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# A HUMMINGBIRD FOR THE $L_2$ LUNAR LIBRATION POINT

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# ABSTRACT

This paper concerns a spacecraft not in a circumlunar orbit, but in a quasi-permanent position in the vicinity of the far-side lunar libration point $L_2$. Such a spacecraft would be a useful communications relay between the back of the moon and the earth. It could be so placed above the libration point as never to be occulted, thus making the communication link a continuous one, independent of time. The spacecraft would be in the lunar gravitational field and thus need permanent thrust to stay in place or to move slowly around a point in space like a hovering hummingbird.

This report shows analytically the accelerations in the vicinity of $L_2$ and the specific impulses needed to keep a spacecraft there economically with a reasonable fuel-to-mass ratio ($m_f / m_0 = 0.05$ to $0.15$). This dictates the kinds of engines needed for such missions, where constant, small accelerations are needed over the lifetime of a spacecraft (in the order of 1 to 3 years).

CONTENTS

## SUMMARY

The suggestions to use the far-side lunar $L_2$ libration point for anchoring a communication satellite assume that this spacecraft will be in an appropriate orbit around this point. The purpose of this paper is to present an analytical investigation of a stationary lunar libration satellite—not an orbiting spacecraft but a humming or hovering craft, stationary with respect to the earth-moon system.

Acceleration expressions are derived so as to acquaint the reader with the situation considered. For example, a spacecraft hovering 3500 km above $L_2$ has an acceleration of about $10^{-2}$ cm/sec$^2$ or $10^{-5}$ g's (earth acceleration). Above $L_2$ means along a perpendicular line from $L_2$ parallel to the earth-moon (barycenter) rotational axis. This, of course, is not a necessity; the spacecraft may also be in the earth-moon plane located on either side of the moon by 3500 km. Actually, the station-keeping requirements may be less than the example quoted. The 3500 km distance (or more) is needed for the spacecraft to observe the earth at all times. This distance will prevent lunar occultation of the spacecraft, thus guaranteeing continuous communication between the back side of the moon and an earth-bound tracking station.

Because this spacecraft is not in motion and is in an accelerating field, continual thrusting is necessary. On the other hand, since the acceleration experienced is extremely small, the use of low-thrust electric space-propulsion systems with high specific impulse seems to be suitable. A 190-kg spacecraft, an ion engine with a 4300-sec specific impulse, 2000-dyne thrust would suffice and would consume only 23 kg of fuel during a year.

Additional thrust will be needed for the antenna pointing since the spacecraft must rotate around its axis once every 27.3 days (lunar month). Because of the extremely small angular lunar motion, $\omega = 2.66 \times 10^{-6}$ sec$^{-1}$, the thrusting requirements are extremely small compared to the station-keeping requirements. Therefore, little rotational control fuel is required.

In summary, this analysis seems to indicate that such a spacecraft would be feasible to build and operate. Launch and guidance operations for lunar orbiting spacecraft are within the state-of-the-art. A spacecraft of this kind would be a necessary extension of the ground tracking network into space. Only with this system (or similar ones) can a communications link be established between the earth and the back side of the moon, a link needed for both unmanned and manned operations on the invisible side of the moon.

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## ACCELERATION EXPERIENCED BY THE HUMMINGBIRD

In order to understand satellite station keeping above the lunar $L_2$ libration point, a simple analytical model of the acceleration as a function of $r$ and $\varphi$ is developed (see Figure 1). Using vector notation as indicated in Figure 1, the acceleration $\vec{x}$ of the satellite mass m* can be written

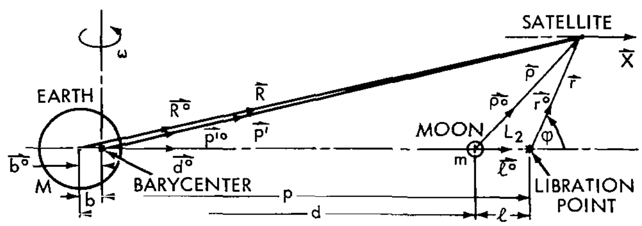


Figure 1—Hummingbird geometry.

$$\vec{x} = \vec{d}^{\,\circ} p\omega^2 - \vec{\rho}^{\,\circ} \frac{\gamma m}{\rho^2} - \vec{R}^{\,\circ} \frac{\gamma M}{R^2}, \tag{1}$$

where

$\vec{d}^{\,\circ}$ = unit vector along the earth-moon axis

$p$ = distance from the barycenter to the spacecraft projection on the earth-moon line

$\omega = 2.66 \times 10^{-6}\ \text{sec}^{-1}$ = moon's angular speed around the earth (Reference 1)

$\vec{\rho}^{\,\circ}$ = unit vector from the moon to the spacecraft

$\rho$ = distance between moon and spacecraft

$\gamma$ = gravitational constant (Reference 1) ($\gamma = 6.668 \times 10^{-8}$ dyn cm$^2$ g$^{-2}$ or cm$^3$ g$^{-1}$ sec$^{-2}$)

$m$ = mass of the moon ($m = 7.350 \times 10^{25}$ g) (Reference 1)

$\vec{R}^{\,\circ}$ = unit vector from the earth center to the satellite

$R$ = distance between earth and satellite

$M$ = mass of the earth ($M = 5.977 \times 10^{27}$ g) (Reference 1).

*m cancels out.

The first term represents the centrifugal acceleration caused by the moon's motion around the earth; the second and third terms represent the moon's attraction of the spacecraft to the moon and the earth, respectively, (thus, the negative signs of the unit vectors $\vec{\rho}^\circ$ and $\vec{R}^\circ$).

Of special interest here is the acceleration $\vec{x}$ as a function of $\vec{r}$, the satellite position vector from the lunar $L_2$ libration point, as shown in Figure 1. From Figure 1 it is also evident that

$$R^2 = (d + \ell + r\cos\varphi)^2 + (r\sin\varphi)^2 ,$$

$$\rho^2 = (\ell + r\cos\varphi)^2 + (r\sin\varphi)^2 ,$$

$$p = (d - b + \ell + r\cos\varphi) .$$

where

$b$ = barycenter distance ($b = d/81 = 4720$ km) (References 2 through 5)

$d$ = earth-moon distance ($d = 3.84 \times 10^5$ km) (Reference 1)

$\ell$ = lunar libration point distance ($\ell \doteq d\sqrt[3]{m/3M} \doteq 6.16 \times 10^4$ km) (Reference 3)

$r$ = distance (magnitude of $\vec{r}$) from the libration point $L_2$ to the spacecraft

$\varphi$ = angle between earth-moon axis and $\vec{r}$

$\vec{r}$ = satellite position vector.

The influence of the sun (only ±10 percent) is considered later.

The magnitude of $x$, namely $x = \sqrt{(\vec{x} \cdot \vec{x})}$, can now be easily calculated from Equation 1. From

$$p\omega^2 = A , \qquad -\frac{\gamma m}{\rho^2} = B , \qquad -\frac{\gamma M}{R^2} = C , \tag{2}$$

one obtains

$$|\vec{x}| = x = \left[A^2 + B^2 + C^2 + 2AB\left(\vec{d}^\circ \cdot \vec{\rho}^\circ\right) + 2AC\left(\vec{d}^\circ \cdot \vec{R}^\circ\right) + 2BC\left(\vec{\rho}^\circ \cdot \vec{R}^\circ\right)\right]^{1/2} . \tag{3}$$

Expressing the vector dot-products in terms of known quantities yields

$$\left(\vec{d}^\circ \cdot \vec{\rho}^\circ\right) = \frac{1}{\rho}(\ell + r\cos\varphi) ,$$

$$\left(\vec{d}^\circ \cdot \vec{R}^\circ\right) = \frac{1}{R}(d + \ell + r\cos\varphi) ,$$

$$\left(\vec{\rho}^{\circ} \cdot \vec{R}^{\circ}\right) = \frac{1}{\rho R}\left[r^2 + \ell^2 + \ell d + r \cos \varphi \, (d + 2\ell)\right] . \qquad (4)$$

Equation 3 now can be used to calculate the acceleration $x$ of the Hummingbird as a function of $r$ and $\varphi$. That is,

$$x = f(\vec{r}) = g(r, \varphi) . \qquad (5)$$

Equations 3 and 5 represent the acceleration of the spacecraft, which must be compensated for by rocket control (ion engines for instance) if one wants to keep the craft hovering over $L_2$ as shown in Figure 1.

In Figure 2, the acceleration $x$ (Equation 3) is shown as a function of $r$ and $\varphi$ in graph form to provide an idea of the acceleration magnitudes involved.

If one considers a spacecraft located at a distance $r$ from the libration point along the earth-moon line ($\varphi = 0$), Equation 1 can be simplified:

$$x = p\omega^2 - \frac{\gamma m}{\rho^2} - \frac{\gamma M}{R^2} . \qquad (6)$$

If $p = (d - b + \ell)$, the spacecraft would be directly located at $L_2$ and would not experience acceleration $x$ (sun's influence excluded). This is the true definition of $L_2$.

What happens when the spacecraft is removed from $L_2$ along the earth-moon line can best be studied by varying Equation 6; this yields

$$\delta x = p\omega^2 \left(\frac{\delta p}{p}\right) + 2 \frac{\gamma m}{\rho^2} \left(\frac{\delta \rho}{\rho}\right) + 2 \frac{\gamma M}{R^2} \left(\frac{\delta R}{R}\right) . \qquad (7)$$

Further, $\delta r = \delta p = \delta \rho = \delta R$ for $\varphi = 0$ as seen from Figure 1 and, therefore, approximately

$$\frac{\delta \rho}{\rho} \doteq \left(\frac{384}{61}\right) \frac{\delta R}{R} , \qquad (8)$$

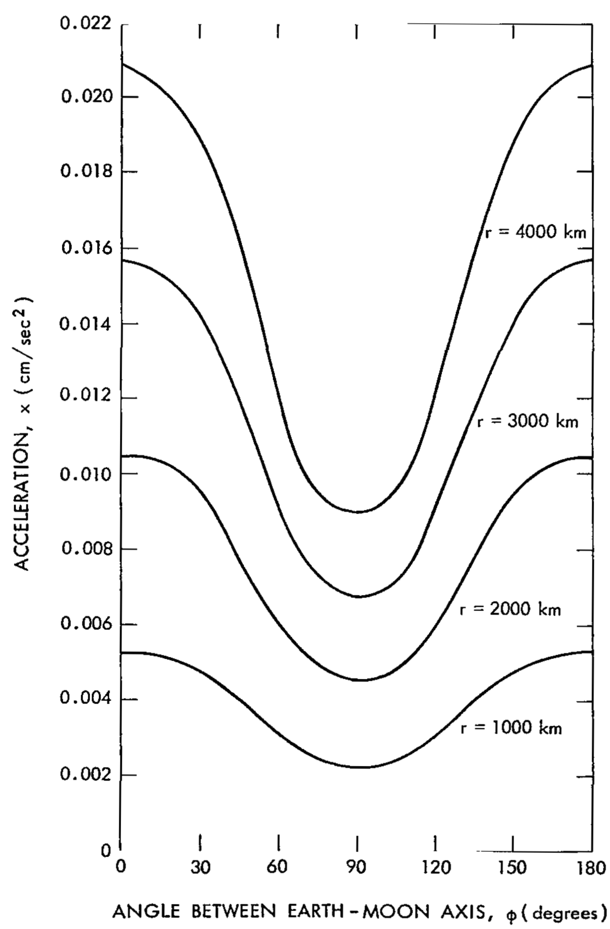


Figure 2—Acceleration of the Hummingbird near $L_2$.

since the value of $\delta\rho$ becomes $\delta r$, $\rho \doteq 61{,}000$ km, and $R \doteq 445{,}000$ km. Introducing Equation 8 into Equation 7 and setting $\delta r = \delta R$ one obtains with $m/M \doteq 1/81$, an approximate equation for the variation $\delta x$ as a function if $\delta r$:

$$\delta x \doteq \delta r \left(\omega^2 + 8\,\frac{\gamma M}{R^3}\right) . \tag{9}$$

As an example, assume

$$\delta r = 3500 \text{ km} = 3.5 \times 10^8 \text{ cm}$$

$$\omega = 2.66 \times 10^{-6} \text{ sec}^{-1}$$

$$\gamma = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ sec}^{-2}$$

$$M = 5.98 \times 10^{27} \text{ g}$$

$$R = (d + \ell) = 4.45 \times 10^{10} \text{ cm.}$$

From in Reference 1, for instance, one obtains

$$\delta x \doteq 1.5 \times 10^{-2} \text{ cm sec}^{-2}$$

or

$$\overline{\delta}x = 1.53 \times 10^{-5}\, g_o \tag{10}$$

in terms of the earth acceleration, where $g_o$ = 981 cm sec$^{-2}$ = the acceleration at the earth surface.

Equation 10 gives a representative number for the acceleration that a spacecraft experiences about 3500 km from $L_2$ along the earth-moon line. This equation also indicates that relatively small accelerations are experienced at these distances which can easily be compensated for by ion engines (References 4 through 7).

The sun's gravitational influence on the Hummingbird, previously neglected, now will be determined approximately. No secondary acceleration influences due to the sun are considered since they are small compared to those discussed (Reference 8). The equilibrium condition for $L_2$ exists only for the case of a rotating earth-moon system as represented by Equation 5 using $\vec{r} = 0$.

When the sun is considered, a nearly sinusoidal perturbation acceleration is superimposed, since $L_2$ rotates around the barycenter (approx. 27.3 days), and thus changes its distance from the sun as shown in Figure 3. Only if $L_2$ lies on the earth orbit is the sun's attractive force approximately

(Reference 8) compensated for by the angular speed $\omega_e$ of the earth (or barycenter) around the sun. The acceleration y of a mass point in a sun orbit is given again by

$$y = \xi\omega_e^2 - \frac{\gamma M_s}{\xi^2} , \tag{11}$$

where

$\xi$ = distance from the sun to the mass point

$\omega_e$ = $1.99 \times 10^{-7}$ $sec^{-1}$ = earth's angular speed around the sun

$\gamma$ = $6.67 \times 10^{-8}$ $cm^3$ $g^{-1}$ $sec^{-2}$ = gravitational constant

$M_s$ = $1.99 \times 10^{33}$ g = mass of the sun.

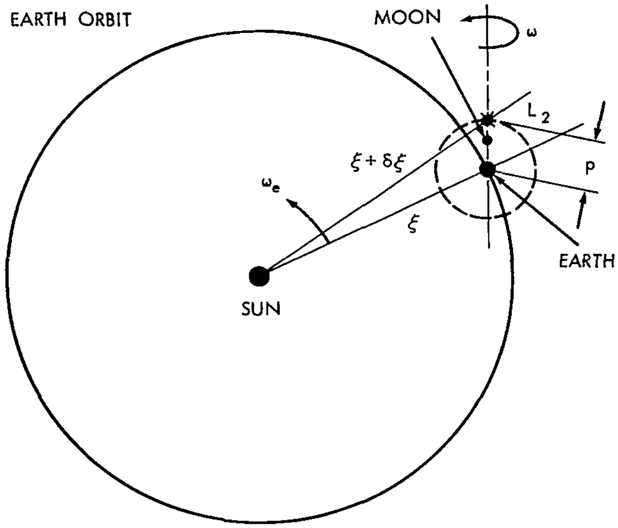


Figure 3—Sun's influence on the acceleration of $L_2$.

If $\xi = \xi_o$ = 1 astronomical unit (AU), then y must be zero.

Varying Equation 11 yields

$$\delta y = \delta\xi\omega_e^2 + 2\gamma M_s \frac{\delta\xi}{\xi^3} . \tag{12}$$

Using $\xi = \xi_o$ = 1 AU from Equation 11, one obtains (since y = 0, as mentioned in this case)

$$\delta y \doteq 3\omega_e^2 \, \delta\xi \tag{13}$$

as the variation for the sun's acceleration due to a change in $\xi$ and $\omega_e$ which is assumed to be constant. For example,

$$\omega_e^2 = 3.96 \times 10^{-14} \ sec^{-2}$$

$$\delta\xi = \rho = (d - b + \ell) = 4.4 \times 10^5 \ km = 4.4 \times 10^{10} \ cm$$

(see also Figures 1 and 3) (moon's orbit assumed circular for simplicity)

then

$$\delta y = 1.74 \times 10^{-3} \ cm \ sec^{-2}$$

or

$$\overline{\delta y} = 1.7 \times 10^{-6} \ g_o \text{ (in terms of earth's acceleration).}$$

Thus, the sun's influence is only approximately 10 percent for the case considered, as shown by Equation 13.

Both Equations 9 and 11 show that the accelerations experienced by hummingbird-type spacecraft are very small indeed (from about 0 to $2 \times 10^{-2}$ cm $\sec^{-2}$ ).

## NECESSARY SPECIFIC IMPULSE FOR ECONOMIC STATION KEEPING

In the foregoing pages, the magnitude of the expected accelerations of a Hummingbird satellite was estimated. The next consideration is the necessary specific impulse $I_{sp}$ needed to keep the spacecraft on station for 1 year using a reasonable fuel-to-spacecraft mass ratio, $m_f/m_o = 0.05$ to 0.10.

The acting force $\vec{F}$ on the spacecraft is given by

$$\vec{F} = m\ddot{\vec{x}} , \tag{14}$$

where $m$ is the total mass, and $\ddot{\vec{x}}$ is the experienced accelerations shown in Equation 5 or $\delta x$ as indicated in Equation 9.

Using the common equation (References 2 and 3) for the force $F$ (magnitude needed only for this consideration) of a rocket, one obtains

$$F = \dot{m} g_o I_{sp} , \tag{15}$$

where $\dot{m}$ is the mass flow (flow rate of exhaust material), $g_o$ is the earth acceleration ($g_o = 981$ cm $\sec^{-2}$) and $I_{sp}$ is the specific impulse. For station keeping, the forces in Equations 14 and 15 must be equal; that is,

$$m\ddot{x} = \dot{m} g_o I_{sp} . \tag{16}$$

Integrating Equation 10 over a time $T$, the useful station-keeping time (say, 1 year $= 3.1 \times 10^7$ sec), one obtains

$$\frac{m_f}{m_o} = \exp\left(\frac{\int_T \ddot{x}\, dt}{g_o I_{sp}}\right) - 1 , \tag{17}$$

where $m = m_f + m_o$ was used; that is, the total flying mass is always the sum of the fuel mass and the spacecraft mass (engine included). Figure 4 shows the ratio $m_f/m_o$ as a function of the specific impulse $I_{sp}$ for a Hummingbird with a station lifetime of 1 year ($\delta x \doteq x \doteq 1.5 \times 10^{-2}$ cm sec$^{-2}$).

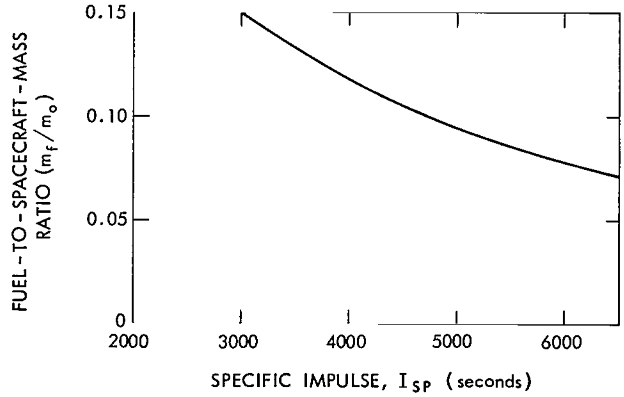


Figure 4—Fuel-to-spacecraft ratio vs. specific impulse (1 year lifetime).

As can be seen from Figure 4, using a rather small fuel-to-spacecraft-mass ratio of say 0.1, specific impulses $I_{sp}$ of the order of 4000 to 5000 seconds are needed. This, coupled with the fact that these control accelerations are really small (approx. $1.5 \times 10^{-2}$ cm sec$^{-2}$), suggests the use of electric space propulsion engines for this kind of operation. Thrust levels, lifetime, power consumption, etc., are all within very reasonable limits for these engines (References 5 through 7). Even deflecting beams can be used for better control purposes.

For instance, Reference 6 cites an engine that was built and tested over hundreds of hours with the following characteristics:

Thrust = $F$ = 1900 dynes

Total power = 500 watts

Power-to-thrust ratio = 260 mw per dyne

Specific impulse = 4330 sec.

A total spacecraft mass, $M = m_f + m_o$, of 190 kg receives an acceleration

$$x = \frac{F}{M} = \frac{1900}{190 \times 10^3} = 10^{-2} \text{ cm sec}^{-2} . \tag{18}$$

This acceleration is well within the control-thrust accelerations needed to keep the spacecraft humming. As can be seen from Figure 4, a favorable fuel-to-spacecraft-mass ratio $m_f/m_o = 0.12$ for a 1-year lifetime also results from the high specific impulse obtainable with ion engines. The same is true for electrical power requirements and engine weight, solar cells may provide the required electrical power for years of space operation.

Another needed control system is one to turn the spacecraft in such a fashion that it will always point its high-gain antenna toward the earth during the rotation of the earth-moon line as shown in Figure 3. This means the spacecraft must rotate once every lunar month around its own axis (see Figures 3 and 5). For this motion, power for ion engines could again be made available.

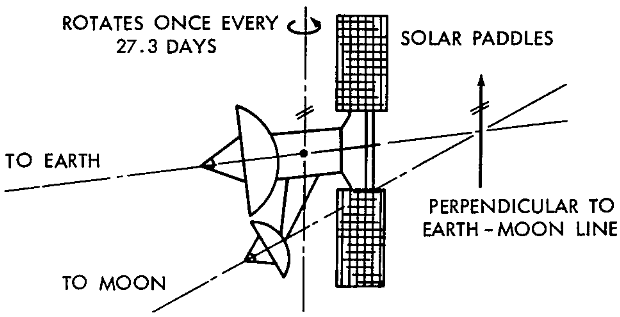


Figure 5—Schematic of the Hummingbird.

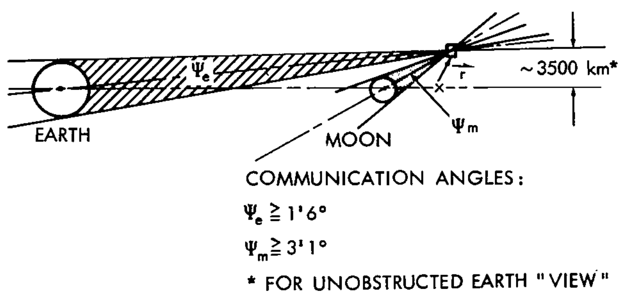


Figure 6—Hummingbird earth and lunar communications coverage.

The control systems must work within certain angular accuracies, depending on the coverage—antenna beams engulfing earth and moon (Figure 6).

To cover the earth and a nearby earth satellite (an orbiting tracking station, for instance) requires:

Earth antenna beam $\psi_e = 3$ degrees

Moon antenna beam $\psi_m = 5$ degrees.

Thus, a correction of ±1 degree would be adequate for spacecraft stabilization. In brief, one must control the rotational motion of the spacecraft during its lunar cycle to this accuracy.

The necessary thrust can be calculated from the basic equation of a system in angular rotation; that is,

$$J\ddot{\varphi} = M = DT , \tag{19}$$

where

$J$ = moment of inertia about the spacecraft rotational axis (see Figure 5)

$\ddot{\varphi}$ = angular acceleration

$M$ = moment about the axis caused by a control engine of thrust $T$ mounted at a distance $D$ from the axis.

Integrating Equation 19 while assuming a constant thrust gives

$$\dot{\varphi} = \frac{DT\,\Delta t}{J} , \tag{20}$$

where $\Delta t$ is the time the thrust is acting, resulting in an angular motion $\dot{\varphi}$ which must equal the moon angular velocity $\omega = 2.66 \times 10^{-6}$ radians sec$^{-1}$. The needed thrust is then from Equation 20:

$$T = \frac{J\omega}{D\,\Delta t} . \tag{21}$$

For example, if

$$J = 1.9 \times 10^9 \text{ g cm}^2 \text{ (equivalent to a dumbbell system using two 95-kg masses 2 meters apart)}$$

$$\omega = 2.66 \times 10^{-6} \text{ radians sec}^{-1}$$

$$D = 100 \text{ cm}$$

$$\Delta t = 1000 \text{ sec (arbitrary),}$$

then

$$T = \frac{(1.9 \times 10^9)(2.66 \times 10^{-6})}{(10^2)(10^3)} = 0.05 \text{ dyne} .$$

This shows that for the rotational motions, a few dynes or a fraction of a dyne of force may be adequate (Sun pressure = 1 dyne/m$^2$ for instance). This is a thrust level much smaller than that needed for station keeping as shown by the example stated previously and therefore should not add substantially to the total spacecraft fuel needed for station keeping.

Goddard Space Flight Center
National Aeronautics and Space Administration
Greenbelt, Maryland, July 26, 1967
311-07-21-51

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