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OUT-OF-THE-ECLIPTIC-PLANE PROBE MISSION EMPLOYING ELECTRIC PROPULSION

by Frank J. Hrach

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SUMMARY

Measurements out of the ecliptic plane are necessary in the study of the Sun and heliocentric space. The present analysis investigates the use of a combined propulsion system to accomplish the out-of-the-ecliptic-plane probe mission. In the combined propulsion system studied herein, energy beyond Earth-escape energy is given to the vehicle by a booster, after which a low-thrust electric propulsion system begins operation. The chemical boosters considered are the Atlas-Agena and the Atlas-Centaur. Three values of electric propulsion system specific powerplant mass are examined, 50, 100, and 150 pounds per kilowatt jet (22.7, 45.4, and 68.0 kg/kW jet). Results are presented for missions to final heliographic inclinations from 25° to 45°. The basic mission profile considered is one in which the distance from the Sun to the vehicle is held constant at a value of 1 astronomical unit (1.49599×10^{11} m), and in which the propulsion is terminated before the argument of latitude of the trajectory reaches 90°. The effects of variations from the basic profile are investigated.

The gross payload capability of the combined propulsion system is larger than that of the booster alone. It is also larger than that of the booster with an additional chemical stage for the more difficult missions, if the specific powerplant mass is low.

The addition of a second propulsion period after the vehicle has coasted through a local maximum in latitude is an attractive variation of the basic mission profile. The value of final mass minus powerplant mass is significantly increased, at the cost of increased propulsion time.

INTRODUCTION

Measurements must be made in the interplanetary medium in order to increase knowledge of the Sun and its processes and of interplanetary space, which is influenced predominantly by the Sun. The physical phenomena to be measured include interplanetary plasma, magnetic fields, cosmic rays, and the zodiacal dust cloud. Since these phenomena are three dimensional, measurements out of the ecliptic plane are necessary.

The ecliptic plane is inclined at an angle of 7.2° with the equatorial plane of the Sun. A reasonable objective for an out-of-the-ecliptic-plane probe mission would be to attain some specified inclination with the Sun's equatorial plane (i.e., a specified heliographic inclination) because the phenomena to be measured, with the exception of the zodiacal dust cloud, depend on heliographic coordinates (ref. 1). A probe in an orbit inclined 90° with the Sun's equatorial plane would provide the most information about heliocentric space outside the ecliptic plane; however, data from the edge of the sunspot belt can be obtained if the orbit is inclined only 30°. From a propulsion standpoint, this mission presents the problem of selecting a propulsion system which will deliver the necessary payload to some specified heliographic inclination within a reasonable period of time.

Out-of-the-ecliptic-plane missions have been analyzed by a number of researchers. Burley (ref. 2) identified those trajectories that yield maximum altitude above the ecliptic plane for a given impulsive energy change. Breakwell (ref. 3) investigated trajectories of vehicles launched perpendicular to the ecliptic plane but with low "hyperbolic excess" speeds relative to the Earth so that the distance from the Earth to the vehicle remains small. Reference 4 contains the characteristic velocity requirements for inclinations to 90° relative to the ecliptic plane for low-thrust electric propulsion vehicles. In reference 4, the vehicle initially has a zero inclination with the ecliptic plane and the thrust is continuous and directed perpendicular to the instantaneous orbital plane until the desired inclination is reached. (The sense of the thrust vector is alternated every half revolution.) An out-of-the-ecliptic-plane probe mission to an inclination of 15°, utilizing an electric propulsion system after Earth-escape energy is reached, is investigated in reference 5. Reference 6 analyzes both the ecliptic and the out-of-the-ecliptic-plane probe missions. The region of interplanetary space that can be explored with a 400-pound (181 kg) spacecraft injected into its heliocentric orbit by presently available chemical boosters is defined as part of the analysis.

The present report investigates the use of a combined propulsion system to accomplish the out-of-the-ecliptic-plane probe mission. In the combined propulsion system studed herein, energy beyond Earth-escape energy is given to the vehicle by the booster after which a constant-thrust electric propulsion system begins operation. This investigation was conducted to estimate gross payloads for the combined system for which the point of starting the electric propulsion system is optimized. The special cases of starting the electric propulsion system from a low Earth orbit after the booster has attained Earth-orbit energy and of starting the electric propulsion system after the booster has imparted exactly Earth-escape energy to the vehicle are examined. Starting the electric propulsion system at low Earth orbit requires a spiral trajectory to escape the Earth and is referred to in this report simply as the electric system. Starting the electric propulsion system at Earth escape is an example of a combined propulsion system for which the starting point of the electric propulsion system is not optimized. The gross payloads achieved by the preceding methods are compared with those of the all-chemical systems. A secondary objective is the determination of the optimum constant-thrust electric propulsion system parameters.

The payload capability of the combined propulsion system for the out-of-the-ecliptic-plane probe mission has already been investigated to a limited extent in reference 7. In that study the power, the specific impulse, and the starting point of the electric propulsion systems are fixed, and thrust is applied normal to the ecliptic plane for 90-day intervals separated by 90-day coast periods until the desired inclination is reached.

The propulsion system is considered to be optimized if, for a particular mission, the final mass of the electric vehicle less the powerplant mass is a maximum; the mass of the propellant tanks, the structure, and the engines must be subtracted from this mass to obtain the payload. If the powerplant will be used both for propulsion and later for operation of other systems of the vehicle, the payload can be increased by that portion of the powerplant required by these systems.

A major assumption in this analysis is that the distance from the Sun to the vehicle remains constant throughout the trajectory. This assumption was made because of mission considerations other than propulsion. These considerations include vehicle guidance, power variation, and communication distance and are discussed in more detail in the ANALYSIS section.

Two boosters were considered for the mission, the Atlas-Agena and the Atlas-Centaur. The electric propulsion system mode of operation studied was that of constant power and constant specific impulse and results in constant thrust. Coasting periods were permitted if they were beneficial. Three values of specific powerplant mass were examined, 50, 100, and 150 pounds per kilowatt jet (22.7, 45.4, and 68.0 kg/kW jet). These values were selected because they are representative of values obtainable through the use of solar cells. Systems utilizing solar cells seem to be the most practical early power source for a small electric propulsion system. Missions were investigated which have as their objective final inclinations in the range from 25° to 45° relative to the Sun's equatorial plane. The requirement that propulsion be terminated before the argument of latitude reaches 90° was imposed on the trajectory, in order that the vehicle reach the point of maximum latitude at a time not much greater than 90 days after entry into heliocentric space. A time of about 90 days would be required by the all-chemical system to reach the point of maximum latitude.

ANALYSIS

General Description of Problem

In this analysis of a combined propulsion system, the particular boosters are specified. However, the low-thrust electric propulsion system parameters are not specified, but optimum values are determined for a particular mission and for particular values of specific powerplant mass.

If the powerplant of the electric vehicle has a lifetime considerably in excess of the required propulsion time, the powerplant can be used to operate the instruments and communication system of the vehicle after the end of the propulsion phase. The payload mass is the final mass of the electric vehicle minus the powerplant mass and the mass of the propellant tanks, the structure, and the engines, plus that portion of the powerplant required to operate the systems of the payload. The payload mass just defined is the gross payload. Systems such as the attitude control system and guidance system are included in this mass.

In equation form, the gross payload is

$$M_l = M_e - M_p - M_w - M_{t+st+en} + K_2 M_w$$

(Symbols are defined in the appendix.) The power conditioning mass is included in the powerplant mass. If the mass of the propellant tanks, the structure, and the engines is assumed to be some fraction of the sum of the propellant and powerplant mass, the payload can be written as follows:

$$M_l = M_e - (M_p + M_w) - K_1(M_p + M_w) + K_2 M_w$$

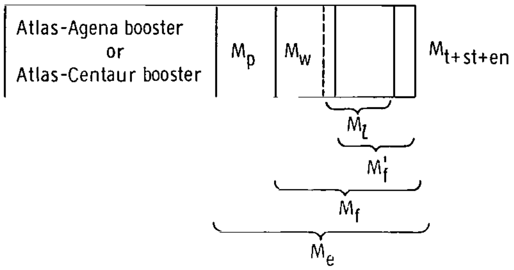
$$= M_e - (1 + K_1)(M_p + M_w) + K_2 M_w$$

For a given electric vehicle mass M_e and a given payload power requirement corresponding to a particular value of $K_2 M_w$, maximum payload is obtained if the sum of the propellant and powerplant mass is a minimum; this is equivalent to maximizing the final mass minus the powerplant mass. This quantity is designated by the symbol M_f':

$$M_f' = M_f - M_w$$

$$= M_e - M_p - M_w$$

The entire vehicle as it would appear on the launch pad is represented schematically in the following sketch.



(a)

The boosters are capable of imparting energy beyond Earth escape to the electric vehicle M_e. The electric propulsion system imparts energy to the final mass M_f sufficient for it to achieve the final heliographic inclination. The propulsive effort must be divided between the booster and electric propulsion system so that M_l is a maximum.

Trajectory Assumptions

It is assumed that the vehicle leaves the Earth's sphere of influence at the intersection of the ecliptic plane and the Sun's equatorial plane in order that the hyperbolic excess speed produced by the booster be most effective in increasing heliographic inclination. There are two opportunities a year at which this is possible. Hereinafter, the term inclination without qualification refers to inclination relative to the Sun's equatorial plane. For example, a vehicle which escapes the Earth with zero hyperbolic velocity has an inclination of 7.2°.

Several other assumptions regarding the vehicle trajectory were made and are listed along with the reasons they were made and the restrictions that result from them. The first assumption is that the distance from the Sun to the vehicle in heliocentric space is held constant at a value of 1 astronomical unit. To realize this requirement, the hyperbolic velocity relative to the Earth V_h must be directed in such a way that, when added to the Earth's velocity V_E, the magnitude of the resultant is the same as that of V_E. Also the low-thrust acceleration a of the electric vehicle must be directed perpendicular to the instantaneous orbit plane. If the thrust is turned off at any time in heliocentric space, the vehicle will be in circular orbit about the Sun. This assumption was made for three reasons:

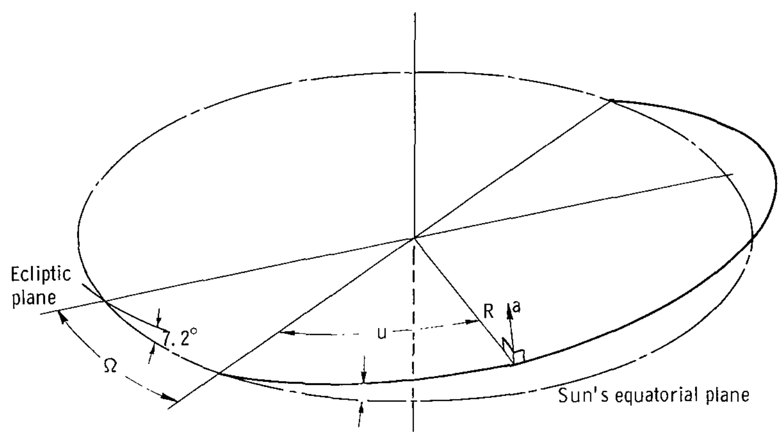
(1) The communication distance between the spacecraft and the Earth is shorter for a constant radius than when the radius varies. Therefore, more information can be received in a given period of time from a transmitter with a fixed power output.

(2) The electric thrustor steering program is simplified. If one axis of the vehicle is alined with the Sun, thrust need be applied only perpendicular to that axis.

(3) If solar cells, which seem to be the most practical early power source for a small electric propulsion system, are used, the output power will remain constant. A constant-power solar-cell system is attractive because the power conditioning and engine systems can be designed for a fixed set of conditions.

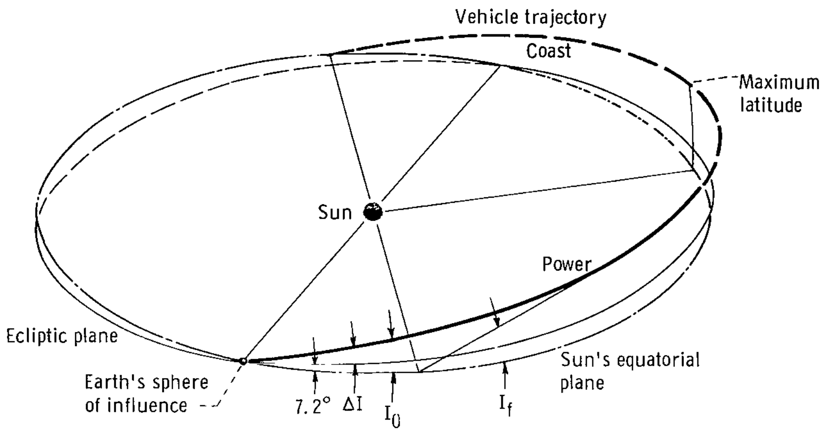
The second assumption is that the gravitational field can be approximated by a succession of single body fields. For motion in the vicinity of the Earth, the effect of the Sun is neglected. At a point sufficiently far from Earth, at which point the velocity of the vehicle relative to the Earth is nearly equal to the hyperbolic velocity, the presence of the Earth in the Sun's field is neglected. It was also assumed for simplicity that low-thrust electric propulsion begins when the vehicle has left the Earth's sphere of influence.

A third assumption is that thrusting is terminated before the angle from the node line to the vehicle radius vector reaches 90°. The angle (u in sketch (b)) is the argument of latitude. This assumption was made to limit the time at which the vehicle reaches the



(b)

maximum latitude point to a value not much greater than that for the all-chemical system. Data from the maximum latitude region of space would be received within a relatively short time after the vehicle was launched. The short propulsion time also results in improved reliability for early electric thrustors. A typical trajectory is shown in sketch (c).



(c)

Basic Equations

Payload equation. - The payload mass of the vehicle is given as follows:

$$M_l = M_e \frac{M_l}{M_e}$$

$$= M_e \left[1 - (1 + K_1)\left(1 - \frac{M_f'}{M_e}\right) + K_2 \frac{M_w}{M_e}\right] \tag{1}$$

More power than is needed by the payload systems will probably be available onboard the spacecraft. Therefore, M_l is a maximum when the quantity $M_e[1 - (1 + K_1)(1 - M_f'/M_e)]$ is a maximum. Also, for simplicity, K_1 is arbitrarily set equal to zero while the combined system is optimized. This procedure implies that $M_e(M_f'/M_e)$ is a maximum for maximum payload. The mass of the propellant tanks, the structure, and the engines can be subtracted from the quantity $M_e(M_f'/M_e)$ and the portion of the powerplant required by the payload systems can be added to this quantity to obtain the payload. The results obtained by this procedure are correct for a value of K_1 equal to zero and are conservative for values of K_1 greater than zero.

The payload mass of the chemical booster M_e (which is also the initial mass of the electric vehicle) is a function of the burnout velocity, which, in turn, can be expressed in terms of the change in velocity ΔV added beyond a particular set of orbit conditions. Curves of payload for the two boosters considered as a function of the ΔV added beyond a 100-nautical-mile (185×10^3 m) circular orbit are shown in figure 1. Also shown in

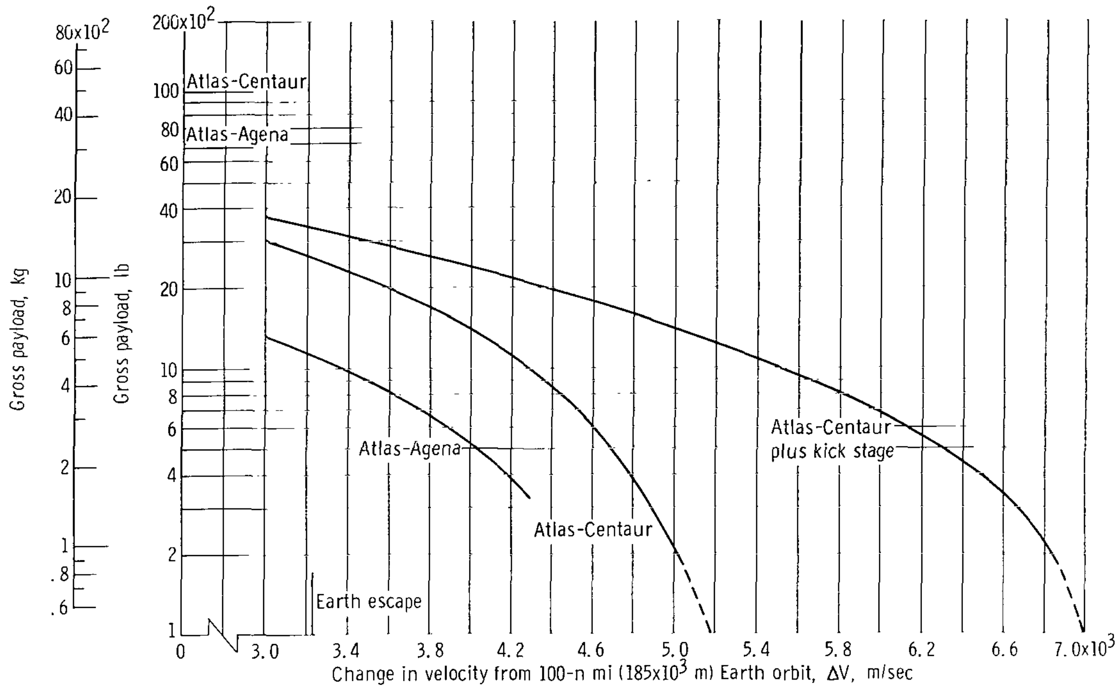


Figure 1. - Booster vehicle performance.

figure 1 is a payload curve for the Atlas-Centaur plus a proposed 8800-pound (3990 kg) additional chemical stage with a specific impulse of 444 seconds and a mass fraction of 0.795. These curves were obtained from reference 8. The payload mass of a chemical system when it is used alone is also a gross payload. Auxiliary systems (e.g., an attitude-control system) are included in this mass. A comparison of the auxiliary system masses for the combined and the all-chemical propulsion systems is not included in this analysis.

The ratio M_f'/M_e can be expressed as follows:

$$\frac{M_f'}{M_e} = 1 - \frac{M_p}{M_e} - \frac{M_w}{M_e} \qquad (2)$$

For constant power and constant specific impulse, the propellant fraction M_p/M_e and the powerplant fraction M_w/M_e can be written as

$$\frac{M_p}{M_e} = \frac{a_0 t P}{I_{sp} g} \qquad (3)$$

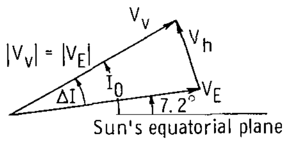
$$\frac{M_w}{M_e} = \alpha \frac{a_0 I_{sp} g}{2} \qquad (4)$$

where α (specific powerplant mass) in equation (4) is the ratio of the powerplant mass to the useful power in the exhaust jet.

The ratio M_f'/M_e can then be written in terms of the propulsion system parameters:

$$\frac{M_f'}{M_e} = 1 - \frac{a_0 t P}{I_{sp} g} - \alpha \frac{a_0 I_{sp} g}{2} \qquad (5)$$

Trajectory equations. - The value of initial inclination relative to the Sun's equatorial plane can be related to the ΔV that must be applied beyond a particular set of orbit conditions as shown in sketch (d). With the assumptions that have been made, I_0 can first



(d)

be written as a function of V_E and V_h:

$$\sin\left(\frac{1}{2}\Delta I\right) = \frac{V_h}{2V_E}$$

$$I_0 = 7.2^\circ + 2 \sin^{-1}\left(\frac{V_h}{2V_E}\right) \qquad (6)$$

The hyperbolic velocity relative to the Earth can be related to the ΔV applied beyond circular orbit and the circular orbit velocity as follows:

$$V_h = \left[(\Delta V + V_c)^2 - 2V_c^2\right]^{1/2} \tag{7}$$

Combining equations (6) and (7) results in the following relation between I_0 and ΔV:

$$I_0 = 7.2^\circ + 2 \sin^{-1}\left\{\frac{\left[(\Delta V + V_c)^2 - 2V_c^2\right]^{1/2}}{2V_E}\right\} \tag{8}$$

The booster payload which depends on the ΔV applied beyond a particular set of orbit conditions can be plotted against I_0 as a result of equation (8). Curves of M_e against I_0 for the boosters considered are given in figure 2.

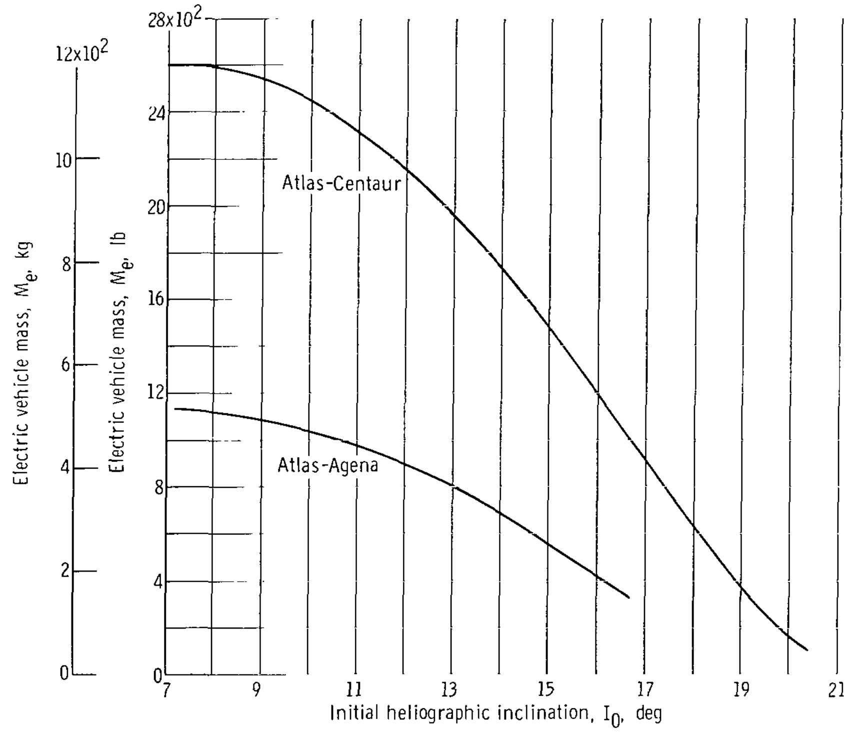


Figure 2. - Electric vehicle mass as function of initial heliographic inclination.

The equations describing the vehicle motion in heliocentric space with the assumed steering program were obtained from reference 9. These equations are given as follows in terms of the vehicle's orbital elements relative to the Sun's equatorial plane and the line of intersection between the Sun's equatorial plane and the ecliptic plane (see sketch (b)):

$$\dot{I} = \frac{a}{V_v} \cos u \tag{9}$$

$$\dot{\Omega} = \frac{a}{V_v} \frac{\sin u}{\sin I} \tag{10}$$

$$\dot{u} = \omega - \dot{\Omega} \cos I \tag{11}$$

From equation (9), it can be seen that, when the argument of latitude u is 90°, the time rate of change of inclination is zero. Thrust acceleration a applied over a time interval when u is near 90° is relatively ineffective in producing a change in inclination.

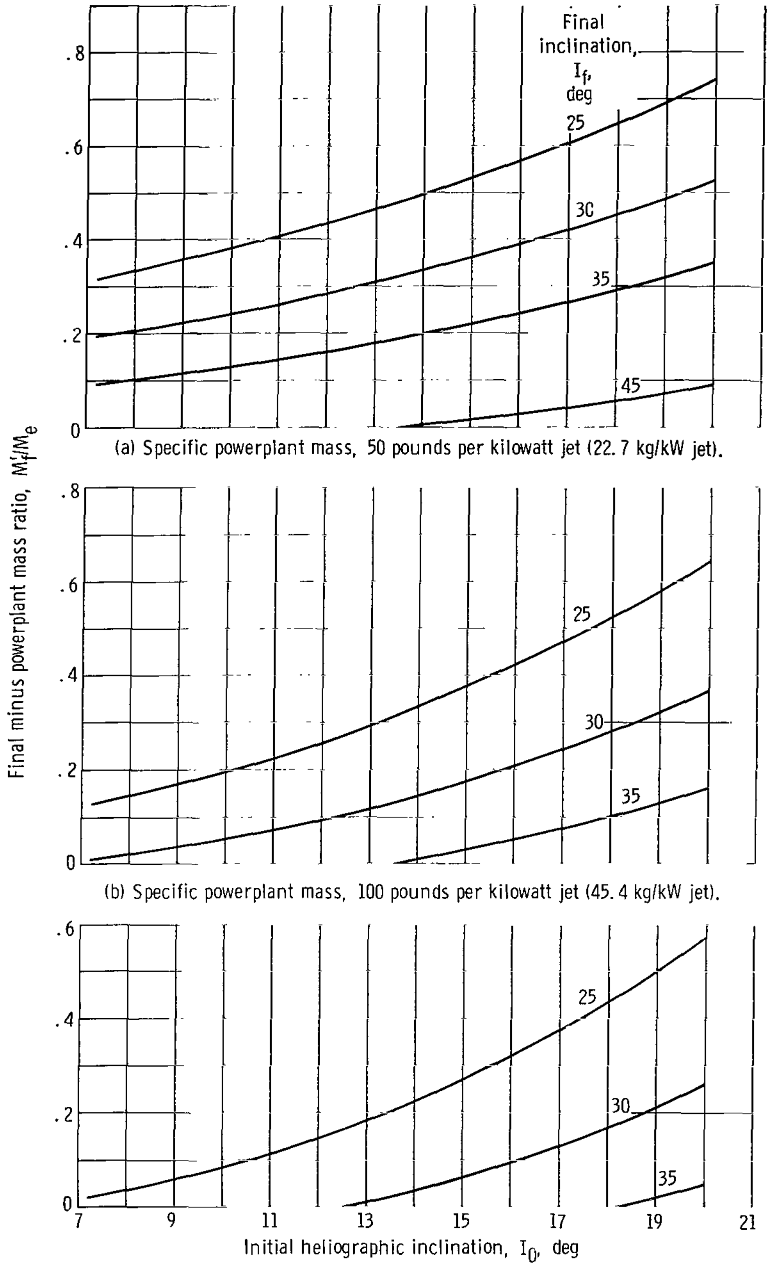
Procedure

The booster payload is a function of the initial inclination in heliocentric space (fig. 2). The low-thrust electric vehicle ratio M_f'/M_e is a function of the initial and desired final inclinations, the specific powerplant mass, and the parameters of the electric propulsion system:

$$M_f' = \left[M_e(I_0)\right]\left[\frac{M_f'}{M_e}(I_0, I_f, \alpha, a_0, I_{sp}, t_P)\right]$$

For a particular value of α and I_f, values of I_0, a_0, I_{sp}, and t_P can be found which maximize M_f'. Examination of equations (9) to (11) reveals that a change in inclination cannot be algebraically added to the initial inclination. The ratio M_f'/M_e depends then on the values of both I_0 and I_f and not on the change in inclination $I_f - I_0$.

The following procedure is used to solve the problem. For a particular α and I_0, the set of propulsion parameters a_0, I_{sp}, and t_P was obtained which would both transfer the vehicle to some desired final inclination and maximize the ratio M_f'/M_e. A modified version of the trajectory code of reference 10 was used to search for the optimum set of propulsion parameters. It was required that propulsion be terminated before the argument of latitude of the trajectory reached 90°. Curves of maximum M_f'/M_e against I_0 for various values of α and I_f were then drawn (fig. 3). These plots along with the curves of M_e against I_0 (fig. 2) were then used to determine that



(c) Specific powerplant mass, 150 pounds per kilowatt jet (68.0 kg/kW jet).

Figure 3. - Final minus powerplant mass ratio as function of initial heliographic inclination for various values of final inclination.

value of I_0 for which M_f' is a maximum. An example of the optimization procedure is given in figure 4. The procedure results in the optimum distribution of propulsive effort for the case in which the booster provides energy beyond Earth escape. Better performance may be obtained if low-thrust propulsion is begun in Earth orbit. This case is examined in the next section.

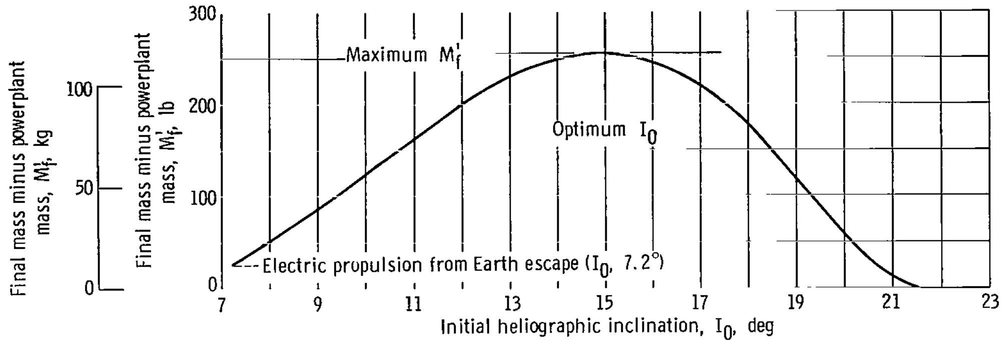


Figure 4. - Example of optimization of initial heliographic inclination. Atlas-Centaur booster; final inclination, 30°; specific powerplant mass, 100 pounds per kilowatt jet (45.4 kg/kW jet).

Low-Thrust Spiral to Escape Energy

A low-thrust tangential escape spiral can be substituted for the high-thrust boost from low Earth orbit conditions in order to reduce the amount of propellant needed to escape from the Earth. The time required for the vehicle to reach the final inclination would be increased, and the reliability requirement on the power generation equipment and the electric thrustors would be more severe. The spacecraft would be exposed to the hazards of trapped radiation about the Earth, and, if solar cells are used as a power source, provision would have to be made for either avoiding the Earth's shadow or supplying power during the time the vehicle is in shadow. These disadvantages might be acceptable if a substantial increase in M_f' were obtained.

The escape point of a typical low-thrust spiral ($a_0 < 10^{-3}$ m/sec) lies at a great distance from the Earth. Even if thrust were continued beyond escape energy until the vehicle leaves the region of the Earth's influence, the hyperbolic velocity relative to the Earth would be small. The initial inclination in heliocentric space would be about 7.2°, and hence this value of I_0 is employed for the analysis of this case.

Previously, the case which optimized the heliocentric phase of the mission was considered. In the present case of the electric system, two phases of the mission utilize electric propulsion, the Earth-escape phase and the heliocentric phase. The problem should be formulated properly to find the values of power and specific impulse for the entire mission, including the spiral, which maximize M_f' for a given amount of propulsion time. The total propulsion time would then be varied over a range of values to obtain a curve of maximum M_f' against propulsion time. Spirals can be added to the trajectories that are optimum for the heliocentric phase only. This procedure is not the best, but the results obtained give an indication of where the electric performance lies. The spirals are obtained by setting the power and specific impulse of the Earth-escape phase equal to those of the heliocentric phase. This method implies that the final acceleration of the spiral trajectory is equal to the initial acceleration of the heliocentric trajectory. With these conditions, the spiral propulsion time and the propellant fraction required for the escape maneuver can be calculated from the following equations obtained from reference 11:

$$\left(t_P\right)_s = \gamma\left(1 - e^{-V_c/I_{sp}g}\right)\frac{I_{sp}g}{a_{00}}$$

$$\left(\frac{M_p}{M_o}\right)_s = \gamma\left(1 - e^{-V_c/I_{sp}g}\right)$$

where a_{00} is the initial acceleration when the vehicle is in low Earth orbit and γ is an empirical function of a_{00} given in reference 11. The quantity a_{00} is related to a_0, the final acceleration and also the initial acceleration in heliocentric space, by the following equation:

$$a_0 = \frac{a_{00}}{1 - \dfrac{a_{00}\left(t_P\right)_s}{I_{sp}g}}$$

The equations from reference 11 are appropriate for constant-thrust spirals obtained with a tangential thrusting program. This program closely approximates the optimum steering program. The final mass minus the powerplant mass at the final inclination is approximately equal to the mass at Earth-escape energy multiplied by the ratio M_f'/M_e which corresponds to an initial inclination of 7.2^o:

$$M_f' \approx M_o\left[1 - \left(\frac{M_p}{M_o}\right)_s\right]\left(\frac{M_f'}{M_e}\right)_{7.2}$$

RESULTS AND DISCUSSION

This section is divided into four major subdivisions. In the first, the values of final mass minus powerplant mass attainable with the combined propulsion system are compared with the payloads of the basic boosters. A comparison is also made between the values of final mass minus powerplant mass of the Atlas-Centaur combined system and the payloads of the Atlas-Centaur with a proposed kickstage. The second section presents values of electric-engine specific impulse and propulsion time for the combined system, which are required to obtain the values of M_f' of the first section. With these values and the data presented in the first section, parameters such as initial acceleration and thrust can be calculated. The effect of off-optimum specific impulse on M_f' is also examined. The third section examines the effect on M_f' caused by variations in the mission profile. The following variations are considered:

(1) Booster acceleration to exactly Earth-escape energy, after which the electric propulsion system begins operation

(2) Substitution of a low-thrust escape spiral for the high-thrust escape from Earth-orbit conditions

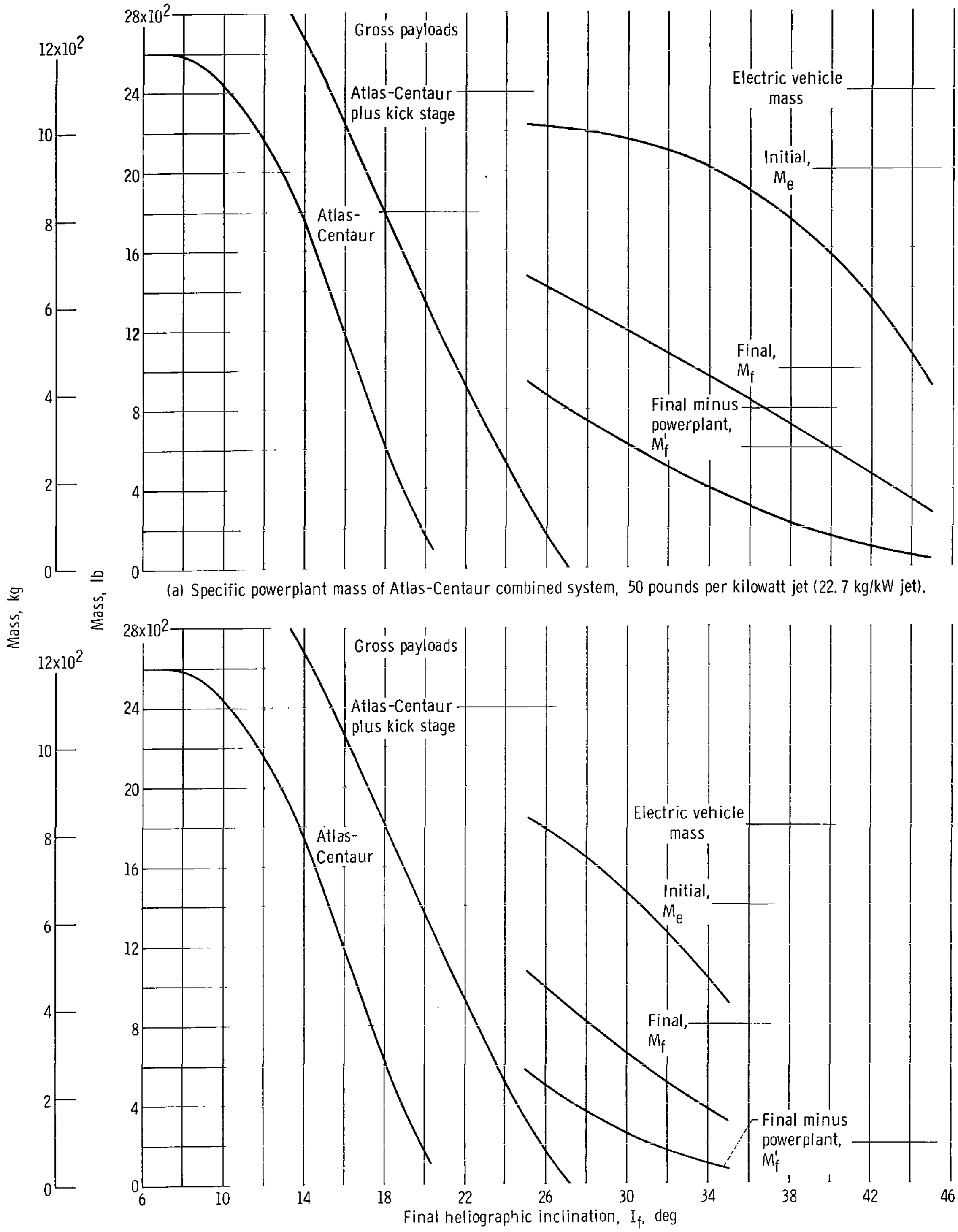
(3) The addition of a second propulsion phase after an intermediate coast period through the region in which thrusting would be ineffective

(4) Relaxation of the requirement that the distance from the Sun to the vehicle remain constant

All these variations result in propulsion times longer than those of the basic profile. The fourth section presents a sample application of the data to obtain a payload comparison between the combined system and the all-chemical systems.

Comparison of Propulsion Systems

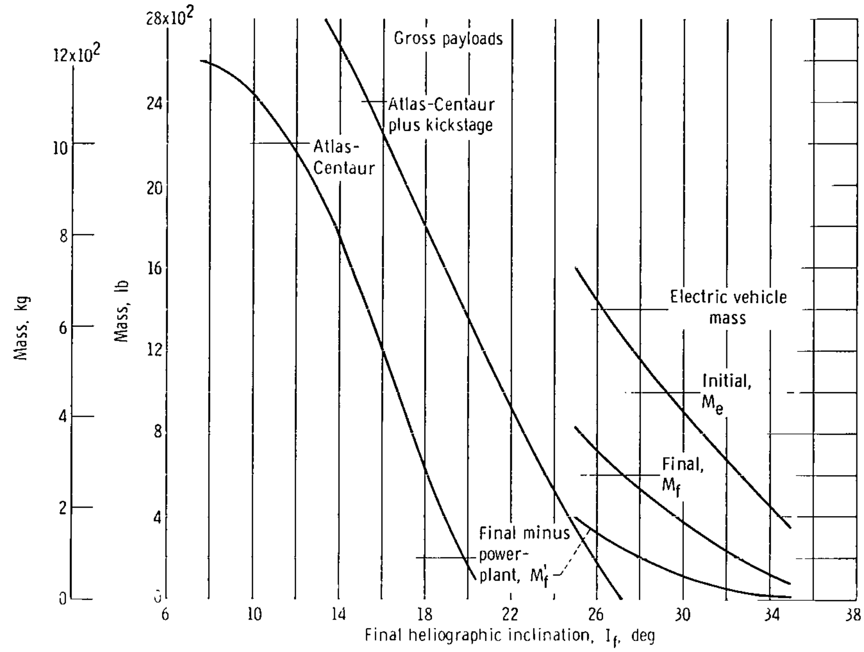
Figures 5 and 6 present curves of final mass minus powerplant mass for the combined systems and payload mass for the chemical systems against final inclination. The Atlas-Centaur, the Atlas-Centaur with the proposed kickstage, and the Atlas-Centaur combined system are compared in figure 5. In figure 6, the Atlas-Agena and Atlas-Agena combined system are compared. The combined system curves represent the payload for the ideal case of no propellant tank mass, no structure mass, and no engine mass. For the real case, the mass of these items must be subtracted from the values plotted. This procedure is illustrated in the section Sample Application. In the present section, however, the mass of tanks and engines has been subtracted from the final mass only for the cases with all-chemical propulsion. Hence, whenever a greater final mass is reported for a combined system it should be viewed merely as a potential advantage



(a) Specific powerplant mass of Atlas-Centaur combined system, 50 pounds per kilowatt jet (22.7 kg/kW jet).

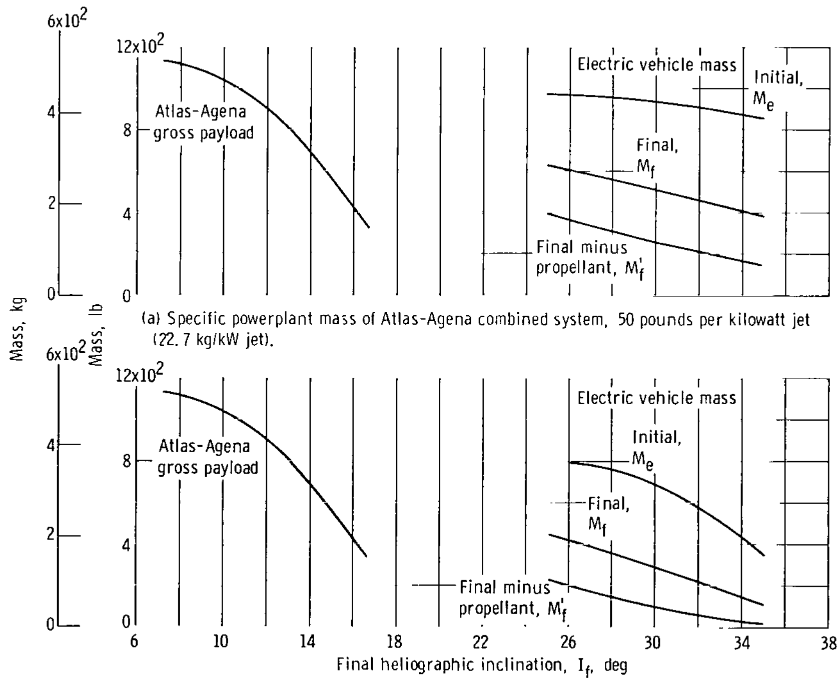
(b) Specific powerplant mass of Atlas-Centaur combined system, 100 pounds per kilowatt jet (45.4 kg/kW jet).

Figure 5. - Comparison of Atlas-Centaur combined system masses with Atlas-Centaur and Atlas-Centaur plus kick stage gross payloads.



(c) Specific powerplant mass of Atlas-Centaur combined system, 150 pounds per kilowatt jet (68.0 kg/kW jet).

Figure 5. - Concluded.



(a) Specific powerplant mass of Atlas-Agena combined system, 50 pounds per kilowatt jet (22.7 kg/kW jet).

(b) Specific powerplant mass of Atlas-Agena combined system, 100 pounds per kilowatt jet (45.4 kg/kW jet).

Figure 6. - Comparison of Atlas-Agena combined system masses with Atlas-Agena gross payload.

that must be verified later. Curves of electric vehicle initial and final mass are also included in figures 5 and 6. From these curves, the value of I_0 associated with a particular combined system can be obtained by reading the value of inclination corresponding to a booster payload mass equal to the initial mass of the electric vehicle.

The following conclusions can be drawn from examination of the figures:

(1) The Atlas-Centaur payload mass curve of figure 5 goes to zero at an inclination of 21°. The Atlas-Agena payload mass curve of figure 6 goes to zero at an inclination of about 19°. Neither booster alone (i.e., with no upper stages) is able to achieve even the lowest value of inclination (25°) considered for the combined systems. An additional propulsion system, either chemical or electric, must be used to achieve an inclination of 25° or more with the boosters considered.

(2) The Atlas-Centaur plus kick stage curve of figure 5 goes to zero at an inclination of about 27°. Therefore, an additional propulsion system for the Atlas-Centaur capable of delivering any payload to an inclination above 27° must have a high specific impulse, such as that provided by electric propulsion.

(3) From examination of figure 5, it can be seen that the value M_f' for a particular value of inclination decreases greatly with increasing specific powerplant mass. The Atlas-Centaur combined system with a specific powerplant mass of 50 pounds per kilowatt jet (22.7 kg/kW jet) can deliver 620 pounds (281 kg) of M_f' to an inclination of 30°. For a specific powerplant mass of 100 pounds per kilowatt jet (45.4 kg/kW jet), the value of M_f' is 250 pounds (113 kg), and for 150 pounds per kilowatt jet (68.0 kg/kW jet), it is 110 pounds (49.9 kg). The Atlas-Agena combined system can deliver 260 pounds (118 kg) of M_f' to an inclination of 30° for an electric stage specific powerplant mass of 50 pounds per kilowatt jet (22.7 kg/kW jet). The system is able to deliver this amount of M_f' to an inclination of only 24° if the specific powerplant mass is 100 pounds per kilowatt jet (45.4 kg/kW jet). An example of how these data for the combined systems can be used to determine payloads is presented in the section Sample Application.

Required Specific Impulses and Propulsion Times

The optimum values of specific impulse are presented in figure 7, and the optimum values of propulsion time in figure 8. The optimum values of specific impulse are nearly equal for the two boosters; the difference cannot be seen in figure 7. For a particular value of final inclination the optimum value of specific impulse decreases with increasing specific powerplant mass. For a constant value of specific powerplant mass, the optimum specific impulse decreases slightly with increasing final inclination. All values of specific impulse lie within the range (<2200 sec) for which electrothermal thrustors presently have the highest efficiency (ref. 12). Propulsion time (fig. 8) decreases mono-

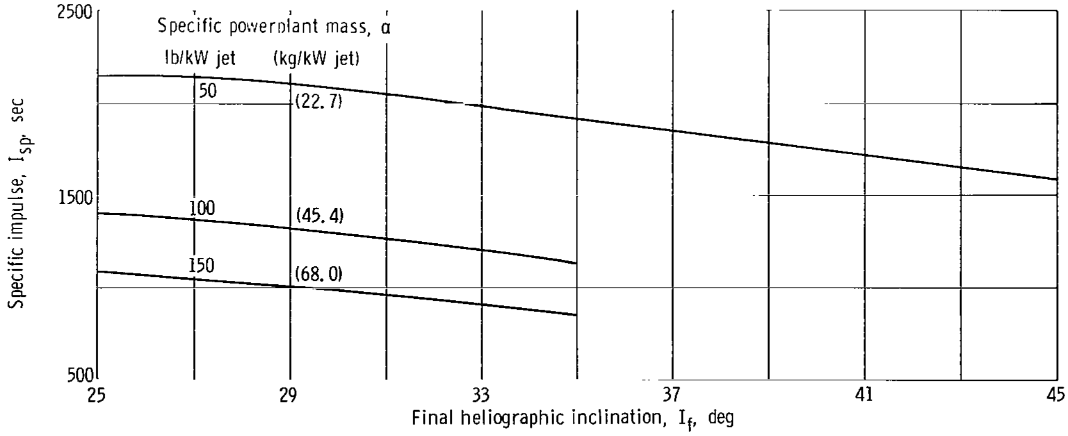


Figure 7. - Optimum electric propulsion system specific impulse for both Atlas-Centaur and Atlas-Agena combined systems. (Specific powerplant masses of 50 and 100 lb/kW jet (22.7 and 45.4 kg/kW jet) examined for Atlas-Agena combined system.)

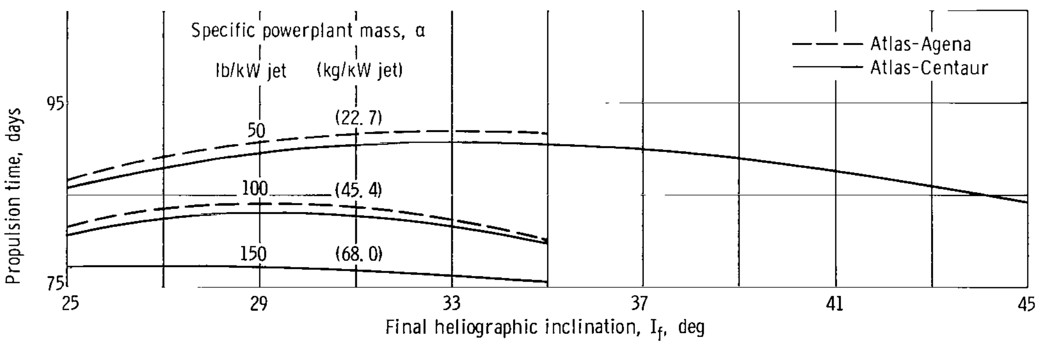


Figure 8. - Optimum electric propulsion time for both Atlas-Centaur and Atlas-Agena combined systems.

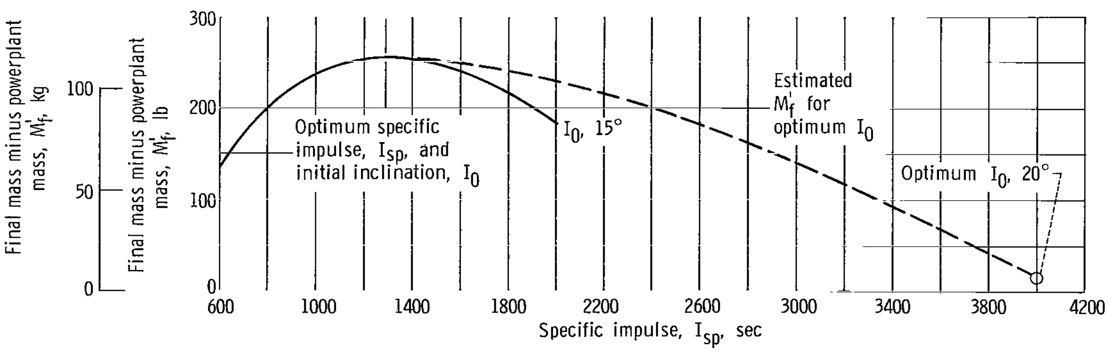


Figure 9. - Effect of variation of specific impulse from optimal value. Atlas-Centaur booster; specific powerplant mass, 100 pounds per kilowatt jet (45.4 kg/kW jet); final inclination, 30°.

tonically with increasing specific powerplant mass and is insensitive to final inclination.

Because of the dependence of efficiency on specific impulse typical of present day electric engines, it might be best to select a specific impulse different from the optimum value for a constant value of specific powerplant mass to improve engine efficiency. When the specific impulse is so selected, the specific powerplant mass (which is equal to the specific mass of the electric power source divided by the engine efficiency) is reduced and the value of M_f' is increased.

Figure 9 illustrates the sensitivity of M_f' to specific impulse. The curves shown are for a mission to a final inclination of 30^o accomplished with an Atlas-Centaur combined system having a specific powerplant mass of 100 pounds per kilowatt jet (45.4 kg/kW jet). For the solid curve, the value of I_0 was held constant at that value which maximized M_f' for the optimum value of specific impulse (i.e., the distribution of propulsive effort between the high-thrust booster and the low-thrust electric propulsion system was the same as that for the optimum specific impulse case). Values of specific impulse were specified and propulsion time was allowed to take on that value which maximized M_f'.

Examination of the solid curve of figure 9 reveals that for this case M_f' varies by approximately 7 percent over a 300-second excursion from the optimum specific impulse. Within this range, the variation could be reduced slightly if I_0 were optimized for each specified value of specific impulse.

The distribution of propulsive effort between the high-thrust booster and the low-thrust electric propulsion system should be reoptimized if an engine with a high value of specific impulse (e.g., 4000 sec) is used. A complete curve of M_f'/M_e against I_0 was obtained for the case of a specific impulse of 4000 seconds, a final inclination of 30^o, and a specific powerplant mass of 100 pounds per kilowatt jet (45.4 kg/kW jet). This curve along with the Atlas-Centaur payload curve was used to determine the optimum value of I_0 and the associated value of M_f'; the value of M_f' is shown in figure 9. The optimum value of I_0 was 20^o. It can be seen that this large increase in specific impulse above the optimal value results in a reduction in M_f' by a factor of 17. The dashed line in figure 9 is the estimated curve of maximum M_f' against specific impulse for which the value of I_0 has been optimized. The increase in M_f' resulting from a reduction in specific powerplant mass must be balanced against the displayed decrease in M_f' to obtain the optimum value of I_{sp}.

Variations of Basic Mission Profile

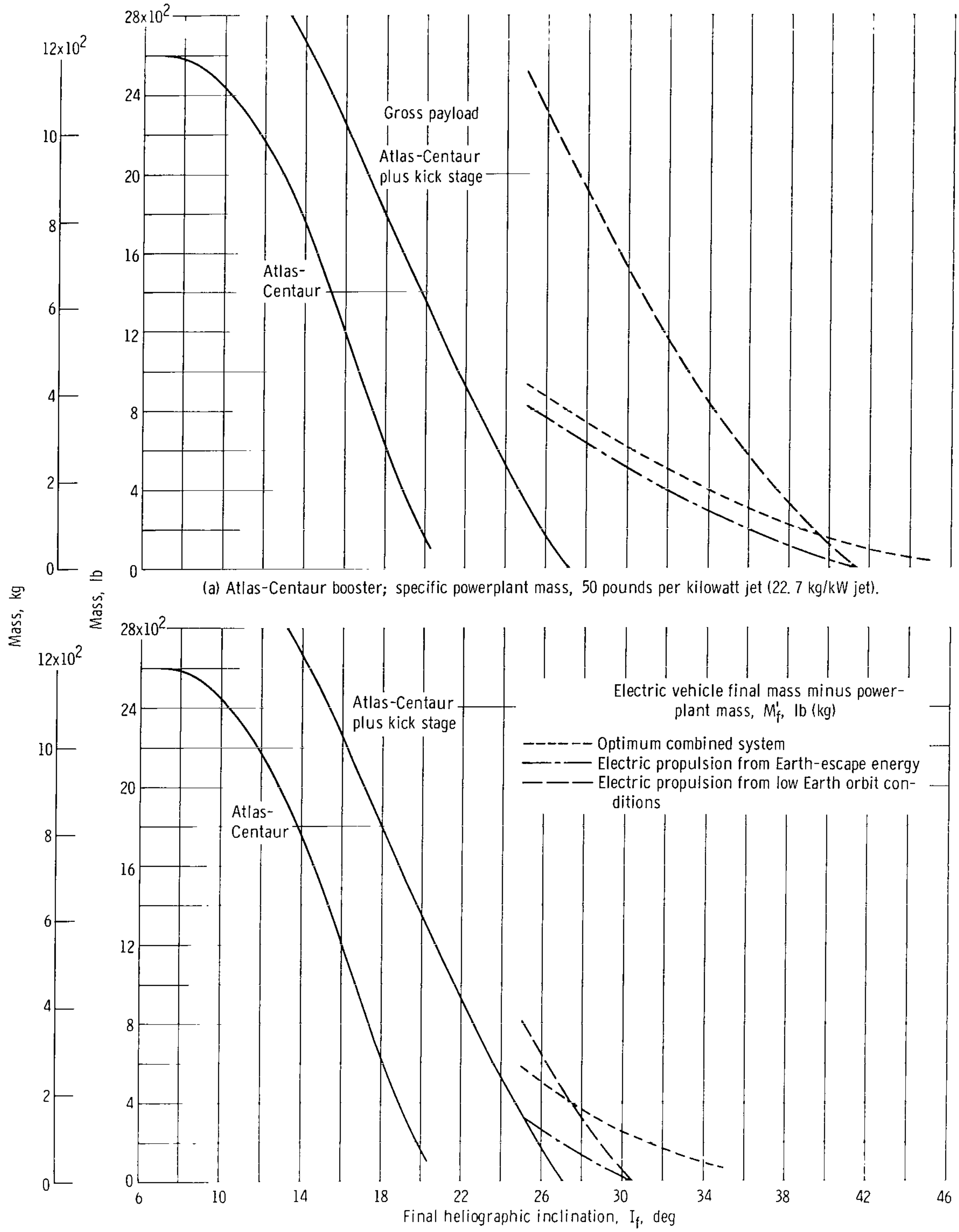
This section examines the effect on the final mass minus the powerplant mass caused by variations from the basic mission profile described in the ANALYSIS section.

Effect of boost to exact escape energy. - Previous investigators have considered the case in which the electric propulsion system begins operation after the vehicle has been brought to Earth-escape energy by a booster. Comparison of these results with the results of this study indicates the advantage of supplying an initial hyperbolic velocity to the vehicle. The advantage for a particular case can be seen in figure 4. The value of M_f' that can be delivered to a final inclination of 30° with an Atlas-Centaur combined system having a specific powerplant mass of 100 pounds per kilowatt jet (45.4 kg/kW jet) is plotted as a function of the initial inclination. In this case, a 10 to 1 advantage is derived from supplying an initial hyperbolic velocity.

A comparison between values of M_f' for the optimized combined system and the combined system for which the hyperbolic velocity is zero is made in figure 10. This figure shows that the optimized combined system performance is considerably better than that of the combined system for which the hyperbolic velocity is zero. This is especially true for the higher values of specific powerplant mass. For example, figure 10(c) indicates that the Atlas-Centaur optimized combined system having a specific powerplant mass of 150 pounds per kilowatt jet (68.0 kg/kW jet) is able to deliver about 330 pounds (150 kg) of M_f' to an inclination of 26°. The combined system with a hyperbolic velocity of zero is unable to deliver any amount of M_f' to that inclination.

Effect of low-thrust spiral to escape energy. - Values of M_f' obtained with the electric system for the two boosters considered and for the three values of specific powerplant mass are given in figure 10. The associated values of total propulsion time range from 180 to 250 days. Although the curves of electric system values of M_f' are conservative, they are indicative of the electric system performance. The curve of M_f' passes through zero at the same inclination as the curve of the combined system for which the vehicle was boosted to exactly escape energy. The important point to be observed is that at this inclination the optimized combined system is still able to deliver values of M_f'. Consider, for example, the case of the Atlas-Centaur booster with a specific powerplant mass of 100 pounds per kilowatt jet (45.4 kg/kW jet) shown in figure 10(b). The electric system curve of M_f' passes through zero at an inclination of about $30\frac{1}{2}^{\circ}$. The optimized combined system is able to deliver about 230 pounds of M_f' to that inclination. At the final inclination of $27\frac{1}{2}^{\circ}$ at which the curves cross, both systems can deliver the same amount of M_f'. However, the hardware mass of the electric system will be much larger than that of the combined system. Even for a specific powerplant mass of 50 pounds per kilowatt jet (22.7 kg/kW jet) (fig. 10(a)) the combined system performance would probably exceed that of the electric system throughout most of the range of inclinations considered because of the high value of hardware mass of the electric system.

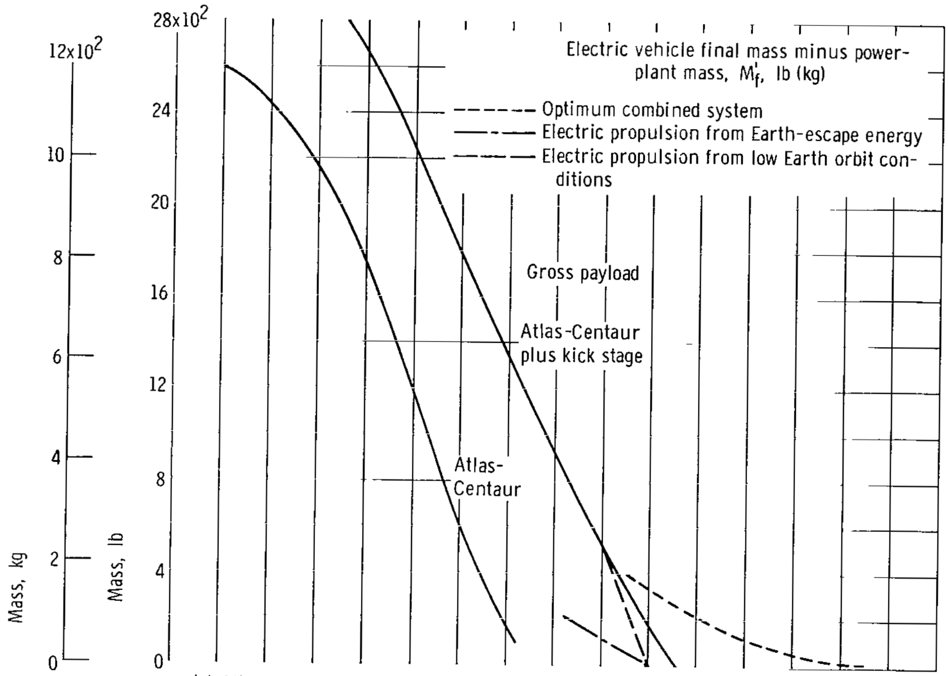
Effect of second propulsion period. - The low-thrust engine can be turned on for a second propulsion period after the vehicle has coasted through a local maximum latitude point. As the vehicle passes through a region of maximum latitude, thrust is ineffective in increasing inclination because the argument of latitude u is nearly 90° (eq. (9)). However, after the vehicle has passed through this region, the inclination can be effectively increased if the direction of the thrust acceleration vector is reversed. Thrust



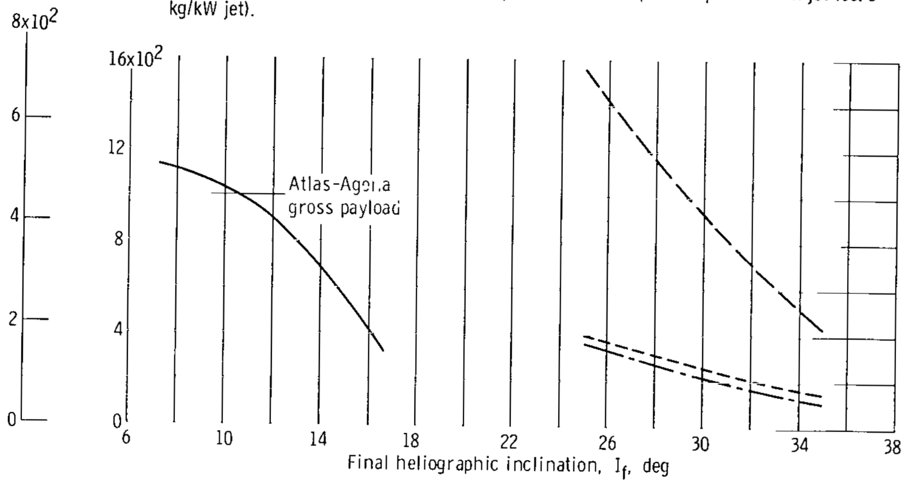
(a) Atlas-Centaur booster; specific powerplant mass, 50 pounds per kilowatt jet (22.7 kg/kW jet).

(b) Atlas-Centaur booster; specific powerplant mass, 100 pounds per kilowatt jet (45.4 kg/kW jet).

Figure 10. - Effect of varying starting point of electric stage

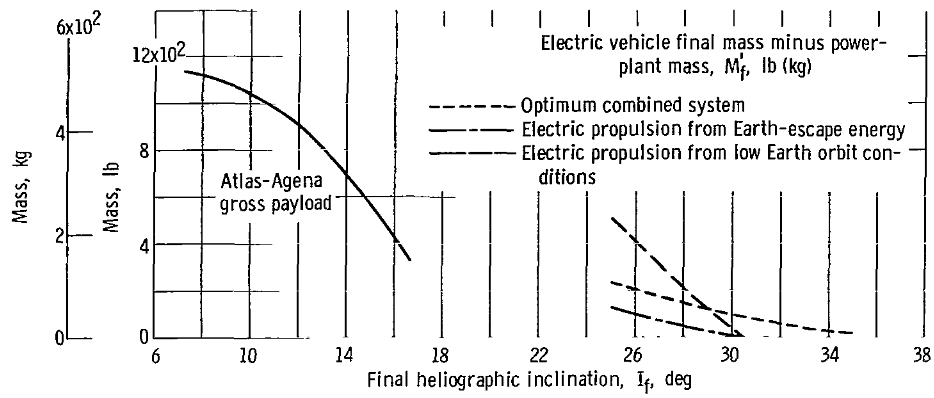


(c) Atlas-Centaur booster; specific powerplant mass, 150 pounds per kilowatt jet (68.0 kg/kW jet).



(d) Atlas-Agena booster; specific powerplant mass, 50 pounds per kilowatt jet (22.7 kg/kW jet).

Figure 10. - Continued.



(e) Atlas-Agena booster; 100 pounds per kilowatt jet (45. 4 kg/kW jet).

Figure 10. - Concluded.

can be applied until u becomes nearly 270°, at which point thrusting again becomes ineffective. This particular thrusting schedule is referred to as the power-coast-power case.

A curve of M_f'/M_e against initial inclination was obtained for the case of a final inclination of 30° and a specific powerplant mass of 100 pounds per kilowatt jet (45.4 kg/kW jet) using the power-coast-power thrusting schedule. The curve was used along with the payload curve of the Atlas-Centaur booster to obtain the optimum value of initial inclination and the associated values of M_e, M_f, and M_f'. Table I compares these quantities and other pertinent parameters with those of the single-propulsion-period case.

The increase in M_f' obtained by utilizing a power-coast-power schedule rather than a single propulsion period is considerable, 710 pounds (322 kg) as compared with 250 pounds (113 kg). The optimum specific impulse is higher for the power-coast-power case because of the longer allowable propulsion time.

If the powerplant is used to operate the instrumentation and communication systems after propulsion is finally terminated, data would not be received from the power-coast-power vehicle until about 270 days have elapsed after the vehicle has entered heliocentric space. However, the onboard power could be used to operate the instrumentation and communication systems during the coast period, and data would be received during this time interval. Because of these periods during which data are received, an increase in allowable propulsion time would be better utilized by using a power-coast-power thrusting schedule rather than employing a spiral trajectory to escape the Earth. The total propulsion time for the power-coast-power example of table I is 241.8 days. The total propulsion time to the same final inclination for the single-propulsion-period case employing a low-thrust escape spiral is 223.9 days. The power-coast-power profile appears attractive for the longer mission times.

Effect of allowing radius to change. - If the restrictions on the direction of the hyperbolic velocity in heliocentric space and on the direction of the low-thrust acceleration vec-

TABLE I. - COMPARISON OF THRUSTING SCHEDULES

[Atlas-Centaur booster; final inclination, I_f, 30°; specific powerplant mass, α, 100 pounds per kilowatt jet (45.4 kg/kW jet).]

	Single propulsion period	Power-coast-power schedule
Electric vehicle final mass minus powerplant mass, M_f', lb; kg	250; 113	710; 322
Electric vehicle final mass, M_f, lb; kg	660; 299	1250; 567
Electric vehicle mass (payload mass of chemical stage), M_e, lb; kg	1460; 662	2150; 975
Initial inclination, I_o, deg	15	12
Electric propulsion system specific impulse, I_{sp}, sec	1290	2340
Low-thrust propulsion time, days	83.1	241.8
Coast time between propulsion periods, days	----	26.8
Total time in heliocentric space before final inclination is reached, days	83.1	268.6

tor were relaxed, a thrusting program could be employed which would improve the performance of the combined-propulsion-system vehicle. An optimum hyperbolic velocity (optimum in both magnitude and direction) could be supplied to the vehicle, after which an optimum low-thrust steering program could be employed to guide the vehicle in heliocentric space. The orbit of the vehicle would not be circular at every instant of time. The eccentricity of the final orbit would, in general, be greater than zero, and the orbital period would, in general, be different from 1 year.

The disadvantages associated with this thrusting program are as follows:

(1) The guidance requirement upon the spacecraft would be more severe than in the constant radius case because the low-thrust steering program would no longer be simple.

(2) If solar cells were used as the power source, provision would have to be made for the variation of available power with distance from the Sun.

(3) Because the period of the final orbit of the vehicle would, in general, be different from that of the Earth, for some intervals during the useful life of the probe, the communication distance could exceed the maximum value for that of a final circular orbit of the same inclination.

The problem of obtaining values of maximum M_f' for the case in which the radius is not constant is considerably more difficult than for the case in which the radius is constant. An additional set of trajectory equations must be integrated to obtain the optimal steering program; a search must be made to determine the initial direction and the change of direction of the low thrust vector; and both the magnitude and the direction of the hyperbolic velocity must be optimized. The hyperbolic velocity was set equal to zero in order to obtain a simple comparison between the constant radius case and the nonconstant radius case. This procedure eliminated the magnitude and the direction of the hyperbolic velocity as variables to be optimized.

In the optimally steered case, no optimum propulsion time exists even if the trajectories are restricted to the class for which propulsion is terminated before the argument of latitude reaches 90^{o}. The more propulsion time allowed, the higher the value of M_f' that can be delivered to a specified inclination, or the higher the final inclination attained with a specified value of M_f'. In the comparison which follows, the propulsion time for the nonconstant radius case was specified to range from 100 to 200 days. The specific powerplant mass was assumed to be 100 pounds per kilowatt jet (45.4 kg/kW jet) and the ratio M_f'/M_e was set at 0.0104. The maximum value of final inclination was then determined over the range of propulsion time. A value of 0.0104 for M_f'/M_e is the maximum value for a mission to a final inclination of 30^{o} from an initial inclination of 7.2^{o} with a constant radius steering program.

The effect of optimal steering for this case is shown in figure 11. A propulsion time

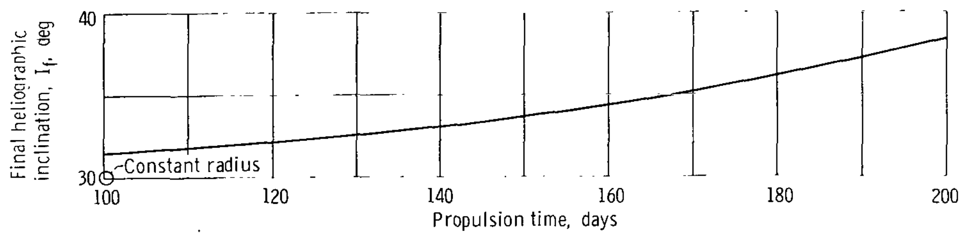


Figure 11. - Effect of optimal steering on final inclination. Ratio of electric vehicle final mass minus powerplant mass to electric vehicle mass, 0.0104; initial inclination, 7.2°; specific powerplant mass, 100 pounds per kilowatt jet (45.4 kg/kW jet)

of 100.6 days is the optimum value for the constant radius case. In this propulsion time, the optimally steered vehicle can attain an inclination of 31.6^{o}. If more propulsion time is allowed, a vehicle with the same amount of M_f' can attain an even higher inclination with an associated increase in optimum specific impulse. The advantage of optimally steering the low-thrust vehicle appears to be significant for the longer propulsion times, but the disadvantages associated with this thrusting schedule might outweight the advantages. The tradeoff between the advantages and disadvantages for the nonconstant radius case is beyond the scope of this report.

If longer propulsion time is permitted, a power-coast-power thrusting schedule would also yield better performance than the single-propulsion-period, constant-radius case but without the disadvantages associated with the optimally steered case. The power-coast-power schedule allows propulsion until the argument of latitude reaches 270^{o}. If optimal steering were used with the power-coast-power schedule, the performance would, of course, be better than that of the constant-radius, power-coast-power case but would have the disadvantages associated with a nonconstant radius.

Sample Application

Suppose that it is desired to send a probe into an orbit inclined an angle of 27^{o} with the Sun's equatorial plane by means of an Atlas-Centaur combined system. And suppose the specific powerplant mass of the electric power source (including power conditioning) is equal to 50 pounds per kilowatt electric (22.7 kg/kW electric). If the engine were 100 percent efficient, the optimum specific impulse would be 2150 seconds (fig. 7). Electric thrustors are presently unable to operate at efficiencies above approximately 45 percent at that specific impulse (ref. 12). If an engine efficiency of 50 percent were postulated, the specific powerplant mass would be 100 pounds per kilowatt jet (45.4 kg/kW jet) and the optimum specific impulse would be 1350 seconds (fig. 7). An efficiency of 50 percent might be attainable at this specific impulse with an electrothermal thrustor. In the following discussion, it is assumed that this is the case.

The mass of the various parts of the electric vehicle can be obtained from figure 5(b). The total mass of the electric vehicle for an inclination of 27° is 1720 pounds (780 kg); the final mass is 900 pounds (408 kg); and the final mass minus the powerplant mass is 440 pounds (200 kg). The powerplant mass is equal to 460 pounds (209 kg). The power required is equal to the powerplant mass divided by the specific powerplant mass of the electric power source 460/50, or 9.2 kilowatts. If the hardware fraction of the electric propulsion system is 10 percent of the mass of the powerplant and the propellant (1280 lb (581 kg)), the hardware mass is 128 pounds (58.1 kg). For an assumed payload power requirement of 500 watts, the mass of the powerplant required by the payload systems is 25 pounds. The gross payload of the probe is then equal to 440 - 128 + 25, or 337 pounds (153 kg). For comparison, the gross payload of the follow-on Pioneer probes is 138 pounds (62.6 kg). This particular vehicle was designed to study particles and fields in the ecliptic plane between 0.8 and 1.2 astronomical units.

The assumed value of 50 pounds per kilowatt electric (22.7 kg/kW electric) may be optimistic, but it is reported that a solar-cell array with this specific mass can be fabricated (refs. 13 and 14). (The specific mass of the power conditioning system must be added to this value to obtain the specific powerplant mass α_{el}.) The assumed value of a 10 percent hardware fraction may be low for this type of propulsion system. If a value of 20 percent is assumed, the value of M_f' will be reduced to 209 pounds (94.8 kg). If, for a given set of electric-vehicle assumptions, the combined system is unable to deliver the required payload, improved performance can be obtained by utilizing a power-coast-power trajectory profile at the cost of longer propulsion time.

The initial acceleration of the electric vehicle for the example given can be determined by use of equation (3):

$$\frac{M_p}{M_e} = \frac{a_0 t_P}{I_{sp} g} \tag{3}$$

The propellant mass is equal to the initial mass (1720 lb, or 780kg) minus the final mass (900 lb, or 408 kg) or 820 pounds (372 kg). The propellant fraction is 820/1720, or 0.477. As shown in figure 8, the propulsion time is about 82 days. Substituting these quantities into the preceding equation, along with the value of 1350 seconds for specific impulse, results in an a_0 of 0.89×10^{-3} meters per second squared. The initial thrust-to-weight ratio is equal to a_0/g or 0.91×10^{-4}. The value of thrust is equal to the thrust-to-weight ratio multiplied by the weight of the vehicle (1720 lb, or 7650 N), or 0.156 pound (0.694 N).

The preceeding example makes use of an approximate procedure for determining the electric propulsion system specific impulse and the associated value of M_f'. If the actual curve of efficiency against specific impulse for the electrothermal engine selected were given, a further optimization could be conducted. With this type of thrustor, a decrease in operating specific impulse produces an increase in engine efficiency. This increase, in turn, produces a decrease in α_{el}/η which tends to increase the value of M_f'. The optimum specific impulse for the new value of α_{el}/η increases. The difference between the new value of optimum specific impulse and the reduced value tends to decrease the value of M_f' because of operation at an off-optimum value of specific impulse. These two effects must be balanced to obtain the value of specific impulse which would maximize the value of M_f'. The magnitude of the increase is dependent on the slope of the engine efficiency characteristic.

SUMMARY OF RESULTS

Values of final mass minus powerplant mass were obtained for missions to inclinations in the range from 25° to 45° with a combined propulsion system. The combined system consisted of a chemical booster and a low-thrust electric propulsion system. The boosters considered were the Atlas-Agena and the Atlas-Centaur. The electric propulsion system specific powerplant masses studied were 50, 100, and 150 pounds per kilowatt jet (22.7, 45.4, and 68.0 kg/kW jet). The basic mission profile was one in which the distance from the Sun to the vehicle remained constant at a value of 1 astronomical unit, and the thrust was terminated before the argument of latitude reached 90° in order to limit propulsion time.

The significant conclusions drawn from this analysis are the following:

1. If high final inclinations (i.e., above 25°) are required for scientific purposes, neither the Atlas-Agena nor Atlas-Centaur booster can deliver any useful payload. The Atlas-Centaur plus the proposed 8800-pound (3990 kg) hydrogen-oxygen kick stage is unable to deliver any payload to an inclination above 27°. The combined system is able to deliver payloads to the higher inclinations if the specific powerplant mass and the

hardware fraction are not too large. For example, the Atlas-Centaur combined system with a specific powerplant mass of 100 pounds per kilowatt jet (45.4 kg/kW jet) and a hardware fraction of 10 percent can deliver 337 pounds (153 kg) of gross payload to an inclination of 27°. The payload includes 25 pounds (11.3 kg) of solar arrays and power conditioning equipment.

2. The optimum values of specific impulse for the basic mission profile decrease with increasing specific powerplant mass and with increasing final inclination. The values are less than 2200 seconds and lie within the range for which electrothermal thrustors presently have the highest efficiency. The optimum propulsion times of 75 to 95 days do not change much with either specific powerplant mass or final inclination. Performance of the mission at slightly off-optimum values of specific impulse does not change the value of electric vehicle final mass minus powerplant mass M_f' significantly.

3. The payload of the optimized combined system is considerably greater than that of the combined system for which the booster provided exactly Earth-escape energy, especially for the higher values of specific powerplant mass. However, the payload of the electric system (system for which the boost from low Earth orbit conditions has been replaced by a low-thrust spiral) is higher than the optimized combined system under some conditions. If the desired final value of inclination is relatively low and the specific powerplant mass and hardware fraction are also low, the electric system is superior.

4. The value of final mass minus powerplant mass delivered to a particular inclination is significantly increased by the introduction of a second propulsion period (power-coast-power schedule) after the vehicle has coasted through a local maximum in latitude. The optimum value of specific impulse is increased because of the longer propulsion time. The disadvantage associated with this case is that data would not be received from the region of maximum latitude until a much later time than in the single-propulsion-period case, in addition to the more stringent demands on thrustor durability.

5. A particular amount of final mass minus powerplant mass can be delivered to a higher inclination if the low-thrust vehicle is optimally steered rather than steered so that the radius is constant. The advantage of optimal steering becomes greater as the allowable propulsion time is increased above the optimum time for the constant radius case. Associated with this advantage is the disadvantage of a final elliptical orbit. A power-coast-power thrusting schedule is more attractive than an optimal steering program because the final coasting orbit in the power-coast-power case would be circular.

In summary, the actual payload can be considerably less than the reported values of final mass minus powerplant mass. Nevertheless, the capability of existing small boosters to deliver payloads to high inclinations out of the ecliptic may be significantly enhanced by the addition of a small electric propulsion system having a solar-cell power source. Whether the scientific value of such missions warrants the development of the electric system is beyond the scope of this study. (Also not considered, but attractive because of the rather limited payloads found herein, is the alternative of using larger boosters.) If the basic mission profile is utilized, the vehicle design would be simple, and propulsion time would be less than 95 days. For a longer permissible propulsion time, the power-coast-power profile appears attractive. The requirement of low specific powerplant mass and hardware fraction is not as strict for this profile.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 15, 1967,
120-26-07-01-22.

APPENDIX - SYMBOLS

a	electric propulsion system thrust acceleration, m/sec^2
a_0	initial electric propulsion system thrust acceleration in heliocentric space, m/sec^2
a_{00}	initial electric propulsion system thrust acceleration when vehicle is at low Earth orbit conditions, m/sec^2
g	acceleration due to gravity, 9.80665 m/sec^2
I	inclination relative to Sun's equatorial plane, deg (rad in eq. (9))
ΔI	change in inclination produced by hyperbolic velocity relative to Earth, deg
I_f	final inclination, deg
I_{sp}	electric propulsion system specific impulse, sec
I_0	initial inclination, deg
K_1	hardware fraction, $M_{t+st+en}/M_e$
K_2	portion of powerplant required by payload systems
M_e	electric vehicle mass (also payload mass of chemical booster), lb (kg)
M_f	electric vehicle final mass, lb (kg)
M_f'	electric vehicle final mass minus powerplant mass, lb (kg)
M_l	electric vehicle gross payload mass, lb (kg)
M_o	mass at low Earth orbit conditions, lb (kg)
M_p	electric propulsion system propellant mass, lb (kg)
$M_{t+st+en}$	mass of electric propulsion system tanks, structure, and engines, lb (kg)
M_w	electric propulsion system powerplant mass (including power conditioning mass), lb (kg)
R	distance from Sun to vehicle, m
t_p	low-thrust propulsion time, sec
u	argument of latitude, rad
ΔV	change in velocity (applied beyond a particular set of Earth orbit conditions), m/sec
V_c	circular orbit velocity, m/sec

V_E velocity of Earth about Sun, m/sec

V_h hyperbolic velocity relative to Earth, m/sec

V_v velocity of vehicle about Sun, m/sec

α specific powerplant mass, lb/kW jet (kg/kW jet)

α_{el} specific powerplant mass of electric power source, lb/kW electric (kg/kW electric)

γ empirical function of a_{00} to compute time and spiral propellant fraction given in ref. 11

η electric engine efficiency

Ω longitude of ascending node, rad

ω angular velocity of Earth about Sun, rad/sec

Subscript:

s spiral

Superscript:

$(\dot{\ })$ derivative with respect to time, d/dt

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