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CONTRIBUTION OF THERMAL NOISE TO THE LINE-WIDTH OF JOSEPHSON RADIATION FROM SUPERCONDUCTING POINT CONTACTS

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The line-width of the Josephson oscillations of a voltage-biased superconducting point contact has been measured between 1.4°K and 8°K, with bias resistors R between 1.7×10^{-10} Ω and 2.6×10^{-5} Ω. Within the experimental accuracy the line-width is proportional to RT, and is consistent with the estimated theoretical value $8kTR/\Phi_0^2$, where k is Boltzmann's constant and Φ_0 is the flux quantum. Line-widths below 0.1 Hz have been observed at 4.2°K for $R = 1.7 \times 10^{-10}$ Ω, providing an experimental upper limit to other noise sources and indicating that this is useful as a voltmeter and thermometer below 10^{-16} V and 10^{-4}°K.

It has been proposed[1] that a voltage-biased superconducting point contact[2,3] might be used as a low temperature thermometer. This proposal was based on the assumption that the main contribution to the line-width of the Josephson radiation from a superconducting point contact comes from the thermal noise voltage on the shunt resistor supplying the bias voltage. A simple argument then predicts that the line-width should be proportional to the shunt resistance R and absolute temperature T of this resistance. We report here an experimental verification of these predictions for values of T between 1.4°K and 8°K and of R between 1.7×10^{-10} Ω and 2.6×10^{-5} Ω.

The experimental arrangement follows that previously described[2-4] and is shown in Fig. 1. Because

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of its small resistance the shunt appears to the point contact as a voltage source[2]

$$V = V_0 + V_N \tag{1}$$

where $V_0 = I_0 R$ and V_N is the random fluctuating component due to thermal noise. We employed two different methods to measure the line-width, referred to as the direct and parametric methods. The direct method consisted of measuring the power output from the point contact with a 27-MHz receiver whose bandwidth was limited to about 1 kHz by a quartz crystal filter as shown in Fig. 1. The bias current I_0 was scanned slowly through the value $\Phi_0 f_D/R$ (where $\Phi_0 f_D = 2.07 \times 10^{-15} \times 27 \times 10^6$ V $= 0.56$ μV) and the receiver output was recorded as a function of I_0. Since this method is limited to line-widths greater than 1 kHz by the crystal filter and will ultimately be limited by the stability of I_0 upon further reducing the bandwidth by conventional heterodyning, we also employed a novel parametric method as shown in Fig. 1. This method, which will be fully described in a separate publication, is related to the oscillating detector mode[4,5] previously employed and requires a 27-MHz signal to be weakly coupled into the Josephson oscillator in addition to the dc bias I_0. Mixing of the Josephson frequency, $f_J = IR/\Phi_0$, and the receiver frequency, $f_D = 27$ MHz, by the nonlinear behavior of the point contact produces signal at $f_D \pm f_J$. For f_J less than the receiver bandwidth (150 kHz) the demodulated signal accurately reproduced f_J, which is then filtered via a (variable) narrow-band audio amplifier. Again as in the direct method above, this filtered signal is rectified and recorded as a function of I_0. The obvious advantage of the parametric over the direct method is that it permits observation of f_J in the audio frequency region and below, where it is relatively easy to measure line-widths of the order of 1 Hz without stringent stability conditions for I_0 and the rf-frequency-determining components.

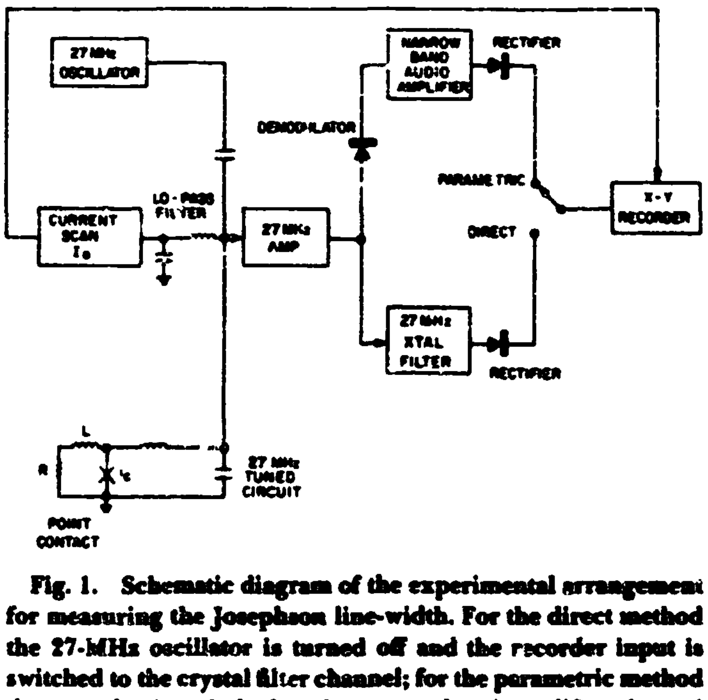

Fig. 1. Schematic diagram of the experimental arrangement for measuring the Josephson line-width. For the direct method the 27-MHz oscillator is turned off and the recorder input is switched to the crystal filter channel; for the parametric method the recorder is switched to the narrow-band amplifier channel and the 27-MHz oscillator is turned on at a low power level.

Some results of the measured line-widths Δf are given in Table I. The accuracy of these data is ±10%. Some inconsistency is anticipated since the observed line-shapes vary with the operating point of the rf detectors. Also the signal-to-noise ratio was not sufficient to permit an accurate line-width determination and a time-averaging method would be desirable. However, in the case of large Δf the direct and parametric measurements give the same result showing that both methods measure the same quantity.

The observed temperature dependence is shown in Fig. 2 for the 26-$\mu\Omega$ resistance using the parametric method. In Fig. 2(*a*) a 10-kHz narrow-band amplifier with a $Q = 25$ was used at nominal temperatures of 2, 4, and 8°K. In order to directly compare the line-widths, Fig. 2(*b*) shows the response where both the frequency scale and the narrow-band amplifier frequency are varied proportional to T. The Josephson line-width from voltage-biased point contacts is shown to be linear in both T and R over the ranges investigated. No evidence of line broadening by other noise sources, e.g., magnetic field fluctuations, receiver input noise feeding back into the cryostat, or shot noise in the junction, has been observed in this study.

We can obtain a simple physical interpretation of these results by considering the random frequency modulation of the Josephson oscillation

Table I. Values of the Josephson Line-width as Functions of Bias Resistance R and Absolute Temperature T.

R (Ω)	T (°K)	Line-width Δf (Hz) Experimental[a]		Theory[b]
2.6×10^{-5}	8	8200	P	5350
	4.2	4500	D	2800
	4.2	3800	P	2800
	2	2100	P	1340
	1.7	1700	D	1140
2.6×10^{-6}	4.2	<1000	D	280
	4.2	380	P	280
1.7×10^{-10}	4.2	<0.1	P	0.018

[a] P – parametric method; D – direct method.
[b] Calculated from Eq. (4).

caused by thermal noise on the bias resistor. We start with Nyquist's formula

$$\langle V_N^2 \rangle = 4kTRf_c \tag{2}$$

where $\langle V_N^2 \rangle$ is the mean square noise voltage and f_c is the width of the band of noise to which the system is sensitive. This extends from zero frequency up to a cutoff which we estimate by considering a single component of the noise spectrum at frequency f_m.

The spectrum of a sine wave which is frequency modulated over a range δf about a center frequency f_0 at a modulation frequency f_m consists of an array of equally spaced side bands at interval f_m. All the side bands of significant amplitude occur in a range $(f_0 - \delta f)$ to $(f_0 + \delta f)$, so that if $f_m > \delta f$ there are no large side bands and a normal receiver would not detect the modulation. This limit corresponds physically to the variation of phase due to the modulation. This limit corresponds physically to the variation of phase due to the modulation becoming less than one cycle.

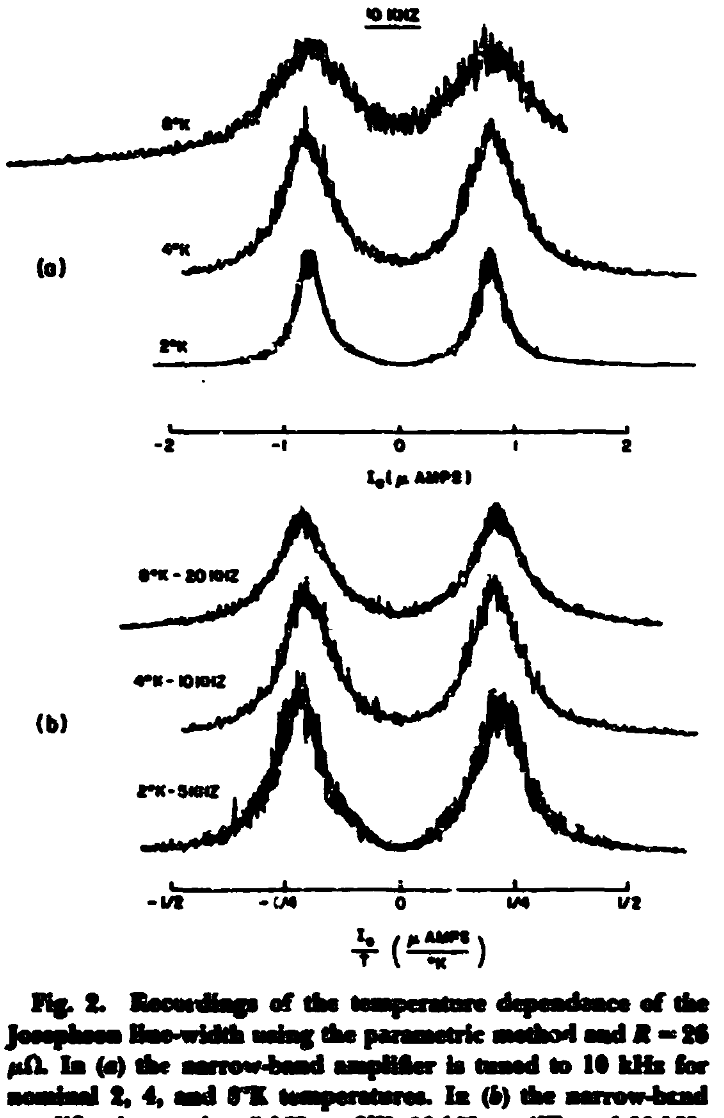

Fig. 2. Recordings of the temperature dependence of the Josephson line-width using the parametric method and $R = 26$ $\mu\Omega$. In (*a*) the narrow-band amplifier is tuned to 10 kHz for nominal 2, 4, and 8°K temperatures. In (*b*) the narrow-band amplifier is tuned to 5 kHz at 2°K, 10 kHz at 4°K, and 20 kHz at 8°K and the scale of I_0 is respectively compressed by a factor of 2.

Using the Josephson relationship between voltage and frequency we find

$$\delta f \approx \langle V_N^2 \rangle^{1/2}/\Phi_0 \tag{3}$$

where $\Phi_0 = h/2e$. Applying the condition that f_m must not exceed δf we find that the cutoff frequency f_c in Eq. (2) is of the same order of magnitude as δf in Eq. (3). Equating f_c and δf, combining Eqs. (2) and (3), and defining the full line-width $\Delta f = 2\delta f$, we have

$$\Delta f \approx 8kTR/\Phi_0^2 = 2.57 \times 10^7\ RT. \tag{4}$$

Equation (4) is consistent with much more rigorous calculations by Scalapino[6] and by Burgess.[7] It predicts correctly the order of magnitude of the observed line-width as well as its linear dependence on R and T.

A corollary of the above discussion is that the line-width is independent of the frequency in agreement with the observations. In the direct method the Josephson oscillator was operated at 27 MHz, while in the parametric method it was operated at audio frequencies and below. The data given in the last line of the table were obtained by operating the Josephson oscillator at a frequency less than 1 Hz, at a bias voltage less than 10^{-15} V. Such a device may be used as a voltmeter, with sensitivity comparable to that reported for other quantum interference techniques.[8]

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