Mass. Inst. tech.
NGR-22-009-250

HYDROSTATIC SEA LEVEL CHANGES

Summary Report by G. Welch

It has long been realized by oceanographers that, because the force of gravity dominates other forces in the oceans, the long-time response of the oceans must be a hydrostatic one. (Water seeks its own level.) It is an open question how short a time the long-time response may be applied to. The hydrostatic response of the ocean may be conveniently divided into four physical effects: glacial melting and rainfall response, steric changes, the inverted barometer, and the equilibrium tide. The equilibrium tide is not included in this paper because the diurnal, semi-diurnal, and lunar fortnightly terms probably do not have hydrostatic response, and the higher terms are unimportant. Changes in the total mass of the oceans, although the largest amplitude single variation in sea surface level, are too slow to be of interest for satellite observations. So I shall treat only the inverted barometer effect and the effect of steric changes.

Consider the Navier-Stokes equation:

$$\frac{d\underline{u}}{dt} = -\frac{1}{\rho}\nabla p - \nabla\Phi + \text{frictional terms and non-conservative forces}$$

On the Earth, Φ gives the sum of gravitational and centrifugal potentials, and there are no major non-conservative body forces. Assume that vertical velocities, accelerations, and frictional forces are negligible, where the unit vector in the vertical direction is defined by

$$\hat{k} = +\frac{\nabla\Phi}{|\nabla\Phi|}$$

Then the vertical equation becomes

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{d\Phi}{dz}$$

Now on the Earth along any plumb line, the magnitude of $\frac{d\Phi}{dz}$ is nearly constant throughout the depth of the ocean. Let $\frac{d\Phi}{dz} = g(x,y)$ or $g(\theta,\varphi)$ on the Earth. Then along a given plumb line $(\theta, \varphi$ CONSTANT$)$, we can write

$$\frac{1}{\rho}\frac{dp}{dz} = -g$$

This is the hydrostatic equation. For our purposes it is more convenient to write it as

$$\alpha\, dp = -g\, dz \quad \text{where } \alpha \equiv \frac{1}{\rho}$$

Now, at a given point on the surface of the ocean, we integrate this equation from a deep reference depth -h to the surface at $+\eta$ and get

$$\int_{p(-h)+P_{ATM}}^{P_{ATM}} \alpha\, dp = -g\int_{-h}^{\eta} dz = -g(\eta + h)$$

In practice, this integral is computed between p(-h) and 0, introducing an absolute error of about 1 cm in height and a relative error of about 10^{-2} cm.

α, the specific volume of sea water, is a function of pressure, salinity, and temperature and may be expressed as the specific volume under standard conditions plus the effects of deviations due to deviations in the three variables. The standard conditions are $p = 0$, $T = 0^{\circ}$C, $S = 35‰$. So

$$\alpha = \alpha(35‰, 0°C, 0 \text{ DECIBARS}) + \delta_p + \delta_s + \delta_T + \delta_{s,T} + \delta_{s,p} + \delta_{T,p} + \delta_{s,T,p}$$

If you choose -h such that p(-h) is a constant in time, then the integral of the first two terms gives a constant which may be used to determine a zero point for sea level

$$\int_{p(-h)}^{0} \left[\alpha(35,0,0) + \delta_p\right] dp = \text{CONST.} = gh$$

The remainder of the terms are called δ, or the steric anomaly. It has been experimentally determined that $\delta_{s,T,p}$ is negligible. So

$$\delta \doteq \delta_s + \delta_T + \delta_{s,T} + \delta_{s,p} + \delta_{T,p}$$

The terms $\delta_s + \delta_T + \delta_{s,T} = \Delta_{s,T}$ are frequently called the thermosteric anomaly.

The result is that

$$\int_{p(-h)}^{0} \delta\, dp = -g\eta \quad \text{OR} \quad \eta = -\frac{1}{g}\int_{p(-h)}^{0} \delta\, dp$$

This gives the formula for experimentally determining the sea level anomaly due to anomalies in the structure of specific volume of the water column. The quantity $g\eta$ is often called the dynamic height anomaly of the sea surface. η can be determined at different times, and variations of steric level computed as a function of time. The data required are values of temperature and salinity vs pressure (depth) at a given deep water station.

The inverted barometer effect is the hydrostatic (steady state) response to a change of atmospheric pressure. Consider a column of fluid of density ρ from the reference depth -h to the free surface at pressure p_{ATM}. In this case, we make a very small error by assuming ρ =constant. The hydrostatic equation now is $\frac{dp}{dz} = -\rho g$

or integrating, $p_{ATM} - p(-h) = -\rho g(\eta + h)$

Changing p_{ATM} by Δp will change η by $\Delta\eta$ such that $\Delta\eta = -\frac{\Delta p}{\rho g}$

To a very close approximation, the sea surface is depressed by 1 cm for every 1 mb. increase in atmospheric pressure. This adjustment is essentially complete in a few days.

The observations of sea level are taken at tide gauges. These are located at the shores of continents and islands. Those of atmospheric pressure are taken at meteorological stations, which are usually located near major cities and airports. The pressure at these stations is then adjusted to sea level. Both of these variables are monitored several times each day at each station, so that monthly averages are likely to be representative of monthly averages of the variables. The temperature profiles are by necessity done in the deep ocean away from the coasts. These are likely to be done at certain stations once every several months with bathythermographs. Temperature and salinity data are taken more seldom with hydrographic casts. These data are sparse enough that monthly averages over many years are about the best that can be done. So the data are scarce in time and unevenly distributed in space, following the geographical distribution of islands and the comfort zones of oceanographers. The results obtained, no doubt, are biased by the unevenness of the grid.

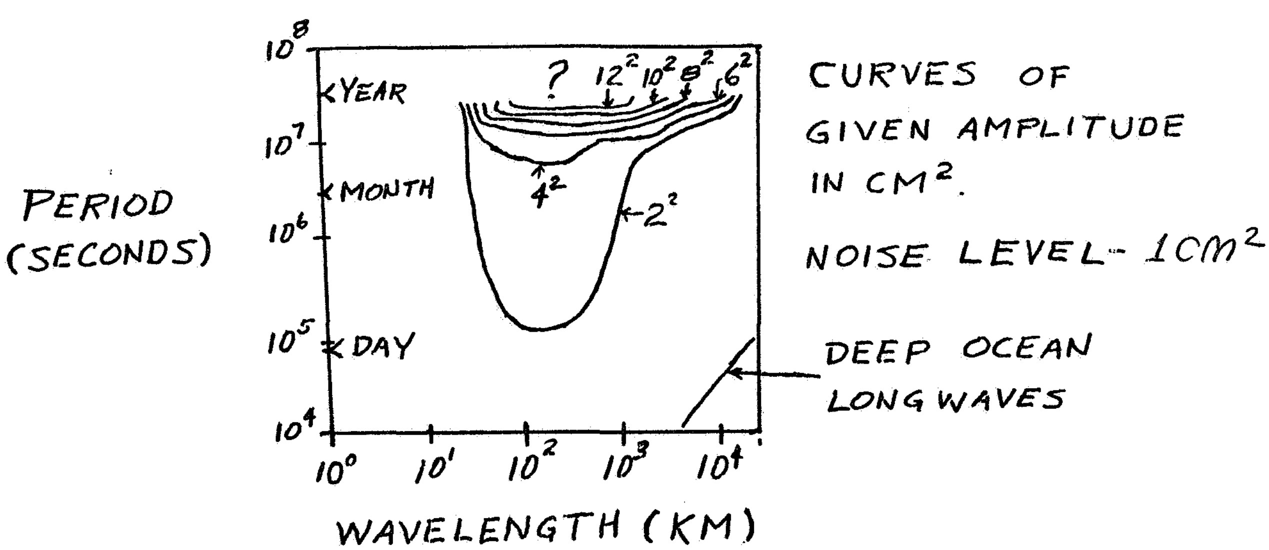
In order to evaluate the pressure term, the annual mean pressure cycle over the ocean must be subtracted from the data before the inverted barometer response is calculated. If the spectrum of pressure variations is calculated, it contains much more high frequency power than the analagous spectra for the other variables. In several cases, it resembles a white noise spectrum. Both seasonal studies and cross-correlation studies show that the pressure effect is most important both annually and non-anually at higher latitudes. Seasonal studies show that in high latitudes, pressure effects are more pronounced in winter than in summer. This reflects what we know about the distribution of pressure variations both in space and in time.

The steric anomalies due to temperature variations are found to be largest in mid-latitudes. They are lower both in the tropics and in the arctic regions. In the mid-latitudes and the tropics, the annual and non-annual variations seem to be of about equal importance. The particularly low value in the arctic regions is somewhat puzzling, as it implies either non-hydrostatic response or a predominately salinity caused variation of the steric anomaly.

In most parts of the oceans, salinity variations have little or no effect on steric level variations. Where they are important, however, they can be surprisingly large. The largest value is in the Bay of Bengal, where there is an annual term of 41 cm. They are also significant (greater than 5 cm.) at some southern hemisphere stations. In all cases, the significant salinity variations are connected with unusually deep temperature variations.

The total hydrostatic response at the time periods under consideration appears to agree very well with recorded changes of sea level over almost all of the oceans. There is a significant difference associated with coastal effects near some coasts. For periods of less than a year, the variations are pressure dominated by latitudes greater than 40° and temperature dominated at lower latitudes. Results are summarized in the following table.

EFFECT		HIGHEST	LOWEST	AVERAGE
SALINITY	annual	41	0	5
	non-annual	?	0	?
TEMPERATURE	annual	12	< 5	8
	non-annual	?	?	?
PRESSURE	annual	12	0	1.6
	non-annual	11	1	5
TOTAL STERIC	annual	40	< 5	11
	non-annual	11	< 5	8



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