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INTERACTION EFFECTS IN
MEASUREMENT SYSTEMS

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INTERACTION EFFECTS IN MEASUREMENT SYSTEMS

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ABSTRACT

Presented is a discussion of the problem of matching an instrument to a system such that a minimum disturbance is generated when the instrument is connected. The concept of loading, as determined by such properties as impedance or stiffness, is used to describe these interaction effects. Practical examples of measurements of electrical quantities, temperature, force, pressure, and flowrate are used as a basis for the discussion.

INTRODUCTION

A fundamental goal in measurement engineering is that the addition of instrumentation to a physical system shall not disturb the system from its noninstrumented state. Strictly speaking, however, this goal is impossible to achieve, since all sensors draw some power or energy from the system to which they are applied.

The effects caused by such disturbances are twofold; the disturbance can: (1) change the value of the variable being measured, or (2) change the operating characteristics of the system. A simple example of the first effect is the loading caused by a low-resistance voltmeter; the voltage appearing at the terminals where the measurement is to be made is reduced

by an amount which is proportional to the current flowing in the voltmeter. An example of the second effect might be that the addition of test instrumentation to an oscillator circuit may cause the oscillator to stop oscillating or to change the frequency of the oscillator. This effect often results in a meaningless experiment. Effects of the first type are the ones that will be discussed in this paper; they are commonly referred to as loading effects.

Since disturbances caused by instrumentation are unavoidable, what is really meant by the original statement is, then, that the disturbance caused by the addition of instrumentation should:

1. produce a negligible effect, or
2. produce an appreciable effect which
   a. can be determined, and
   b. can be corrected for with a negligible uncertainty.

A further characteristic of instrument interaction effects worth noting at this point is that these effects occur throughout a measurement system. Figure 1 illustrates this point. A resistance thermometer is used to measure the temperature of a fluid. The system consists of a sensor, a power supply and signal conditioner, an amplifier, and one or more readout instruments. Interaction problems can arise at the various interconnections in the system as noted on the figure. Many of these problems are made negligible by proper choice of the electrical input and output impedances of the system components. Some of the remaining effects can be corrected by system calibration. The "thermal connection" of the sensor to the experiment is a most important effect and is often overlooked because of its nonelectrical nature. Interaction effects of this type will be discussed in some detail later in the paper.

This paper will discuss some examples of loading effects and methods of minimizing them. The discussion will center first on problems in electrical measurements and then go on to problems in the measurement of temperature, force, pressure, and flow.

EXAMPLES OF INTERACTION EFFECTS

Loading Effects in Electrical Measurements

Figure 2 shows some simple examples of loading effects in electrical measurements. The first of these (a), measuring the emf of a standard cell with a voltmeter, is an obvious case of bad practice. The error in emf measurement for this case will be 33 percent and the amount of current drawn will probably damage the cell. To make this measurement properly, one should use a measuring instrument that draws negligible current, i.e., a null-potentiometer.

The circuit shown in 2(b) is commonly used with strain gage transducers in order to obtain a standardized full-scale output signal. In this case, a loading error will result, even if the voltmeter has an infinitely high input resistance, because of the shunting effect of the span adjusting network.

Figure 2(c) shows the measurement of a d-c current using a volt-ohm-milliameter type test set. In this case, the loading error results from the resistance of the milliameter circuit, which, for this type of meter, is quite high. A common test set of this type with a 20,000 Ω/volt sensitivity (a 50 μa meter movement) has a sufficiently high resistance that a 1/4 volt drop appears across the meter terminals for full scale current flow.

Figure 2(d) illustrates the point that the shunt capacitance across the input terminals of a measuring instrument must be considered when making a-c measurements. For the case shown, the oscilloscope with a 1 megohm input resistance shunted by a 50 pF capacitor has an input impedance of about 8000Ω at 400 kHz. The loading error for the case shown will then be about 7 percent is Z_s is capacitive.

Calculation of Loading Effects

Methods of calculating the effect of loading in both voltage and current measurements are given in figure 3. For simplicity these methods are given for d-c measurements. For a-c measurements the resistances of the source, meter, and load must be replaced with their respective a-c impedances. An extension of these methods provides a way of correcting for voltmeter- and ammeter-loading errors by making two measurements. These techniques are described in figures 4 and 5. In each case, a second measurement is made with externally added loading resistors (R_{L3} and R_{M3} in figs. 4 and 5, respectively). The choice of the values used for R_{L3} and R_{M3} is dependent on the source resistance (R_S) in the circuits being measured; the object is to change the loading effect enough to get a measurable change in the indicated variables ($e_2 - e_1$) and ($i_2 - i_1$). Care must be taken, however, to insure that the sources are not damaged by excessive loading. In each case it is also possible to calculate the actual source resistance; these equations are also presented in the figures.

Determination of Load and Source Resistances

In order to evaluate loading effects, it is necessary to know the input resistances (or impedances) of the instrumentation and the source resistance (or impedance) of the signals to be measured. In the case of commercial instruments, this information can usually be obtained from the manufacturer's literature. Also, these values can be obtained experimentally; however, care must be exercised to insure

that shunt capacitances or series inductances are not overlooked if a-c or transient measurements are being made. For sensitive galvanometers, the measurement of coil resistance must be done carefully to avoid damage to the galvanometer; a recommended method for making this measurement is given in figure 6. Source resistances can be measured by the techniques previously mentioned and shown in figures 4 and 5.

A further caution should be noted in the case of null-balance instruments. Such instruments are characterized by a very high input resistance at null. At off-null conditions, however, the input resistance is variable and has a minimum value much lower than that at null. Figure 7 shows two common types of null-balance instruments, the servo-balanced potentiometer and a potentiometer-type amplifier. The effective input resistances at null and at open loop conditions are shown in the figure. Typical values are also listed. The lower values of off-null input resistance are not commonly specified by manufacturers but are often implied by a recommended maximum source resistance to be used with the instrument. It is the off-null resistance, not the resistance at null, that determines dead-zone, offset, signal-to-noise ratio, reproducibility, or related factors that characterize instrument quality.

Loading Caused by Paralleled Readout Instruments

In many measuring systems, it is advantageous to use more than one readout instrument. A measurement may be displayed on an indicator in a control room to aid the experiment operator and also routed to recorders and to a digital data system to prepare

it for easy access to a computer. In such cases, loading errors may arise due to the reduced load resistance resulting from having a number of readout instruments in parallel. The lead resistance of long lines between the signal source and the readout instruments will further affect this problem. Figure 8 shows two methods for connecting readout instruments in parallel and states the required conditions for negligible loading errors. This problem is especially critical if one or more of the readout instruments is a null-balance type instrument, because of the lower input resistance of these devices at off-null conditions. In this case, it is usually desirable to add external resistors to increase the isolation of these instruments from the rest of the system. Such resistors are shown in the lower part of figure 8.

The order of magnitude of the loading error in the circuits of figure 8 is given by the ratio of the terms on the left-hand and right-hand sides of the inequalities shown as "required conditions".

Considerations in Choosing Load Resistance

From the preceding discussion on loading effects, it is apparent that minimum loading is achieved by making $R_L = \infty$ for voltage measurements and $R_M = 0$ for current measurements. However, choosing readout instruments solely on this basis may not result in an optimum measuring system. In the case of voltage measurements, choosing $R_L \to \infty$ will result in minimum loading effects; it also results in maximum noise voltage and maximum hum voltage. Conversely, choosing $R_L = R_S$ results in appreciable loading effects but also provides maximum signal-to-noise ratio and maximum signal power transfer. In the case of galvanometer type detectors, higher signal power delivered to the detector results in

a more rugged, more reliable, less position-sensitive, and more accurate instrument. It is for these reasons that manufacturers of volt-ohm-milliameter type test sets use relatively high resistance meter movements. It is also for these reasons that an appropriate compromise in the choice of R_L is recommended in designing a measuring system.

Loading Effects in Temperature Measurement

It was previously stated that interaction effects occur throughout a measuring system. Nonelectrical effects involving the connection of an instrument to an experiment must also be considered. An example of this, the "thermal connection" of a temperature sensor to an experiment, was mentioned earlier. This effect can be thought of as a "thermal loading" effect which is determined by the flow of heat between the experiment, the sensor, and the surrounding environment. Two examples of thermal loading are shown in figures 9 and 10.

Figure 9 shows a temperature sensor used to measure the temperature of a fluid flowing through a duct. In this case heat is transferred from the fluid to the sensor by convection and conduction. Electrical self-heating of a resistance thermometer may be another source of heat transferred to the sensor. Heat is transferred away from the sensor by conduction along the sensor support to the duct wall (assuming the wall is colder than the fluid) and also by conduction along the wires connecting the sensor to the rest of the measuring system. Thermal radiation may also provide a mode of significant heat flow away from the sensor. A schematic diagram of these heat flows is shown at the upper right in figure 9. If the resistance to heat flow between the fluid and the sensor is small compared to the resistance to heat flow between the sensor and the walls and lead wires, the temperature of the sensor will approach the temperature of the fluid and the error due to thermal loading will be minimized.

Some methods of decreasing the thermal loading effect are shown at the bottom of figure 9. Increasing the immersion depth of the sensor increases the length of the support and therefore increases the thermal resistance to heat flow away from the sensor. Greater immersion depth also increases the surface area of the support in contact with the fluid, promoting heat transfer from the fluid to the support and decreasing the temperature gradient along the support.

If a thermocouple may be electrically grounded, welding it to its sheath will improve heat transfer from the surrounding fluid. If the sensor may not be grounded, but must be sheathed for mechanical protection, appropriately-located holes in the sheath will promote convective heat transfer.

Figure 10 shows some examples of surface temperature measurements. At the top of the figure are two methods of mounting thermocouples such that heat conduction along the wires away from the measuring junction is minimized. In each case the wires are routed along an isotherm and with the wires in thermal (but not electrical) contact with the surface. At the bottom are shown two methods of increasing the thermal contact between the surface and the sensor. It should be noted that in the case on the left, the flattened thermocouple junction, the emf of the thermocouple represents the temperature at the point where the two alloys first come into contact, and not the average temperature over the area of the flattened junction, unless the flattened junction is very massive.

Loading Effects in Force Measurement

Figure 11 shows the loading effect in force measurements using, as an example, the measurement of the weight of a tank and its contents. The sources of the loading effect in this case are the pipes and supports connected to the tank. As shown in the schematic diagram on the right, these pipes and supports act as springs which exert a restraining force

on the tank. The stiffnesses K_p and K_s shown on the diagram are the stiffnesses of these springs for deflection in the vertical direction. The stiffness K_M is the stiffness of the load cell and the force coupling members which connect the load cell to the tank. As shown in the equation at the bottom, the loading effect will be minimized if $(K_p + K_s) \ll K_M$. It should be noted that minimizing the loading effect by making K_M very high results in a measuring system with a greater sensitivity to high-frequency components of building vibration. This result is comparable to making a voltage measurement with a very high R_L; the signal-to-noise ratio is decreased.

One further comment is of interest in relation to this example: the loading effect in this case is due to energy storage rather than to power dissipation, as in the other examples discussed earlier. In this case, potential energy (force times deflection) is stored in the load cell and the restraining springs in the system. In the earlier examples, power was lost, either as i^2R losses in the instrument and source resistances or as heat flow away from the experiment.

Loading Effect in Measuring Fluctuating Gas Pressure

As an example of a loading effect in pressure measurement, figure 12 shows a duct with a gas flowing through it, where the pressure of the gas is fluctuating sinusoidally around some average level. A pressure trans-ducer is connected via a tube to this duct in order to measure this fluctu-ating pressure. A loading effect occurs if the system consisting of the connecting tube and transducer volume is unable to accurately transmit the fluctuating pressure to the diaphragm of the transducer. If the damp-ing in this tube-volume system is low, the system can be described as a second-order dynamic system whose resonant frequency is f_n. The equa-tion for f_n in figure 12 takes into account both the distributed volume of the tube and the volume in the transducer; if the volume in the transducer

were made zero (V = 0), the resonant frequency is that of a quarter-wave-length "organ pipe." If the damping in the system is zero, the measured pressure fluctuation amplitude P_M is related to the true pressure P_T by

$$P_M = P_T \left[\frac{1}{1 - \left(\frac{f}{f_n}\right)^2} \right]$$

If $f/f_n = 0.2$, the error due to this loading will then be about 4 percent, and the measured pressure fluctuation amplitude P_M will be greater than P_T.

Conversely, if the damping in the system is high (that is, the tube acts like a capillary), the system can be described as a first-order dynamic system characterized by a time constant, τ. It can be seen from the equation for τ in figure 12 that increasing the volume V increases the time constant. The relationship between P_M and P_T then becomes:

$$P_M = P_T \frac{1}{\sqrt{1 + (2\pi f)^2 \tau^2}}$$

The loading effect will be minimized if

$$(2\pi f)^2 \tau^2 \ll 1$$

and the effect of loading will always be that P_M is less than P_T. In both of these cases, it is evident that the loading effect will be minimized by choosing a pressure transducer whose internal volume V is small compared to the volume v of the tube.

Loading Effects in Flow Measurement

An example of a loading effect in flow measurement is shown in figure 13. In this case, a pump supplies a flow rate Q_T to a load such as a heat exchanger through a closed piping system which, for simplicity, is assumed to have no pressure loss. The head supplied by the pump is $P_S\big|_{Q_T}$, and the load has a pressure drop of $\Delta P_L\big|_{Q_T}$. The notation emphasizes that P_S and ΔP_L will be different if Q is different. Obviously, $P_S\big|_{Q_T} = \Delta P_L\big|_{Q_T}$. Now if a flowmeter is added to the system, the pressure drop across the meter causes the flow rate to decrease to Q_M. The pressure drop across the flowmeter at Q_M is ΔP_M, and for this flow rate a flowmeter resistance term is defined which is $R_M = (\Delta P_M/Q_M)$. In order to calculate the loading effect of the flowmeter, the changes in pump head $P_S\big|_{Q_T} - P_S\big|_{Q_M}$ and load pressure drop $\Delta P_L\big|_{Q_T} - \Delta P_L\big|_{Q_M}$ must be determined. Since characteristics of pumps and of many flow systems are nonlinear functions, the changes in P_S and ΔP_L are functions of the operating point. If the pump and load characteristic curves are plotted, these pressure changes can be related to the slopes of the characteristic curves at a flow rate Q_T (assuming $(Q_T - Q_M) < Q_T$). The pump head can then be written as:

$$P_S = P_S\bigg|_{Q_T} - \frac{\partial P_S}{\partial Q}\bigg|_{Q_T} \cdot (Q_T - Q_M)$$

and the load pressure drop is:

$$\Delta P_L = \Delta P_L \Big|_{Q_T} - \frac{\partial \Delta P_L}{\partial Q}\Big|_{Q_T} \cdot (Q_T - Q_M)$$

Combining these equations with that for the pressure drop across the meter results in an equation for the error due to loading:

$$Q_M = Q_T \left[1 - \frac{R_M}{R_M + \frac{\partial \Delta P_L}{\partial Q}\Big|_{Q_T} - \frac{\partial P_s}{\partial Q}\Big|_{Q_T}} \right]$$

It should be noted that the slope of the pump characteristic $\frac{\partial P_s}{\partial Q}\Big|_{Q_T}$ will be negative; that is, the pump head will increase for a decrease in flow. The result is that Q_M will be less than Q_T, and the fractional error due to loading will be the second term inside the square bracket.

Conclusion

In the preceding discussion, the effects of loading in measurement systems have been described. Methods for determining the loading effect quantitatively and for minimizing such effects have been described. An effort has been made to use practical measurement problems as a basis for the discussion. Two points made in this discussion should be emphasized:

1. Loading effects can occur at <u>all</u> of the interfaces of a measurement system <u>including</u> the interface between the primary sensor and the experiment.

2. Errors due to loading can be minimized by making the effect negligible or by making a correction for an appreciable loading effect. In many cases, the latter approach leads to a more desirable measuring system because of practical considerations such as signal-to-noise ratio.

FIG. 1   INTERACTION EFFECTS CAN ARISE THROUGHOUT

A MEASURING SYSTEM
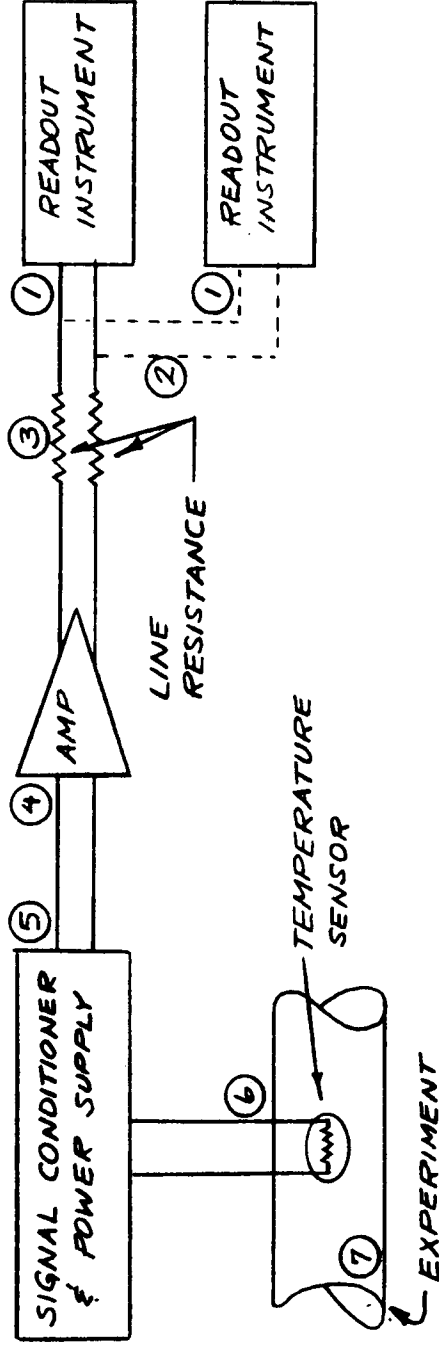
POSSIBLE INTERACTION PROBLEMS

① CONNECTION OF READOUT INSTRUMENTS

② EFFECTS BETWEEN PARALLELED READOUT INSTRUMENTS

③ EFFECTS OF LINE RESISTANCE

④ CONNECTION OF AMPLIFIER

⑤ EFFECTS OF SIGNAL CONDITIONING CIRCUITS

⑥ ELECTRICAL HEATING OF SENSOR

⑦ "THERMAL CONNECTION" OF SENSOR TO EXPERIMENT

FIG. 2   LOADING EFFECTS IN ELECTRICAL MEASUREMENTS

- EXAMPLES

1000Ω/VOLT

1 VOLT
500Ω
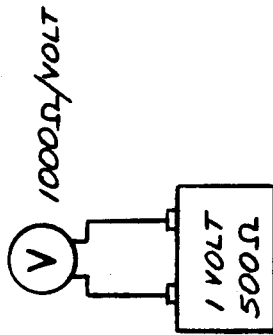
a) STANDARD CELL
EMF MEASUREMENT
WITH A VOLTMETER

500 ma
¼ VOLT DROP

ma

2Ω

i

1 VOLT
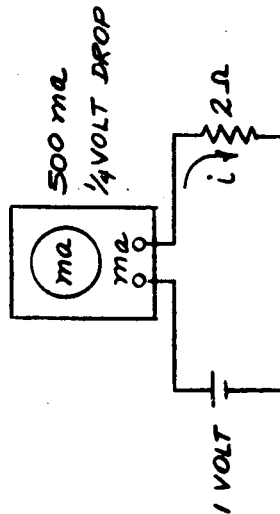
c) MEASUREMENT OF CURRENT
USING A VOLT-OHM-MILLIAMMETER
TEST SET

2000Ω

350Ω
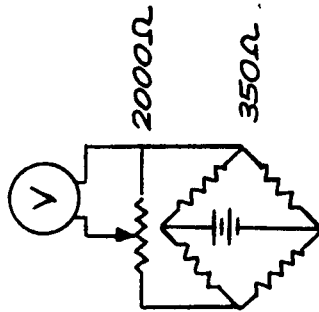
b) USE OF A SPAN ADJUSTING
NETWORK ON A STRAIN
GAGE BRIDGE CIRCUIT

50 pf

1MΩ

SCOPE

400 KHz

Z_s 600Ω

AC SIGNAL SOURCE
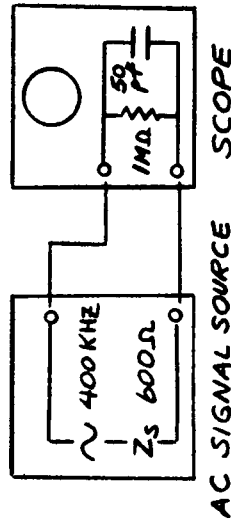
d) AC MEASUREMENTS - EFFECT OF
SHUNT CAPACITANCE ON LOAD
IMPEDANCE

FIG. 3

MAGNITUDE OF LOADING EFFECT
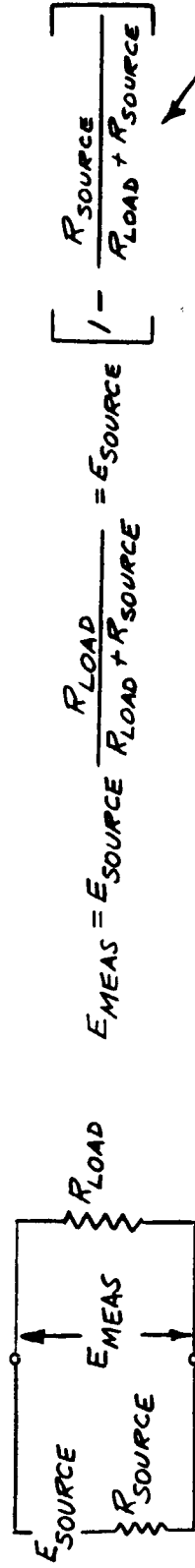
$$E_{MEAS} = E_{SOURCE} \frac{R_{LOAD}}{R_{LOAD} + R_{SOURCE}} = E_{SOURCE} \left[1 - \frac{R_{SOURCE}}{R_{LOAD} + R_{SOURCE}} \right]$$

fractional error due to loading

VOLTMETER LOADING
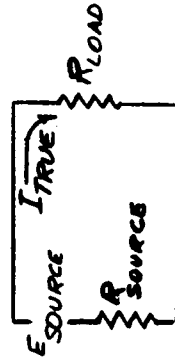
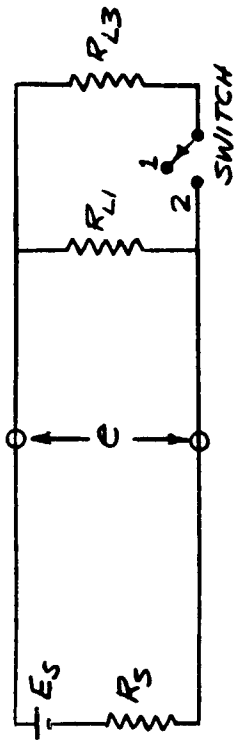$$I_{TRUE} = \frac{E_{SOURCE}}{R_{LOAD} + R_{SOURCE}}$$
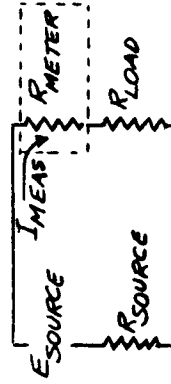
$$I_{MEAS} = \frac{E_{SOURCE}}{R_{LOAD} + R_{SOURCE} + R_{METER}} = I_{TRUE} \left[1 - \frac{R_{METER}}{R_{LOAD} + R_{SOURCE} + R_{METER}} \right]$$

fractional error due to loading

AMMETER LOADING

FIG. 4 A METHOD OF CORRECTING FOR VOLTMETER LOADING ERROR

R_{L1} = VOLTMETER RESISTANCE

$$R_{L2} = \frac{R_{L1} R_{L3}}{R_{L1} + R_{L3}}$$

WITH SWITCH OPEN (1): $\quad e_1 = \dfrac{E_S R_{L1}}{R_S + R_{L1}}$
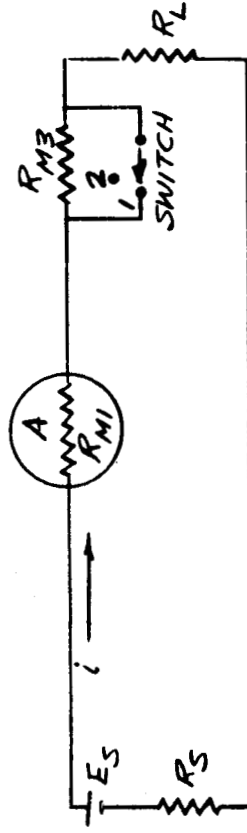
WITH SWITCH CLOSED (2): $\quad e_2 = \dfrac{E_S R_{L2}}{R_S + R_{L2}}$

THEN:

$$E_S = e_2 \left[1 + \frac{(e_2 - e_1)}{\dfrac{R_{L2}}{R_{L1}} e_1 - e_2} \right]$$

$$R_S = \frac{e_2 - e_1}{e_1 / R_{L1} - e_2 / R_{L2}}$$

FIG.5   A METHOD OF CORRECTING FOR AMMETER LOADING ERROR

$R_{M1} = $ AMMETER RESISTANCE

$R_{M2} = R_{M1} + R_{M3}$

WITH SWITCH CLOSED (1):

$$i_1 = \frac{E_S}{R_S + R_L + R_{M1}}$$

WITH SWITCH OPEN (2):

$$i_2 = \frac{E_S}{R_S + R_L + R_{M2}}$$

$$i_T = \frac{E_S}{R_S + R_L}$$

WITH SWITCH CLOSED (1)
AND NO AMMETER:

$$i_T = i_2 \left[1 + \frac{\frac{i_2 - i_1}{R_{M1}}}{\frac{i_1 - i_2}{R_{M2}}} \right]$$

THEN:

$$R_S = \frac{i_1 (R_{M1} + R_L) - i_2 (R_{M2} + R_L)}{i_2 - i_1}$$

FIG. 6   MEASURING THE COIL RESISTANCE OF A

SENSITIVE GALVANOMETER

CONSTANT CURRENT
GENERATOR

R_S

DECADE
RESISTANCE
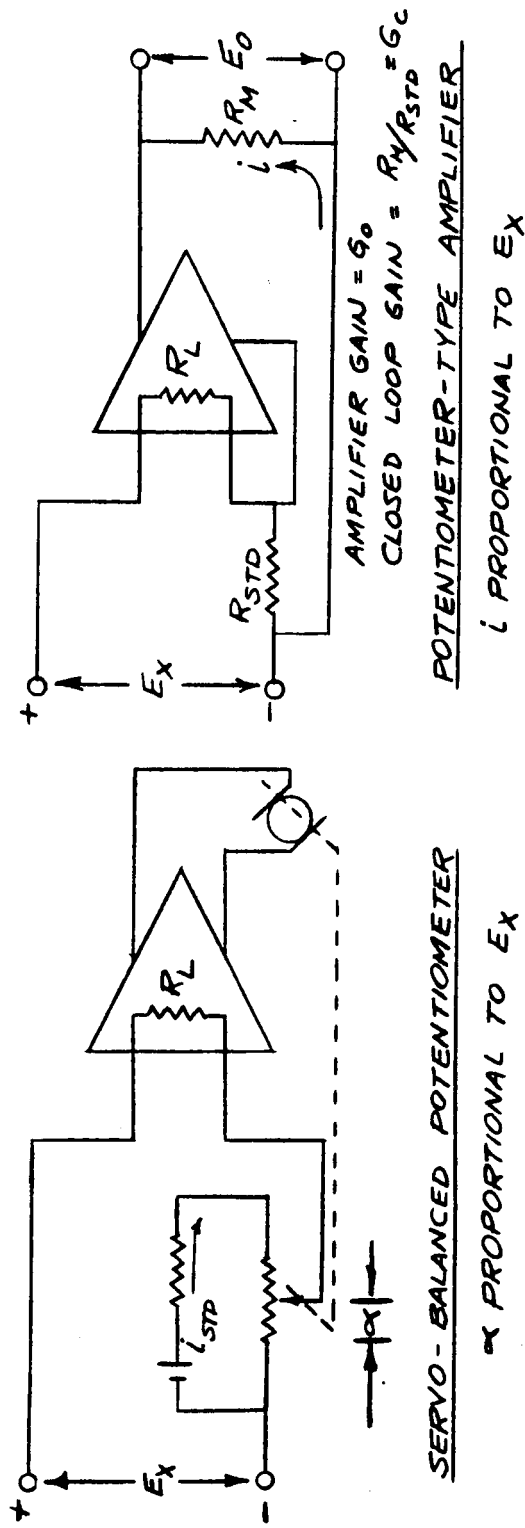BOX

R_P

SWITCH

M

1   2
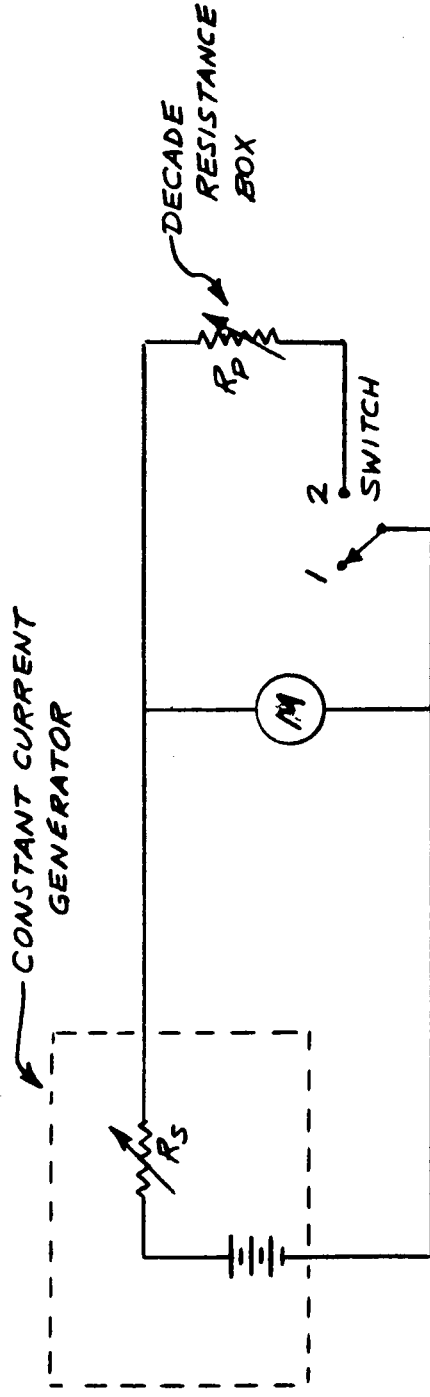
$R_S > 100 R_P$ FOR ERROR $< 1\%$

WITH SWITCH AT 1: ADJUST R_S UNTIL METER READS FULL SCALE

WITH SWITCH AT 2: ADJUST R_P UNTIL METER READS HALF SCALE

THEN, R_P = COIL RESISTANCE

FIG. 7

NULL - BALANCE INSTRUMENTS

SERVO - BALANCED POTENTIOMETER

$\propto$ PROPORTIONAL TO E_x

AMPLIFIER GAIN = G_0

CLOSED LOOP GAIN = R_M/R_{STD} = G_c

POTENTIOMETER-TYPE AMPLIFIER

i PROPORTIONAL TO E_x

EFFECTIVE INPUT RESISTANCE

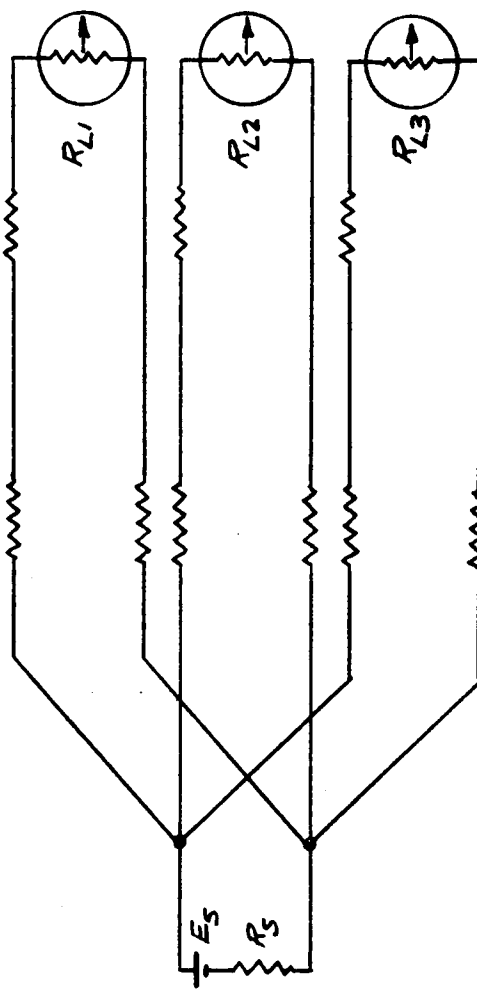|		
AT NULL	$\approx \infty$	$\approx \dfrac{R_L G_0}{1 + G_c}$ ($\sim 10M\Omega$)
OPEN LOOP (OFF-NULL)	$\approx R_L$ (400 TO 4000Ω)	$\approx R_L$ (0.1 $M\Omega$)

FIG. 8 LOADING CAUSED BY PARALLELED READOUT INSTRUMENTS

LEAD RESISTANCE

R_{L1} R_{L2} R_{L3}

E_S R_S
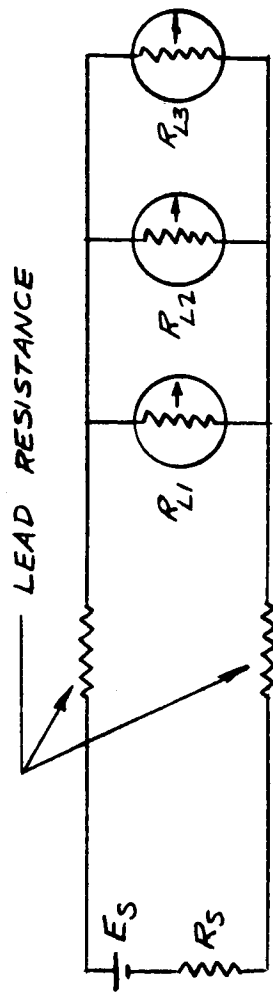
REQUIRED CONDITION:

$$(R_S + R_{LEAD}) \ll \frac{1}{\sum_i \frac{1}{R_{L_i}}}$$

EXTERNAL RESISTANCES — IF REQUIRED

LEAD RESISTANCES
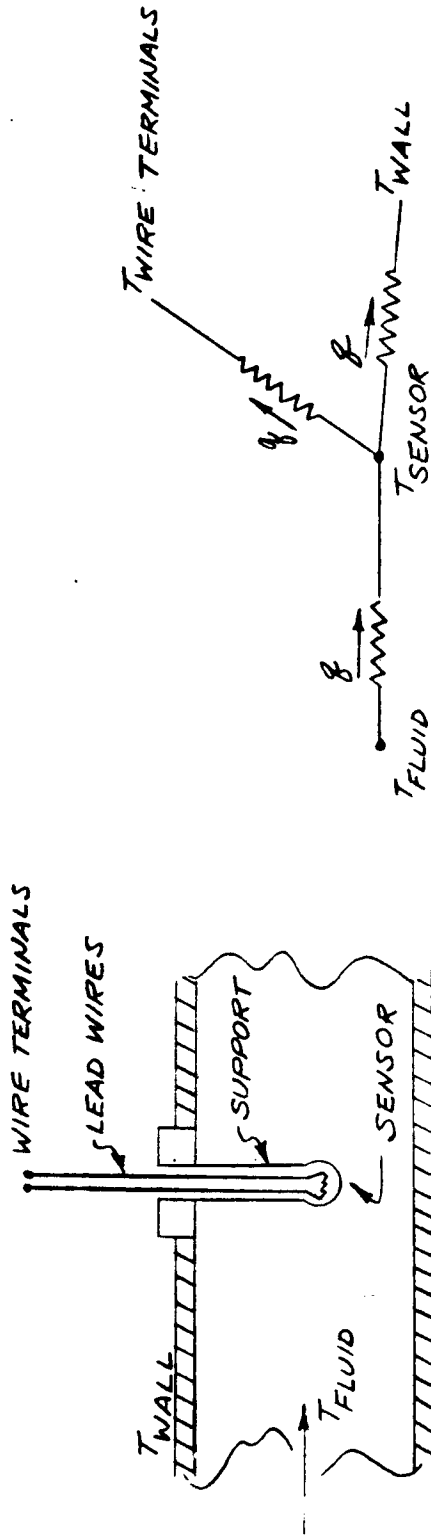
R_{L1} R_{L2} R_{L3}

E_S R_S

REQUIRED CONDITION:

$$R_S \ll \frac{1}{\sum_i \frac{1}{R_i}}$$

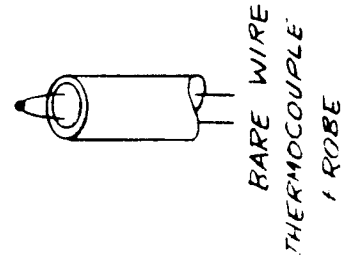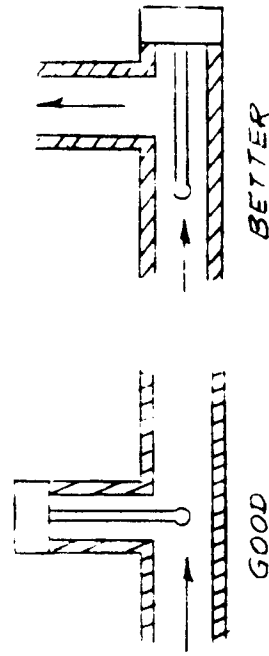R_i = TOTAL RESISTANCE OF EACH LOOP, EXCLUDING R_S

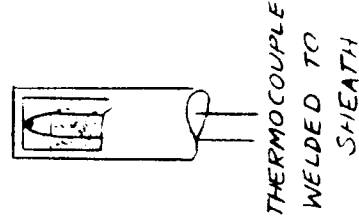# FIG. 9 FLUID TEMPERATURE MEASUREMENTS – "THERMAL LOADING"

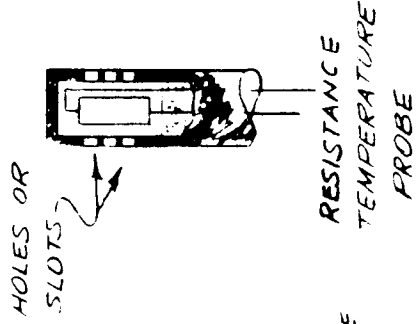IMPROVEMENTS: TO DECREASE THERMAL LOADING

– INCREASE IMMERSION DEPTH

– IMPROVE "THERMAL CONTACT" OF SENSOR

FIG. 10 SURFACE TEMPERATURE MEASUREMENTS—"THERMAL LOADING"

THERMAL GRADIENT

JUNCTION

WIRES ROUTED ALONG ISOTHERM—ELECTRICALLY INSULATED BUT IN THERMAL CONTACT WITH PIPE

LEAD WIRES

HIGH CONDUCTIVITY SHEET WRAPPED AROUND SENSOR & SPOT WELDED TO SURFACE

RESISTANCE TEMPERATURE SENSOR

WIRES ELECTRICALLY INSULATED BUT IN THERMAL CONTACT WITH SURFACE

THERMAL GRADIENT

JUNCTION

$T_{SURFACE}$

T_{SENSOR}

WIRE TERMINALS

THERMOCOUPLE WIRES

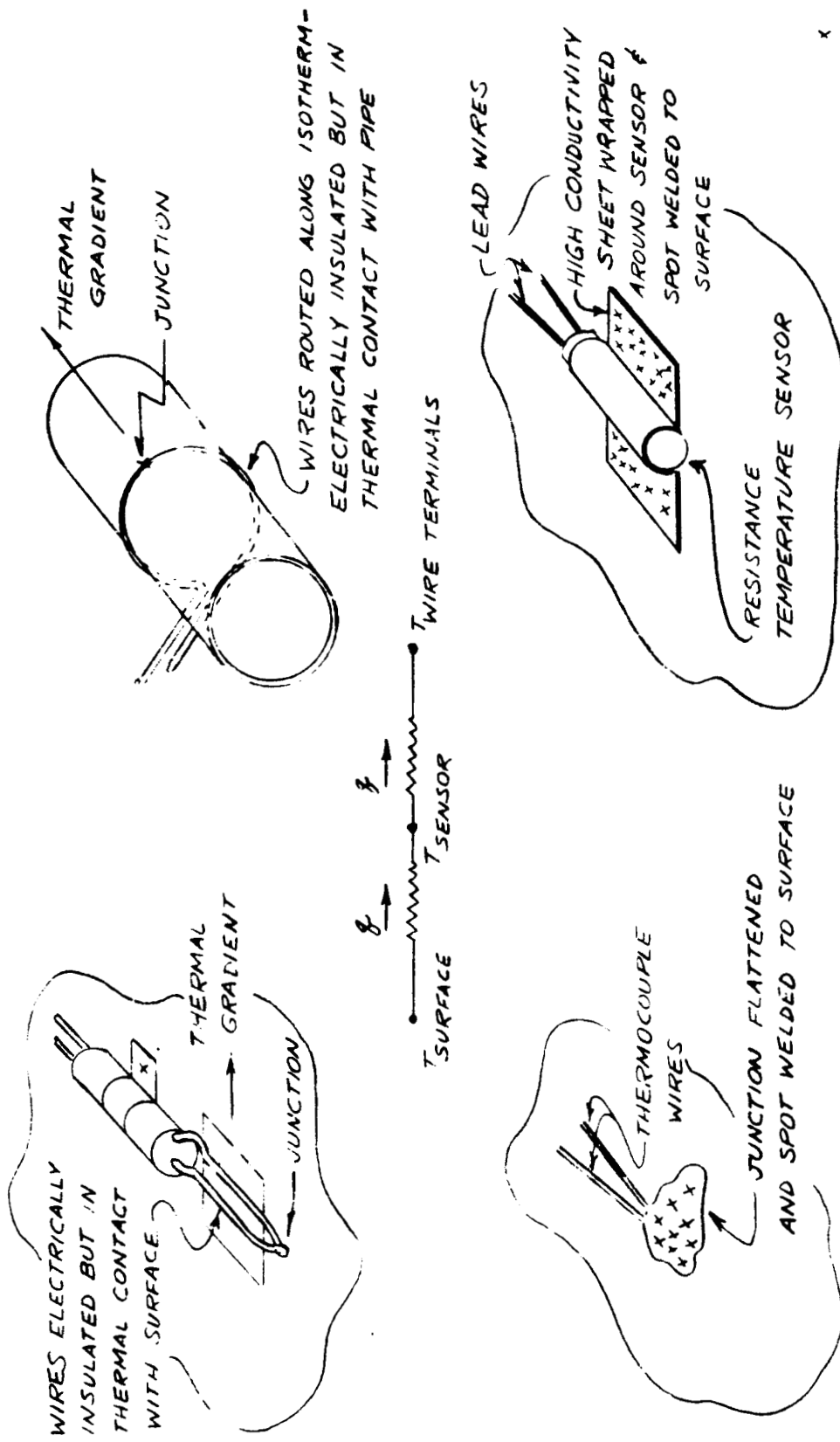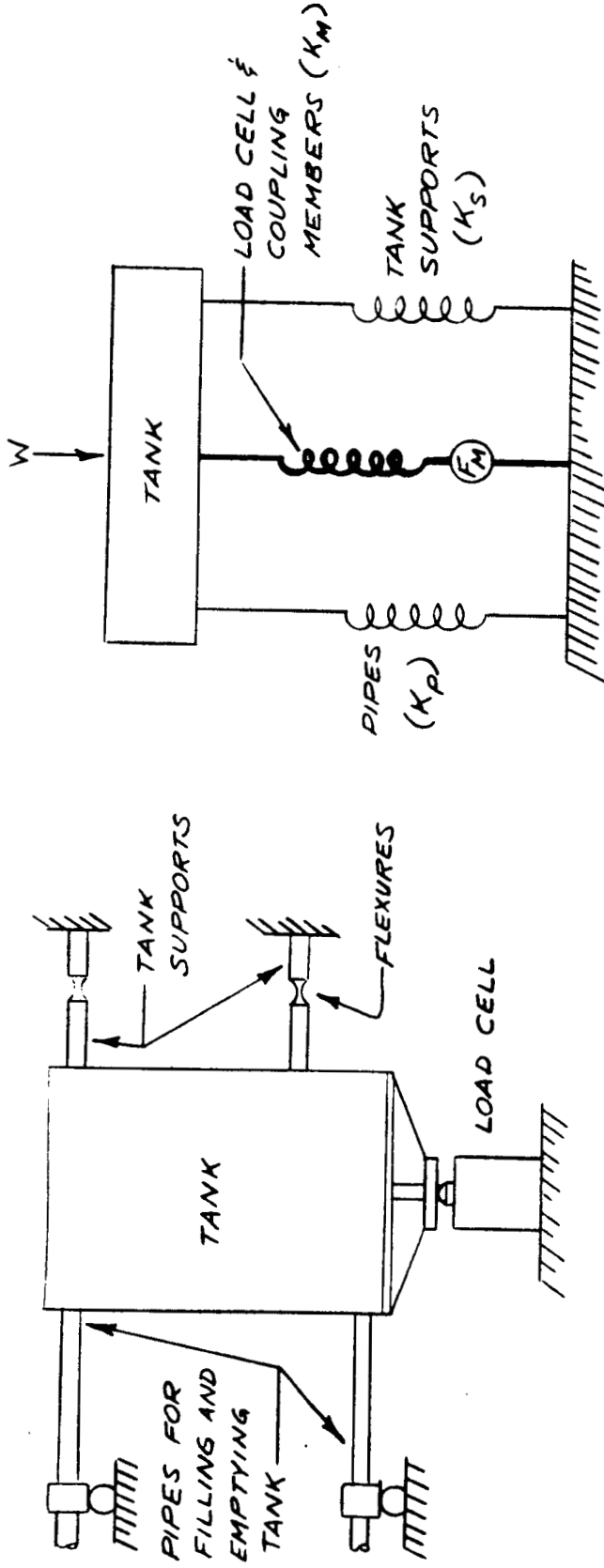JUNCTION FLATTENED AND SPOT WELDED TO SURFACE

FIG. 11   LOADING EFFECTS IN FORCE MEASUREMENT
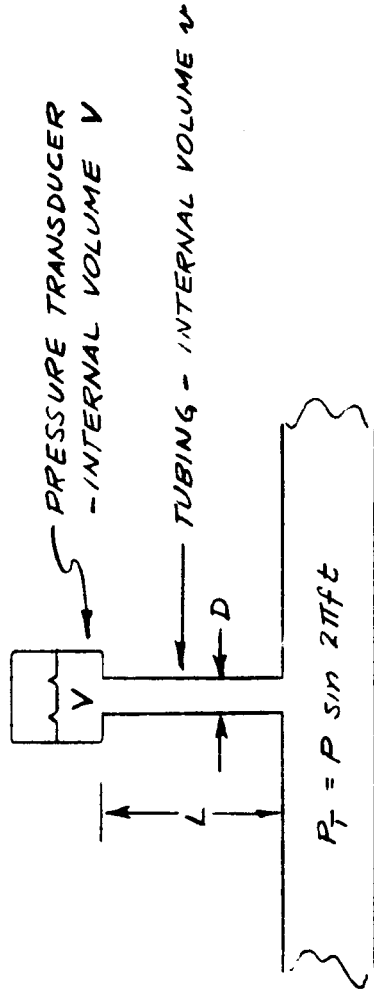
EXAMPLE: WEIGHING THE CONTENTS OF A TANK

$$F_M = W\left(\frac{K_M}{K_M + K_P + K_S}\right) = W\left[1 - \frac{K_P + K_S}{K_M + K_P + K_S}\right]$$

fractional error
due to loading

FIG. 12.

LOADING EFFECT IN MEASURING

FLUCTUATING GAS PRESSURE

PRESSURE TRANSDUCER — INTERNAL VOLUME V

TUBING — INTERNAL VOLUME v

$P_T = P \sin 2\pi f t$

IF THE DAMPING IN THE TUBE IS LOW:

RESONANT FREQUENCY OF THE TUBE & VOLUME $f_n = \dfrac{C}{4L} \dfrac{1}{\sqrt{1 + \dfrac{\pi^2}{4} \dfrac{V}{v}}}$

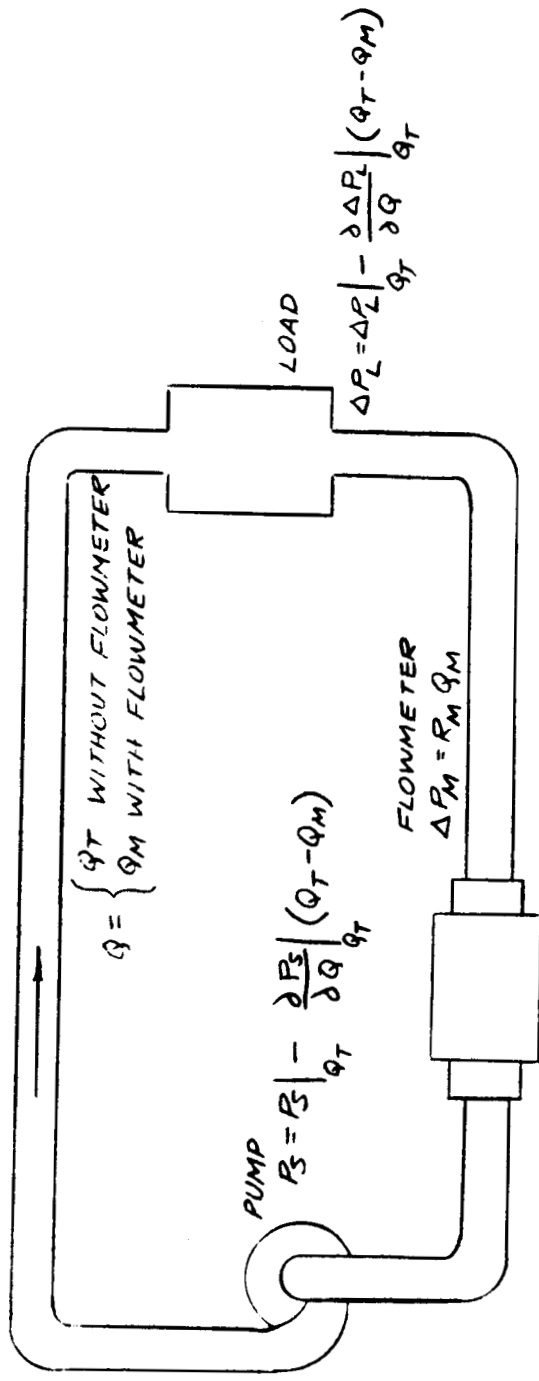$$P_M = P_T \left[\frac{1}{1 - \left(\frac{f}{f_n}\right)^2} \right]$$

IF THE DAMPING IN THE TUBE IS HIGH (CAPILLARY TUBE):

TIME CONSTANT OF THE TUBE & VOLUME $T = \dfrac{128\,v}{\pi^2 C^2} \left(\dfrac{L}{D}\right)^2 \left(1 + \dfrac{\pi^2}{4} \dfrac{V}{v}\right)$

$$P_M = P_T \frac{1}{\sqrt{1 + (2\pi f)^2 T^2}}$$

C = VELOCITY OF SOUND
v = KINEMATIC VISCOSITY

FIG. 13   LOADING EFFECT IN FLOW MEASUREMENT

$$Q = \begin{cases} Q_T & \text{WITHOUT FLOWMETER} \\ Q_M & \text{WITH FLOWMETER} \end{cases}$$

PUMP
$$P_S = P_S\big|_{Q_T} - \frac{\partial P_S}{\partial Q}\bigg|_{Q_T}(Q_T - Q_M)$$

FLOWMETER
$$\Delta P_M = R_M Q_M$$

LOAD
$$\Delta P_L = \Delta P_L\big|_{Q_T} - \frac{\partial \Delta P_L}{\partial Q}\bigg|_{Q_T}(Q_T - Q_M)$$

SINCE $P_S = \Delta P_L$ WITHOUT FLOWMETER

$P_S = \Delta P_L + \Delta P_M$ WITH FLOWMETER

$$Q_M = Q_T\left[1 - \frac{R_M}{R_M + \frac{\partial \Delta P_L}{\partial Q}\big|_{Q_T} - \frac{\partial P_S}{\partial Q}\big|_{Q_T}}\right]$$

fractional error due to loading