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INVESTIGATION OF AIR-FLOW VELOCITY BY LASER BACKSCATTER

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SUMMARY

An investigation of laser light backscatter properties from an atmosphere emphasized the effect of frequency shift. The detection scheme described is based on this effect and proved successful in the laboratory determination of flow velocity of a contaminated atmosphere. The results agree well with measurements taken with a hot wire anemometer. This investigation used a continuous wave laser radiating in the visible region. The velocity of an air stream containing a small concentration of contaminants was measured. Using this technique to detect clear air turbulence would require that Mie scattering predominate in the turbulent region. This technique does not presently appear practical for airborne detection of clear air turbulence considering the available laser transmitters and detectors, and the uncertain knowledge of the contaminating particle content in a turbulent region.

INTRODUCTION

Aircraft most often encounter clear air turbulence (CAT) near the jet stream at altitudes of 20,000 to 40,000 feet. The region of turbulence is small compared with the overall air volume; it is usually less than 3,000 feet in vertical depth, 20 miles in width, and 50 or more miles in length along the direction of the wind (ref. 1). This localized nature of CAT makes it very difficult to predict from normal meteorological observations. Consequently, many devices have been studied for remotely detecting and determining the severity of CAT regions; as yet, however, no suitable devices have been found. Several different approaches to the detection of turbulent regions are presently being pursued. Some devices rely on the detection of temperature or electric field gradients in the vicinity of turbulence (ref. 1). However, since these effects occur quite near CAT, there would be too little advance warning for the pilot to avoid the area. A fundamental requirement of a CAT detector is that the device give the pilot enough advance warning to avoid the turbulence.

The present situation regarding CAT has been concisely summarized in the following three statements by Dr. Paul Rosenberg in the introduction to reference 1:

"1. The physics and meteorology of CAT are still poorly understood. More research is needed on the mesoscale and microscale causes of CAT and on the mechanisms of its formation.

"2. Forecasting of CAT is still in a rudimentary stage of development.

"3. No method or device has yet been proven able to detect CAT and warn the pilot with a confidence level which is high enough for practical, operational use."

The above conclusions are still valid, and research is required in all three areas. This report deals only with item 3, that of developing techniques to detect CAT.

The success of conventional radar in detecting storms has led to consideration of its use in detecting turbulence. However, the extreme ratio of the wavelength to particle size, and excessive power and antenna size requirements, make its effective use on aircraft appear extremely doubtful. An optical radar using a laser appears more promising because of its much shorter wavelength. The amplitude of the backscattered signal is proportional to the fourth power of the wave number (eq. (B1)). The backscattered signal, hence, also varies directly as the fourth power of frequency. For example, a laser system operating at 10^{14} Hz using the same parameters as a 10^{9} Hz microwave radar system would receive a signal greater by a factor of 10^{20} due to its higher frequency. Another advantage of using a higher frequency is that the diffraction limited beam angle is directly proportional to the ratio of wavelength to aperture diameter. For this reason, a 10^{14} Hz laser system using the same parameters as a 10^{9} Hz radar system would have a 10^{5} smaller transmitter aperture for the same beam divergence angle.

This report covers a series of laboratory experiments that were performed to determine the feasibility of using an on-board laser as a probe for detecting CAT. The investigation was made to compare the properties of laser radiation backscattered from a moving and from a static atmosphere. Laboratory experiments rather than aircraft flight experiments were chosen in order to work with an atmosphere whose velocity and contaminants could be controlled. The feasibility of detecting localized velocity in an air stream using laser backscatter was investigated because clear air turbulence in some instances is identified by a localized high velocity unidirectional air stream. Wind velocity of this type in a plane normal to aircraft flight path can cause shear forces destructive to the aircraft structure.

SCATTERING MECHANISM AND LASER PROPERTIES

This investigation of laser detection of CAT involved a preliminary study of the scattering phenomena and the properties of the laser beam. From this study the frequency property of an atmospherically backscattered beam was selected for detailed investigation.

Atmospheric scattering consists of three phenomena: Rayleigh scattering from gas molecules and microscopic particles that are small compared with the wavelength of light, Mie scattering from dust particles that are comparable in size to the wavelength of light, and refractive scattering due to inhomogeneities of the refractive index. An analytical study of the latter phenomenon has revealed it to be insignificant compared with the first two (ref. 2).

A study of the effects of turbulence on the characteristics of backscattered radiation requires an understanding of the properties of the backscattered laser light. Important properties to be considered are amplitude, polarization, coherence, and frequency. The amplitude is an indication of the power in the beam. Polarization of the wave indicates the orientation of the electric-field vector as a function of time in a plane perpendicular to the direction of propagation. It can be visualized as a lissajous figure described by the tip of the electric-field vector in the plane. For example, a plane-polarized wave forms a straight-line lissajous figure. One of the most important laser characteristics is coherence which allows the laser to be radiated in a narrow beam and focused into an intense spot. It is also this property that makes light incident on a surface appear granular. There are two types of coherence, spatial and temporal. The two coherence characteristics can be thought of as one three-dimensional coherence function; spatial coherence describes phase correlation across the wave front, and temporal coherence describes phase correlation of the radiation in the direction of propagation. Temporal coherence indicates the frequency stability. The frequency of the laser electromagnetic wave is of the order of 10^{14} Hz. The laser output can be composed of one or several discrete frequencies depending on the optical cavity length and laser power input.

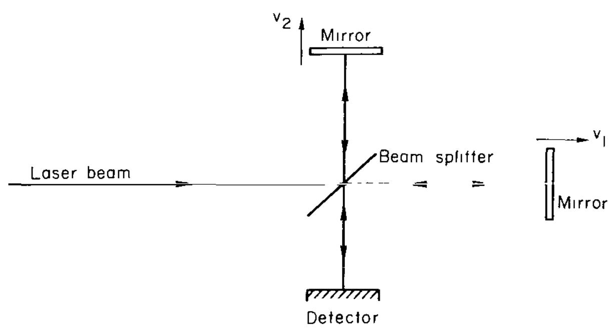
Preliminary experiments and calculations based on fluctuations in the local refractive index showed that the amplitude of the returned signal was not significantly modified in a turbulent region. On the other hand, experiments and calculations have shown that if the turbulent region contains significantly greater particle concentrations than the surrounding nonturbulent region, a change of amplitude could be detected (ref. 3). However, even in this case, the amplitude probably would not be a reliable property to use since absorption and scattering from clouds, and phenomena other than CAT, would affect the amplitude during transmission. There are several efforts currently directed toward determining the feasibility of detecting CAT by observing the amplitude change of backscattered laser light. Some research programs are currently using instrumented aircraft to investigate the amplitude change in the light backscattered from a turbulent region. One of these showed only marginal potential for amplitude detection.

It was therefore decided to concentrate the present study on the frequency effects in a backscattered laser beam. This choice was further supported because of the direct relationship between laser light frequency and particle velocity. Furthermore, it was felt that a frequency shift proportional to the severity of the turbulence could be predicted. An analysis of the detection problem from the standpoint of the frequency property indicates that some particulate matter must be present in CAT in order to derive a detectable backscattered signal with current laser technology because the mean air molecular or particle velocities must be detected in the presence of Brownian motion. Brownian motion, the random movement of molecules and particles due to temperature, as shown in appendix C, spreads the frequency spectrum of the return signal. This spread is larger for molecules than for particles, and would make detection of the mean molecule velocity difficult. For example, an atmosphere moving away from the laser source at 6 m/sec and unaffected by Brownian motion, would cause a frequency shift in laser backscatter of about 20 MHz for a laser beam frequency of 4.8×10^{14} Hz. For a normal atmosphere Brownian motion causes a half power bandwidth frequency spread of about 940 MHz from molecules, and a frequency spread of only 2.3 kHz from particles, such as dust of 2 micron diameter. For this example, the frequency spread of laser backscatter from molecules is far greater than the frequency shift due to the mean atmospheric velocity; from particles the converse is true. Thus, it can be seen that Brownian motion should not cause detection problems for particle scattering, but would be a very severe problem for molecular scattering.

Recent meteorological data and speculation by various authorities in the field indicate that a turbulent region may contain significant particulate matter (refs. 4-6).

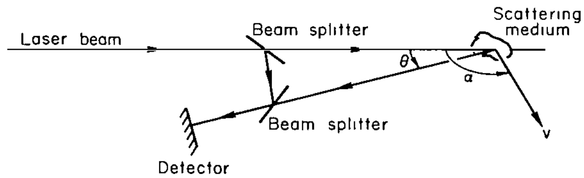
Experimental Analysis

A number of methods were investigated to detect the frequency shift due to particle motion. The frequency shift can be described by considering the Michelson interferometer of sketch (a). As can be seen, there are two paths over which the laser beam can travel to reach the detector. The difference in length of these two paths is called the optical path difference (OPD). If the OPD is a multiple of a wavelength, constructive interference occurs at the detector. As one mirror is moved relative to the other, the OPD changes. When the OPD is an odd multiple of one-half wavelength, destructive interference occurs. If the interferometer is modified by imparting a velocity to each mirror, a sequence of interference effects will be observed at the detector. This sequence of constructive and destructive interferences is a result of the Doppler shift of the laser beam frequency from each mirror. If the mirror velocities are different, the difference in the corresponding Doppler frequencies can be detected. If one considers the mirrors to be moving with velocities V_1 and V_2, the OPD will be $2(V_2 - V_1)t$ after a time t assuming each mirror is an equal distance from the beam splitter at $t = 0$. The phase difference varies directly as the OPD and cycles through multiples of 2π. The time required for one cycle defines the difference frequency (f_d) of the interference change (the difference between the two Doppler frequencies). Therefore, $(4\pi/\lambda)(V_2 - V_1)/f_d = 2\pi$ which yields $f_d = 2(V_2 - V_1)/\lambda$ where λ is the wavelength of the transmitted light.



Sketch (a)

Two interferometer detection techniques can be derived for determining the Doppler frequency shift from a scattering medium. In the first technique one mirror is stationary and the other is replaced by the scattering medium. The stationary mirror provides a reference source which serves the same purpose as a local oscillator in standard heterodyne circuits. The technique will be referred to as the head-wind det.‿tion technique. A system of this type is shown in sketch (b). Because of the moving particles in the medium a continually changing optical path difference exists between the reference source and scattering source. The Doppler frequency shift as derived in appendix D is given by the following equation.

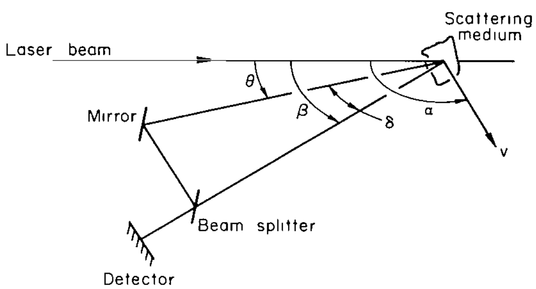


Sketch (b)

$$f_d = \frac{2V \cos\left(\alpha - \frac{\theta}{2}\right) \cos \frac{\theta}{2}}{\lambda}$$

It can be seen that the system detects the Doppler frequency shift due to velocity along the receiver line of sight (for small values of θ).

An alternate technique which will be referred to as the cross-wind detection technique relies on scattered radiation from two slightly different scattering angles (sketch (c)). The radiation from two slightly different scattering angles is photomixed thus eliminating the use of a reference source. The Doppler frequency equation (1) derived in appendix E shows that the



Sketch (c)

$$f_d = \frac{4V}{\lambda} \sin \frac{\delta}{2} \sin \left[\alpha - \left(\theta + \frac{\delta}{2}\right)\right] \quad (1)$$

frequency is a function of the velocity component normal to the bisector of the angle subtended between receivers.

In summary the two detection techniques described differ in two ways. The first difference is the velocity detected. The component of velocity detected by the head-wind technique is parallel to the receiver line of sight. Any relative motion along this line of sight will cause a Doppler shift. Such a technique applied to the detection of air velocity ahead of an aircraft would also register aircraft velocity. The cross-wind technique measures the velocity component normal to the bisector of the angle subtended by the two receivers. Cross or vertical winds could therefore be detected ahead of an aircraft independently of aircraft velocity. The second difference is the use of a reference source in the head wind case and not in the alternate. Thus intensity must be adjusted in the head wind case. In the cross-wind case the intensity of laser backscatter collected from each receiver optics is inherently of equal magnitude. Because of these two differences, the cross-wind technique appeared to be more useful for detecting clear air turbulence and was implemented for experimental feasibility. However, it can be seen that the combination of the two detection techniques described can be used to detect a three-dimensional velocity vector.

Experimental Results

A laboratory detection system was constructed in which a laser beam was scattered from an air stream. A very slight amount of smoke, not visible under ordinary illumination, was added to the air stream to produce Mie scattering from which a velocity-dependent backscattered difference frequency was detected. The experimental arrangement is shown pictorially in figure 1, and

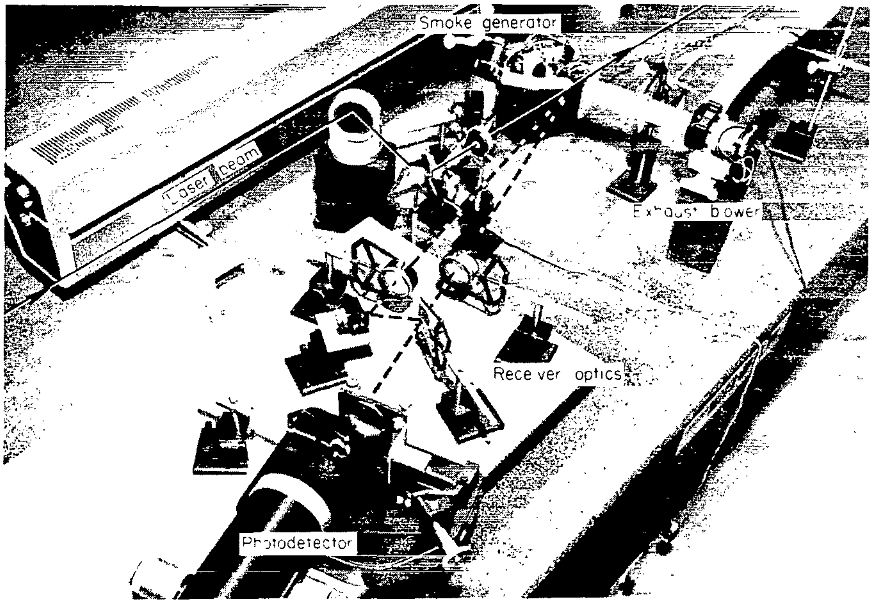


Figure 1.- Experimental arrangement of alternate technique to detect velocity from laser backscatter.

diagrammatically in sketch (d). A continuous wave helium neon laser operating at a wavelength of 6328 Å and 80 mW output was used as the illuminating source. Particles were produced by heating smoke pellets.[1] The smoke was sent to the air intake of a variable speed blower which controlled the velocity of the particles through the test region.

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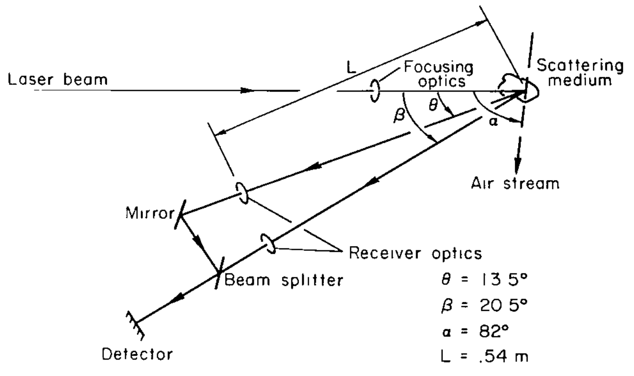
[1]Lionel toy electric train smoke pellets with particles in the 0.03 to 1 micron range and approximately 10^4 particles/cm^3.

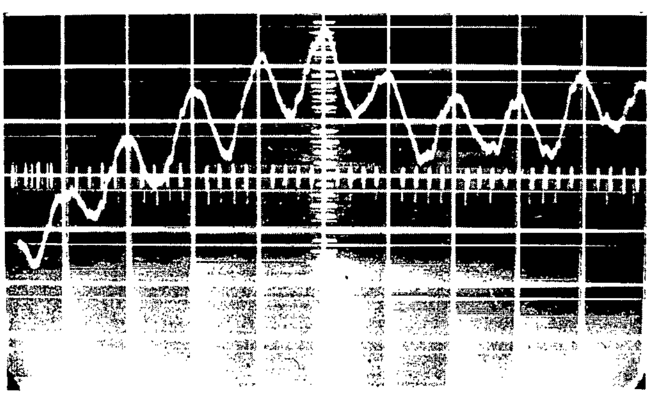


Sketch (d)

The laser beam was focused on the air stream to provide highly localized backscatter. The field of view of each optical path was limited so that only the small area about the laser focal point was imaged on the detector. For optimum superposition of images the optical magnifications were made equal; that is, the ratios of object to image distance were the same. This then allowed particle images to remain superimposed for a maximum time. This condition is necessary for photo-mixing which requires the wave fronts of the two beams to be plane and parallel at the detector in order to avoid a variation in phase of the difference frequency signal over the photocathode surface.

The required conditions for optical alinement were satisfied utilizing a diffusely reflecting target placed at the point where the air stream was to be investigated. The ease of visually detecting the images from the two optical receivers simplified the superposition procedure. Slight vibrations of the target caused a sinusoidal difference frequency signal at the output of the detector which was viewed on an oscilloscope. The alinement was verified when the displayed signal could be interrupted by blocking either channel.

The velocity of the stream was measured with a hot wire anemometer. The detected signal developed across the anode load of the photomultiplier was displayed directly on an oscilloscope. A typical trace is shown in sketch (e).



Sketch (e)

The frequency of the signal is a measure of particle velocity; the random amplitude variation is a result of particle density fluctuation. The velocities determined from the frequencies of many such signal measurements are plotted versus the velocity determined from the anemometer in figure 2. The straight line shows one to one correspondence; the experimental variation from a straight line is within the accuracy of the measurements. For example, for the conditions of sketch (e) the difference frequency was measured to be 100 kHz. The air-stream velocity was calculated from equation (1) rearranged below as

$$V = \frac{\lambda f_d}{4 \sin \frac{\zeta}{2} \sin\left[\alpha - \left(\theta + \frac{\zeta}{2}\right)\right]}$$

$$V = 28.6 \text{ cm/sec}$$

evaluated for:

$$\lambda = 0.6328 \times 10^{-4} \text{ cm} \qquad \zeta = 7^{\circ}$$

$$f_d = 10^5 \text{ Hz} \qquad \theta = 13.5^{\circ}$$

$$\alpha = 82^{\circ}$$

The corresponding anemometer reading was V = 29.5 cm/sec. The velocities investigated were low in order to keep the difference frequency within the band pass of the oscilloscope; however, there is no reason why higher velocities could not be detected.

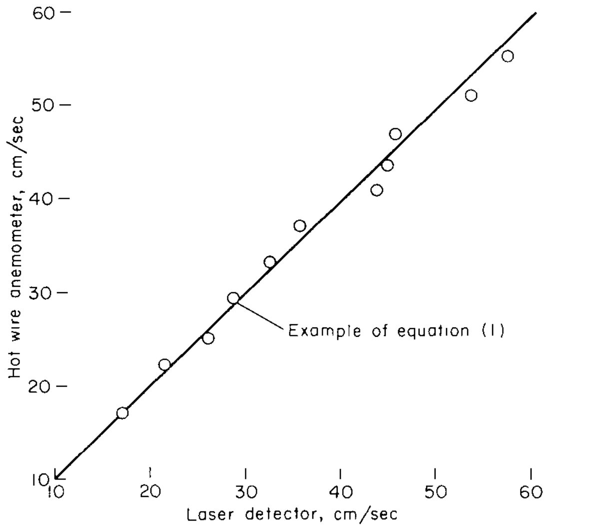


Figure 2.- Comparison of air jet velocity measured by hot-wire anemometer and by backscattered laser detection technique.

For an aircraft CAT detection system using a dual optical detection technique, the angular separation of the collecting optics would be very small for long ranges. Because of this small angle, the velocity difference as seen by the optical systems would be very small. However, because of the high frequency of laser radiation, the difference frequency would still be significant. For example, if the receiving optics were separated by a distance of 1.5 meters, and observed a volume of air 24 km away that was moving normal to the direction of laser propagation at 7.5 m/sec, the difference frequency would be approximately 1 kHz. Detection of frequencies above 1 kHz should not present any problem since fluctuations in amplitude of backscatter due to atmospheric scintillation are below 1 kHz (ref. 7). In concept, the detection system described appears feasible as a CAT detector; however, the applicability of the system depends on laser power output and the existence of particulate matter in the air volume to be investigated. An upper bound on power requirements can be determined if Rayleigh scattering is considered (particle size small with respect to the wavelength of light); a value of 180 MW is computed in appendix B for a pulsed laser. The same calculation for a continuous wave laser with a detection bandwidth of 10 kHz would yield 18 kW. Power required for Mie scattering would be substantially less than the above value, depending on the particle size and concentration.

Utilization of present pulsed lasers which meet the power requirements is limited because of the low repetition rates and short pulse duration. Present continuous wave lasers are also limited by low output power capability.

The power required and the detectability of the difference frequency signal depends on particulate matter concentrations in a turbulent region. Data on such concentrations is essentially nonexistent. Results of experiments presently being made to determine such concentrations should shed more light on the feasibility of the detection system presented.

CONCLUSIONS

Two different techniques for using laser backscatter to detect the velocity of air flow were investigated. The primary difference between the two techniques is that one uses a conventional local reference source and the other does not. Detection schemes that use a local reference source are maximally sensitive to velocities in the direction of the receiver line of sight. The alternate method developed at Ames Research Center is sensitive to velocities normal to receiver line of sight. This technique detects a difference frequency between radiation backscatter as viewed from slightly different directions. This technique was successfully used to detect the velocity of a slightly contaminated air stream. Furthermore, the difference frequency was linearly proportional to the velocity. The presence of some particles is necessary in order that the Brownian frequency spectrum of the backscatter be small compared to the frequency shift caused by the mean velocity. Detecting clear air turbulence with the technique presented does not appear feasible at present because of the high laser power required and the meager knowledge of the particle content in a CAT region.

Areas where the technique may be applicable with presently available lasers are in the detection of air velocity in wind tunnels, relative air speed on aircraft, cloud velocity, and liquid flow.

Ames Research Center
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APPENDIX A

NOMENCLATURE

Quantity	Description	Units	Representative value[1]
A	area of laser beam at range R	cm^2	
A_r	collecting area of the receiver	cm^2	7.06×10^2
$B = D_r/2R$	1/2 angle subtended by the receiver at range R	radians	6.34×10^{-6}
D_r	diameter of the receiver	cm	30.5 (12 in.)
K	Boltzmann's constant	erg/deg	1.38×10^{-16}
M	mass of a molecule or particle	g	
N	molecular density at range R	$molecules/cm^3$	10^{19}
N_λ	spectral radiance at 7000	$W/cm^2\ sr\text{-}\mu$	10^{-3}
P_o	laser power output	W	
P_r	power received	W	
P_s	scattered intensity	W/cm^2	
$P_t = P_o/A$	transmitted incident intensity at range R	W/cm^2	
R	range to the region of interest (region being probed for turbulence)	cm	2.4×10^6 (15 miles)
T	temperature	^oK	
$V = A \cdot (\Delta t)c$	scattering volume at range R	cm^3	

[1]See footnote on page 11.

NOMENCLATURE - Concluded

Quantity	Description	Units	Representative value[1]
c	velocity of light	cm/sec	3.0×10^{10}
Δf	electrical bandwidth	Hz	10^{8}
h	Planck's constant	W sec^2	6.6×10^{-34}
$k = 2\pi/\lambda$	wave number	cm^{-1}	10^{5}
q	electron charge	coulomb	1.6×10^{-19}
Δt	laser pulse length	sec	1.0×10^{-8}
α	polarizability	cm^3	1.8×10^{-24}
η	quantum efficiency of the photomultiplier tube	electrons/photon	0.1
θ_d	angle between dipole oscillation and propagation of the scattered wave	radians	
θ_r	receiver field of view	radians	10^{-5}
θ_t	transmitter field of view	radians	
$\lambda = c/\gamma$	optical wavelength	cm	6.9×10^{-5}
$\Delta\lambda$	optical filter bandwidth	μ	2×10^{-4}
γ	optical frequency	Hz	4.8×10^{14}
$p = \eta q/h\gamma$	cathode sensitivity	A/W	2.9×10^{-2}

[1]Representative values are approximate values used in appendix B that are realistic for a pulsed laser transmitter system probing a region of interest at a distance of 24 km. 24 km is a realistic minimum distance required to maneuver a subsonic jet in order to avoid the region of interest.

APPENDIX B

TRANSMITTER POWER REQUIREMENTS

The magnitude of the laser transmitter power required to detect backscattered radiation from the atmosphere at a given range is dependent on the presence of both air molecules and particulate matter at the range in question. The information available on the density and characteristics of particulate matter in the atmosphere is limited and insufficient to arrive at a meaningful number for the power required. However, an upper bound for this power can be derived on the basis of a purely molecular atmosphere (Rayleigh scattering). For a representative calculation a conventional pulsed system is first considered with a wide bandwidth receiver having representative values, as described in the nomenclature, to enable detection of the backscatter signal for a signal-to-noise ratio of 10. Following this, calculations for a continuous wave laser are presented.

The signal-to-noise ratio can be represented as $S/N = I^2/i_n^2$ where I is the photomultiplier cathode signal current and i_n is the photomultiplier cathode noise current. Both I^2 and i_n^2 are functions of the power received and differ depending on whether coherent or noncoherent detection is used.

The Rayleigh equation for scattered intensity is given as

$$P_s = \frac{Nk^4\alpha^2 P_t V \sin^2 \theta_d}{R^2}$$

The above equation is based on the following conditions:

(1) Isotropic polarizability

(2) Linear dipole scattering

(3) Spherical particles

Attenuation due to scattering and absorption of the laser beam is neglected since it is small compared to the ratio of backscattered return signal to the laser output power.

Consider the single scattering particle shown in figure 3 and let the laser beam be linearly polarized in the z direction and propagating along the y axis. The radiation backscattered within the cone angle B is collected in the receiver.

If the area of the receiver is small and the distance R large (true for most applications) then it can be assumed that the intensity is uniform over the area of the receiver. This then gives for the power received:

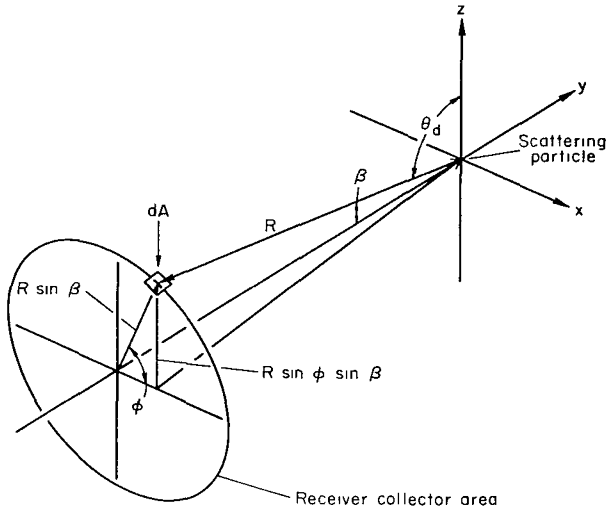


Figure 3.- Single scattering particle.

$$P_r = \frac{Nk^4\alpha^2P_tV\ \sin^2\ \theta_d\pi(D_r/2)^2}{R^2}$$

Also, based on the above assumption

$$B = \frac{D_r}{2R}$$

therefore,

$$P_r = \pi Nk^4\alpha^2P_tV\ \sin^2\ \theta_dB^2$$

For backscatter $\theta_d = 90^o$, P_r can be simplified as

$$P_r = \pi Nk^4\alpha^2P_tVB^2 \tag{B1}$$

For direct detection the photocathode signal current is $I_s = pP_r$ where p is the cathode sensitivity. The total mean square noise current is $i_n{}^2 = i_d{}^2 + i_b{}^2 + i_s{}^2$ where $i_d{}^2$ is mean square dark noise current, $i_b{}^2$ is mean square background current, and $i_s{}^2$ is mean square signal noise current. The conventional relationship between a shot noise current and the dc current that generates it is $i = \sqrt{2q\Delta fI}$; hence $i_n{}^2 = 2q\Delta f(I_d + I_b + I_s)$ where $I_s = pP_r$, $I_b = pP_b$. Therefore

$$\frac{S}{N} = \frac{p^2P_r{}^2}{2qp\Delta f\left(\frac{I_d}{p} + P_b + P_r\right)} \tag{B2}$$

For these calculations a value of $S/N = 10$ was used. The components of the noise current in the expression for S/N can be evaluated for the representative values shown in the nomenclature.

$$P_b = \frac{\pi}{4}\ (N_\lambda\Delta\lambda\theta_r{}^2A_r)$$

$$P_b = 11.1\times10^{-15}\ W$$

The equivalent dark current power input for a 9558B photomultiplier is

$$P_d = \frac{I_d}{p} = 5.2\times10^{-14}$$

Therefore,

$$\frac{S}{N} = \frac{2.9\times10^{-2}P_r^2}{2\times1.6\times10^{-19}\times10^8(5.2\times10^{-14} + 1.11\times10^{-14} + P_r)} = 10$$

which yields

$$P_r = 1.1\times10^{-8}\ \mathrm{W} \tag{B3}$$

This value of P_r is much greater than the background and equivalent dark current powers; thus the signal-noise ratio obtained is very nearly the theoretical optimum for direct detection

$$\frac{S}{N} = \frac{pP_r}{2q\Delta f} \tag{B4}$$

The total pulsed laser power required is determined from equations (B1) and (B3) and is $P_r = 1.1\times10^{-8} = \pi N k^4 \alpha^2 (P_o/A) V B^2$ which yields $P_o = 90\times10^6$ watts required for direct detection with $S/N = 10$ at the range and viewing angle assumed.

The signal-noise ratio for coherent heterodyne detection can be represented as $S/N = (I_{IF})^2/i_n^2$ where I_{IF} is the difference frequency signal current between the two frequencies being detected (one frequency usually being the local oscillator).

$$I_{IF}^2 = 2I_s I_{LO} = 2p^2 P_r P_{LO}$$

where P_{LO} is the local oscillator power

$$i_n^2 = i_d^2 + i_b^2 + i_s^2 + i_{LO}^2$$

where i_{LO}^2 is the mean square local oscillator noise.

$$i_n^2 = 2q\Delta f(I_d + I_b + I_s + I_{LO}) = 2qp\Delta f\left(\frac{I_d}{p} + P_B + P_r + P_{LO}\right)$$

and the signal-to-noise ratio is

$$\frac{S}{N} = \frac{2p^2 P_r P_{LO}}{2qp\Delta f\left(\frac{I_d}{p} + P_B + P_r + P_{LO}\right)} = \frac{pP_r}{q\Delta f\left(1 + \frac{P_r}{P_{LO}} + \frac{P_B}{P_{LO}} + \frac{I_d}{pP_{LO}}\right)} \tag{B5}$$

Rather than estimate actual powers required it is more meaningful to compare the above equation with equation (B4) for direct detection. If the local oscillator power is very large (so that the bracketed factor becomes unity) the signal-to-noise ratio for heterodyne detection is twice as large as that for direct detection. If signals received from slightly different scattering angles from the same source are mixed as described in the report so that equivalently $P_{LO} = P_r$ = half the total received signal, thus the bracket in equation (B5) becomes 2 and the numerator power is reduced by half and twice as much transmitted power would be required as for direct detection or 180 megawatts for $S/N = 10$.

For a continuous wave (cw) laser equation (B5) and equation (B1) can again be used for calculating the transmitted power required for a signal-to-noise ratio of 10. For this case, the bandwidth can be reduced from 10^2 MH to 10 kHz (the bandwidth can be reduced for the cw laser since the large bandwidth required for the pulsed laser was needed to resolve the very fast rise time pulses) and the power transmitter requirement can correspondingly be reduced by a factor of 10^4. Therefore, the cw transmitter power required equals 18 kW.

APPENDIX C

FREQUENCY BANDWIDTH DUE TO BROWNIAN MOTION

The Maxwell-Boltzmann distribution which characterizes Brownian motion has the form

$$dn = n \sqrt{\frac{n}{2\pi KT}}\, e^{-\frac{M_v^2}{2KT}}\, dv$$

where dn represents the number of atoms of mass M which, among a total of n, have a velocity component between v and v + dv. In order to determine the frequency bandwidth, the Doppler frequency relation $f_d = (2v\gamma/c)$ must be used so that the distribution is a function of f_d instead of v; the function then becomes

$$dn = n \sqrt{\frac{n}{2\pi KT}}\, e^{-\frac{M_c^2 f_d^2}{8KT\gamma^2}}\, \frac{c}{2\gamma}\, df_d$$

Since the backscatter intensity is proportional to the number of atoms, it can be written as

$$P = Ae^{-\frac{M_c^2 f_d^2}{8KT\gamma^2}}$$

where A is a constant. The frequency f_d for which P = A/2 determines the half bandwidth; therefore

$$e^{-\frac{M_c^2 f_d^2}{8KT\gamma^2}} = \frac{1}{2}$$

or

$$\frac{M_c^2 f_d^2}{8KT\gamma^2} = \ln 2$$

and

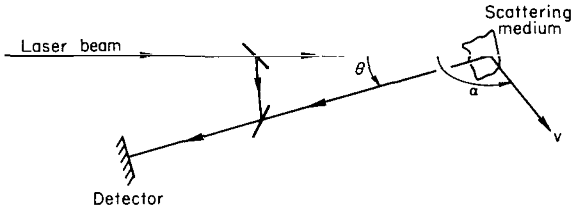
$$f_d = \frac{\gamma}{c}\sqrt{\frac{8KT\ \ln 2}{M}}$$

For typical air molecules with $M = 5\times10^{-23}$ gram, the half power bandwidth is $f_d = 938$ MHz. For a 2 micron silt particle of mass $M = 8.4\times10^{-12}$ gram the half bandwidth is $f_d = 2.3$ kHz.

APPENDIX D

DERIVATION OF DOPPLER FREQUENCY FOR HEAD-WIND TECHNIQUE

The Doppler frequency shift (due to velocity) for the conventional technique can be derived using the optical path difference concept. Consider the detection geometry shown in sketch (f). The change in optical path length is due to the velocity which can be expressed as two components, one parallel and the other perpendicular to the direction of the laser beam propagation. The parallel component of velocity $-V \cos \alpha$ results in a change of optical path after a time t of $-Vt \cos \alpha(1 + \cos \theta)$. The perpendicular velocity component $V \sin \alpha$ gives an optical path change of $-Vt \sin \theta \sin \alpha$ for the same time t. The total optical path change after a time t is then



Sketch (f)

$$Vt[-\cos \alpha(1 + \cos \theta) - \sin \alpha \sin \theta]$$

simplifying

$$-2Vt \cos\left(\alpha - \frac{\theta}{2}\right) \cos \frac{\theta}{2}$$

An optical path change equal to one wavelength corresponds to one cycle of the Doppler frequency. Therefore,

$$f_d = \frac{2V}{\lambda} \cos\left(\alpha - \frac{\theta}{2}\right) \cos \frac{\theta}{2}$$

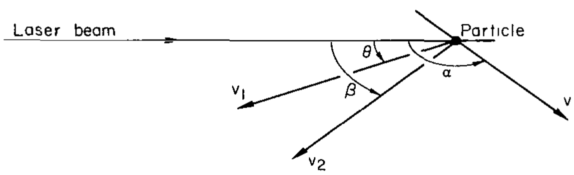
APPENDIX E

DERIVATION OF THE DOPPLER FREQUENCY FOR CROSS-WIND TECHNIQUE

The equation for the difference frequency f_d for the alternate detection technique is identical to that derived for the interferometer; that is,

$$f_d = \frac{2(v_2 - v_1)}{\lambda}$$

To evaluate f_d for a velocity vector at an angle α with respect to the direction of laser propagation, consider the geometry as shown in sketch (g).



Sketch (g)

The angles θ and B define the direction of backscatter observed by each optical collector. It is the difference in particle velocity as seen from these directions that is needed to evaluate f_d. The component of particle velocity along the direction θ is $V \cos(\alpha - \theta)$ and that along B is $V \cos(\alpha - B)$. The differential velocity is then

$$v_2 - v_1 = V[\cos(\alpha - B) - \cos(\alpha - \theta)]$$

Expanding

$$v_2 - v_1 = V[\cos B \cos \alpha + \sin B \sin \alpha - \cos \theta \cos \alpha - \sin \theta \sin \alpha]$$

$$= V[\cos \alpha(\cos B - \cos \theta) + \sin \alpha(\sin B - \sin \theta)]$$

Using the trigonometric identities

$$\cos B - \cos \theta = -2 \sin \frac{1}{2} (\theta + B) \sin \frac{1}{2} (B - \theta)$$

$$\sin B - \sin \theta = 2 \sin \frac{1}{2} (B - \theta) \cos \frac{1}{2} (\theta + B)$$

yields

$$v_2 - v_1 = V \cos \alpha \left[-2 \sin \frac{1}{2} (\theta + B) \sin \frac{1}{2} (B - \theta)\right]$$

$$+ V \sin \alpha \left[2 \sin \frac{1}{2} (B - \theta) \cos \frac{1}{2} (\theta + B)\right]$$

$$= 2V \sin \frac{1}{2} (B - \theta)\left[\sin \alpha \cos \frac{1}{2} (\theta + B) - \cos \alpha \sin \frac{1}{2} (\theta + B)\right]$$

$$= 2V \sin \frac{1}{2} (B - \theta) \sin\left[\alpha - \frac{1}{2} (\theta + B)\right]$$

The difference frequency is then

$$f_d = \frac{4V}{\lambda} \sin \frac{1}{2} (B - \theta) \sin\left[\alpha - \frac{1}{2} (\theta + B)\right]$$

Letting

$$\zeta = B - \theta$$

gives

$$f_d = \frac{4V}{\lambda} \sin \frac{\zeta}{2} \sin\left[\alpha - \left(\theta + \frac{\zeta}{2}\right)\right]$$

When the optics are symmetrically placed about the laser line of sight, $\theta = -B$ and f_d reduces to $(4V/\lambda)\sin \theta \sin \alpha$ where $V \sin \alpha$ is the component of velocity perpendicular to laser propagation.

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