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RADIANT-INTERCHANGE CONFIGURATION FACTORS FOR SPHERICAL AND CONICAL SURFACES TO SPHERES

by James P. Campbell and Dudley G. McConnell

Lewis Research Center
Cleveland, Ohio

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SUMMARY

Geometric configuration view factors have been calculated for a sphere and for cones radiating to a sphere. A computer program was set up to give configuration view factors not only for the entire radiating body but, also, for bands of the radiating body which permit the determination of the configuration view factor between any part of the radiating body and the sphere.

The range of configurations consisted of equal radii spheres, and right-circular cones of semiapex angles of 15°, 30°, 45°, and 60° radiating to a sphere on the cone axis with radius equal to the cone base radius. Separations for all cases ranged from zero to 10 sphere radii.

Curves are presented that give the configuration view factor for any configuration within the limits of the study.

INTRODUCTION

Many chemical- and nuclear-rocket propulsion systems use cryogenic liquids for propellants. As a result, on long-time interplanetary missions, thermal protection of these liquids is imperative to prevent excessive boil off.

During the planet transfer portion of the flight, the largest external heat source will be the solar heat flux. Smolak, Knoll, and Wallner (ref. 1) have shown that shadow shields can greatly reduce solar heating under such conditions. In particular, inflated shadow shields promise low structural weight and, therefore, high shielding efficiency in terms of boiloff weight to shield weight.

In the analysis of the radiant heat transfer between a shadow shield and a cryogen storage tank, the configuration view factor is needed. The equation for the configuration view factor between finite areas is

$$A_1F_{A1-A2} = \int_{A_1}\int_{A_2} \frac{\cos\psi_1 \cos\psi_2}{\pi L^2} dA_1\, dA_2 \tag{1}$$

For the differential area view factor from an area element to a finite area, the equation is

$$F_{dA1-A2} \equiv \frac{d(A_1F_{A1-A2})}{dA_1} = \int_{A_2} \frac{\cos\psi_1 \cos\psi_2}{\pi L^2} dA_2 \tag{2}$$

Solutions of these equations are available in closed form for only a few simple configurations.

Hamilton and Morgan (ref. 2) compiled the configuration view factors that had been previously published and added several for different geometrical shapes. They were limited to a few simple configurations consisting of flat surfaces, concentric bodies, or infinite parallel surfaces. Since then, the digital computer has made more complex surfaces amenable to analysis. Consequently, configuration view factors for three-dimensional surfaces, such as spheres, cones, cylinders, etc., are now appearing in the literature.

In setting up an experimental program to determine the effectiveness of shadow shielding in a space environment, shapes that were attractive, because they afforded intuitively small view factors and could be easily fabricated, were the cone and the sphere. However, a review of the literature showed that many of the desired configuration view factors were not available.

Of the few available sphere-sphere (ref. 3) and the cone-sphere (ref. 4) configuration view factors, the pertinent parameters investigated were limited and none were useful for analyzing the effect of applying high emittance zones or bands (ref. 5) and for variation of temperature over the radiating surface. A computer program was therefore set up to give configuration view factors from a sphere, or any part of a sphere, to a sphere and from a cone, or any part of a cone, to a sphere over a range of separations. The computer program was set up by use of the method presented in the appendix.

This paper is a compilation of the configuration view factors which resulted from this computing program. Although the work presented herein does not include every possible sphere-sphere and cone-sphere configuration, the actual cases studied are quite complete. In addition, because configuration view factors are presented for any part of a radiating body, radiant heat-transfer calculations can be made for cases where the emittance and temperature are not constant over the entire surface.

SYMBOLS

A	surface area
F	radiative heat-transfer configuration view factor
i, j, k	unit vectors along x, y, z axes, respectively
L	distance between dA_1 and dA_2
$\vec{n}$	unit vector normal to dA
R	radius of sphere
R_c	base of cone
r	distance from center of sphere to elemental surface area or distance from apex of cone to elemental surface area
s	separation between two spheres or cone and sphere
x, y, z	axes in Cartesian coordinate system
θ	angle from z-axis to position vector $\vec{r}$ and cone semiapex angle
φ	angle in the x-y plane
ψ	angle between the normal to an elemental area and a line connecting this elemental area to another (angle between $\vec{n}$ and $\vec{L}$)

Subscripts:

1	shield
2	tank

DESCRIPTION OF CONFIGURATIONS

Sphere-Sphere Configuration

The sphere-sphere configuration view factors were determined for separation distances varying from zero to 10 tank radii (in steps of one tank radius) and were limited to equal radii spheres. At each step, the configuration view factor was determined for the complete range of spherical-cap half angles from zero to 90° as shown in figure 1.

Cone-Sphere Configuration

The configuration view factor for four different half-angle (15°, 30°, 45°, and 60°)

right-circular cones radiating to a colinear sphere were determined. The cone-sphere separations varied from zero to 10 tank radii. The cone base radius ran the complete range from zero to the sphere radius as shown in figure 2.

DESCRIPTION OF CHARTS

Sphere-Sphere

The values for both F_{A1-A2} and A_1F_{A1-A2} are shown in figure 3 for the whole sphere radiating to a sphere of equal radius. The configuration view factors for this particular case ($R_1 = R_2$) was one of those reported by Jones (ref. 3). However, the values for F_{A1-A2} that come out of this study are twice those of Jones, because he took the entire surface area of the sphere for A_1, while this study considered only that part of the sphere that was radiating to the other sphere. This same criterion for establishing the value of A_1 was used for all the values in the present report.

The results, when only a part of the first sphere is radiating to the second sphere, may be determined by use of figure 4. The curves shown indicate the value of A_1F_{A1-A2} for the spherical caps, but the value of A_1F_{A1-A2} between any two values of θ is simply the difference between the values for each θ. The differential area view factor F_{dA1-A2} for any point on the surface of the radiating sphere is given in figure 5. If the configuration view factor for an extremely narrow band is desired, greater accuracy should result from the use of the differential area view factor over the desired area.

Cone-Sphere

The values for A_1F_{A1-A2} and F_{A1-A2} for right-circular cones of semiapex angles of 15°, 30°, 45° and 60° radiating to a spherical body with the same radius as the cone base are shown in figure 6 over a range of separations from zero to 10 tank radii. The configuration view factors determined by Nothwang, Arvesson, and Hamaker (ref. 4) for configurations that matched those of the present study are indicated by points on the appropriate curve.

The configuration view factors, for cases where only part of the radiating cone is considered, may be determined by use of figure 7. These curves indicate the value of A_1F_{A1-A2} for the cone caps as indicated, but, as in the case of the sphere-sphere configuration, the value of A_1F_{A1-A2} for bands around the cone is simply the difference between the values of A_1F_{A1-A2} for the two cones with base radii equal to the radii of the bottom and top of the band. As in the sphere-sphere configuration, the differential area view factors F_{dA1-A2} for cones have been included (fig. 8) for greater accuracy for narrow bands. This view factor is given for cones with semiapex angles of 15°, 30°, 45°, and 60°.

Plots can be drawn that would permit interpolation of the configuration view factors for cones with semiapex angles different from the 15°, 30°, 45°, and 60° reported. These have not been included because of the great number of combinations of base radius and separation possible with the reported data.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 24, 1967,
124-09-05-09-22.

APPENDIX - CONFIGURATION FACTORS

Configuration Factor Between Two Spheres

The calculation for the configuration view factors reported herein were performed with a high-speed digital computer. The necessary equations for these computations are derived in this appendix.

The method is that of Nothwang, Arvesen, and Hamaker (ref. 4) and Jones (ref. 3) in which the coordinate positions of points on each body, the distance between the points, and the unit normals to the body surface at the points are expressed in vector notation and applied to the equation for the diffuse radiation configuration view factor.

In the sphere-sphere case, the geometry is as shown in figure 9 and the vectors are written as

$$\vec{r}_1 = r_1(\vec{i} \sin \theta_1 \cos \varphi_1 + \vec{j} \sin \theta_1 \sin \varphi_1 + \vec{k} \cos \theta_1)$$

$$\vec{r}_2 = r_2(\vec{i} \sin \theta_2 \cos \varphi_2 + \vec{j} \sin \theta_2 \sin \varphi_2 + \vec{k} \cos \theta_2)$$

$$\vec{n}_1 = \vec{i} \sin \theta_1 \cos \varphi_1 + \vec{j} \sin \theta_1 \sin \varphi_1 + \vec{k} \cos \theta_1$$

$$\vec{n}_2 = \vec{i} \sin \theta_2 \cos \varphi_2 + \vec{j} \sin \theta_2 \sin \varphi_2 + \vec{k} \cos \theta_2$$

$$\vec{R}_1 = \vec{k} r_1$$

$$\vec{R}_2 + \vec{k} r_2$$

$$\vec{S} = \vec{k} s$$

The distance vector between the two points is

$$\vec{L} = \vec{r}_2 - \vec{R}_2 - \vec{S} - \vec{R}_1 - \vec{r}_1$$

The angles ψ_1 and ψ_2 between the distance vector and the normals to the surfaces may be obtained from the scalar product relations:

$$\cos \psi_1 = \frac{\vec{n}_1 \cdot \vec{L}}{|n_1| \; |L|}$$

$$\cos \psi_2 = - \frac{\vec{n}_2 \cdot \vec{L}}{|n_2| \; |L|}$$

The elemental areas located at $\vec{r}_1$ and $\vec{r}_2$ are given by

$$dA_1 = r_1^2 \sin \theta_1 \, d\theta_1 \, d\varphi_1$$

$$dA_2 = r_2^2 \sin \theta_2 \, d\theta_2 \, d\varphi_2$$

Substituting into

$$A_1 F_{A1\text{-}A2} = \int_{A_1} \int_{A_2} \frac{\cos \psi_1 \cos \psi_2 \, dA_1 \, dA_2}{\pi L^2}$$

yields the configuration view factor.

Configuration Factor Between a Cone And a Sphere

In the cone-sphere case the geometry is as shown in figure 10 and the vectors are written the same as in the sphere-sphere case except for the normal to the conical surface which is

$$\vec{n}_1 = \vec{i} \cos \theta_1 \cos \varphi_1 + \vec{j} \cos \theta_1 \sin \varphi_1 - \vec{k} \sin \theta$$

The distance vector between the two points is

$$\vec{L} = \vec{r}_2 - \vec{R}_2 - \vec{S} - \vec{r}_1$$

The angles ψ_1 and ψ_2 and the elemental area dA_2 are the same as in the sphere-sphere case, and the elemental area of the conical surface is

$$dA_1 + r_1 \sin \theta_1 \, dr_1 \, d\varphi_1$$

The configuration view factor is then found with the same substitutions as before.

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1. Smolak, George R.; Knoll, Richard H.; and Wallner, Lewis E.: Analysis of Thermal-Protection Systems for Space-Vehicle Cryogenic-Propellant Tanks. NASA TR R-130, 1962.
2. Hamilton, D. C.; and Morgan, W. R.: Radiant-Interchange Configuration Factors. NACA TN-2836, 1952.
3. Jones, L. R.: Diffuse Radiation View Factors Between Two Spheres. J. Heat Transfer, vol. 87, no. 3, Aug. 1965, pp. 421-422.
4. Nothwang, George J.; Arvesen, John C.; and Hamaker, Frank M.: Analysis of Solar-Radiation Shields for Temperature Control of Space Vehicles Subjected to Large Changes in Solar Energy. NASA TN D-1209, 1962.
5. Jones, L. R.; and Barry, D. G.: Lightweight Inflatable Shadow Shields for Cryogenic Space Vehicles. J. Spacecraft and Rockets, vol. 3, no. 5, May 1966, pp. 722-728.

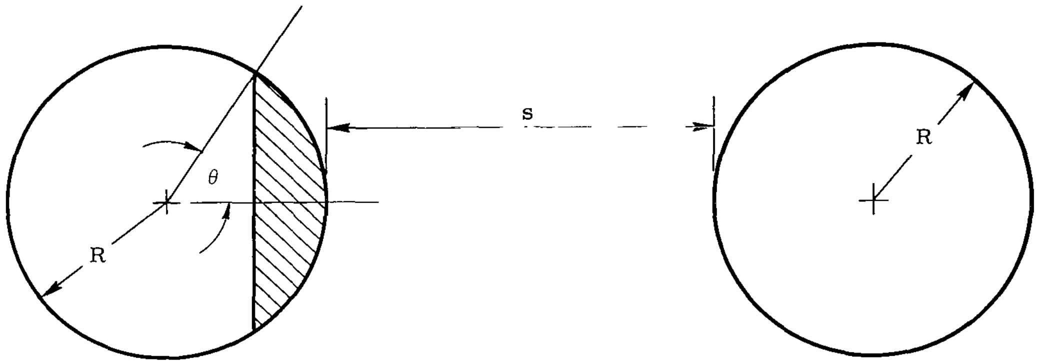


Figure 1. - Sphere-cap - sphere configuration. Range of angles from z-axis to position vector, 0° to 90°; sphere-sphere separation, 0 to 10 sphere radii (in increments of 1 radius).

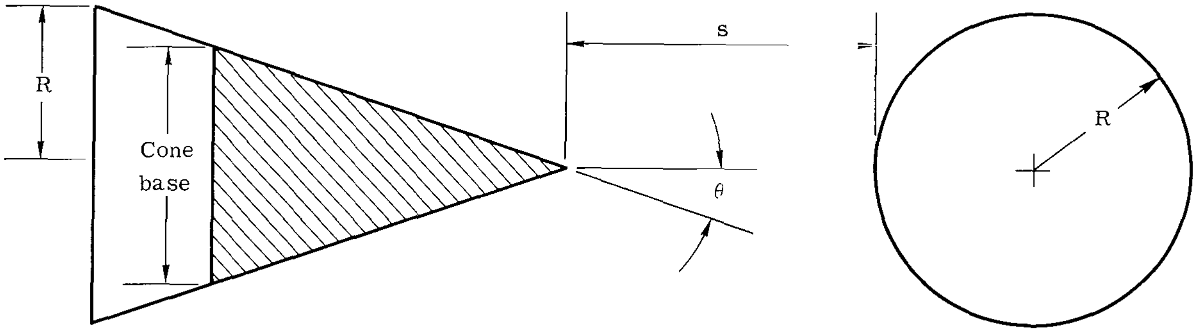


Figure 2. - Cone-sphere configuration. Range of cone semiapex angles, 15°, 30°, 45°, and 60°; cone base radius, 0 to radius of sphere; cone-sphere separation, 0, 1, 2, 4, 6, 8, and 10 sphere radii.

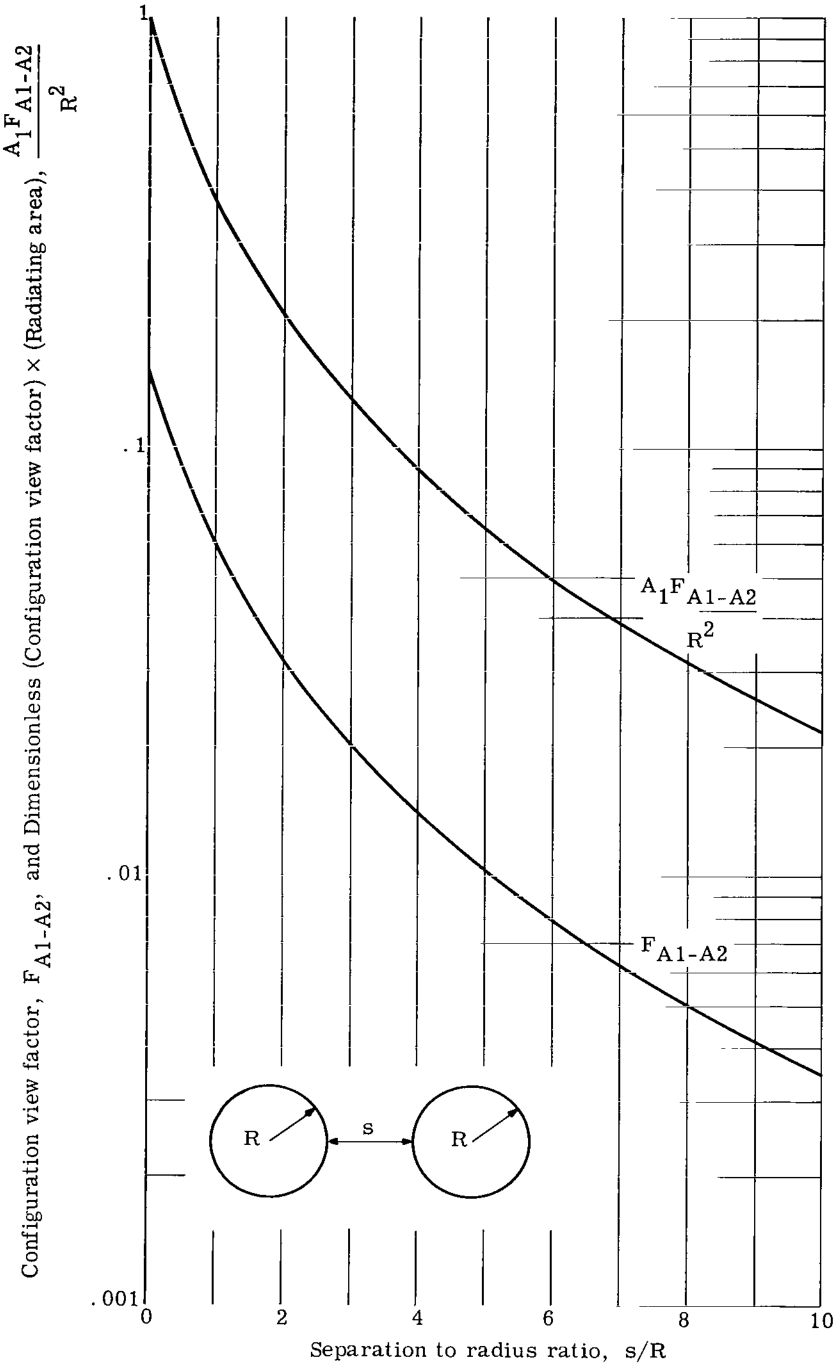


Figure 3. - Configuration view factor and dimensionless configuration view factor times radiating area for sphere to sphere configuration.

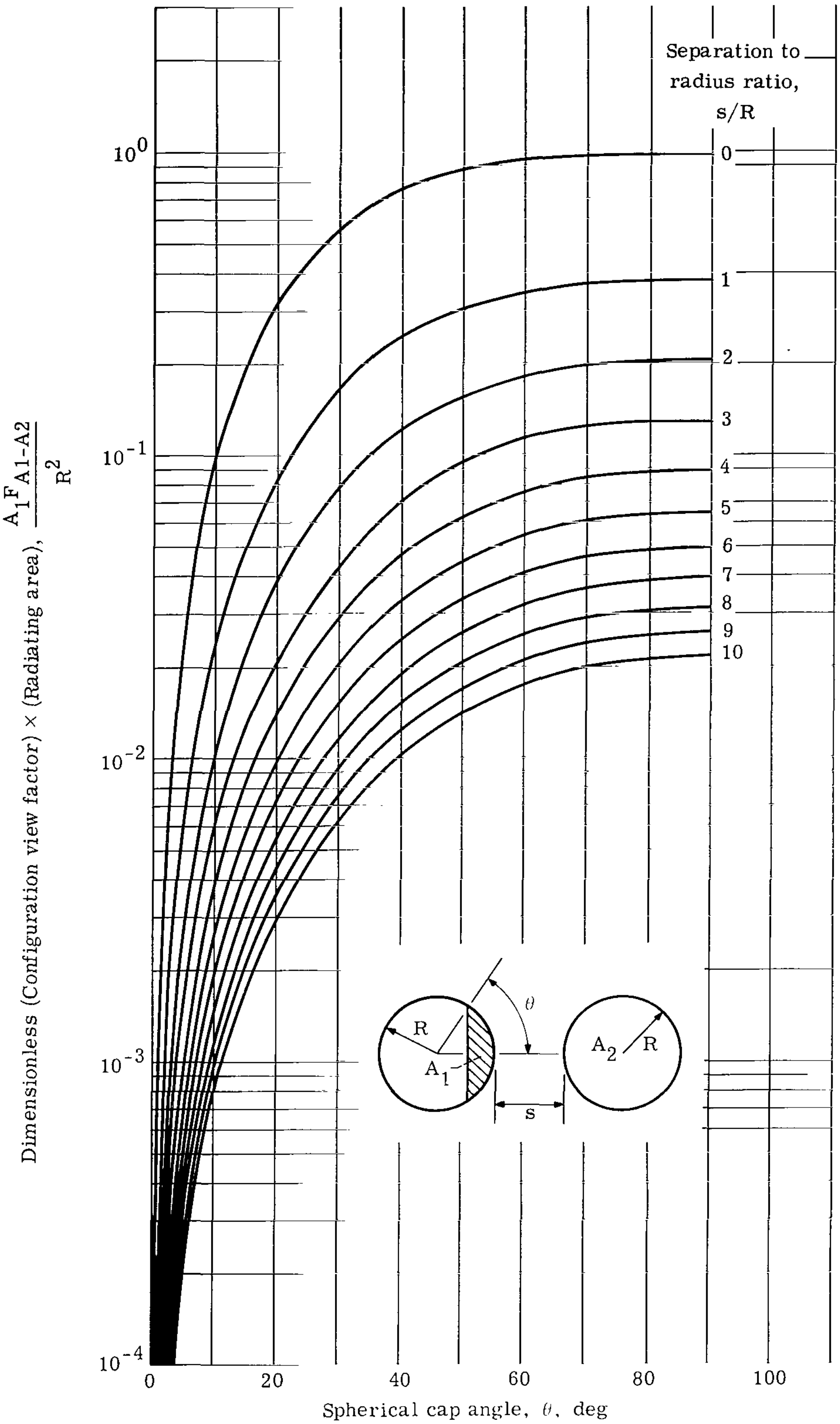


Figure 4. - Dimensionless configuration view factor times radiating area for sphere cap to sphere.

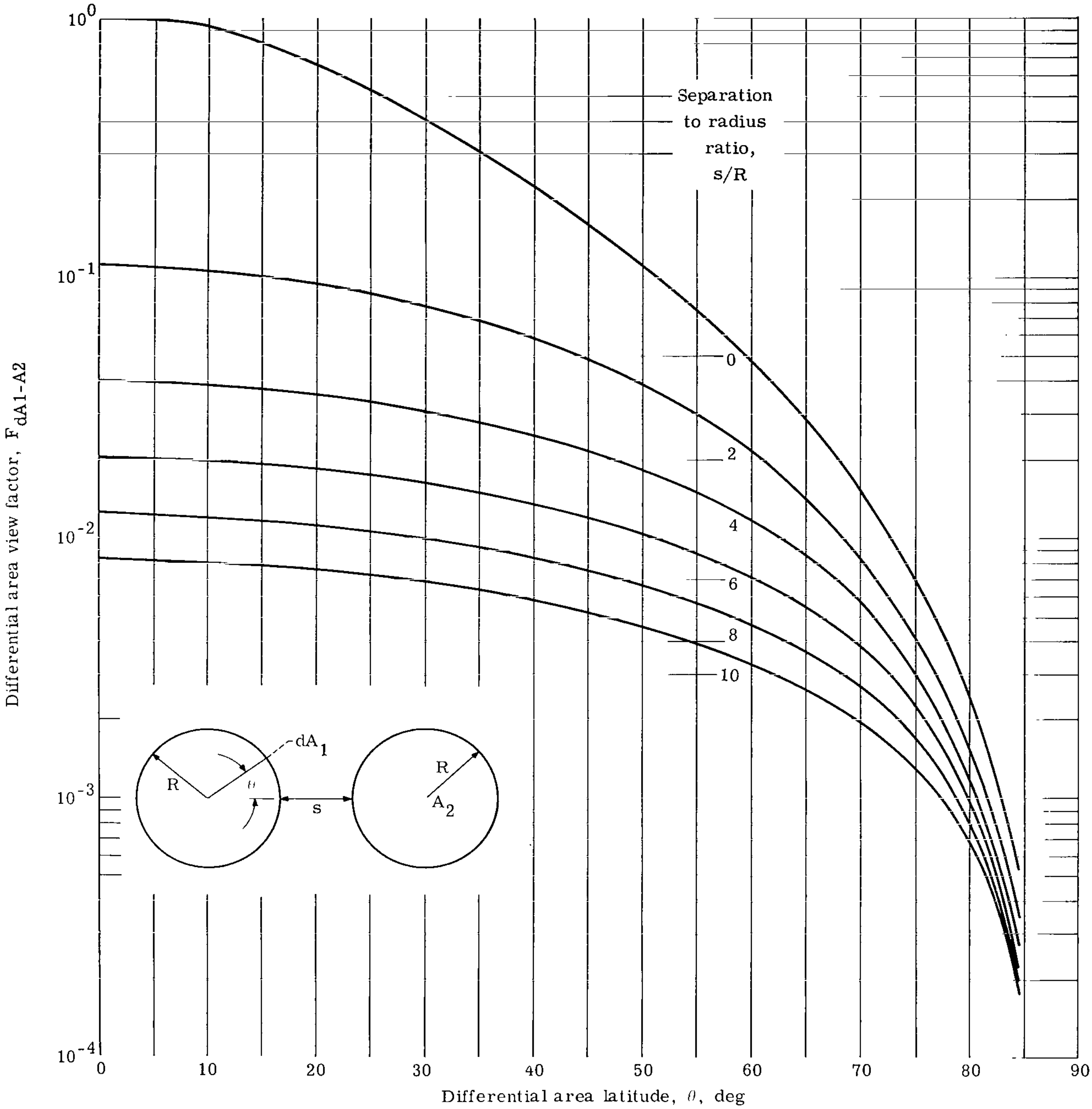


Figure 5. - Differential area view factor of any point on surface of sphere to sphere of equal radius.

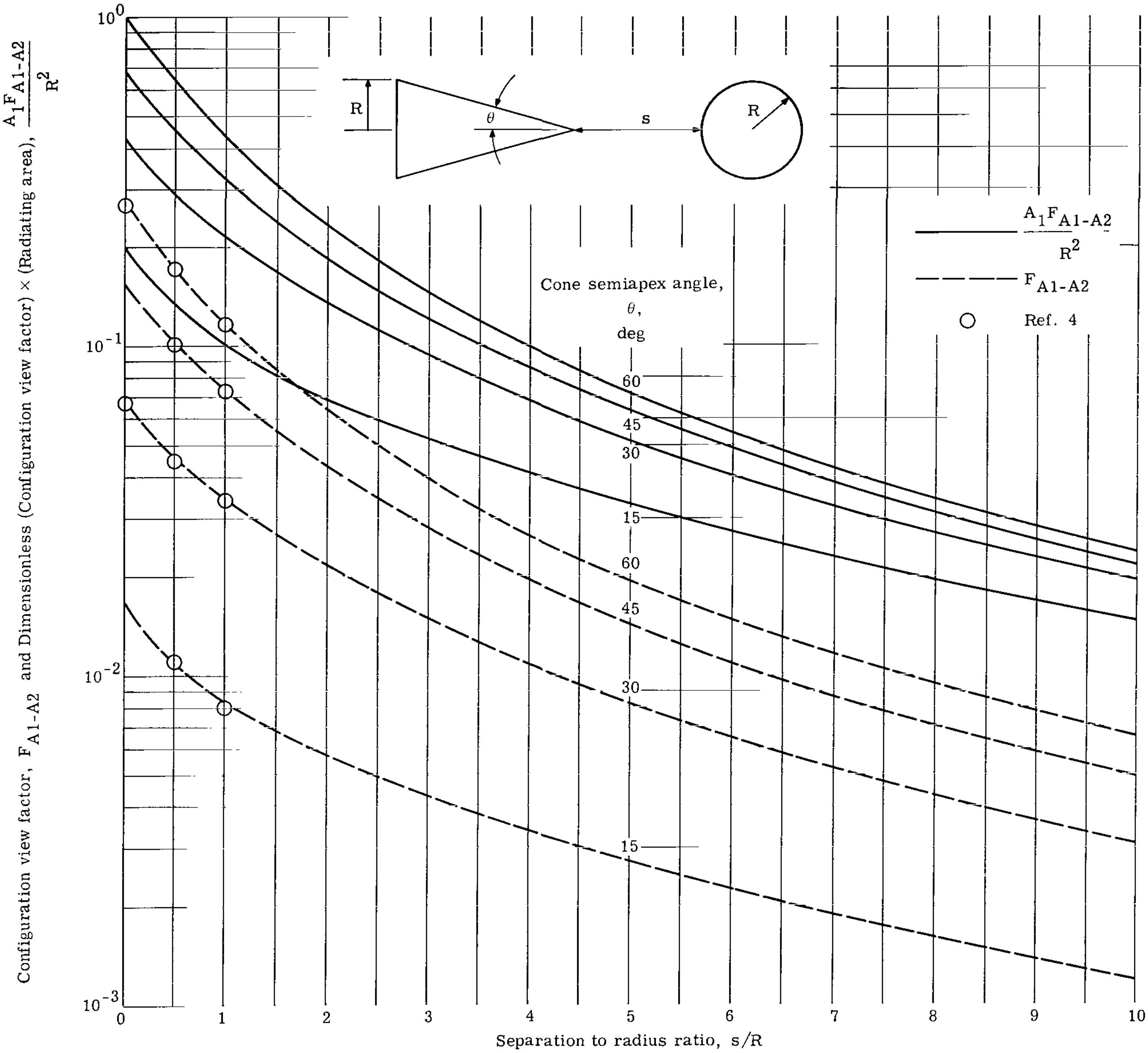
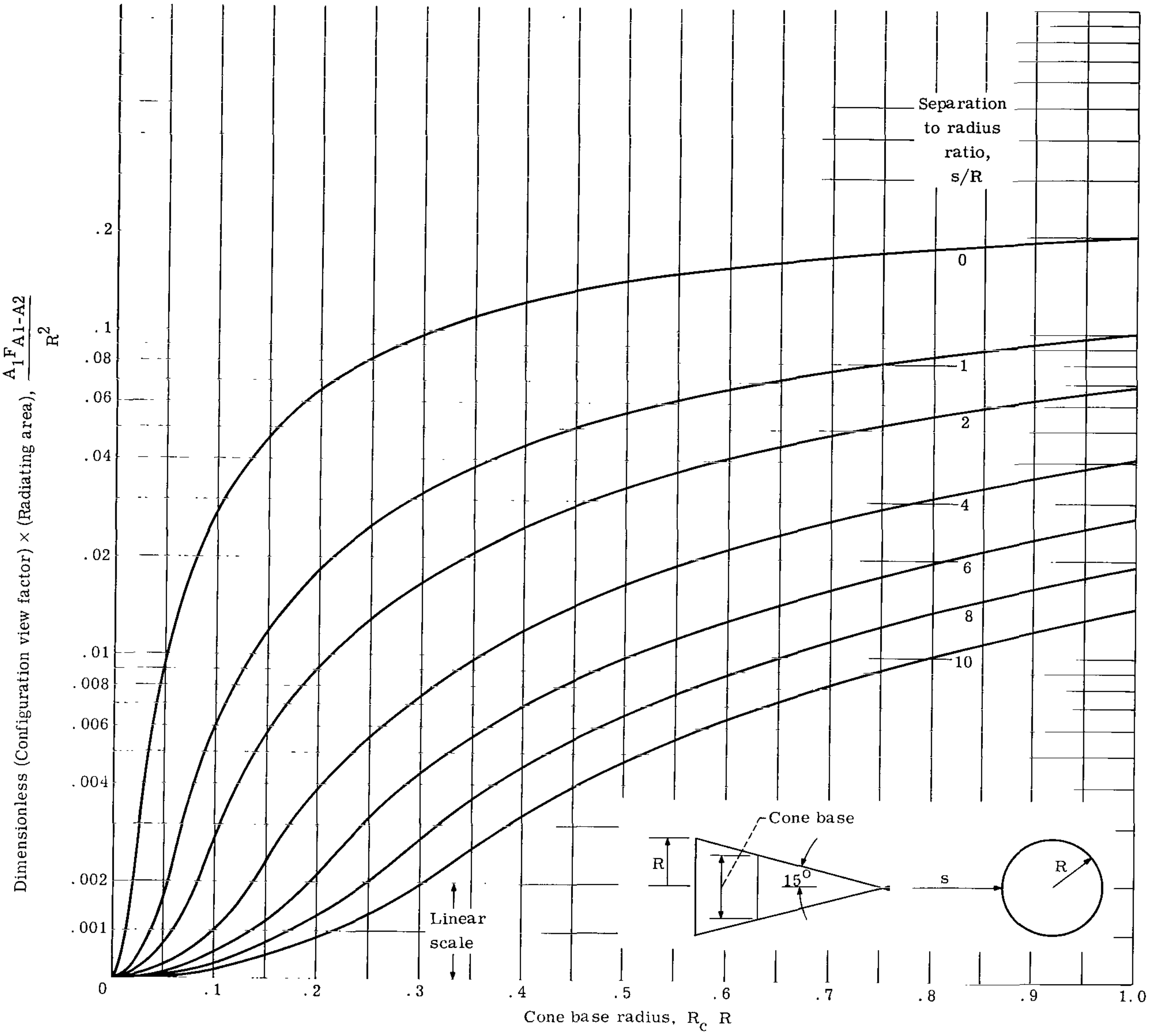
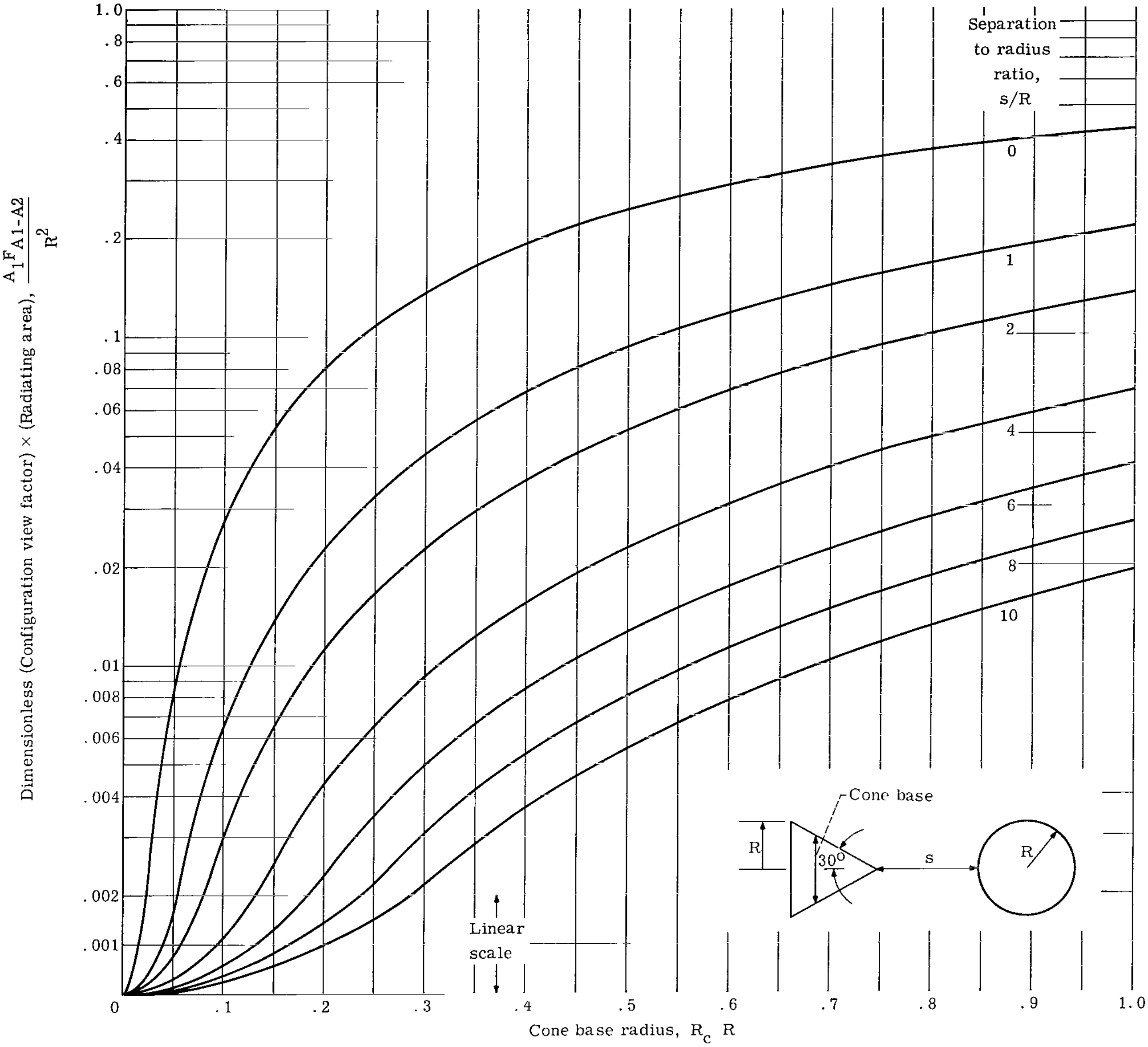


Figure 6. - Configuration view factor and dimensionless configuration view factor times radiating area for cone to equal radius sphere on cone axis.



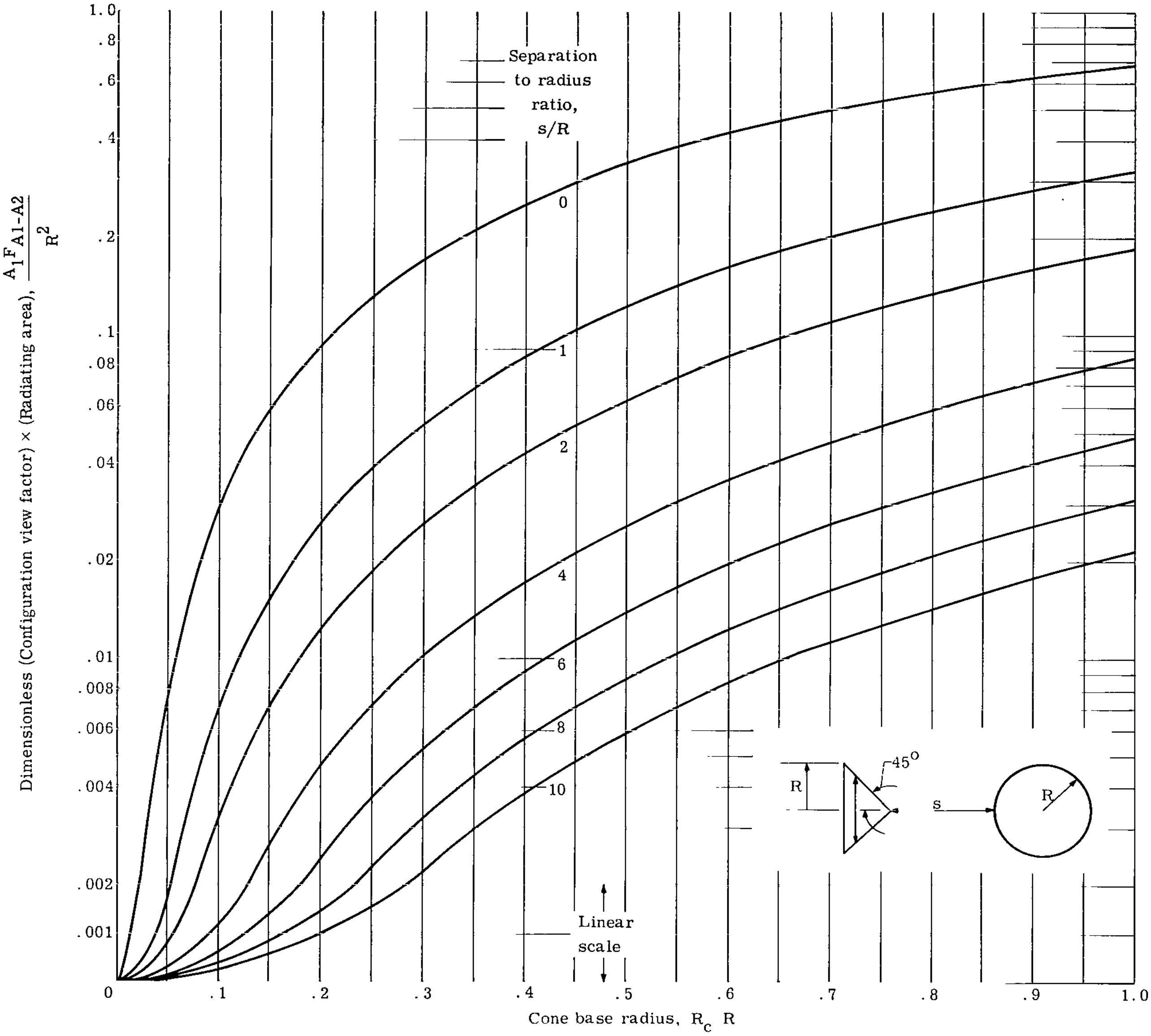
(a) 15^{o} cone.

Figure 7. - Dimensionless configuration view factor times radiating area for cone to sphere on cone axis.



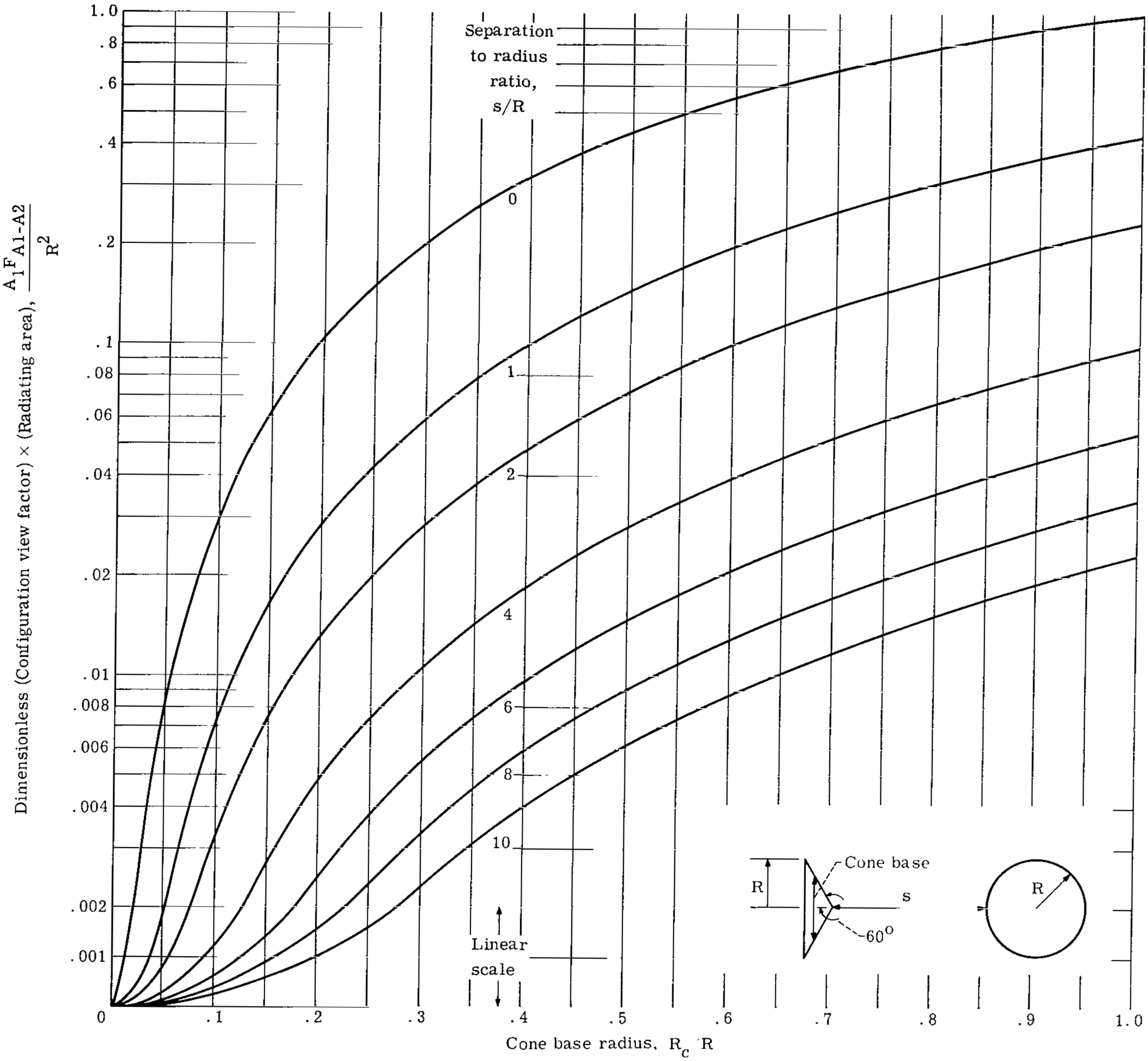
(b) 30° cone.

Figure 7. - Continued.



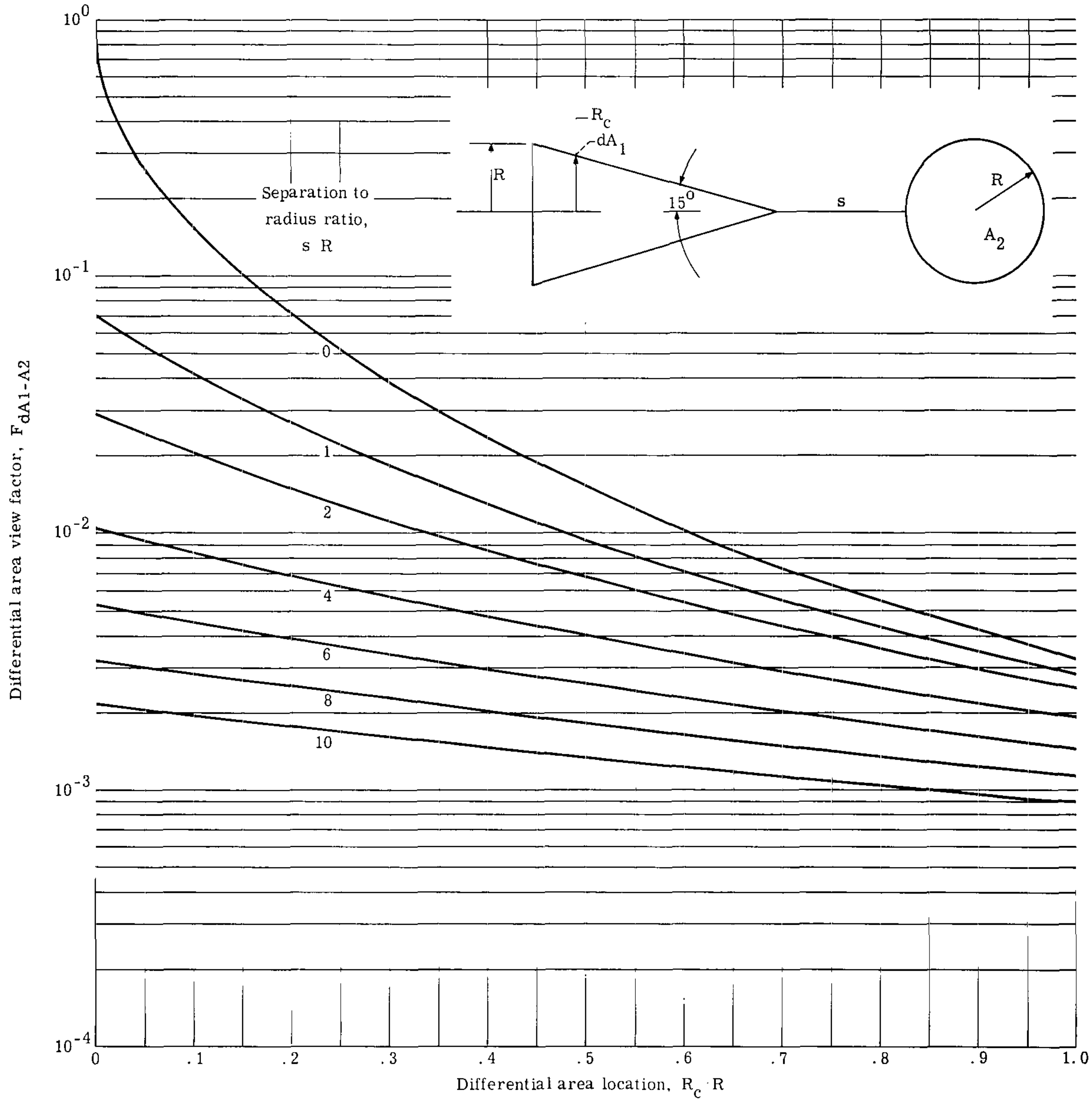
(c) 45° cone.

Figure 7. - Continued.



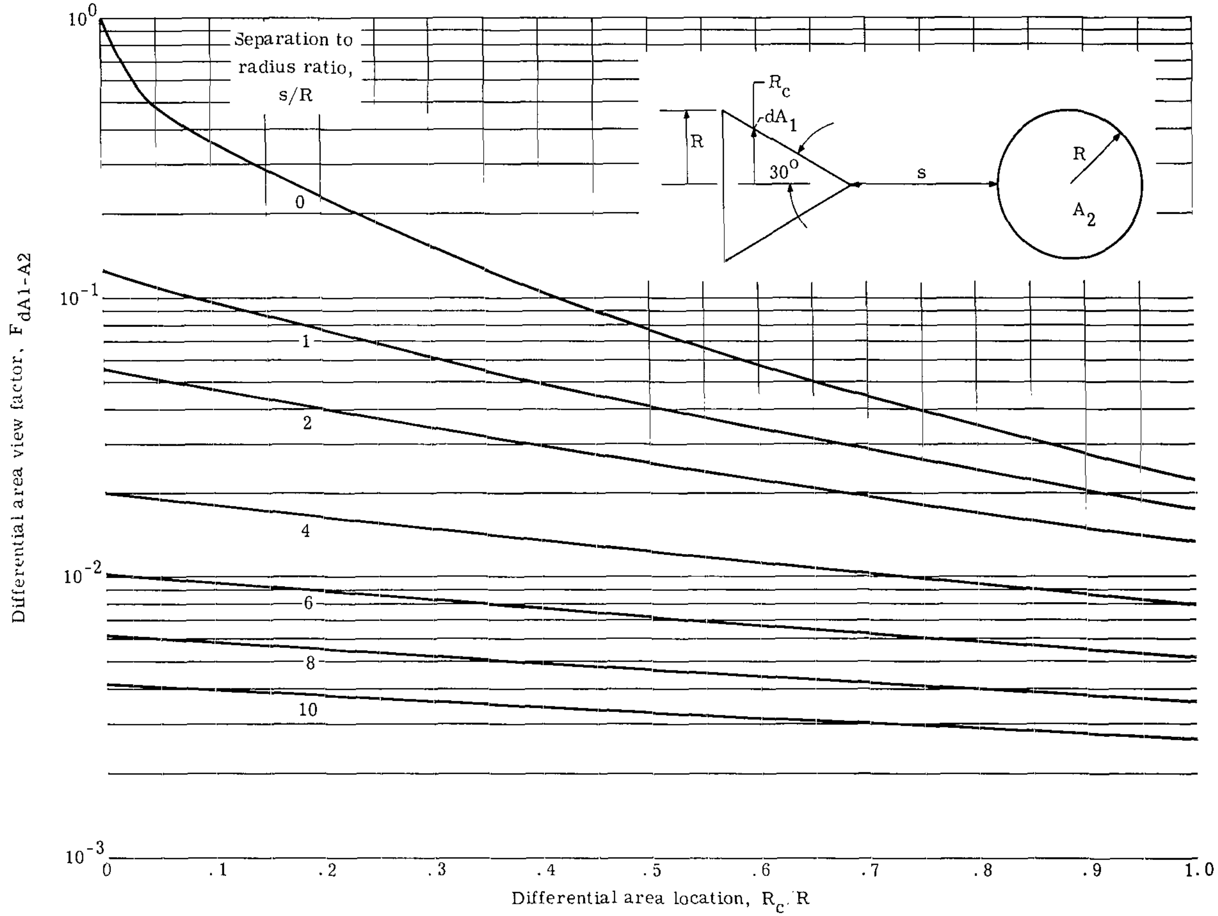
(d) 60^o cone.

Figure 7. - Concluded.



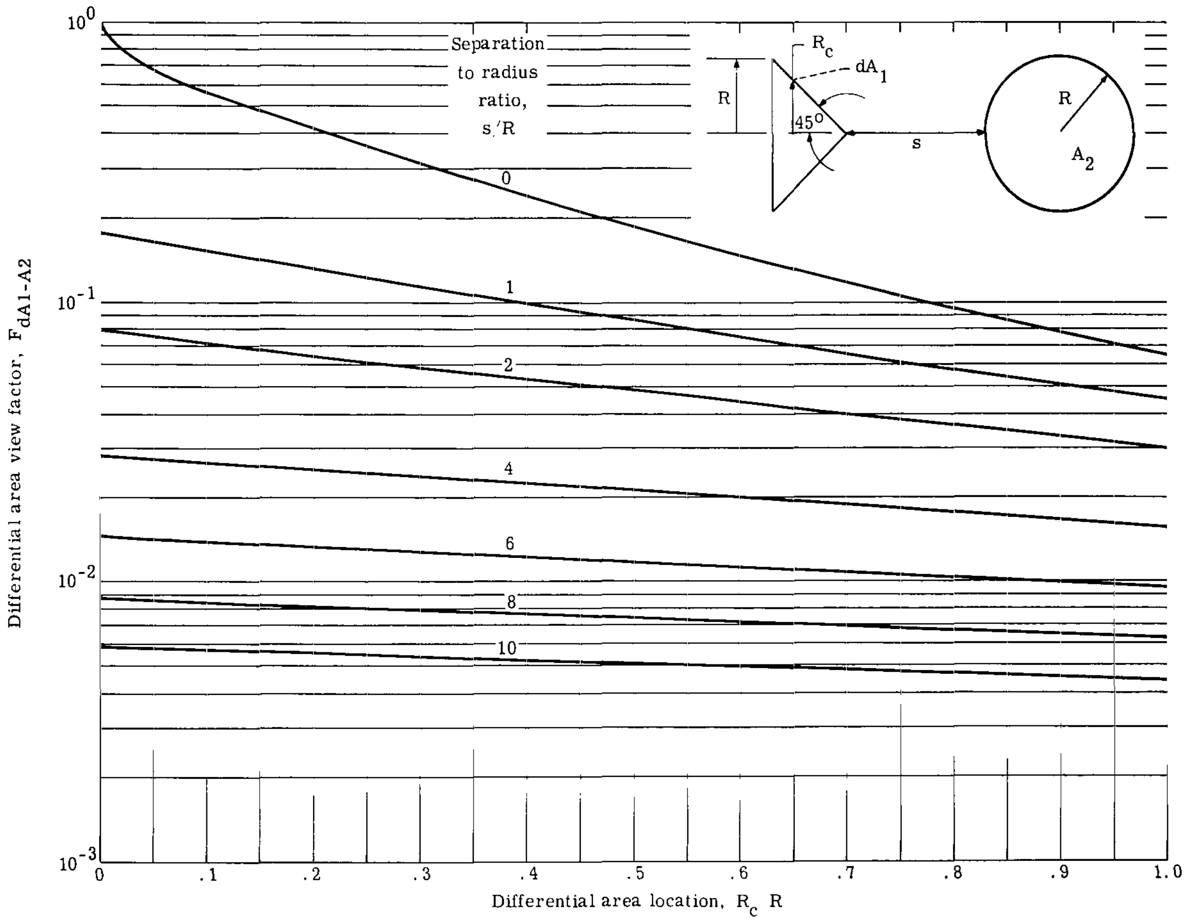
(a) 15^0 cone.

Figure 8. - Differential area view factor of any point on surface of cone to sphere on cone axis.



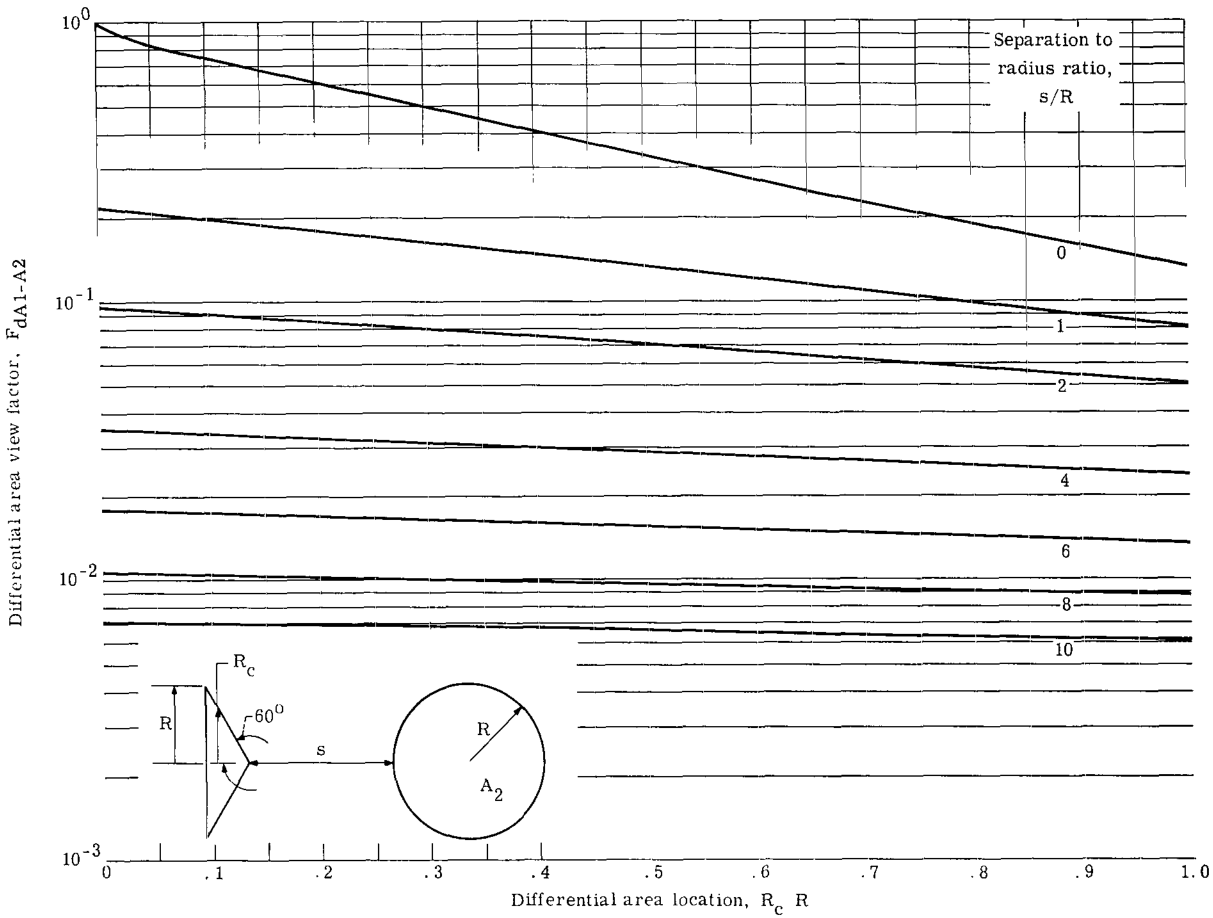
(b) 30° cone.

Figure 8. - Continued.



(c) 45^o cone.

Figure 8. - Continued.



(d) 60° cone.

Figure 8. - Concluded.

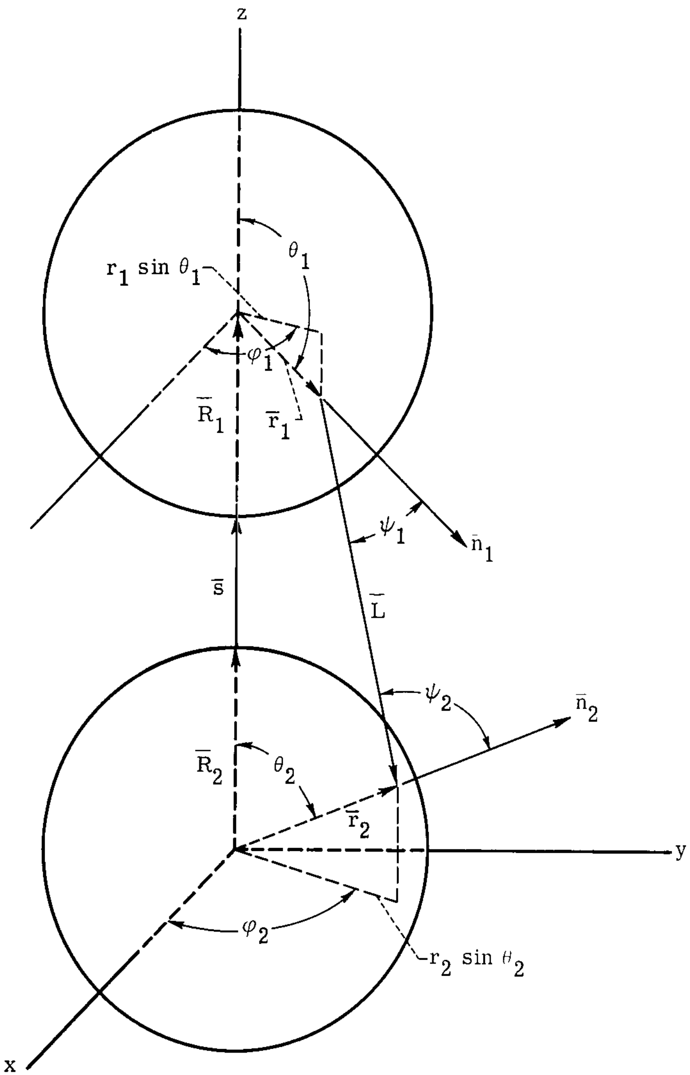


Figure 9. - Sphere-sphere geometry.

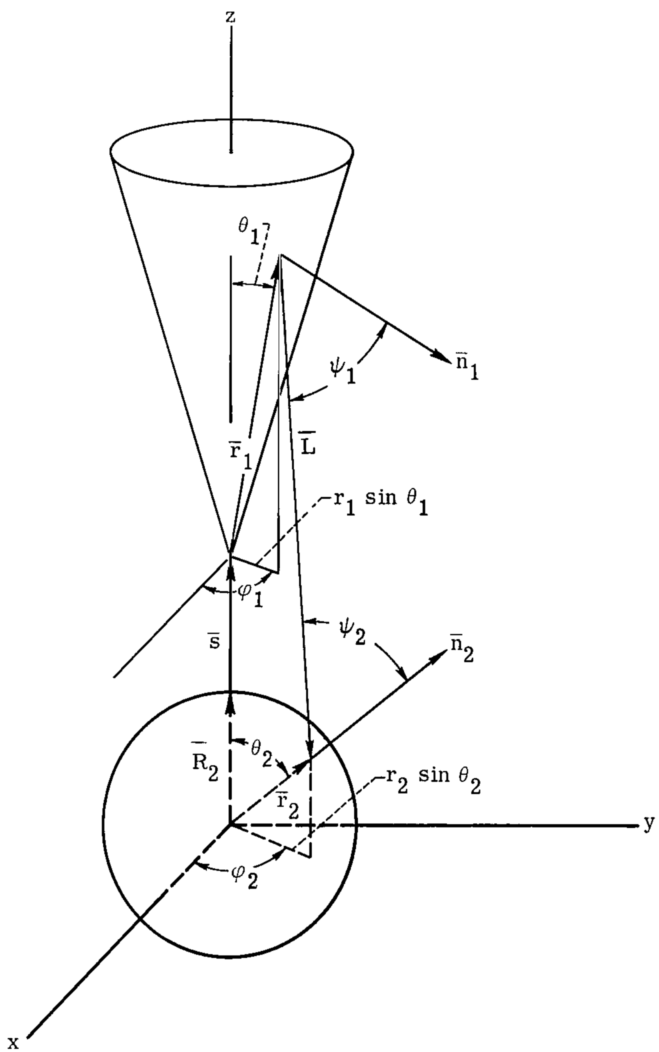


Figure 10. - Cone-sphere geometry.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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