00 | 0. | 0. | 0. | 0. |
| 11 | -0.2500000E-00 | 0.4333333E-00 | 0. | 0. | 0. | 0. |
| 12 | 0.1210719E-07 | -0.1117587E-07 | 0. | 0. | 0. | 0. |
| 13 | 0.2500000E-00 | 0.4333333E-00 | 0. | 0. | 0. | 0. |
| 14 | -0.2793968E-08 | 0.4999999E-00 | 0. | 0. | 0. | 0. |

## Table VIII-2 (contd)

(21)

```
DEFLECTION LINK TOOK   1.78                                                                                   0.10
```

(22)

```
          STRESSES AT THE NODES OF TWO- OR THREE-DIMENSIONAL CONTINUUM BY BEST FIT STRAIN TENSORS
    ALL QUANTITIES ARE IN OVERALL SYSTEM, UNLESS ** APPEARS INDICATING DATA IN KSI, ETA AND ZTA LOCAL SYSTEM
                                      * INDICATES NODE ON BOUNDARY

NODE  MAT CLAS       FIRST  COMP    SECOND COMP     THIRD  COMP    FOURTH COMP     FIFTH  COMP     SIXTH  COMP

  1 DR.COSINS     KSI  -0.8064-0.5914 0.            ETA   0.5914-0.8064-0.          ZTA   0.      0.      1.0000
  1 DEFLECTNS**     0.13329E-01   -0.97746E-02      0.              0.             -0.               0.
 1*    1    1**     1.00000E 00    1.00000E 00      0.63636E 00     0.12016E-07     0.               0.
 1*    1    1       1.00000E 00    1.00000E 00      0.63636E 00     0.10803E-06     0.               0.
  2 DR.COSINS     KSI   0.     -1.0000 0.           ETA   1.0000 0.     -0.         ZTA   0.      0.      1.0000
  2 DEFLECTNS**     0.            -0.82645E-02      0.              0.             -0.               0.
 2*    1    1**     1.00000E 00    1.00000E 00      0.63636E 00     0.13970E-08     0.               0.
 2*    1    1       1.00000E 00    1.00000E 00      0.63636E 00    -0.13970E-08     0.               0.
  3 DR.COSINS     KSI   0.     -1.0000 0.           ETA   1.0000 0.     -0.         ZTA   0.      0.      1.0000
  3 DEFLECTNS**     0.            -0.               0.              0.             -0.               0.
 3*    1    1**     1.00000E 00    1.00000E 00      0.63636E 00    -0.83353E-07     0.               0.
 3*    1    1       1.00000E 00    1.00000E 00      0.63636E 00     0.83353E-07     0.               0.
  4 DR.COSINS     KSI   0.     -1.0000 0.           ETA   1.0000 0.     -0.         ZTA   0.      0.      1.0000
  4 DEFLECTNS**     0.             0.82645E-02      0.              0.             -0.               0.
 4*    1    1**     0.10000E 01    1.00000E 00      0.63636E 00    -0.93132E-08     0.               0.
 4*    1    1       1.00000E 00    0.10000E 01      0.63636E 00     0.93132E-08     0.               0.
  5 DR.COSINS     KSI   0.8064-0.5914 0.            ETA   0.5914 0.8064-0.          ZTA   0.      0.      1.0000
  5 DEFLECTNS**     0.13329E-01    0.97746E-02      0.              0.             -0.               0.
 5*    1    1**     1.00000E 00    0.10000E 01      0.63636E 00     0.60082E-08     0.               0.
 5*    1    1       0.10000E 01    0.10000E 01      0.63636E 00     0.20862E-06     0.               0.
  6 DR.COSINS     KSI  -0.8662 0.4997-0.            ETA  -0.4997-0.8662 0.          ZTA   0.      0.      1.0000
  6 DEFLECTNS**     0.16538E-01   -0.40745E-09      0.             -0.              0.               0.
 6*    1    1**     1.00000E 00    1.00000E 00      0.63636E 00    -0.74464E-08     0.               0.
 6*    1    1       1.00000E 00    1.00000E 00      0.63636E 00     0.14901E-07     0.               0.
 7     1    1       1.00000E 00    1.00000E 00      0.63636E 00     0.10904E-07     0.               0.
 8     1    1       0.10000E 01    1.00000E 00      0.63636E 00     0.93881E-08     0.               0.
 9     1    1       0.10000E 01    0.10000E 01      0.63636E 00     0.19991E-07     0.               0.
 10 DR.COSINS     KSI   0.8662 0.4997-0.            ETA  -0.4997 0.8662 0.          ZTA   0.      0.      1.0000
 10 DEFLECTNS**     0.16538E-01   -0.29104E-09      0.             -0.              0.               0.
10*    1    1**     1.00000E 00    1.00000E 00      0.63636E 00     0.14893E-07     0.               0.
10*    1    1       1.00000E 00    1.00000E 00      0.63636E 00     0.11176E-07     0.               0.
 11 DR.COSINS     KSI  -0.4997 0.8662-0.            ETA  -0.8662-0.4997 0.          ZTA   0.      0.      1.0000
 11 DEFLECTNS**     0.16538E-01   -0.58208E-09      0.             -0.              0.               0.
11*    1    1**     0.10000E 01    1.00000E 00      0.63636E 00     0.37232E-08     0.               0.
11*    1    1       1.00000E 00    1.00000E 00      0.63636E 00    -0.18626E-07     0.               0.
12     1    1       0.10000E 01    1.00000E 00      0.63636E 00     0.11262E-07     0.               0.
 13 DR.COSINS     KSI   0.4997 0.8662-0.            ETA  -0.8662 0.4997 0.          ZTA   0.      0.      1.0000
 13 DEFLECTNS**     0.16538E-01   -0.93132E-09      0.             -0.              0.               0.
13*    1    1**     0.10000E 01    0.10000E 01      0.63636E 00     0.37232E-08     0.               0.
13*    1    1       1.00000E 00    1.00000E 00      0.63636E 00     0.              0.               0.
 14 DR.COSINS     KSI   0.      1.0000-0.           ETA  -1.0000 0.      0.         ZTA   0.      0.      1.0000
 14 DEFLECTNS**     0.16529E-01   -0.10287E-08      0.             -0.              0.               0.
14*    1    1**     1.00000E 00    1.00000E 00      0.63636E 00     0.55879E-08     0.               0.
14*    1    1       1.00000E 00    1.00000E 00      0.63636E 00    -0.55879E-08     0.               0.
```

(25)

```
STRESS LINK TOOK   1.70  SECONDS.
```

**Table VIII-3. Listing of punched card output of circular cylinder problem**

```
 3   1   1   5   6   6   2   8   7   9  11   9  14  10   8  12  12  12  13  13
 9  14   4  14  10  14   5  14
```

**Table VIII-4. List of input cards of the prism problem (encircled numbers on the right are the input item numbers, see Section IV)**

```
PRISM SUBJECTED TO PRESSURE AT ONE END SUPPORTED WITHOUT FRICTION AT THE OTHER   (1)
  12    23 1 8     8          1 1                        41      23               (2)
 1 10.67E6 4.    E6                                                               (3)
 1 1.                                                                             (4)
   1                                          2               2.
   3  4.                                      4  4.           2.
   5                            5.            6               2.          5.     (14)
   7  4.                        5.            8  4.           2.          5.
   9                           10.           10               2.         10.
  11  4.                       10.           12  4.           2.         10.
   11    11        12    12          13    13          21    21        23    23
   32    32        33    33          43    43                                   (15)
 -  11001      5   6   8   7   4   2   1   3
 -  21001 100  9  10  12  11   8   6   5   7                                    (16)
                                                                         END    (19)
```

**Table VIII-5. Computer printouts of the prism problem (encircled numbers at the left are the output item numbers, see Section VI)**

(1)

```
                              LINEAR ELASTICITY PROBLEM

 PRISM SUBJECTED TO PRESSURE AT ONE END SUPPORTED WITHOUT FRICTION AT THE OTHER

TOTAL NUMBER OF NODES                         12
TOTAL NUMBER OF FINITE ELEMENTS                2
DEGREES OF FREEDOM AT A NODE                   3
ITYPE VALUE                                   -0         0 FOR ISOTROPIC, 1 FOR ORTHOTROPIC, 2 FOR GENERAL MATERIAL
IGEM VALUE                                     1         0 FOR 2-, 1 FOR 3-DIMENSIONAL STRUCTURES
ISTR VALUE                                    -0         1 FOR PLANE STRAIN CASE, 0 OTHERWISE
MAXIMUM NUMBER OF CONTACTS IN AN ELEMENT       8
CONTACT NUMBER INCLUDING DUMMIES              10
IBN VALUE                                      8         NUMBER OF SUPRESSED DEGREES OF FREEDOM IF NO MULTIPLE CONNECTIONS
TOTAL NUMBER OF CONCENTRATED LOADS            -0
PRESSURE TYPES                                 1
MATERIAL TYPES                                 1
THICKNESS TYPES                               -0
TEMPERATURE CHANGE TYPES                      -0
TEMPERATURE GRADIENT TYPES ALONG Y            -0
TEMPERATURE GRADIENT TYPES ALONG Z            -0
AREA TYPES                                    -0
TORSION CONSTANT TYPES                        -0
Y MOMENT OF INERTIA TYPES                     -0
Z MOMENT OF INERTIA TYPES                     -0
NUMBER OF ANGLES FIXING PRINCIPAL AXES        -0
INX VALUE                                      4         NUMBER OF THE LINK FROM WHICH RETURN-TO-BEGINNING IS MADE
INP VALUE                                      1         0 MINIMUM PRINT, 1 PARTIAL PRINT, 2 COMPLETE PRINT
ISHUF VALUE                                   -0         0 NO RELABELLING, 1 RELABEL, 2 OR 3 READ CARDS FOR RELABELLING
ICOR VALUE                                    -0         0 READ CARDS, 1 CALL SUBROUTINE CORG FOR COORDINATES
IBUN VALUE                                    -0         0 READ CARDS, 1 CALL SUBROUTINE BUNG FOR BOUNDARY CONDITIONS
IMES VALUE                                    -0         0 READ CARDS, 1 CALL SUBROUTINE MESG FOR MESH TOPOLOGY
IPIR VALUE FOR SHELL LOCAL NODAL AXES         -0         0 ASSUME ZERO, 1 COMPUTE AS PRINCIPAL, 2 READ AS INPUT
CHAIN PROGRAM TAPE NUMBER                      2
SCRATCH TAPE NUMBER                            3         0 OR - DO NOT COMPUTE RESIDUALS, OTHERWISE COMPUTE
ACCELERATION*UNIT MASS                       -0.
DIRECTION COSINES OF ACCELERATION            -0.         -0.          -0.
```

(2)

```
 MATERIAL PROPERTIES

TP     E              G              ALPHA                  TP     E              G              ALPHA

 1    0.10670E 08    0.40000E 07   -0.


 PRESSURE TYPES

 1    0.10000E 01
```

(3)

```
                                          NODAL COORDINATES

NODE      X              Y              Z                  NODE      X              Y              Z

   1  -0.           -0.            -0.                       2  -0.            0.20000E 01   -0.
   3   0.40000E 01  -0.            -0.                       4   0.40000E 01   0.20000E 01   -0.
   5  -0.           -0.             0.50000E 01              6  -0.            0.20000E 01    0.50000E 01
   7   0.40000E 01  -0.             0.50000E 01              8   0.40000E 01   0.20000E 01    0.50000E 01
   9  -0.           -0.             0.10000E 02             10  -0.            0.20000E 01    0.10000E 02
  11   0.40000E 01  -0.             0.10000E 02             12   0.40000E 01   0.20000E 01    0.10000E 02
```

## Table VIII-5 (contd)

(4)

MESH TOPOLOGY

| EL NO | ND-1 | ND-2 | ND-3 | ND-4 | ND-5 | ND-6 | ND-7 | ND-8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 6 | 8 | 7 | 4 | 2 | 1 | 3 |
| 2 | 9 | 10 | 12 | 11 | 8 | 6 | 5 | 7 |

ELEMENT PROPERTY TYPES

| ELMT | PRES | MTRL | THCK | DTMP | TGDY | TGDZ | AREA | I-XX | I-YY | I-ZZ | FI-Y |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | -0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 1 | 1 | 0 | -0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(6)

TOPOLOGY OF THE REDUCED STIFFNESS MATRIX

NUMBER OF ELEMENTS RETAINED AT EACH RCW OF UPPER STIFFNESS MATRIX (DIAGONAL INCLUDED)

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 16 | 2 | 15 | 3 | 14 | 4 | 13 | 5 | 24 | 6 | 23 | 7 | 22 | 8 | 21 | 9 | 20 | 10 | 19 |
| 11 | 18 | 12 | 17 | 13 | 16 | 14 | 15 | 15 | 14 | 16 | 13 | 17 | 12 | 18 | 11 | 19 | 10 | 20 | 9 |
| 21 | 8 | 22 | 7 | 23 | 6 | 24 | 5 | 25 | 4 | 26 | 3 | 27 | 2 | 28 | 1 | | | | |

(7)

STIFFNESS MATRIX REQUIRES (DECIMAL) 358. STORAGE LOCATIONS

(8)

TOTAL COMMCN LENGTH IS (DECIMAL) 1115. STORAGE LOCATIONS

(9)

COUNT OF MAIN DIAGONAL ELEMENTS OF ROW LISTED STIFFNESS MATRIX

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 17 | 3 | 32 | 4 | 46 | 5 | 59 | 6 | 83 | 7 | 106 | 8 | 128 | 9 | 149 | 10 | 169 |
| 11 | 188 | 12 | 206 | 13 | 223 | 14 | 239 | 15 | 254 | 16 | 268 | 17 | 281 | 18 | 293 | 19 | 304 | 20 | 314 |
| 21 | 323 | 22 | 331 | 23 | 338 | 24 | 344 | 25 | 349 | 26 | 353 | 27 | 356 | 28 | 358 | 29 | 359 | | |

(10)

FORCE AND DISPLACEMENT BOUNDARY CONDITIONS IN DIRECTION 1

| NODE | P | IBC | IBB | C | NODE | P | IBU | IBB | C |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0. | C | 37 | -0. | 2 | 0. | 0 | 37 | -0. |
| 3 | 0. | -1 | 2 | 0.1000E 01 | 4 | 0. | -1 | 3 | 0.1000E 01 |
| 5 | 0. | -1 | 5 | 0.1000E 01 | 6 | 0. | -1 | 8 | 0.1000E 01 |
| 7 | 0. | -1 | 11 | 0.1000E 01 | 8 | 0. | -1 | 14 | 0.1000E 01 |
| 9 | 0. | -1 | 17 | 0.1000E 01 | 10 | 0. | -1 | 20 | 0.1000E 01 |
| 11 | 0. | -1 | 23 | 0.1000E 01 | 12 | 0. | -1 | 26 | 0.1000E 01 |

(10)

FORCE AND DISPLACEMENT BOUNDARY CONDITIONS IN DIRECTION 2

| NODE | P | IBC | IBB | C | NODE | P | IBO | IBB | C |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0. | C | 37 | -0. | 2 | 0. | -1 | 1 | 0.1000E 01 |
| 3 | 0. | C | 37 | -0. | 4 | 0. | -1 | 4 | 0.1000E 01 |
| 5 | 0. | -1 | 6 | 0.1000E 01 | 6 | 0. | -1 | 9 | 0.1000E 01 |
| 7 | 0. | -1 | 12 | 0.1000E 01 | 8 | 0. | -1 | 15 | 0.1000E 01 |
| 9 | 0. | -1 | 18 | 0.1000E 01 | 10 | 0. | -1 | 21 | 0.1000E 01 |
| 11 | 0. | -1 | 24 | 0.1000E 01 | 12 | 0. | -1 | 27 | 0.1000E 01 |

## Table VIII-5 (contd)

(10)

FORCE AND DISPLACEMENT BOUNDARY CONDITICNS IN DIRECTION 3

| NODE | P | IBC | IBB | C | NODE | P | IBO | IBB | C |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0. | C | 37 | -0. | 2 | 0. | 0 | 37 | -0. |
| 3 | 0. | C | 37 | -0. | 4 | 0. | 0 | 37 | -0. |
| 5 | 0. | -1 | 7 | 0.1000E 01 | 6 | 0. | -1 | 10 | 0.1000E 01 |
| 7 | 0. | -1 | 13 | 0.1C00E 01 | 8 | 0. | -1 | 16 | 0.1000E 01 |
| 9 | 0. | -1 | 19 | 0.1000E 01 | 10 | 0. | -1 | 22 | 0.1000E 01 |
| 11 | 0. | -1 | 25 | 0.1000E 01 | 12 | C. | -1 | 28 | 0.100CE 01 |

(13)

INPUT LINK TOCK 1.27 SECONDS.

(17)

GENERATION LINK TOOK 2.8C SECCNDS.

(19)

NODAL DEFLECTICNS

| NODE | DISP. ALONG X | DISP. ALONG Y | DISP. ALCNG Z | ROTA. ABCUT X | ROTA. ABOUT Y | ROTA. ABOUT Z |
|---|---|---|---|---|---|---|
| 1 | -0. | -0. | -0. | 0. | 0. | 0. |
| 2 | -C. | -0.6255856E-07 | -0. | 0. | 0. | 0. |
| 3 | -C.1251171E-06 | -0. | -0. | 0. | 0. | 0. |
| 4 | -C.1251172E-06 | -0.6255858E-07 | -0. | 0. | 0. | 0. |
| 5 | -C.7881021E-13 | -0.9664425E-13 | 0.4686035E-06 | 0. | 0. | 0. |
| 6 | -C.6920656E-13 | -0.6255867E-07 | 0.4686036E-06 | 0. | 0. | 0. |
| 7 | -C.1251172E-06 | -0.1004255E-12 | 0.4686036E-06 | 0. | 0. | 0. |
| 8 | -C.1251172E-06 | -0.6255869E-07 | 0.4686037E-06 | 0. | 0. | 0. |
| 9 | -0.2402223E-12 | -0.3185637E-12 | 0.9372071E-06 | 0. | 0. | 0. |
| 10 | -0.2252380E-12 | -0.6255889E-07 | 0.9372072E-06 | 0. | 0. | 0. |
| 11 | -C.1251174E-06 | -0.3336843E-12 | 0.9372072E-06 | 0. | 0. | 0. |
| 12 | -C.1251174E-06 | -0.6255891E-07 | 0.9372073E-06 | 0. | 0. | 0. |

(20)

FORCES ACTING AT THE NODES

| NODE | FORCE ALONG X | FORCE ALONG Y | FORCE ALCNG Z | MOMENT ABOUT X | MOMENT ABOUT Y | MOMENT ABOUT Z |
|---|---|---|---|---|---|---|
| 1 | C.1871958E-06 | 0.5215406E-07 | -0.1999999E 01 | 0. | 0. | 0. |
| 2 | -C.2011657E-06 | 0.6193295E-07 | -0.200000CE 01 | 0. | 0. | 0. |
| 3 | 0.8288771E-07 | -0.149C116E-06 | -0.200000CE 01 | 0. | 0. | 0. |
| 4 | 0.3725290E-07 | 0.4237518E-07 | -0.2000001E 01 | 0. | 0. | 0. |
| 5 | 0.1433011E-06 | 0.3421886E-06 | -0.9387732E-06 | 0. | 0. | 0. |
| 6 | C.2082253E-06 | -0.2905726E-06 | -0.3576279E-06 | 0. | 0. | 0. |
| 7 | 0.3445894E-07 | 0.3744813E-06 | -0.7003546E-06 | 0. | 0. | 0. |
| 8 | -C.3143525E-06 | -0.2677552E-06 | 0.4991889E-06 | 0. | 0. | 0. |
| 9 | -0.2803281E-06 | -0.358C659E-06 | 0.200000CE 01 | 0. | 0. | 0. |
| 10 | -C.9632517E-07 | 0.3241003E-06 | 0.2000000E 01 | 0. | 0. | 0. |
| 11 | 0.1676381E-06 | -0.2968257E-06 | 0.200000CE 01 | 0. | 0. | 0. |
| 12 | 0.8760281E-09 | 0.2165325E-06 | 0.200000CE 01 | 0. | 0. | 0. |

## Table VIII-5 (contd)

(21)

```
DEFLECTION LINK TOOK   2.00                                                                                   0.45
```

(22)

```
          STRESSES AT THE NODES OF TWO- OR THREE-DIMENSIONAL CONTINUUM BY BEST FIT STRAIN TENSORS
   ALL QUANTITIES ARE IN OVERALL SYSTEM, UNLESS ** APPEARS INDICATING DATA IN KSI, ETA AND ZTA LOCAL SYSTEM
                                          * INDICATES NODE ON BOUNDARY

NODE  MAT CLAS       FIRST  COMP     SECOND COMP     THIRD  COMP     FOURTH COMP      FIFTH  COMP     SIXTH  COMP

   1 DR.COSINS     KSI  -0.4211-0.8422-0.3369     ETA  -0.8944 0.4472-0.          ZTA   0.1506 0.3013-0.9416
   1 DEFLECTNS**    0.              0.             0.              -0.              -0.              -0.
   1*   1    4**    0.63804E-01     0.16391E-06    0.88652E 00    -0.13648E-07      0.17834E-00     0.19005E-06
   1*   1    4      0.88070E-02     0.35228E-01    0.90629E 00     0.17614E-01     -0.55043E-01    -0.11009E-00
   2 DR.COSINS     KSI  -0.4211 0.8422-0.3369     ETA   0.8944 0.4472-0.          ZTA   0.1506-0.3013-0.9416
   2 DEFLECTNS**   -0.52684E-07    -0.27977E-07    0.18849E-07    -0.              -0.              -0.
   2*   1    4**    0.63804E-01     0.10431E-06    0.88653E 00    -0.14417E-07      0.17834E-00    -0.19935E-06
   2*   1    4      0.88070E-02     0.35228E-01    0.90629E 00    -0.17614E-01     -0.55043E-01     0.11009E-00
   3 DR.COSINS     KSI   0.4211-0.8422-0.3369     ETA  -0.8944-0.4472 0.          ZTA  -0.1506 0.3013-0.9416
   3 DEFLECTNS**   -0.52684E-07     0.11191E-06    0.18849E-07    -0.               0.              -0.
   3*   1    4**    0.63804E-01     0.14901E-07    0.88653E 00    -0.70999E-09      0.17834E-00    -0.30884E-06
   3*   1    4      0.88069E-02     0.35228E-01    0.90629E 00    -0.17614E-01      0.55043E-01    -0.11009E-00
   4 DR.COSINS     KSI   0.4211 0.8422-0.3369     ETA   0.8944-0.4472 0.          ZTA  -0.1506-0.3013-0.9416
   4 DEFLECTNS**   -0.10537E-06    -0.83931E-07    0.37697E-07    -0.               0.              -0.
   4*   1    4**    0.63804E-01     0.21607E-06    0.88653E 00     0.13608E-08      0.17834E-00     0.25926E-06
   4*   1    4      0.88071E-02     0.35228E-01    0.90630E 00     0.17614E-01      0.55043E-01     0.11009E-00
   5 DR.COSINS     KSI  -0.4472-0.8944-0.         ETA  -0.8944 0.4472-0.          ZTA   0.      0.     -1.0000
   5 DEFLECTNS**    0.12169E-12     0.27269E-13   -0.46860E-06    -0.              -0.              -0.
   5*   1    4**   -0.29456E-07    -0.26077E-06    1.00000E 00     0.19782E-08      0.74705E-07    -0.21613E-07
   5*   1    4     -0.21292E-06    -0.77301E-07    1.00000E 00     0.93713E-07      0.14078E-07     0.76484E-07
   6 DR.COSINS     KSI  -0.4472 0.8944-0.         ETA   0.8944 0.4472-0.          ZTA  -0.     -0.     -1.0000
   6 DEFLECTNS**   -0.55954E-07    -0.27977E-07   -0.46860E-06    -0.              -0.              -0.
   6*   1    4**   -0.28092E-07     0.14901E-07    1.00000E 00     0.44798E-08      0.28459E-07     0.67295E-07
   6*   1    4      0.27187E-08    -0.15910E-07    1.00000E 00     0.19885E-07     -0.47464E-07    -0.55550E-07
   7 DR.COSINS     KSI   0.4472-0.8944-0.         ETA  -0.8944-0.4472 0.          ZTA  -0.      0.     -1.0000
   7 DEFLECTNS**   -0.55954E-07     0.11191E-06   -0.46860E-06    -0.               0.              -0.
   7*   1    4**   -0.25428E-07    -0.11176E-06    1.00000E 00    -0.15780E-07      0.55732E-07    -0.53367E-07
   7*   1    4     -0.81869E-07    -0.55318E-07    1.00000E 00    -0.44000E-07     -0.72657E-07     0.25982E-07
   8 DR.COSINS     KSI   0.4472 0.8944-0.         ETA   0.8944-0.4472 0.          ZTA   0.     -0.     -1.0000
   8 DEFLECTNS**   -0.11191E-06    -0.83931E-07   -0.46860E-06    -0.               0.              -0.
   8*   1    4**   -0.30245E-07     0.18626E-06    0.10000E 01    -0.12846E-07     -0.39724E-07     0.11828E-07
   8*   1    4      0.13269E-06     0.23333E-07    0.10000E 01    -0.94311E-07      0.71859E-08     0.40820E-07
   9 DR.COSINS     KSI  -0.4211-0.8422 0.3369     ETA  -0.8944 0.4472-0.          ZTA  -0.1506-0.3013-0.9416
   9 DEFLECTNS**    0.31571E-06     0.72395E-13   -0.88243E-06     0.              -0.              -0.
   9*   1    4**    0.63804E-01     0.74506E-07    0.88652E 00     0.85803E-08     -0.17834E-00     0.17559E-07
   9*   1    4      0.88070E-02     0.35228E-01    0.90629E 00     0.17614E-01      0.55043E-01     0.11009E-00
  10 DR.COSINS     KSI  -0.4211 0.8422 0.3369     ETA   0.8944 0.4472-0.          ZTA  -0.1506 0.3013-0.9416
  10 DEFLECTNS**    0.26302E-06    -0.27977E-07   -0.90128E-06     0.              -0.              -0.
  10*   1    4**    0.63804E-01    -0.22352E-07    0.88652E 00     0.55673E-08     -0.17834E-00     0.30987E-08
  10*   1    4      0.88069E-02     0.35228E-01    0.90629E 00    -0.17614E-01      0.55043E-01    -0.11009E-00
  11 DR.COSINS     KSI   0.4211-0.8422 0.3369     ETA  -0.8944-0.4472 0.          ZTA   0.1506-0.3013-0.9416
  11 DEFLECTNS**    0.26302E-06     0.11191E-06   -0.90128E-06     0.               0.              -0.
  11*   1    4**    0.63804E-01     0.37253E-07    0.88652E 00    -0.16285E-08     -0.17834E-00    -0.85730E-07
  11*   1    4      0.88070E-02     0.35228E-01    0.90629E 00    -0.17614E-01     -0.55043E-01     0.11009E-00
  12 DR.COSINS     KSI   0.4211 0.8422 0.3369     ETA   0.8944-0.4472 0.          ZTA   0.1506 0.3013-0.9416
  12 DEFLECTNS**    0.21034E-06    -0.83931E-07   -0.92013E-06     0.               0.              -0.
  12*   1    4**    0.63804E-01    -0.37253E-07    0.88652E 00    -0.90966E-08     -0.17834E-00     0.44415E-07
  12*   1    4      0.88069E-02     0.35228E-01    0.90629E 00     0.17614E-01     -0.55043E-01    -0.11009E-00
```

(25)

```
STRESS LINK TOOK   2.58  SECONDS.
```

# Appendix

## Program Tape, Modification for Other Hardware, and Error Handling

### I. Program Tape

The standard ELAS program with the data of sample problems of Section VIII may be obtained from COSMIC* on a magnetic tape generated with 800 bits/in. density. On the tape there are two files. The first file contains the source program and the data ready to be compiled and executed. The straight listing of this file is provided in Volume II of this report. The second file on the tape is the binary program and the data cards of the sample problems, ready for execution. The listing and the punched cards may be obtained directly from the tape if so desired.

### II. How to Modify the Standard ELAS for *36-Bit-Word Machines of Larger Core Memory*

The standard ELAS program is designed for 32K core memory machines with 36-binary-bit word length. However, it may be easily adapted for machines with larger than 32K memory and 36-bit word length, in different levels, as described in the following paragraphs.

The program may be compiled in the bigger core machine by using the first file of the program tape described above. The binary cards produced by the compilation may be used as the object deck by inserting the XEQ and CHAIN cards. Such a procedure will cause Error Message 5 to appear erroneously in larger problems. To prevent the unwarranted appearance of Error Message 5, the user should increment the constant 19810. in the card E1SRT145 of subroutine SRAT of Link 1 by the additional core in excess of 32768.

The standard ELAS program is designed for a maximum of 540 nodes. In order to increase this, the following changes should be made in subroutines SRAT, ARAN, EXCH, OUTPT, and the main program of Link 1 before the compilation:

(1) Dimensions of ISIR, IMAX, IMIN vectors and ABIN matrix should be raised from 540 to the desired value. Note that the column dimension of ABIN matrix is obtained by dividing the row dimension by the word length 36.

(2) The equivalence statements of IMAX, IMIN, and ISIR should be adjusted in subroutines SRAT, ARAN, OUTPT, and the main program of Link 1 so that ISIR, IMAX, and IMIN vectors follow ABIN matrix immediately in the DUMMY area of COMMON.

(3) In subroutine OUTPT, matrix ABIN is assumed to consist of, at the most, 15 columns. ISUR value used in this subroutine is generated by subroutine SRAT, and it represents the number of columns of the current ABIN matrix. Provisions should be made in subroutine OUTPT for the cases where ISUR is larger than 15.

(4) If the desired value of the maximum number of nodes is larger than 999, the row order of BB matrix on card ELAS3032 of the main program of Link 3 should be changed accordingly.

The maximum number of unknown deflections in the standard ELAS is confined to 10,000. This limit may be relaxed by changing the base number 10,000 appearing in Table VI-2, which implies that the constant 10,000 in the main program of Link 1 on cards ELAS1393, ELAS1396, ELAS1405, ELAS1411, ELAS1418, ELAS1420, ELAS1424, ELAS1426, ELAS1491 in subroutine DARN on card E2DRN049, and in the main program of Link 3 on cards ELAS3062 and ELAS3078 should be changed to the desired value.

The standard ELAS program assumes that the descriptive information of the problem being solved is not more than 9,000 words in the upper COMMON for Links 1 and 3 and 14,000 words in Link 4 (no assumption is made in Link 2 on this matter). Error Messages 7 and/or 23 will automatically appear when these assumptions are violated. To safeguard against the appearance of these messages in large problems being run in larger than 32K machines, the equivalence statement of DUMMY in all Link 1 programs, the equivalence statement of BB in the main program of Link 3, and the equivalence statement of FF in all Link 4 programs should be changed to allow larger than 9,000, 9,000, and 14,000 locations, respectively, in the upper COMMON for the problem description.

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## III. How to Modify Standard ELAS for Machines With a Word Length Other Than 36 Bits

The standard ELAS program assumes that the word length is 36 bits. However, the standard ELAS may be modified for machines with a word length other than 36 bits, as follows:

(1) The column dimension of matrix ABIN should be computed by dividing the row dimension by the word length, and the dimension statements of matrix ABIN in subroutines SRAT, ARAN, EXCH, OUTPT and the main program of Link 1 should be corrected accordingly. This correction also implies correction in the equivalence statements of IMAX, IMIN, and ISIR vectors in subroutines SRAT, ARAN, OUTPT and the main program of Link 1, since these vectors follow ABIN matrix immediately in the DUMMY area of COMMON.

(2) Subprograms LEBIN and SEBIN should be rewritten for the new word length.

(3) ISUR, JJ, and JBIT values in subroutine SRAT on cards E1SRT048, E1SRT084, and E1SRT086 should be computed with the new word length.

(4) In subroutine ARAN on cards E1ARN064, E1ARN069, E1ARN074, E1ARN075, E1ARN080, E1ARN136, E1ARN138, E1ARN171, E1ARN174, E1ARN178, E1ARN187, E1ARN190, E1ARN221, E1ARN223, E1ARN234, and E1ARN236, the constant 36 should be replaced with the new word length.

(5) In subroutine EXCH on cards E1EXC007, E1EXC008, E1EXC013, E1EXC014, E1EXC015, and E1EXC016, the constant 36 should be replaced with the new word length.

(6) In subroutine OUTPT, on cards E1OPT044, E1OPT047, the octal formats 013 are for 12 octal digits per word. These formats should be changed to fit the new word length.

(7) Subroutine TICK should be rewritten for the new word length and replaced in all four links.

## IV. How to Modify Standard ELAS If the Time Counter in the Core Memory Is Not at Absolute Location 5

The time spent in the execution of each link of the ELAS program is determined by subroutine TICK, which appears in each link. Subroutine TICK is also used in determining the net equation inversion time in Link 3, and in the control of Output Item 5 in Link 1. This subroutine assumes that the time is available in 1/60-second units in absolute core location 5 as a binary integer. If the computer hardware is not compatible with this, subroutine TICK should be rewritten and replaced in each link.

## V. Handling of Possible Errors in the Program

Different versions of the ELAS program have been operational at the Jet Propulsion Laboratory since the Fall of 1966. From time to time errors were encountered and corrected, and it is anticipated that other errors may be discovered in further use of the program. The authors wish to learn of program errors encountered by other users, and suggest that a letter describing the error be sent to the Applied Mechanics Section of the Jet Propulsion Laboratory. It may be possible to help the user in correcting the errors. Also, as errors are corrected, it may be possible to issue a revised version of the program through COSMIC when appropriate.