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DETERMINING THE SPIN AXIS OF A SPINNING SATELLITE

by F. O. Vonbun

Goddard Space Flight Center
Greenbelt, Md.

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ABSTRACT

Many satellites sent into orbit during the last few years are spinning to maintain stabilization, or for other reasons. The purpose of this paper is to determine the spin axis, a unit vector $\vec{s}^{\circ}$, of the satellite using ground tracking information. It will be shown that range rate information is adequate for this purpose. The determination of $\vec{s}^{\circ}$ is necessary to check the onboard sensors once the spacecraft is in orbit. In the case of a slight onboard malfunction, the spin axis could not be determined otherwise.

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INTRODUCTION

Often it is desirable or necessary to determine the spin axis of a rotating satellite in space in order to check onboard sensors or determine the change in spin axis because of a malfunction. In the latter case many of the experiments onboard can still be evaluated as long as the spin axis is known. Determining the spin axis by the method outlined here requires the radiating antenna to be displaced from the spin axis center. This condition is necessary since the Doppler modulation imposed on the range rate of the rotating satellite is used for the analysis. The range rate of a satellite in orbit can be determined with great precision (References 1 through 4); errors in the order of cm/sec have been achieved with CW-ranging systems. This fact will be utilized to determine the spin axis (unit vector $\vec{s}^{\circ}$)* of a rotating satellite. The only assumptions made here are that the antenna of the satellite is displaced from its axis of symmetry, that the spacecraft is spinning only, and that the satellite orbit can be, or has been, determined.

RANGE RATE MODULATION DUE TO SATELLITE SPIN

Some of our satellites are spinning in space with the antenna displaced from its spin axis.† This paper shows how the range rate $\dot{r}$ is modulated by the spinning motion of the spacecraft. Figures 1 and 2 depict the vector diagram of the antenna motion in detail as well as the angle ψ = arc cos $(\vec{r}^{\circ} \cdot \vec{s}^{\circ})$ between the satellite spin axis $\vec{s}^{\circ}$ and the satellite position vector $\vec{r}$ (or $\vec{r}^{\circ}$). This angle, as shown later, will be used for the determination of $\vec{s}^{\circ}$. Using vector notation, the antenna position vector $\vec{d}$ can be written as

$$\vec{d} = \vec{d}_0 \cos \omega\Delta t + d\left(\vec{s}^{\circ} \times \vec{d}_0^{\circ}\right) \sin \omega\Delta t \quad , \tag{1}$$

*A unit vector will always be designated with a zero superscript; that is, $\vec{x}^{\circ} = \vec{x}/|\vec{x}|$.

†See Reference 5 for centered turnstile antenna.

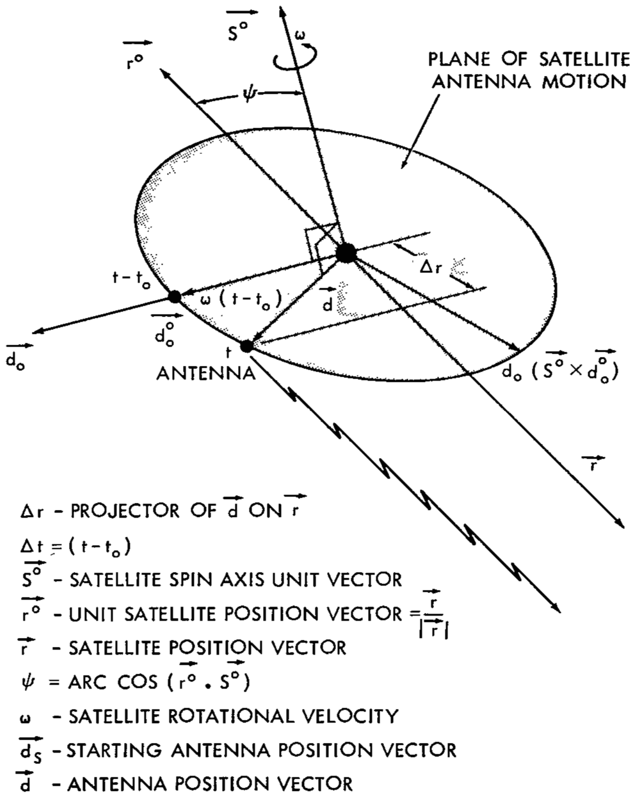

Figure 1—Antenna motion vector diagram.

Also, differentiating Equation 1 with respect to time and introducing it into Equation 3 yields, after some manipulation,

$$\Delta\dot{r} = -\omega d\left(\vec{d}_0^\circ \cdot \vec{r}^\circ\right) \sin \omega\Delta t + \omega d\left(\vec{d}_0^\circ \cdot \left(\vec{r}^\circ \times \vec{s}^\circ\right)\right)\cos\omega\Delta t + \left(\frac{d}{r}\right)\left(\vec{d}^\circ \cdot \dot{\vec{r}}\right) - \left(\frac{d}{r}\right)\dot{r}\left(\vec{d}^\circ \cdot \vec{r}^\circ\right) \quad . \qquad (4)$$

Inspecting Equation 4 shows that as long as

$$\frac{|\dot{\vec{r}}|}{r} << \omega \quad \text{and} \quad \frac{\dot{r}}{r} << \omega \ , \qquad (5)$$

the third and fourth terms of Equation 4 can be neglected.*

where $\vec{d}^\circ = d \cdot \vec{d}_0^\circ$ is the starting position of the spacecraft antenna at $t = t_0$ or $\Delta t = 0$. Equation 1 follows directly from Figure 1, as can be seen. Projecting $\vec{d}$ onto the range vector $\vec{r}$ results in:

$$\Delta r = \left(\vec{d} \cdot \vec{r}^\circ\right) = \frac{1}{r}\left(\vec{d} \cdot \vec{r}\right) \ . \qquad (2)$$

Since the spacecraft is spinning, the value Δr changes with time (and with orbital motion, which is assumed to be negligible as shown later); the following is obtained by differentiating Equation 2 with respect to time:

$$\Delta\dot{r} = \frac{1}{r^2}\left\{r\left[\left(\dot{\vec{d}} \cdot \vec{r}\right) + \left(\vec{d} \cdot \dot{\vec{r}}\right)\right] - \left(\vec{d} \cdot \vec{r}\right)\dot{r}\right\} \cdot \qquad (3)$$

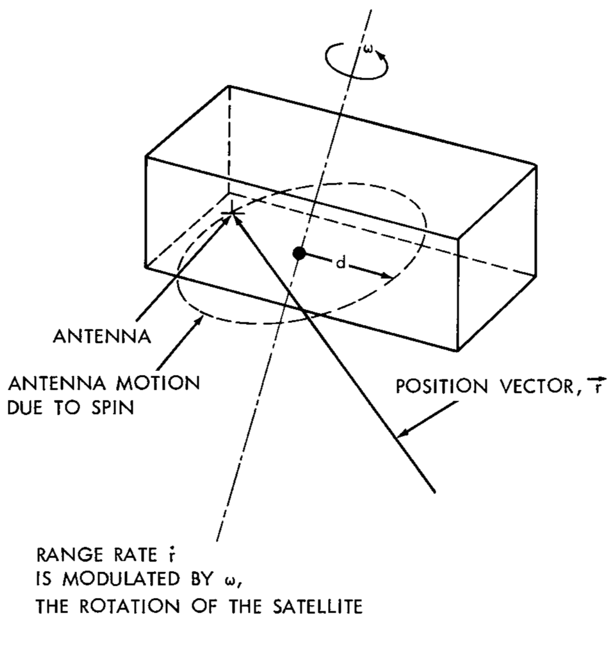

Figure 2—Satellite and antenna details where d equals antenna distance from spin axis.

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*Example: $|\dot{\vec{r}}| \doteq |\vec{v}|$ (neglect earth rotation) $\doteq$ 8000 m/sec
n = 9 revolutions per min.; $\omega = 2\pi n/60$ radians/sec = 1 sec^{-1} (approx.)
$r \doteq$ 400 km (slant range)
than: $|\dot{\vec{r}}|/r \doteq 2.10^{-2}$ sec^{-1} << 1 sec^{-1}.

Since $\vec{d}_0^\circ$ is an arbitrary starting vector (unit vector) it will be chosen so that

$$\left(\vec{d}_0^\circ \cdot \vec{r}^\circ\right) = 0 , \tag{6}$$

or

$$\vec{d}_0^\circ = \frac{1}{\sin\psi}\left(\vec{r}^\circ \times \vec{s}^\circ\right) .$$

Introducing Equation 6 into Equation 4, neglecting the terms just mentioned, yields

$$\Delta\dot{r} = (\omega d \sin\psi) \cos\omega\Delta t , \tag{7}$$

(with $v/r \ll \omega$, $\dot{r}/r \ll \omega$) as the range rate modulation caused by the spinning motion of a satellite.

If the range rate modulation $\Delta\dot{r}_i$ from the i^{th} ground tracking station is given as shown in Figure 3, the angle ψ_i between the spin axis and the position vector $\vec{r}_i$ (or $\vec{r}_i^\circ$) can be determined from Equation 7; that is,

$$\psi_i = \arc \sin \left(\frac{\Delta\dot{r}_{i\,max}}{\omega_i\, d}\right) , \tag{8}$$

since $\omega = 2\pi/T$ is given by the same process. A better determination of ω can be obtained from the power spectrum of the range-rate information as shown in Figure 4. Angle ψ_i will now be used to determine the components of the unit vector $\vec{s}^\circ$, the satellite spin axis.

DETERMINING COMPONENTS OF SATELLITE SPIN AXIS UNIT VECTOR $\vec{s}^\circ$ AND THEIR ERRORS

Adapting the legend in Figure 1 (see also Figure 5) gives:

$$\cos\psi_i = \left(\vec{r}_i^\circ \cdot \vec{s}^\circ\right) , \tag{9}$$

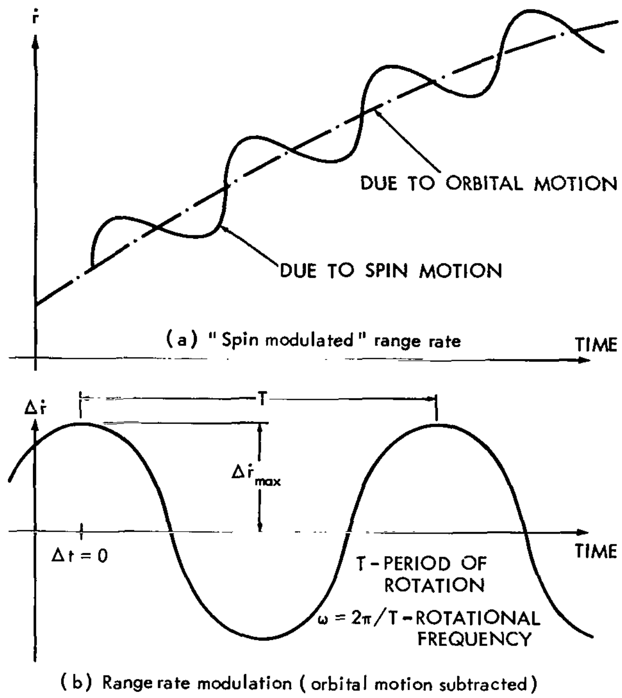

Figure 3—Range rate modulation.

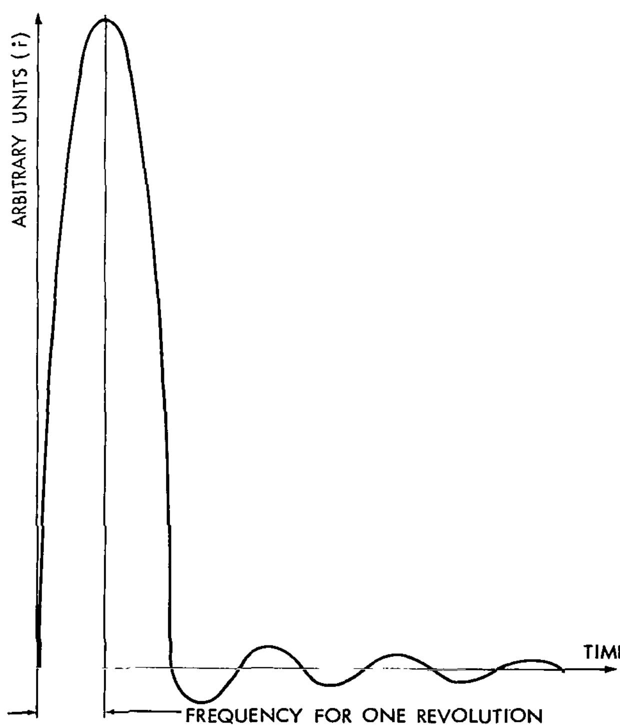

Figure 4—Power density spectrum.

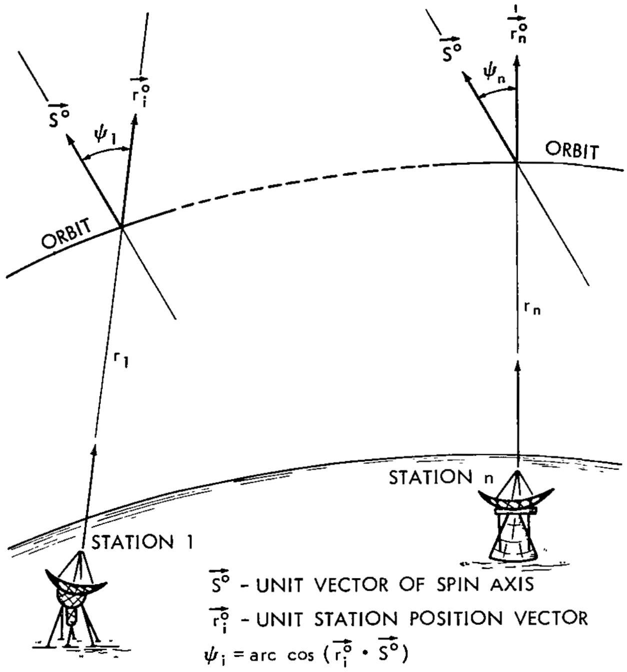

Figure 5—Satellite spin axis in relation to ground-station line of sight.

where $\vec{r}_i^\circ$ is the unit satellite position vector of the i^{th} ground station. From Equation 8, the angle ψ_i can be determined; thus only $\vec{s}^\circ$ is unknown in Equation 9. A straight-forward solution of Equation 9 would be to measure the $\Delta\dot{r}_i$ from three stations and determine the components s_1, s_2, s_3 from Equation 9.

However, a much better solution can be obtained by using all available information; that is, by using much more than three measurements. In practice, many ground stations take a large number of range rate measurements. This in turn dictates a least-square solution, since more than the necessary three equations (measurements) are available.

The dot product of Equation 9 can be written also in component form as

$$\cos\psi_i = r_{i1} s_1 + r_{i2} s_2 + r_{i3} s_3 , \tag{10}$$

where the values r_{i1}, r_{i2}, r_{i3} and s_1, s_2, s_3 are the components of the unit vectors $\vec{r}_i^\circ$ and $\vec{s}^\circ$, respectively.

Using many measurements of $\Delta\dot{r}_i$ and ω_i from which ψ_i can be calculated from Equation 8, one has a set of linear equations at hand. From Equation 10 one obtains for n measurements of $\Delta\dot{r}_i$ and ψ_i the following:

$$\begin{aligned} a_1 &= \cos\psi_1 = r_{11} s_1 + r_{12} s_2 + r_{13} s_3 \\ a_2 &= \cos\psi_2 = r_{21} s_1 + r_{22} s_2 + r_{23} s_3 \\ &\vdots \\ a_n &= \cos\psi_n = r_{n1} s_1 + r_{n2} s_2 + r_{n3} s_3 . \end{aligned} \tag{11}$$

Equation 11 suggests a matrix treatment which will be followed from this point on. In matrix form Equation 11 reads

$$A_{(n\times 1)} = R_{(n\times 3)} \cdot S_{(3\times 1)} , \tag{12}$$

where the matrices A, R, and S (the unit vector of the satellite spin axis) are as follows:

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}_{(A\times 1)} , \qquad R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ \vdots & \vdots & \vdots \\ r_{n1} & r_{n2} & r_{n3} \end{bmatrix}_{(n\times 3)} , \qquad S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}_{(3\times 1)} .$$

To solve Equation 12 in the least-square sense, standard procedures are being applied (References 6 through 12). Since in practice Equation 12 as such will never exist, because of enhanced errors in the measurements as well as in the analyses, one must write instead:

$$A_{(n\times 1)} = R_{(n\times 3)} \cdot S_{(3\times 1)} + \epsilon_{(n\times 1)} , \tag{13}$$

where ϵ is an error matrix with expectation

$$E(\epsilon) = 0 \tag{14}$$

and the dispersion matrix

$$V(s)_{(n\times n)} = E\left(\epsilon\epsilon^T\right) = \sigma_{ai}^2 \, I_{(n\times n)} , \tag{15}$$

where $I_{(n\times n)}$ is a unit matrix and σ_{ai}^2 is the variance of the measurements made. This means that in each of Equations 11 an error term must be added to correspond with actual conditions.

Equation 13 must be used to determine the matrix S or the unit vector $\vec{s}_i^{\,\circ}$. The principle of least squares requires that S be solved for in such a way that the sum of the errors squared $\left(\epsilon^T \epsilon\right)$ is a minimum (ϵ^T is the transpose of ϵ). A necessary condition for this is that

$$\frac{\partial}{\partial S} \left(\epsilon^T \epsilon\right) = 0 ; \tag{16}$$

or, using Equation 13, one obtains from Equation 16

$$\frac{\partial}{\partial S} \left[\left(A^T - S^T R^T\right) \cdot (A - RS)\right] = 0 ,$$

which yields, after some calculations, the best estimate for S; that is,

$$\hat{S}_{(3\times 1)} = \left(R^T R\right)^{-1}_{(3\times 3)} R^T_{(3\times n)} A_{(n\times 1)} . \tag{17}$$

Equation 17 represents the solution to the problem of determining satellite spin axis.

In order to estimate the errors made in the determination of $\vec{s}^{\,\circ}$, its dispersion matrix must be calculated. From Equations 17 and 15 one obtains:

$$\hat{S} = \left(R^T R\right)^{-1} R^T (RS + \epsilon) . \tag{18}$$

Using Equation 15 and introducing Equation 18, one obtains,

$$V(\hat{S}) = E\left(\epsilon\epsilon^{T}\right) = \sigma_{ai}^{2}\left(R^{T}R\right)^{-1} \tag{19}$$

for the dispersion matrix of $\hat{S}$.

ESTIMATING MEASUREMENT VARIANCE

In order to evaluate the dispersion matrix $V(\hat{S})_{(3\times 1)}$ the measurement variance must be estimated also. From Equation 8 $\Delta\dot{r}_1$ and ω_i are measured quantities which in turn have errors associated with them. These can be estimated from the noise of the data (this is to be assumed here as known). From Equations 8 and 11 one can write

$$\sin\psi_i = \frac{1}{d}\left(\frac{\Delta\dot{r}_{i\,max}}{\omega_i}\right), \tag{20}$$

and

$$\cos\psi_i = a_i \; .$$

Varying both equations, one obtains for the variation in a_i

$$\delta a_i = \frac{\Delta\dot{r}_{i\,max}}{\left(d\omega_i\right)^2\sqrt{1-\left(\dfrac{\Delta\dot{r}_{i\,max}}{d\omega_i}\right)^2}}\left[\delta\left(\Delta\dot{r}_{i\,max}\right) - \Delta\dot{r}_{i\,max}\left(\frac{\delta\omega_i}{\omega_i}\right)\right] . \tag{21}$$

Assuming now that the errors are uncorrelated and replacing the value $\Delta\dot{r}_{i\,max}$ by $\left(\Delta\dot{r}_{i\,max} + \sigma\Delta\dot{r}_{i\,max}\right)$— since it can never be zero, because of the inherent errors in its determination—one obtains from Equation 21:

$$\sigma_{ai} = \frac{\Delta\dot{r}_{i\,max} + \sigma\Delta\dot{r}_{i\,max}}{\left(\omega_i\, d\right)^2\sqrt{1-\left(\dfrac{\Delta\dot{r}_{i\,max} + \sigma\Delta\dot{r}_{i\,max}}{\omega_i\, d}\right)^2}}\left[\sigma\Delta\dot{r}_{i\,max}^{2} + \Delta\dot{r}_{i\,max}^{2}\left(\frac{\sigma\omega_i}{\omega_i}\right)^2\right]^{1/2} . \tag{22}$$

With the measurement variance σ_{ai}^2 and the matrix R known, the disposition matrix $V(\hat{S})$ given by Equation 19 can be evaluated. Equation 22 shows also that the limitations for σ_{ai}; $\left(\Delta\dot{r}_{i\,max} + \sigma\Delta\dot{r}_{i\,max}/\omega_i\, d\right)$ must be smaller than 1 to prevent excessive errors.

CONCLUSIONS

It has been shown how the range-rate modulation of a spinning satellite with a displaced antenna can be used to determine its spin axis (unit vector) in space. An estimate of the spin axis errors associated with this method has also been presented. It is interesting to note that no additional equipment is necessary on the satellite or on the ground. This method may well be suited for determining the spin axis of communications and navigation satellites. If necessary, it could also be used to check the axis of spin-stabilized kick stages by adding a Doppler transponder and antenna to the stage.

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National Aeronautics and Space Administration
Greenbelt, Maryland, July 27, 1967
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