NASA TECHNICAL NOTE

NASA TN D-4484

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NASA TN D-4484

APPROXIMATE RELATIONS FOR LAMINAR HEAT-TRANSFER AND SHEAR-STRESS FUNCTIONS IN EQUILIBRIUM DISSOCIATED AIR

by Ernest V. Zoby

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Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1968

ERRATA

NASA Technical Note D-4484

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April 1968

Completed 17 Sep 69 SEU

Page 7: Under the statement "The ranges of boundary conditions for equations (13) and (14) were," the third equation should be $0.00603 \leqq \zeta_w \leqq 0.313$ instead of $0.00603 \leqq \zeta_w \leqq 0.1$.

Page 8, second line: Delete the phrase "without reference to the stagnation point."

Pages 11 and 12: In equations (B1) and (B4) of appendix B, the enthalpy difference should be $(H_e - h_w)$ instead of $(H_{aw} - h_w)$.

Since there were no exact solutions available for wall enthalpies near the adiabatic-wall value for the blunt axisymmetric cases, the use of equations (16) and (B4) should be restricted to $\zeta_w \leqq 0.5$ when t_e is not close to 1.

Issued 10-24-68

NASA-Langley, 1968

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APPROXIMATE RELATIONS FOR LAMINAR HEAT-TRANSFER AND SHEAR-STRESS FUNCTIONS IN EQUILIBRIUM DISSOCIATED AIR

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SUMMARY

Simple, approximate equations have been developed for computing the normal derivatives of enthalpy and velocity evaluated at the surface of a flat plate or cone and of a blunt axisymmetric body. These approximate equations were developed by correlating exact similar solutions to the laminar boundary-layer equations for equilibrium dissociated air. The results of these approximate equations represent the exact solutions within ±10 percent.

INTRODUCTION

Local laminar heat-transfer and shear-stress values are functions of the normal derivatives of the enthalpy (temperature) and velocity profiles, respectively, evaluated at the body surface. These normal derivatives are functions of the boundary conditions and the boundary-layer chemistry, and their computation can be very involved.

For a perfect gas these normal derivatives have been computed by several techniques, and the results of these computations have been presented for wide ranges of boundary conditions and fluid-property variations in publications such as references 1 to 9. Exact and approximate analyses have been presented in references 10, 11, and 12 for reacting boundary layers. However, solutions for the normal derivatives by these methods are time consuming, and extensive parametric studies such as have been made for the perfect gas are not available. Also, some of the approximations are tedious to evaluate and usually result in a loss of accuracy or are not generally applicable to a wide range of flow conditions or both. Because of the existing problems involved with the computation of the derivatives (and thereby the heat-transfer or shear-stress values), simple, accurate methods are desirable. This paper presents approximate methods which have been obtained by correlating similar solutions of the boundary-layer equations for equilibrium dissociated air. The results of these equations represent the exact solutions within ±10 percent. The development of the methods is given in appendix A. In appendix B these methods are related to expressions for computing the heat transfer and shear stress on flat plates, cones, and blunt axisymmetric bodies.

SYMBOLS

c_f	skin-friction coefficient
f	similar stream function
H	total enthalpy
h	static enthalpy
h_E	reference enthalpy $\left(1.97 \times 10^4 \frac{\text{joules}}{\text{gram}}\right)$
h^*	Eckert's reference enthalpy
K	constant used in equation (12) to compute $\zeta'_{w,s}$
m	exponent used in equation (12) to compute $\zeta'_{w,s}$
N_{Pr}	Prandtl number
n	shape parameter in equations (3) and (4)
p	pressure
$\dot{q}$	heating rate
R_{eff}	effective nose radius
r	radius of body of revolution
t	static-enthalpy ratio, $\frac{h}{H_e}$
u	velocity component along x-axis
x,y	boundary-layer coordinates in physical system
β	pressure-gradient parameter
ζ	total-enthalpy ratio, $\frac{H}{H_e}$

μ viscosity

ξ,η similarity coordinates

ρ density

τ shear stress

Subscripts:

aw adiabatic wall

E evaluated at reference enthalpy h_E and local pressure

e local conditions external to boundary layer

s stagnation condition

w wall conditions

Superscripts:

* evaluated at reference enthalpy h^*

' first derivative with respect to η

'' second derivative with respect to η

ANALYSIS AND DISCUSSION

Theory

The governing boundary-layer equations which constitute a system of nonlinear, partial differential equations can be reduced to ordinary differential equations by a transformation of the coordinate system and the assumption of local similarity. (Even if similarity is not assumed, the system of equations is related to a transformed coordinate system.) Because of this approach for solving the boundary-layer equations, the convective heating rate and the aerodynamic shear stress at the wall are given by

$$-\dot{q}_w = \left(\frac{\mu_w}{N_{Pr,w}}\right)\left(\frac{\partial h}{\partial \eta}\right)_w\left(\frac{\partial \eta}{\partial y}\right)_w \tag{1}$$

and

$$\tau_w = \mu_w\left(\frac{\partial u}{\partial \eta}\right)_w\left(\frac{\partial \eta}{\partial y}\right)_w \tag{2}$$

where η is the transformed y coordinate.

With the aid of the Howarth and Mangler transformations

$$\xi = \int_0^x \rho_w \mu_w u_e r^{2n} dx \tag{3}$$

and

$$\eta = \frac{u_e r^n}{\sqrt{2\xi}} \int_0^y \rho \, dy \tag{4}$$

equations (1) and (2) can be written for a flat plate (n = 0) as

$$-\dot{q}_w = \frac{H_e}{\sqrt{2}\, N_{Pr,w}}\left(\frac{\rho_w \mu_w u_e}{x}\right)^{0.5} \zeta'_w \tag{5}$$

and

$$\tau_w = \frac{u_e}{\sqrt{2}}\left(\frac{\rho_w \mu_w u_e}{x}\right)^{0.5} f''_w \tag{6}$$

and for a blunt axisymmetric body (n = 1) as

$$-\dot{q}_w = \frac{H_e \rho_w \mu_w u_e r}{N_{Pr,w}\sqrt{2\xi}} \zeta'_w \tag{7}$$

and

$$\tau_w = \frac{u_e^2 \rho_w \mu_w r}{\sqrt{2\xi}} f''_w \tag{8}$$

where

$$\zeta'_w = \left[\frac{\partial\left(\frac{H}{H_e}\right)}{\partial \eta}\right]_w \tag{9}$$

and

$$f''_w = \left[\frac{\partial\left(\frac{u}{u_e}\right)}{\partial \eta}\right]_w \tag{10}$$

The heat-transfer and shear-stress values on cones can be computed by using the Mangler transformation with equations (5) and (6), respectively.

In this investigation, the thermodynamic and transport properties of equilibrium dissociated air are assumed to be known, and the problem is to evaluate the normal derivatives in equations (5) to (8). The exact method (ref. 10) of determining these derivatives is a solution of the equations for the compressible laminar boundary layer on a high-speed digital computer. This approach is the only good method presently available for the computation of f''_w on a blunt axisymmetric body. However, since the complete solution of the boundary-layer equations is not always desirable and can be time consuming, approximate methods (refs. 11 and 12) which are acceptable for engineering applications have been developed.

Approximate Methods

For flat plates, the normal velocity derivative is approximated in references 11 and 13 as

$$f''_w = 0.47\left(\frac{\rho^*\mu^*}{\rho_w\mu_w}\right)^{0.5} \tag{11}$$

where Eckert's reference-enthalpy method is used to evaluate the reference conditions. Equation (11) is used with equation (6) to compute the shear-stress or skin-friction coefficient c_f; the skin-friction coefficient is then related to the heat-transfer rate through a modified form of Reynolds analogy. Therefore, this procedure does not require a direct approximation for the normal enthalpy derivative to compute heat-transfer rates. In reference 11, this approach for computing the heat-transfer and shear-stress values is shown to compare very well with the exact solutions.

For blunt axisymmetric bodies, equations of the form

$$\zeta'_{w,s} = K\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^m\left(1 - \zeta_w\right) \tag{12}$$

for approximating the normal enthalpy derivative at the stagnation point are given in references 11 and 14. The results from these references were obtained by correlating exact solutions of the stagnation-point boundary-layer equations and can be used with equation (7) to compute the stagnation-point heat-transfer rate. For calculations away from the stagnation point, equation (61) in reference 11 and equation (28) in reference 12 are approximations for $\zeta'_w/\zeta'_{w,s}$ $\left(\text{shown as } \dfrac{g_{nw}/(1 - g_w)}{g_{nws}/(1 - g_{ws})} \text{ in ref. 12}\right)$ that can be used with an equation similar to equation (23) in reference 11 to compute $\dot{q}_w/\dot{q}_{w,s}$. Although equation (61) in reference 11 is more general than that given in reference 12, it is difficult to evaluate and represents exact similar solutions within only 15 percent. The relation given in reference 12 is based on high wall cooling conditions and constant Prandtl number and explicitly neglects dissipation effects. In addition, neither relation allows for the direct evaluation of ζ'_w and, thereby, of $\dot{q}_w$. No accurate, practical methods are known for the approximation of f''_w and, thereby, of τ_w on a blunt axisymmetric body.

From the preceding discussion, the desirability for simple, accurate methods for evaluating the normal derivatives of enthalpy and velocity in a reacting boundary layer is evident. These methods have been obtained by correlating the exact similar solutions given in reference 11 for equilibrium dissociated air with a unit Lewis number. In reference 11, a nonunit Lewis number for equilibrium dissociated air is shown to have a negligible effect on the shear stress and heat transfer. The relations developed (as given in appendix A) for the present methods are as follows:

For a flat plate or cone

$$f''_w = 0.47\left(\frac{\rho^*\mu^*}{\rho_w\mu_w}\right)^{0.475} \tag{13}$$

and

$$\zeta'_w = 0.47\left(\frac{\rho^*\mu^*}{\rho_w\mu_w}\right)^{0.475}\left[1 - \left(1 - N_{Pr,w}\right)\left(1 - t_e\right)\right]\left(1 - \zeta_w\right) \tag{14}$$

For a blunt axisymmetric body

$$f''_w = 0.47\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.475}\left(1 + \sqrt{\beta t_e}\right)e^{\beta\zeta_w/6} \tag{15}$$

and

$$\zeta'_w = 0.47\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.475}\left(1 + 0.1\sqrt{\beta t_e}\right)\left[1 - \left(1 - N_{Pr,w}\right)\left(1 - t_e\right)\right]\left(1 - \zeta_w\right) \tag{16}$$

where β for a body of revolution at an angle of attack of 0^o is obtained from reference 11 as

$$\beta = \frac{2\left(\frac{du}{dx}\right)_e}{u_e^2 t_e \rho_w \mu_w r^2}\int_0^x \rho_w \mu_w u_e r^2 dx \tag{17}$$

at the stagnation point $\beta_s \equiv 1/2$ and $t_e = 1$. The present solutions (eqs. (13) to (16)) as well as the exact solutions of reference 11 are based on the transport-property data given in reference 15.

In table I and figures 1 to 4 results from equations (13) to (16) are compared with a representative number of exact similar solutions from table III of reference 11. (The exact solutions in table III of reference 11 are only for conditions up to and including a fully dissociated boundary layer.) The results of the equations compare within ±7 percent for a majority of the solutions and within ±10 percent for all the solutions. In addition to the simplicity and accuracy of equations (13) to (16), the results with which the equations were correlated cover a wide range of boundary conditions. The ranges of boundary conditions for equations (13) and (14) were

$$0.2505 \leqq \frac{\rho_e \mu_e}{\rho_w \mu_w} \leqq 1.969$$

$$0.00794 \leqq t_e \leqq 0.8$$

and

$$0.00603 \leqq \zeta_w \leqq 0.313$$

The ranges of boundary conditions for equations (15) and (16) were

$$0.1835 \leqq \frac{\rho_e \mu_e}{\rho_w \mu_w} \leqq 0.9367$$

$$0.2 \leqq t_e \leqq 1.0$$

$$0.0076 \leqq \zeta_w \leqq 0.75$$

and

$$0.5 \leqq \beta \leqq 3.5$$

As previously stated, the present relations and the exact solutions of reference 11 are based on the transport-property data in reference 15. Erroneous results would be obtained from these relations (eqs. (13) to (16)) by using transport-property data significantly different from that of reference 15. In addition, results from the present relations obtained by using the transport-property data of reference 15 are expected to be in good agreement with other exact solutions wherein different high-temperature transport data have been used. This good agreement is expected because the authors of reference 16 showed that changing the values of the transport properties in the outer portion of the boundary layer by a factor of 3 has a negligible effect on an exact solution for f''_w and ζ'_w.

With equations (14) and (16) the heating rates on flat plates and blunt axisymmetric bodies, respectively, can be computed directly ~~without reference to the stagnation point~~. Equation (15) provides a simple, accurate method for computing f_w'' and, thereby, the shear stress on a blunt axisymmetric body. The application of equations (13) to (16) to expressions for computing the heat transfer and shear stress on flat plates, cones, and blunt axisymmetric bodies is given in appendix B.

CONCLUDING REMARKS

The convective heating rate and aerodynamic shear stress are functions of the normal derivatives of the enthalpy and velocity, respectively, evaluated at the surface of a body. Simple, approximate equations have been developed for determining these derivatives for a laminar boundary layer in equilibrium dissociated air. These equations were developed by correlating exact similar solutions to the boundary-layer equations over wide ranges of boundary conditions for flat plates or cones and blunt axisymmetric bodies. The results of the approximate equations represent the exact solutions within ±10 percent.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., December 22, 1967,
129-01-03-08-23.

APPENDIX A

DEVELOPMENT OF APPROXIMATE RELATIONS

For a flat plate or cone the incompressible zero-pressure-gradient relation for the velocity and enthalpy derivatives is obtained from reference 17 (p. 487) as

$$f''_w = \frac{\zeta'_w}{1 - \zeta_w} = 0.47 \tag{A1}$$

Compressibility effects were accounted for in the present investigation by evaluating the ratio $\frac{\rho\mu}{\rho_w\mu_w}$ at a reference condition given by Eckert's reference-enthalpy technique. The relation for f''_w then obtained (eq. (13)) was

$$f''_w = 0.47\left(\frac{\rho^*\mu^*}{\rho_w\mu_w}\right)^{0.475}$$

The exponent 0.475 was used rather than 0.5, which was used in references 11 and 13 (p. 136), since it gave a better fit to most of the data. The approximate relation for ζ'_w was found to correlate the exact solutions better if the term $\left[1 - \left(1 - N_{Pr,w}\right)\left(1 - t_e\right)\right]$ was used rather than the factor $N_{Pr,w}{}^{1/3}$ which is given in reference 18 (p. 264). In reference 6, the author shows that $N_{Pr,w}{}^{1/3}$ does not account for variable Prandtl number effects on ζ'_w. The present term, which is part of an expression for computing the zero-pressure gradient ζ'_w in reference 13, allows for the variation in the Prandtl number and the dissipation parameter t_e. The resulting relation for ζ'_w (eq. (14)) was

$$\zeta'_w = 0.47\left(\frac{\rho^*\mu^*}{\rho_w\mu_w}\right)^{0.475}\left[1 - \left(1 - N_{Pr,w}\right)\left(1 - t_e\right)\right]\left(1 - \zeta_w\right)$$

For the blunt axisymmetric body, the effect of pressure gradient on ζ'_w was accounted for by the term $\left(1 + 0.1\sqrt{\beta t_e}\right)$ which is similar to the expression given in reference 12. In addition, the data of reference 11 were correlated better by evaluation of the $\rho\mu$ product at the conditions external to the boundary layer rather than at the reference conditions. The resulting expression for ζ'_w on a blunt body (eq. (16)) was

$$\zeta'_w = 0.47\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.475}\left(1 + 0.1\sqrt{\beta t_e}\right)\left[1 - \left(1 - N_{Pr,w}\right)\left(1 - t_e\right)\right]\left(1 - \zeta_w\right)$$

In reference 3, the pressure-gradient parameter β was shown to have a greater effect on f''_w than on ζ'_w (S' in ref. 3). For the calculation of f''_w, the relation

$$f_w'' = 0.47\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.475}\left(1 + \sqrt{\beta t_e}\right) \tag{A2}$$

was used in an attempt to correlate the exact solutions. However, for increasing values of β at values of $\zeta_w \geqq 0.2$, equation (A2) underpredicted the exact solutions. The exact solutions of reference 11 were normalized with the corresponding results of equation (A2), and the ratio was plotted as a function of the product of $\beta\zeta_w$. The deviation from unity appeared to vary exponentially with increasing values of $\beta\zeta_w$. The exponential $e^{\frac{\beta\zeta_w}{6}}$ was found to fit the deviation. Therefore, the resulting relation for f_w'' on a blunt axisymmetric body (eq. (15)) was

$$f_w'' = 0.47\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.475}\left(1 + \sqrt{\beta t_e}\right)e^{\frac{\beta\zeta_w}{6}}$$

APPENDIX B

APPLICATION OF APPROXIMATE RELATIONS

The approximate equations (13) to (16) can be used with equations (5) and (6) and equations (7) and (8) to compute heat-transfer and shear-stress values on flat plates or sharp cones and blunt axisymmetric bodies, respectively. After proper substitution of the present results in equations (5) to (8), the resulting expressions for the heat transfer and shear stress are as follows:

For a flat plate

$$-\dot{q}_w = \frac{0.332(H_e - h_w)}{N_{Pr,w}}\left(\frac{\rho_w \mu_w u_e}{x}\right)^{0.5}\left(\frac{\rho^* \mu^*}{\rho_w \mu_w}\right)^{0.475}\left[1 - \left(1 - N_{Pr,w}\right)\left(1 - t_e\right)\right] \tag{B1}$$

and

$$\tau_w = 0.332 u_e \left(\frac{\rho_w \mu_w u_e}{x}\right)^{0.5}\left(\frac{\rho^* \mu^*}{\rho_w \mu_w}\right)^{0.475} \tag{B2}$$

For a cone, multiply the right-hand side of equations (B1) and (B2) by the $\sqrt{3}$ (Mangler's transformation for a cone and flat plate).

For a blunt axisymmetric body (at the stagnation point)

$$-\dot{q}_{w,s} = \frac{0.711}{N_{Pr,w,s}}\left(H_e - h_w\right)_s \left(\rho_w \mu_w\right)_s^{0.025}\left(\rho_e \mu_e\right)_s^{0.475}\left(\frac{du}{dx}\right)_s^{0.5} \tag{B3}$$

where

$$r = x$$

$$u_e = x\,\frac{du}{dx}$$

$$\beta_s \equiv \frac{1}{2}$$

$$t_e = 1$$

and, as in reference 19

$$\left(\frac{du}{dx}\right)_s \approx \frac{1}{R_{eff}}\sqrt{\frac{2p_s}{\rho_s}}$$

APPENDIX B

For a blunt body (away from the stagnation point)

$$-\dot{q}_w = \frac{0.47\left(H_{e} - h_w\right)}{N_{Pr,w}}\sqrt{\frac{\rho_w\mu_w}{\beta t_e}\left(\frac{du}{dx}\right)_e}\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.475}\left(1 + 0.1\sqrt{\beta t_e}\right)\left[1 - \left(1 - N_{Pr,w}\right)\left(1 - t_e\right)\right] \tag{B4}$$ *

$$\tau_w = 0.47\sqrt{\frac{\left(\rho_w\mu_w\right)u_e^2}{\beta t_e}\left(\frac{du_e}{dx}\right)}\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.475}\left(1 + \sqrt{\beta t_e}\right)e^{\frac{\beta\zeta_w}{6}} \tag{B5}$$

* see errata sheet

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TABLE I.- COMPARISON OF SOLUTIONS OF CORRELATION EQUATIONS WITH EXACT SIMILAR SOLUTIONS FOR EQUILIBRIUM DISSOCIATED AIR

t_E	t_e	ζ_w	N_{Pr}	$\frac{\rho_e \mu_e}{\rho_w \mu_w}$	β	f''_w Present solutions	f''_w Exact solutions (ref. 11)	ζ'_w Present solutions	ζ'_w Exact solutions (ref. 11)
3.131	0.0476	0.0476	0.709	1.0	0	0.362	0.3683	0.249	0.2609
3.131	.0476	.313	.768	1.969	0	.460	.4695	.246	.2387
.952	.0476	.0145	.708	.6535	0	.298	.3037	.212	.2222
.476	.0476	.0476	.768	1.009	0	.364	.3708	.270	.2815
1.43	.02173	.0715	.699	1.529	0	.388	.3944	.254	.2630
1.086	.02173	.109	.768	1.785	0	.411	.4199	.283	.2924
.4346	.02173	.0435	.768	1.286	0	.361	.3695	.267	.2805
.8125	.01235	.0812	.768	1.969	0	.396	.4054	.280	.2923
.6175	.01235	.00938	.709	.9053	0	.283	.2867	.200	.2092
.5224	.00794	.0522	.768	1.969	0	.372	.3819	.272	.2856
.397	.00794	.00603	.709	.9052	0	.264	.2670	.187	.1954
10.0	1.0	.152	.709	.5122	.5	.591	.5410	.310	.3055
					1.0	.702	.6464	.319	.3160
					1.8	.838	.7718	.329	.3260
					2.2	.898	.8235	.333	.3296
					3.0	1.008	.9135	.340	.3351
					3.5	1.073	.9632	.344	.3377
10.0	1.0	.30	.680	.6419	.5	.666	.6407	.285	.2646
					1.0	.800	.7939	.293	.2753
					1.8	.976	.9797	.302	.2854
					2.2	1.055	1.057	.306	.2890
					3.0	1.028	1.194	.312	.2944
					3.5	1.302	1.271	.316	.2971
10.0	1.0	.75	.735	.9149	.5	.819	.8517	.121	.1141
					1.0	1.021	1.114	.124	.1199
					1.8	1.321	1.436	.128	.1254
					2.2	1.473	1.571	.129	.1272
					3.0	1.791	1.811	.132	.1302
					3.5	2.003	1.946	.134	.1316
10.0	.8	.50	.699	.8468	.5	.739	.7442	.217	.1987
					1.0	.894	.9468	.222	.2074
					1.8	1.110	1.195	.228	.2156
					2.2	1.214	1.300	.231	.2185
					3.0	1.422	1.485	.236	.2229
10.0	.6	.50	.699	.9367	.5	.735	.7411	.211	.1882
					1.0	.879	.9387	.216	.1959
					1.8	1.079	1.182	.221	.2032
					2.2	1.176	1.284	.223	.2058
					3.0	1.370	1.466	.227	.2098
					3.5	1.494	1.568	.229	.2118
10.0	1.0	.20	.690	.5568	.5	.618	.5756	.305	.2909
10.0		.50	.699	.7833	.5	.745	.7481	.224	.2079
2.0		.0304	.709	.2940	.5	.450	.4182	.273	.2625
					1.0	.528	.4971	.280	.2718
					1.8	.621	.5894	.289	.2807
					2.2	.660	.6269	.292	.2839
					3.0	.729	.6916	.299	.2888
					3.5	.768	.7269	.302	.2912

TABLE I.- COMPARISON OF SOLUTIONS OF CORRELATION EQUATIONS WITH EXACT SIMILAR SOLUTIONS FOR EQUILIBRIUM DISSOCIATED AIR – Continued

t_E	t_e	ζ_w	N_{Pr}	$\frac{\rho_e\mu_e}{\rho_w\mu_w}$	β	f''_w Present solutions	f''_w Exact solutions (ref. 11)	ζ'_w Present solutions	ζ'_w Exact solutions (ref. 11)
2.0	1.0	0.20	0.768	0.5790	0.5	0.629	0.6205	0.310	0.3155
					1.0	.750	.7705	.319	.3285
					1.8	.901	.9526	.329	.3410
					2.2	.969	1.029	.333	.3454
					3.0	1.095	1.162	.340	.3521
					3.5	1.170	1.237	.344	.3555
2.0	1.0	.05	.681	.3448	.5	.486	.4569	.288	.2656
2.0	1.0	.075	.685	.4014	.5	.523	.4978	.302	.2783
1.0	1.0	.0152	.709	.2322	.5	.402	.3752	.248	.2399
					1.0	.471	.4470	.254	.2485
					1.8	.552	.5314	.262	.2567
					2.2	.586	.5657	.266	.2596
					3.0	.647	.6248	.271	.2641
1.0	1.0	.10	.768	.4571	.5	.558	.5422	.312	.3257
					1.0	.659	.6633	.321	.3383
					1.8	.782	.8092	.331	.3504
					2.2	.835	.8698	.335	.3547
					3.0	.931	.9758	.342	.3613
					3.5	.986	1.035	.346	.3646
1.0	.8	.0152	.709	.2505	0	.268	.2703	.248	.2206
					.5	.398	.3733	.240	.2365
					1.0	.462	.4415	.246	.2449
					1.8	.538	.5218	.253	.2531
					2.2	.570	.5544	.256	.2560
					3.0	.626	.6107	.261	.2606
1.0	.2	.0152	.709	.4028	0	.293	.2941	.221	.2200
					.5	.402	.3701	.238	.2339
					1.0	.443	.4241	.241	.2427
					1.8	.490	.4895	.244	.2522
					2.2	.510	.5164	.246	.2558
					3.0	.546	.5629	.248	.2617
1.0	.8	.10	.768	.4932	0	.364	.3671	.312	.2955
					.5	.553	.5377	.306	.3191
					1.0	.647	.6530	.314	.3314
					1.8	.762	.7925	.323	.3433
					2.2	.811	.8504	.326	.3476
					3.0	.900	.9519	.333	.3542
1.0	.4	.10	.768	.6249	0	.381	.3840	.296	.2891
					.5	.549	.5280	.304	.3099
					1.0	.624	.6291	.310	.3219
					1.8	.716	.7528	.316	.3342
					2.2	.756	.8044	.318	.3388
					3.0	.828	.8948	.323	.3461
					3.5	.870	.9450	.326	.3498

TABLE I.- COMPARISON OF SOLUTIONS OF CORRELATION EQUATIONS WITH EXACT SIMILAR SOLUTIONS FOR EQUILIBRIUM DISSOCIATED AIR – Concluded

t_E	t_e	ζ_w	N_{Pr}	$\frac{\rho_e \mu_e}{\rho_w \mu_w}$	β	f''_w Present solutions	f''_w Exact solutions (ref. 11)	ζ'_w Present solutions	ζ'_w Exact solutions (ref. 11)
1.0	0.2	0.10	0.768	0.7931	0	0.393	0.3965	0.288	0.2889
					.5	.559	.5218	.318	.3086
					1.0	.620	.6126	.322	.3209
					1.8	.694	.7250	.327	.3343
					2.2	.726	.7721	.329	.3394
					3.0	.785	.8549	.332	.3476
					3.5	.820	.9009	.334	.3518
1.0	1.0	.03	.680	.2909	.5	.447	.4227	.271	.2522
	.4			.3977	0	.314	.3153	.246	.2286
					.5	.440	.4163	.248	.2439
					1.0	.498	.4862	.253	.2528
					1.8	.566	.5701	.258	.2621
					3.0	.645	.6645	.264	.2710
1.0	.2	.03	.680	.5048	0	.325	.3263	.235	.2298
					.5	.448	.4146	.253	.2446
					1.0	.494	.4778	.256	.2539
					1.8	.548	.5546	.260	.2640
					3.0	.612	.6417	.264	.2741
.5	1.0	.0076	.709	.1835	.5	.359	.3357	.223	.2190
					1.0	.421	.4008	.229	.2268
					1.8	.493	.4775	.236	.2343
					2.2	.523	.5088	.239	.2369
					3.0	.576	.5627	.244	.2410
.5	1.0	.05	.768	.3613	.5	.497	.4784	.295	.3150
					1.0	.584	.5798	.303	.3267
					1.8	.689	.7012	.312	.3380
					2.2	.733	.7513	.316	.3419
					3.0	.812	.8388	.323	.3480
					3.5	.856	.8872	.327	.3511
.5	.6	.05	.768	.4297	0	.336	.3389	.289	.2829
					.5	.489	.4710	.286	.3031
					1.0	.563	.5617	.292	.3142
					1.8	.651	.6713	.299	.3253
					2.2	.689	.7167	.302	.3293
					3.0	.755	.7960	.308	.3357
.5	.4	.05	.768	.4932	0	.345	.3477	.282	.2800
					.5	.488	.4672	.287	.2988
					1.0	.553	.5511	.292	.3098
					1.8	.630	.6532	.298	.3213
					2.2	.663	.6957	.300	.3256
					3.0	.722	.7698	.305	.3324
.5	.2	.05	.768	.6249	0	.356	.3595	.275	.2794
					.5	.497	.4623	.300	.2971
					1.0	.549	.5369	.304	.3084
					1.8	.610	.6291	.308	.3208
					2.2	.637	.6676	.310	.3255
.5	.2	.05	.768	.6249	3.0	.684	.7351	.313	.3332
					3.5	.711	.7724	.315	.3373

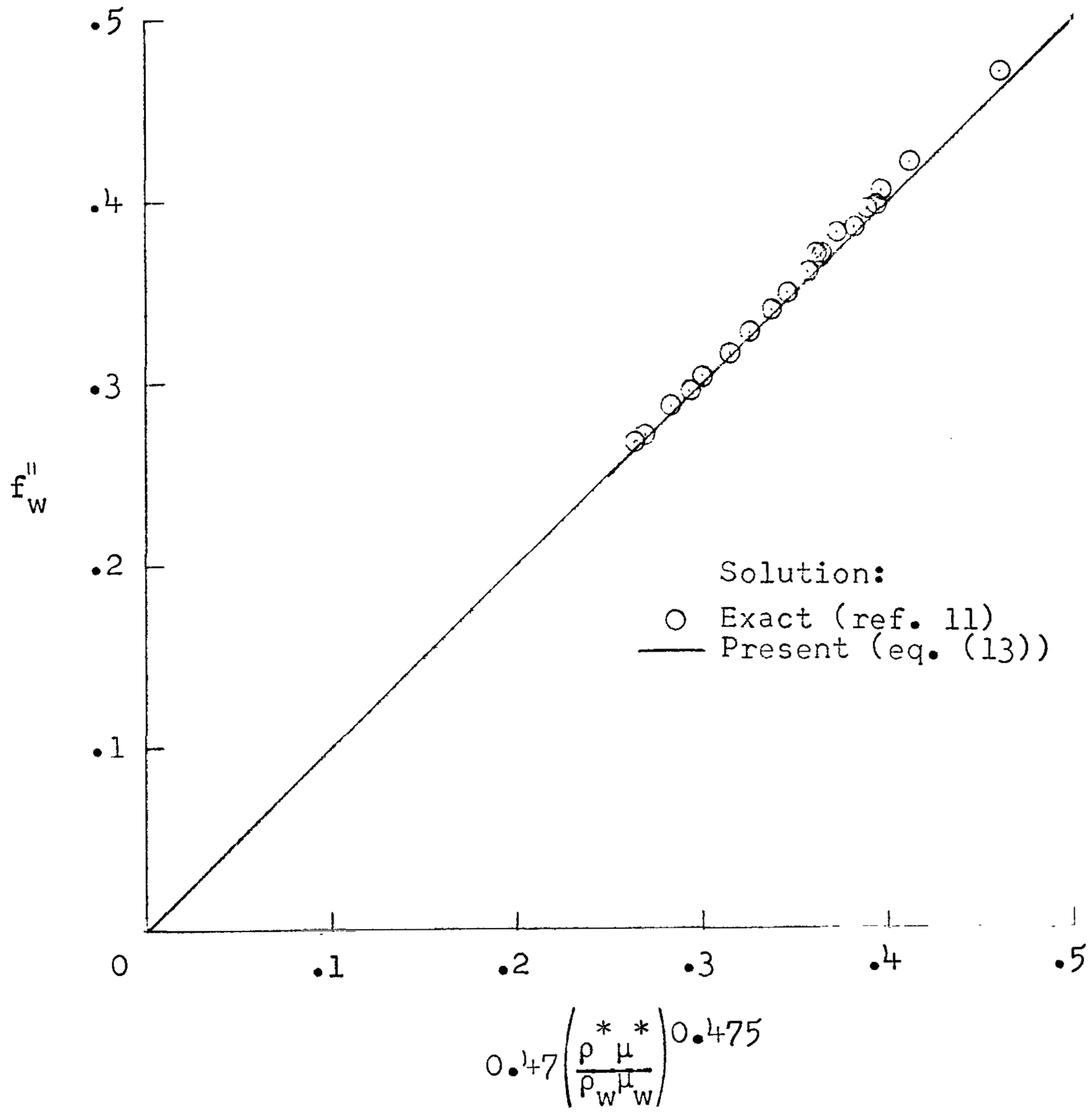

Figure 1.- Comparison of present results with exact solutions for velocity derivative for cone or flat plate.

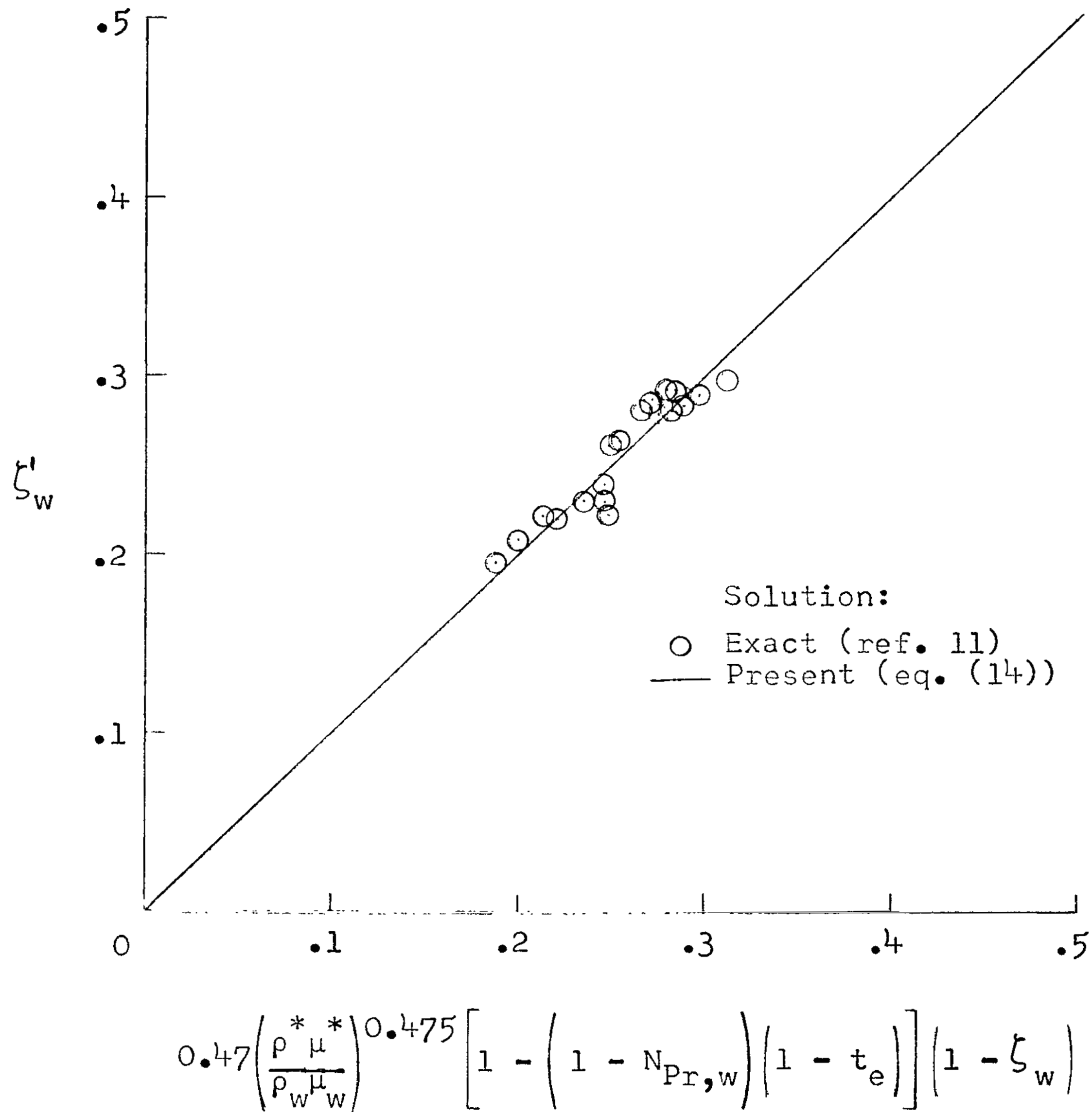

Figure 2.- Comparison of present results with exact solutions for enthalpy derivative for cone or flat plate.

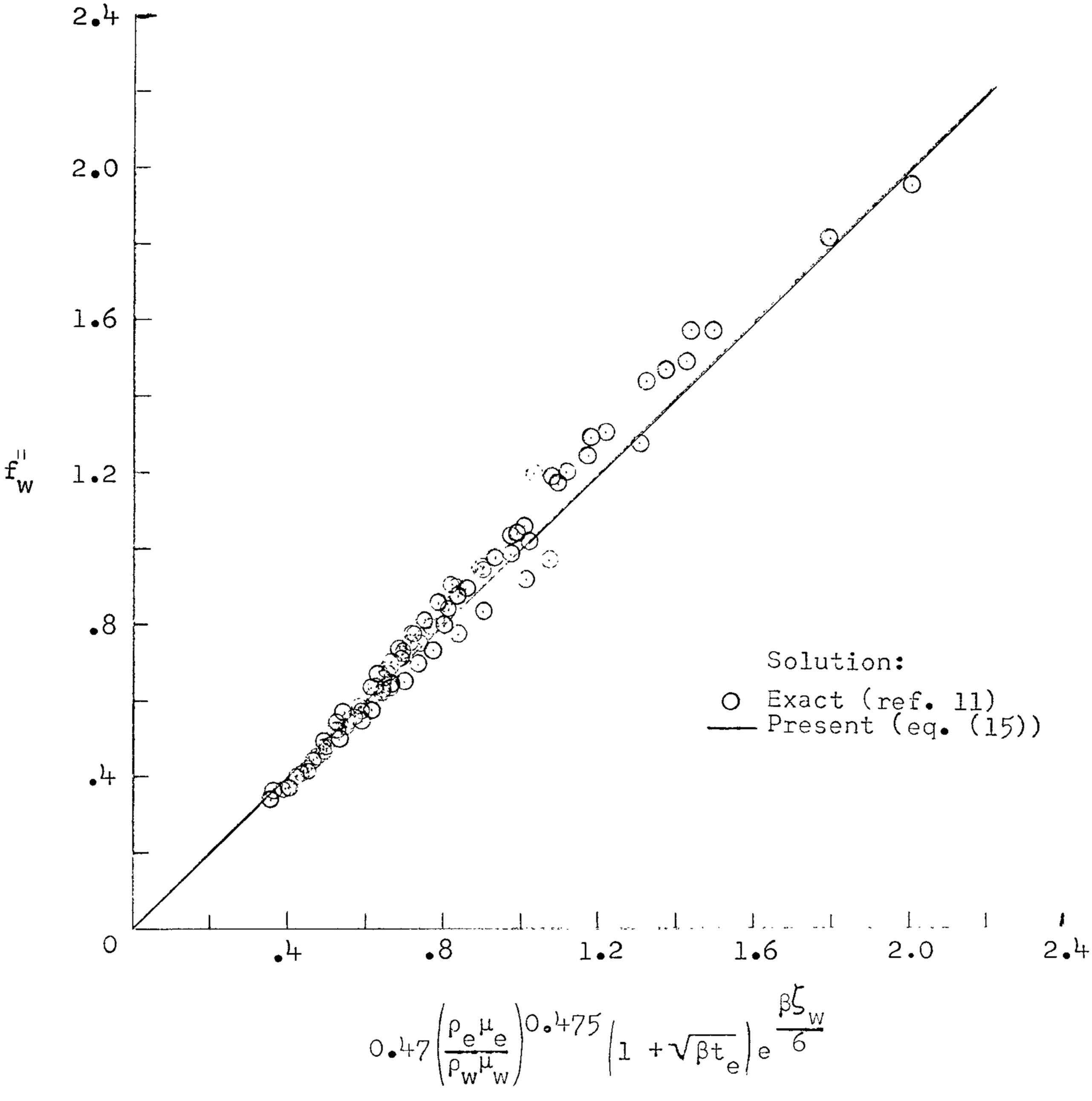

Figure 3.- Comparison of present results with exact solutions for velocity derivative for blunt axisymmetric body.

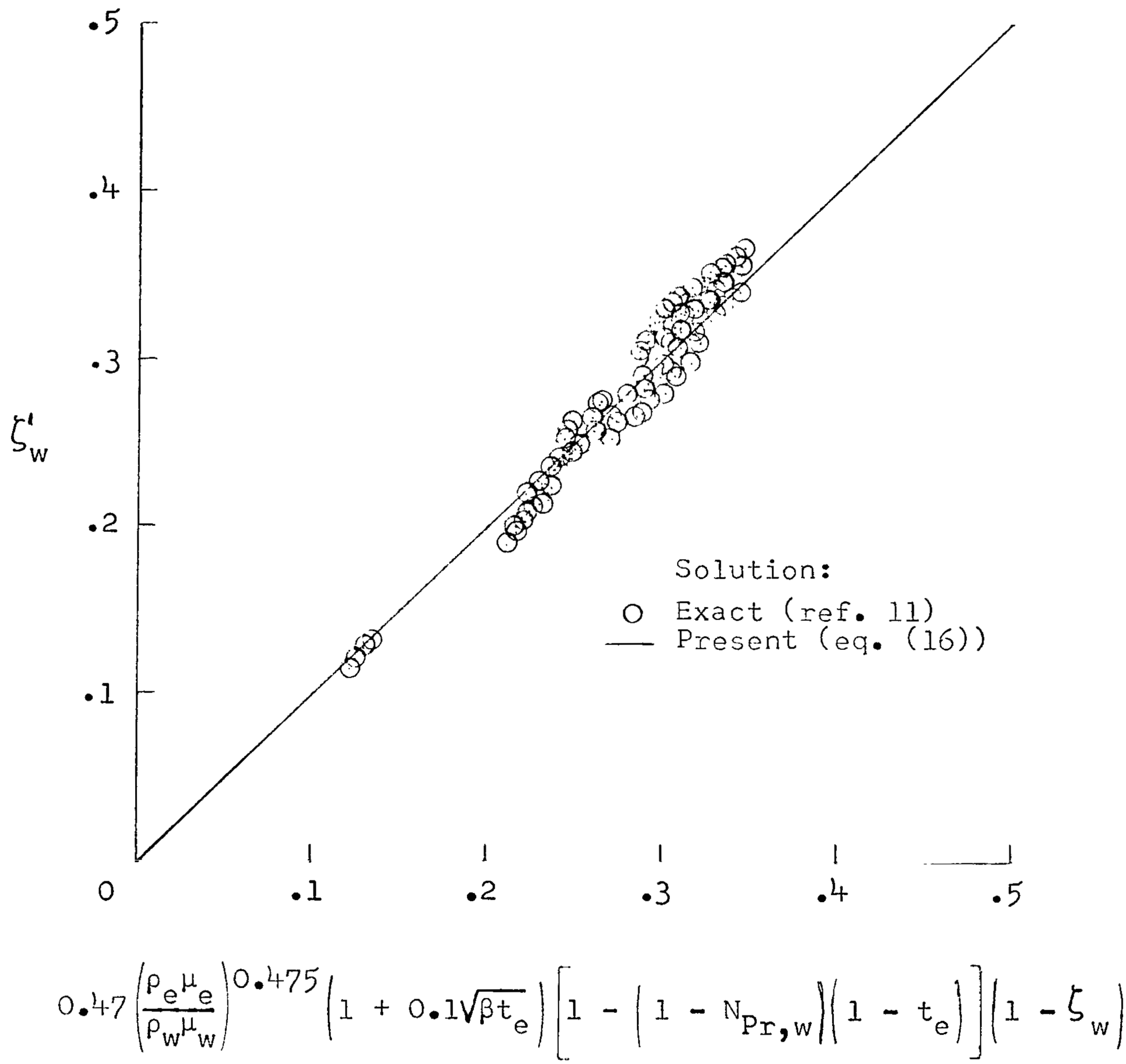

Figure 4.- Comparison of present results with exact solutions for enthalpy derivative for blunt axisymmetric body.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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