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ABSTRACT

A general formula is derived for calculating the gamma-ray spectrum resulting from the annihilation of cosmic-ray positrons. This formula is used to calculate annihilation-gamma-ray spectra from various equilibrium spectra of secondary galactic positrons. These spectra are then compared with the gamma-ray spectra produced by other astrophysical processes.

Particular attention is payed to the form of the gamma-ray spectrum resulting from the annihilation of positrons having kinetic energies below 5 keV. It is found that for mean leakage times out of the galaxy of less than 400 million years, most of the positrons annihilating near rest come from the beta decay of unstable nuclei produced in cosmic-ray p-C^{12}, p-N^{14}, and p-O^{16} interactions, rather than from pi-meson decay. It is further found that the large majority of these positrons will annihilate from an S state of positronium and that 3/4 of these will produce a three-photon annihilation continuum rather than the two-photon line spectrum at 0.51 MeV. The results of numerical calculations of the gamma-ray fluxes from these processes are given.

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I. INTRODUCTION

The annihilation of cosmic-ray positrons has for some time been recognized as a potential source of cosmic gamma rays. Gamma-ray fluxes from cosmic positron annihilation have been estimated and discussed by various authors (Pollack and Fazio, 1963; Hayakawa, et. al., 1964; Ginzburg and Syrovatskii, 1964a,b). Pollack and Fazio have discussed the possible relationship between the present flux of 0.5 MeV gamma-radiation from positron annihilation and the cosmic-ray intensity and galactic gas density 10^9 years ago. Ginzburg and Syrovatskii have pointed out that the intensity of the 0.5 MeV line may be a sensitive measure of the leakage rate of cosmic-ray positrons from the galaxy. These authors have also given an approximate formula for the calculation of the gamma-ray spectrum from the annihilation of high-energy positrons.

It has therefore become apparent that the cosmic-gamma ray spectrum from cosmic-ray positron annihilation may contain much potential information reflecting various astrophysical conditions in our own galaxy and in possible

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cosmic-ray sources, both galactic and extragalactic. It is for this reason, that the author had recently undertaken a more detailed investigation of the various aspects of the cosmic positron annihilation problem (Stecker, 1967b). That work has now lead to this further treatment pointing out the potential richness and complexity of the problem.

The point of departure for our discussion will be a derivation of the general formula for calculating the annihilation-gamma-ray spectrum (AGS) from cosmic-ray positrons valid at all energies. This formula is then used to calculate AGS from various equilibrium spectra of secondary cosmic-ray positrons. The results of various numerical calculations will be given and discussed.

The significance of direct measurement of the positron equilibrium spectrum itself will be discussed as a possible sensitive indicator of the diffusion and leakage of cosmic-ray leptons out of the galaxy.

II. THE ANNIHILATION-GAMMA-RAY SPECTRUM

The cross section for positron annihilation as a function of energy was first deduced by Dirac (1930). An excellent presentation of the theory is given by Heitler (1960). The most important annihilation mode of the free electron-positron system is the annihilation

$$e^{+} + e^{-} \rightarrow \gamma + \gamma \tag{1}$$

The frequency of this annihilation is 372 times greater than that of the free three-photon annihilation and we may neglect all but the two-photon mode in considering the gamma-ray spectrum from free $e^+ - e^-$ annihilation. (However, as we shall see later, the three-photon mode becomes important when we consider the effect of positronium formation by positrons of energies less than 5 keV.)

The differential cross section for gamma-ray production in the collision c.m.s. of a free two-photon annihilation may be written as

$$d\sigma = \frac{\sigma_0}{2\gamma_c^2 \beta_c} \phi(\chi; \gamma) \, d\chi \tag{2}$$

where $\sigma_0 = \pi r_0^2$, $r_0 = e^2/Mc^2$ is the classical radius of the electron, γ is the Larentz factor of the positron, $\beta = \sqrt{1 - 1/\gamma^2}$, the c.m.s. Lorentz factor and velocity are given by

$$\gamma_c = \sqrt{\frac{\gamma + 1}{2}} \qquad \text{and} \qquad \beta_c = \sqrt{\frac{\gamma - 1}{\gamma + 1}} , \tag{3}$$

χ is the cosine of the angle between the incoming positron and the outgoing gamma-ray in the c.m.s., and the angular distribution function, $\phi(\chi; \gamma)$ is defined as

$$\phi(\chi; \gamma) = \frac{1 + \beta_c^2 \left(2 - \chi^2\right)}{1 - \beta_c^2 \chi^2} - \frac{2\beta_c^4 \left(1 - \chi^2\right)^2}{\left(1 - \beta_c^2 \chi^2\right)^2} \tag{4}$$

The energy of an annihilation-gamma-ray in the laboratory system is given by the Doppler relation as

$$E_\gamma = \left(Mc^2\right) \gamma_c^2 \left(1 + \beta_c \chi\right) \tag{5}$$

If we now define the dimensionless energy $\eta = E_\gamma/Mc^2$, we may use Eqs. (3) and (5) to determine the normalized distribution function uniquely relating χ to the laboratory Lorentz factor of the positron and the laboratory energy of the gamma-ray as follows:

$$f(\chi;\, \gamma,\, \eta) = \frac{2}{\sqrt{\gamma^2 - 1}}\, \delta\left[\chi - \chi_0\,(\eta,\, \gamma)\right]$$

where

$$\chi_0\,(\eta,\, \gamma) \equiv \frac{(2\eta - 1) - \gamma}{\sqrt{\gamma^2 - 1}} \quad . \tag{6}$$

The total number of gamma-rays produced per second by a positron of energy γMc^2 is

$$Q_{TOT}\,(\gamma) = \frac{2\sigma_0\, n_e\, \beta c}{2\gamma_c^{\,2}\, \beta_c} \int_{-1}^{1} d\chi\, \phi(\chi;\, \gamma)$$

$$= \frac{2\sigma_0\, n_e\, c}{\gamma} \int_{-1}^{1} d\chi\, \phi(\chi;\, \gamma)$$

where n_e is the number density of electrons in the medium.

Therefore, the gamma-ray source spectrum from the annihilation of cosmic-ray positrons with a density $n(\gamma)\ \mathrm{cm}^{-3}\gamma^{-1}$ is given by

$$Q(\eta) = 4 n_e \sigma_0 c \int_1^{\infty} d\gamma \frac{n(\gamma)}{\gamma\sqrt{\gamma^2 - 1}}$$

$$\times \int_{-1}^{1} \phi(\chi; \gamma)\, \delta\left[\chi - \chi_0(\eta, \gamma)\right] \tag{7}$$

We now wish to integrate over the delta function in Eq. (7). The result of this integration is to replace the last integral in Eq. (7) by a function $\Phi(\eta, \gamma)$ such that

$$\Phi(\eta, \gamma) = \begin{cases} \phi\left(\chi_0(\eta, \gamma), \gamma\right) & \text{for} \quad |\chi_0| \leq 1 \\ 0 & \text{for} \quad |\chi_0| > 1 \end{cases} . \tag{8}$$

We may, therefore, replace the integral over $d\chi$ by the algebraic function $\phi\left(\chi_0(\eta, \gamma), \gamma\right)$ provided the limiting condition on χ_0 is transformed into a limiting condition on the integration over $d\gamma$. The limiting conditions $\chi_0 = \pm 1$ correspond to the relations

$$\eta_{\pm} = \gamma_c^2 \left(1 \pm \beta_c\right) \tag{9}$$

It follows from Eq. (9) that the product

$$\eta_+ \eta_- = \gamma_c^4 \left(1 - \beta_c^2\right) = \gamma_c^2 = \frac{\gamma + 1}{2} \tag{10}$$

and the sum

$$\eta_+ + \eta_- = 2\gamma_c^2 = \gamma + 1 \tag{11}$$

Therefore, η_+ and η_- are the roots of the quadratic equation

$$\eta^2 - 2\gamma_c^2 \eta + \gamma_c^2 = 0 \tag{12}$$

which can be solved for γ_c in terms of η, yielding

$$\gamma_c^2 = \frac{\eta^2}{2\eta - 1} = \frac{\gamma + 1}{2} \tag{13}$$

or

$$\gamma = \frac{\eta^2 + (\eta - 1)^2}{2\eta - 1} \tag{14}$$

which is finite and positive for $\eta > 1/2$ and has a minimum of $\gamma = 1$ at $\eta = 1$.

Figure 1 shows the curve defined by Eq. (14) split into the branch defined by the physical extremes $\chi = \pm 1$. Also shown are the asymptotes $\eta = 1/2$ and $\eta = \gamma + 1/2$, the line defining $\chi = 0$ and the shaded region corresponding to the physical values of $|\chi| \leq 1$. It can be seen from Fig. 1 that the physical range of γ defined by the shaded region $\left(|\chi| \leq 1\right)$ is bounded on the bottom by the curve of Eq. (14) and is unbounded on the top.

Figure 1 indicates that no gamma rays can be produced from the annihilation process having energies less than or equal to $Mc^2/2$. This physical restriction may be seen more clearly as a direct consequence of Eq. (9) by noting that

$$\eta_- \leq \eta \leq \eta_+ \tag{15}$$

where

$$\eta_- = \gamma_c^2 \left(1 - \beta_c\right) = \frac{1 - \beta_c}{1 - \beta_c^2} = \frac{1}{1 + \beta_c} > \frac{1}{2} \tag{16}$$

so that η_- may approach, but never reach 1/2 as $\gamma_c \to \infty$. On the other hand

$$\eta_+ = \gamma_c^2 \left(1 + \beta_c\right) = \frac{1}{1 - \beta_c} \tag{17}$$

which increases without bound as $\gamma_c \to \infty$.

The general restriction on the range of η may be designated by the introduction of the Heavyside step function, $H_+ \left(\eta_0\right)$ which is defined by the relation

$$H_+ \left(\eta_0\right) = \begin{cases} 1 & \text{for} \quad \eta > \eta_0 \\ 0 & \text{for} \quad \eta \leq \eta_0 \end{cases} \tag{18}$$

We may therefore rewrite Eq. (7) in the form

$$Q(\eta) = 4H_+ \left(\frac{1}{2}\right) n_{e-} \sigma_0 c$$

$$\times \int_{G(\eta)}^{\infty} d\gamma \frac{n(\gamma)}{\gamma\sqrt{\gamma^2 - 1}} \phi\left(\chi_0 \left(\eta, \gamma\right), \gamma\right) \tag{19}$$

where

$$G(\eta) = \frac{\eta^2 + (\eta - 1)^2}{2\eta - 1}$$

From the general results which we have obtained, we may immediately arrive at formulas for asymptotic spectra which may be used as guides in examining the results of numerical calculations. These are obtained as follows:

A) For two-photon annihilations at rest, it follows from Eq. (5) that the AGS is simply a line at energy $\eta = 1$ (0.51 MeV). This is, of course a familiar and expected result.

B) The AGS from the two-photon annihilation of an ultrarelativistic positron may be obtained from a consideration of the angular distribution function, $\phi(\chi; \gamma)$ given by Eq. (4). At ultrarelativistic energies, the angular distribution function peaks sharply at $\chi = \pm 1$ so that the gamma-rays resulting from the annihilation lie close to the asymptotes of Fig. 1, viz. $\eta = 1/2$ and $\eta = \gamma + 1/2$. This result, as pointed out by Heitler (op. cit.) may be discussed physically as follows. In an ultrarelativistic annihilation, the resulting photons are emitted in a sharply backward and sharply forward direction in the collision c.m.s. respectively. In the laboratory system, the forward photon carries off almost all the energy of the collision while the backward photon carries off an energy between about 0.25 and 0.5 MeV. Therefore, the AGS for $\eta >> 1$ may be obtained by using the approximate production-cross-section

$$\sigma_A(\eta; \gamma) \simeq \sigma_A(\gamma)\,\delta(\eta - \gamma) \qquad \text{for} \quad \eta >> 1 \tag{20}$$

The two-photon annihilation cross-section at ultrarelativistic energies has the asymptotic form

$$\sigma_A(\gamma) \simeq \frac{\sigma_0}{\gamma}\left[\ell n(2\gamma) - 1\right], \qquad \gamma >> 1 \tag{21}$$

Therefore, the ultrarelativistic asymptotic form of the AGS is given by the expression

$$Q(\eta) \simeq n_{e^-}\,\sigma_0\, c\, \frac{n(\eta)\left[\ell n(2\eta) - 1\right]}{\eta} \tag{22}$$

as has been previously noted by Ginzburg and Syrovatskii. These asymptotic formulas should be kept in mind when examining the numerical evaluations of the AGS obtained from the exact formula given by Eq. (19).

III. THE EQUILIBRIUM SPECTRUM OF SECONDARY GALACTIC POSITRONS FROM PI-MESON DECAY

In order to calculate plausible annihilation-gamma-ray spectra from our own galaxy, it will be assumed that the only source of galactic cosmic-ray positrons above a few MeV energy is the result of primary cosmic-rays colliding with atoms of interstellar hydrogen and helium gas in the galaxy. These high-energy collisions are known to produce positive pi-mesons which rapidly decay into positrons and neutrinos. Much accelerator data is available on the production of pi-mesons in interactions up to 30 GeV/c and many cosmic-ray studies have been made of higher energy interactions. Various calculations have been made using this data to calculate positron source spectra in the galaxy (Pollack and Fazio (1963); Ginzburg and Syrovatskii (1964b); Hayakawa, et al. (1964); Jones (1963); Ramaty and Lingenfelter (1966)). In these calculations, it is usually assumed that the primary cosmic-ray spectrum is essentially the same as that observed at the earth. Ramaty and Lingenfelter have shown that in the energy range where most of the pions are produced (above 500 MeV) the primary galactic cosmic-ray spectrum observed at the earth is little affected by local conditions in the solar system and that local effects induce very little error in the positron flux calculation. We therefore take for our source spectrum of cosmic-ray positrons, the one derived by Ramaty and Lingenfelter. (This spectrum is shown in Fig. 2.)

We further assume that these positrons have reached a quasi-equilibrium condition in the galaxy determined by leakage out of the halo, annihilation, and energy loss by ionization and coulomb interactions, bremsstrahlung, Compton collisions and synchrotron radiation. The equilibrium density spectrum, $n(\gamma)$, which will be used in Eq. (19), is then taken to be determined by the continuity equation

$$\frac{\partial}{\partial \gamma}\left[n(\gamma)\, r(\gamma)\right] = q_{+}(\gamma) - \frac{n(\gamma)}{\tau_s(\gamma)} \tag{23}$$

where $q_+(\gamma)$ is the positron source spectrum from pi-meson decay, $r(\gamma)$ is the total energy loss rate per unit time and $\tau_s(\gamma)$ is the effective positron survival time given by

$$\frac{1}{\tau_s(\gamma)} = \frac{1}{\tau_A(\gamma)} + \frac{1}{\tau_\ell(\gamma)} \tag{24}$$

with $\tau_A(\gamma)$ and $\tau_\ell(\gamma)$ being the annihilation time and mean leakage time for positrons of energy γMc^2. These quantities are given by

$$\frac{1}{\tau_A(\gamma)} = \frac{n_{e^-}\sigma_0 c}{\gamma}\sqrt{\frac{\gamma-1}{\gamma+1}}\left[\frac{-\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right. \tag{25}$$

and

$$\frac{1}{\tau_\ell(\gamma)} = \frac{\sqrt{\gamma^2-1}}{\gamma T_\ell}, \qquad T_\ell = \text{constant} \tag{26}$$

where we have assumed a mean leakage time inversely proportional to velocity.

Most of the interstellar gas in our galaxy is unionized with the exception of the so-called HII regions near the very hot O and B stars which are powerful sources of ionizing ultraviolet radiation. Allen (1963) gives the proportion of space near the galactic plane occupied by clouds of interstellar gas and dust as 7% and that occupied by ionized clouds (HII regions) as 0.4%. We will therefore assume that the galactic gas is entirely neutral for the purposes of these calculations and take for the energy loss rate from ionization the expression

$$r_I(\gamma) = \frac{8}{3}\sigma_0 n_{e^-} c \frac{\gamma}{\sqrt{\gamma^2 - 1}}\left[22 + \ln\left\{\gamma(\gamma - 1)\left(\gamma^2 - 1\right)\right\} - 1.695\left(\frac{\gamma^2 - 1}{\gamma^2}\right) - \frac{1.39}{\gamma}\right] \tag{27}$$

(See Heitler (1960); Morrison (1961)).

The energy loss rate from bremsstrahlung may be taken as

$$r_B(\gamma) = 7.3 \times 10^{-16} n_e \gamma \tag{28}$$

based on radiation lengths for hydrogen and helium given by Dovzhcnko and Pomanskii (1964).

The loss rate from synchrotron radiation and Compton collisions may be taken as

$$r_{s+c}(\gamma) = 1.3 \times 10^{-9}\left(H^2 + 3 \times 10^{-11} \rho_\gamma\right)\gamma^2 \tag{29}$$

where H is given in gauss and the radiation density, ρ_γ, is given in ev/cm^3 (Ramaty and Lingenfelter (1964)).

The total positron energy-loss rate is taken to be the sum of Eqs. (27) - (29) and is simply denoted by $r(\gamma)$. Equation (23) may then be solved to yield the equilibrium positron density spectrum in the form

$$n(\gamma) = \frac{1}{|r(\gamma)|}\int_\gamma^\infty dz\, q_+(z) \exp\left[-\int_\gamma^z \frac{dw}{|r(w)|\, \tau_s(w)}\right] \tag{30}$$

(Stecker, (1967b)).

Equation (30) will be used to obtain numerical solutions for $n(\gamma)$, the positron equilibrium flux

$$I_+(\gamma) = \frac{c}{4\pi} \frac{n(\gamma)\sqrt{\gamma^2 - 1}}{\gamma} \tag{31}$$

and the resulting AGS of

$$Q(\eta) = 4H_+ \left(\frac{1}{2}\right) n_{e^-} \sigma_0 c \int_{G(\eta)}^{\infty} d\gamma \frac{\phi(\chi_0(\eta, \gamma), \gamma)}{\gamma\sqrt{\gamma^2 - 1}\,|r(\gamma)|}$$

$$\times \int_{\gamma}^{\infty} dz\, q_y(z) \exp\left[-\int_{\gamma}^{z} \frac{dw}{|r(w)|\,\tau_s(w)}\right]. \tag{32}$$

In order to utilize Eqs. (30) - (32) to determine the positron equilibrium flux and AGS in the galaxy, we chose typical average values for the quantities n_e, H, ρ_γ and T_ℓ for both the galactic disc and the galactic halo. These values are given in Table 1. The radiation density, ρ_γ, includes the contribution of 0.25 eV/cm^3 from the 2.7°K universal microwave field (Stokes, Partridge and Wilkinson, 1967)

Using the values given in Table 1 for the galactic halo to numerically integrate Eqs. (30) - (32), typical galactic spectra are obtained for $I_+(\gamma)$ and $Q(\eta)$. The results are given in Figs. (3) and (4) for various values of mean path length X(g/cm^2) and corresponding T_ℓ. The flux of annihilation-gamma-radiation observed at the earth, $I_A(\eta)$, is given by

$$I_A(\eta) = \frac{L_{eff}}{4\pi} Q(\eta) \qquad (33)$$

where L_{eff} is the effective path length for gamma-ray production.

For leakage times less than 10 million years, the equilibrium positron flux has roughly the same characteristics of the source spectrum of Fig. 2 and its magnitude is proportional to the leakage time. In the case of longer leakage times, the positrons are trapped in the galaxy for a sufficient time for the energy loss processes, particularly ionization loss, to affect the spectrum by progressively flattening it below 30 MeV. Studies of spallating cosmic-ray nuclei yield a mean path length, $X = 4 \pm 1$ g/cm^2 for cosmic-ray nuclei, corresponding to a mean leakage time of the order of 100 million years. The curve in Fig. 3 corresponding to $X = 5$ g/cm^2 is in agreement with the cosmic-ray positron measurements of Hartman (1967). It may be noted that measurements of the galactic positron flux below 30 MeV would yield a more sensitive check on the galactic mean leakage time.

Figure 4 shows the annihilation-gamma-ray spectra obtained using the positron equilibrium fluxes of Fig. 3. The spectra shown are from annihilations-in-flight of positrons having energies greater than 5 keV. AGS from positrons annihilating with energies below 5 keV will be discussed later. It can be seen that the peaks of these spectra lie in the region $1/2 < \eta < 1$. This effect is due to a pile-up of those gamma-rays from the annihilation of relativistic positrons which are emitted in the backward direction in the c.m.s.

In Fig. 8, this flux is compared with gamma-ray spectra from neutral pi-meson production from galactic cosmic-ray collisions (Stecker, 1967) and with the positron's own bremsstrahlung spectrum ($I_\pi(E_\gamma)$ and $I_B(E_\gamma)$ respectively). It should be noted that these spectra are rigidly related since the ultimate source of $I_\pi(E_\gamma)$ is the same cosmic-ray collision process generating the positrons and $I_B(E_\gamma)$ is completely determined by $I_+(\gamma)$ and n_e. The total galactic bremsstrahlung spectrum is of course determined by the sum of the cosmic-ray positron and electron fluxes in the galaxy and may be expected to be at least twice as large as the $I_B(\eta)$ flux shown in Fig. 8. We have also shown the expected bremsstrahlung gamma-ray flux from the observed cosmic-ray electron spectrum.

Recent measurements have indicated that for $\eta \leq 2$ the observed isotropic gamma-ray spectrum follows a power law of the form $I_{obs}(\eta) \sim \eta^{-2.3}$ (See review paper by Gould, 1967). An extrapolation of the observed isotropic spectrum is also plotted in Fig. 8 along with the conjectured extrapolation of Shen and Berkey (1968). Various authors have suggested that this flux may be due to Compton collisions between intergalactic positrons and electrons and the universal thermal microwave radiation (Hoyle, 1965; Gould, 1965; Felten, 1965; Fazio, Stecker and Wright, 1966; Felten and Morrison, 1966 or background from external galaxies (Gould and Burbidge, 1963; Silk, 1968). If this is indeed the case, it may place severe restrictions on observations of the galactic AGS.

IV. THE ANNIHILATION-GAMMA-RAY SPECTRUM FROM COSMIC-RAY POSITRONS ANNIHILATING NEAR REST

We now come to the problem of determining the AGS from cosmic-ray positrons annihilating near rest. Because some aspects of this determination seem deceptively simple, there has been a tendency to oversimplify this problem in the literature. For this reason, I will first list various aspects of the problem essential to an accurate analysis before proceding to treat them.

A. In considering annihilations near rest, one must consider the possibility of the intermediate formation of the bound electron-positron system, i.e., the positronium atom. At low energies, the cross-section for positronium formation becomes much greater than the cross-section for free annihilation.

B. Once formed in interstellar space, a positronium atom will annihilate 75% of the time into three photons. This situation contrasts sharply with the case of free annihilations where three-photon annihilations occur with a probability of less than 1/2%. Therefore, the three-photon annihilation process, which produces a continuous spectrum from 0-0.5 MeV, must be considered along with the two-photon line-annihilations. Their relative importance depends directly on the fraction of positrons which ultimately form positronium.

C. The most important source of the cosmic-ray positrons having energies greater than a few MeV is the source we have been considering, the decay of secondary charged pi-mesons. There are, however, sources of relatively low energy positrons (less than a few MeV) which, as we shall see, may have a greater probability of being trapped inside the galaxy until they annihilate near rest. These sources are the beta-emitters, which are produced predominantly by low energy cosmic-ray interactions involving carbon, nitrogen and oxygen. Therefore, any observable 0.5 MeV line radiation from the galaxy may be primarily due to these beta emitters and that an observation of the intensity of the 0.5 MeV line could supply information on the intensity of low-energy cosmic-radiation in the galaxy.

V. GALACTIC POSITRONIUM FORMATION

The cross-section for positronium formation by fast positrons in atomic hydrogen has been calculated by Cheshire (1964) and will be used as an approximation for the interstellar medium. The ratio of positronium formation to free annihilation is only significant at non-relativistic energies and may be approximated by

$$\frac{\sigma_{pos}(\gamma)}{\sigma_A(\gamma)} \equiv S(\gamma) = \begin{cases} 0 & \text{for} \quad (\gamma-1) > 10^{-2} \\ 10^{-6}(\gamma-1)^{-3} & 10^{-4} < (\gamma-1) < 10^{-2} \\ 10^{-2}(\gamma-1)^{-2} & 2.5 \times 10^{-5} < (\gamma-1) < 10^{-4} \\ 6.4 \times 10^{11}(\gamma-1) & 10^{-5} < (\gamma-1) < 2.5 \times 10^{-5} \\ 0 & (\gamma-1) < 10^{-5} \end{cases} \quad (34)$$

At energies of the order of the hydrogen binding energy, the probability for positronium formation may be determined as follows (Deutsch, 1953):

Whereas free electrons and positrons may readily combine to form positronium with a binding energy just half that of hydrogen (the effective Bohr radius being $2a_0$), a positron needs a certain threshold energy in order to break apart an atom so that it may combine with its electron to form positronium. The binding energy of hydrogen is 13.6 eV; the binding energy of positronium is 6.8 eV. If we consider a fast positron impinging on a hydrogen medium and being slowed down by ionizing collisions, and if we assume that the last ionizing collision is equally likely to leave a positron with any kinetic energy between zero and V_i (V_i being thc ionization energy of the medium), then the fraction

$$f_u \equiv \frac{V_i - 6.8}{V_i} \tag{35}$$

of positrons may form positronium.

We must also take into account the possibility of inelastic collisions rapidly reducing the energy of the positron, not through ionization of the atom but through excitation. Therefore, if the first excitation level of the atom is V_1, and if $V_1 > (V_i - 6.8)$, then the fraction

$$f_\ell \equiv \frac{V_1 + 6.8 - V_i}{V_i} \tag{36}$$

should be a lower limit for the probability of positron formation. For a hydrogen medium

$$V_i = 13.6 \text{ eV} \qquad \text{and} \qquad V_1 = 10.2 \text{ eV}, \tag{37}$$

and therefore the probability of positronium formation, P_{Π}, is

$$0.25 < P_{\Pi} < 0.50 \ . \tag{38}$$

For practical purposes, we may therefore use

$$P_{\Pi} = \frac{3}{8} \ . \tag{39}$$

By a similar argument, we can show that the probability of the formation of positronium in its first excited state (n = 2, binding energy of only 1.7 eV) is quite small.

VI. GAMMA-RAY SPECTRA FROM TWO- AND THREE-PHOTON ANNIHILATIONS AT REST

The cross-section for free annihilation into ζ photons, i.e., for the process

$$e^+ - e^- \rightarrow \zeta\gamma \ , \tag{40}$$

is of the order

$$\sigma_{A,\zeta\gamma} \sim \alpha^{(\zeta-2)} \sigma_{A,2\gamma} \ , \tag{41}$$

where

$$\alpha = \frac{e^2}{\hbar c} \simeq \frac{1}{137}^{*}. \tag{42}$$

Since we have seen that the positronium formation process should also be considered, we must also consider reactions of the type

$$e^+ + e^- \rightarrow \Pi \;\; \llcorner\!\!\rightarrow \zeta\gamma \tag{43}$$

where the capital pi symbol will stand for positronium. Here again, we can neglect processes involving $\zeta > 3$.

The positronium annihilations, unlike the free annihilations, obey selection rules since positronium is in an eigenstate of charge conjugation C, and C is conserved in electromagnetic interactions.

If ζ photons are produced in the final state, then

$$C = (-1)^{\zeta} . \tag{44}$$

It can be shown that under particle exchange, positronium obeys a kind of generalized Pauli principle and changes sign. The exchange of particles involves

*Positrons may annihilate with bound electrons producing a single photon, with momentum being conserved by the binding nucleus. However, the cross section for this process is always less than $\pi r_0^2 Z^5/(137)^4$, where the atomic number Z is almost always 1 or 2 under astrophysical conditions. Therefore, the single-photon annihilation can be neglected in the following discussion (Heitler, 1960).

the exchange of the product of the space, spin, and charge-conjugation parts of the wave function. Therefore,

$$(-1)^{\ell}(-1)^{S+1}C = -1 , \tag{45}$$

where ℓ is the orbital angular-momentum quantum number and S is the spin quantum number. By combining Eqs. (44) and (45), we obtain the selection rule

$$\ell + S = \zeta . \tag{46}$$

We may therefore specify the processes given by Eq. (43) as

$$e^{+} + e^{-} \rightarrow \Pi(^{1}S_{0}) \rightarrow 2\gamma . \tag{47}$$

and

$$e^{+} + e^{-} \rightarrow \Pi(^{3}S_{1}) \rightarrow 3\gamma . \tag{48}$$

Processes involving $\ell \neq 0$ can be neglected (Deutsch, 1953).

The lifetime for positronium annihilation into two gamma-rays is given by

$$\begin{aligned} \tau_{2\gamma}^{-1} &= 4\sigma_{A,2\gamma}\, v|\psi(0)|^{2} , \\ &= 4\pi r_{0}^{2}\,\frac{c}{v}\,v\,\frac{1}{\pi}\left(\frac{1}{2a_{0}n}\right)^{3} , \\ &= \frac{r_{0}^{2}c}{2a_{0}^{3}n^{3}} = \frac{\alpha^{6}c}{2r_{0}n^{3}} , \end{aligned} \tag{49}$$

or

$$\tau_{^1S_0 \to 2\gamma} = 1.25 \times 10^{-10} n^3 \text{ sec} ,$$

where n is the principal quantum number of the positronium state, a_0 is the Bohr radius given by

$$a_0 = \alpha^{-2} r_0 , \tag{50}$$

v is the electron velocity, and $|\psi(0)|$ is the absolute magnitude of the wave function at the origin of the system. The effective Bohr radius for positronium is twice that of hydrogen, and this is taken into account. The factor of 4 in Eq. (49) takes into account the fact that for the two-photon decay of positronium, we know we are concerned with the "singlet" state, whereas in the case of free annihilation the probability of a singlet interaction (spins antiparallel) is 1/4 and the probability of a triplet interaction (spins parallel) is 3/4. The factor $|\psi(0)|^2$ is just the effective electron density seen by the positron.

The lifetime for positronium annihilation into three gamma rays is given by (Ore and Powell, 1949)

$$\tau^{-1}_{^3S_1 \to 3\gamma} = \frac{2}{9\pi}\left(\pi^2 - 9\right) \frac{\alpha^7 c}{r_0 n^3} \tag{51}$$

or

$$\tau_{^3S_1 \to 3\gamma} = 1.4 \times 10^{-7} n^3 \text{ sec} . \tag{52}$$

The lifetimes for both the two- and three-photon annihilations of positronium are therefore so short that for considerations of galactic gamma-ray production, we may consider the annihilation to take place effectively at the time that positronium is formed. Therefore, we conclude that 3/4 of the positronium formed in the galaxy annihilates into three-photons.

The energy spectrum of the two-photon annihilation in the center-of-momentum system (c.m.s.) of the electron-positron pair is single line at $E_\gamma \simeq 0.51$ MeV ($\eta = 1$), as can easily be seen from considerations of conservation of energy and momentum. The natural width of this line due to the uncertainty principle is small, being on the order of

$$\Delta E = \frac{\hbar}{\tau} \simeq 5.3 \times 10^{-12} \text{ MeV} . \tag{53}$$

Dominant broadening can be expected to be due to the Doppler effect, and is of the order

$$\frac{\Delta E}{E} = \frac{\Delta E}{Mc^2} = \beta . \tag{54}$$

The effect of astrophysical conditions on the broadening of the 0.51-MeV line from two-photon annihilation can be determined as follows:

In free $e^+ - e^-$ annihilations, we may consider a gas or plasma at temperature T. Then the distribution of the component of particle velocity along the line of sight of the observed gamma ray is of the form

$$f\left(\beta_{\|}\right) d\beta_{\|} = \left(\frac{b}{\pi}\right)^{1/2} e^{-b\beta_{\|}^{2}} d\beta_{\|} \; , \tag{55}$$

where

$$b \equiv \frac{Mc^2}{2kT} \; . \tag{56}$$

It then follows from Eqs. (23) and (36) that the spectral-line shape has the Gaussian form

$$f(\eta)\, d\eta = \sqrt{\frac{b}{\pi}}\, e^{-b\left[(\eta-1)/\eta\right]^2} d\eta \tag{57}$$

Equation (57) has the same form as Eq. (55), since the number of collisions involving velocity v is proportional to $\sigma v \simeq \sigma_0 c$ independent of v.

The order of magnitude of the broadening is of the order of

$$\Delta\eta = \left(\frac{\Delta E_\gamma}{M}\right) \simeq b^{-1/2} \simeq 1.8 \times 10^{-5}\, T^{1/2} \; , \tag{58}$$

so that for T = 100°K, $\Delta E \simeq 10^{-1}$ keV. The Doppler width for two-photon positronium annihilation will be discussed later.

The energy spectrum from the three-photon annihilation is continuous, as allowed by conservation of momentum. It has been calculated by Ore and

Powell (1949) to be of the form

$$F(\eta) = \frac{2}{\pi^2 - 9}\left[\frac{\eta(1-\eta)}{(2-\eta)^2} - \frac{2(1-\eta)^2}{(2-\eta)^3}\ \ell n(1-\eta) + \frac{2-\eta}{\eta} + \frac{2(1-\eta)}{\eta^2}\ \ell n(1-\eta)\right], \quad (59)$$

The function $F(\eta)$ is shown in Fig. 5; $F(\eta)$ is normalized so that

$$\int_0^1 F(\eta)\, d\eta = 1 . \quad (60)$$

VII. THE NUMBER OF COSMIC-RAY POSITRONS ANNIHILATING NEAR REST

We now turn our discussion to the problem of the number of cosmic-ray positrons annihilating near rest. These positrons most likely come from two source:

A) Positrons from the decay of secondary charged pi-mesons which were created at low enough energies to be trapped for a sufficiently long time in the galaxy to lose essentially all their energy before either annihilating in flight or escaping from the galaxy.

B) Positrons from the decay of beta-emitting nuclei formed in collisions of low-energy cosmic-rays involving nuclei of carbon, nitrogen and oxygen.

The fraction of the original positron flux from the decay of secondary pi-mesons which annihilate near rest, f_+, is given by

$$f_+ = \frac{\int_1^\infty dz\, q_+(z) \exp\left[-\int_1^z \frac{dw}{|r(w)|\, \tau_s(w)}\right]}{\int_1^\infty dz\, q_+(z)} \qquad (61)$$

This fraction was calculated numerically using Eqs. (24) - (29) and is given in Table 2 and Fig. 6 for various possible mean leakage times, T_ℓ. Table 3 gives the corresponding values of $Q_{rest,\pi}$, the total number of positrons from pi-meson decay per cm^3 per second annihilating below 5 keV for the halo model of Table 1 ($Q_{rest,\pi}$ being defined by the numerator of Eq. (61) and the resulting gamma-ray fluxes).

In the case of an infinite leakage time (all positrons being trapped and annihilating in the galaxy) we find that 80% of the positrons produced annihilate near rest, a figure which is in perfect agreement with that given by Heitler (1960) as an asymptotic value for the annihilation of ultrarelativistic positrons when the dominant energy loss comes from ionization. However, for the leakage time usually considered as plausible for the galactic halo, 10^8 years (corresponding to $\chi \simeq 5$ gm/cm^2), only 1-2% of these positrons annihilate near rest.

We next consider the positrons produced in the decay of beta-emitting nuclei produced in low-energy cosmic-ray collisions. The important reactions to be considered are listed in Table 3 (Audouze, et al., 1967). Whereas the majority of pi-mesons are produced by cosmic-rays having energies over 500 MeV, beta-emitting nuclei can be produced by cosmic-rays with energies down to below 20 MeV. The positrons produced have energies less than 5 MeV with the singular exception of those produced from the decay of N^{12}. Using the cross-section data given in Table 3, and the quiet-sun observations of the cosmic-ray spectrum from 20-1000 MeV/nucleon (Comstock, et. al., 1966). We obtain the value for the fluxes given in Table 4. This value probably represents a lower limit to the true astrophysical value since over 95% of these beta-ray positrons will remain trapped in the galaxy and annihilate near rest. For mean leakage times less than about 400 million years most of the positrons annihilating near rest come from the beta-emitting unstable nuclei produced in cosmic-ray p-C,N,O interactions.

Equation (34) may now be used to determine the amount of positronium being formed by positrons combining with electrons as a function of energy. The result of this calculation is shown in Fig. 7. This figure shows the percentage of positrons which, after having survived to reach a 10 keV kinetic energy, survive to reach lower energies. The dashed line indicates the survival fraction for free annihilation only; the solid line indicates the survival fraction when positronium formation is taken into account. Figure 5 shows that almost all of the positrons annihilating near rest do so through intermediate positronium formation with an average energy of about 35 keV. Therefore 75% of the resultant annihilations occur via the three-photon channel and 25% produce a two-photon line with a Doppler width of about 5 keV.

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Table 1

Average Astrophysical Parameters for the Galactic Disc and Halo.

	n_e (cm^{-3})	H(μg)	ρ_γ(eV/cm^3)	T_ℓ (10^6 yrs.)	L_{eff} (cm)
Disc	0.5–1	6	0.45	2–4	3×10^{22}
Halo	$1\text{–}3 \times 10^{-2}$	3	0.65	100–300	5×10^{22}

Table 2

Fraction of Positrons from π^+-decay which Survive to

Annihilate Near Rest ($T_+ \lesssim 5$ keV).

T_ℓ (10^6 yrs.)	f_+ (%)
∞	80
600	20
300	9.3
100	1.6
60	0.61
30	0.14
10	0.022
6	0.0065
3	0.0025

Table 3

Principal Reactions Leading to Production of β-Emitting Nuclei

(See Audouze, et al. (1967)).

Reaction	σ (mb)	Decay	Halflife	Positron Energy (MeV)
$C^{12}(p,3p2n)B^8$	?	$B^8(\beta^+)Be^8$	0.78 sec	1.4
$C^{12}(p,p2n)C^{10}$	~3	$C^{10}(\beta^+)B^{10}$	19 sec	1.9
$N^{14}(p,2p3n)C^{10}$	?			
$O^{16}(p,3p,4n)C^{10}$	<10			
$C^{12}(p,pn)C^{11}$	~70	$C^{11}(\beta^+)B^{11}$	20.5 min	0.97
$N^{14}(p,2p2n)C^{11}$	~30			
$O^{16}(p,3p3n)C^{11}$	~10			
$C^{12}(p,n)N^{12}$	?	$N^{12}(\beta^+)C^{12}$	0.011 sec	16.4
$N^{14}(p,p2n)N^{12}$	?			
$O^{16}(p,2p3n)N^{12}$	?			
$N^{14}(p,pn)N^{13}$	~15	$N^{13}(\beta^+)C^{13}$	10.0 min	1.19
$O^{16}(p,2p2n)N^{13}$	~10			
$N^{14}(p,n)O^{14}$	~50*	$O^{14}(\beta^+)N^{14}$	71 sec	1.18,4.14
$O^{16}(p,p2n)O^{14}$	<10			
$O^{16}(p,pn)O^{15}$	~50	$O^{15}(\beta^+)N^{15}$	2.06 min	1.74

? - Unknown and not included in estimate of positron production.

*This cross section has a value of about 100 mb at energies below 12 MeV but is negligible above 150 MeV.

Table 4

Positron Annihilation Rate and Gamma-Ray Flux from

Positrons Annihilating Near Rest.

Source		$Q_{T,rest}$ (cm^{-3} sec^{-1})	Flux* (2γ-line) (cm^{-2} sec^{-1} sr^{-1})	Flux* (3γ-continuum) (cm^{-2} sec^{-1} sr^{-1})
π^+	T_ℓ (10^6 yrs.)			
	∞	1.4×10^{-27}	2×10^{-5}	1×10^{-4}
	600	3.5×10^{-28}	5×10^{-6}	2.5×10^{-5}
	300	1.6×10^{-28}	2.5×10^{-6}	1×10^{-5}
	100	3×10^{-29}	5×10^{-7}	2×10^{-6}
	60	1×10^{-29}	1.5×10^{-7}	7×10^{-7}
	30	2.5×10^{-30}	4×10^{-8}	2×10^{-7}
	10	3.8×10^{-31}	6×10^{-9}	2.5×10^{-8}
	6	1.1×10^{-31}	1.5×10^{-9}	7.5×10^{-9}
	3	4.4×10^{-32}	7×10^{-10}	3×10^{-9}
p-C,N,O		2.5×10^{-28}	4×10^{-6}	1.6×10^{-5}

*Assuming $\langle nL_{eff} \rangle = 3 \times 10^{21}$ cm^{-2}.

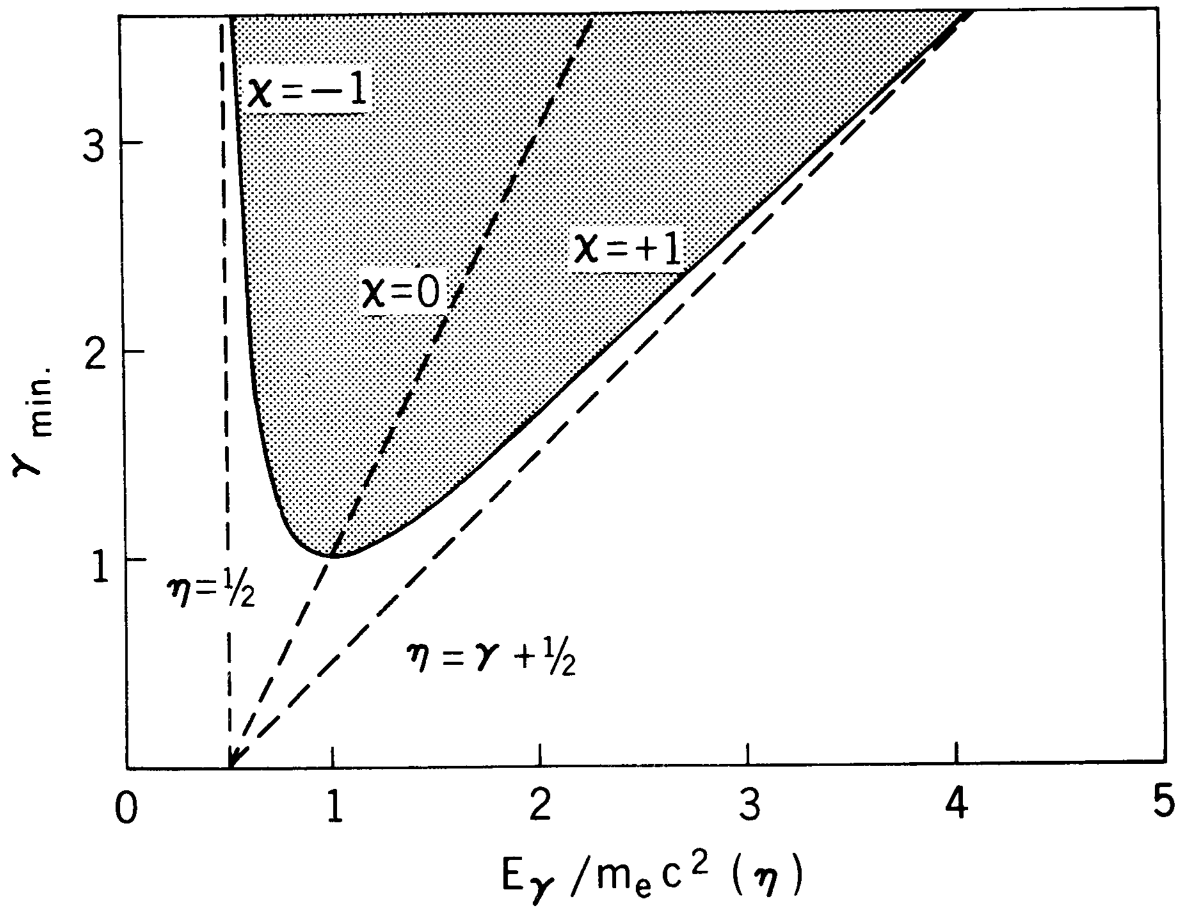

Figure 1. The kinematic limits on the positron Lorentz factor involved in the determination of the laboratory-energies of the annihilation-gamma-rays produced.

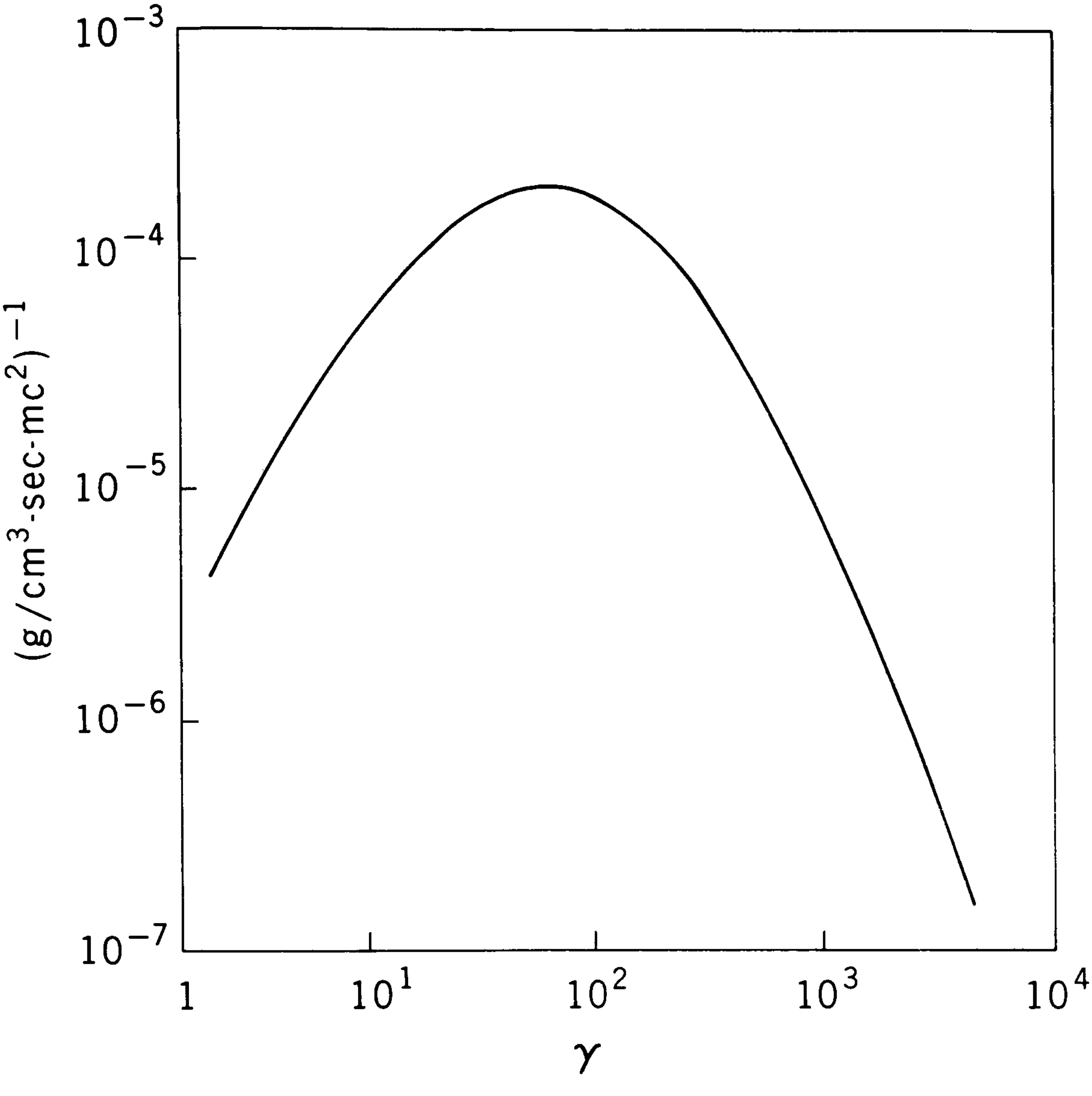

Figure 2. The source spectrum of positrons produced from the decay of the positive pi-mesons formed in cosmic-ray collisions (Ramaty and Lingenfelter; 1967).

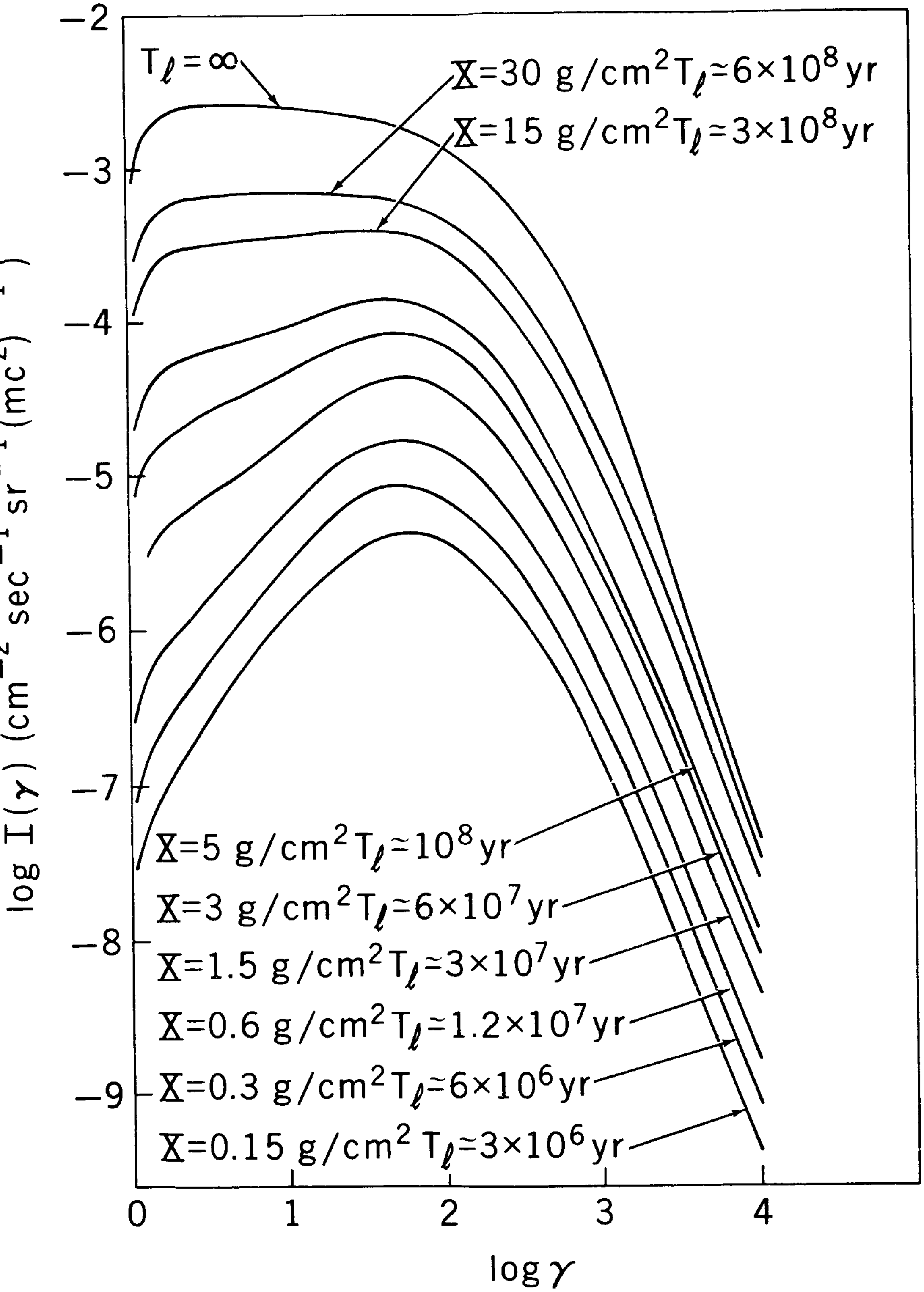

Figure 3. Various positron equilibrium fluxes for the halo model of the galaxy given for various approximate mean path lengths and mean leakage times.

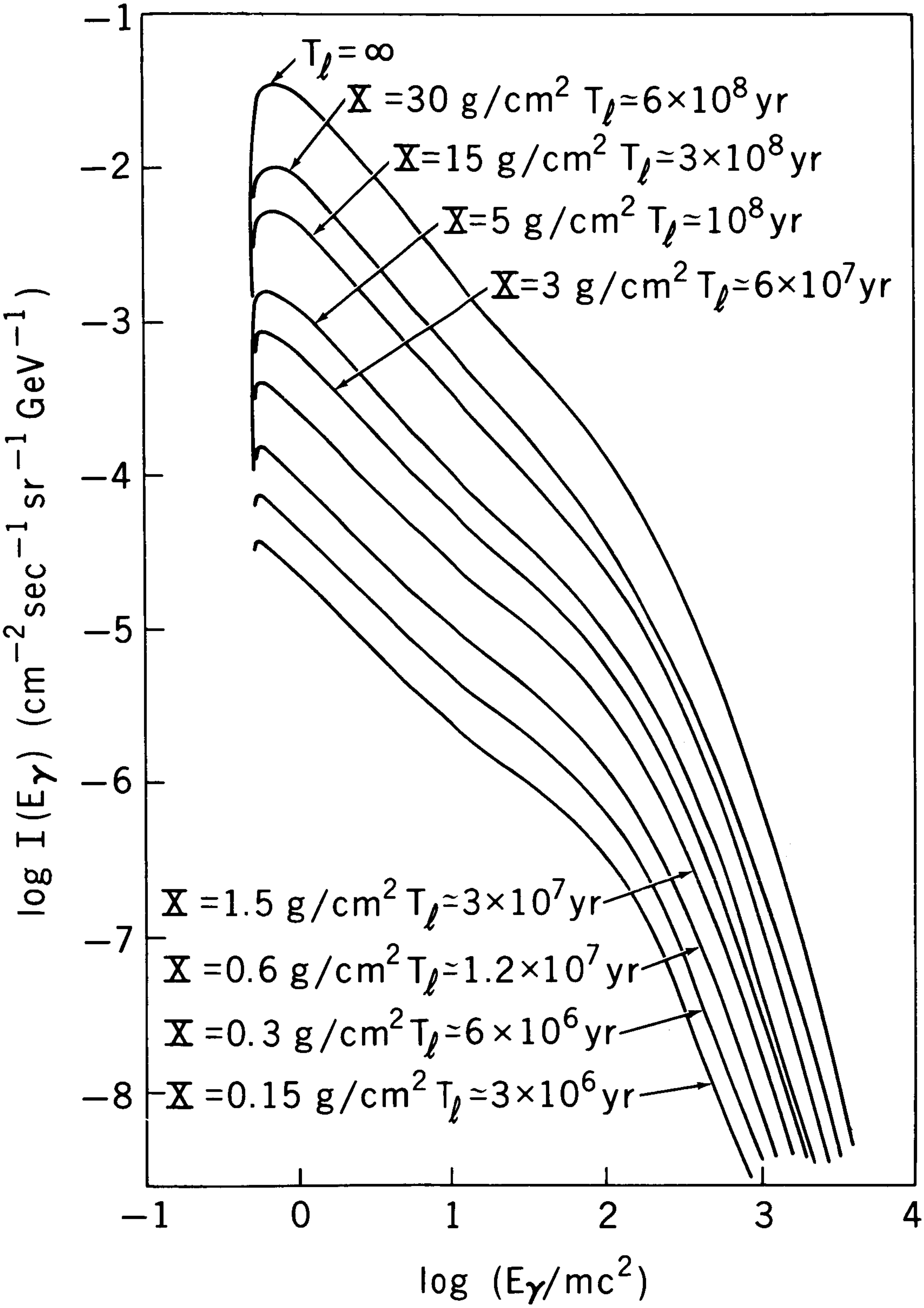

Figure 4. The annihilation-gamma-ray flux spectra from the positron equilibrium fluxes given in Fig. 3 resulting from annihilations-in-flight of positrons having energies greater than 5 keV.

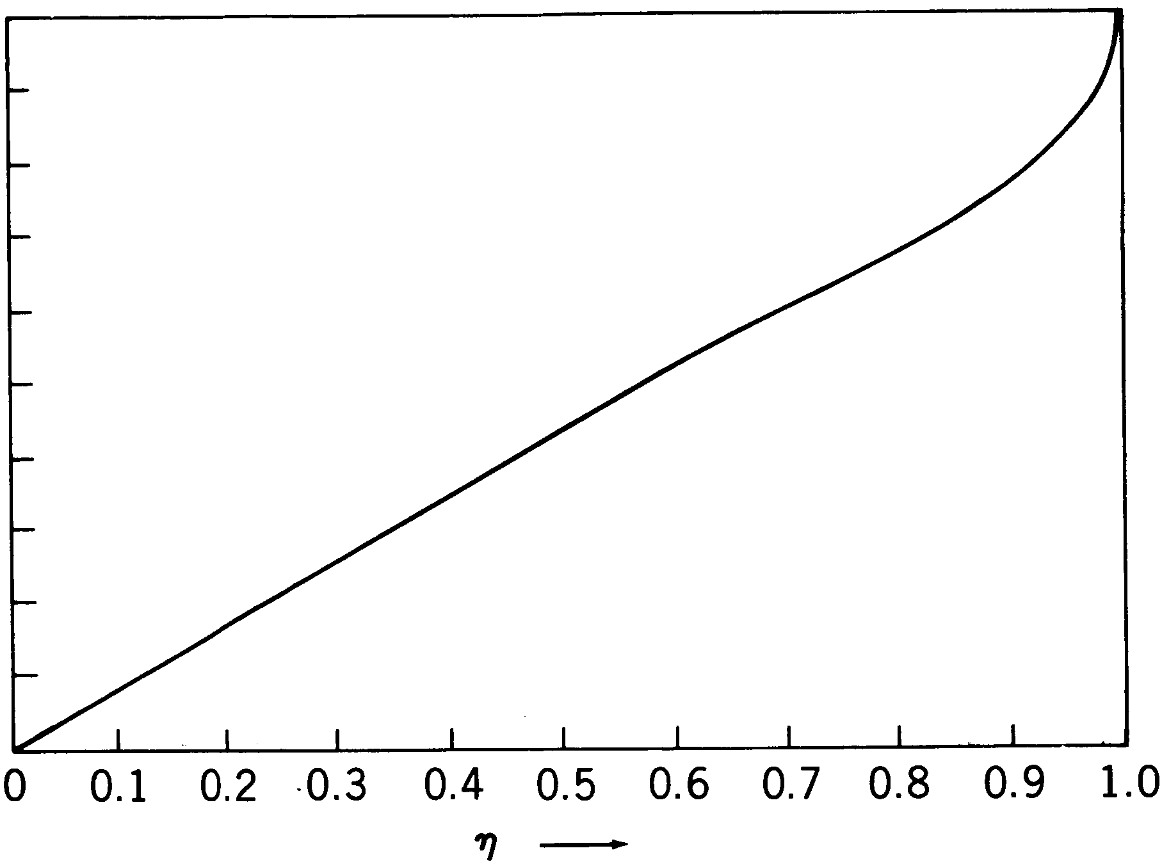

Figure 5. Energy spectrum of gamma rays resulting from the three-photon annihilation of an electron and a positron (from Ore and Powell, 1949).

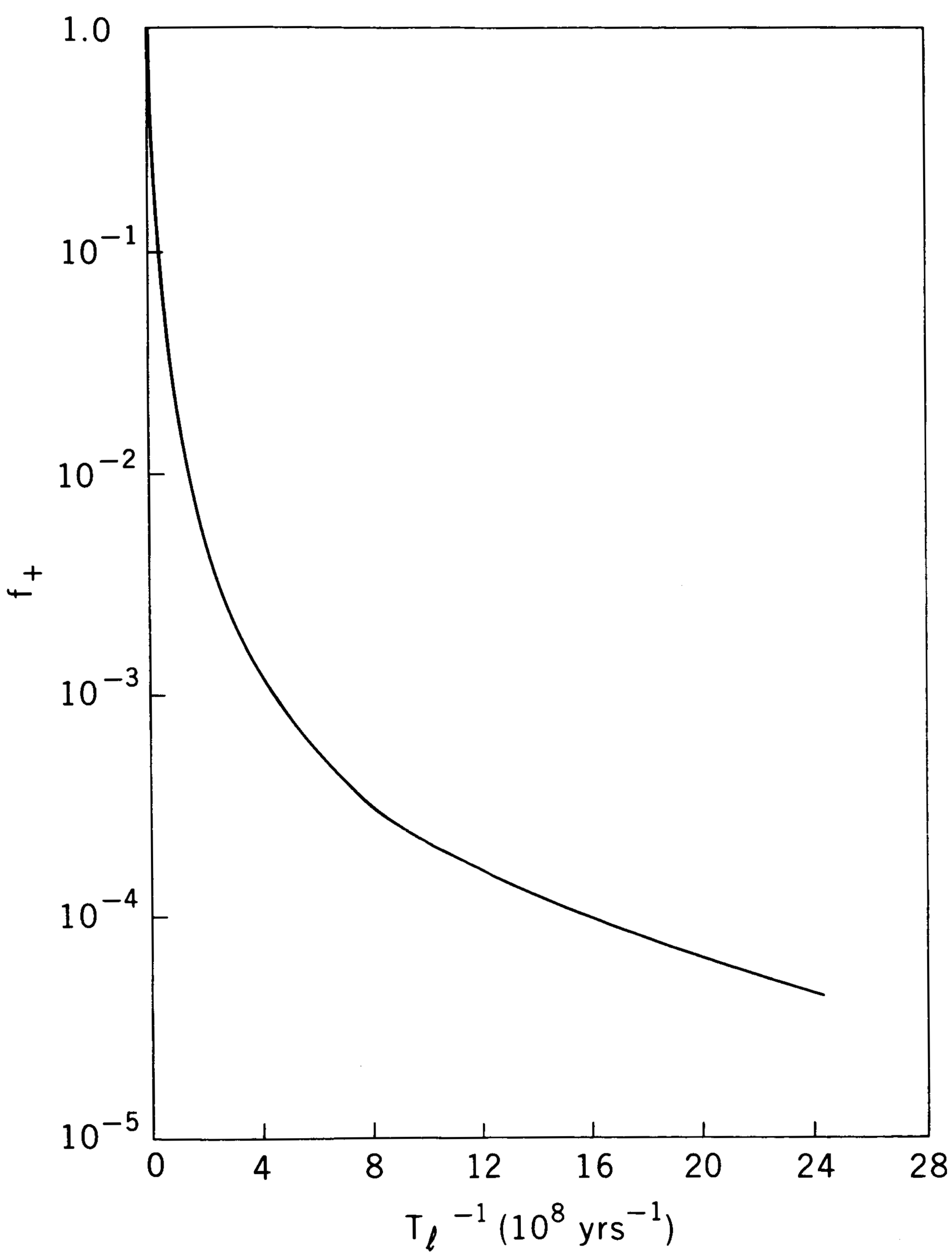

Figure 6. The fraction of pi-meson produced positrons which annihilate at energies less than 5 keV as a function of inverse-mean-leakage-time.

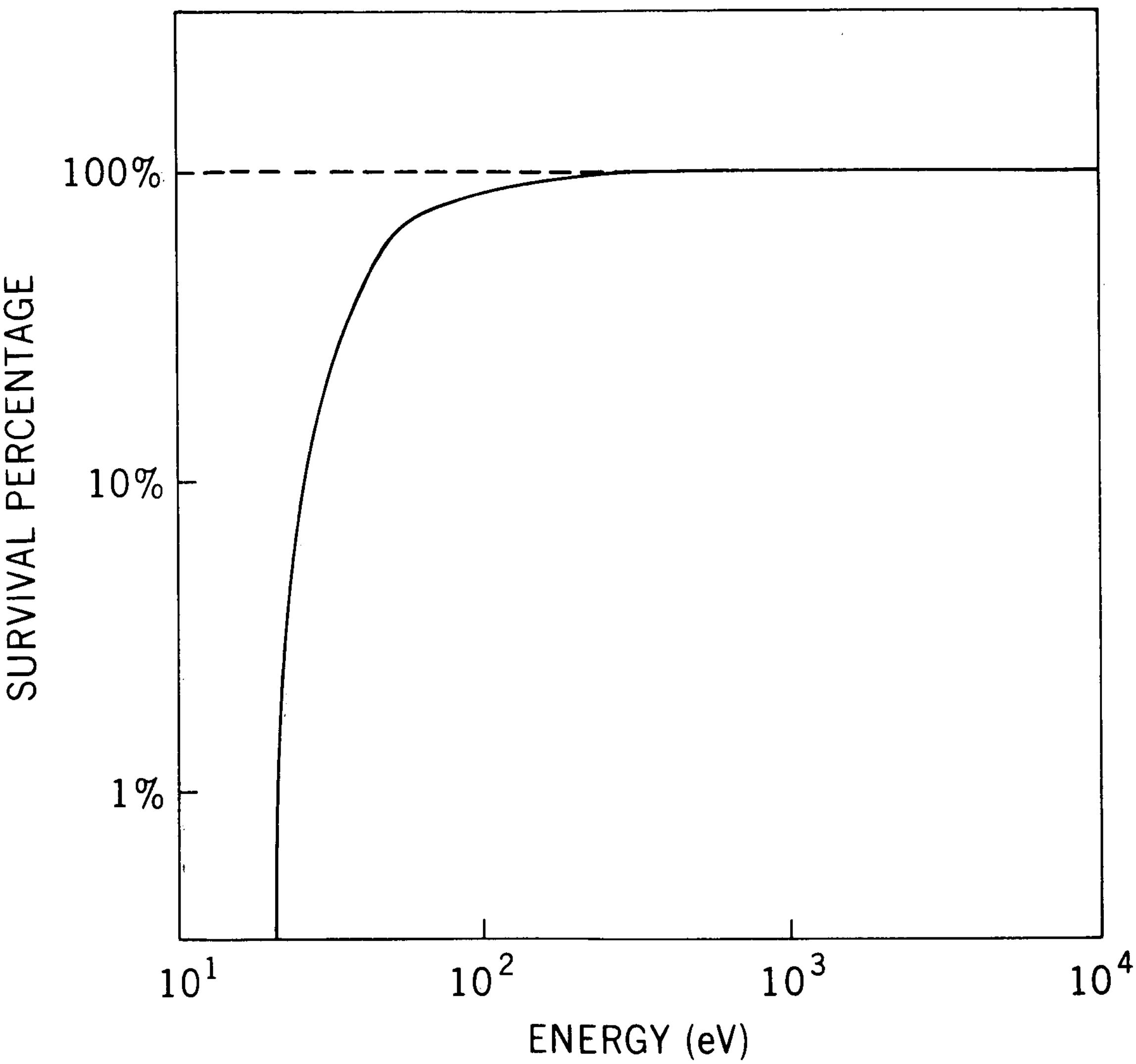

Figure 7. The percentage of positrons which, after having survived to reach an energy of 10 keV, survive to reach lower energies. The dashed line indicates the survival fraction found by taking into account free annihilation only; the solid line indicates the survival fraction when positronium formation is also taken into account.

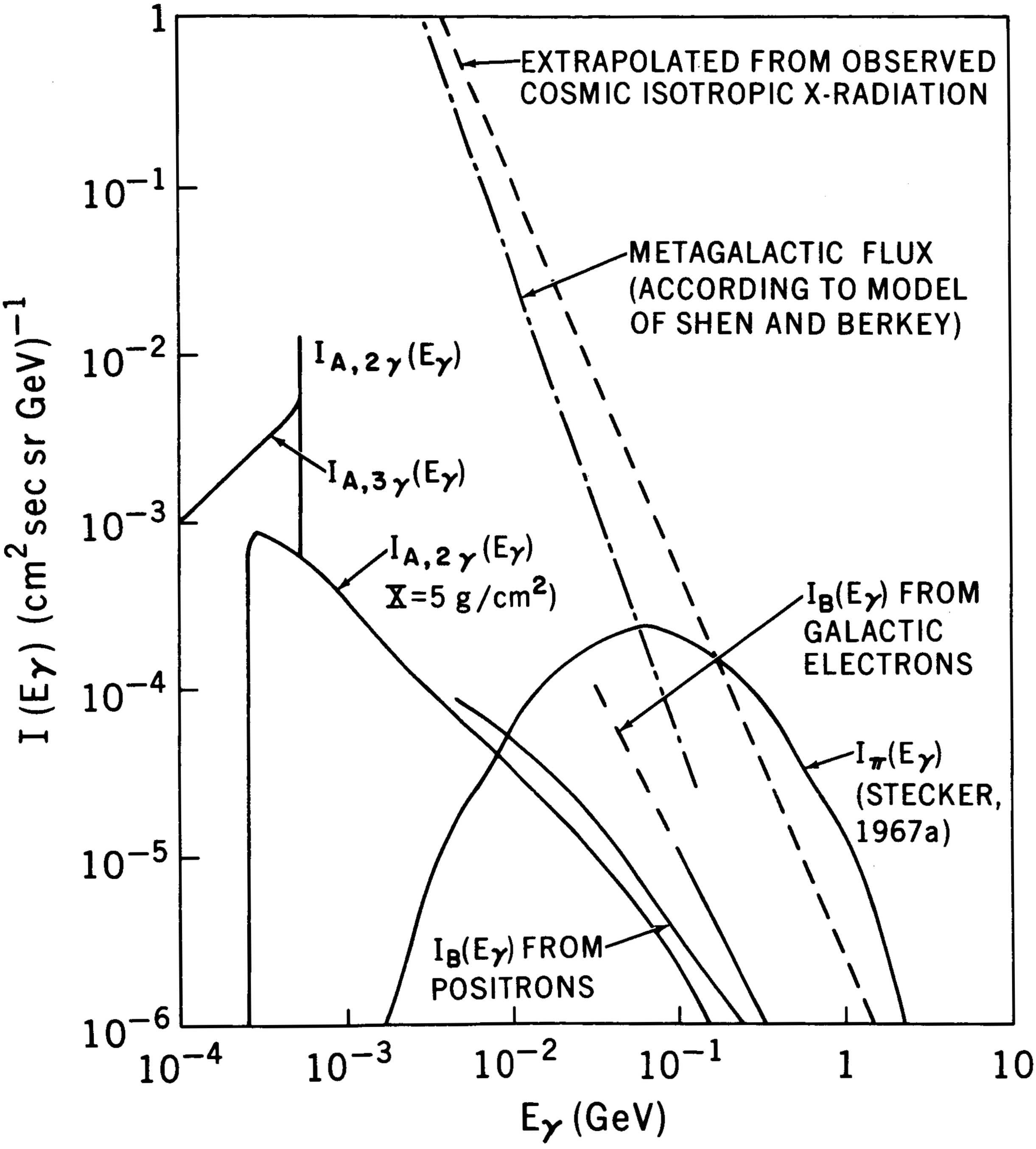

Figure 8. Isotropic gamma-ray spectra from various astrophysical processes.