DESCRIPTION OF AN ANALYTIC METHOD FOR THE DETERMINATION OF SPACECRAFT WINDOW-INDUCED NAVIGATION SIGHTING ERRORS

by

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ABSTRACT

Navigation measurements made through a spacecraft window with an optical instrument are subject to window-induced errors. The primary sources of these errors are nonflat window surfaces, nonparallel window surfaces, and window surfaces deformed due to pressure loading. Another source of measurement error is the index of refraction difference between the air inside the spacecraft and the space vacuum outside. An analytic method for the determination of these window-induced errors has been developed. Given the spacecraft and window configuration: (1) Window surface pressure deformations are determined either analytically or experimentally; (2) Window surface anomalies of manufacture are obtained experimentally; (3) Pressure deformations and anomalies of manufacture are used to obtain a math model of the window surface shapes; and (4) Light rays are traced through the window to determine the errors. The method has been applied to the analysis of errors associated with a Gemini window and the results compared with experimental data. The results agree quite well indicating the analytic method does permit accurate prediction of window-induced navigation measurement errors. The method appears to be applicable to the analysis of errors associated with other spacecraft or aircraft windows.

INTRODUCTION

On-board navigation systems for manned interplanetary or translunar flight may rely on navigation measurements made with an optical instrument through a spacecraft window. For example, the hand-held space sextant has been proposed for use in a back-up navigation system for the midcourse, Earth-orbital, and orbit rendezvous phases of manned interplanetary or translunar flight. The navigation measurement of interest here is the angle between two celestial bodies such as a star and a planet for the midcourse phase shown in Figure 1, and between a star and the target spacecraft in the rendezvous. These angular measurements must necessarily be accurate to within a few seconds of arc to insure mission success. However, navigation measurements made through a spacecraft window are subject to errors because deviations in the line of sight are induced by the window.

The effect of these navigation errors on a particular space mission is illustrated in Figure 2. The arrival error in perigee altitude is shown as a function of random navigation sighting error for a vehicle returning from an interplanetary Mars mission with Venus swingby. Navigation measurements are made with a sextant. Data are presented for sightings corrected for window-induced errors (indicated by the lower boundary of the shaded area) and for sightings uncorrected for window-induced errors (indicated by the shaded area). The upper boundary of the shaded area corresponds to a window-induced error of 15 arcsec which can be expected for a particular window. Sightings uncorrected for the window-induced error result in altitude errors at perigee arrival of up to 5 km greater than those for sightings corrected for window-induced errors. Thus, the window-induced sighting error significantly reduces the probability of arriving in a safe entry corridor, and thereby reduces the probability of mission success. For example, the probability of reaching a safe entry corridor with a random sighting error of 20 arcsec is reduced by about 16 percent if the sightings are not corrected for a 15 arcsec window error. It is imperative then to know these window-induced errors to within a few seconds of arc and to compensate for them when making navigation sightings. Window-induced line-of-sight deviations could be determined experimentally. In fact, deviations associated with a particular window have been measured experimentally at Ames for a limited number of conditions (Ref. 1). However, an experimental study to obtain accurate detailed mapping of line-of-sight deviations associated with any given window would be a huge task. Furthermore, a generalized experimental study of the line-of-sight deviations associated with spacecraft windows of various sizes and shapes would be even more costly.

An alternate approach initiated by Ames was a theoretical study of spacecraft window-induced line-of-sight deviations. The goal of the study was to develop an analytic method that would permit deviations associated with any type of spacecraft window to be predicted accurately to within a few seconds of arc. The objectives of this paper are to: (1) discuss the sources of window-induced line-of-sight deviations, (2) describe an analytic method for computing these deviations, (3) apply the method to the analysis of line-of-sight deviations associated with a particular spacecraft window, as an example, and (4) illustrate the accuracy of the analytic method by comparison with experimental data.

SOURCES OF LINE-OF-SIGHT DEVIATIONS

The deviation of a line of sight is illustrated in Figure 3 by examining Snell's Law or the Law of Refraction. In words, Snell's Law states that the index of refraction n_1 of medium 1 times the sine of the angle θ_1 which the ray makes with the normal to the interface between two mediums is equal to the index of refraction n_2 of medium 2 times the sine of the angle θ_2 which the ray makes with the normal to the interface in medium 2. In equation form, $n_1 \sin \theta_1 = n_2 \sin \theta_2$. If n_1 and n_2 are different, the light ray will refract since θ_1 and θ_2 must be different to satisfy Snell's Law as shown in the left-hand sketch of Figure 3. The source of this type of deviation in the case of a spacecraft, for example, is the index of refraction difference between the air inside the spacecraft and the space vacuum outside. The window is also a source of deviation if the ray intersects the window faces at points where the normals to the faces are different as shown in the right-hand sketch of Figure 3. Here even if n_1 and n_2 are equal,

θ_1 and θ_2 are still not equal because the normals are different. The primary sources of this type of deviation are nonflat window surfaces, nonparallel window surfaces, and window surfaces deformed by a pressure differential across the windowpane.

DESCRIPTION OF ANALYTIC METHOD

Line-of-sight deviations are determined analytically by tracing the line of sight through the window and comparing the line of sight outside the window with that inside. This ray trace analysis must necessarily be very accurate for line-of-sight deviations to be predicted to within seconds of arc. A precise mathematical model of the window surface shapes is required for the ray trace analysis. The window surface shapes are determined from accurate knowledge of window surface anomalies which are a result of the manufacturing process and window surface deformations resulting from pressure differentials across the window. Surface anomalies from the manufacturing process are determined experimentally, whereas the pressure deformations are determined either analytically or experimentally. An analytic method for accurately predicting window-induced line-of-sight deviations requires an accurate ray trace program and precise knowledge of window surface shapes. The method developed for determining line-of-sight deviations induced by spacecraft windows is illustrated by the flow diagram in Figure 4. The most important step, the math modeling, requires accurate measurements of the window surface anomalies and a knowledge of the window surface deformations under pressure. The spacecraft internal and external pressure and temperature cause structural deformations. In this study, window deformations associated with temperature differentials across the window were not considered since preliminary calculations indicated they were negligible in comparison with pressure deformations. If the window edge conditions are known, a structural analysis digital computer program (Ref. 2) is used to determine pressure deformations. Required inputs to the structural analysis program are the pressure differential across the window, window shape and thickness, edge conditions, and physical properties of the window material.

If the window edge conditions are not known, the window surface pressure deformations are determined by analyzing interference photographs of the window. The experimental apparatus is also shown in Figure 4. An actual window in its frames is mounted in a pressure tank to simulate the actual in-flight pressure environment. The test equipment, an interferometer, consisting of a gas laser, mirrors, and other related instrumentation, is placed on a granite table. A typical interference photograph and the technique used to determine deformations from it will be discussed later in this paper. Both the experimental and analytical procedures determine window surface pressure deformations at discrete points. These deformations are then input to a program that provides a math model of the surface shapes. The window anomalies, the nonflatness of window surfaces and the angle between the nonparallel surfaces of a window pane, are obtained from interference photos of the windows.

In the math modeling program the deformed window surface is described by a function approximation technique. The output of the math modeling program is the matrix of coefficients used to describe the window surfaces in the digital-computer ray-trace program.

The digital computer program traces a line of sight (i.e., light ray) through the spacecraft window to determine line-of-sight deviations. Program inputs are the matrices of coefficients used to describe the window surfaces, the index of refraction of each medium through which the line of sight passes, the number of windowpanes, the distances between the panes, and the thickness of panes.

To summarize, the analytic method for determining line-of-sight deviations is as follows: Given the spacecraft and window configuration,

(1) Window surface pressure deformations are determined either analytically or experimentally;

(2) Window surface anomalies of manufacture are obtained experimentally;

(3) Pressure deformations and anomalies of manufacture are used to obtain a math model of the window surface shapes; and

(4) Light rays corresponding to lines of sight are traced through the window to determine line-of-sight deviations.

At this point, some of the major elements of the analytic method will be discussed in more detail. Figure 5 shows interference photographs obtained experimentally. The photograph at the top is typical of those used to determine window surface nonflatness as a result of manufacture. (The small circles are not relevant to the discussion and should be ignored.) The magnitude of the deviation from flat is a function of the number of fringe lines that cross a particular straight line drawn on the interference pattern. The angle between the nonparallel surfaces of a windowpane is obtained from the spacing of the fringes rather than their straightness. The bottom photograph in Figure 5 represents a Gemini window subjected to a pressure load. The window surface pressure deformations are determined by analyzing a fringe pattern of this type. The large increase in the number of fringes when the pane is subjected to pressure indicates the pane is appreciably deformed. The ring near the center of the fringe pattern corresponds to the maximum deformation, and the outer rings represent contours of constant deformation or elevation. This fringe pattern is similar to a ground contour map where the change in elevation between contours is 11 millionths of an inch. The pressure deformations are obtained by counting the fringes as a function of position relative to the fringe near the center. Counting fringes near the edges is tedious because the fringes are very close together and difficult to discern; thus good magnification and photo contrast are very important. Despite these difficulties, photographs like these have been used successfully.

As previously indicated, an alternate method of determining the window surface pressure deformations is a structural analysis program which is used if the window-edge conditions are known. Its accuracy is illustrated in Figure 6. The pressure deformations obtained by this method for an elliptic window (dashed line) are compared with deformations computed from the known closed-form solution for the same window (solid line). The deformations are given as a function of position on a vertical axis as indicated by the sketch in the figure. The results agree to better than 8 hundred-thousandths of an inch; the slopes of the deformed window surfaces are predicted to better than a second of arc. The structural analysis program should provide equally good prediction of deformations for window shapes other than ellipses if the window edge conditions are known.

After the window surface pressure deformations and anomalies of manufacture are obtained, they are described mathematically for selected window areas by the math-modeling program. The math-modeling program uses a technique that describes the deformations and anomalies as a function of deflections from a reference plane. The function used is a least squares fit which described the deflections from the reference plane in terms of a fourth-order polynomial. (Any technique which provides an accurate math model of the surface shapes could be used.) As previously noted, the math models obtained are then input to the digital computer ray trace program in the form of matrix coefficients.

The ray trace program is specifically designed to trace light rays corresponding to lines of sight through a spacecraft window system. The program permits consideration of any type of window with no restriction on size, shape, material, or number of panes. The necessary equations are written in vector matrix form for mathematical convenience and ease in computation. The program computes single line-of-sight deviations which can be easily combined to compute errors for a two-line-of-sight instrument such as the sextant. The ray trace program differs from other ray trace programs used for purposes of lens design. These programs usually require rotationally symmetric lens systems whereas our program is designed to permit study of spacecraft window systems that are not, in general, rotationally symmetric.

APPLICATION OF METHOD TO GEMINI WINDOW ANALYSIS

In order to illustrate the ability of the analytical method to accurately predict window-induced line-of-sight deviations, the method was used to compute the deviations associated with a Gemini type spacecraft window. The Gemini window was chosen because it was available and was of interest to Ames because of the Ames T002 sextant-sighting experiment on the Gemini XII flight. In-flight navigation measurements from on board the spacecraft were made with a hand-held sextant. The effects of the Gemini window on these measurements had to be known in order to evaluate the results of the experiment. The Gemini window (Fig. 7) consists of three panes of fused silica glass. The inner two panes are 0.38 inch thick and the outer pane is 0.33 inch thick. The pressure environment is 5.5 psi in the spacecraft cabin, 14.7 psi between the inner panes and the middle pane, 0 psi between the middle and outer panes, and of course, space vacuum outside the spacecraft. Math models of the window surfaces were obtained from experimentally determined pressure deformations. Line-of-sight deviations were computed for the Gemini window for lines of sight which intersect the window at several points with various incidence angles and in several planes of incidence. The points, incidence angle and incidence plane, are illustrated in Figure 7. Predicted line-of-sight deviations are shown by the solid lines in Figure 8 for lines of sight in a vertical plane of incidence perpendicular to the window along the vertical or y axis. The deviations are shown as a function of the vertical line-of-sight position for incidence angles of 15° and 30°. Deviations up to about 15 arcsec for 30° incidence angles are predicted by the analytic method. These analytically predicted line-of-sight deviations are compared with line-of-sight deviations measured experimentally which are indicated by the dashed lines. These data were obtained at Ames in an experimental study of the optical effects of the Gemini window. Obviously, analytic and experimental results agree quite well. Similarly, good agreement is obtained for other incidence angles and for lines of sight in different incidence planes.

CONCLUDING REMARKS

An analytic method for predicting spacecraft window-induced line-of-sight deviations has been described. The method has been applied to the analysis of deviations associated with a Gemini spacecraft window. The analytic method permits accurate prediction of window-induced deviations as evidenced by comparison with experimental data. The analytic method is believed to be applicable to the analysis of line-of-sight deviations associated with many different spacecraft or aircraft windows.

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2. Melosh, R. J.; Diether, P. A.; and Brennan, M.: Structural Analysis System Usage Report. WDL-EM073. Philco Western Development Laboratories, Contract No. 950321. NASA CR-60975, 1963.

FIGURE TITLES

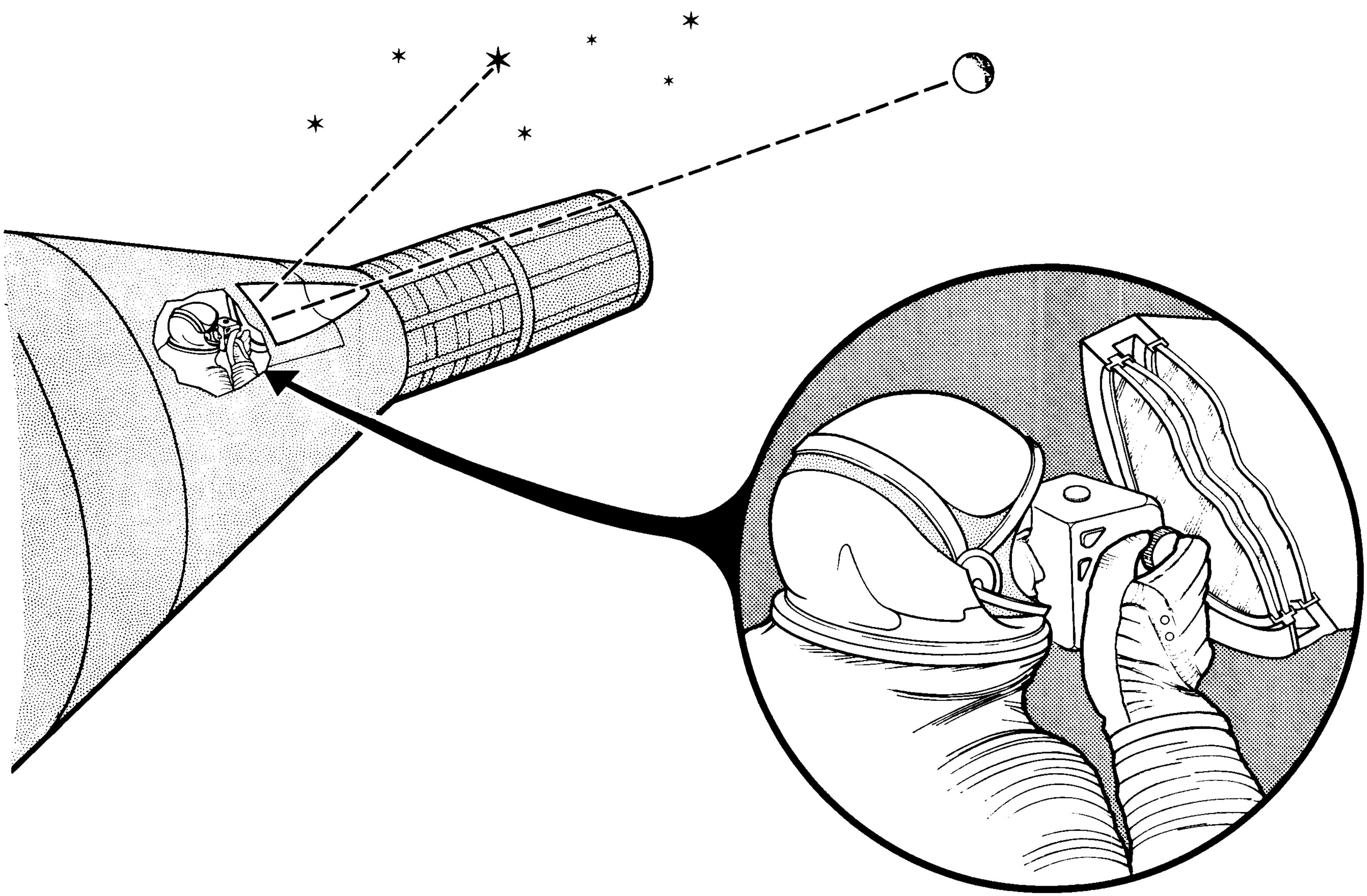

Figure 1

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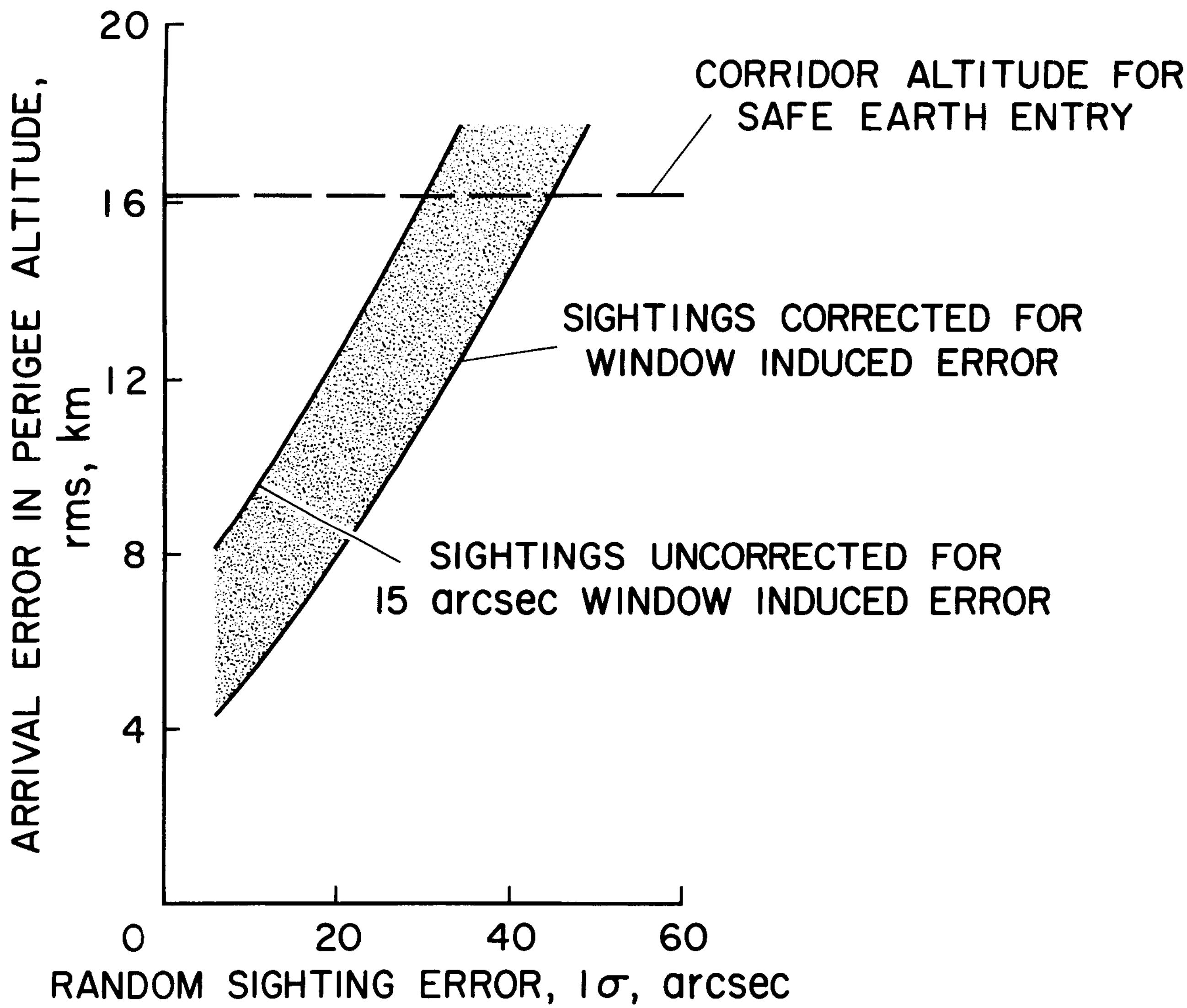

Figure 2

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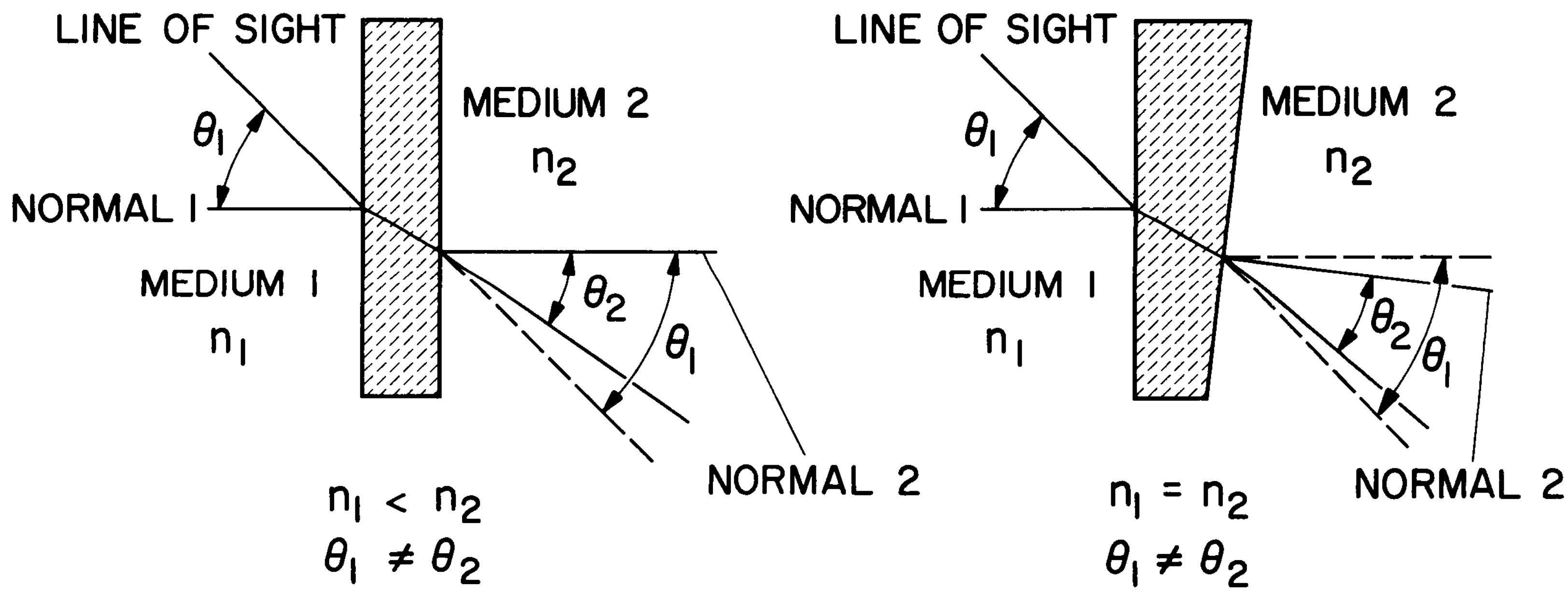

Figure 3

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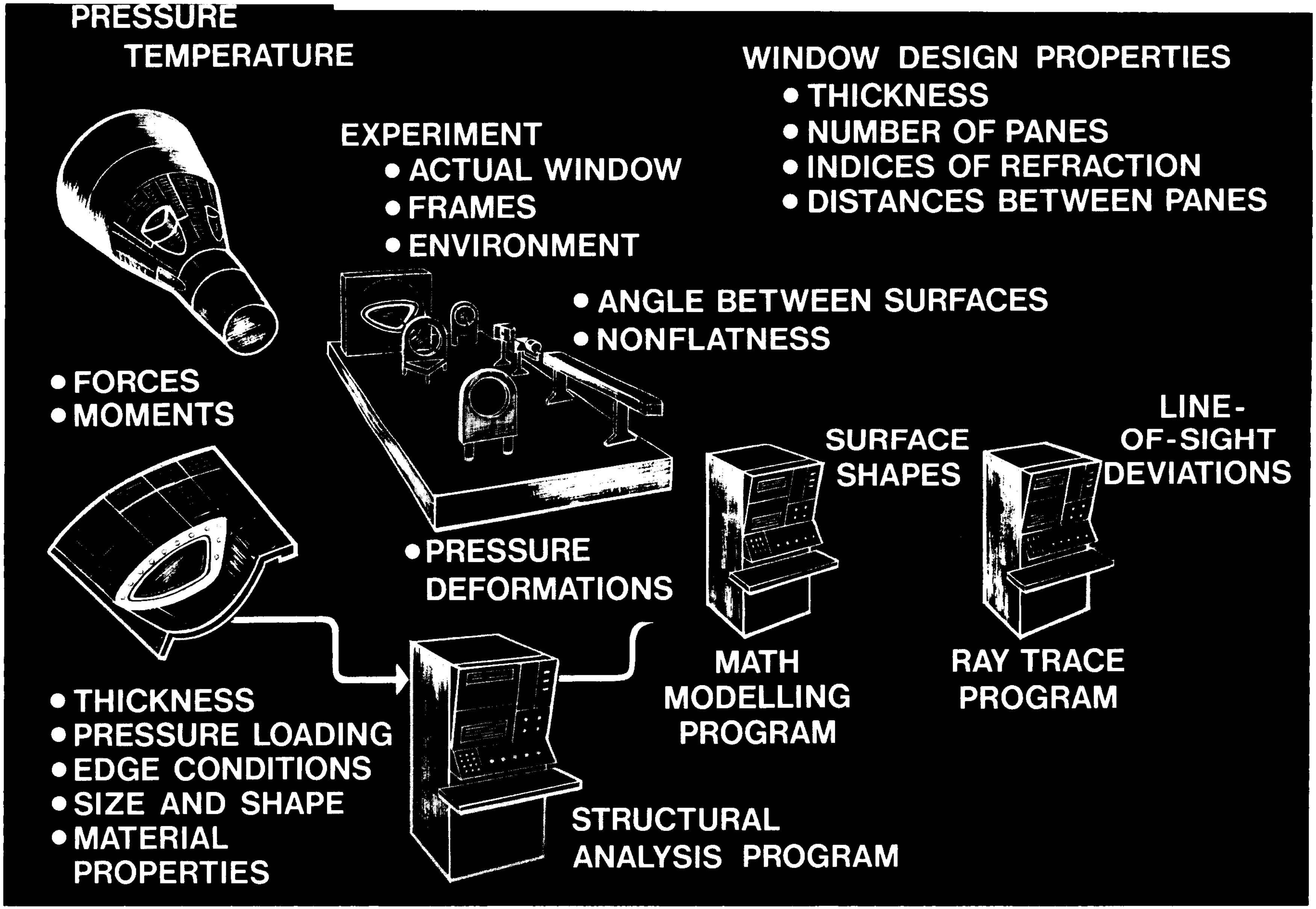

PRESSURE
TEMPERATURE
WINDOW DESIGN PROPERTIES
• THICKNESS
• NUMBER OF PANES
• INDICES OF REFRACTION
• DISTANCES BETWEEN PANES
EXPERIMENT
• ACTUAL WINDOW
• FRAMES
• ENVIRONMENT
• ANGLE BETWEEN SURFACES
• NONFLATNESS
• FORCES
• MOMENTS
SURFACE SHAPES
LINE-OF-SIGHT DEVIATIONS
• PRESSURE DEFORMATIONS
MATH MODELLING PROGRAM
RAY TRACE PROGRAM
• THICKNESS
• PRESSURE LOADING
• EDGE CONDITIONS
• SIZE AND SHAPE
• MATERIAL PROPERTIES
STRUCTURAL ANALYSIS PROGRAM

Figure 4

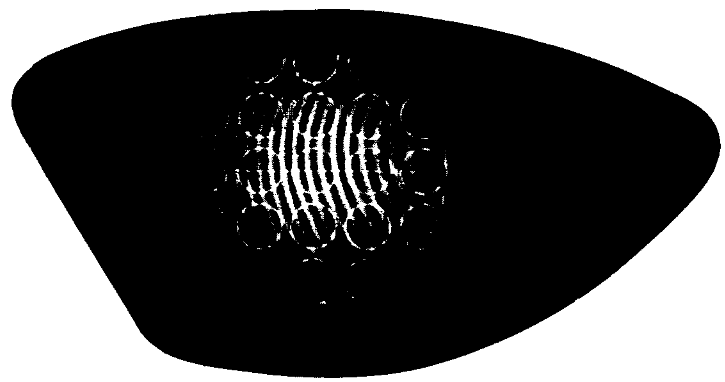

PHOTO USED TO DETERMINE
NONFLATNESS OF
WINDOW SURFACES

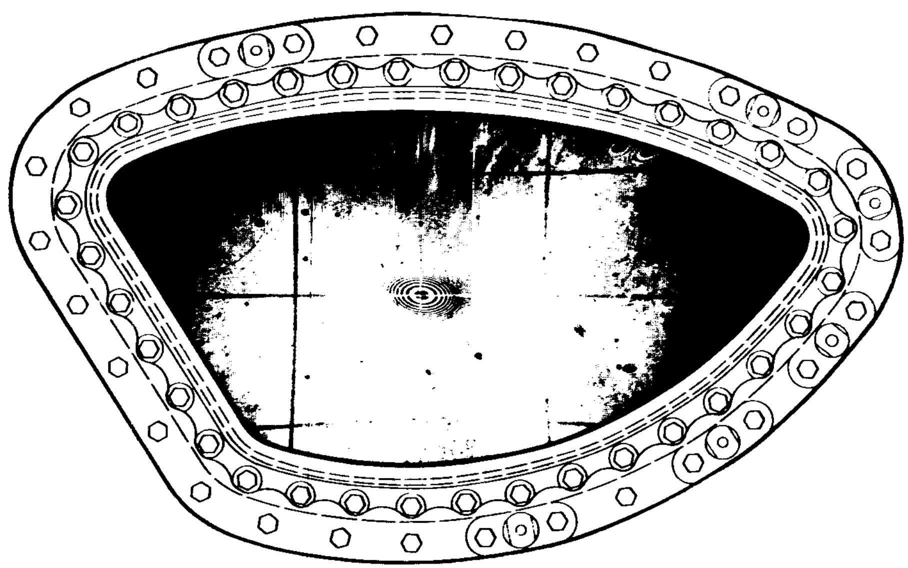

PHOTO USED TO DETERMINE
WINDOW SURFACE
PRESSURE DEFORMATIONS

Figure 5

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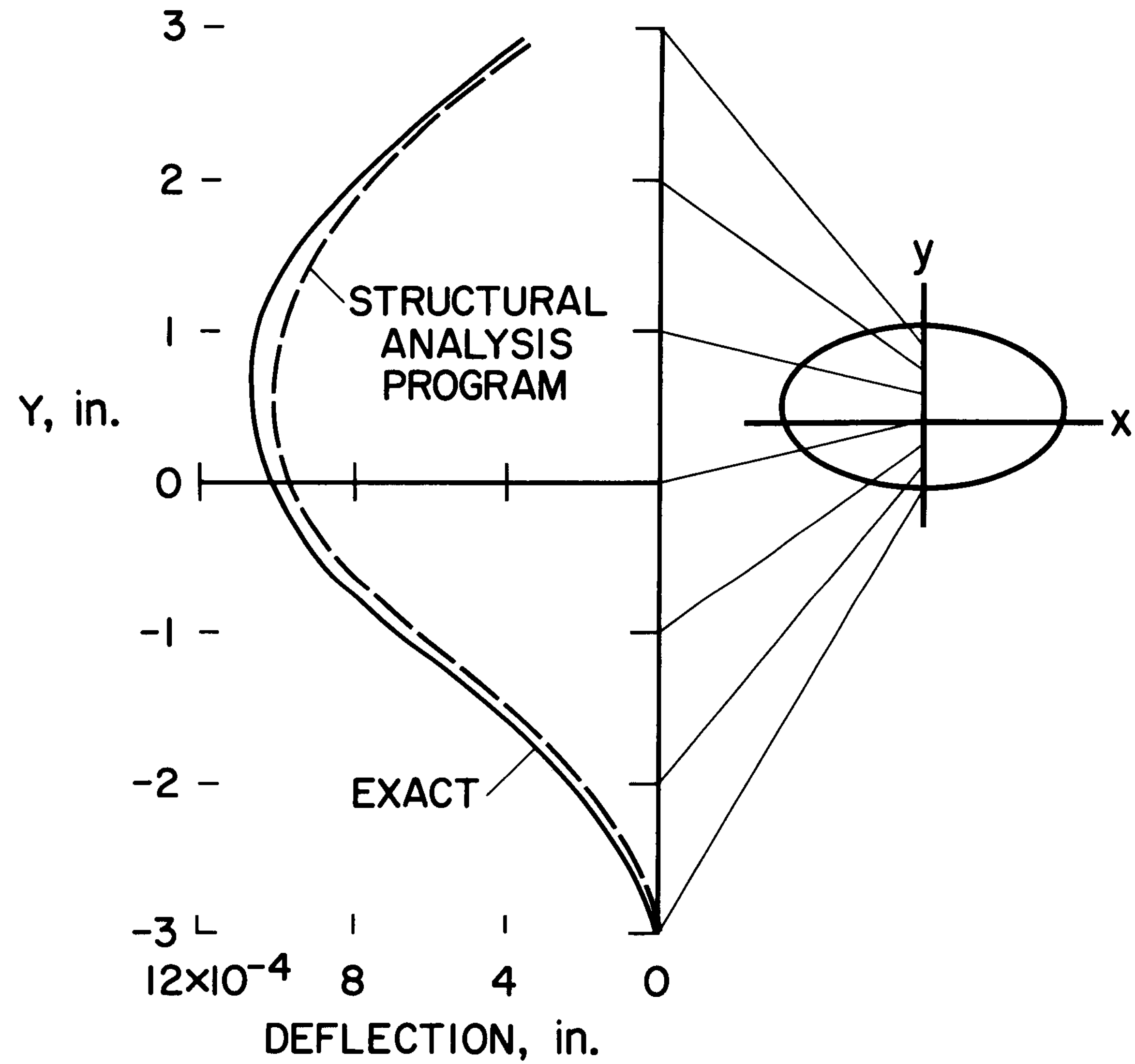
3
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ANALYSIS
PROGRAM
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Figure 6

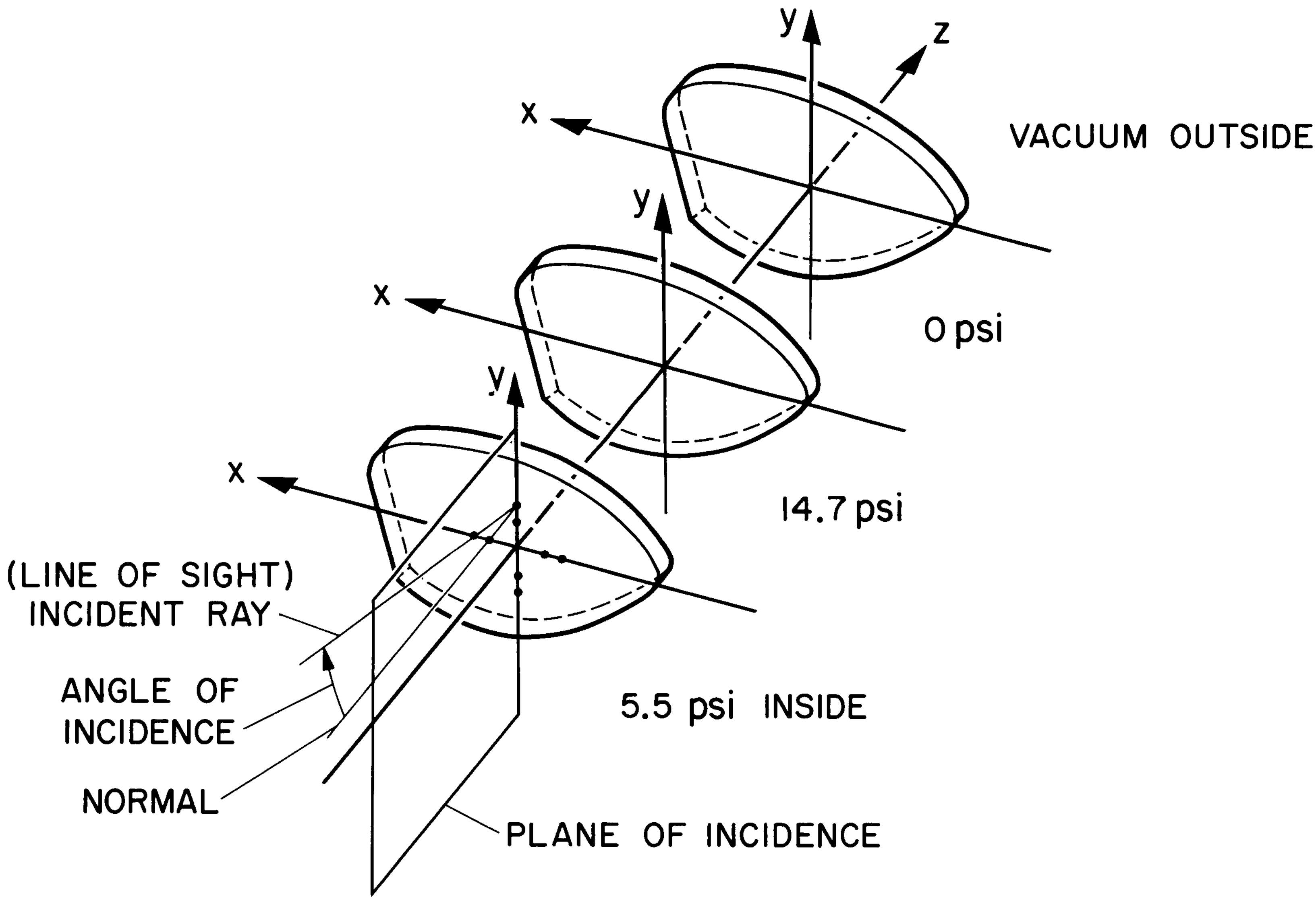

Figure 7

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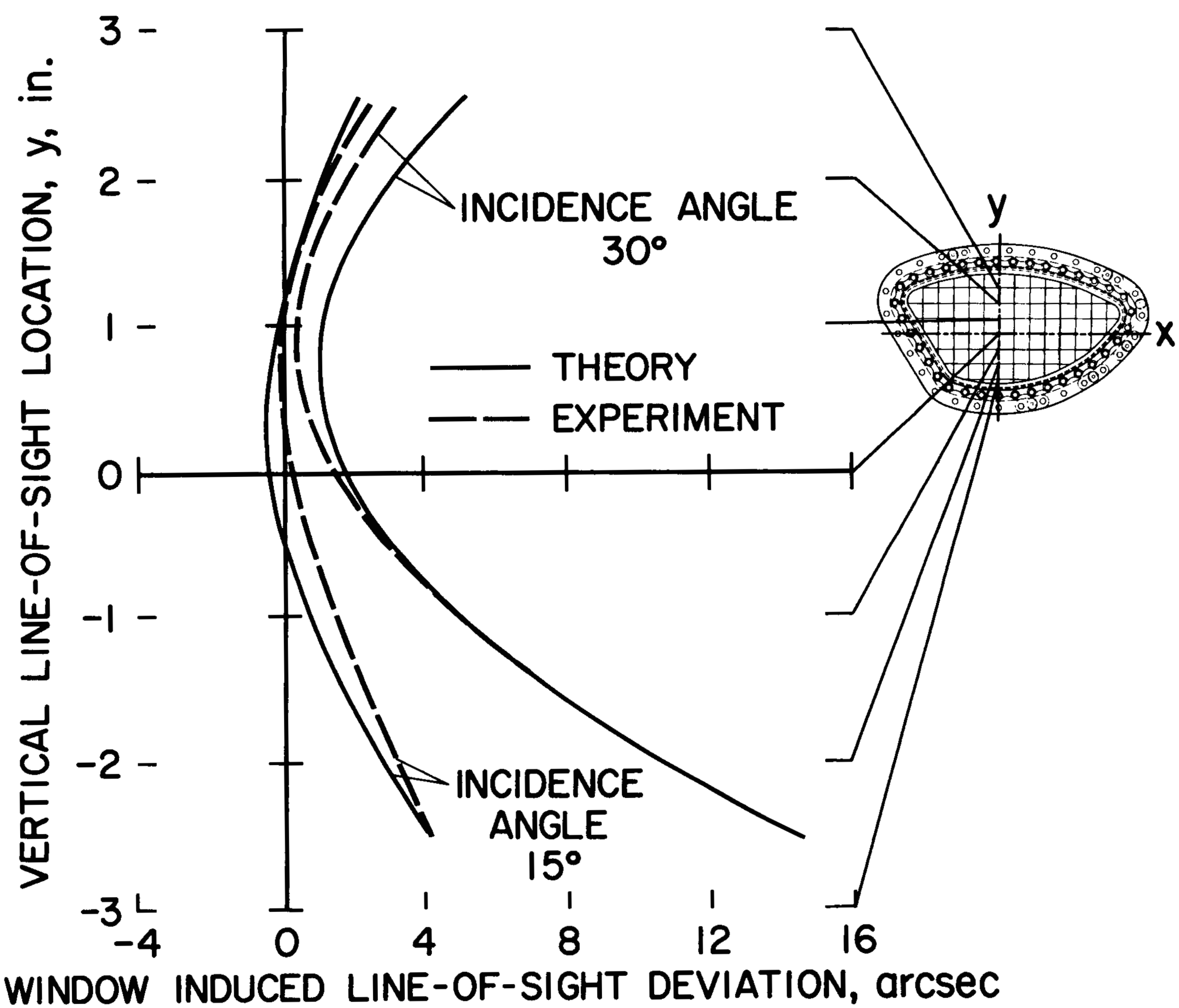

Figure 8

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