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CORRECTION OF BALLOON X-RAY ASTRONOMY DATA FOR THE EFFECTS OF ATMOSPHERIC ATTENUATION, K X-RAY ESCAPE, AND ENERGY RESOLUTION

by James W. Overbeck

Prepared by
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Mass.

for

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Abstract

An x-ray flux from outside the atmosphere determines the counting rate of an x-ray detector beneath several g/cm^2 of atmosphere, but a unique inverse relationship does not exist. Therefore x-ray astronomy data analysis requires the computer simulation of the processes which transform assumed x-ray energy spectra into detectors' counting rates. The programs used by the author, E. Allen Womack, Jr., and Harvey D. Tananbaum are presented and are applied to a specific example. A set of conventions for presenting one's results as an x-ray spectrum above the atmosphere is also explained.

I. Introduction

There is no one-to-one correspondence between the counting rate of an x-ray detector beneath several g/cm^2 of atmosphere and the x-ray flux at the top of the atmosphere. This is primarily due to two effects which cause pulse heights from a detector not to be uniquely related to the energies of the x-rays which produced them. The first is the sum of statistical fluctuations in light photon and photoelectron production, usually lumped under the familiar term "energy resolution". The second, the escape of K x-rays from such high Z elements as Iodine or Xenon in the detector, causes a pulse to indicate about 30 keV less energy than the original photon actually had. Results of calculations on this effect have been presented by Stein and Lewin[1].

Another effect of the same sort is the partial deposition of energy by x-rays which Compton scatter in the detector and then leave. This is important above 100 keV and is extremely important for energies of a few MeV. This effect is neglected here because we are mainly concerned with energies below 100 keV. A more important effect is the increase in the detector's total absorption probability (referred to here as efficiency) due to the increased path length of an x-ray which has Compton scattered. The data[2,3] of the author, E. Allen Womack, Jr., and Harvey D. Tananbaum have had such large statistical errors above 50 keV (where this effect is most important) that we have neglected this effect for the time being. Calibration with radioactive isotopes having known intensity ratios for their gamma ray lines will be used when the need arises.

In reference 3 the author and Harvey D. Tananbaum reported a significant increase in the x-ray flux from Cygnus XR-1 over a period of eight months. One of the main purposes of this report is to allow other investigators to compare their data with ours with the assurance that agreement or disagreement is not due to differences in data analysis or data presentation.

This report has four parts. The first part is a description and listing of a program, DETMD, which generates thermal bremsstrahlung and power law x-ray spectra, attenuates them by the atmosphere, multiplies them by the detector's efficiency, shifts x-rays in energy to account for K x-ray escape, and smears the result with a gaussian, energy dependent resolution function. The inputs to DETMD are tables of detector efficiency, K x-ray escape probabilities, attenuation coefficients for air, x-ray spectrum parameters, g/cm^2 of air considered, detector energy resolution parameters, and pulse height analyzer energy thresholds. Its purpose is to allow the user to calculate the ratio of x-ray fluxes in several energy bands to counting rates in the corresponding pulse height bands.

The second part consists of the programs we used to generate the tables of air attenuation coefficients, efficiency, and K x-ray escape probabilities. The efficiency and K x-ray escape program, NAIEF, applies only to our 1 mm thick NaI crystal with a .005 inch beryllium window. The user will have to modify it or start from scratch to generate efficiency tables for his own detector. The program is included here to allow others to check our analysis and to generate sample input data for program DETMD.

In the third part is presented a self consistent set of conventions for presenting one's results as a graph of an x-ray source's spectrum (above the atmosphere), thereby allowing accurate visual comparison of different experimenters' results.

The computer programs were written in Fortran IV for the IBM 7044, but they should run equally well on an IBM 7094 equipped with the operating system IBSYS. For operation on the IBM 360 some numbers will have to be truncated to 7 decimal digits. The Fortran logical designations of the input-output devices are: 5, card reader; 6, printer; and 7, card punch. As originally written the programs contained many statements which caused the generation of graphs of the results on a Calcomp digital incremental plotter. This is the cause of some structures in the programs which seem to be awkward or unnecessary. It is also the cause of the small step size (0.1 keV) used in all the calculations.

II. Program DETMD

The assumed differential number spectra above the atmosphere are

$$\left(\frac{dn}{dE}\right)_{\circ} = A\left(\frac{E_{\circ}}{E}\right)e^{-(E-E_{\circ})/kT} \quad \underline{\text{or}} \quad A\left(\frac{E_{\circ}}{E}\right)^{N} \quad \underline{\text{if}}$$

NTYPE = 1 or 2 respectively.
A = ANORM, $E_{\circ}$ = EREF, kT = EKT, N = POWER

The spectrum at the detector is $\left(\frac{dn}{dE}\right)_1 = \left(\frac{dn}{dE}\right)_{\circ} e^{-\mu(E)X}$

μ = the attenuation coefficient of air = ATTEN
X = g/cm^2 of air along line of sight = GRAMS.

The spectrum of energy losses in the detector is

$$\left(\frac{dn}{dE}(E)\right)_2 = \left(\frac{dn}{dE}(E)\right)_1 [\varepsilon(E) - p_{\alpha}(E) - p_{\beta}(E)]$$
$$+\left(\frac{dn}{dE}(E + E_{K\alpha})\right)_1 \; p_{\alpha}(E + E_{K\alpha})$$
$$+\left(\frac{dn}{dE}(E + E_{K\beta})\right)_1 \; p_{\beta}(E + E_{K\beta})$$

ε = the efficiency of the detector, i.e. the probability that an incident photon will interact = EFFIC

p_{α} = the probability that an incident photon will give rise to a K_{α} photon which escapes from the detector = ESCKA

p_{β} = the probability that an incident photon will give rise to a K_{β} photon which escapes from the detector = ESCKB

$E_{K\alpha}$, $E_{K\beta}$ = energies of K_{α} and K_{β} photons = 32.5, 28.6 keV for iodine.

The pulse height spectrum is

$$\left(\frac{dn}{dE}(E)\right)_3 = \int_0^\infty \left(\frac{dn}{dE}(E')\right)_2 \frac{e^{-\frac{(E'-E)^2}{2\sigma(E')^2}}}{\sqrt{2\pi}\,\sigma(E')}\, dE'$$

where $\sigma(E') = aE' + b\sqrt{E'}$

a = ASIG, b = BSIG

The four spectra are printed on four separate pages. Each is integrated over 1 keV bands, 5 keV bands, and eight other bands whose boundaries are the inputs CALIB*THRESH. Following the listing of program DETMD are some of its results for kT = 4.3 keV and the author's detector parameters. The first page of results is $\left(\frac{dn}{dE}\right)_o$ for A = 1.0 and E_o = 20 keV. The next three pages are the corresponding $\left(\frac{dn}{dE}\right)_1$, $\left(\frac{dn}{dE}\right)_2$, and $\left(\frac{dn}{dE}\right)_3$ for 4.0 g/cm^2 of air along the line of sight. The last three pages are $\left(\frac{dn}{dE}\right)_3$ for 4.5, 5.0, and 5.5 g/cm^2 of air. The following example illustrates how these results are used. Absorption by 5.0 g/cm^2 of air along the line of sight is assumed. One counts n_{ab} = .002 pulses/ cm^{2}sec which have pulse heights corresponding to energies between E_a = 23.4 and E_b = 46.7 keV. If one assumes the spectrum above the atmosphere had the form $\frac{K}{E}\, e^{-E/kT}$ with kT = 4.3 keV then the number of incident photons with energies between E_a and E_b was $N_{ab}(kT) = R_{ab}(kT)\ \ n_{ab}$ = .01666, where

$$R_{ab} = \frac{\int_{E_a}^{E_b} \left(\frac{dn}{dE}\right)_o dE}{\int_{E_a}^{E_b} \left(\frac{dn}{dE}\right)_3 dE} = \frac{\underline{1.19006} + \underline{.19147} + \underline{.03155}}{\underline{.11938} + \underline{.04034} + \underline{.00999}} = 8.33$$

(see underlined numbers in the computer printout)

The best value of the coefficient K is

$$K = \frac{A\,E_{\circ}e^{E_{\circ}/kT}\,n_{ab}}{\int_{E_a}^{E_b}\left(\frac{dn}{dE}\right)_{3} dE} = \frac{1 \times 20\,e^{\frac{20}{4.3}} \times .002}{.16971} = 24.68$$

FORTRAN SOURCE LIST

```
ISN        SOURCE STATEMENT

  0 $IBFTC DETMD
  1          DIMENSION ATTEN(1900), EFFIC(2000), ESCKA(2000), ESCKB(2000),
            1 GAUSS(2000), COUNT(2000), CTSA(40), CTSB(8), THRESH(9), Y(2000)
  2          NPOINT = 2000
  3          DO 10 INIT = 1, 2000
  4          GAUSS(INIT) = EXP(-((FLOAT(INIT - 1)/500.0)**2)/2.0)
  5   10     CONTINUE
  7          DO 12 JFIX = 1, 100
 10          EFFIC(JFIX) = 1.0
 11          ESCKA(JFIX) = 0.0
 12          ESCKB(JFIX) = 0.0
 13   12     CONTINUE
 15          READ(5, 15) (EFFIC(JA), JA = 101, 2000), (ESCKA(JB), JB = 101,
            1 2000), (ESCKB(JC), JC = 101, 2000)
 32   15     FORMAT(10F7.5)
 33          READ(5, 20) ATTEN
 34   20     FORMAT(10F7.4)
 35   50     READ(5, 52) NDSPEC, NDATM, NDRESL, NDTRSH, NDCALB, NDQUIT
 44   52     FORMAT(6I1)
 45          IF(NDQUIT.NE.0) GO TO 1000
 50          IF(NDSPEC.NE.1) GO TO 60
 53          READ(5, 54) NTYPE, EREF, ANORM, TORPWR
 55   54     FORMAT(I1, 3F7.3)
 56   60     IF(NDATM.NE.1) GO TO 65
 61          READ(5, 63) GRAMS
 62   63     FORMAT(F7.4)
 63   65     IF(NDRESL.NE.1) GO TO 75
 66          READ(5, 68) ASIG, BSIG
 67   68     FORMAT(2F10.5)
 70   75     IF(NDTRSH.NE.1) GO TO 80
 73          READ(5, 78) THRESH
 74    78    FORMAT(6F10.4/3F10.4)
 75   80     IF(NDCALB.NE.1) GO TO 85
100          READ(5, 83) CALIB
101   83     FORMAT(F10.5)
102   85     CONTINUE
103          IF(NTYPE.EQ.2) GO TO 110
106          EKT = TORPWR
107          DO 105 JD=2,NPOINT
110          EN = FLOAT(JD - 1)/10.0
111          Y(JD) = ANORM*EXP(-(EN-EREF)/EKT)/(EN/EREF)
112  105     CONTINUE
114          Y(1)=0.0
115          GO TO 140
116  110     POWER = -TORPWR
117          DO 115 JE=2,NPOINT
120          EN = FLOAT(JE-1)/10.0
121          Y(JE) = ANORM*((EN/EREF)**POWER)
122  115     CONTINUE
124          Y(1)=0.0
125  140     CONTINUE
126          GM = 0.0
127          ASSIGN 145 TO JRETRN
130          JRET = 145
131          GO TO 900
```

```
ISN        SOURCE STATEMENT

132   145   CONTINUE
133         GM = GRAMS
134         DO 155 JG = 1, 100
135         Y(JG) = 0.0
136   155   CONTINUE
140         DO 160 JH = 101, NPOINT
141         Y(JH) = Y(JH)*EXP(-GRAMS*ATTEN(JH-100))
142   16C   CONTINUE
144         ASSIGN 190 TO JRETRN
145         JRET = 190
146         GO TO 900
147   19C   CONTINUE
150         DO 210 JI = 1, NPOINT
151         IF(JI-286.LT.1) GO TO 205
154         Y(JI-286) = Y(JI-286) + ESCKA(JI)*Y(JI)
155         IF(JI-325.LT.1) GO TO 205
160         Y(JI-325) = Y(JI-325) + ESCKB(JI)*Y(JI)
161   205   Y(JI) = Y(JI)*(EFFIC(JI) - ESCKA(JI) - ESCKB(JI))
162   21C   CONTINUE
164   235   CONTINUE
165         ASSIGN 240 TO JRETRN
166         JRET = 240
167         GC TO 900
170   24C   CONTINUE
171         DO 255 JCT = 1, NPOINT
172         COUNT(JCT) = 0.0
173   255   CONTINUE
175         DO 270 KE = 1, NPOINT
176         E = FLOAT(KE-1)/10.0
177         SIGMA=ASIG*E+BSIG*SQRT(E)
200         QKSIG = 50.0/SIGMA
201         YADJ = 0.1*Y(KE)/(2.5065*SIGMA)
202         DO 265 KEPRIM = 1, NPOINT
203         NGAUS = IFIX(FLOAT(IABS(KE-KEPRIM))*QKSIG) + 1
204         IF(NGAUS.GT.2000) GO TO 260
207         COUNT(KEPRIM) = COUNT(KEPRIM) + YADJ*GAUSS(NGAUS)
210         GO TO 265
211   26C   IF(KEPRIM.GT.KE) GC TC 270
214   265   CONTINUE
216   27C   CONTINUE
220         DO 275 JJ = 1, NPOINT
221         Y(JJ) = CCUNT(JJ)
222   275   CONTINUE
224         ASSIGN 305 TO JRETRN
225         JRET = 305
226         GO TO 900
227   305   CONTINUE
23C         GO TO 50
231   90C   IF(NTYPE.EQ.2) GO TO 905
234         WRITE(6, 901) TCRPWR
235         GO TO 906
236   901   FORMAT(1H1,  40HTHERMAL BREMSSTRAHLUNG SOURCE WITH KT =    , F7.4,
           1 4H KEV   )
237   903   FORMAT(1H ,  21HAFTER ATTENUATION BY    , F7.4,  15H G/CM**2 OF AIR
           1   / 1H ,   32HINCIDENT SPECTRUM NORMALIZED TO    , F7.4,
```

```
ISN       SOURCE STATEMENT

        2 12H PER KEV AT    , F7.4,   4H KEV    )
240  905  WRITE(6, 904) TCRPWR
241  904  FORMAT(1H1,   48HPOWER LAW DIFFERENTIAL SPECTRUM WITH EXPONENT =
        1   , F7.4)
242  906  WRITE(6, 903) GM, ANORM, EREF
243       IF(JRET.EQ.145.OR.JRET.EQ.190)  WRITE(6, 908)
246       IF(JRET.EQ.240)  WRITE(6, 910)
251       IF(JRET.EQ.305)  WRITE(6, 912)
254       IF(JRET.EQ.305)  WRITE(6, 914) ASIG, BSIG
257  908  FORMAT(1H0,  46HBEFORE CORRECTIONS FOR K ESCAPE AND RESOLUTION    )
260  910  FORMAT(1H0,  29HAFTER CORRECTION FOR K ESCAPE    )
261  912  FORMAT(1H0,  45HAFTER CORRECTIONS FOR K ESCAPE AND RESOLUTION    )
262   914 FORMAT(1H ,8HSIGMA = ,F7.5,5H*E + ,F8.5,14H*SQRT(E)    KEV)
263       DO 915 KBIN=1,8
264  915  CTSB(KBIN) = 0.0
266       DO 916 JBIN = 1, 40
267  916  CTSA(JBIN) = 0.0
271       MBIN = 1
272       DO 918 KNRG = 1, NPOINT
273       NBIN = (KNRG-1)/50 + 1
274       CTSA(NBIN) = CTSA(NBIN) + 0.1*Y(KNRG)
275       ENERGY = 0.1*FLOAT(KNRG-1)
276       IF(ENERGY.LT.CALIB*THRESH(1)) GO TO 918
301       IF(ENERGY.GE.CALIB*THRESH(MBIN+1)) MBIN = MBIN + 1
304       CTSB(MBIN) = CTSB(MBIN) + 0.1*Y(KNRG)
305  918  CONTINUE
307       WRITE(6, 920)
310  920  FORMAT(1H0,  6HENERGY  )
311       LINMX = (NPOINT-1)/100 + 1
312       DO 921 LIN = 1, LINMX
313       LENRGY = 10*(LIN-1)
314       LSTART = 1 + 100*(LIN-1)
315       LSTOP = LSTART + 90
316       WRITE(6, 919) LENRGY, (Y(LA), LA = LSTART, LSTOP, 10)
323  919  FORMAT(1H , I3, 10F10.5)
324  921  CONTINUE
326       WRITE(6, 123) CTSA, CALIB, THRESH, CTSB
327  123  FORMAT(1H0,  20HCOUNTS IN 5 KEV BINS  /1H0, 10F10.5 /1H ,
        1 10F10.5/1H , 10F10.5/1H ,10F10.5/  1H0,  33HCOUNTS IN BINS WITH T
        2HRESHOLDS =      , F7.4, 6H TIMES   /1H , 9F8.3/1H0,8F8.5)
330       GO TO JRETRN, (145, 190, 240, 305)
331  1000 CONTINUE
332       END
```

INPUT DATA CARDS FOR PROGRAM DETMD (FOR SAMPLE RESULTS PRESENTED IN THIS REPORT)

```
190 CARDS CONTAINING EFFIC FROM 10.0 TO 199.9 KEV
190 CARDS CONTAINING ESCKA FROM 10.0 TO 199.9 KEV
190 CARDS CONTAINING ESCKB FROM 10.0 TO 199.9 KEV
190 CARDS CONTAINING ATTEN FROM 10.0 TO 199.9 KEV
111110
1 20.000  1.000  4.30
4.00
0.07      0.29
     16.60     23.40     30.30     37.50     46.70     54.20
     66.50     81.00    101.00
1.0000
010000
4.50
010000
5.00
010000
5.50
000001
```

THERMAL BREMSSTRAHLUNG SOURCE WITH KT = 4.3000 KEV
AFTER ATTENUATION BY 0.0000 G/CM**2 OF AIR
INCIDENT SPECTRUM NORMALIZED TO 1.0000 PER KEV AT 20.0000 KEV

BEFORE CORRECTIONS FOR K ESCAPE AND RESOLUTION

ENERGY										
0	0.00000	1659.60828	657.62284	347.44568	206.51398	130.93047	86.46907	58.73747	40.73096	28.69283
10	20.46525	14.74435	10.71121	7.83570	5.76626	4.26513	3.16887	2.36362	1.76911	1.32824
20	1.00000	0.75477	0.57097	0.43282	0.32872	0.25009	0.19057	0.14544	0.11114	0.08504
30	0.06515	0.04997	0.03836	0.02948	0.02268	0.01746	0.01345	0.01037	0.00800	0.00618
40	0.00478	0.00369	0.00286	0.00221	0.00171	0.00133	0.00103	0.00080	0.00062	0.00048
50	0.00037	0.00029	0.00023	0.00018	0.00014	0.00011	0.00008	0.00006	0.00005	0.00004
60	0.00003	0.00002	0.00002	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000
70	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
80	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
90	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
110	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
130	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
140	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
160	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
170	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
190	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN 5 KEV BINS

1542.19000	291.76567	51.77771	11.34383	2.73215	0.69488	0.18312	0.04948	0.01362	0.00380
0.00107	0.00031	0.00009	0.00003	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN BINS WITH THRESHOLDS = 1.0000 TIMES

16.600	23.400	30.300	37.500	46.700	54.200	66.500	81.000	101.000

8.09347	1.19006	0.19147	0.03155	0.00294	0.00049	0.00002	0.00000

THERMAL BREMSSTRAHLUNG SOURCE WITH KT = 4.3000 KEV
AFTER ATTENUATION BY 4.0000 G/CM**2 OF AIR
INCIDENT SPECTRUM NORMALIZED TO 1.0000 PER KEV AT 20.0000 KEV

BEFORE CORRECTIONS FOR K ESCAPE AND RESOLUTION

ENERGY										
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
10	0.00000	0.00001	0.00011	0.00087	0.00364	0.00980	0.01913	0.03011	0.04055	0.04866
20	0.05350	0.05460	0.05299	0.04944	0.04474	0.03948	0.03417	0.02908	0.02444	0.02031
30	0.01672	0.01362	0.01103	0.00889	0.00713	0.00571	0.00455	0.00362	0.00287	0.00228
40	0.00180	0.00143	0.00113	0.00089	0.00070	0.00055	0.00043	0.00034	0.00027	0.00021
50	0.00017	0.00013	0.00010	0.00008	0.00006	0.00005	0.00004	0.00003	0.00002	0.00002
60	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
70	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
80	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
90	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
110	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
130	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
140	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
160	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
170	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
190	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN 5 KEV BINS

0.00000	0.00000	0.00837	0.16831	0.24961	0.13708	0.05227	0.01720	0.00535	0.00162
0.00048	0.00014	0.00004	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN BINS WITH THRESHOLDS = 1.0000 TIMES

16.600 23.400 30.300 37.500 46.700 54.200 66.500 81.000 101.000

0.32554 0.20907 0.05711 0.01200 0.00129 0.00023 0.00001 0.00000

THERMAL BREMSSTRAHLUNG SOURCE WITH KT = 4.3000 KEV
AFTER ATTENUATION BY 4.0000 G/CM**2 OF AIR
INCIDENT SPECTRUM NORMALIZED TO 1.0000 PER KEV AT 20.0000 KEV

AFTER CORRECTION FOR K ESCAPE

ENERGY										
0	0.00000	0.00044	0.00035	0.00028	0.00022	0.00159	0.00125	0.00098	0.00076	0.00059
10	0.00046	0.00036	0.00039	0.00108	0.00381	0.00993	0.01923	0.03018	0.04061	0.04868
20	0.05347	0.05446	0.05269	0.04891	0.04393	0.03838	0.03278	0.02746	0.02264	0.01840
30	0.01478	0.01171	0.00921	0.00719	0.00528	0.00426	0.00342	0.00275	0.00219	0.00175
40	0.00139	0.00111	0.00088	0.00070	0.00055	0.00044	0.00034	0.00027	0.00021	0.00017
50	0.00013	0.00010	0.00008	0.00006	0.00005	0.00004	0.00003	0.00002	0.00002	0.00001
60	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
70	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
80	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
90	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
110	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
130	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
140	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
160	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
170	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
190	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN 5 KEV BINS

0.00169	0.00465	0.00969	0.16862	0.24733	0.12886	0.04308	0.01303	0.00418	0.00129
0.00038	0.00011	0.00003	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN BINS WITH THRESHOLDS = 1.0000 TIMES

16.600	23.400	30.300	37.500	46.700	54.200	66.500	81.000	101.000

0.32470	0.19878	0.04590	0.00930	0.00102	0.00017	0.00001	0.00000

THERMAL BREMSSTRAHLUNG SOURCE WITH KT = 4.3000 KEV
AFTER ATTENUATION BY 4.0000 G/CM**2 OF AIR
INCIDENT SPECTRUM NORMALIZED TO 1.0000 PER KEV AT 20.0000 KEV

AFTER CORRECTIONS FOR K ESCAPE AND RESOLUTION
SIGMA = 0.07000*E + 0.29000*SQRT(E) KEV

ENERGY										
0	0.00001	0.00036	0.00035	0.00035	0.00060	0.00101	0.00112	0.00096	0.00079	0.00076
10	0.00101	0.00182	0.00360	0.00676	0.01148	0.01761	0.02458	0.03157	0.03771	0.04233
20	0.04505	0.04584	0.04487	0.04251	0.03917	0.03525	0.03108	0.02693	0.02298	0.01936
30	0.01612	0.01330	0.01088	0.00884	0.00714	0.00575	0.00462	0.00371	0.00298	0.00239
40	0.00192	0.00155	0.00125	0.00101	0.00082	0.00066	0.00053	0.00043	0.00035	0.00028
50	0.00022	0.00018	0.00015	0.00012	0.00009	0.00008	0.00006	0.00005	0.00004	0.00003
60	0.00002	0.00002	0.00002	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000
70	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
80	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
90	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
110	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
130	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
140	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
160	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
170	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
190	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN 5 KEV BINS

0.00207	0.00463	0.03163	0.16655	0.21351	0.12690	0.05147	0.01766	0.00596	0.00204
0.00069	0.00023	0.00008	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN BINS WITH THRESHOLDS = 1.0000 TIMES
16.600 23.400 30.300 37.500 46.700 54.200 66.500 81.000 101.000

0.28597 0.19074 0.05672 0.01304 0.00173 0.00037 0.00002 0.00000

THERMAL BREMSSTRAHLUNG SOURCE WITH KT = 4.3000 KEV
AFTER ATTENUATION BY 4.5000 G/CM**2 OF AIR
INCIDENT SPECTRUM NORMALIZED TO 1.0000 PER KEV AT 20.0000 KEV

AFTER CORRECTIONS FOR K ESCAPE AND RESOLUTION
SIGMA = 0.07000*E + 0.29000*SQRT(E) KEV

ENERGY										
0	0.00001	0.00031	0.00030	0.00030	0.00052	0.00087	0.00097	0.00083	0.00068	0.00060
10	0.00067	0.00105	0.00199	0.00376	0.00659	0.01048	0.01517	0.02018	0.02493	0.02885
20	0.03158	0.03294	0.03299	0.03190	0.02993	0.02738	0.02449	0.02150	0.01857	0.01581
30	0.01329	0.01105	0.00911	0.00746	0.00606	0.00491	0.00397	0.00320	0.00258	0.00208
40	0.00168	0.00136	0.00110	0.00089	0.00072	0.00058	0.00047	0.00038	0.00031	0.00025
50	0.00020	0.00016	0.00013	0.00010	0.00008	0.00007	0.00005	0.00004	0.00003	0.00003
60	0.00002	0.00002	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000
70	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
80	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
90	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
110	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
130	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
140	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
160	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
170	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
190	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN 5 KEV BINS

0.00180	0.00387	0.01812	0.10929	0.15785	0.10138	0.04310	0.01523	0.00524	0.00182
0.00062	0.00021	0.00007	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN BINS WITH THRESHOLDS = 1.0000 TIMES

16.600 23.400 30.300 37.500 46.700 54.200 66.500 81.000 101.000

0.20063 0.15068 0.04782 0.01141 0.00154 0.00034 0.00002 0.00000

THERMAL BREMSSTRAHLUNG SOURCE WITH KT = 4.3000 KEV
AFTER ATTENUATION BY 5.0000 G/CM**2 OF AIR
INCIDENT SPECTRUM NORMALIZED TO 1.0000 PER KEV AT 20.0000 KEV

AFTER CORRECTIONS FOR K ESCAPE AND RESOLUTION
SIGMA = 0.07000*E + 0.29000*SQRT(E)   KEV

ENERGY										
0	0.00001	0.00027	0.00026	0.00026	0.00045	0.00070	0.00085	0.00073	0.00059	0.00049
10	0.00049	0.00066	0.00116	0.00218	0.00389	0.00638	0.00954	0.01310	0.01668	0.01986
20	0.02231	0.02383	0.02439	0.02404	0.02296	0.02133	0.01935	0.01721	0.01503	0.01292
30	0.01097	0.00920	0.00764	0.00629	0.00515	0.00419	0.00341	0.00276	0.00224	0.00181
40	0.00147	0.00119	0.00097	0.00079	0.00064	0.00052	0.00042	0.00034	0.00028	0.00022
50	0.00018	0.00015	0.00012	0.00009	0.00008	0.00006	0.00005	0.00004	0.00003	0.00003
60	0.00002	0.00002	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000
70	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
80	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
90	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
110	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
130	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
140	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
160	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
170	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
190	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN 5 KEV BINS

0.00156	0.00330	0.01080	0.07279	0.11739	0.08118	0.03612	0.01314	0.00461	0.00162
0.00056	0.00019	0.00006	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN BINS WITH THRESHOLDS = 1.0000 TIMES
16.600 23.400 30.300 37.500 46.700 54.200 66.500 81.000 101.000

0.14226 0.11938 0.04034 0.00999 0.00138 0.00030 0.00002 0.00000

```
THERMAL BREMSSTRAHLUNG SOURCE WITH KT =  4.3000 KEV
AFTER ATTENUATION BY  5.5000 G/CM**2 OF AIR
INCIDENT SPECTRUM NORMALIZED TO  1.0000 PER KEV AT 20.0000 KEV

AFTER CORRECTIONS FOR K ESCAPE AND RESOLUTION
SIGMA = 0.07000*E +  0.29000*SQRT(E)   KEV
```

ENERGY										
0	0.00000	0.00023	0.00023	0.00023	0.00039	0.00066	0.00074	0.00063	0.00051	0.00042
10	0.00038	0.00046	0.00072	0.00131	0.00236	0.00397	0.00610	0.00862	0.01129	0.01380
20	0.01588	0.01735	0.01811	0.01819	0.01766	0.01666	0.01532	0.01379	0.01218	0.01058
30	0.00906	0.00766	0.00641	0.00531	0.00438	0.00358	0.00293	0.00238	0.00194	0.00158
40	0.00128	0.00104	0.00085	0.00069	0.00056	0.00046	0.00037	0.00030	0.00025	0.00020
50	0.00016	0.00013	0.00010	0.00008	0.00007	0.00005	0.00004	0.00004	0.00003	0.00002
60	0.00002	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000
70	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
80	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
90	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
110	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
130	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
140	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
160	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
170	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
190	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COUNTS IN 5 KEV BINS

0.00135	0.00285	0.00669	0.04914	0.08779	0.06514	0.03029	0.01134	0.00405	0.00144
0.00050	0.00017	0.00006	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

```
COUNTS IN BINS WITH THRESHOLDS =  1.0000 TIMES
  16.600  23.400  30.300  37.500  46.700  54.200  66.500  81.000 101.000

0.10184 0.09485 0.03407 0.00874 0.00123 0.00028 0.00002 0.00000
```

III. Programs Used to Generate Inputs to DETMD

AIRAT generates a 1900 entry table of attenuation coefficients for air between 10.0 and 199.9 keV. This table is called TABLE within AIRAT and is called ATTEN when used as input to DETMD. The inputs to AIRAT are energies and Compton attenuation coefficients for nitrogen, oxygen, and argon at these energies. The constituents of air correspond to the indices 1, 2, and 3 respectively.

INTERP allows interpolation between three consecutive points on a smooth curve by finding coefficients A, B, and C of the quadratic $Ax^2 + Bx + C$ which fits the three points. It is used by AIRAT to find Compton attenuation coefficients in between the tabulated values it receives as input.

E1 generates values of the exponential integral $E_1(x) = \int_x^\infty e^{-t} \frac{dt}{t}$ using approximation formulae given in Reference 4.

E1 is used by NAIEF in calculating the probability that an isotropically emitted iodine K x-ray escapes from the uniformly thick sheet of NaI.

NAIEF calculates ε, p_α, and p_β for x-rays between 10 and 199.9 keV normally incident on a 1 mm. thick NaI crystal. The formulae used are

$$\varepsilon(E) = 1 - e^{-\mu x_o} \quad \text{where } x_o = .367 \text{ g/cm}^2 \text{ and}$$

$$\mu = 124\left(\frac{E}{10}\right)^{-2.7778} \quad \text{for } 10 \leqslant E \leqslant 33.23 \text{ keV}$$

$$\mu = 30.8\left(\frac{E}{33.23}\right)^{-2.6596} \quad \text{for } 33.23 < E < 199.9 \text{ keV}$$

$$p_\alpha(E) = \int_0^{x_o} dx \; \mu e^{-\mu x} \quad \tfrac{1}{2}\delta\omega f_\alpha\Big(e^{-\mu_\alpha x} + e^{-\mu_\alpha(x_o-x)} - \mu_\alpha x E_1(\mu_\alpha x) - \mu_\alpha(x_o-x)E_1\big(\mu_\alpha(x_o-x)\big)\Big)$$

$$p_\beta(E) = \int_0^{x_\circ} dx\mu e^{-\mu x} \quad \tfrac{1}{2}\delta\omega f_\beta \Big(e^{-\mu_\beta x} + e^{-\mu_\beta(x_\circ - x)} -$$

$$\mu_\beta x E_1(\mu_\beta x) - \mu_\beta(x_\circ - x)E_1\big(\mu_\beta(x_\circ - x)\big)\Big)$$

δ = DELTA = fraction of photoelectric processes taking place in the K shell = .875

ω = OMEGA = K fluorescence yield of iodine = .84

μ_α = AATTEN = $\mu(E_{K\alpha})$ = 6.62 cm^2/g

μ_β = BATTEN = $\mu(E_{K\beta})$ = 4.66 cm^2/g

f_α = ABRANC = fraction of K x-rays which are K_α x-rays = .7937

f_β = BBRANC = fraction of K x-rays which are K_β x-rays = .2063

FORTRAN SOURCE LIST

ISN SOURCE STATEMENT

```
 $IBFTC AIRAT
       DIMENSION ETAB(11), SIGMA(3, 11), EPLOT(1900), TABLE(1900),
      1 A(3), B(3), C(3), FRAC(3), CONV(3), POWER(3), COEFF(3)
       READ(5, 25) ETAB, ((SIGMA(J,K), K = 1, 11), J = 1, 3)
 25    FORMAT(11F6.1/11F6.2/11F6.2/11F6.2)
       FRAC(1) = .7556
       FRAC(2) = .2315
       FRAC(3) = .0129
       CONV(1) = .04301
       CONV(2) = .03765
       CONV(3) = .01508
       POWER(1) = -3.28443
       POWER(2) = -3.30707
       POWER(3) = -3.071145
       COEFF(1) = 77.0
       COEFF(2) = 142.0
       COEFF(3) = 4230.0
       DE = 0.1
       J = 1
       DO 500 N = 1, 1900
       EN = N
       E = (EN - 1.0)*DE + ETAB(1)
       EPLOT(N) = E
       IF(E.LT.ETAB(J)) GO TO 35
       DO 30 K = 1, 3
       CALL INTERP(ETAB(J), ETAB(J+1), ETAB(J+2), SIGMA(K,J),
      1 SIGMA(K, J+1), SIGMA(K, J+2), A(K), B(K), C(K))
 30    CONTINUE
       J = J+1
 35    CONTINUE
       TABLE(N) = 0.0
       DO 100 M = 1, 3
       TABLE(N) = TABLE(N) + FRAC(M)*CONV(M)*(COEFF(M)*(E/ETAB(1))
      1 **POWER(M) + A(M)*E**2 + B(M)*E + C(M))
 100   CONTINUE
 500   CONTINUE
       WRITE(6, 510)
 510   FORMAT(1H1, 55HPHOTON ATTENUATION COEFFICIENTS IN CM**2/GM FOR DRY
      1 AIR    / 1H0, 6HENERGY    )
       DO 550 NA = 1, 1900, 10
       IF(MOD(NA, 500).EQ.1.AND.NA.NE.1) WRITE(6, 515)
       CONTINUE
 515   FORMAT(1H1)
       NB = NA + 9
       WRITE(6, 520) EPLOT(NA), (TABLE(JA), JA = NA, NB)
 520   FORMAT(1H , F4.0, 10F8.4)
 550   CONTINUE
       WRITE(7, 560) (TABLE(JB), JB = 1, 1900)
 560   FORMAT(10F7.4)
       CALL EXIT
       END
```

INPUT DATA CARDS FOR PROGRAM AIRAT (ENERGIES AND COMPTON ATTENUATION COEFFICIENTS OF NITROGEN, OXYGEN, AND ARGON AT THESE ENERGIES)

```
  10.0  15.0  20.0  30.0  40.0  50.0  60.0  80.0 100.0 150.0 200.0
  8.96  6.72  5.73  4.84  4.45  4.14  3.98  3.73  3.54  3.15  2.87
 11.50  8.28  6.95  5.77  5.18  4.80  4.61  4.30  4.06  3.61  3.29
 56.00 36.00 28.00 19.00 15.80 13.60 12.40 10.80  9.85  8.43  7.57
```

ISN SOURCE STATEMENT

```
  $IBFTC NAIEF
        DIMENSION EXPTAB(2000), EFFIC(1900), ESCKA(1900), ESCKB(1900),
       1 FRACKA(200), FRACKB(200)
        DO 5 JEX = 1, 2000
        X = FLOAT(JEX - 1)/200.0
        EXPTAB(JEX) = EXP(-X)
    5   CONTINUE
  C   ATTENUATION COEFFICIENTS FOR IODINE K ALPHA AND BETA
        AATTEN = 6.62
        BATTEN = 4.66
        DELTA = .875
        OMEGA = .84
  C   RELATIVE NUMBERS OF K ALPHA AND BETA PHOTONS PRODUCED
        ABRANC = .7937
        BBRANC = .2063
        DO 20 JDEPTH = 1, 200
        DEPTH = .001835*FLOAT(JDEPTH)
  C   .367 = THICKNESS OF NAI CRYSTAL CONSIDERED (IN G/CM**2)
        HEIGHT = .367 - DEPTH
        XFA = AMAX1(.000001, DEPTH*AATTEN)
        XFB = AMAX1(.000001, DEPTH*BATTEN)
        XRA = AMAX1(.000001, HEIGHT*AATTEN)
        XRB = AMAX1(.000001, HEIGHT*BATTEN)
        FRACKA(JDEPTH) = 0.5*DELTA*OMEGA*ABRANC*(EXP(-XFA) + EXP(-XRA) -
       1 XFA*E1(XFA) - XRA*E1(XRA))
        FRACKB(JDEPTH) = 0.5*DELTA*OMEGA*BBRANC*(EXP(-XFB) + EXP(-XRB) -
       1 XFB*E1(XFB) - XRB*E1(XRB))
   20   CONTINUE
        DO 100 JENRGY = 1, 1900
        ENRGY = 10.0 + FLOAT(JENRGY - 1)/10.0
        EFFIC(JENRGY) = 0.0
        ESCKA(JENRGY) = 0.0
        ESCKB(JENRGY) = 0.0
        IF(ENRGY.GT.33.23) GO TO 25
        ATTEN = 124.0*(ENRGY/10.0)**(-2.7778)
        EFFIC(JENRGY) = 1.0 - EXP(-.367*ATTEN)
        GO TO 100
   25   CONTINUE
        ATTEN = 30.8*(ENRGY/33.23)**(-2.6596)
        DATTEN = .001835*ATTEN
        DO 80 KDEPTH = 1, 200
        DEPTH = .001835*FLOAT(KDEPTH)
        NLAMDA = DEPTH*ATTEN*200.0 + 1.0
        IF(NLAMDA.GT.2000) GO TO 35
        FLUX = EXPTAB(NLAMDA)
        DEFFIC = DATTEN*FLUX
        EFFIC(JENRGY) = EFFIC(JENRGY) + DEFFIC
        ESCKA(JENRGY) = ESCKA(JENRGY) + DEFFIC*FRACKA(KDEPTH)
        ESCKB(JENRGY) = ESCKB(JENRGY) + DEFFIC*FRACKB(KDEPTH)
   35   CONTINUE
   80   CONTINUE
  100   CONTINUE
        WRITE(6, 105)
  105   FORMAT(1H1, 67HDEC 6, 1966      PHOTOELECTRIC EFFICIENCY OF 1 MM. T
       1HICK NAI CRYSTAL     /1H , 75H(FRACTION OF NORMALLY INCIDENT PHOTO
```

FORTRAN SOURCE LIST NAIEF

```
ISN        SOURCE STATEMENT

          2NS THAT ARE ABSORBED PHOTOELECTRICALLY)      /1HO, 6HENERGY    )
 70         DO 120 NLINE = 1, 190
 71         IF(NLINE.EQ.51.OR.NLINE.EQ.101.OR.NLINE.EQ.151) WRITE(6,110)
 74         CONTINUE
 75   110   FORMAT(1H1)
 76         NENRGY = NLINE + 9
 77         NSTART = (NLINE - 1)*10 + 1
100         NSTOP = NSTART + 9
101         WRITE(6, 115) NENRGY, (EFFIC(NA), NA = NSTART, NSTOP)
106   115   FORMAT(1H , I3, 10F7.4)
107   120   CONTINUE
111         WRITE(7, 121) EFFIC
112   121   FORMAT(10F7.5)
113         WRITE(6, 125)
114   125   FORMAT(1H1, 88HDEC 6, 1966      PROBABILITY THAT A PHOTON INCIDENT
          1NORMALLY ON A 1 MM. THICK NAI CRYSTAL         )
115         WRITE(6, 126)
116   126   FORMAT(1H , 79H                   PRODUCES AN IODINE K ALPHA X-RAY WH
          1ICH ESCAPES FROM THE CRYSTAL      /1HO,   6HENERGY  )
117         DO 130 MLINE = 1, 190
120         IF(MLINE.EQ.51.OR.MLINE.EQ.101.OR.MLINE.EQ.151) WRITE(6,110)
123         MENRGY = MLINE + 9
124         MSTART = (MLINE - 1)*10 + 1
125         MSTOP = MSTART + 9
126         WRITE(6, 115) MENRGY, (ESCKA(MA), MA = MSTART, MSTOP)
133   130   CONTINUE
135         WRITE(7, 121) ESCKA
136         WRITE(6, 125)
137         WRITE(6, 136)
140   136   FORMAT(1H , 78H                   PRODUCES AN IODINE K BETA X-RAY WHI
          1CH ESCAPES FROM THE CRYSTAL     /1HO, 6HENERGY    )
141         DO 140 LLINE = 1, 190
142         IF(LLINE.EQ.51.OR.LLINE.EQ.101.OR.LLINE.EQ.151) WRITE(6,110)
145         LENRGY = LLINE + 9
146         LSTART = (LLINE - 1)*10 + 1
147         LSTOP = LSTART + 9
150         WRITE(6, 115) LENRGY, (ESCKB(LA), LA = LSTART, LSTOP)
155   140   CONTINUE
157         WRITE(7, 121) ESCKB
160         CALL EXIT
161         END
```

```
                                        FORTRAN SOURCE LIST
ISN        SOURCE STATEMENT

  0 $IBFTC E1
  1        FUNCTION E1(X)
  2        Y = ABS(X)
  3        X2 = Y*Y
  4        X3 = X2*Y
  5        IF(Y.GT.1.0) GO TO 10
 10        X4 = X2*X2
 11        X5 = X4*Y
 12        E1 = -ALOG(X) - .57721566 + .99999193*Y - .24991055*X2 +
          1 .05519968*X3 - .00976004*X4 + .00107857*X5
 13        RETURN
 14   10   IF(Y.GT.10.0) GO TO 20
 17        E1 = EXP(-Y)*(X2 + 2.334733*Y + .250621)/(X3 + 3.330657*X2 +
          1 1.681534*Y)
 20        RETURN
 21   20   E1 = EXP(-Y)*(X2 + 4.03640*Y + 1.15198)/(X3 + 5.03637*X2 +
          1 4.19160*Y)
 22        RETURN
 23        END
```

```
                                        FORTRAN SOURCE LIST
ISN        SOURCE STATEMENT

  0 $IBFTC INTERP
  1        SUBROUTINE INTERP(X1, X2, X3, Y1, Y2, Y3, A, B, C)
  2        X12 = X1**2
  3        X22 = X2**2
  4        X32 = X3**2
  5        DET = X12*(X2 - X3) - X1*(X22 - X32) + (X3*X22 - X2*X32)
  6        A = ((X2-X3)*Y1 + (X3-X1)*Y2 + (X1-X2)*Y3)/DET
  7        B = ((X32-X22)*Y1 + (X12-X32)*Y2 + (X22-X12)*Y3)/DET
 10        C = ((X3*X22-X2*X32)*Y1 + (X1*X32-X3*X12)*Y2 + (X2*X12-X1*X22)*Y3)
          1 /DET
 11        RETURN
 12        END
```

IV Data Presentation

A logically complete way to present the results of the foregoing analysis is to give the coefficient K found for each energy interval as a function of kT or N or other parameters. This is also a correct way if for some one value of the parameters K is nearly the same for all energy intervals. This is essentially the procedure followed by Clark[5] in his measurement of the spectrum of the Crab Nebula between 15 and 62 keV. Since then, however, results of x-ray observations from balloons have been presented as energy spectra characteristic of the x-ray sources alone, with the effects of finite energy resolution and K x-ray escape supposedly removed.

Our method of spectrum presentation described below is one which approaches correctness as the range of K values for different energy bands approaches zero for some value of a spectrum parameter such as kT or N. Let us refer to this "best" value of the parameter as $k\hat{T}$. The ordinates in a graph of the x-ray differential number spectrum, $\frac{dn}{dEdAdt}$, are then taken to be $\frac{N_{ab}(k\hat{T})}{E_b-E_a}$, $\frac{N_{bc}(k\hat{T})}{E_c-E_b}$, etc. where $N_{ab}(kT)$ is as defined in section II of this report. For the abscissa corresponding to an energy band from E_j to E_k we pick the energy E' such that

$$K_{jk}\frac{e^{-E'/k\hat{T}}}{E'} = \frac{N_{jk}(k\hat{T})}{E_k-E_j}$$

$$\text{where } K_{jk} = \frac{AE_o\, e^{E_o/k\hat{T}}\, n_{jk}}{\int_{E_j}^{E_k}\left(\frac{dn}{dE}(k\hat{T})\right)_3 dE}$$

Thus E' is the energy at which the spectrum equals its average value between E_j and E_k. In terms of K_{jk} the ordinate is

$$\frac{K_{jk}}{E_k - E_j} \int_{E_j}^{E_k} e^{-E/k\hat{T}} \frac{dE}{E}$$

Similarly in a graph of the differential energy spectrum, $E \frac{dn}{dEdAdt}$, the ordinate is

$$\frac{K_{jk}}{E_k - E_j} \int_{E_j}^{E_k} e^{-E/k\hat{T}} dE$$

and the abscissa is the E' such that

$$K_{jk} e^{-E'/k\hat{T}} = \frac{K_{jk}}{E_k - E_j} \int_{E_j}^{E_k} e^{-E/k\hat{T}} dE$$

Through the point defined by ordinate and abscissa pass a horizontal bar extending from E_j to E_k and a vertical bar whose length represents the error in the determination of N_{jk}, atmospheric depth, and detector properties. The horizontal bar is not to be interpreted as an error bar or as a measure of energy resolution, but just shows the width of the pulse height channel which yielded the data. This is our intent in references 2 and 3 and we believe it is the intent of the other workers in this field. Our vertical error bar does not reflect any error in the process of finding $k\hat{T}$ or finding which parameter, kT, N, or some other, is most appropriate.

If we assume $k\hat{T}$ = 4.3 keV and one standard deviation in n_{ab} = .002 pulses/cm^{2}sec is .0008 pulses/cm^{2}sec then the ordinate in a graph of $\frac{dn}{dEdAdt}$ (Figure 1a) is

$$\frac{N_{ab}(\hat{kT})}{E_b - E_a} = \frac{.01666}{23.3} = 7.15 \times 10^{-4} \text{ photons/cm}^2\text{sec keV}$$

and the vertical error bar extends from 0.6 to 1.4 times 7.15 x 10^{-4}. So $K_{ab} = 24.68 \pm 9.87$ and the abscissa, E' = 30.3 keV. Note that $\hat{kT}$ = 4.3 keV implies that the flux at the middle of the energy band, 35 keV, is very different from that at 30.3 keV. The curved line in Figure 1a is the assumed spectrum at the top of the atmosphere. Figure 2 shows this and the pulse height spectra our detector would produce in case there were 4.0 and 5.5 g/cm^2 of air along the line of sight. Figure 1B shows the differential energy spectrum $E \frac{dn}{dEdAdt}$. The ordinate is 1.97 x 10^{-2} keV/cm^{2}sec keV and the abscissa is 30.7 keV.

Of course the one datum considered in this example in no way justifies the choice of $\hat{kT}$ = 4.3 keV. The example was taken from an observation of Scorpius X-1. Previous observations of this source by other groups had established the value of $\hat{kT}$. The example was chosen because the steepness of the spectrum necessitates careful presentation of the spectral data and the weakness of the source necessitated consolidation of data over a wide energy range.

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4. M. Abramowitz and I.A. Stegun, editors, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Dover Publications, Inc., New York (1965), page 231.

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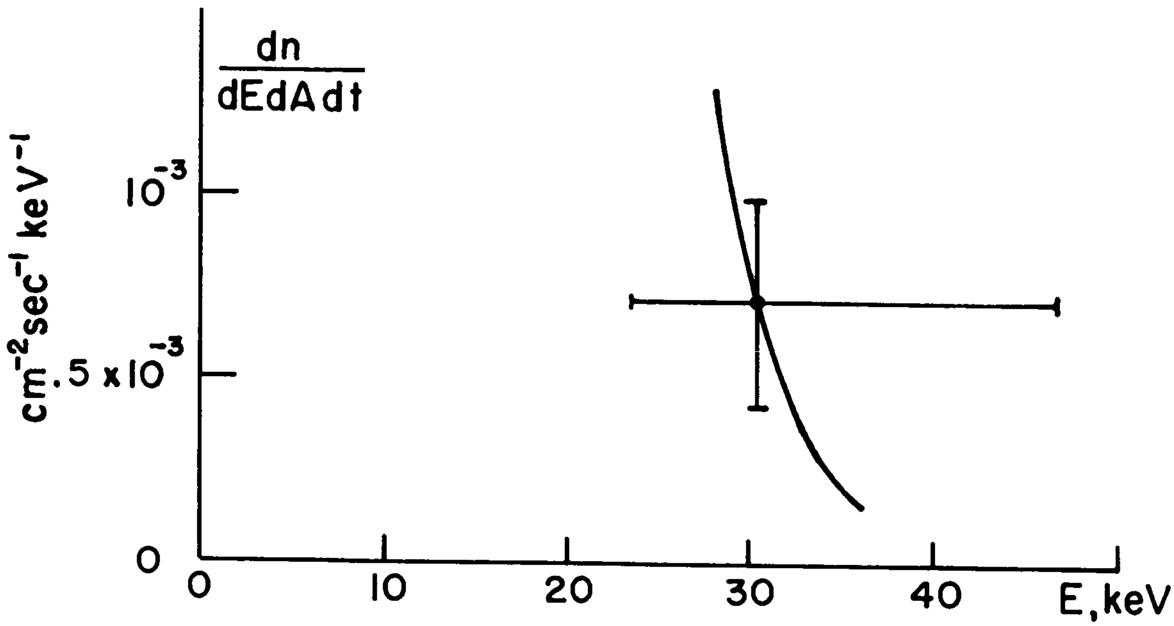


Figure 1a

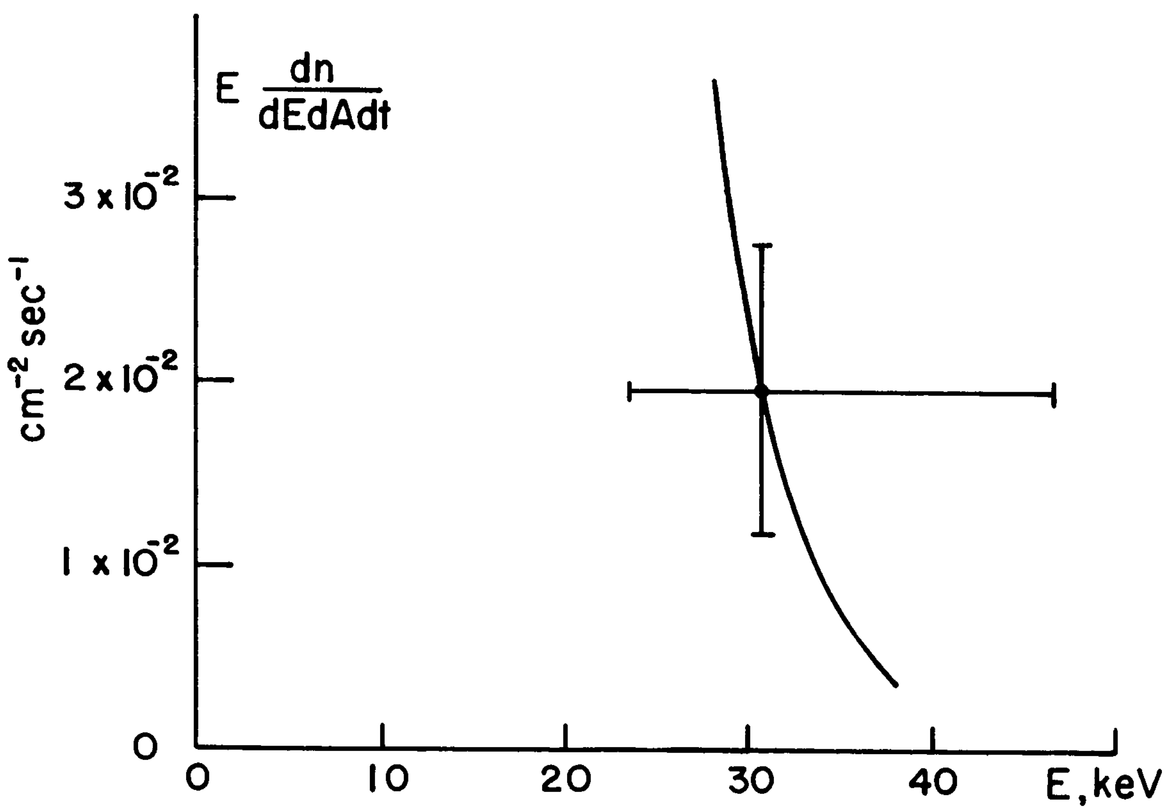
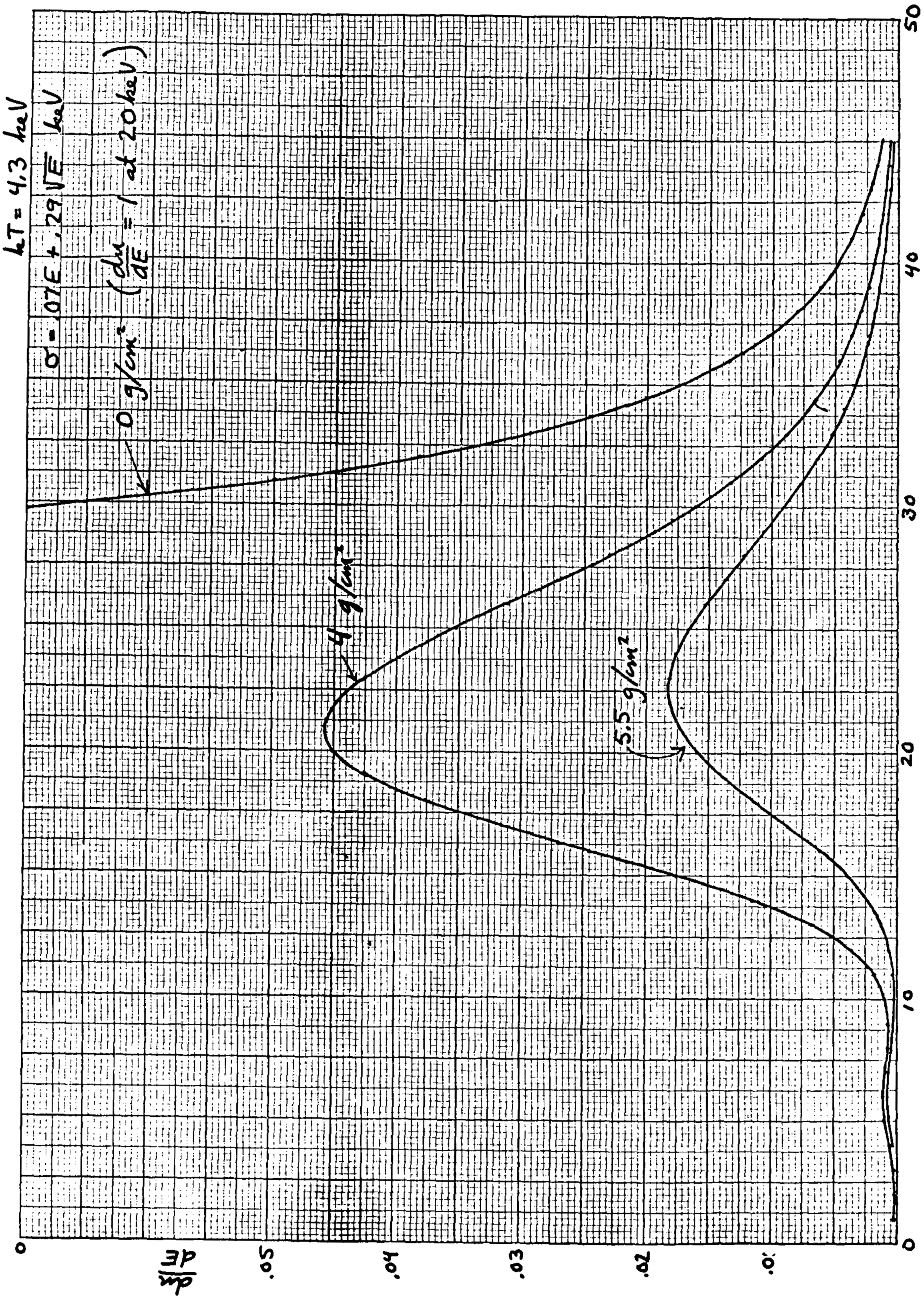


Figure 1b



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