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THERMODYNAMICS OF MHD ENERGY CONVERSION

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Previous theoretical and experimental work is reviewed with an over-all view of assessing the accuracy to which the performance of MHD Energy Conversion devices can be predicted. Both open- and closed-cycle linear magnetohydrodynamic generators are discussed. The recent progress made in open-cycle generators, with combustion reactions serving as the heat source, is described. Equilibrium thermodynamics is used to determine the theoretical performance of these generators. Particular attention is given to the loss mechanisms at the edges of the field regions and through the wall and electrode boundary layers. Existing experimental results are compared with the theoretical predictions. Major engineering problems are pointed out and discussed.

In the case of closed-cycle generators, successful operation depends upon achievement of adequate electrical conductivity at relatively low temperatures. The most promising approach is through non-equilibrium ionization. The thermodynamic energy exchange processes which govern the extent of non-equilibrium ionization are discussed. Future prospects for development and key research areas pertinent to closed-cycle MHD power generation are described.

Les travaux antérieurs, théoriques ou expérimentaux, sont rappelés en vue d'en dégager une indication quant à la précision avec laquelle on peut prédire la performance d'un convertisseur MHD. La thermodynamique des générateurs MHD linéaires, à cycle ouvert ou fermé, est étudiée. Les progrès récemment accomplis dans la conception des générateurs à cycle ouvert utilisant les réactions de combustion comme source de chaleur, sont présentés. La thermodynamique de l'état d'équilibre est utilisée pour la détermination théorique de la performance de ces générateurs. On étudie en particulier les mécanismes de pertes dues aux effets de bord et ceux dûs aux couches limites pariétales et de l'électrode. Les résultats des déterminations expérimentales sont comparés aux prédictions théoriques et l'évolution prévisible des performances est indiquée. Il est démontré que la réalisation des générateurs

MHD à l'échelle commerciale dépend entièrement de facteurs économiques dont les éléments clés sont le rendement thermodynamique et le coût des matériaux. Les problèmes de réalisation pratique sont présentés et discutés.

Dans les générateurs à cycle fermé on obtient un bon rendement sous réserve d'une conductivité adéquate à des températures relativement basses. La méthode la plus intéressante consiste à provoquer une ionisation hors équilibre. Les processus thermodynamiques d'échange d'énergie qui régissent l'étendue de l'ionisation hors équilibre sont discutés. Les perspectives de développement et les domaines de recherche les plus importants pour la production d'énergie MHD en cycle fermé sont décrits.

INTRODUCTION

First, a word about the title and content of this paper. The subject is not concerned exclusively with thermodynamics nor is the spectrum of problems associated with MHD energy conversion completely covered. Primarily, this is a discussion of the phenomena that occur in the generator and the design knowledge which has been generated by a host of investigators in recent years. Thermodynamics most certainly plays a significant role and the generator is the key, or at least the unique, element in this method of direct energy conversion. In this sense, the title is appropriate for what follows.

In any attempt to estimate the technical feasibility of MHD energy conversion, it must be admitted that the entire evidence is not available to a single appraiser. In the United States the centers of development are engaged in a competitive contest in which the winner may reap rather prodigious rewards. As a result, the most recently published information can hardly be considered completely descriptive of the present state of the art. For example, significant status reports were published by Rosa (Ref. 9) of the Avco-Everett Research Laboratory, Sutton (Ref. 19) of the General Electric Company, Blackman (Ref. 34) of MHD Research Inc., and Way (Ref. 3) of the Westinghouse Research Labs. in 1961. Very little general information has come to light since, despite the fact that research and development effort has continued at an intensive level.*

The authors of this report are confident that progress has been considerable. It seems clear, however, that formidable technical problems, particularly in the development of long-life components and superconducting magnets, still remain. In addition, design methods relating to the determination of conductivity under equilibrium and non-equilibrium conditions, turbulent boundary layer heat transfer and skin friction, and the influence of the Hall parameter in the presence of non-uniformities, still lack suitable precision. These factors are considered in more detail in a subsequent section of this paper.

Special note must be made of the intensive analytical effort which has been devoted to MHD energy conversion. As far as contributions from the United States are concerned, personnel of the General Electric Company

* See Ref. 40 for the most recent progress report.

and the Avco-Everett Research Laboratories deserve to be singled out. However, they are not alone by any means and the reader should at least scan the lengthy reference list at the end of this study to gain full appreciation of the many individuals, institutions, and corporations who have added to the store-house of design information.

It is hoped that this paper will add some perspective to the existing design information and methods. The evolution of a competitive, long-lived, reliable MHD generator requires a thorough understanding of what is now recognized to be a complex problem involving interaction in fluid mechanics, electromagnetics, and heat transfer. Simplified analysis methods reveal the basic nature of the MHD generator. They do not, however, offer sufficient precision on which to base a large-scale engineering development effort. It is proposed, therefore, to retrogress to a somewhat fundamental viewpoint from which it may be possible to emphasize the complicating factors in this design problem. This involves a discussion of nomenclature, geometry, and the basic, dependent, variables involved. In addition, one is concerned with the appropriate formulation of the governing systems of differential equations since this insures that a systematic simplification to tractable form will clearly identify those aspects of the problem which are either being ignored or approximated. Of special importance is an accurate description of the boundary conditions involved since, in an MHD generator, many of the inherent complexities and most severe engineering problems can be traced directly to the boundaries of the device. In this paper, as in others, the macroscopic approach is employed. This means that phenomenological relations are involved, such as Ohm's Law and the Stokes stress-strain formula. These must be appropriate to the problem, hence some attention is directed to their proper selection. Finally, from within the more or less rigorous formulation of the problem, the designer is faced with the necessity of systematically simplifying his mathematics to the point where analytical methods, empirical data, and computer programs may be assembled which will suffice as the basis for a logically sufficient design procedure. As in other instances, one is aided in this step by an appropriate assessment of the pertinent similarity parameters.

With the above formulation as a background, it is then possible to arrive at preliminary design analysis methods; to identify and describe the significant loss mechanisms; and to compare the results of such investigations with existing experimental evidence. In this fashion, the major engineering problems remaining receive additional emphasis.

Part A of this report is an attempt to carry out the above formulation as it applies to open-cycle, linear MHD generators. Part B then deals with closed-cycle MHD generators wherein special attention is focused on the significance of non-equilibrium ionization and the associated conductivity.

PART A

OPEN CYCLE, LINEAR, MHD GENERATORS

THE DESIGN ANALYSIS PROBLEM

Definition of the Problem

1. *Geometry and Nomenclature* The analysis problem associated with an open-cycle, linear, MHD generator channel can be stated as follows. Consider the flow of a slightly ionized gas of conductivity, σ, through a rectangular channel as depicted in Fig. 1. Take the origin of a rectangular

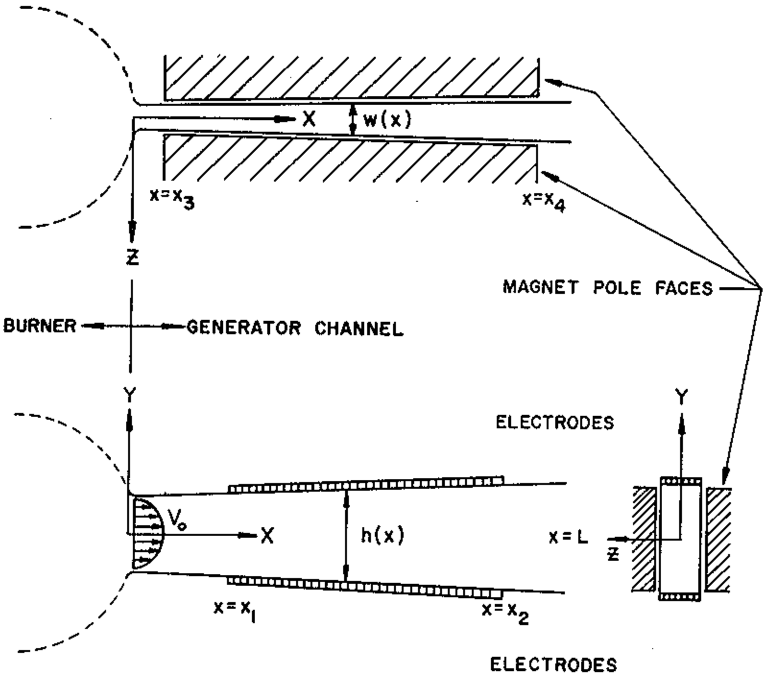


Fig. 1. Geometry and nomenclature: open cycle, linear, MHD generator.

coordinate system (x y, z) at the interface between the burner section and the generator channel, the latter of length, L. The height and width of the channel, not necessarily constant, are given by $h(x)$ and $w(x)$, respectively. An externally applied magnetic field is supplied through pole faces in regions specified by:

$$z = \pm \frac{w}{2}; \quad -\frac{h}{2} \leq y \leq \frac{h}{2}; \quad x_3 \leq x \leq x_4$$

Electrodes, through which the generated voltage will be applied to an external load, occupy the regions defined by:

$$y = \pm \frac{h}{2}; \quad -\frac{w}{2} \leq z \leq \frac{w}{2}; \quad x_1 \leq x \leq x_2$$

Presumed known is the thermodynamic state and the velocity distribution, $\vec{V}_0(y, z)$, of the gas at the entrance plane, $x = 0$. Further, the exit pressure, p_L, at $x = L$ will have a stated value. The resistance in the external load, R_L, is a parameter in the design problem. In addition, the strength of the externally applied magnetic field, $\vec{B}$, is at one's discretion. An important design consideration is whether or not the electrodes are segmented.

2. *The Basic Dependent Variables* As is typical in the design of power generation systems, one is interested in developing expressions for the load characteristics (voltage-amperage curves), the power generated, and the efficiency. These can readily be calculated if the following dependent variables are determined:

(a) The electric field, $\vec{E}$
(b) The electric field, $\vec{B}$
(c) The current density, $\vec{J}$
(d) The velocity distribution, $\vec{V}$
(e) The thermodynamic state variables, e.g., p, ρ, T (pressure, density, temperature)

These dependent variables are equivalent to fifteen scalar quantities. The mathematical system which is adequate to determine the values of these variables is described in the next section.

Basic Mathematical Laws of MHD

Expressed in general terms, the objective is to identify and describe the basic mathematical laws which suffice to calculate the motion of a fluid continuum which is viscous, compressible, heat conducting, and electrically conducting. This involves the general equations of fluid mechanics suitably modified for electromagnetic effects plus appropriate forms of Maxwell's equations for a moving, deformable medium.

The status of this mathematical system is in reasonably good order and most certainly is adequate for engineering purposes. For detailed discussions on the macroscopic derivation of these equations, the reader is referred to texts such as those by Panofsky, Cambel, etc. In what follows here, the basic validity of the mathematics will be accepted and attention will be focused on applicability to the MHD energy conversion problems as stated above, the simplifications necessary to arrive at design information, and the adequacy of the design calculations which result from the simplified equations.

The following discussion is broken down as follows. First, the fluid mechanics laws are enumerated. Second, some comments on Maxwell's equations are provided. Third, the applicable constitutive relations are discussed and, finally, those steps necessary to determine the phenomenological constants are briefly mentioned. The equations themselves are presented in the Appendix for those readers with an interest in the details.

1. *Conversion of Mass* Within the non-relativistic framework, matter can neither be created nor destroyed. In a reacting medium, or in one that is

undergoing dissociation, recombination, or ionization, there may be a transfer between species but, in total, the principle of conservation of mass is not altered from its usual form in fluid mechanics. This means that one is left with the familiar relation involving specific density, ρ, and the macroscopic velocity vector, $\vec{V}$. Equation (A.1).

2. *Conservation of Momentum* In MHD, as in "ordinary" fluid mechanics (OFM), the rate of change of momentum of a fluid element is equated to the force acting on the fluid element. (Equation (A.2).) In OFM, these forces consist of surface contributions (normal pressure and viscous shear) and body forces (gravitational). The gravitational body force is generally neglected in both OFM and MHD unless liquids or natural convection are involved.

MHD contributes two additional body force contributions. The first of these is electrostatic and appears when a net space charge distribution, q_v, exists in the presence of an electric field, $\vec{E}$, and the second is the Lorentz body force, $\vec{J} \times \vec{B}$, which appears when non-parallel current density and magnetic field vectors are present. In MHD energy conversion, the Lorentz body force is the basic phenomenon which is exploited. It is generally assumed that the space charge distribution is either zero or negligible although, as will be noted later, this may not be a valid simplification within MHD boundary layers.

One of the analysis problems which must be anticipated is that of properly specifying the stress tensor, or dyadic, $\overleftrightarrow{\tau}$. This question will be treated in some detail under the heading of Constitutive Relations.

3. *Conservation of Energy* In OFM, consideration of the principle of conservation of energy involves nothing more or less than an appropriate expression of the first law of thermodynamics which requires that heat added to a fluid element must appear as a change in enthalpy in the fluid element and/or work done by the normal forces acting at the surface of the fluid element. Heat addition can arise from radiation, chemical reaction, conduction, or viscous dissipation.

In the case of MHD, one simply adds the Joule Heating equation (A.3). It should be noted, however, that this implies that the electrostatic and magnetostatic energy carried by the fluid have been ignored. The validity of this simplification can be readily established for MHD energy conversion devices. The electromagnetic energy contributions can be expressed in terms of the Poynting Vector, $\vec{N} = \vec{E} \times \vec{H}$, which obeys the "electrodynamic" energy equation

$$\nabla \cdot \vec{N} = -\left[(\vec{E} \cdot \vec{I}) + \frac{\partial}{\partial t}(U_m + U_E)\right]$$

where $U_m = \frac{1}{2}\vec{H} \cdot \vec{B}$ is the magnetostatic energy

$U_E = \frac{1}{2}\vec{E} \cdot \vec{D}$ is the electrostatic energy

But, for a linear, homogeneous, isotropic medium the magnetic induction vector, $\vec{B}$, and the magnetic field vector, $\vec{H}$, are given by the constitutive relation, $\vec{B} = \mu\mu_0\vec{H}$, so that

$$U_m = \frac{B^2}{2\mu\mu_0}$$

where μ is the dimensionless permeability of the medium and μ_0 is the permeability in a vacuum. Similarly, $\vec{E}$ and the electric displacement vector, $\vec{D}$, are related by $\vec{D} = \kappa\kappa_0 \vec{E}$ so that

$$U_E = \frac{\kappa\kappa_0 E^2}{2}$$

where κ is the dimensionless dielectric constant of the medium and κ_0 is the permittivity of free space.

Now, U_m and U_E must appear in the MHD energy equation if $\vec{B}$, $\vec{E}$, μ and κ are either strongly spatially or time dependent. This is not felt to be the case in open-cycle, linear MHD generators although it should be stated that no careful appraisal of this question has appeared in the open literature available to the present authors.

The principle of conservation of energy can be stated in alternative form in terms of the specific entropy, S equation (A.4). From this expression it is clear that the irreversible, or loss, mechanisms within an MHD generator are due to the internal volume heating (radiation, chemical reactions), heat conductivity, viscous dissipation, and electrical dissipation. Since these phenomena will occur predominantly near the boundaries of the generator it is clear that the boundary conditions on the design problem represent the determining factors on the efficiency of the device. More attention will be given to this when the boundary conditions are discussed.

4. *Equation of State* The equations of OFM are closed by the inclusion of an equation of state which describes the fluid in terms of its thermodynamic properties. Thus, ρ = constant is the equation of state for a liquid and, $p = \rho RT$ is the equation of state for a perfect gas. The equation of state must be selected so as to be appropriate for the fluid used in the generator and the temperature range (ionization) involved. For the present purposes it will suffice to use the functional relations, $p = p(\rho, T)$. The reader is cautioned, however, to treat this somewhat innocuous part of MHD analysis with some care since even in an MHD generator the ionization level may vary sufficiently in a spatial sense so as to necessitate a careful appraisal of the *local* form of the equation of state.

The preceeding discussion has been concerned with the macroscopic equations of fluid mechanics and their modification to fit the problem of MHD energy conversion. Similar action with the equations of electromagnetics is also necessary. This involves stating Maxwell's equations in a form applicable to moving, deforming, media. To make the problem reasonably tractable, consideration will be limited to working fluids which are linear, homogeneous, and isotropic. It should be noted, however, that this does not require that the phenomenological coefficients be constants; rather, it demands that they be representable as scalars. The Hall effect can be handled within the framework of this simplification.

The basis for Maxwell's equations can be arrived at from several points of view, one being that they are a mathematical statement of the *experimental* results of Ampere and Faraday. The interpretation then results that the equations are valid for circumstances in which the medium is stationary with respect to the laboratory frame of reference and the electric and magnetic vector quantities are as observed from the same coordinate system. In the MHD problem, one may wish to refer quantities to a coordinate system which moves locally with the distorting medium. In fact, this is precisely the interpretation of the convective derivative. The question then is, are Maxwell's equations identical in both frames of reference, and how does one interpret the electric and magnet vector quantities? The answers to these questions seem fairly well established. For example, Dr. Sydney Goldstein in a series of lectures at Boulder, Colorado, U.S.A., has set forth an anlysis which can be interpreted as follows:

If $\vec{E}$, $\vec{V}$ and $\vec{B}$ are all measured relative to the same frame of reference then the electric field measured by an observer moving with a velocity $\vec{V}$ is

$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B}$$

If Faraday's Law is to be taken as a universal law this means that

$$\nabla \times \vec{E}' = -\frac{\partial \vec{B}'}{\partial t'}$$

and

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Consistent with the above transformation for the electric field and the MHD approximation (R_{em} small), then $\vec{B} = \vec{B}$ and $t' = t$ so that

$$\nabla \times \vec{E}' = \nabla \times \vec{E} + \nabla \times (\vec{V} \times \vec{B}) = -\frac{\partial \vec{B}}{\partial t}$$

or

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = -(\vec{B} \cdot \nabla)\vec{V} + \vec{B}(\nabla \cdot \vec{V})$$

This appears to state that Faraday's Law is of the same form to a fixed and moving observer only if $\vec{V} =$ constant (a non-deformable medium moving in pure translation). From this one can proceed to the conclusion that the transformation $\vec{E}' = \vec{E} + \vec{V} \times \vec{B}$ has only local validity and changes form from point to point when the medium is undergoing distortion. Fortunately, in MHD generators this point is largely academic because the distortions are negligible when measured on a relativistic scale and one can use Maxwell's equations in an unmodified form. The lesson should be, however, that the mathematics of MHD bears close scrutiny whenever new areas of application are under investigation.

5. *Ampere's Law* In 1819, Oersted demonstrated that a current-carrying conductor could exert a mechanical force on a magnet through a distance with the intervening space devoid of all matter. During the period 1820–25, Ampere showed experimentally that similar mechanical forces exist between carriers of steady currents. He was able to establish the nature of these forces and thus identified the Lorentz Force, $\vec{J} \times \vec{B}$. At the heart of Ampere's results is the fact that a steady current carrier gives rise to magnetic field, $\vec{H}$. The technique for calculating this effect is credited to Biot-Savart. By use of integral transformations it is eventually possible to arrive at

$$\nabla \times \vec{H} = \vec{I}$$

as the law by which the magnetic field is determined from the flow of a steady conduction current, $\vec{I}$. There is one very fundamental change required in the above expression. If the divergence is taken we have $\nabla . \vec{I} = 0$. That is, it would appear that the conduction current is a solenoidal vector. However, from considerations of conservation of charge, it can be shown that the sum, $\vec{I} + \partial\vec{D}/\partial t$, where $\vec{D}$ is the electric displacement vector, is a solonoidal vector. That is, $\vec{I}$ is *not* a solenoidal vector. This fact was recognized by Maxwell who corrected the mathematical formulation of Ampere's Law so as to read

$$\nabla \times \vec{H} = \vec{I} + \frac{\partial \vec{D}}{\partial t}$$

Finally, it is recognized that in MHD the steady current may be due not only to the conduction current, $\vec{I}$, but may also contain a contribution arising from the macroscopic transport of space charge density, $q_v\vec{V}$. Hence, equation (A.6) is the suitable form of Ampere's Law for study of MHD generators.

$$\nabla \times \vec{H} = g_v\vec{V} + \vec{I} + \frac{\partial \vec{D}}{\partial t} \tag{A.6}$$

Now conservation of charge as applied to the problem stated for MHD analysis requires that

$$\nabla \cdot (q_v\vec{V} + \vec{I}) = -\frac{\partial q_v}{\partial t}$$

and, from equation (A.6),

$$\nabla \cdot (q_v\vec{V} + \vec{I}) = -\frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$

It can be concluded, therefore, that

$$\nabla \cdot \vec{D} = q_v$$

That is, the space charge density acts as a "source" for the electric displacement, $\vec{D}$. If the space charge density is identically zero, then $\vec{D}$ is a solenoidal vector. Further, if for linear, homogeneous medium, the dielectric factor is also a constant, one concludes that the electric field vector, $\vec{E}$, is also solenoidal. If it is then possible to express, $\vec{E}$, as the gradient of a scalar function, ϕ (from Faraday's Law), it follows that the potential, ϕ, satisfies Laplace's equation and has values within the generator which are determined by conditions on the boundaries of the generator.

In nearly all instances, these simplifications are utilized in MHD generator analysis and design with a precision which is undoubtedly suitable for preliminary purposes. However, this may not be sufficient for the *detailed* study of loss mechanisms, particularly those arising from boundary layer phenomena, and it is to be hoped that detailed attention will be devoted to such questions in future studies.

6. *Faraday's Law* Oersted and Ampere discovered the electromagnetic reactions arising in conducting media carrying *steady* currents. In a sense, the discoveries of Faraday were closely related to those of Oersted and Ampere; in fact, Helmholtz and Thomson showed that Faraday's results could be mathematically deduced from Ampere's Laws by proper application of the principle of conservation of energy.

Specifically, Faraday's experiments led to the conclusion that induced currents are generated by a time rate of change of magnetic flux. If this conclusion is stated in mathematical form and integral transformations employed, there results equation (A.7):

$$\nabla == \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{A.7}$$

It follows, then, that if the magnetic induction, $\vec{B}$, is independent of time, the electric field vector, $\vec{E}$, is irrotational and one may define an electric potential, ϕ. This circumstance almost certainly holds throughout an MHD generator. To begin with, the applied magnetic induction, $\vec{B}_0$, will have some constant value. In addition, the "induced" magnetic induction will be small compared to $\vec{B}_0$. This is equivalent to an uncoupling of the fluid mechanics and electromagnetics and the similarity parameters which determine the degree of this uncoupling are pointed out later. In essence, then, the form of Faraday's Law pertinent to MHD generators is

$$\nabla \times \vec{E} = 0$$

One further comment. If one takes the divergence of equation (A.7), the conclusion is that $\nabla \cdot \vec{B}$ is independent of time. Since magnetic sources do not exist, it follows that

$$\nabla \cdot \vec{B} = 0$$

In other words, the magnetic induction is a solenoidal vector and for linear, homogeneous media of constant permeability the magnetic field intensity, $\vec{H}$, is also a solenoidal vector.

If the principle of conservation of charge, equation (A.8) is included in the above mathematical system, at one's disposal is thirteen scalar relations to be utilized in calculating pertinent components of the basic variables, $\vec{E}, \vec{B}, \vec{J}, \vec{V}, p, \rho$ and T. The writing of these equations from phenomenological principles has, however, introduced additional variables principally being the stress tensor, $\overleftrightarrow{\tau}$; the heat flux vector, $\vec{Q}_c$; the magnetic field intensity, $\vec{H}$; the electrical displacement, $\vec{D}$; and the conduction current, $\vec{I}$. To determine these quantities requires appropriate set of cause-effect, or constitutive, relations. The correct selection of these auxiliary equations constitutes one of the most sensitive areas of MHD generator analysis and design.

Phenomenological Equations

It is with the study of phenomenological, or constitutive, equations that one reaches the interface between the macroscopic and microscopic analysis of MHD phenomena. In the latter instance the methods of statistical mechanics are the central tools and the objective is a detailed understanding of transport phenomena. As has been seen, the macroscopic approach has left the necessity of deriving a suitable number of cause-effect equations. These may be derived from microscopic principles, or they may be *defined* in linear form on the basis of experimental evidence and suitable reasoning from simplified transport theories. The latter course is generally favored under the press of engineering necessity and the former is certainly more satisfying from the viewpoint of fundamental understanding. The course adopted here is to state the constitutive relations as they appear from the phenomenological viewpoint but to discuss them on a microscopic basis where this is helpful from the design standpoint.

1. *Ohm's Law* This is perhaps the most important constitutive relation in MHD generator analysis since it describes the current, $\vec{I}$, (effect) arising from the various electric fields (cause). The proper form of the law must be based upon the medium utilized within the generator. For example, the following assumptions are made. The medium is only slightly ionized and is everywhere locally neutral ($q_v = 0$; $\vec{J} = \vec{I}$). The latter part of this statement may be subject to some question in the neighborhood of electrode sheaths. It is customary to exclude consideration of electron pressure from Ohm's Law in applications to MHD generator analysis. In addition, it is assumed that the density of the medium is sufficiently large and the applied magnetic field sufficiently small so that ion slip can be neglected. Provision for Hall current, however, is included. With these assumptions, Ohm's Law can be written as given in equation (A.9).

Several comments are in order, First, equation (A.9) contains our basic dependent variables from which one auxiliary vector, $\vec{I}$, can be calculated (or eliminated from the basic equations). It introduces, however, the electron density, n_e, and a phenomenological constant, the electrical conductivity, σ, which is temperature and pressure dependent and which must either be calculated separately or determined experimentally. From either approach, the determination of σ has proven exceedingly difficult and remains one of the central problems in calculating the characteristics of MHD generators. In addition, one is left with the question as to whether this form of Ohm's Law *accurately* applies to MHD generators. The fact is that, *a priori*, one cannot be sure. In the absence of a truly rigorous microscopic theory, one can only test the validity of this Ohm's Law by including it in a suitably broad range of analyses of MHD generators and then performing a detailed comparison of calculations with experimental results. Such comparisons have been attempted but so far the myriad of complexities involved do not permit the true validation of this constitutive relation.

2. *Magnetic Permeability* Ampere's Law is written in terms of the magnetic field vector, $\vec{H}$, which is a "causal" variable. The Lorentz force, however, is due to the magnetic induction, $\vec{B}$, which may be thought of as an "effect" variable. One therefore requires a constitutive relation between these two variables which is accurately descriptive of the phenomenon involved within the MHD generator working fluid. Determination of the appropriate form for this relation requires a detailed study of the atomic structure of the medium since what is involved is the magnetic moment associated with the orbiting electrons. For some materials this leads to a complicated tensorial relation between $\vec{H}$ and $\vec{B}$ and a permeability coefficient which is a tensor quantity. Fortunately, there is no reason to believe that, in this instance at least, the medium cannot be treated as linear, homogeneous, and isotropic, in which case the constitutive relation becomes the simple form presented as equation (A.10). Furthermore, one should be free to treat the dimension less permeability, μ, as a constant without concern. Put in different terms, one may feel free to neglect amperian currents in MHD generator analysis and design.

3. *Electrical Permittivity* In this instance the cause-and-effect variables are, respectively, the electric field, $\vec{E}$, and the electric displacement, $\vec{D}$. In anisotropic materials, these quantities are related tensorially and are not parallel. It is again fortunate, however, in that conditions in an MHD generator are such as to give little reason for not assuming the medium to be linear and isotropic. The electric field, $\vec{E}$, and the electric displacement, $\vec{D}$, can then be related as in equation (A.11) wherein the dimensionless dielectric parameter, κ, can be taken to be a constant.

4. *Stress-Strain Law* Here the situation in MHD should be viewed as completely analogous to that in OFM. That is, the effect variable, τ, must be related to the cause variables which in this instance are strains, or derivatives of the fluid velocity. Both the stress and the strain are tensors but if it is assumed that the medium is linear and isotropic, *and in laminar motion*, the familiar Stokes stress-strain Law may be employed (equation A.12)

and only one phenomenological constant, the viscosity coefficient, λ, need be determined. Of course, λ is strongly temperature dependent.

The real difficulty stems from the fact that the motion in an MHD generator will most probably be turbulent and that the important boundary phenomena will take place in a turbulent boundary layer. One may modify the Stokes stress-strain Law to incorporate the Reynolds stresses as given in equation (A.13) (or equivalent) but since this procedure has fallen somewhat short of engineering goals in OFM, there is little reason to hope that such efforts will meet with greater success in MHD.

None of this should be construed to imply that study of laminar, MHD, boundary layers has no value in generator design. Quite the contrary, it may be one of the few ways by which a greater understanding of the loss phenomena can be gained. Such analyses will not, however, suffice to yield the specific design information required to proceed with confidence in large-scale, large-power, MHD generators. What will be needed is considerably more experimental data on turbulent viscous losses and turbulent heat transfer under conditions which realistically approximate those to be found in full-scale generators. In other words, the time will come, and soon, when experimental apparatus must be put into operation for the specific purpose of "generating" the required empirical data. In this sense, the development of MHD generators is no different than for any other large, and fairly unique, system.

5. *Heat Conduction Law* If the medium is linear, homogeneous, and isotropic and if molecular energy transport predominates, then one may rely on the simple heat conduction law given in equation (A.14). Unfortunately, turbulence plays a large role here just as it does in the stress-strain relation. The result is that correlation of heat transfer data in OFM under conditions of turbulent flow is highly empirical and strong reliance is placed on the Reynolds analogy in which the local heat transfer coefficient is made proportional to the skin friction coefficient with the Prandtl number as a parameter. There seems to be little hope that one can do better than this in MHD, at least for the present.

This bears repeating—two of the principal loss mechanisms in MHD generators, namely, frictional pressure drop and heat transfer, seemingly can be approached only on an empirical basis. The designer must therefore substitute ingenuity for mathematics. But it would appear to the present authors that this is more often than not the case in engineering. It is possible therefore to be quite hopeful that these difficulties will be circumvented and that development of competitive MHD energy conversion will not be hindered.

As a final note in this discussion on constitutive relations, it should be emphasized that the phenomenological approach requires the independent determination of the phenomenological constants. In MHD generator design, the constants of prime significance are

the electrical conductivity
the viscosity coefficient
the heat transfer coefficient

In addition, detailed calculations will also require that one have values for the permeability, μ, and the dielectric constant, κ. Of these, the conductivity, σ, plays the largest role and, unfortunately, it is one of the more difficult to determine. From the experimental data available to the writers of this report, it would appear that an uncertainty of about 30 per cent exists in the data for σ which are applicable to MHD generators although some isolated results appear to be better than this. The point is, however, that in the process of developing an MHD generator, the designer cannot have an *a priori* confidence in the data for σ which is higher than that indicated above. This, therefore, clearly identifies one of the principal design problems, viz., arriving at techniques for predicting the electrical conductivity, σ, with greater precision.

Within the limitations introduced, the mathematical system described above is that which should be employed in the design of MHD generators. Such a design is a boundary value problem, however, the solution of which cannot be gained unless the appropriate boundary conditions are specified. As has been indicated several times, the more interesting phenomena in an MHD generator take place near the boundaries, which leads one to conclude, and correctly so, that a careful and precise statement of the boundary conditions is essential to the design process. An amplification of this point is attempted in the next section.

Boundary Conditions

The statements which follow are not intended to be a mathematical discussion of the admissible and inadmissible boundary conditions, or the required number to match the mathematical system previously defined. There are several reasons for this. First, it is recognized that simplifications will be required in the mathematics before it can be applied to the design problem which means that the number and type of boundary conditions must be determined, at least partially, on the basis of the simplifications introduced. Second, the statement of boundary conditions is merely an enumeration of the physical "constraints" in the problem which means that first and foremost one must understand the device in order to intelligently select and mathematically state the appropriate boundary conditions. What follows, therefore, is an attempt to describe those considerations which will directly influence the selection of boundary conditions. It is not pretended that the topics selected are either unique or exhaustive. It is believed, however, that they are the most important.

1. *Entrance Conditions* The state of the fluid upon entering the generator must be specified. This includes the thermodynamic state, the velocity distribution, and the conductivity. Hopefully, these will be spatially uniform in the $x = 0$ plane. If they are not, the actual performance of the generator may be considerably different from that calculated for a uniform entering state. For example, non-uniformities in temperature (conductivity) and velocity will lead to non-uniformities in the Lorentz body force which will in turn give rise to eddy current losses. This boundary condition is then

perhaps stated more correctly as a design objective. That is, the burner, seed injection, and nozzle system should be designed so as to insure to the maximum practicable extent the uniformity of the entering state of the working fluid.

2. *Fluid velocity on the generator walls* An appropriate treatment of this boundary condition is not as straightforward as first thoughts would indicate. The complication arises primarily from the presence of the turbulent boundary layers and the adverse static pressure gradient. Thus the principal question is where to apply to the boundary condition. Put another way, the flow in the central core must be appropriately matched to the boundary layer. Here, one may take a cue from OFM in which iterative methods are employed. That is, a first design is carried out as though the boundary layer were not present and then one calculates the boundary layer which would result. The design is then repeated with this boundary layer present, and a new layer distribution is calculated. Hopefully, the process converges and this is quite likely in large generators with length-to-diameter ratios such that fully developed channel flow will not be reached. In any event, turbulent boundary layer methods must be employed and this limits the precision of the calculations. It should be obvious that the channel walls should include as few protuberances as possible, otherwise the boundary layer problem moves from the difficult category to higher-order complexities.

It should also be noted that since the velocity varies rapidly through the boundary layer from its free stream value to zero at the generator wall, there is an equally rapid change in the potential across the boundary layer. This means that the actual loading curves for the generator will vary from those calculated from ideal, one-dimensional methods by an amount dependent on the extent of the boundary layer. Also, since this represents a non-uniformity in the Lorentz body force, it will generate eddy currents and concomitant losses. There are ample reasons, therefore, to strive for an accurate prediction of the boundary layer with respect to both its extent and the velocity profile.

3. *Temperature and temperature gradients on the generator wall* If freedom of choice were available, one would choose to thermally insulate the walls of an MHD generator to keep heat transfer losses low and conductivity high. It is well known, however, that even reasonable values of conductivity under equilibrium conditions require such a high working fluid temperature as to be in excess of present materials limitations unless some wall cooling is provided. Thus the boundary condition on temperature can be simply stated in terms of the highest allowable constant value consistent with long operating life. A trade-off is clearly implied. If the wall is allowed to run too hot, replacement of wall materials will be a major problem. If the walls are excessively cooled, electrode emission will be supressed, heat transfer losses will be excessive, and non-uniformities in conductivity near the walls will give rise to additional eddy current losses.

Suppose that the wall temperature has been selected on the basis of the materials question. One has still to calculate the heat transfer since this will determine the coolant flow required and the associated generator losses. Since the boundary layer is turbulent, appropriate empirical relations must be utilized. To repeat, however, one would hope to be able to calculate the temperature profile within the boundary layer since from this the non-uniformity in conductivity and the resultant eddy current losses are determined.

4. *Region and intensity of the applied magnetic field* Since the applied magnetic field has a beginning and an end in an MHD generator which gives rise to distortions in the Lorentz body force and results in eddy current losses, it is important to include this fact in the boundary conditions. Fortunately, this characteristic of MHD generators is now well understood largely through the efforts of George Sutton and his co-workers at the General Electric Company who have shown that proper tailoring of the magnetic field and/or vanes will substantially reduce this effect. It is important to note, however, that the solution directly involves a change in the boundary conditions.

Finally, one should recognize that the external circuitry, in which one may include the geometry and resistance of the electrodes, plays an important role. This becomes particularly true when segmented electrodes are employed to minimize Hall reduction of conductivity. In addition, the appropriate level of exit pressure is to be specified, this being complicated only by the inclusion of a pre-heater in open-cycle generators.

Similarity Parameters

The central design question in MHD generator development involves the selection of an appropriate model from within the complex arena that has been described. As is often the case, this has been accomplished largely through studies of the similarity parameters involved. The following covers the more important of these parameters, the choices which they assist in making, and the ranges of values which they will probably have in most MHD generators of the type under consideration here.

1. *Reynolds number* This parameter plays basically the same role in MHD as in OFM. That is, for Reynolds numbers above a critical value it is necessary to calculate boundary layer profiles, thickness, skin friction and heat transfer on the basis of turbulent boundary layer theory. This will be the case in even fairly small units. For example, for a mass flow rate at 1·0 megawatt of 0·11 kg/sec and a reference length of only 0·1 meter, the Reynolds number is approximately 6×10^5 (Ref. 34). In addition, length-to-hydraulic-radius ratios of 10 or less are typical of current design proposals which means that one cannot expect the flow to be fully developed. Thus, the suggestion that MHD generators be analyzed on the basis of inviscid one-dimensional core flow and turbulent boundary layers seems to represent the most logical procedure. This conclusion is also supported by experiment evidence quoted in Ref. 7.

2. *Magnetic Reynolds number* By definition, the magnetic Reynolds number is

$$R_{e_m} = \sigma \mu L V$$

If this parameter is of order larger than one, there will be effective coupling between the flow field and the electromagnetic field and large induced field effects will result. In typical designs, the electrical conductivity is modest, the permeability is small and V lies in the high subsonic regime. The characteristic length, L, cannot be unduly large otherwise the device becomes impractical. It is found necessary to seed the working fluid for enhanced conductivity at temperatures which do not exceed material limits.

What implications does this have in terms of the mathematical system? If the magnetic Reynolds number is very small, induced magnetic fields can be neglected and the fluid mechanics equations and electromagnetics are uncoupled. However, for R_{e_m} of the order of unity this appears not to be the case (Ref. 6) so that care must be exercised in applying simplified versions of the equations of MHD to generator analysis and design.

3. *The Hartman number* This parameter measures the relative importance of magnetic and viscous effect. It is defined by

$$H_a{}^2 = \frac{\sigma B^2 L^2}{\lambda}$$

If it is significantly large, then the pressure losses caused by viscous friction should be negligible by comparison with the conversion of pressure drop by electromagnetic means into useful power. In fact, in Ref. 6 it is demonstrated that viscous losses may be neglected when $H_a{}^2$ exceeds 1000. It would appear that the best way to accomplish this is to increase the magnetic field; however, this increases the losses in the magnet and until one has superconducting magnets, the optimum value of B is definitely limited.

One can rewrite the definition of the Hartman number as

$$H_a{}^2 = \frac{\sigma B^2 L^2}{\rho V L} R_e = S R_e$$

where $S = \dfrac{\sigma B^2 L}{\rho V}$ is the interaction number which represents the ratio of magnetic effects to inertia effects. In MHD generators the goal is to make this parameter of order unity or greater. Hence, one can conclude that $H_a{}^2$ is of the order of the Reynolds number in the regime of design interest and in the free stream, we should be free to ignore viscous effects *in so far as frictional losses are concerned.* However, a word of caution is in order. The presence of the boundary layer not only yields friction losses, it also reduces the effective cross-sectional area of the generator and significantly influences the heat transfer. Thus one must take into account the presence of the boundary layer even though the direct frictional losses may be negligible.

4. *The Hall parameter* This quantity may be written in alternative form as

$$N_H = \omega\tau = WB\left(\frac{\rho_0}{\rho}\right)$$

where ω = electron cyclotron frequency, τ = mean time of collisions between electrons and other particles, B, is the strength of the applied magnetic induction, ρ/ρ_0 is the density in the generator in atmospheres, and W is a monotonically decreasing function of temperature and density.

Since this parameter is treated in some detail in a subsequent section of this report, discussion is deferred accordingly. Suffice to say that the parameter essentially determines the degree to which the current density vector, $\vec{J}$, and the resultant electric field, $\vec{E} + \vec{V} \times \vec{B}$, are non-parallel. Thus, large values of $\omega\tau$ are interpreted as reducing the "effective" conductivity. The parameter can be reduced by lowering, $\vec{B}$, but this lowers the interaction parameter, hence the effectiveness of the generator. It can also be reduced by increasing the density but this aggravates the heat transfer problem. One, therefore, finds that Hall parameters in excess of one are incorporated in typical generator designs with segmented electrodes utilized to reduce the losses. This point will receive additional amplification in the treatment on losses.

With the above as background, a discussion of preliminary design considerations follows.

PRELIMINARY DESIGN ANALYSIS

As stated, it is proposed to discuss preliminary design of MHD generators on the hypothesis of a one-dimensional, inviscid, core in the presence of turbulent boundary layers on both the electrode and insulator walls. The one-dimensional model yields the ideal characteristics of the generator and the spurious losses are associated with the boundary layer. This part is concerned solely with the analysis of the inviscid core.

Assumptions and Restrictions

The geometry taken is that of Fig. 1 except that the height and width, h and w, each differ from the true dimensions by twice the boundary layer thickness which is to be determined. It is assumed that there is only one macroscopic velocity component, u, in the x-direction and that it is independent of y and z. Further, the space charge density is taken to be zero ($q_v = 0$), hence the total current density is identical with the conduction current, $\vec{J} = \vec{I}$. Viscosity effects are considered to be negligible, a redundant assumption if no velocity gradients are presumed to exist. Heat transfer due to radiation and heat generation due to chemical reactions are both ignored. It is also assumed that *all* of the basic dependent variables are stationary $\left(\frac{\partial}{\partial t} = 0\right)$. Since the working fluid is a gas and thermal convection is negligible, the gravitational body force vector, $\vec{F}$, may be neglected. Heat conduction is not considered in the core flow. In one of the more critical areas, it is assumed that the Hall currents in the core are suppressed completely by segmented electrodes so that the applicable form of Ohm's Law is

$$\vec{I} = \sigma[\vec{E} + \vec{V} \times \vec{B}]$$

The governing equations for the core flow, applicable within the above assumptions, are written in the next section

Mathematical Relations for Inviscid, One-dimensional, Core Flow

On the basis of the preceding assumptions coupled with the hypotheses that the applied magnetic field far exceeds the induced fields and the electric field lies only in the electrode (y) direction,

$$v = w = I_x = I_z = B_x = B_y = E_x = E_z = 0$$

The governing equations of the Appendix, applicable to the core flow problem then are

$$\frac{\mathrm{d}}{\mathrm{d}x}(\rho u) = 0 \qquad \textit{Conservation of Mass } (1)$$

$$\rho u \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{\mathrm{d}p}{\mathrm{d}x} + I_y B_z \qquad \textit{Conservation of Momentum } (2)$$

$$\rho u \frac{\mathrm{d}h}{\mathrm{d}x} - u\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{I_y^2}{\sigma} \qquad \textit{Conservation of Energy } (3)$$

$$I_y = \sigma[E_y - uB_z] \qquad \textit{Ohm's Law } (4)$$

$$I_y = -\frac{1}{\mu\mu_0}\frac{\mathrm{d}B_z}{\mathrm{d}x} \qquad \textit{Ampere's Law } (5)$$

$$E_y = -\frac{\mathrm{d}\phi}{\mathrm{d}x} \qquad \textit{From Faraday's Law } (6)$$

The ideal performance of an MHD generator, based on the above, can now be obtained without solving the equations. The procedure is as follows. As is customary, the loading factor of the generator is defined by

$$\eta = \frac{E_y}{uB_z}$$

and is less than unity for generators. To generate a current in the positive y direction, it is required that the magnetic induction be in the negative z direction, $(-B_z)$. Then

$$I_y = \sigma u(-B_z)(1 - \eta) \tag{7}$$

is the local current density. The electrical, specific, power output is

$$P_0 = E_y I_y = \sigma u^2(-B_z)^2\eta(1 - \eta) \tag{8}$$

The maximum specific power output then occurs at $\eta = \frac{1}{2}$ and has the value

$$P_{0_{\max}} = \tfrac{1}{4}\sigma u^2(-B_z)^2 \tag{9}$$

On the basis of the simplifications which have been introduced, the only loss is associated with the Joullian dissipation as given in equation (3). The specific value of this power loss is

$$P_L = \frac{I_y^2}{\sigma} = \sigma u^2(-B_z)^2(1 - \eta)^2 \tag{10}$$

It follows, then, that the efficiency is

$$\varepsilon_0 = \frac{P_0}{P_0 + P_L} = \eta \tag{11}$$

These results for the ideal generator, which have been pointed out by a number of contributors, are plotted in Fig. 2. Obviously with so many

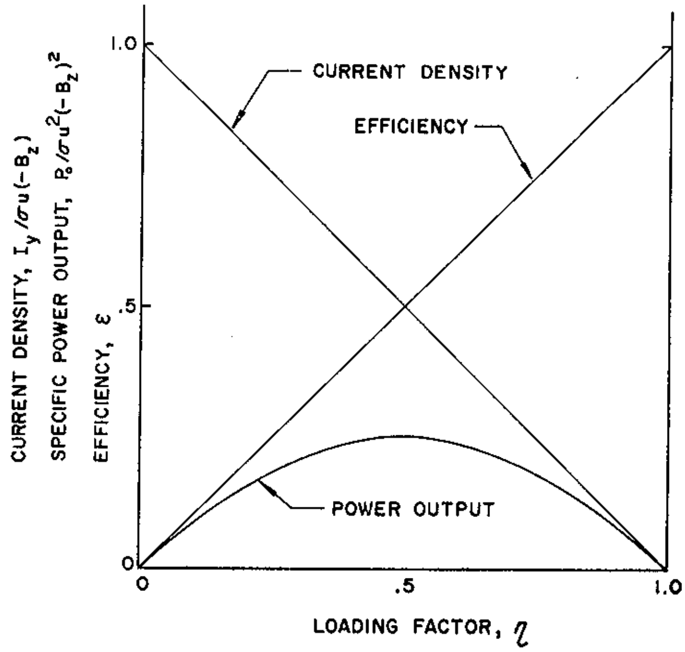


Fig. 2. Performance ideal MHD generator.

simplifications and assumptions these results are not sufficient as a foundation for design and the various details must be concluded. In the next section of this report, the detail design problems are discussed on the basis of the various loss mechanisms involved.

ANALYSIS OF LOSS MECHANISMS

The losses which will be specifically dealt with here can be classified as follows:

End Effects
Aerodynamic Losses

Heat Transfer Losses
Electrode Losses
Hall Effects

Each of these phenomena can, and has been, the subject of specialized papers. It is not intended, therefore, to treat them in detail. However, the status of the art of estimating these losses will be appraised and their significance in the design problem will be pointed out.

End Effects (*Finite imposed magnetic field region*)

This problem has been very thoroughly studied by Sutton *et al.* (Refs. 13, 25, 26) who have shown that significant losses occur when the magnetic field terminates at the ends of the electrodes. (In Fig. 1, this corresponds roughly to $x_1 = x_3$ and $x_2 = x_4$.) They also demonstrate that these eddy losses can be reduced by extending the magnetic field beyond the electrodes and by the use of upstream and downstream vanes. Since the purpose here is to demonstrate the significance of this loss, the results for the worst case will be discussed, the inference then being that proper design will be in the direction of the ideal performance.

Thus, for the severest case in which the magnetic field extends only over the length of the electrodes, Sutton (Ref. 26) has shown that the ratio of actual power density, P_a, to ideal power P_0, consistent with the assumption and simplifications of the second section, is given by (approximately)

$$\frac{P_a}{P_0} = 1 - \frac{1}{A^*}\frac{\eta}{1-\eta} \tag{12}$$

where A^* is the reduced electrode aspect ratio as defined by

$$A^* = \frac{\pi}{2\ln 2}\frac{X_E}{h} \tag{13}$$

with X_E the length of the electrode section in the x-direction.

Two very significant conclusions can be drawn. First, the device ceases to be a generator and becomes a pump at a transition value of the loading factor given by

$$\eta_{tr.} = \frac{A^*}{1 + A^*} \tag{14}$$

which means that the high-efficiency end of the MHD performance curve is significantly modified. Second, the value of the loading factor for which maximum specific power output is attained is now no longer, 1/2, but instead is given by

$$\eta_{P_{a_{\max}}} = \frac{A^*}{2(1 + A^*)} \tag{15}$$

and the value of the actual specific power at this value of loading factor is

$$\left(\frac{P_a}{P_0}\right)_{P_{a_{\max}}} = 1 - \frac{1}{2 + A^*} \tag{16}$$

The actual efficiency, ε_a, also differs from the ideal efficiency, ε_0, when end effects are included. Since the ideal efficiency is given by

$$\varepsilon_0 = \frac{P_0}{P_{\text{in}}} = \eta$$

it follows that the actual efficiency has the value

$$\varepsilon_a = \eta \left[1 - \frac{1}{A^*} \frac{\eta}{1-\eta}\right] \tag{17}$$

This efficiency now has a maximum when plotted versus the loading factor, η, which occurs at

$$\eta_{\varepsilon_{a_{\max}}} = 1 - \frac{1}{\sqrt{1 + A^*}} \tag{18}$$

These results are plotted in Fig. 3. It is to be noted that the actual power

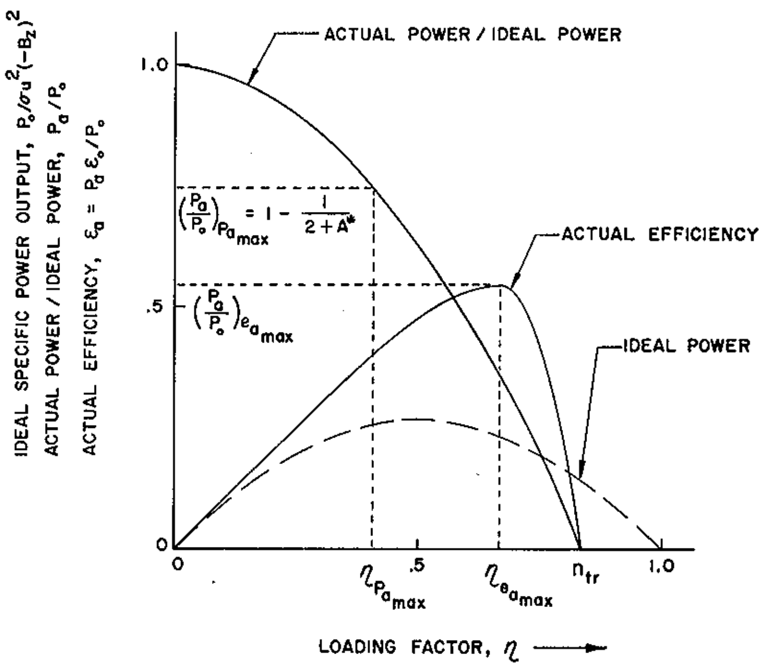


Fig. 3. End loss due to terminating magnetic field.

is, of course, always less than the ideal power. The loading factor for maximum power is less than one-half. The efficiency, on the other hand, will in general have a maximum at values of loading factor in excess of one-half. A trade-off study is necessary to determine the weighting of maximum power versus maximum efficiency operation so that a design optimum can be selected.

Finally, it should be noted that the important parameter in this loss mechanism is the electrode aspect ratio, X_E/h. The obvious conclusion is that when this ratio gets to the point where A^* is of the order of ten, then end effects will tend to be negligible.

Aerodynamic Losses

It was indicated earlier that aerodynamic losses in an MHD generator should be based on turbulent boundary layers in view of the fairly large Reynolds numbers. The truth is, of course, that turbulent boundary layer analysis including electromagnetic effects is not abundant. Some data were published by Brouillette and Lykoudis (Ref. 39) but these were obtained for fully developed channel flow with mercury as a working fluid. At the moment, therefore, the designer is probably forced to calculate the pressure drop due to turbulent skin friction and the thickness of the turbulent boundary layer on the basis of existing non-electromagnetic, empirical, methods. It may be possible to introduce corrections as indicated by laminar, electromagnetic, boundary layer theory although to the knowledge of the present authors, no one has yet attempted to form a judicious combination of these rather separate areas for design purposes. The reader is referred to Refs. 32 and 33 as more thorough investigations into the behaviour of the laminar, MHD, boundary layers.

To proceed with the empirical approach, one must evaluate the frictional loss

$$\vec{\tau}:(\nabla\vec{V})$$

in equation (A.3) in a fashion similar to the calculation of the Joullian loss, Iy^2/σ, for the ideal generator. On the basis of a one-dimensional core flow, this means that the specific power loss arising due to friction at the walls is

$$P_{L_f} = \tau_0 \frac{\bar{u}_w}{\delta} \tag{19}$$

where τ_0 is the frictional stress at the wall and where the slope of the velocity profile at the wall is taken to be the average boundary layer velocity divided by the boundary layer thickness. Then, if as is customary, the friction coefficient is defined by

$$f = \frac{2\tau_0}{\rho u^2}$$

The power loss due to friction becomes

$$P_{L_f} = f\frac{\rho u^2}{2}\frac{u}{\delta} \times \text{(Boundary Layer Volume)} \tag{20}$$

It follows that the ratio of frictional power loss to maximum ideal power output is

$$\frac{P_{L_f}}{P_{0_{\max}}} = \frac{2f\rho u}{\delta\sigma(-B_z)^2} \times \frac{\text{(Boundary Layer Volume)}}{\text{(Generator Volume)}}$$

or

$$\frac{P_{Lf}}{P_{0\max}} = \frac{2f}{S}\frac{L}{D} \tag{21}$$

where $S = \dfrac{\sigma B^2 L}{\rho u}$ is the interaction parameter and is of order unity for MHD generators. From turbulent boundary layer theories, the friction coefficient is estimated to be approximately

$$f = 0{\cdot}06\, R_e^{-\frac{1}{4}}$$

This means that for Reynolds numbers of the order of 10^5 and L/D ratios of the order of 10, the loss due to friction is such that

$$\frac{P_{Lf}}{P_{0\max}} \approx 10 \text{ to } 12 \text{ per cent}$$

It would appear, therefore, that frictional losses, although modest, are certainly not negligible. One would conclude that this matter should therefore be given more attention in the detail design of large MHD generators.

One should also strive for a better understanding of electromagnetic effects in turbulent boundary layers. Some incentive for this comes from existing studies of laminar, MHD, boundary layers (Refs. 32 and 33). There are Hall factors which would lead one to look for spurious electromagnetic effects in the boundary layer which, in turn, could couple with the fluid phenomena and lead to significant differences between MHD boundary layers and OFM boundary layers. The first of these arises from the fact that the reduced velocity in the boundary layer lowers the Lorentz force, hence locally changes the current-voltage characteristic. Second, if the walls are cooled, then the working fluid near the walls will also have a lower temperature and its conductivity will be lowered there. This will obviously also change the current-voltage characteristic. Both of these first two factors together will, in general, give rise to an electrode sheath which may allow Hall current in the boundary layer even though segmentation is introduced to prevent this occurrence. Under such circumstances, determination of aerodynamics and heat transfer losses will be far more complicated than presently supposed. Results from laminar, MHD, boundary layer theory would indicate that the layers are significantly thicker than in OFM and that the friction coefficient can also be increased by at least a factor of two, particularly under non-equilibrium conditions.

To repeat, the problem of the turbulent, MHD, boundary layer is an important facet of MHD generator design and it is hoped that this problem will attract considerable research attention.

Heat Transfer Losses

As mentioned previously, the problem of evaluating heat transfer losses is currently best handled by recourse to empirical, turbulent, boundary layer heat transfer methods. Thus, in equation (A.3) the loss associated with the conduction term $(-\nabla \cdot Q_c)$ is replaced by an appropriate, experimentally determined, equation for the heat transfer rate. In OFM such equations have been verified over a rather wide range of the basic variables involved. In MHD work, the amount of applicable data is severely limited. It follows, then, that the question of which heat transfer expression to use from OFM is not too cogent and, in the present paper, Ref. 34 is followed, and the heat transfer rate per unit area is expressed by

$$Q_c = \frac{0{\cdot}04\, \rho u h_c(1 - h_w/h_s)}{P_r^{\frac{2}{3}} R_e^{\frac{1}{5}}} \tag{22}$$

where P_r = Prandtl number $\approx 0{\cdot}7$
R_e = Reynolds number based on channel length

It follows then that the ratio of the heat transfer loss to the maximum power density of the ideal generator is given by

$$\frac{(Q_c/D)}{P_{0\max}} = \frac{0{\cdot}32(L)}{P_r^{\frac{2}{3}} R_e^{\frac{1}{5}}(\gamma - 1)SM_c^2} \, \frac{1 - h_w/h_s}{(D)} \tag{23}$$

where γ = ratio of specific heats $\approx 1{\cdot}2$
M_c^2 = Mach number of the core flow $\approx 1{\cdot}0$

If it is assumed that a Reynolds number of 6×10^5, then $R_e^{\frac{1}{5}} = 15$, and equation (23) has the value

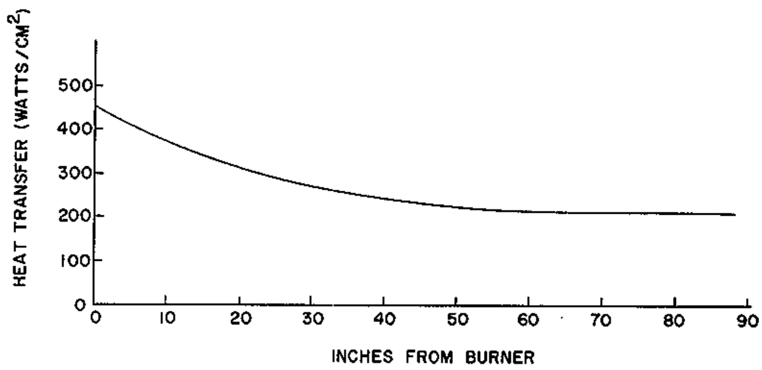
$$\frac{(Q_c/D)}{P_{0\max}} = 0{\cdot}135 \, \frac{1 - h_w/h_s}{(D/L)} \tag{24}$$

One can conclude, therefore, that for small generators $(L/D \rightarrow 1{\cdot}0)$ and large cooling rates $(h_w/h_s \ll 1)$, the loss due to heat transfer can be as much as 13 to 14 per cent of the maximum power density. Clearly, this is reduced for large generators due to the area-volume relationship and by allowing the walls to operate as hot as materials' limits will permit. It should be remembered, however, that these conclusions do not allow for any electromagnetic effects in the boundary layer and results from laminar, MHD, boundary layer theory (Ref. 33) indicate that heat transfer may be augmented to a significant degree. It is felt, therefore, that considerably more research on heat transfer in turbulent, MHD, boundary layers would be very valuable.

It must be admitted, however, that existing experimental data from generators run at low power levels and modest magnetic field levels do not indicate that electromagnetic effects play a critical role in heat transfer in MHD generators. For example, Fig. 4 (taken from Ref. 7) is a comparison between experimental heat transfer rates and theoretical calculations of the sort indicated above. The agreement is exceptional but the question remains as to whether such results will hold true in instances where the electromagnetic parameters are typical of large-scale, large-power-density generators.

Electrode Losses

In attempting to find a consensus in the literature relative to this loss mechanism, it rapidly becomes apparent that a variety of opinions exist. To some extent, at least, this can be traced to the basis on which the various investigators have arrived at their conclusions. One method is to apply a battery voltage at zero magnetic field across a hot, seeded, flowing gas and to measure the resulting current flow (Ref. 3). The resistance calculated from such data is attributed to the boundary layer drop and this is in turn identified as the magnitude of the electrode drop. It is then assumed that this drop is reasonably constant during generator operation. As one might



FROM: REF. 7, LOUIS, LOTHROP, BROGAN; AVCO-EVERETT

GAS ENTHALPY	2250 cal/gm
MAGNETIC FIELD	3.2 Wb/m^2
INLET HYDRAULIC DIA.	6 inches
INTERACTION NUMBER	1.0

Fig. 4. Heat transfer in MHD generator.

suspect, such experiments usually lead to the conclusions that the electrode drop is fairly small.

Another, more direct technique, is to obtain the electrode drop from actual generator operation by measuring the potential distribution across the channel. The procedure involves installation of terminals in the insulator wall and the measurement of their potential relative to ground. (See e.g. Ref. 7 and Ref. 21, the latter conducted with an accelerator rather than a generator.) In these instances, the electrode drops are fairly large (11 to 32 volts, Ref. 7) and depend upon the loading factor.

The detail mechanism associated with these electrode losses is undoubtedly related to the velocity decrease in the boundary layer, the extent of the boundary layer, and the decrease in conductivity. The following, elementary analysis is intended to show the influence of the first two of these factors.

It contributes little to design methods and is not directly related to the mathematical derivation presented in previous sections. It does, however, support the conclusion that electrode losses can be considerable.

The open-circuit voltage of an MHD generator is uB_xh. Also, if *no* boundary layer phenomena occurred in the layer of thickness, δ, then the voltage difference across that region would be $uB_x\delta$. However, the velocity decreases in the boundary layer, so that the actual change in voltage is $B_z\delta\bar{u}_w$, where $\bar{u}_w$ is a suitable average of the velocity in the boundary layer. One can, therefore, state that the voltage drop due to the electrode boundary layer is

$$\Delta\phi = B_z\delta(\bar{u} - u_w)$$

This means that the percentage drop is

$$\frac{\Delta\phi}{\phi} = \frac{\delta}{L}\left(1 - \frac{\bar{u}_w}{u}\right)\frac{L}{h}$$

Or, since the boundary layer is assumed turbulent,

$$\frac{\Delta\phi}{\phi} = \frac{0{\cdot}37(1 - \bar{u}_w/u)L/h}{R_e^{\frac{1}{5}}} \tag{25}$$

One can check this expression against experimental data, but before doing so it is instructive to relate the electrode drop to an associated power loss. The ideal power of the generator per unit length is

$$P_0 = E_yI_yhw$$

The power lost per unit length of generator as a result of electrode drop is

$$P_L = 2B_z\delta(u - \bar{u}_w)I_yw$$

It follows then, that

$$\frac{P_a}{P_0} = \frac{P_0 - P_L}{P_0} = 1 - \frac{2(u - \bar{u}_w)B_z}{E_y}\frac{\delta}{h}$$

Or, alternatively,

$$\frac{P_a}{P_0} = 1 - \frac{0{\cdot}74(L/h)(1 - \bar{u}_w/u)}{\eta R_e^{\frac{1}{5}}} \tag{26}$$

It is interesting to insert representative numbers into equations (25) and (26). For this purpose, take $\eta = \frac{1}{2}$ (point of maximum ideal power operation), $R_e = 6 \times 10^5 (R_e^{\frac{1}{5}} = 15)$, $\bar{u}_w/u = \frac{1}{2}$, and $L/h = 10$. Then, $\Delta\phi/\phi = 12$ per cent and $P_a/P_0 = 51$ per cent. In Ref. 7, the voltage drops corresponded to $\Delta\phi/\phi$ of about 13 per cent in an open-circuit test. Thus, although the problem of estimating electrode drops cannot be considered solved, the above expressions do give some feel for the importance of the problem to MHD generator design.

The Hall Effect

In the appendix, Ohm's Law equation (A.9) is written so as to include the Hall voltage

$$\frac{1}{en_e}(\vec{I} \times \vec{B})$$

which when multiplied by the scalar electrical conductivity, σ, is identified as the Hall current. Since this phenomenon has been amply described in the past, there is no necessity in discussing its physical significance here. It is important, however. to identify the magnitude of the loss associated with this parameter.

The point of view adopted in Ref. 9 is a most useful one. It is the following. Assuming that no core current can flow in the Z direction because the walls at $Z = \pm \frac{w}{2}$ are electrically insulated, the scalar components of equation (A.9) are

$$I_x = \frac{\sigma}{1 + (\omega\tau)^2}[E_x - \omega\tau(E_y - uB_z)] \tag{27}$$

$$I_y = \frac{\sigma}{1 + (\omega\tau)^2}[E_y - uB_z + \omega\tau E_x] \tag{28}$$

where $N_H = \omega\tau = \sigma B_z/e\eta_e$ is the Hall parameter, or Hall coefficient. I_x is called the Hall current and in the type of generator under consideration here represents a loss mechanism since it flows in the axial direction rather than into the electrodes directly. An "effective" conductivity is therefore introduced which is defined by

$$\frac{\sigma_{\text{eff}}}{\sigma} = \frac{I_y}{I_y(I_x = 0)} = \frac{I_y}{\sigma(E_y - uB_z)} \tag{29}$$

From this definition, then

$$\frac{\sigma_{\text{eff}}}{\sigma} = \frac{1}{1 + (\omega\tau)^2}\left[1 + (\omega\tau)\frac{E_x}{E_y - uB_z}\right] \tag{30}$$

Under ideal circumstances the Hall current is zero and the concomitant losses are also zero. This is the basis for segmented electrodes which have been proven effective in reducing the Hall current losses. On the other hand, the significance of the Hall parameter becomes obvious when full Hall current is admitted. Then, $E_x = 0$, and

$$\frac{\sigma_{\text{eff}}}{\sigma}(E_x = 0) = \frac{1}{1 + (\omega\tau)^2} \tag{31}$$

Equation (31) is plotted in Fig. 5 and since the power generated is directly proportional to the conductivity it is obvious that the Hall losses, if unimpeded, would be catastrophic. Incidentally, the range of interesting values of N_H exceeds two at which point $\sigma_{\text{eff}}/\sigma = 20$ per cent.

The incentive for segmented electrodes is quite evident. However, such procedure does not completely eliminate the Hall current loss and since even higher Hall parameters are desirable in order to increase the interaction number, this is a matter for some concern. There are several factors which limit the effectiveness of segmented electrodes, among these being the difficulty in determining correct spacing and, of course, the fact that infinite segmentation cannot be attained. There is another consideration, however, that may be equally important and it has to do with the boundary layer. The Hall parameter depends, among other things, on the electron density. This density is different in the boundary layer than it is in the core flow. Therefore, even if the Hall current is suppressed in the core flow, it can still exist in

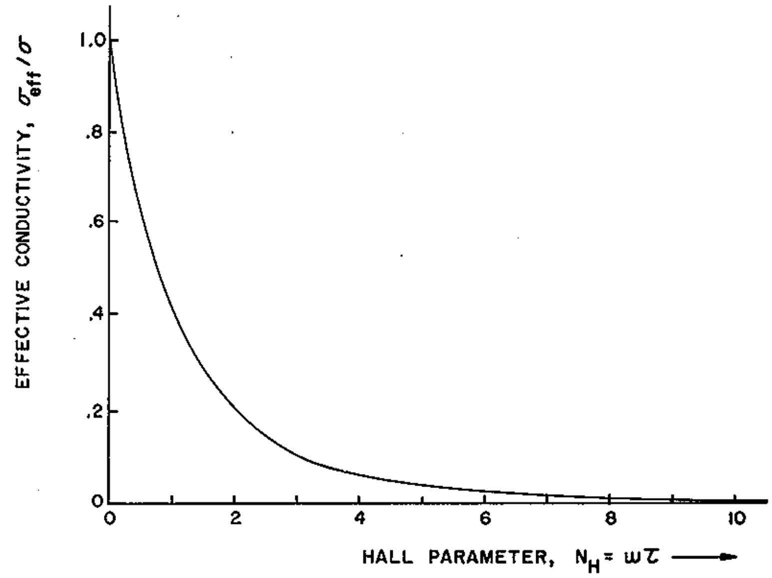


Fig. 5. Effect of Hall parameter on conductivity.

the boundary layer and short out across the electrodes. This phenomenon has been analyzed in Ref. 32 for laminar, MHD, boundary layers wherein it is found that the increasing Hall parameter thins the boundary layer which in turn increases heat transfer, friction, and electrical losses. The necessity for extending such studies, both analytically and experimentally, to turbulent, MHD, boundary layers is, therefore, identified. Meanwhile, the losses due to Hall effects with segmented electrodes, must largely be based on experimental data from existing generators at modest values of the Hall parameter.

Summary of Losses

The preceding methods allow one to estimate the power output and generator efficiency within the limitations which have been described.

For convenience and illustrative purposes, a summary is provided of the loss formulas and a hypothetical numerical example is presented. The losses which have been included can be written in the following fashion.

End Losses

$$\frac{P_{L_1}}{P_0} = a_1 \frac{\eta}{1-\eta}$$

where P_{L_1} = the power loss due to end effects
P_0 = the ideal power as given by equation (8)

$$a_1 = \frac{1}{A^*}$$

$$A^* = \frac{\pi}{2 \ln 2} \frac{X_E}{h}$$

h = depth of generator section

Friction and Heat Transfer Losses

$$\frac{P_{L_2}}{P_0} = a_2 \frac{1}{\eta(1-\eta)}$$

where P_{L_2} is the power loss due to friction and heat transfer

$$a_2 = a_{2_f} + a_{2_{hit}}$$
$$= \frac{0{\cdot}03(L/D)}{R_e^{\frac{1}{5}}S} + \frac{0{\cdot}08(1 - h_w/h_s)L}{P_r^{\frac{2}{3}}R_e^{\frac{1}{5}}(\gamma - 1)SM_e^2D}$$

L = generator channel length
R_e = Reynolds number based on L

Electrode Losses

$$\frac{P_{L_3}}{P_0} = a_3 \frac{1}{\eta}$$

where P_{L_3} = the power loss due to the electrode boundary layer phenomena

$$a_3 = \frac{0{\cdot}74(L/h)(1 - \bar{u}_w/u)}{R_e^{\frac{1}{5}}}$$

$\bar{u}_w$ = "average" velocity in the turbulent boundary layer

Hall Current Losses

$$\frac{P_{L_4}}{P_0} = a_4$$

where P_{L_4} = the Hall eddy current power loss

$$a_4 = 1 - \frac{e_E}{1 + (\omega\tau)^2}$$

e_E = empirical factor which describes effectiveness of electrode segmentation
$\omega\tau = N_H$ = Hall parameter

The principal additional loss which has not been included in the above list, and which could be charged logically to the generator, is the loss associated with the magnet. To date this has been a very significant factor in the efficiency of MHD generators to the extent that self-excitation has only recently been attained (Ref. 40). However, progress on superconducting magnets has also recently been attained (Ref. 41) which promises to diminish this detrimental aspect of MHD generator design. In view of these events, and the fact that here primary concern is with phenomena internal to the generator, the magnet coil losses are omitted. The expression for the "actual" turbine efficiency of the MHD generator can then be written in the form,

$$\varepsilon = \frac{\eta}{1 + \eta\left[a_4 \dfrac{\eta}{1-\eta} + a_2 \dfrac{1}{\eta(1-\eta)} + \dfrac{a_3}{\eta} + a_4\right]} \tag{32}$$

which takes into account the fact that the ideal generator efficiency is equal to the loading factor, η. Likewise, if the "actual" power output is denoted by P_a, where this is taken to be the "ideal" power minus the power losses, then

$$\frac{P_a}{\sigma u^2(-B_z)^2} = \varepsilon(1-\eta) \tag{33}$$

In order to compare equations (32) and (33) with their equivalents for the ideal system equations (8) and (11), it is necessary to assign numerical values to the power loss coefficients a_1, a_2, a_3 and a_4. This means that one must select a system. The following numerical factors are therefore selected, not on the basis of any existing system, but rather for the purpose of demonstrating the relative significance of the loss factors. Thus,

$$\begin{array}{rlrl}
A^* &= 20 & \gamma &= 1{\cdot}2 \\
R_e &= 6 \times 10^5 & D &= 3{\cdot}0 \\
\dfrac{h_w}{h_s} &= 0{\cdot}8 & \dfrac{L}{D} &= 8{\cdot}0 \\
P_r &= 0{\cdot}7 & u_w/u &= 0{\cdot}8 \\
S &= 1{\cdot}0 & \omega &= 2{\cdot}0 \\
M_e &= 1{\cdot}0 & \varepsilon_E &= 4{\cdot}0
\end{array}$$

From these it follows that

$$a_1 = 0{\cdot}05, \quad a_2 = 0{\cdot}018, \quad a_3 = 0{\cdot}079, \quad a_4 = 0{\cdot}200$$

Equations (32) and (33) are then plotted in Fig. 6 and compared with the corresponding curves for the ideal generator. To be noted is the fact that the efficiency now has a maximum at $\eta \approx 0{\cdot}8$ with the maximum in actual power occurring at η slightly below 0·5. Optimum design of the generator will therefore involve a trade-off between efficiency and maximum power output.

This example also points out the fact that the end losses, electrode losses, and Hall current losses are more limiting than frictional and heat transfer considerations, assuming, of course, that our appraisal is accurate. The end losses are amenable to reduction by vanes and tailoring of the magnetic field. The electrode and Hall current losses represent a more severe design problem and will have to be approached with great care. The size factor should reduce frictional and heat losses to secondary importance in large-scale generators.

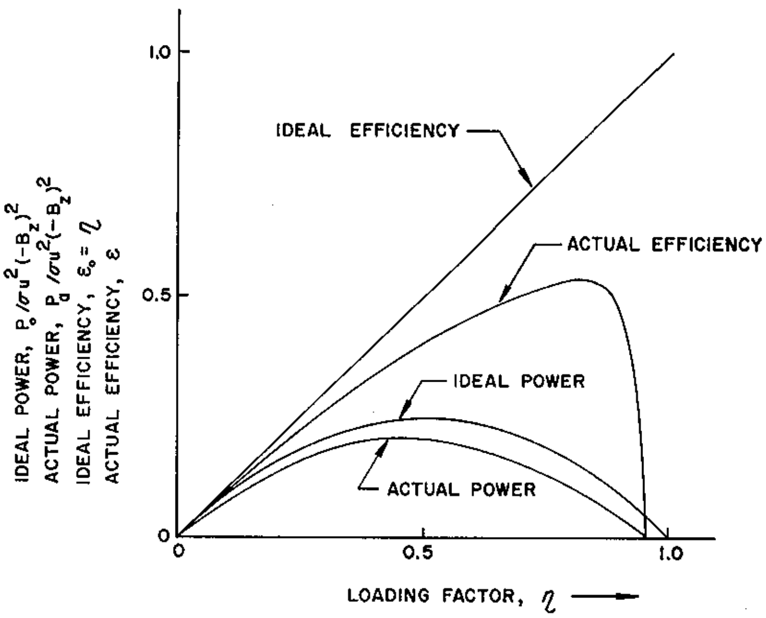


Fig. 6. Effect of losses on generator power and efficiency.

RESULTS FROM EXPERIMENTAL GENERATORS

The test of any analytical design procedure is how well it stands in comparison with experimental results. Considerable data have been published against which to make such a check. Results are available from the Avco-Everett Research Laboratories (Refs. 7, 9 and 36), MHD Research, Inc. (Ref. 34), and the Westinghouse Research Laboratories (Ref. 3). The last reference contains a very inclusive listing of system parameters and therefore the results given therein have been chosen as the basis for the following correlation.

Figure 7 is a plot of current and power as a function of the loading factor, η, as reported from the experiments of Ref. 3 and as calculated for the ideal generator by use of equations (7) and (8). In arriving at this plot the following data, as quoted in the reference, were employed.

Core velocity $= u =$ 803 meters/sec
Magnetic field $= B_z =$ 1·0 Webers/sq meter
Channel width $= w = 4{\cdot}13 \times 10^{-2}$ meters
Channel height $= h = 12{\cdot}4 \times 10^{-2}$ meters
Channel length $= L = 40{\cdot}7 \times 10^{-2}$ meters
Conductivity $= \sigma =$ 29·5 mhos/meter (calculated)

Using these data, the theoretical value of the open circuit voltage is

$$\phi_{o.c.} = 99{\cdot}5 \text{ volts}$$

The corresponding experimental value of the open circuit voltage was found to be 85 volts. Similarly the calculated value of the short-circuit current is 398 amps whereas the experimental value is 155 amps. Maximum power recorded was about 3·5 kilowatts whereas the ideal theory predicts 9·9 kilowatts. Clearly the losses involved in this experiment were considerable, hence it serves as a good vehicle against which to check a theoretical prediction of losses.

Such a check can be made by comparing experimental values of efficiency and power with the same quantities as computed by means of equations (32)

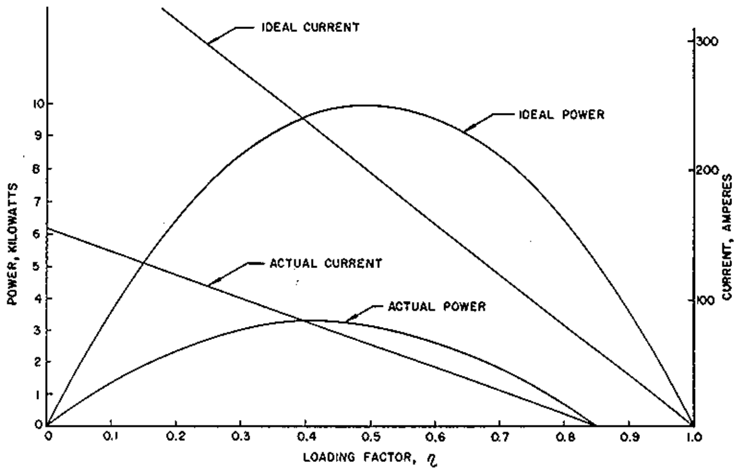


Fig. 7. MHD generator performance characteristics.
(Based on the data of S. Way, Westinghouse Research Labs, ref. 3.)

and (33). To accomplish this, we evaluate the loss factors for the particular experiment which is being analyzed here.

For the end losses, the loss factor is given by $a_1 = \frac{1}{A^*}$. For the geometry involved, $A^* = 7{\cdot}45$ so that $a_1 = 0{\cdot}134$. However, this generator is designed such that the magnetic field extends beyond the electrodes which has been shown by Sutton to reduce the loss by at least 50 per cent. Therefore we take $a_1 = 0{\cdot}07$.

The generator section was uncooled so that only friction losses are included here. (This should be "conservative" since some cooling did result simply from conduction through the walls and into the laboratory.) The friction loss factor is given by:

$$a_{2_f} = \frac{0{\cdot}03(L/D)}{R_e^{\frac{1}{4}}S}$$

The Reynolds number is based on a mass flow rate of 0·481 kg/sec and a viscosity coefficient of 30×10^{-6} Newton-sec/sq meter. This gives

$$R_e = 12{\cdot}7 \times 10^5$$

The interaction parameter, $S = \sigma B^2 L/\rho u$ has the value 0·13 which accounts, at least in part, for the "poor" performance of this generator. The corresponding value of the friction loss factor then is $a_{2_f} = 0{\cdot}340$. It should be noted that the length-to-hydraulic-diameter ratio, L/D, of this generator is 26.

The electrode loss parameter includes the ratio of average velocity in the boundary layer to the core velocity. This ratio has been assumed to have a value of 0·8, again on a "conservative" basis. The corresponding value of the loss parameter is $a_3 = 0{\cdot}027$.

Estimation of the Hall current loss parameter is the most difficult of all. Since interest here is in the fit between theory and experiment, this loss parameter is estimated directly from the experimental data through observance of the fact that, based on the theory of this report, the Hall current loss is independent of the loading parameter, η. This means that the Hall current loss factor determines the value of η for which the actual power vanishes. In the experiments of Ref. 3, this occurred as $\eta = 0{\cdot}85$ for which the corresponding value of the Hall loss factor is $a_4 = 0{\cdot}177$.

With the loss factors as given above, the equation for predicted efficiency becomes

$$\varepsilon = \eta \left\{ 1 + \eta \left[0{\cdot}07 \frac{\eta}{1-\eta} + 0{\cdot}340 \frac{1}{\eta(1-\eta)} + 0{\cdot}027 \frac{1}{\eta} + 1{\cdot}177 \right] \right\}^{-1}$$

where η is the efficiency of the ideal generator. The predicted power is then calculated from equation (33) using the value of the predicted efficiency as noted above.

The results of these calculations have been plotted in Fig. 8 along with the experimental values of power density and efficiency. The correlation is reasonably satisfactory especially when it is recognized that the heat transfer loss has been ignored and when one recalls the simplifications that have been introduced into the calculations. Nevertheless, the design and construction of a large, central, MHD power station will require even greater precision if one is to meet design objectives at competitive installation costs. It seems fair to conclude, therefore, that the various efforts at analytical and experimental refinement called for in this paper should command the attention of all those workers interested in the successful development of MHD power generation.

CONCLUSIONS AND RECOMMENDATIONS

In this part, an attempt has been made to describe the analytical tools at the disposal of the designer of open cycle, linear, MHD generators and to define the limitations of the theory. While recognizing that significant progress has been made on the part of a number of investigators, it is felt, nonetheless, that even more precision is required in analytical design methods if the development of large, central, MHD power plants is to be justified. In particular, the following problem areas deserve additional intensive investigation.

1. Methods for calculating, or measuring, the electrical conductivity of hot, seeded gases must be improved.

2. Techniques for estimating skin friction and heat transfer in turbulent,

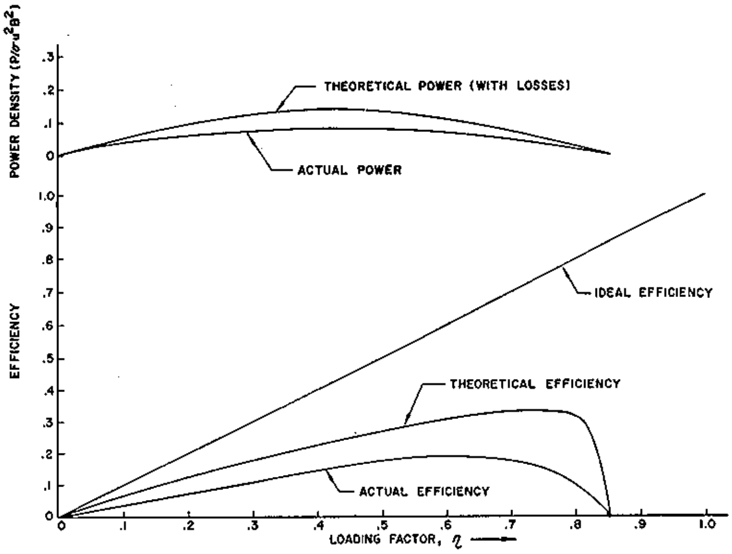


Fig. 8. Comparison of theoretical and experimental MHD generator characteristics. (Based on the data of S. Way, Westinghouse Research Labs, ref. 3.)

electrically conducting, boundary layers in the presence of large magnetic fields should be developed.

3. The problem of estimating Hall current losses in the presence of flow non-uniformities requires additional attention.

4. Methods should be developed which will allow greater precision in the estimation of electrode losses.

APPENDIX

THE MATHEMATICAL RELATIONS OF MHD

The equations stated here are based upon the macroscopic, or phenomenological, point of view. They apply to the continuum regime and significant variations are required for low-density phenomena. Further, the summary is restricted to gases although many of the relations apply to liquids as well. Relativistic effects are not included. With these restrictions, MHD phenomena of broad variety will be governed by the following.

Conservation of Mass

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \vec{V} = 0 \tag{A.1}$$

ρ = specific density of the fluid
$\vec{V}$ = velocity of the fluid relative to a reference frame fixed in the laboratory
$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{V} \cdot \nabla$ is the fluid, or convective derivative

Conservation of Momentum

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \nabla \cdot \overleftrightarrow{\tau} + \rho \vec{F} + q_v \vec{E} + \vec{J} \times \vec{B} \tag{A.2}$$

p = fluid static pressure
$\overleftrightarrow{\tau}$ = surface stress tensor or dyadic dependent upon velocity gradients
$\vec{F}$ = body force vector in the fluid, e.g., arising from gravity
$\vec{J} = \vec{I} + q_v \vec{V}$

Conservation of Energy

$$\rho \frac{Dh}{Dt} - \frac{Dp}{Dt} = Q_R - \nabla \cdot \vec{Q}_C + \overleftrightarrow{\tau} : (\nabla \vec{V}) + \frac{I^2}{\sigma} \tag{A.3}$$

$h = I' + P/\rho$ = specific enthalpy = $h(\rho, T)$
I' = specific internal energy
Q_R = volume heating due to radiation, combustion, etc.
$\vec{Q}_C$ = heat flux vector; dependent upon temperature gradients

Alternative Form of Conservation of Energy

$$\rho \frac{DS}{Dt} + \nabla \cdot \left(\frac{\vec{Q}_C}{T} \right) = \frac{Q_R}{T} + \frac{k(\nabla T) \cdot (\nabla T)}{T^2} + \frac{\overleftarrow{\tau} \vdots (\nabla \vec{V})}{T} + \frac{I^2}{\sigma T} \quad \text{(A.4)}$$

S = specific entropy

Equation of State

$$P = P(\rho, T) \quad \text{(A.5)}$$

Ampere's Law

$$\nabla \times \vec{H} = q_v \vec{V} + \vec{I} + \frac{\partial D}{\partial t} \quad \text{(A.6)}$$

Faraday's Law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(A.7)}$$

Conservation of Charge

$$\nabla \cdot (\vec{I} + q_v \vec{V}) = -\frac{\partial q_v}{\partial t} \quad \text{(A.8)}$$

The above constitute thirteen scalar relations. They are, therefore, sufficient in number to determine thirteen of the thirty-four dependent unknowns. The writing of these equations from phenomenological principles has introduced, however, cause-effect relationships for which subsidiary, or constitutive, relations for the medium must be written. With restrictions and assumptions as indicated, these are as follows.

Ohm's Law Nelgecting effect of electron partial pressure and assuming that applied magnetic field is not sufficiently large to cause ion slip, we get

$$\vec{I} = \nabla \left[\vec{E} + \vec{V} \times \vec{B} - \frac{1}{en_e} (\vec{I} \times \vec{B}) \right] \quad \text{(A.9)}$$

σ = scalar electrical conductivity

$\frac{1}{en_e}$ = Hall coefficient

e = charge on an electron

n_e = electron number density

Magnetic Permeability (Linear, homogeneous, isotropic, medium)

$$\vec{B} = \mu \mu_0 \vec{H} \quad \text{(A.10)}$$

Electrical Permittivity

$$\vec{D} = \mathrm{KK}_0 \vec{E} \quad \text{(A.11)}$$

Stress-Strain Law (Linear, homogeneous, isotropic medium)

$$\overleftrightarrow{\tau} = \lambda$$

$$\begin{bmatrix} \left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right) & \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \left(\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right) & \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) & \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) & \left(\frac{4}{3}\frac{\partial w}{\partial z} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right) \end{bmatrix} \tag{A.12}$$

(*Note*: The above is correct only for laminar motion. In the case of turbulence, the additional transport must be included by adding in the Reynolds stress components.)

$$\rho \begin{bmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{u'v'} & \overline{v'^2} & \overline{v'w'} \\ \overline{u'w'} & \overline{v'w'} & \overline{w'^2} \end{bmatrix} \tag{A.13}$$

where $u, v, w = x, y, z$ components of velocity vector, $\vec{V}$
u', v', w' = velocity perturbation due to turbulence

Heat Conduction Law (Linear, homogeneous, isotropic medium)

$$\vec{Q}_c = -k\nabla T \tag{A.14}$$

To our system of thirteen scalar "laws" we have now added the equivalent of twenty-one scalar "constitutive" relations which, in principle, allow us to calculate the components of:

$$\overleftrightarrow{\tau}, \vec{Q}_c, \vec{H}, \vec{D}, \vec{I}$$

Finally, we must be able to specify values for the phenomenological constants σ, k, μ, κ and λ. These in general are temperature and pressure dependent for a given medium and values for them are obtained from experiment or statistical mechanics.

SYMBOLS

A^* reduced electrode aspect ratio
$\vec{B}$ magnetic induction vector
D hydraulic diameter
D electric displacement vector
$\vec{E}$ electric field vector
$\vec{H}$ magnetic field vector

H_a	Hartman Number
$\vec{I}$	Conduction current density vector
J	current density vector
M_c	Mach number of the core flow
$\vec{N}$	Poynting vector
N_H	Hall parameter
P_0	specific power output
P_r	Prandtl number
$\vec{Q}_c$	heat flux vector
R	gas constant
R_{e_m}	Magnetic Reynolds Number
S	interaction number
T	absolute temperature
U_E	electrostatic energy
U_m	magnetostatic energy
$\vec{V}$	velocity distribution vector
X_E	electrode length
f	friction coefficient
h	height of specific enthalpy
h_c	enthalpy of the core flow
h_s	gas stagnation enthalpy
h_w	gas enthalpy at the wall
k	coefficient of thermal conductivity
n_e	electron density
p	pressure
q_v	net charge distribution
γ	ratio of specific heats
δ	boundary layer thickness
ε_a	actual efficiency
ε_0	ideal efficiency
η	loading factor
κ	dimensionless permeability
κ_0	dimensionless permeability of free space
λ	viscosity coefficient
μ	dimensionless permeability
μ_0	dimensionless permeability of free space
ρ	density
σ	electrical conductivity
$\overleftrightarrow{\tau}$	mean time of collisions
τ_0	frictional stress
$\overleftrightarrow{\tau}$	stress tensor
ϕ	potential
ω	electron cyclotron frequency

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PART B

CLOSED-CYCLE GENERATORS

Introduction

The critical role of thermodynamics in determining the future of MHD as a large-scale power generation technique has been demonstrated in Part A of this paper. The increased thermodynamic efficiency offered by the higher operating temperatures of the MHD device (relative to turbo-generators) may be regarded as the prime motivation for pursuing development of commercial MHD power generators. Similarly, the thermodynamic losses, via wall and electrode heat transfer and electrical power dissipation, loom large among the limiting factors which restrict application of open-cycle MHD devices to large-scale power generation (or which define the minimum size of practical devices).

On the other hand, the experimental results discussed in Part A show sufficient correlation with the theoretical predictions to lend confidence to the validity of the now-existing thermodynamic performance analyses. The problems remaining in the development of open-cycle commercial MHD power generation are not fundamental or basically thermodynamic in nature; instead, the problems are of an engineering nature—related to the development of suitable long-life large-scale components and the reduction of losses.

The status of closed-cycle MHD power generators stands in marked contrast to the open-cycle situation, however. The fundamental feasibility of the nuclear reactor-coupled closed-cycle generator remains to be demonstrated. As we will see, thermodynamic non-equilibrium processes, chiefly physical energy exchange processes in the gas, hold the key to successful application of the closed MHD cycle to power generation.

Let us first consider the possible applications of closed-cycle MHD. By closed-cycle, of course, we mean those devices in which the working fluid is continuously recirculated. As presently envisioned, the heat source in such a closed loop would be a nuclear fission reactor. Compared with other competing energy conversion schemes, MHD offers the potential advantages of high power capability, relatively high power density and an absence of moving parts exposed to high temperatures in addition to the already-mentioned advantage associated with high temperature levels of operation, i.e., high thermo-dynamic efficiency. For spacecraft power, the high operating temperatures of MHD devices are particularly important because of the fourth power dependence of heat rejection radiator area on cycle temperature. Also, the MHD generator should be considerably more tolerant of nuclear radiation than are competitive energy conversion systems.

Thus, closed-cycle MHD generators have potential application wherever nuclear reactors are used as thermal sources in the generation of relatively large amounts of power. Application of closed-cycle MHD to long duration spacecraft power supplies will probably precede application to commercial power because of the aforementioned particular advantages for space power and because of the widely recognized necessity of using nuclear energy in long-term space operations.

What problems, then, limit the present-day feasibility of coupling a nuclear reactor to a MHD generator channel? The all-important problem is the gap between the minimum temperature needed to achieve adequate gas ionization and conductivity in the MHD generator and the maximum temperature output of a nuclear reactor.

If ionization at thermodynamic equilibrium is to be used to achieve adequate electrical conductivity, even with the addition of easily ionized seed elements, static temperatures in the range of 2000–3000°K must be achieved in the working fluid. Current reactor technology, on the other hand, deals in peak (stagnation) temperatures of around 1000°K with future expectations of 1500–2000°K. Non-equilibrium ionization of one form or another is the commonly proposed means of bridging this gap.

Methods of producing non-equilibrium ionization and conductivity fall into three broad areas: (1) use of left-over ions and electrons from a preceding ionizing process (rapid expansion through a nozzle, r.f. ionization ahead of the generator electrode, etc.) (2) external maintenance of ionization in the generator section (through electron beams, r.f. fields, photoionization, etc.) and (3) maintenance of non-equilibrium ionization by elevation of the electron temperature (by induced or applied electric fields).

In all of these methods for achieving non-equilibrium conductivity, the MHD generator designer is attempting to make the conductivity-producing electrons persist against a natural tendency to recombine. To examine the feasibility of these schemes, we now compare the required duration of electron existence with the time needed for recombination to occur.

A Performance-Based Criterion for Non-Equilibrium Effects

As discussed in Part A of this paper, the interaction number or ratio of electromagnetic force to inertial force for a useful generator must be of the order unity. That is

$$\frac{\sigma B^2 L}{\rho U} \approx 1 \tag{1}$$

Letting τ_f, the flow time through the generator, be L/U, equation (1) becomes

$$\tau_f \approx \frac{\rho}{\sigma B^2} \tag{2}$$

The power output per unit volume of an ideally loaded, ideally segmented generator is (neglecting ion slip)

$$\frac{P}{V} = \frac{\sigma U^2 B^2}{4}$$

To minimize electron-ion recombination, the generator should be as short as possible. However, magnetic field requirements dictate an aspect ratio at least of order unity. For development of a suitable criterion, we choose a generator volume equal to L^3, i.e., cross-sectional area equal to L^2. Note that use of a more practical length-area ratio will simply insert a numerical factor in the criterion. Then the power output can be written as

$$P = \frac{\sigma U^2 B^2 L^3}{4} = \frac{\sigma U^5 B^2}{4} \tau_f^3 \tag{3}$$

If electron-atom collisions predominate (that is, for degrees of ionization below about 10^{-3}), the conductivity, σ, is

$$\sigma = \frac{n_e \, \mathrm{e}^2}{m_e n_a c_e Q_{ea}}$$

which we can write as

$$\sigma = K \frac{n_e}{\rho} \tag{4}$$

where

$$K = \frac{\mathrm{e}^2}{c_e} \frac{m_a}{m_e Q_{ea}} \quad \text{and} \quad c_e = \left(\frac{8kT_e}{\pi m_e}\right)^{\frac{1}{2}}$$

Thus, K is independent of the generator operating parameters except for the square root dependence on electron temperature.

The generator power can now be written as

$$P = \frac{Kn_e}{4\rho} U^5 B^2 \tau_f^3 \tag{5}$$

and by eliminating ρ between (2) and (5), our expression for flow time (2) becomes

$$\tau_f \approx \frac{1}{U^2} \left(\frac{16P^2}{Kn_e B^2}\right)^{\frac{1}{4}} \tag{6}$$

This equation may be regarded as defining the minimum size of device required to produce a given amount of power.

It is now recognized (Refs. 1, 6, 7, 8 and 14) that electron-ion recombination in monatomic plasmas occurs primarily via three-body (electron-electron-ion) collisions for the electron densities and temperatures of interest in MHD power generation. Thus, we can define a recombination coefficient, β, by the equation

$$\frac{\mathrm{d}n_e}{\mathrm{d}t} = -\beta n_e^3 \tag{7}$$

Integrating (7) over a time interval τ_r, we get

$$\beta \tau_r = \frac{1}{2}\left[\frac{1}{(n_{e,2})^2} - \frac{1}{(n_{e,1})^2}\right]$$

An upper bound on the allowable β can be obtained by permitting, say, an order of magnitude decrease in n_e through the time interval, i.e., by neglecting the second term of the right-hand side compared with the first. Then the recombination time is

$$\tau_r = \frac{1}{2\beta n_e^2} \tag{8}$$

If the electrons are to persist through the length L of the generator section, then we must have

$$\tau_r \geq \tau_f$$

or, inserting (6) and (8), we get the following criterion to be satisfied:

$$\frac{1}{2\beta n_e^2} \geq \tau_f \approx \frac{1}{U^2}\left(\frac{16P^2}{Kn_eB^2}\right)^{\frac{1}{5}} \tag{9}$$

which can be rewritten as a dimensionless number

$$N \equiv \frac{U^2K^{0\cdot 2}B^{0\cdot 4}}{3\cdot 5\beta n_e^{1\cdot 8}P^{0\cdot 4}} \geq 1 \tag{10a}$$

or, for computational ease

$$N^5 = \frac{U^{10}KB^2}{512\beta^5 n_e^9 P^2} \geq 1 \tag{10b}$$

The effects of varying the adjustable parameters are evident upon examining the criterion (10). The velocity, recombination coefficient and electron density play major roles, while the magnetic field strength, over-all power level and conductivity parameter (K) play lesser roles. Although electron temperature does not appear explicitly in (10), we note that the recombination coefficient drops steeply with T_e in the electron temperature range of interest. β goes like $T_e^{-4\cdot 5}$ in the temperature range $1000°K \leq T_e \leq 2500°K$ (Ref. 1). The square root dependence of K on T_e hardly affects matters at all. Thus, the criterion (10) demonstrates clearly the considerable benefit of elevating the electron temperature and thereby lowering the recombination coefficient.

However, as will be discussed later, the feasibility of elevating the electron temperature with the required uniformity and without encountering excessive losses has not yet been demonstrated. Our criterion then shows the primary alternatives available to the generator designer, i.e., raising the velocity or lowering the electron density. The velocity is fixed by the thermodynamics of nozzle expansion, i.e., it is limited by the stagnation temperature. The advantage of using a low molecular weight gas is evident.

At first glance, it would appear that lowering the power level, i.e., the size of the device, is a way of satisfying the criterion. However, systems considerations, namely that the generated power must exceed the magnet power plus losses, dictates a minimum practical power level.

If we consider the power level fixed at some minimum value, magnetic field fixed at some maximum obtainable value, velocity fixed by a stagnation temperature limit, then our criterion defines an electron density-electron temperature relation. For a given electron temperature, a maximum electron density is defined by criterion (10) and the relationship between recombination coefficient and electron temperature.

To take a numerical example, we choose the minimum practical power as 1 megawatt, the magnetic field as 3·3 webers/m^2, the flow velocity as 1000 m/sec, the electron-atom cross section as 10^{-15} cm^2 and the working fluid as helium. Upon inserting the numbers in the criterion (10), we find that use of an electron density of 10^{14} cm^{-3} requires that

$$\beta \leq 10^{-25} \text{ cm}^6/\text{sec}$$

Using Byron's (Ref. 1) calculation of β, we find the minimum electron temperature to be approximately 4500°K.

Alternatively, if we limit the electron temperature to a value compatible with allowable material temperatures, ($T_e \approx 1800$°K) we find that the criterion (10) is satisfied for $n_e \leq 10^{13}$ cm^{-3}. To see if this sort of cold electron-low electron density generator is of any practicality, we now calculate the other characteristics. Using the previously presented formulae (3), (4), (5), we compute a flow time, τ_f (= recombination time), of 5×10^{-4} sec. The generator characteristic length $L = U\tau_f$ then is 0·5 meters. The power required to produce the assumed magnetic field in the volume L^3, using Bitter's (Ref. 21) graphs and equations, comes out to be 0·9 megawatts for room temperature copper coils (and half this for copper cooled to -120°C, a tenth this for copper cooled to -200°C). Thus, with room-temperature copper, this cold-electron generator, operating ideally, would just about become self-excited at a power level of one megawatt. Of course, larger power levels would give more favorable efficiency. The gas conductivity for this example comes out to be 4·9 mhos/m. The neutral density is 0·016 kg/m^3 corresponding to $n_a \approx 2 \times 10^{18}$ cm^{-3}. The degree of ionization is 10^{-5} and the current density is about $\frac{1}{3}$ amp/cm^2.

These values raise the question of whether successful operation can be obtained at such low electron densities (and conductivity) in high magnetic fields. (Also, the relatively low densities are disadvantageous to minimization of the size and weight of magnets and heat exchangers.) The Hall coefficient ($\omega_e\tau_e$) is given by

$$\omega_e\,\tau_e = \frac{m_a e}{m_e c_e Q_{ea}}\,(\tau K n_e)^{-\frac{1}{2}} = \frac{eB}{m_e c_e n_a Q_{ea}}$$

and comes out to be 7·8 for the above example. Thus the real question is whether this high $\omega_e\tau_e$ mode of operation can be realized at these low current densities with achievable uniformity levels.

Rosa (Ref. 15) has analyzed the consequences of axial non-uniformities by considering alternating strips of fluid with uniform properties in each strip. He finds a very marked effect of such non-uniformities, particularly at high values of $\omega_e\tau_e$. The necessity for performing appropriate experiments at high $\omega_e\tau_e$ is indicated clearly.

Now our criterion as developed above is directly applicable to the rention of previously, produced electrons, i.e., the first of the mentioned methods of securing non-equilibrium conductivity. We can show quickly, however, that essentially the same criterion is applicable to other schemes for achieving non-equilibrium ionization.

If ionization is maintained by external means, then the ionizing power added per unit volume to make up for recombination losses must be substantially less than the power generated per unit volume. The power criterion is that

$$P_{\text{loss}}\ (\text{recombination}) < P_{\text{gen}} \tag{11}$$

Using (3) and (7), the criterion (11) becomes

$$n_e^{\,3} < \frac{U^2B^2}{4E_{\text{ion}}\beta} \tag{12a}$$

or

$$n_e^{\,2} < \frac{KU^2B^2}{4\rho E_{\text{ion}}\beta} \tag{12b}$$

Again, this criterion can be thought of as defining a relation between electron density and electron temperature. Inserting the same numbers used previously we find for $T_e \approx 1800°\text{K}$, $n_e < 10^{14}\ \text{cm}^{-3}$ and for $T_e \approx 4500°\text{K}$, $n_e < 10^{15}\ \text{cm}^{-3}$. Thus, using the power criterion, i.e., letting external ionization replenish the electrons which are lost by recombination, permits an increase in the allowable electron density by about an order of magnitude compared with use of left-over electrons. However, as seen from equation (9), the associated size of the device (τ_f) decreases only slightly. Thus, the power criterion is nearly the same as the previously derived residence-time criterion.

The third method of achieving non-equilibrium ionization is to use the power dissipated in the generator, i.e., the motion-induced electric field, to heat the electron gas. The elevated electron temperature causes an increased electron-impact ionization rate and a decreased recombination rate. In an ideally loaded generator, the power generated per unit volume equals the power dissipated in the gas by joule heating, i.e., the external resistance equals the internal resistance. The dissipated power in turn serves to elevate the temperature of the electron gas, is radiated away, is lost to the neutrals, or is lost to the walls.

If a quasi-steady degree of ionization is to be maintained through joule heating of the electron gas, then the dissipated power (which equals the generated power) must at least exceed the recombination losses (in the electron gas) plus the energy lost in electron-atom (or electron-ion for higher degrees of ionization) elastic collisions. In addition, non-ionizing inelastic (excitation) collisions may consume some of the dissipated power, but collisional de-excitation of radiation-excited energy levels can return energy to the electron gas.

For simplicity, if we assume that most of the recombination energy is lost to the electron gas (as will be shown below to be a reasonable approximation for these conditions), then the requirement that dissipated power exceed recombination losses just gives us, again, criterion (12). The additional requirement that dissipated power exceed elastic energy losses gives us an additional criterion, namely,

$$P_{\text{elastic}} < P_{\text{gen}} = P_{\text{diss}}$$

or

$$3n_a n_e Q_{ea} c_e \frac{m_e}{m_a} \cdot k(T_e - T_a) < \frac{\sigma U^2 B^2}{4}$$

where the elastic energy loss correction factor (δ) is taken as unity. ($\delta > 1$ for polyatomic molecules; δ can be less than unity for seeded low molecular weight gases.) This criterion can be written as

$$n_a^2(T_e - T_a) < \frac{KU^2B^2}{12 Q_{ea} c_e m_e k} \tag{13}$$

which illustrates a limit to the working fluid density and the amount of elevation of the electron temperature independent of the electron density and recombination rate. Using the same numbers used previously, we get

$$n_a^2(T_e - T_a) < 2 \times 10^{40}\ {}^\circ\text{K} - \text{cm}^{-6}$$

which limits the electron temperature rise to less than 5000°K for $n_a = 2 \times 10^{18}\ \text{cm}^{-3}$. These calculations illustrate the conflicting requirements which dictate a rather narrow range of possible operating conditions for non-equilibrium MHD generators. The key to achieving a finite range of possible operating conditions lies in the rates of the energy exchange processes, including ionization and recombination.

Energy Exchange Processes

The value of maintaining an elevated electron temperature and thus reducing the recombination rate is demonstrated emphatically by criterion (10) or (12) and the above calculations. The necessity here is to add sufficient energy to the electron gas to maintain its temperature significantly above the atom temperature in the presence of energy losses by elastic collisions, inelastic excitation and ionization collisions, thermal conduction to walls or colder gas, radiation, and energy gain in the electron gas from the electron-ion recombination process.

Because of the complexities of the processes involved, and because of our lack of knowledge of the appropriate energy exchange collision cross-sections, a truly definitive analysis of electron heating has not appeared. Hurwitz, Sutton and Tamor (Ref. 3) delineated the effects of electron heating on overall generator performance, while Ben Daniel *et al.* (Refs. 4 and 5) gave attention to the physical processes involved. Byron and his colleagues (Ref. 1) applied an approximate method of determining the collision cross-sections to calculation of electric field heating in an MHD generator. Robben (Ref. 6) has summarized the physical processes. The degree of uncertainty in these calculations is indicated by the differences between Byron's (Ref. 1) and Ben Daniel's (Ref. 4) estimates of the excitation cross-sections for cesium and potassium.

At this point, we can discuss certain aspects of these processes which have recently been brought to light by the abovementioned investigators and, especially, through the work of Bates *et al.* (Refs. 7 and 8). However, it appears that determination of the feasibility of electron-heating as a *practical* means of achieving ionization in MHD generators will have to be accomplished experimentally.

It was already mentioned that the electron gas loses energy by elastic collisions with the ions and atoms, through inelastic excitation and ionization collisions and by radiation and conduction, while energy is gained from the induced (or if desired, from an externally applied) electric field and from collisional recombination reactions.

The elastic collisional energy exchange rates probably are known more accurately than any of the other mentioned effects. References 9, 10, 11 and 12 give information on some of the applicable cross-sections. Uncertainties exist, however, due to possible charge exchange effects and non-Maxwellian electron distribution functions. For the fields and electron densities considered above, a near-Maxwellian energy distribution is expected (Ref. 22). However, in regions where n_e drops below about 10^{12} cm^{-3}, departures from the Maxwell distribution can be expected.

It is in the inelastic collisions that the large uncertainties exist, however. The energy losses and gains involved in excitation, ionization, radiation and recombination are inexorably interrelated. Although the dominant role of collisional processes in ionization and in relatively dense plasma was long recognized, it is only in the past few years that the importance of combined collisional-radiative processes in plasmas of the density of interest in MHD power generation was made clear.

A semi-classical method, developed by Gryzinski (Ref. 13), for computing electron impact excitation and de-excitation cross-sections showed that the cross-sections in a hydrogenic atom are inversely proportional to the square of the energy gap between the initial and final excited states. Thus, the cross-sections for transitions among the closely spaced upper levels are large compared with the cross-sections for lower-level transitions. On the other hand, the radiative de-excitation probability decreases rapidly with increasing quantum level. The net de-excitation probability, from higher to immediately adjacent lower states, which is the sum of the radiative plus the collisional probability, thus will exhibit a minimum. As recognized by Byron (Ref. 14), this minimum will be the rate-limiting step in the successive de-excitation of a recombining ion and electron. Quantum levels above the level at which the minimum occurs will be in equilibrium with the free electrons.

As the electrons and ions recombine, only the energy associated with the quantum levels above the minimum will be returned to the electron gas. Since, for electron densities of interest in MHD power generation, the minimum occurs at quantum level 3 or 4 (Ref. 6), only a small part of the recombination energy will be transferred back to the electron gas. The rest of the energy will be radiated. While, in some cases, this radiation will be reabsorbed in the atom gas, it is lost to the electron gas in any event. Since ionization still occurs primarily through electron impact, even if no net change in electron density takes place in the generator section, the electron gas loses energy as a result of these inelastic collisional and radiative recombination processes. For the electron densities of interest here, continuum radiation from the electron gas is a negligible loss. However, whether or not the line or resonance de-excitation radiation is trapped in the device will affect the electron temperature, since, if the radiation is trapped, ordinarily radiating levels may be de-excited by electron collisions, which would return the energy to the electrons.

Clearly, a satisfactory analysis of the degree of elevation of the electron temperature must include all of these effects. Since it is not yet known how well Gryzinski's semi-classical method (Ref. 13) predicts the cross-sections for atoms more complicated than hydrogen (or helium), even a thorough detailed analysis of electron heating must be viewed with some skepticism.

An additional effect not considered in existing analyses of electron heating is thermal conduction in the electron gas. This conduction can be parallel to the flow (from regions of high electron temperature to regions of lower electron temperature) or transverse to the flow, to the walls or colder electron gas near the walls. Because of the large disparity between the electron mass and the atom-ion mass, the rate of energy exchange between electrons and heavy particles is relatively slow (and it must be if the electron temperature is to be elevated above the heavy particle temperature). However, electrons effectively transport thermal energy because of their high mobility. Thus, although the energy content of the electron gas may be very small (because of the low degree of ionization) and the over-all heat transfer may be little affected, the thermal boundary layer (or region of conduction effects near the walls) in the electron gas can be much thicker than the thermal (or fluid mechanic) boundary layer in the atom-ion gas.

The non-equilibrium slightly ionized boundary layer remains to be solved. However, just noting that the ratio of electron gas-atom gas thermal boundary layer thicknesses is approximately $(Pr_e/Pr_a)^{-\frac{1}{2}} \approx 16$ (for helium), we see that the effect of cold walls on the electron temperature might extend over a significant portion of the flow. Note that the low densities required for non-equilibrium operation make the boundary layers already relatively thick. The deleterious effect of reduced electron temperature (increased recombination rate) has already been discussed.

Because of all these uncertainties and effects, it seems that experiments will be the only way to achieve a definitive answer to the question of whether non-equilibrium operation through self-induced electron heating can be achieved in a practical device. Unfortunately, the required experiments are very difficult to scale down in size (and cost). For one thing, the effects of non-uniformities, being spatially sensitive, cannot be simulated with confidence in less than a full-size experiment. A substantial scale of experiment is indicated also if the electrode sheath voltages are not to be large compared with the core voltages. Fluid mechanically, the appropriate Reynolds number must be achieved; on the other hand, the classical wind-tunnel technique of using small scale with high pressure is inapplicable because non-equilibrium ionization will occur only at low pressures. Note also that the relatively simple and economical shock tube experiments can contribute very little information on the effect of non-uniformities.

For these reasons, closed-loop experiments which attempt to establish non-equilibrium motion-induced ionization are underway in several laboratories. Preliminary results from these experiments are just beginning to appear. Talaat (Ref. 16) has reported the continuous generation of 20–40 milliwatts of power through the establishment of a conductivity of about 10^{-3} mho/m in cesium-seeded helium at a temperature of 850°K. While highly encouraging, since this conductivity is about two or more orders of magnitude higher than the equilibrium conductivity, these experiments are still a long way from demonstrating the practicality of the technique (the conductivity achieved here is also two or more orders of magnitude below a useful conductivity for power generation).

Other qualitative demonstrations of non-equilibrium motion-induced conductivity have been accomplished in a shock tube (Ref. 17). Shock-heated xenon exhibited apparent five-fold increases in conductivity due to a combination of electron heating and magnetic compression. The test section configuration was such that quantitative separation of the effects was not possible. In other experiments (Ref. 18), arc-heated potassium-seeded argon was expanded in a nozzle into a segmented electrode test section. Although data obtained without a magnetic field present showed evidence of non-equilibrium conductivity, the electrode sheath voltage drops with a magnetic field present were large enough to obscure the core gas conductivity. Still other recent experiments (Refs. 19 and 20), again with no magnetic field, have demonstrated non-equilibrium conductivity, but not associated with self-induced fields. It is worth noting that all of these experiments have exhibited conductivities below that associated with collisional equilibration of the electron gas with all levels of excitation, i.e., the aforementioned combined radiative-collisional de-excitation processes and thermal conduction effects have to be considered.

To summarize, then, we see that the most promising means to achieve adequate conductivity in a closed-cycle MHD generator depend on maintenance of an elevated electron temperature or operation at low electron densities. Over-all systems considerations favor elevation of the electron temperature. The presently most-favored means of maintaining an elevated electron temperature is through heating by motion-induced electric fields. While the method seems promising and preliminary experiments are encouraging, the feasibility of the scheme under actual MHD generator conditions remains to be established. Major uncertainties with any non-equilibrium scheme are the rates of energy losses from the electron gas, especially because of inelastic collisions, and the effects of non-uniformities. Solution of these problems, along with advances in nuclear reactor technology towards operation at higher temperatures and lower working fluid pressures, should make closed-cycle MHD a valuable source of electrical power, both in space flight and terrestrial applications.

Promising Research Directions

Foremost among the research problems involved in closed-cycle MHD power generation is demonstration of the feasibility of using motion-induced fields to maintain the electron temperature at a sufficiently high level that recombination losses are tolerable. Analytical attempts to demonstrate this feasibility, while valuable in selecting experimental conditions and working fluids, cannot yet be definitive because of a lack of knowledge of the cross-sections of energy exchange processes for complicated atoms.

Thus, one promising avenue of research is the study of the basic energy exchange processes and determination of the rates and cross-sections involved. Until theoretical predictions can be made with confidence, and until the effect of non-uniformities is established, a second important area (which is already being pursued actively) is experimental studies of motion-induced ionization effects at relatively high values of $\omega_e\tau_e$. Of possible decisive importance here is the effect of non-uniformities at low densities and at high values of $\omega_e\tau_e$. Again careful experimental studies are indicated. Finally, a research subject that will be badly in need of attention if successful work proceeds in the other areas is the development of nuclear reactors especially suited to coupling with MHD generators.

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SYMBOLS

- B magnetic field strength
- c_e electron thermal velocity
- E_{icn} energy of ionization
- e electronic charge
- k Boltzmann constant
- L characteristic length
- m_a atom mass
- m_e electron mass
- N dimensionless number defined by equation (10)
- n_a atom density
- n_e electron density
- P power
- Pr Prandtl number
- Q_{ea} electron-atom collision cross-section
- T_a atom temperature
- T_e electron temperature
- U flow velocity
- V volume
- β recombination coefficient

δ elastic energy loss parameter
ρ gas density
σ electrical conductivity
τ_e electron collision time
τ_f L/U—flow time
τ_r recombination time
ω_e electron gyro frequency

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DISCUSSION

Helmut Burkhart (Germany):

The title of this paper, "Thermodynamics of MHD-Energy Conversion", is somewhat misleading. It is written very clearly, and therefore in my opinion it is an excellent introduction to the general problems faced with Faraday-generators. The paper develops an overall picture of the factors to be considered in designing a linear MHD Energy converter.

One remark I would like to make on the fundamental set of equations used. Since we often deal with mixtures as a working medium, we ought to use equations of state not only as a function of two independent variables of state as it is the case for homogeneous, uniform, one-component media. Thus in general the thermic equation of state should read

$$p = p(\rho,\ T,\ c_i)$$

where c_i are the concentrations of the mixture components. Furthermore the authors ought to include the caloric equation of state into their fundamental set of equations, which is equally important. This could either be the enthalpy

$$h = h(\rho,\ T,\ c_i)$$

the entropy, or the inner energy as a function of the state variables.

Some further comments I would like to make on the results and conclusions of the authors.

1. Concerning electrode losses
   In this paper the voltage which is not generated, because of the smaller velocity of the boundary layer, is declared as electrode drops. But this does exist even if there is no current flowing. It is an effect of the boundary layer of the flow, i.e. of the velocity profile. In addition to this, there is a voltage loss which is commonly referred to as electrode drops. This is linked to the current flowing and it has to be separated into a cathode drop and an anode drop. Both have a different origin and depend on the mode in which the current is drawn. According to a theory of Dr. Schoeck (Ref. 1), the anode drop stems from a temperature profile which leads to an electrical conductivity profile. The cathode drop has its origin in the extraction process of electrons from the surface of the cathode. This part of the cathode drop may vanish, if there is a thermally emitting cathode. It is in the order of 15 to 20 volts, if there is a contracted cathode spot which delivers the electrons for the electric current. We are preparing a paper on this subject which is to be presented in September this year at the WGLR—Meeting in Berlin (Ref. 2).
2. Concerning Hall current losses I would like to point to an extensive study of this subject by Celinski who is going to present a paper on this subject in July this year at the Paris conference on MHD (Ref. 3).
3. Some information concerning turbulent hydromagnetic flow may be found in a booklet on this subject by Harris, which was edited in New York. Generally a magnetic field dampens turbulence (Ref. 4).
4. I fully agree with the authors on the necessity of more precise measurements of electric conductivity in the working fluid. At the same time I would like to add that at high-temperature flow there is a need for precision measurements of velocity profiles. Perhaps there are some further comments to these problems from the audience.

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Ralph Roberts (Office of Naval Research, U.S.A.):

The open-cycle MHD system, which is the one of principal concern to the authors, has little value to space applications, the theme of the meeting, where only closed-cycle systems are of real consideration. In general the paper is a good review of the internal losses in the MHD device and offers semiquantitative methods for estimating some of these.

Unavailable to the authors at the time of the meeting was the recent paper of the AVCO group describing their work on fluid dynamics of the open cycle generator (Ref. 1). They point out that a large linear MHD generator may be operated with a Hall coefficient as high as 2·5 to 3 without the presence of a first-order Hall current by using the principle of segmented electrodes. Thus, it appears that the problem of Hall losses in a properly designed generator is a minor one.

In addition, these workers have shown that at fields as high as 33,000G the heat-transfer profile along a channel approximates that expected for a flat plate in a turbulent flow without MHD effects. Likewise, the determined friction coefficients were appropriate to a conventional turbulent flow. The explanation the authors present for the lack of these effects is the relatively low electrical conductivity at the walls.

The magnitude of these effects has not yet been determined for non-equilibrium closed-cycle MHD systems, which have much more promise for space applicability. However, the general problem has been treated theoretically by Hale and Kerrebrock (Ref. 2). They conclude that for the non-equilibrium case, there are large current concentrations that increase the wall shear to three or four times and the heat transfer to ten times their values for normal boundaries. They further conclude that the effect of increasing the Hall parameter is to increase the average current in the boundary layer, leading to thinner boundary layers, increased heat transfer and wall shear and increased electrical losses. In addition, the current concentrations near the wall indicate the possibility of electrical instabilities in the non-equilibrium case.

The comments are those of the author and are not to be construed as representing the views of the Office of Naval Research.

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