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COMPARATIVE STUDY OF MIXED- AND ISOLATED-FLOW METHODS FOR COOLED-TURBINE PERFORMANCE ANALYSES

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ABSTRACT

The two methods are described and then compared by using two simplified example turbines, one being a two-stage impulse type and the other a two-stage reaction turbine. The agreement in efficiency predicted by the two methods is within ± 0.01 for total coolant fractions up to 0.156 and within ± 0.013 at the highest total coolant fraction considered, which was 0.22.

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SUMMARY

This report describes and compares two analytical methods for determining the effect of coolant air on turbine performance. One of these methods uses a mixed flow model wherein the coolant flow is assumed to be completely mixed with the gas stream resulting in homogeneous flow conditions at the outlet of every blade row. The other method uses an isolated flow model in which the coolant and the primary air are assumed to be entirely uninfluenced by one another. Both methods are used to determine a primary-air efficiency of the cooled turbine. In this efficiency concept the total ideal work output is based on the primary air flow and its specific ideal work output.

The two analytical methods were applied to two simplified example turbines, one of which was a two-stage impulse type, and the other, a two-stage reaction type. The effect of coolant on performance was obtained for ranges of coolant flow and ratio of coolant temperature to turbine inlet temperature and for two levels of coolant total pressure recovery. The performance results are expressed as the variation of primary-air efficiency compared with the efficiency of the uncooled turbine.

Both analytical methods indicated that the trend of efficiency with coolant flow was positive, almost zero, or negative; the trend depended on coolant temperature ratio and coolant pressure coefficient or coolant velocity coefficient. As any of these parameters was increased, turbine efficiency for a given coolant fraction improved because of the increased work output of the coolant flow. The trends of efficiency with coolant flow predicted by the two performance estimation procedures were similar. The efficiency levels obtained from the two procedures were also in reasonably good agreement, being within ±0.01 for total coolant fractions up to 0.156 and within ±0.013 at a total coolant fraction of 0.22, which was the highest coolant flow considered.

INTRODUCTION

Many advanced aircraft applications require increased engine cycle temperatures to achieve their performance goals. This requirement necessitates cooling of the turbine blading to maintain blade strength and in some cases oxidation resistance. The cooling method commonly considered for these high-temperature engines utilizes air bled from the compressor, ducted through the turbine-blade cooling passages, and discharged into the turbine gas stream. The work output of the cooled turbine is affected by the amount of coolant, the location of the injection point of the coolant, and the temperature and total pressure of the coolant in relation to that of the turbine gas stream.

Some analyses have been made concerning the effect of cooling on turbine performance (refs. 1 to 4). These analyses, however, only considered the change in performance due to the heat transferred to the coolant and made no attempt to determine the effect of discharging the coolant into the main gas stream. Furthermore, experimental data on the effect of cooling air on turbine performance are scarce and are difficult to interpret or correlate because of the varied forms of efficiency that are used. Therefore, the development of analytical means of estimating the effect of cooling air on turbine performance is considered of interest.

This report describes and compares two such analytical methods. The first method, termed "mixed flow," uses a model that assumes that complete mixing of the coolant and gas stream occurs in the injection blade row. The second method, termed "isolated flow," uses a model that assumes that the coolant and primary air are completely independent and do not influence one another. Both methods were applied to an example two-stage impulse turbine and a two-stage reaction turbine for ranges of coolant flow and ratio of coolant temperature to turbine inlet temperature and for two levels of coolant total-pressure recovery. The performance results of the cooled turbines were determined in terms of a primary-air efficiency in which the total ideal work output is based on the primary air and its specific ideal work output. The results of these calculations are also presented to provide a comparison of the two methods.

DESCRIPTION OF ANALYTICAL METHODS

The coolant flow that bypasses the combustor represents a thermodynamic loss that can be considered in the cycle calculations. The problem, then, in determining the effect of coolant flow on engine performance is the evaluation of the primary-air efficiency of the cooled turbine. The primary-air efficiency as defined herein uses the product of the primary-air flow and its ideal specific work output as the total ideal work output. Therefore, the variation in primary-air efficiency directly reflects the variation in net turbine work output. The coolant flow can affect the net work output in different ways. The rotor coolant flows require pumping work, which detracts from the net work output. In addition, most of the coolant flows enter the turbine gas stream with some potential capacity to increase the net work output of the turbine, since in most cases there would be a pressure drop available between the coolant injection point and the turbine outlet static pressure. Furthermore, the coolant entering a blade row would have considerably less momentum than the gas stream and would therefore reduce the momentum of the gas stream. All these effects must be accounted for in the estimation of the performance of a cooled turbine.

One method of performance estimation, representing a boundary condition, is the mixed-flow analysis. In this procedure the coolant flow is assumed to be completely mixed with the gas stream in the injection blade row. The net work output is evaluated by determining the aftermixed conditions out of each blade row. In this concept, momentum of the gas stream is imparted to the coolant such that homogeneous flow conditions are attained at the blade row outlet. Thus, the interaction effect, coolant work output, and primary-air work output are evaluated by determining the aftermixed flow conditions.

Another procedure that can be used assumes that the coolant expands from its injection point to the turbine outlet, entirely uninfluenced by the primary flow. It is also assumed that the primary air is uninfluenced by the coolant; therefore, its work output is the same as that of the uncooled turbine. Thus, the evaluation of net work output with coolant involves the determination of the incremental net work outputs of the individual coolant flows. This procedure also represents a boundary condition, with respect to the degree of mixing between the primary flow and the coolant, and is termed the isolated-flow analysis.

General Procedure Assumptions

Both of the analytical procedures being considered require a means of estimating the effective total pressure of the coolant at the outlet of the injection blade row before the effect of mixing is considered. The static pressure of the coolant at its point of injection must be at least equal to the stream static pressure. Furthermore, in accelerating blade rows there is a net pressure drop that could accelerate some of the coolant flow between its injection point and the blade outlet. Thus, at the blade outlet of accelerating blade rows the coolant total pressure would be expected to equal the static pressure plus some fraction of the available dynamic pressure. This fraction, or factor, is termed k_p herein. (All symbols are defined in appendix A.) With impulse blade rows there would be no overall pressure drop in the flow passage to accelerate the coolant. Therefore, the effective coolant total pressure in the injection blade row is estimated by means of a coolant velocity coefficient k_V. This coefficient is defined as the coolant critical velocity ratio expressed as a fraction of the gas-stream critical velocity ratio before mixing.

Both analytical procedures are two-dimensional and assume the turbine to have flexible radial boundaries. Thus, the static pressure levels through the turbine blading are the same as for the uncooled turbine regardless of the coolant additions.

Mixed-Flow Analysis

The pertinent feature of the mixed-flow analytical model is that the flow is uniform at the outlet of each blade row with respect to temperature, pressure, and velocity. The gas stream entering any blade row consists of the primary flow entering the first-stage stator and all the coolant flows that have been added up to that point. The procedure for determining the aftermixed conditions of the coolant and the gas stream is simplified by assuming that temperature uniformity is achieved before the two streams mix. The aftermixed critical velocity ratio is then taken as the momentum average of the two streams, as expressed algebraically in equation (B2) of appendix B for a first-stage stator blade row. A comparison of this method of obtaining the aftermixed velocity with an adaptation of the procedure of appendix C of reference 5 showed the two methods to be in close agreement.

Another key assumption of the mixed-flow analysis is that the total-pressure loss across a blade row of the gas stream (before mixing) is the same as that of the uncooled turbine. This assumption, along with the specification that the static pressures are the same as for the uncooled turbine, simplifies the solution for the gas stream total pressure before mixing at a blade outlet. This simplification is illustrated by equation (B9) of appendix B. Use of the total-pressure loss assumption and the static-pressure assumption also makes the results independent of the actual pressure ratio, and therefore the efficiency, of the uncooled turbine.

As would be expected, the mixture velocity is decreased from that of the uncooled turbine. If the blade inlet angles of the uncooled turbine were used for the cooled turbine, the velocity degradation would cause the flow to enter the downstream stage with incidence angles and blade entry losses. It is assumed herein that the inlet angles can be adjusted to the proper flow angle; therefore, incidence losses are not incurred in the mixed-flow analytical model.

The overall performance estimation involves a step-by-step procedure of calculating the velocities of the mixture at the various stations from inlet to outlet. The net work output is obtained by summing the products of $(w_c + w_p)UV_u$ into and out of the rotor blades. The ratio of the primary-air efficiency of the cooled turbine to that of the uncooled turbine is identical to the net work output ratio, as mentioned previously in this section. The detailed equations for this method are given in appendix B.

Isolated-Flow Analysis

The pertinent feature of the isolated-flow analysis method is that the primary air is unaffected by the coolant. Thus, the specific work output of the primary flow is fixed, and the analytical procedure is concerned with the determination of the incremental work output contributed by the various cooling flows. The coolant flows are considered individually, and their flow and state conditions are first evaluated, by means of a k_p or k_v value, at the outlet of the blade row in which they are introduced. From the injection blade row to the turbine outlet the flow conditions are calculated for the individual coolant flows by assuming they experience the same static-pressure variation and encounter the same blade angles as for the uncooled turbine. The net work output of the coolant flows can then be determined and added to the uncooled turbine work output to obtain the cooled turbine work output and efficiency.

For the mixed-flow analysis, the blade inlet angles are assumed to be adjustable to the inlet gas angle. In the isolated-flow model it is assumed that the blade inlet angles are fixed to the uncooled turbine velocity diagram angles and that the coolant flows are therefore subjected to incidence losses. These losses are evaluated by assuming that the normal component of blade entry velocity is lost with respect to total pressure. Another assumption used in the isolated-flow model is that the loss total-pressure ratio of the coolant across any blade row downstream of the injection blade row is the same as for the primary flow. Thus, as in the mixed-flow procedure, the results of the performance estimation method are independent of the actual pressure level and efficiency of the uncooled turbine.

One notable difference between the two performance estimation methods is that the mixed-flow model requires the coolant flow rates as input data. The isolated-flow model is not dependent on coolant flow rates in the determination of coolant-stage work coefficients. These work coefficients depend only on the coolant temperature ratio and the k_p and k_v assumptions. Once the coolant-stage work coefficients have been evaluated, any coolant schedule can be applied, and the turbine efficiency variation can be determined. The detailed equations used for this method are given in appendix C.

COMPARATIVE ANALYSIS OF EXAMPLE TURBINES

The two analytical performance estimation methods were each applied to two representative example turbines. In this section the example turbines are described and analysis results are presented.

Description of Example Turbines

The turbine designs selected are both two-stage, one being an impulse type and the other a reaction type. The simplified velocity diagrams of the uncooled turbines are shown in figure 1. The impulse diagram has a stator angle of 65° and a tangential momentum change $\Delta V_u = 2U$, and the reaction diagram has a stator angle of 60° and a tangential momentum change $\Delta V_u = U$. Both diagrams have constant blade speed equal to one-half the turbine inlet critical velocity, constant axial velocity, and zero exit whirl. With these simplifying features all the temperatures throughout the turbine can be related to turbine inlet temperature, and all the velocities can be related to the appropriate critical velocity. The specific work output of the impulse diagram is obviously twice that of the reaction diagram, since the impulse diagram has twice the tangential momentum change at the same blade speed.

The assumed coolant flow schedule that is used in applying the methods to the example turbines is shown in table I. The schedule assumed is arbitrary; however, the linear decrease of coolant per blade row from inlet to outlet represents a reasonable simplified approximation of an actual coolant schedule distribution.

The effect of coolant temperature is considered by expressing it in the form of the ratio of coolant temperature to turbine inlet temperature. The values used for this ratio are 0.3, 0.45, and 0.6, and these are assumed to cover the range of conditions that might be encountered. The other quantities assumed as input parameters are the k_p and k_v values. Two levels of these coefficients are assumed for both turbine types. For the impulse turbine, these levels are $k_p = 0.5$ with $k_v = 0.5$ and $k_p = 0.25$ with $k_v = 0$. For the reaction turbine the two levels are $k_p = 0.5$ and $k_p = 0.25$.

In the application of both methods nondimensional equations are used throughout, as can be seen in appendixes B and C. Thus, the results of both methods are independent of temperature except for the second-order effect that the specific-heat ratio γ would have on the various local pressure ratios. A specific-heat ratio γ of 1.3 was used herein.

Analysis Results

The two analytical methods are applied to the two example turbine types to obtain their efficiency - coolant-flow characteristics. The results indicated by the two methods are then compared with each other. This comparison is of interest because the two methods represent widely different means of accounting for the overall effect of the coolant and the primary air in passing through the turbine. Finally, the significance of the stage work coefficients that are obtained as an intrinsic part of the isolated flow procedure is discussed briefly.

Effect of coolant on performance. - The overall effect of coolant is shown in figures 2 and 3 for the impulse turbine and the reaction turbine, respectively, as the variation of primary air efficiency compared with that of the uncooled turbine model. The abscissa in both figures is the coolant fraction of the first-stage stator row. The coolant fraction of the other blade rows can be obtained by referring to table I. Both figures 2 and 3 show that the effect of coolant on efficiency can be positive, almost zero, or negative, depending on coolant temperature ratio and coolant pressure coefficient or coolant velocity coefficient. Also, the efficiency level for a given coolant flow increases with increasing coolant temperature ratio and with increasing values of k_p and k_v. This effect might be expected, since increasing values of these parameters represent an increased potential of the coolant flow to produce useful work in passing through the blading.

Comparison of analytical methods. - The similarity of the trends of efficiency with coolant flow as obtained by the two methods is evident in figures 2 and 3. The levels of efficiency variation predicted by the two methods agree very well in some cases, while in others they tend to diverge somewhat with increased coolant flow. A direct comparison of the efficiencies obtained by the two performance estimation methods is made in figures 4 and 5 with the mixed flow procedure results used as a base. This comparison shows that the fractional deviation in efficiency ranges from +0.013 to -0.013. These differences occur at the highest coolant schedule considered, which corresponds to a total coolant fraction of 0.22. If a more moderate or realistic coolant fraction of 0.12 is assumed (first-stage stator coolant fraction of 0.045), the difference indicated by the two methods is 0.008 or less. Thus, the agreement between the two performance estimation procedures was reasonably good, being within ±0.01 for total coolant fractions up to 0.156.

Stage work coefficients. - The stage work coefficients obtained in the isolated flow procedure are listed in table II. In addition to the overall effect on efficiency as shown in figures 2 and 3, the stage work coefficients show the contribution of the individual coolant flows in the two stages as well as their overall contribution. The overall stage work coefficient for the primary flow would be 2 for both example turbines since both had two stages of equal work output. The highest value of 0.92 indicates that this stator coolant flow produced somewhat less than half the specific work output of the primary flow. The value of -0.5 listed for $K_{f,B}$ and $K_{s,D}$ for the impulse turbine at a k_v of 0 represents the pumping work done on the coolant. The corresponding values for the reaction turbine would have been -1.000 if a k_p of 0 were assumed. The change in the stage work coefficients can also be noted as the coolant recovery coefficient or the coolant temperature ratio is changed. Thus, these coefficients represent additional information that is available from the isolated-flow procedure.

CONCLUDING REMARKS

Two analytical methods to determine the effect of coolant air on turbine performance are described herein. The methods are applied to two simplified example turbines, one being a two-stage impulse turbine and the other a two-stage reaction turbine, and the results are compared. Both methods indicate the effect of coolant on efficiency to be positive, almost zero, or negative, depending on coolant temperature ratio and coolant pressure coefficient or coolant velocity coefficient. Turbine efficiency, for a given coolant fraction, is shown to increase as any of these parameters is increased because of an increased recovery of coolant work output. The trends of efficiency with coolant fraction predicted by the two methods are similar. The level of efficiency obtained by the two methods is in reasonably good agreement, being within ±0.01 for total coolant fractions up to 0.156 and within ±0.013 at the highest coolant flow rate, corresponding to a total coolant fraction of 0.22.

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APPENDIX A

SYMBOLS

A first-stage stator coolant flow expressed as fraction of primary flow, dimensionless

B first-stage rotor coolant flow expressed as fraction of primary flow, dimensionless

C second-stage stator coolant flow expressed as fraction of primary flow, dimensionless

D second-stage rotor coolant flow expressed as fraction of primary flow, dimensionless

Δh_f actual enthalpy drop across first stage of uncooled turbine, Btu/lb (J/kg)

Δh_s actual enthalpy drop across second stage of uncooled turbine, Btu/lb (J/kg)

i incidence angle (difference between flow angle, α or β, and blade entry angle), deg

K coolant-stage work coefficient, ratio of stage specific work output of coolant to stage specific work of uncooled turbine

k_p coolant pressure coefficient, ratio of dynamic pressure of coolant to dynamic head available across blade row $\left(\text{for first-stage stator coolant flow } Aw_p,\ k_p = (p'_{c,1a} - p_1)/(p'_0 - p_1)\right)$

k_v coolant velocity coefficient, coolant critical velocity ratio divided by gas-stream critical velocity ratio $\left(\text{for first-stage rotor, } k_v = (W/W_{cr})_{2a,c} \Big/ (W/W_{cr})_{2a,fs}\right)$

p absolute pressure, lb/(sq ft) (N/m^2)

T absolute temperature, ^oR (^oK)

$T_{c,0}$ coolant supply temperature, ^oR (^oK)

U blade velocity, ft/sec (m/sec)

V absolute gas velocity, ft/sec (m/sec)

V_{cr} velocity of sound at Mach 1 based on absolute total state, ft/sec (m/sec)

W gas velocity relative to rotor blade, ft/sec (m/sec)

W_{cr} velocity of sound at Mach 1 based on total state relative to moving blade row, ft/sec (m/sec)

w	mass flow rate, lb/sec (kg/sec)
w_p	mass flow entering first-stage stator, lb/sec (kg/sec)
α	absolute flow angle measured from axial direction, deg (angle positive when tangential component of velocity is in direction of blade velocity vector U)
β	relative flow angle measured from axial direction, deg (angle positive when tangential component of velocity is in direction of blade velocity vector U)
γ	ratio of specific heats
η	turbine efficiency based on total- to static-pressure ratio

Subscripts:

A, B, C, or D	coolant fraction A, B, C, or D
a	used with station number 1, 2, 3, or 4 to denote equivalent condition of gas stream or coolant before mixing at that station
c	coolant
f	first stage
fs	gas stream at blade outlet before mixing with coolant
s	second stage
u	tangential component of velocity (positive in direction of blade velocity vector U)
un	uncooled turbine
x	axial component of velocity
0	station at turbine inlet (see fig. 1(c))
1	station at first-stage stator outlet
2	station at first-stage rotor outlet
3	station at second-stage stator outlet
4	station at turbine outlet

Superscripts:

'	total state absolute
''	total state relative to moving blade row

APPENDIX B

MIXED-FLOW PERFORMANCE ESTIMATION METHOD

As discussed in the Mixed-Flow Analysis section, the mixed-flow performance estimation procedure involves a step-by-step solution of the flow conditions through the turbine including the effect of incorporating coolant flow into the gas stream. All quantities are expressed nondimensionally. The procedure is presented for the two-stage turbine example.

First-Stage Stator Outlet

Since critical velocity is a function of total temperature, the ratio of critical velocity across the first-stage stator is given by

$$\frac{V_{cr,1}}{V_{cr,0}} = \left(\frac{1 + A\dfrac{T_{c,0}}{T'_0}}{1 + A}\right)^{1/2} \tag{B1}$$

The velocity out of the first-stage stator after mixing is obtained by the following equation:

$$\left(\frac{V}{V_{cr}}\right)_1 = \frac{\left(\dfrac{V}{V_{cr}}\right)_{1a,fs} + A\left(\dfrac{V}{V_{cr}}\right)_{1a,c}}{1 + A} \tag{B2}$$

The quantity $\left(V/V_{cr}\right)_{1a,fs}$ is the same as $\left(V/V_{cr}\right)_{1,un}$, and $\left(V/V_{cr}\right)_{1a,c}$ is determined as follows. The pressure ratio of the coolant is given by

$$\left(\frac{p}{p'}\right)_{1a,c} = \frac{1}{1 + k_p\left(\dfrac{1}{\dfrac{p_{1a,fs}}{p'_0}} - 1\right)}$$

where $p_{1a,fs}/p'_0$ depends on $\left(V/V_{cr}\right)_{1a,fs}$ and on a loss assumption relating $p'_{1a,fs}$ to p'_0. The coolant critical-velocity ratio is then obtained as

$$\left(\frac{V}{V_{cr}}\right)_{1a,c} = \left\{\frac{\gamma + 1}{\gamma - 1}\left[1 - \left(\frac{p}{p'}\right)_{1a,c}^{(\gamma-1)/\gamma}\right]\right\}^{1/2}$$

The stator outlet velocity in terms of inlet critical velocity is then obtained from combining equations (B1) and (B2)

$$\frac{V_1}{V_{cr,0}} = \left(\frac{V}{V_{cr}}\right)_1 \frac{V_{cr,1}}{V_{cr,0}} \tag{B3}$$

First-Stage Rotor Inlet

The ratio of relative to absolute critical velocity is obtained from the general equation

$$\frac{W_{cr,1}}{V_{cr,1}} = \left\{1 - \frac{\gamma - 1}{\gamma + 1}\left[2 \frac{U}{V_{cr,1}}\left(\frac{V_u}{V_{cr}}\right)_1 - \left(\frac{U}{V_{cr,1}}\right)^2\right]\right\}^{1/2} \tag{B4}$$

Since $U/V_{cr,0}$ is known, $U/V_{cr,1}$ can be obtained by using equation (B1) as

$$\frac{U}{V_{cr,1}} = \frac{U}{V_{cr,0}} \frac{V_{cr,0}}{V_{cr,1}}$$

The quantity $\left(V_u/V_{cr}\right)_1$ can be obtained from equation (B2) and the velocity-diagram stator outlet angle α_1. The blade inlet relative velocity is obtained from the following equation:

$$\frac{W_1}{V_{cr,1}} = \left\{\left[\left(\frac{V}{V_{cr}}\right)_1 \sin \alpha_1 - \frac{U}{V_{cr,1}}\right]^2 + \left[\left(\frac{V}{V_{cr}}\right)_1 \cos \alpha_1\right]^2\right\}^{1/2} \tag{B5}$$

The blade inlet relative critical velocity ratio $\left(W/W_{cr}\right)_1$ is obtained by combining equations (B4) and (B5); the static- to total-pressure ratio at the blade inlet can then be computed from

$$\left(\frac{p}{p''}\right)_1 = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{W}{W_{cr}}\right)_1^2\right]^{\gamma/(\gamma-1)} \tag{B6}$$

First-Stage Rotor Outlet

In passing through the rotor, the rotor coolant is incorporated into the gas stream. The rise in temperature of the rotor-coolant caused by the rotor pumping work is given by

$$\frac{T''_{c,0}}{T_{c,0}} = \left[1 + \frac{\gamma - 1}{\gamma + 1}\left(\frac{U}{V_{cr,0}}\right)^2 \frac{1}{\left(\dfrac{T_{c,0}}{T'_u}\right)}\right]$$

Multiplying this equation by $T_{c,0}/T'_0$ yields the following equation for the ratio of coolant critical velocity to the critical velocity at the turbine inlet:

$$\frac{W_{cr,2a,c}}{V_{cr,0}} = \left[\frac{T_{c,0}}{T'_0} + \frac{\gamma - 1}{\gamma + 1}\left(\frac{U}{V_{cr,0}}\right)^2\right]^{1/2} \tag{B7}$$

The critical velocity of the flow at the rotor outlet relative to that at the turbine inlet is

$$\frac{W_{cr,2}}{V_{cr,0}} = \left[\frac{(1 + A)\left(\dfrac{W_{cr,1}}{V_{cr,1}}\right)^2\left(\dfrac{V_{cr,1}}{V_{cr,0}}\right)^2 + B\left(\dfrac{W_{cr,2a,c}}{V_{cr,0}}\right)^2}{1 + A + B}\right]^{1/2} \tag{B8}$$

which can be evaluated by using equations (B4), (B3), and (B7).

The blade-outlet static- to total-pressure ratio for the gas stream is obtained by using the assumption regarding blade-loss pressure ratio mentioned in the Mixed-Flow Analysis section. The equation is

$$\frac{p_2}{p''_{2a,fs}} = \frac{p_1}{p''_1} \frac{\left(\frac{p_2}{p''_2}\right)_{un}}{\left(\frac{p_1}{p''_1}\right)_{un}} \tag{B9}$$

The two pressure ratios for the uncooled turbine are readily determinable from the velocity diagram. The critical velocity ratio of the gas stream at the blade outlet is then determined from the pressure ratio of equation (B9):

$$\left(\frac{W}{W_{cr}}\right)_{2a,fs} = \left\{\frac{\gamma + 1}{\gamma - 1}\left[1 - \left(\frac{p_2}{p''_{2a,fs}}\right)^{(\gamma-1)/\gamma}\right]\right\}^{1/2} \tag{B10}$$

For the rotor blade rows of the example impulse turbine the two pressure ratios $\left(p_2/p''_2\right)_{un}$ and $\left(p_1/p''_1\right)_{un}$ are equal, and $p_2/p''_{2a,fs}$ in equation (B9) is equal to p_1/p''_1. Thus, $\left(W/W_{cr}\right)_{2a,fs}$ is equal to $\left(W/W_{cr}\right)_1$.

The coolant critical velocity ratio at the blade outlet is obtained from the gas-stream critical velocity ratio, the assumed k_p value, and the procedure presented following equation (B2). For the impulse rotor blade row the coolant critical velocity ratio is obtained by using the k_v value as

$$\left(\frac{W}{W_{cr}}\right)_{2a,c} = k_v\left(\frac{W}{W_{cr}}\right)_{2a,fs}$$

The aftermixed critical velocity ratio at the blade outlet is then

$$\left(\frac{W}{W_{cr}}\right)_2 = \frac{(1 + A)\left(\frac{W}{W_{cr}}\right)_{2a,fs} + B\left(\frac{W}{W_{cr}}\right)_{2a,c}}{1 + A + B} \tag{B11}$$

The ratio of absolute critical velocity to relative critical velocity at the blade outlet is given by

$$\left(\frac{V_{cr}}{W_{cr}}\right)_2 = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{W_u}{W_{cr}}\right)_2^2 + \frac{\gamma - 1}{\gamma + 1}\left(\frac{V_u}{W_{cr}}\right)_2^2\right]^{1/2} \tag{B12}$$

where

$$\left(\frac{W_u}{W_{cr}}\right)_2 = \left(\frac{W}{W_{cr}}\right)_2 \sin \beta_2$$

and

$$\left(\frac{V_u}{W_{cr}}\right)_2 = \left(\frac{W}{W_{cr}}\right)_2 \sin \beta_2 + \frac{\dfrac{U}{V_{cr,0}}}{\dfrac{W_{cr,2}}{V_{cr,0}}}$$

The ratio of critical velocity across the first stage is obtained from equations (B12) and (B8)

$$\frac{V_{cr,2}}{V_{cr,0}} = \frac{W_{cr,2}}{V_{cr,0}}\left(\frac{V_{cr}}{W_{cr}}\right)_2$$

Second-Stage Stator

The critical velocity ratio entering the second stage stator is obtained from

$$\left(\frac{V}{V_{cr}}\right)_2 = \left\{\left[\frac{\left(\dfrac{V_u}{W_{cr}}\right)_2}{\left(\dfrac{V_{cr}}{W_{cr}}\right)_2}\right]^2 + \left[\frac{\left(\dfrac{V_x}{W_{cr}}\right)_2}{\left(\dfrac{V_{cr}}{W_{cr}}\right)_2}\right]^2\right\}^{1/2} \tag{B13}$$

where $\left(V_{cr}/W_{cr}\right)_2$ is given by equation (B12), $\left(V_u/W_{cr}\right)_2$ is expressed in known quantities after equation (B12), and $\left(V_x/W_{cr}\right)_2 = \left(W/W_{cr}\right)_2 \cos \beta_2$. From this critical velocity ratio the static- to total-pressure ratio is evaluated as follows:

$$\left(\frac{p}{p'}\right)_2 = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_2^2\right]^{\gamma/(\gamma-1)} \tag{B14}$$

By using the uncooled turbine local pressure ratios, the stator outlet pressure ratio for the gas stream is then determined as

$$\left(\frac{p}{p'}\right)_{3a,fs} = \left(\frac{p}{p'}\right)_2 \frac{\left(\frac{p}{p'}\right)_{3,un}}{\left(\frac{p}{p'}\right)_{2,un}} \tag{B15}$$

The gas-stream critical velocity ratio at the stator outlet is calculated from

$$\left(\frac{V}{V_{cr}}\right)_{3a,fs} = \left\{\frac{\gamma + 1}{\gamma - 1}\left[1 - \left(\frac{p}{p'}\right)_{3a,fs}^{(\gamma-1)/\gamma}\right]\right\}^{1/2} \tag{B16}$$

The critical velocity of the stator coolant entering the stream at station 3 is obtained by using the $\left(V/V_{cr}\right)_{3a,fs}$ value from equation (B16), the assumed k_p value, and the procedure following equation (B2). The aftermixed critical velocity ratio at the stator outlet is then determined as

$$\left(\frac{V}{V_{cr}}\right)_3 = \left[\frac{(1 + A + B)\left(\frac{V}{V_{cr}}\right)_{3a,fs} + C\left(\frac{V}{V_{cr}}\right)_{3a,c}}{1 + A + B + C}\right] \tag{B17}$$

The critical velocity at the stator outlet, related to that at the turbine inlet, is expressed as

$$\frac{V_{cr,3}}{V_{cr,0}} = \left[\frac{(1 + A + B)\left(\frac{V_{cr,2}}{V_{cr,0}}\right)^2 + C\left(\frac{T_{c,0}}{T'_0}\right)}{1 + A + B + C}\right]^{1/2} \tag{B18}$$

where $V_{cr,2}/V_{cr,0}$ is expressed in known quantities following equation (B12).

Second-Stage Rotor Inlet

The equation for relative- to absolute-critical-velocity ratio at station 3 is similar to equation (B4):

$$\left(\frac{W_{cr}}{V_{cr}}\right)_3 = \left\{1 - \frac{\gamma - 1}{\gamma + 1}\left[2\frac{U}{V_{cr,3}}\left(\frac{V_u}{V_{cr}}\right)_3 - \left(\frac{U}{V_{cr,3}}\right)^2\right]\right\}^{1/2} \tag{B19}$$

The value of $\left(V_u/V_{cr}\right)_3$ can be obtained from $\left(V/V_{cr}\right)_3$ of equation (B17) and $\sin \alpha_3$. The value of $U/V_{cr,3}$ can be determined since $U/V_{cr,0}$ is known and $V_{cr,3}/V_{cr,0}$ is given by equation (B18). The blade-inlet relative critical velocity is obtained from the following equation, which is similar to equation (B5):

$$\left(\frac{W}{V_{cr}}\right)_3 = \left\{\left[\left(\frac{V}{V_{cr}}\right)_3 \sin \alpha_3 - \frac{U}{V_{cr,3}}\right]^2 + \left[\left(\frac{V}{V_{cr}}\right)_3 \cos \alpha_3\right]^2\right\}^{1/2} \tag{B20}$$

The relative critical velocity ratio at the blade inlet $\left(W/W_{cr}\right)_3$ is obtained by combining equations (B19) and (B20). The inlet static- to total-pressure ratio is then determined as follows:

$$\left(\frac{p}{p''}\right)_3 = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{W}{W_{cr}}\right)_3^2\right]^{\gamma/(\gamma-1)} \tag{B21}$$

Second-Stage Rotor Outlet

The relative critical velocity of the second-stage rotor coolant as compared with critical velocity at the turbine inlet is given by

$$\frac{W_{cr,4a,c}}{V_{cr,0}} = \left[\frac{T_{c,0}}{T'_0} + \frac{\gamma - 1}{\gamma + 1}\left(\frac{U}{V_{cr,0}}\right)^2\right]^{1/2} \tag{B22}$$

which is equivalent to equation (B7). The relative critical velocity of the gas stream at station 4 is obtained from

$$\frac{W_{cr,4}}{V_{cr,0}} = \left[\frac{(1 + A + B + C)\left(\frac{W_{cr}}{V_{cr}}\right)_3^2\left(\frac{V_{cr,3}}{V_{cr,0}}\right)^2 + D\left(\frac{W_{cr,4a,c}}{V_{cr,0}}\right)^2}{1 + A + B + C + D}\right]^{1/2} \tag{B23}$$

where $\left(W_{cr}/V_{cr}\right)_3$ is given by equation (B19), $V_{cr,3}/V_{cr,0}$ by equation (B18), and $W_{cr,4a,c}/V_{cr,0}$ by equation (B22).

The static- to total-pressure ratio of the gas stream at the rotor outlet before mixing was obtained from the following equation:

$$\frac{p_4}{p''_{4a,fs}} = \left(\frac{p}{p''}\right)_3 \frac{\left(\frac{p}{p''}\right)_{4,un}}{\left(\frac{p}{p''}\right)_{3,un}} \tag{B24}$$

The value of $(p/p'')_3$ is given by equation (B21), and the uncooled turbine pressure ratios are obtainable from the uncooled turbine velocity diagram. The gas-stream critical velocity ratio at the blade outlet is then evaluated from this pressure ratio:

$$\left(\frac{W}{W_{cr}}\right)_{4a,fs} = \left\{\frac{\gamma + 1}{\gamma - 1}\left[1 - \left(\frac{p_4}{p''_{4a,fs}}\right)^{(\gamma-1)/\gamma}\right]\right\}^{1/2} \tag{B25}$$

For the example impulse rotor blade rows, $\left(W/W_{cr}\right)_{4a,fs}$ is equal to $\left(W/W_{cr}\right)_3$, as described following equation (B10). The coolant blade outlet critical velocity ratio is then obtained from the gas-stream critical velocity ratio by using the assumed k_p value and the procedure discussed following equation (B2). By using an assumed k_v value, this critical velocity ratio for the impulse blade rows is obtained as

$$\left(\frac{W}{W_{cr}}\right)_{4a,c} = k_v\left(\frac{W}{W_{cr}}\right)_{4a,fs}$$

The aftermixed critical velocity ratio at the blade outlet is then obtained from

$$\left(\frac{W}{W_{cr}}\right)_4 = \frac{(1 + A + B + C)\left(\frac{W}{W_{cr}}\right)_{4a,fs} + D\left(\frac{W}{W_{cr}}\right)_{4a,c}}{1 + A + B + C} \tag{B26}$$

Cooled-Turbine Work Output

From the determination of the velocity conditions through the turbine, the work output of the cooled turbine can be evaluated and compared to that of the uncooled turbine. The specific work output is expressed in nondimensional form as $U\,\Delta V_u/V_{cr,0}^2$, and the weight flow is normalized by the primary weight flow. Thus, the work output for the first stage is

$$\sum \frac{wU\,\Delta V_u}{w_p V_{cr,0}^2} = \frac{U}{V_{cr,0}}\left\{(1 + A)\left(\frac{V_{u,1}}{V_{cr,0}}\right) - (1 + A + B)\left[\left(\frac{W}{W_{cr}}\right)_2 \frac{W_{cr,2}}{V_{cr,0}} \sin\beta_2 + \frac{U}{V_{cr,0}}\right]\right\} \tag{B27}$$

where $V_{u,1}/V_{cr,0}$ is determinable from equation (B3) and $\sin\alpha_3$, $\left(W/W_{cr}\right)_2$ is determinable from equation (B11), and $W_{cr,2}/V_{cr,0}$ is given in equation (B8).

The second-stage work output is

$$\sum \frac{wU\,\Delta V_u}{w_p V_{cr,0}^2} = \frac{U}{V_{cr,0}}\left\{(1 + A + B + C)\left(\frac{V}{V_{cr}}\right)_3 \frac{V_{cr,3}}{V_{cr,0}} \sin \alpha_3 - (1 + A + B + C + D)\left[\left(\frac{W}{W_{cr}}\right)_4 \frac{W_{cr,4}}{V_{cr,0}} \sin \beta_4 + \frac{U}{V_{cr,0}}\right]\right\} \tag{B28}$$

where $\left(V/V_{cr}\right)_3$ is obtained from equation (B17), $V_{cr,3}/V_{cr,0}$ from equation (B18), $\left(W/W_{cr}\right)_4$ from equation (B26), and $W_{cr,4}/V_{cr,0}$ from equation (B23).

Since the efficiency is based only on primary air, the ratio of work output of the cooled turbine to that of the uncooled turbine is the same as the efficiency ratio. The general equation for efficiency variation of the cooled turbine relative to the uncooled turbine efficiency is

$$\frac{\Delta\eta}{\eta_{un}} = \frac{\text{eq. (B27)} + \text{eq. (B28)} - \left(\sum \dfrac{U\,\Delta V_u}{V_{cr,0}^2}\right)_{un}}{\left(\sum \dfrac{U\,\Delta V_u}{V_{cr,0}^2}\right)_{un}}$$

For the example turbines considered, the work output of the uncooled turbine expressed in this nondimensional form is 0.5 for the reaction turbine and 1.0 for the impulse turbine. Thus, the efficiency variation of the cooled turbine compared to the efficiency of the uncooled turbine is

$$\frac{\Delta\eta}{\eta_{un}} = \frac{\text{eq. (B27)} + \text{eq. (B28)} - 0.5}{0.5} \tag{B29}$$

for the reaction turbine and

$$\frac{\Delta\eta}{\eta_{un}} = \frac{\text{eq. (B27)} + \text{eq. (B28)} - 1.0}{1.0} \tag{B30}$$

for the impulse turbine.

APPENDIX C

ISOLATED-FLOW PERFORMANCE ESTIMATION METHOD

As discussed in the Isolated-Flow Analysis section, this procedure involves the determination of the incremental work output of the coolant flows. The work output of each individual coolant flow is evaluated separately. The velocities are expressed in terms of coolant critical velocities. Again, the procedure is presented for the two-stage turbine.

First-Stage Stator Coolant

The coolant critical velocity ratio out of the first-stage stator $\left(V/V_{cr}\right)_{1,c}$ is obtained, as in the mixed-flow analysis, from $\left(V/V_{cr}\right)_{1,un}$ and from the assumed value of k_p by using the procedure described following equation (B2). The blade speed is determined in terms of coolant critical velocity by the equation

$$\frac{U}{V_{cr,1c}} = \frac{\dfrac{U}{V_{cr,0}}}{\left(\dfrac{T_{0,c}}{T_0'}\right)^{1/2}} \tag{C1}$$

since

$$\frac{V_{cr,1c}}{V_{cr,0}} = \left(\frac{T_{0,c}}{T_0'}\right)^{1/2}$$

The ratio of relative to absolute critical velocity of the coolant is obtained by using the following equation:

$$\left(\frac{W_{cr}}{V_{cr}}\right)_{1,c} = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V_u}{V_{cr}}\right)_{1,c}^2 + \frac{\gamma - 1}{\gamma + 1}\left(\frac{W_u}{V_{cr}}\right)_{1,c}^2\right]^{1/2} \tag{C2}$$

where

$$\left(\frac{V_u}{V_{cr}}\right)_{1,c} = \left(\frac{V}{V_{cr}}\right)_{1,c} \sin \alpha_1$$

and

$$\left(\frac{W_u}{V_{cr}}\right)_{1,c} = \left(\frac{V_u}{V_{cr}}\right)_{1,c} - \frac{U}{V_{cr,1c}}$$

Since only the parallel component of relative velocity at the blade entry was assumed to contribute to the relative total pressure, the static- to total-pressure ratio at the rotor inlet is given by

$$\left(\frac{p}{p''}\right)_{1,c} = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{W}{W_{cr}}\right)_{1,c}^2 \cos^2 i_1\right]^{\gamma/(\gamma-1)} \tag{C3}$$

where i is the incidence angle, or the difference between the coolant flow angle β_1 and the blade entry angle. The coolant flow angle is given by

$$\beta_1 = \tan^{-1} \frac{\left(\frac{W_u}{V_{cr}}\right)_{1,c}}{\left(\frac{V_x}{V_{cr}}\right)_{1,c}}$$

where

$$\left(\frac{V_x}{V_{cr}}\right)_{1,c} = \left(\frac{V}{V_{cr}}\right)_{1,c} \cos \alpha_1$$

The coolant relative critical velocity ratio is expressed in known quantities as

$$\left(\frac{W}{W_{cr}}\right)^2_{1,c} = \frac{\left(\frac{W_u}{V_{cr}}\right)^2_{1,c} + \left(\frac{V_x}{V_{cr}}\right)^2_{1,c}}{\left(\frac{W_{cr}}{V_{cr}}\right)^2_{1,c}}$$

The pressure ratio at the blade outlet is obtained by using the assumption that the overall blade-loss pressure ratio is the same for the coolant as for the primary flow:

$$\left(\frac{p}{p''}\right)_{2,c} = \left(\frac{p}{p''}\right)_{1,c} \frac{\left(\frac{p}{p''}\right)_{2,un}}{\left(\frac{p}{p''}\right)_{1,un}} \tag{C4}$$

The critical velocity ratio of the coolant at the blade outlet is then obtained from this pressure ratio and the relation

$$\left(\frac{W}{W_{cr}}\right)_{2,c} = \left\{\frac{\gamma + 1}{\gamma - 1}\left[1 - \left(\frac{p}{p''}\right)_{2,c}^{(\gamma-1)/\gamma}\right]\right\}^{1/2} \tag{C5}$$

The tangential component of rotor outlet velocity is then determined as

$$\frac{V_{u,2,c}}{V_{cr,1,c}} = \left(\frac{W}{W_{cr}}\right)_{2,c} \sin\beta_2\left(\frac{W_{cr}}{V_{cr}}\right)_{1,c} + \left(\frac{U}{V_{cr}}\right)_{1,c} \tag{C6}$$

The first-stage coefficient is then

$$K_{f,A} = \frac{\frac{U}{V_{cr,1,c}}\left[\left(\frac{V_u}{V_{cr}}\right)_{1,c} - \frac{V_{u,2,c}}{V_{cr,1,c}}\right]\frac{T_{0,c}}{T_0'}}{\left(\frac{U\,\Delta V_u}{V^2_{cr,0}}\right)_{un(0-2)}} \tag{C7}$$

In the particular case of an impulse rotor the outlet relative velocity is equal to the effective inlet relative velocity, $W_{2,c} = W_{1,c} \cos i_1$, and the procedure is simplified since it is not necessary to evaluate state conditions relative to the rotor by using the procedure involving equations (C3) to (C5).

The temperature ratio of the coolant across the first stage is obtained from the following equation:

$$\left(\frac{T'_2}{T'_1}\right)_c = 1 - 2\,\frac{\gamma - 1}{\gamma + 1}\,\frac{U}{V_{cr,1,c}}\left[\left(\frac{V_{u,1}}{V_{cr,1}}\right)_c - \left(\frac{V_{u,2}}{V_{cr,1}}\right)_c\right] \tag{C8}$$

where $\left(V_{u,2}/V_{cr,1}\right)_c$ is given by equation (C6) and $\left(V_{u,1}/V_{cr,1}\right)_c$ is given following equation (C2). The static- to total-pressure ratio entering the second-stage stator would in general be based on the effective critical velocity ratio as

$$\left(\frac{p}{p'}\right)_{2,c} = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_{2,c}^2 \cos^2 i_2\right]^{\gamma/(\gamma-1)}$$

For the example turbines used herein the stator entry direction is axial and the effective critical velocity ratio is then the axial component

$$\left(\frac{V_x}{V_{cr}}\right)_{2,c} = \frac{\left(\frac{W}{W_{cr}}\right)_{2,c} \cos \beta_2 \left(\frac{W_{cr}}{V_{cr}}\right)_{1,c}}{\left(\frac{T'_2}{T'_1}\right)_c^{1/2}} \tag{C9}$$

where $\left(W/W_{cr}\right)_{2,c}$ is given by equation (C5), $\left(W_{cr}/V_{cr}\right)_{1,c}$ by equation (C2), and $\left(T'_2/T'_1\right)_c$ by equation (C8).

From this critical velocity ratio the static- to total-pressure ratio is obtained:

$$\left(\frac{p}{p'}\right)_{2,c} = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V_x}{V_{cr}}\right)_{2,c}^2\right]^{\gamma/(\gamma-1)} \tag{C10}$$

The static- to total-pressure ratio at the stator outlet is then determined:

$$\left(\frac{p}{p'}\right)_{3,c} = \left(\frac{p}{p'}\right)_{2,c} \frac{\left(\frac{p}{p'}\right)_{3,un}}{\left(\frac{p}{p'}\right)_{2,un}} \tag{C11}$$

The stator outlet critical velocity is obtained from this pressure ratio:

$$\left(\frac{V}{V_{cr}}\right)_{3,c} = \left\{\frac{\gamma + 1}{\gamma - 1}\left[1 - \left(\frac{p}{p'}\right)_{3,c}^{(\gamma-1)/\gamma}\right]\right\}^{1/2} \tag{C12}$$

The rotor blade speed in terms of coolant critical velocity is

$$\frac{U}{V_{cr,3,c}} = \frac{\frac{U}{V_{cr,1,c}}}{\left(\frac{T_2'}{T_1'}\right)_c^{1/2}} \tag{C13}$$

where $\left(T_2'/T_1'\right)_c$ is given by equation (C8). The procedure for the evaluation of the work done in the second stage by the first-stage stator coolant is similar to the procedure for the first-stage work. The equations are therefore listed without discussion. Second-stage relative to absolute critical velocity ratio is obtained as

$$\left(\frac{W_{cr}}{V_{cr}}\right)_{3,c} = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V_u}{V_{cr}}\right)_{3,c}^2 + \frac{\gamma - 1}{\gamma + 1}\left(\frac{W_u}{V_{cr}}\right)_{3,c}^2\right]^{1/2} \tag{C14}$$

where

$$\left(\frac{V_u}{V_{cr}}\right)_{3,c} = \left(\frac{V}{V_{cr}}\right)_{3,c} \sin \alpha_3$$

and

$$\left(\frac{W_u}{V_{cr}}\right)_{3,c} = \left(\frac{V_u}{V_{cr}}\right)_{3,c} - \frac{U}{V_{cr,3,c}}$$

Rotor-inlet static- to total-pressure ratio is

$$\left(\frac{p}{p''}\right)_{3,c} = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{W}{W_{cr}}\right)^2_{3,c} \cos^2 i_3\right]^{\gamma/(\gamma-1)} \tag{C15}$$

where i_3 is the incidence angle of the flow entering the second-stage rotor. The coolant relative flow angle is given by

$$\beta_3 = \tan^{-1} \frac{\left(\frac{W_u}{V_{cr}}\right)_{3,c}}{\left(\frac{V_x}{V_{cr}}\right)_{3,c}}$$

where

$$\left(\frac{V_x}{V_{cr}}\right)_{3,c} = \left(\frac{V}{V_{cr}}\right)_{3,c} \cos \alpha_3$$

The coolant relative critical velocity ratio is determined from known quantities as

$$\left(\frac{W}{W_{cr}}\right)^2_{3,c} = \frac{\left(\frac{W_u}{V_{cr}}\right)^2_{3,c} + \left(\frac{V_x}{V_{cr}}\right)^2_{3,c}}{\left(\frac{W_{cr}}{V_{cr}}\right)^2_{3,c}}$$

Rotor-outlet static- to total-pressure ratio is

$$\left(\frac{p}{p''}\right)_{4,c} = \left(\frac{p}{p''}\right)_{3,c} \frac{\left(\frac{p}{p''}\right)_{4,un}}{\left(\frac{p}{p''}\right)_{3,un}} \tag{C16}$$

Rotor-outlet critical velocity ratio is

$$\left(\frac{W}{W_{cr}}\right)_{4,c} = \left\{\frac{\gamma + 1}{\gamma - 1}\left[1 - \left(\frac{p}{p''}\right)_{4,c}^{(\gamma-1)/\gamma}\right]\right\}^{1/2} \tag{C17}$$

Second-stage outlet tangential velocity is

$$\left(\frac{V_{u,4}}{V_{cr,3}}\right)_c = \left(\frac{W}{W_{cr}}\right)_{4,c} \sin \beta_4 \left(\frac{W_{cr}}{V_{cr}}\right)_{3,c} + \frac{U}{V_{cr,3,c}} \tag{C18}$$

Second-stage work coefficient is

$$K_{s,A} = \frac{\frac{U}{V_{cr,3,c}}\left[\left(\frac{V_{u,3}}{V_{cr,3}}\right)_c - \left(\frac{V_{u,4}}{V_{cr,3}}\right)_c\right]\frac{T_{0,c}}{T_0'}\left(\frac{T_2'}{T_1'}\right)_c}{\left(\frac{U\,\Delta V_u}{V_{cr,0}^2}\right)_{un,2-4}} \tag{C19}$$

As mentioned for the work output in the first-stage rotor, the procedure for the case of an impulse rotor is simplified. Since there is no expansion across the rotor and since $W_{4,c} = W_{3,c} \cos i_3$, the procedure involving equations (C15) to (C17) for evaluating total-state conditions relative to the rotor is not required.

First-Stage Rotor Coolant

The relative critical velocity of the first-stage rotor coolant out of the first-stage rotor is obtained from the uncooled-turbine-outlet critical velocity ratio $\left(W/W_{cr}\right)_{2,un}$ and the assumed k_p value by using the procedure described following equation (B2). In the case of the impulse rotor, where a k_v assumption is used, the coolant critical velocity ratio is

$$\left(\frac{W}{W_{cr}}\right)_{2,c} = k_v\left(\frac{W}{W_{cr}}\right)_{2,un}$$

The critical velocity of the coolant relative to the turbine inlet critical velocity $(W_{cr,2,c}/V_{cr,0})$ is given by equation (B7). The first-stage work coefficient is

$$K_{f,B} = -\frac{\dfrac{U}{V_{cr,0}}\dfrac{V_{u,2,c}}{V_{cr,0}}}{\left(\dfrac{U\,\Delta V_u}{V_{cr,0}^2}\right)_{un,0-2}} \tag{C20}$$

where

$$\frac{V_{u,2,c}}{V_{cr,0}} = \left(\frac{W}{W_{cr}}\right)_{2,c} \sin\beta_2 \frac{W_{cr,2,c}}{V_{cr,0}} + \frac{U}{V_{cr,0}}$$

The temperature ratio across the first stage of the rotor coolant is obtained by the equation

$$\left(\frac{T_2'}{T_0}\right)_c = \left[1 - 2\left(\frac{\gamma-1}{\gamma+1}\right)\frac{U}{V_{cr,0}}\,\frac{\dfrac{V_{u,2,c}}{V_{cr,0}}}{\dfrac{T_{0,c}}{T_0'}}\right] \tag{C21}$$

Since the blade entry direction is axial, the effective critical velocity ratio entering the second-stage stator is evaluated with the following equation for the example turbines

$$\left(\frac{V_x}{V_{cr}}\right)_{2,c} = \frac{\left(\frac{W}{W_{cr}}\right)_{2,c} \cos \beta_2 \frac{W_{cr,2,c}}{V_{cr,0}}}{\left(\frac{T_2'}{T_0}\right)_c^{1/2} \left(\frac{T_{0,c}}{T_0'}\right)^{1/2}} \tag{C22}$$

The work of the first-stage rotor coolant in the second stage is then calculated by using the same equations as were used for the first-stage stator coolant (eqs. (C10) to (C19)); thus, $K_{s,B}$ is evaluated for the rotor coolant.

Second-Stage Stator Coolant

The stator coolant can do work only in the second stage. The procedure for evaluating $K_{s,C}$ is identical to that of the first-stage coolant determination of $K_{f,A}$. The value of $K_{s,C}$ for the second-stage stator coolant is close to that of $K_{f,A}$ of the first-stage stator coolant for the example turbines because the total temperatures of the two coolant flows are the same, as are the blade speeds and the velocity diagrams. The slight difference occurs because of the increased critical velocity ratios in the second-stage rotor.

Second-Stage Rotor Coolant

The procedure for this coolant-flow work estimation is the same as that for first-stage rotor coolant work done in the first stage. The value of $K_{s,D}$ for the second-stage rotor coolant is also close to that of $K_{f,B}$ of the first-stage rotor coolant for the example turbines. This similarity is again because the total temperatures of the two coolant flows are the same, as are the blade speeds and the velocity diagrams. The slight difference in values occurs because of the increased critical velocity ratios in the second-stage rotor.

Turbine Efficiency Variation

The stage work coefficients described in the preceding sections are listed in table II. A coolant schedule can be applied to the coefficients to obtain the cooled-turbine performance variation. Based on these coefficients and a coolant schedule, the general equation for the efficiency variation is

$$\frac{\Delta\eta}{\eta_{un}} = \frac{A(K_{f,A}\,\Delta h_f + K_{s,A}\,\Delta h_s) + B(K_{f,B}\,\Delta h_f + K_{s,B}\,\Delta h_s) + CK_{s,C}\,\Delta h_s + DK_{s,D}\,\Delta h_s}{h_f + h_s}$$

For the examples considered, the work split between stages is equal for the uncooled turbines ($\Delta h_f = \Delta h_s$), and the equation for efficiency variation is

$$\frac{\Delta\eta}{\eta_{un}} = \frac{A(K_{f,A} + K_{s,A}) + B(K_{f,B} + K_{s,B}) + CK_{s,C} + DK_{s,D}}{2} \tag{C23}$$

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TABLE I. - ASSUMED COOLANT FLOW SCHEDULES

USED FOR EXAMPLE TURBINE ANALYSIS

First-stage stator coolant fraction, A	First-stage rotor coolant fraction, B	Second-stage stator coolant fraction, C	Second-stage rotor coolant fraction, D
0.02	0.01	----	----
.03	.02	0.01	----
.04	.03	.02	0.01
.05	.04	.03	.02
.06	.05	.04	.03
.07	.06	.05	.04

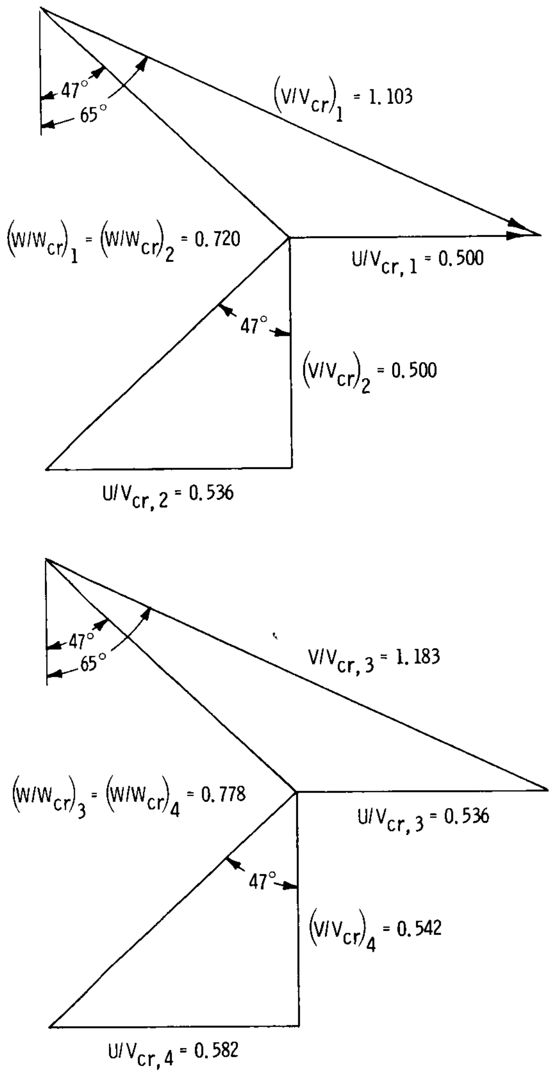
TABLE II. - STAGE WORK COEFFICIENTS FROM ISOLATED-FLOW ANALYSIS

(a) Impulse turbine

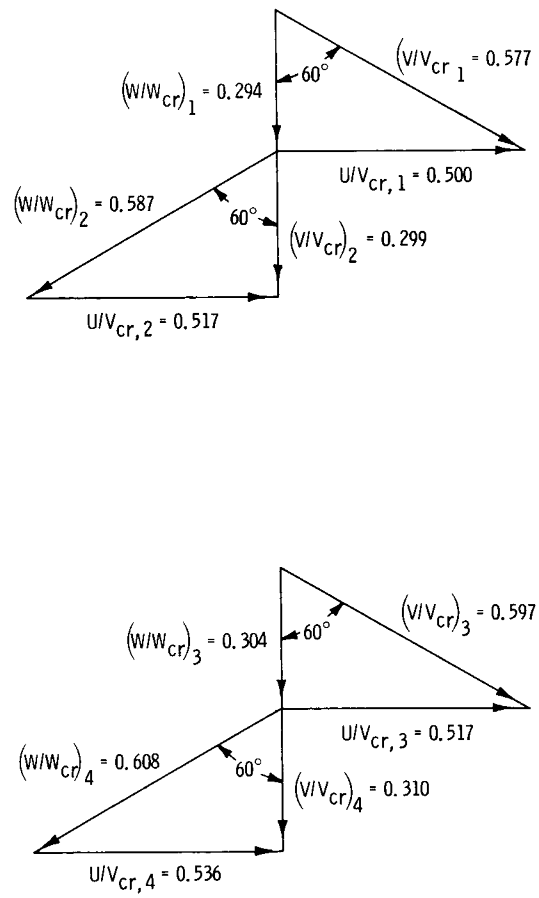
Coolant temperature ratio, $T_{c,0}/T'_0$	Coolant fraction	Coolant-stage work coefficient for first stage, K_f	Coolant-stage work coefficient for second stage, K_s	Overall stage work coefficient, ΣK
Coolant pressure coefficient, k_p, 0.25; impulse rotor velocity coefficient, k_v, 0				
0.3	A	-0.160	0.222	0.062
	B	-.500	.288	-.212
	C	------	-.104	-.104
	D	------	-.500	-.500
0.45	A	-0.030	0.411	0.381
	B	-.500	.486	-.014
	C	------	.049	.049
	D	------	-.500	-.500
0.6	A	0.084	0.580	0.664
	B	-.500	.657	.157
	C	------	.171	.171
	D	------	-.500	-.500
Coolant pressure coefficient, k_p, 0.5; impulse rotor velocity coefficient, k_v, 0.5				
0.3	A	0.027	0.190	0.217
	B	-.348	.280	-.068
	C	------	.098	.098
	D	------	-.336	-.336
0.45	A	0.205	0.388	0.593
	B	-.317	.482	.165
	C	------	.292	.292
	D	------	-.302	-.302
0.6	A	0.356	0.567	0.923
	B	-.291	.657	.366
	C	------	.456	.456
	D	------	-.274	-.274

(b) Reaction turbine

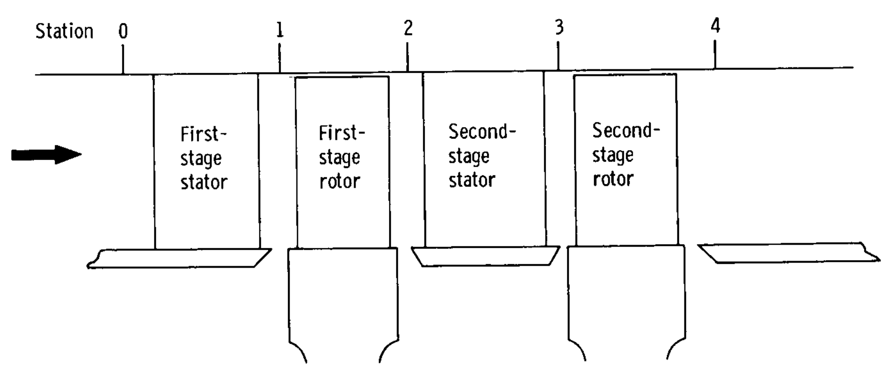
Coolant temperature ratio, $T_{c,0}/T'_0$	Coolant fraction	Coolant-stage work coefficient for first stage, K_f	Coolant-stage work coefficient for second stage, K_s	Overall stage work coefficient, ΣK
Coolant pressure coefficient, k_p, 0.25				
0.3	A	-0.184	0.143	-0.041
	B	-.679	.142	-.537
	C	------	-.155	-.155
	D	------	-.667	-.667
0.45	A	-0.002	0.371	0.369
	B	-.613	.371	-.242
	C	------	.028	.028
	D	------	-.599	-.599
0.6	A	0.143	0.569	0.712
	B	-.558	.562	.004
	C	------	.184	.184
	D	------	-.541	-.541
Coolant pressure coefficient, k_p, 0.5				
0.3	A	-0.053	0.133	0.080
	B	-.555	.163	-.392
	C	------	-.020	-.020
	D	------	-.539	-.539
0.45	A	0.154	0.362	0.516
	B	-.464	.388	-.076
	C	------	.195	.195
	D	------	-.444	-.444
0.6	A	0.330	0.561	0.891
	B	-.386	.583	.197
	C	------	.377	.377
	D	------	-.364	-.364



(a) Impulse turbine.

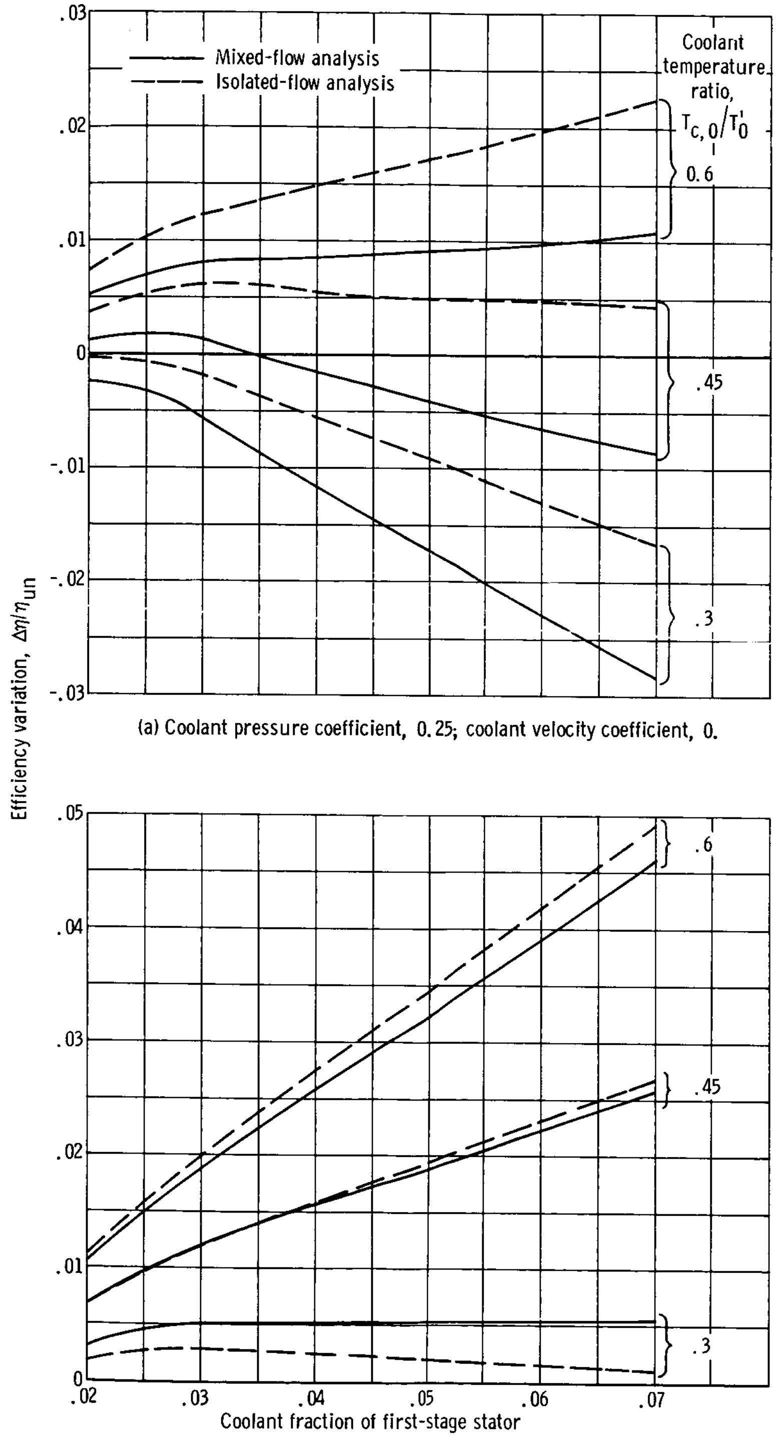


(b) Reaction turbine.



(c) Station locations.

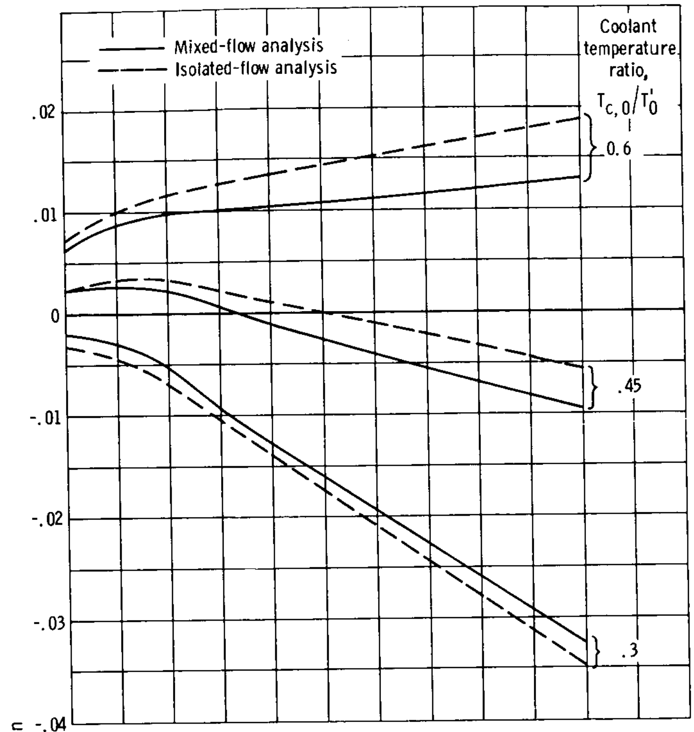
Figure 1. - Velocity diagrams and stations used in analysis. Velocity diagrams shown are for uncooled turbine.



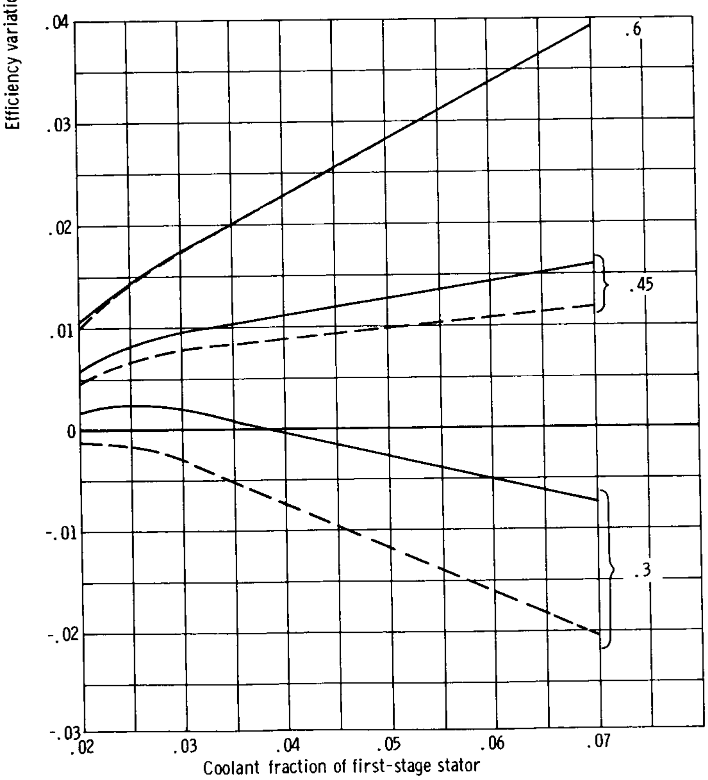
(a) Coolant pressure coefficient, 0.25; coolant velocity coefficient, 0.

(b) Coolant pressure coefficient, 0.5; coolant velocity coefficient, 0.5.

Figure 2. - Effect of coolant flow on primary-air efficiency of impulse turbine.

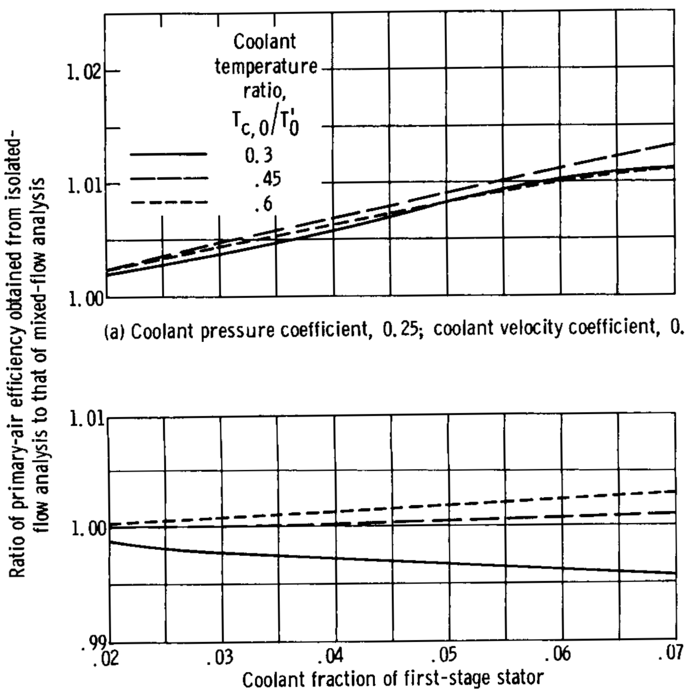


(a) Coolant pressure coefficient, 0.25.



(b) Coolant pressure coefficient, 0.5.

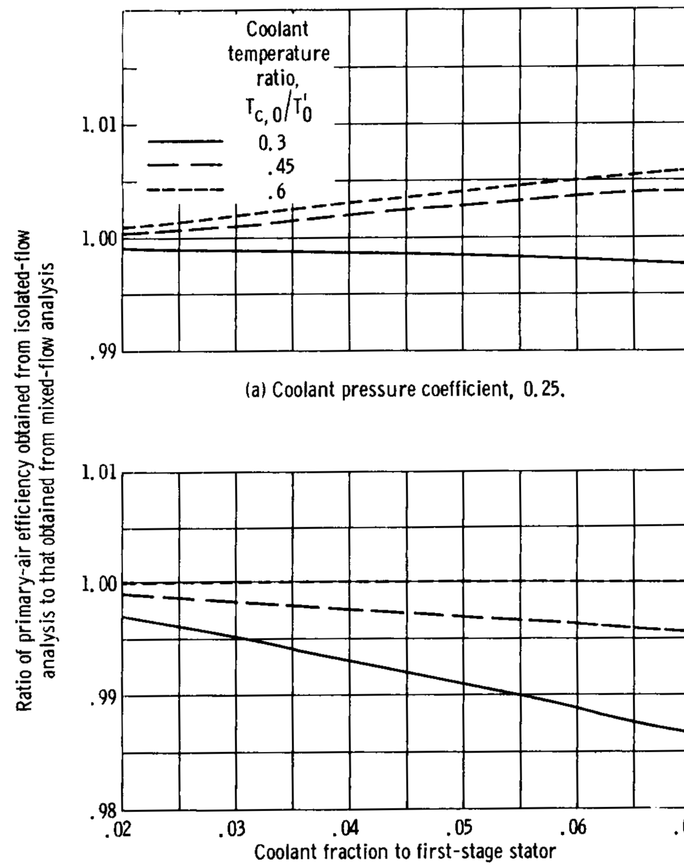
Figure 3. - Effect of coolant flow on primary-air efficiency of reaction turbine.



(a) Coolant pressure coefficient, 0.25; coolant velocity coefficient, 0.

(b) Coolant pressure coefficient, 0.5; coolant velocity coefficient, 0.5.

Figure 4. - Comparison of primary-air efficiency predicted by two performance-estimation methods for impulse turbine.



(a) Coolant pressure coefficient, 0.25.

(b) Coolant pressure coefficient, 0.5.

Figure 5. - Comparison of primary-air efficiency predicted by two performance-estimation methods for reaction turbine.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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