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ANALYSIS OF THREE-FLUID, CROSSFLOW HEAT EXCHANGERS

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Manned Spacecraft Center
Houston, Texas

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ABSTRACT

The detailed behavior of three-fluid, crossflow heat exchangers has been investigated. The equations governing the two-dimensional temperature distributions of the three fluids have been derived and nondimentionalized. Performance characteristics have been determined for a wide range of operating parameters for single-pass heat exchangers. The performance of two-pass heat exchangers for both cocurrent and countercurrent flow has been studied for selected operating conditions. Results have been presented graphically in terms of the temperature effectiveness of the two outer fluids as functions of heat-exchanger size for sets of fixed operating conditions. Nondimensional operating parameters have been defined which allow an efficient presentation of the large volume of performance data required to represent a practical range of operating conditions. Sample problems are included to illustrate the use of the performance graphs for design applications.

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SUMMARY

The detailed behavior of three-fluid, crossflow heat exchangers has been investigated. The equations governing two-dimensional temperature distributions of the three fluids have been derived and nondimensionalized. The performance characteristics have been determined for a wide range of operating parameters for single-pass heat exchangers. Performance of two-pass heat exchangers for both cocurrent and countercurrent flow has been studied for selected operating conditions. Results have been presented graphically in terms of the temperature effectiveness of the two outer fluids as functions of heat-exchanger size for sets of fixed operating conditions. Interpolation techniques have been used to obtain performance data for intermediate values. Nondimensionalized operating parameters have been defined which allow an efficient presentation of the large volume of performance data required to represent a practical range of operating conditions. Sample problems are included to illustrate the use of the performance curves and the interpolation techniques.

An expression for overall effectiveness has been derived which compares the heat transferred by a particular exchanger with that transferred by one of infinite size. Isolated cases, corresponding to poor design, are cited in which the overall effectiveness may be greater than unity. This effect emphasizes the importance of using the temperature effectiveness of the two outer fluids as the primary design variables and the overall effectiveness as an auxiliary parameter.

A computer program has been developed for the study of both single- and multiple-pass heat exchangers. Output options are available for detailed studies of temperature distributions within a particular exchanger and for generation of performance data for a large number of heat exchangers.

INTRODUCTION

Considerable effort has been expended in previous investigations to define performance characteristics of heat exchangers involving energy transfer between two fluids. Now that industrial processes have been developed which require simultaneous heat exchange between more than two fluids, analytical techniques are needed to describe the performance of multifluid heat exchangers. One example of such a process is the large-scale production of oxygen in an air separation plant which requires heat exchange between oxygen, nitrogen, and air at very low temperatures. There are also possibilities for combining several separate two-fluid heat-exchanging operations more economically in a single multifluid arrangement.

Several investigators (refs. 1 to 5) have pursued the problem of multifluid heat exchangers in parallel or counterflow in which only one physical dimension of the exchanger is considered. The purpose of this investigation is the detailed study of three-fluid heat exchangers in crossflow as a completely two-dimensional problem.

In this study, the performance of three-fluid crossflow heat exchangers is determined and presented graphically in terms of the temperature effectiveness of two of the fluids referred to the third fluid. The effectiveness is determined as a function of heat-exchanger size for sets of fixed operating conditions. The introduction of nondimensional operating variables reduces the volume of data required to represent a practical range of operating conditions. The number of boundary conditions for the temperatures is reduced from three to one by the introduction of a nondimensional inlet temperature parameter.

An expression for overall effectiveness is derived which compares the performance of a heat exchanger to that of an infinitely large exchanger operating at the same conditions. A study of the two-dimensional temperature distributions reveals circumstances in which the overall effectiveness may be greater than unity. This result implies that in a three-fluid, crossflow heat exchanger, total heat transfer is not always maximized by increasing the size of the exchanger.

Effectiveness factors are determined for a wide range of operating parameters for single-pass, three-fluid heat exchangers. Performance of multiple-pass, three-fluid heat exchangers for both cocurrent and countercurrent flow is studied for selected operating conditions.

Sample problems are used to illustrate the application of the effectiveness curves to heat-exchanger design problems. Since some of the performance data can be explained only in terms of the two-dimensional variation of the temperatures of each fluid, these problem solutions are also used to provide insight into the detailed behavior of the fluids within the heat exchangers.

The basic differential equations for the spatial distribution of the temperatures of the three fluids were solved numerically using a digital computer. A program is available for both single- and multiple-pass calculations. An automatic-integration step-size control was developed through the consideration of overall conservation of energy so that multiple cases may be run continuously with the optimum step size used for each individual case. Output options are available for a detailed study of spatial temperature distribution within the exchanger or for the determination of the overall performance using only the average exit temperatures and effectiveness values.

SYMBOLS

A	$\frac{u_{1,2}x_o y_o}{\dot{m}_1 c_{p,1}}$
B	$\frac{u_{2,3}x_o y_o}{\dot{m}_3 c_{p,3}}$
C	$\frac{u_{1,2}x_o y_o}{\dot{m}_2 c_{p,2}}$
c_p	specific heat of fluid
$c_{p,j}$	specific heat of fluid (j)
D	$\frac{u_{2,3}x_o y_o}{\dot{m}_2 c_{p,2}}$
E	overall effectiveness, $\frac{Q_a}{Q_\infty}$
K_1	$\frac{\dot{m}_1 c_{p,1}}{\dot{m}_2 c_{p,2}}$, capacity rate ratio for fluid (1)
K_3	$\frac{\dot{m}_3 c_{p,3}}{\dot{m}_2 c_{p,2}}$, capacity rate ratio for fluid (3)
M	largest value of the set (A, B, C, D)
$\dot{m}$	mass flow rate of fluid
NTU_1	number of transfer units of a heat exchanger referred to fluid (1) (equal to A)
NTU_3	number of transfer units of a heat exchanger referred to fluid (3) (equal to B)
$\overline{Q}$	nondimensional heat-transfer rate, $\frac{Q}{\dot{m}_2 c_{p,2}(t_{1,i} - t_{2,i})}$
Q_a	total heat-transfer rate in a heat exchanger of finite size

Q_∞ total heat-transfer rate in a counterflow heat exchanger of infinite size

q_j heat transferred to fluid (j) per unit time

T nondimensional temperature, $\frac{t - t_{2,i}}{t_{1,i} - t_{2,i}}$ (all subscripts listed for t are applied to T also)

t local temperature of fluid

$t_{1,3,mix}$ inlet mixing temperature of fluids (1) and (3), $\frac{\dot{m}_1 c_{p,1} t_{1,i} + \dot{m}_3 c_{p,3} t_{3,i}}{\dot{m}_1 c_{p,1} + \dot{m}_3 c_{p,3}}$

$t_{2,e,\infty}$ exit temperature of fluid (2) for an infinitely large heat exchanger

$t_{2,max}(X=0)$ maximum value of t_2 along the y-axis for an infinitely large heat exchanger, $\frac{Ut_{1,i} + t_{3,i}}{U + 1}$

t_j local temperature of fluid (j)

$t_{j,e}$ exit temperature of fluid (j)

$t_{j,em}$ average exit temperature of fluid (j)

$t_{j,i}$ inlet temperature of fluid (j)

U conductance ratio, $\frac{u_{1,2}}{u_{2,3}}$

$u_{1,2}$ overall conductance between fluids (1) and (2)

$u_{2,3}$ overall conductance between fluids (2) and (3)

X nondimensional coordinate of heat-exchanger surface, $\frac{x}{x_o}$

x coordinate of heat-exchanger surface

x_o x-dimension of heat exchanger

Y	nondimensional coordinate of heat-exchanger surface, $\frac{y}{y_o}$
y	coordinate of heat-exchanger surface
y_o	y-dimension of heat exchanger
Δt_i	inlet temperature parameter, $\frac{t_{1,i} - t_{2,i}}{t_{3,i} - t_{2,i}}$
θ_1	temperature effectiveness of fluid (1), $\frac{t_{1,i} - t_{1,em}}{t_{1,i} - t_{2,i}}$
θ_3	temperature effectiveness of fluid (3), $\frac{t_{3,i} - t_{3,em}}{t_{3,i} - t_{2,i}}$

Subscripts:

1	fluid (1)
2	fluid (2)
3	fluid (3)
c	cold fluid
e	exit (temperature)
em	average exit (temperature)
h	hot fluid
i	inlet
ident	identical order
inv	inverted order
j	fluid (j); general or typical reference to fluid (1), (2), or (3)
m	average value (of temperature normal to fluid flow direction (appendix A))
max	maximum value
min	minimum value

mix	inlet mixing (temperature)
N	number of integration step in x direction (appendix A)

Superscripts:

c	corrected
p	predicted

PROBLEM FORMULATION

Derivation of the Governing Equations for Three-Fluid, Crossflow Heat Exchangers

Figure 1 is a schematic representation of a single-pass, three-fluid, crossflow heat exchanger. Heat is transferred between the center fluid (2) and outer fluids (1) and (3); however, there is no heat directly transferred between the two outer fluids. The immediate objective is to determine the temperature distributions of the three fluids in the heat exchanger for a given size and operating condition. Once the temperature distributions are known, the heat transferred to each fluid may be calculated from average exit temperatures; subsequently, the performance of the heat exchanger may be evaluated.

In this investigation, the following simplifying assumptions have been made to reduce the complexity of the equations.

1. Steady flow exists for the three fluids.

2. Fluid properties are constant. This assumption is adequate where large differences do not exist between the temperatures of the fluid and the heat-transfer surface. In most cases, evaluation of properties at a mixed mean temperature is sufficient to correct for property variations. A discussion of the effect of temperature-dependent fluid properties may be found in reference 6.

3. For a particular surface, the local conductance is constant and equal to the overall conductance. This assumption is consistent with steady flow and constant fluid properties if entrance effects are neglected.

4. The heat exchanger is considered to be adiabatic. If the performance of any heat exchanger is degraded because of heat exchange with the surroundings, it can be insulated. Effects caused by such heat exchange are not of interest in this investigation.

5. The effects of longitudinal and lateral conduction in the heat exchanger are neglected. These effects are important if large temperature gradients exist and will reduce heat-exchanger effectiveness.

6. There is no lateral mixing in any fluid. This behavior is closely approximated in plate-fin heat exchangers and when flows are not baffled. This assumption preserves the two-dimensional character of the problem. If mixing were assumed, the analysis would be simplified considerably.

Under the previous assumptions, the governing equations for the temperature distributions in three-fluid, crossflow heat exchangers will be derived. Figure 2 represents the heat-transfer surface between fluids (1) and (2). For a properly designed exchanger, outer fluids (1) and (3) will be either both hotter or both colder than the center fluid (2). For purposes of discussion during this derivation, the center fluid is arbitrarily assumed to be hotter than the two outer fluids; however, the resulting equations are independent of the assumed temperature levels.

In figure 2 the heat transferred per unit time into fluid (1) from fluid (2) across the elemental area $dx\,dy$ is

$$dq_1 = u_{1,2}(t_2 - t_1)dx\,dy \tag{1}$$

where t_1 and t_2 are both functions of x and y.

This expression may be equated to the energy increase per unit time of the element of fluid (1) between y and $y + dy$ as it moves from x to $x + dx$, which is

$$dq_1 = c_{p,1}\left(\dot{m}_1 \frac{dy}{y_o}\right)\frac{\partial t_1}{\partial x}\,dx \tag{2}$$

Introducing nondimensional coordinates $X = \frac{x}{x_o}$ and $Y = \frac{y}{y_o}$ and equating the above expressions for dq_1, the resulting differential equation is

$$\frac{\partial t_1}{\partial X} = \frac{u_{1,2}x_o y_o}{\dot{m}_1 c_{p,1}}\left(t_2 - t_1\right) \tag{3}$$

Following a similar procedure for fluid (3) gives

$$\frac{\partial t_3}{\partial X} = \frac{u_{2,3}x_o y_o}{\dot{m}_3 c_{p,3}}\left(t_2 - t_3\right) \tag{4}$$

A differential volume element of fluid (2) is bounded by two surfaces and is in thermal communication with both fluids (1) and (3). The energy transferred to this element of fluid (2) from the outer fluids is

$$dq_2 = \left[u_{1,2}\left(t_1 - t_2\right) + u_{2,3}\left(t_3 - t_2\right)\right] dx\, dy \tag{5}$$

As the elemental volume of fluid (2) between x and x + dx moves from y to y + dy, its thermal energy increase may be expressed as

$$dq_2 = c_{p,2}\left(\dot{m}_2 \frac{dx}{x_o}\right) \frac{\partial t_2}{\partial y}\, dy \tag{6}$$

Equating these two expressions and introducing the nondimensional coordinates yields

$$\frac{\partial t_2}{\partial Y} = \frac{u_{1,2}x_o y_o}{\dot{m}_2 c_{p,2}}\left(t_1 - t_2\right) + \frac{u_{2,3}x_o y_o}{\dot{m}_2 c_{p,2}}\left(t_3 - t_2\right) \tag{7}$$

Three simultaneous partial differential equations have been derived which define the temperatures of each of the three fluids as functions of both space coordinates x and y. The resulting equations are

$$\left.\begin{aligned} \frac{\partial t_1}{\partial X} &= A\left(t_2 - t_1\right) \\ \frac{\partial t_2}{\partial Y} &= C\left(t_1 - t_2\right) + D\left(t_3 - t_2\right) \\ \frac{\partial t_3}{\partial X} &= B\left(t_2 - t_3\right) \end{aligned}\right\} \tag{8}$$

where

$$
\left.
\begin{aligned}
A &= \frac{u_{1,2}x_o y_o}{\dot{m}_1 c_{p,1}} & \qquad C &= \frac{u_{1,2}x_o y_o}{\dot{m}_2 c_{p,2}} \\
B &= \frac{u_{2,3}x_o y_o}{\dot{m}_3 c_{p,3}} & \qquad D &= \frac{u_{2,3}x_o y_o}{\dot{m}_2 c_{p,2}}
\end{aligned}
\right\} \tag{9}
$$

and

$$
t_j = t_j\,(x, y) \tag{10}
$$

Under the assumptions of the problem, the nondimensional terms A, B, C, and D are constants which depend upon the heat-capacity rates of the fluids, the heat-exchanger dimensions, and the values of conductance at the two heat-transfer surfaces. When A, B, C, and D are specified, along with the inlet temperatures of the three fluids, the equations may be solved for t_1, t_2, and t_3 as functions of position throughout the heat exchanger.

Reduction of the number of boundary conditions. - To nondimensionalize the basic equations with respect to temperature t_j, all temperatures can be referred to the inlet temperature of the center fluid $t_{2,i}$ by subtracting $t_{2,i}$ from the temperature t_j and dividing this quantity by the difference between the inlet temperatures of fluids (1) and (2), or by $t_{1,i} - t_{2,i}$. The resulting equations are

$$
\left.
\begin{aligned}
\frac{\partial T_1}{\partial X} &= A\left(T_2 - T_1\right) \\
\frac{\partial T_2}{\partial Y} &= C\left(T_1 - T_2\right) + D\left(T_3 - T_2\right) \\
\frac{\partial T_3}{\partial X} &= B\left(T_2 - T_3\right)
\end{aligned}
\right\} \tag{11}
$$

where

$$\left.\begin{aligned} T_1 &= \frac{t_1 - t_{2,i}}{t_{1,i} - t_{2,i}} \\ T_2 &= \frac{t_2 - t_{2,i}}{t_{1,i} - t_{2,i}} \\ T_3 &= \frac{t_3 - t_{2,i}}{t_{1,i} - t_{2,i}} \end{aligned}\right\} \tag{12}$$

The advantage of the above formulation is apparent when the boundary conditions are examined. They become

$$\left.\begin{aligned} T_1(X=0) &= 1 \\ T_2(Y=0) &= 0 \\ T_3(X=0) &= \frac{t_{3,i} - t_{2,i}}{t_{1,i} - t_{2,i}} \end{aligned}\right\} \tag{13}$$

The boundary conditions for any problem may be specified by a single quantity called the inlet temperature parameter

$$\Delta t_i = \frac{t_{1,i} - t_{2,i}}{t_{3,i} - t_{2,i}} \tag{14}$$

This parameter is the ratio of the temperature levels of the two outer fluids referred to the temperature level of the center fluid. The third boundary condition becomes $T_{3,i} = \frac{1}{\Delta t_i}$. The outer fluids may be numbered so that Δt_i varies between zero and unity. As an example, in the case of both outer fluids being hotter than the center fluid, fluid (1) is always the colder of the two hot fluids. When Δt_i is unity, $t_{1,i}$ equals $t_{3,i}$; and when Δt_i is very small, $t_{3,i}$ is considerably greater than $t_{1,i}$.

For example, consider two sets of inlet temperatures

$$\left.\begin{aligned} t_{1,i} &= 300^\circ \text{ F} \\ t_{2,i} &= 100^\circ \text{ F} \\ t_{3,i} &= 500^\circ \text{ F} \end{aligned}\right\} \tag{15}$$

and

$$\left.\begin{aligned} t_{1,i} &= 50^\circ \text{ F} \\ t_{2,i} &= 0^\circ \text{ F} \\ t_{3,i} &= 100^\circ \text{ F} \end{aligned}\right\} \tag{16}$$

In both cases, Δt_i equals 0.5, and the problems are equivalent in the foregoing nondimensional formulation.

Now that the specification of the inlet temperatures has been reduced to a single parameter, any problem may be completely defined by the five quantities A, B, C, D, and Δt_i.

Multiple-pass, three-fluid, crossflow heat exchangers. - The use of multiple passes has long been recognized as a possible method of improving heat-exchanger performance. The previous investigations reported in references 7 and 8 have considered the problem of multiple-pass, two-fluid, crossflow heat exchangers.

While the previous derivation and discussion has been applied to determining the temperature distributions in a single-pass heat exchanger, the basic procedure can also be applied to each pass of a multiple-pass heat exchanger. The extension of the analysis to multiple passes involves only the proper specification of boundary conditions for each pass for the several possible flow arrangements. The configuration for a two-pass, three-fluid, crossflow heat exchanger is illustrated in figure 3. This particular arrangement is called countercurrent. A cocurrent arrangement is obtained by reversing the indicated direction of fluid (2). The flow detail for each pass is the same as that illustrated in figure 1.

The solution of the cocurrent case is straightforward; the outlet temperatures of the first pass become inlet temperatures of the second pass, and so on, for all subsequent passes. The solution in any pass is independent of the solutions of all subsequent passes. The temperature distribution in any pass is found in the same manner as in the single-pass heat exchanger, with the exception that the initial temperature generally will not be constant along the inlet boundaries after the first pass.

The problem is somewhat more difficult for the countercurrent heat exchanger. Figure 4 represents the mathematical configuration for this case. The difficulty arises because the initial temperature distributions are not completely known for either pass. Using the terminology in reference 7, there are "outer" boundaries where the initial conditions for the whole exchanger are given in "inner" boundaries which are effectively common to the two passes. The inlet temperature for fluid (2) in pass 1 along an inner boundary is dependent upon the solution in pass 2. Since it is evident that the solution in pass 2 is dependent in turn on pass 1, some iterative scheme is suggested using assumed distributions along inner boundaries.

Following the basic scheme outlined in reference 7, an inlet value of T_2 is assumed for pass 1, and the resulting exit values of T_1 and T_3 are used as input for pass 2. The exit value of T_2 for pass 2 is used in the second calculation for pass 1. This iterative procedure is continued until the average exit values of T_2 for pass 2 (for consecutive iterations) differ by less than a given convergence criterion. For the calculations in this study, the convergence criterion selected was that the average values of T_2 for consecutive iterations would differ by less than 1 percent. Depending upon the particular case, the number of iterations required for convergence was from three to five.

For more than two passes, values of T_2 would have to be assumed on all inner boundaries where needed, the number of such boundaries being one less than the number of passes. Numerical calculations in this investigation were confined to two-pass heat exchangers.

Discussion of the numerical solution of the basic equations. - The equations have been solved numerically by a first-order, predictor-corrector integration scheme. A FORTRAN program has been developed for use with the IBM 7094 or Univac 1107/1108 computers. The program can handle single-pass and multiple-pass calculations. An automatic step-size control, governed by the values of certain input parameters, was developed so that a large number of cases could be run continuously with the optimum step size used for each individual case. Output options are available which allow the user to study either detailed temperature distributions or overall performance characteristics. The details of the numerical procedure are described in appendix A, and the computer program is discussed in appendix B.

RESULTS AND DISCUSSION

The performance of a two-fluid heat exchanger may be expressed by a single dependent variable which is a function of two independent variables. For example, the overall effectiveness can be expressed as a function of the number of transfer units NTU of the heat exchanger and the capacity rate ratios of the two fluids. An investigation of three-fluid heat exchangers requires consideration of two dependent variables and five independent variables.

Nondimensional Independent Variables

In the foregoing section entitled "Problem Formulation," it was noted that the five independent parameters which can be used to define a specific problem are A, B, C, D, and Δt_i. However, these are not easily associated with the physical variables of the problem.

To specify a particular problem in terms of quantities which are more useful to a designer, the quantities A, B, C, and D may be combined into a new set of parameters which are more amenable to physical interpretation. The new parameters are

$$NTU_1 = A = \frac{u_{1,2} x_o y_o}{\dot{m}_1 c_{p,1}} \tag{17}$$

$$U = \frac{C}{D} = \frac{u_{1,2}}{u_{2,3}} \tag{18}$$

$$K_1 = \frac{C}{A} = \frac{\dot{m}_1 c_{p,1}}{\dot{m}_2 c_{p,2}} \tag{19}$$

and

$$K_3 = \frac{D}{B} = \frac{\dot{m}_3 c_{p,3}}{\dot{m}_2 c_{p,2}} \tag{20}$$

The constant A is retained as the basic size parameter and is called NTU_1, or the number of transfer units of the heat exchanger referred to fluid (1). This parameter represents the ability of the heat exchanger to change the temperature of fluid (1). A large value of NTU_1 can result from a large physical size $x_o y_o$, a high conductance between fluids (1) and (2) $u_{1,2}$, and a small capacity rate for fluid (1) $\dot{m}_1 c_{p,1}$. All of these factors would make fluid (1) relatively easy to heat or cool. Therefore, the nondimensional input parameter NTU_1 is a good representation of the size of the exchanger. Since a similar parameter based on either fluid (2) or (3) also could have been defined, the choice of fluid (1) as a reference for size is arbitrary. For example, a size parameter based on fluid (3) could have been defined as $NTU_3 = \frac{u_{2,3} x_o y_o}{\dot{m}_3 c_{p,3}}$, which is equal to the quantity B.

The parameters K_1 and K_3 are nondimensional heat-capacity flow rates of the outer fluids (1) and (3) referred to the center fluid (2) and will be called capacity rate ratios. For a well-designed heat exchanger, the combined heat-capacity flow rates of the outer two fluids should not be significantly different from the capacity flow rate of the center fluid. This implies that $K_1 + K_3$ should be near unity for proper design.

The parameter U is called the conductance ratio, and indicates the relative ability of fluids (1) and (3) to transfer heat to fluid (2).

Problems may be specified now by the five independent parameters NTU_1, K_1, K_3, U, and Δt_i.

Discussion of Nondimensional Dependent Variables

The solution of the basic equations provides two-dimensional distributions of the temperatures of all three fluids throughout the heat exchangers. Some of the phenomena which occur in three-fluid heat exchangers can be explained only by a study of these detailed distributions. Particular examples will be discussed in later sections. However, the designer is interested mainly in the overall performance characteristics of the heat exchanger, for example, the average exit temperatures of the fluids.

The dependent variables chosen to represent the performance of three-fluid heat exchangers are the temperature effectivenesses of the two outer fluids. These variables are defined by the following expressions.

$$\left.\begin{aligned} \theta_1 &= \left(\frac{t_{1,i} - t_{1,em}}{t_{1,i} - t_{2,i}}\right) \times 100 \text{ percent} \\ \theta_3 &= \left(\frac{t_{3,i} - t_{3,em}}{t_{3,i} - t_{2,i}}\right) \times 100 \text{ percent} \end{aligned}\right\} \tag{21}$$

The quantities $t_{1,em}$ and $t_{3,em}$ are the average exit temperatures of fluids (1) and (3). They are obtained by averaging the local values for the exit temperatures obtained from the two-dimensional numerical integration of the basic equations.

The variables θ_1 and θ_3 represent the degree to which the temperatures of the outer fluids have approached the inlet temperature of the center fluid when they leave the heat exchanger. The effectiveness θ_1 or θ_3 will be 100 percent when the average exit temperature of fluid (1) or fluid (3) equals the inlet temperature of fluid (2). The effectiveness will be zero when there is no change in temperature. There are circumstances when one of the temperature effectivenesses actually can be negative. It has been assumed that for proper design, the center fluid (2) will either heat or cool both outer fluids. Consider the case for which $t_{1,i}$ and $t_{3,i}$ are both greater than $t_{2,i}$ and for which the function of the heat exchanger is to cool fluids (1) and (3). If the inlet temperature and heat capacity rates of fluid (3) are considerably greater than those of fluid (1), heat will be transferred through fluid (2) to fluid (1) in a large part of the heat exchanger. Fluid (1) will leave the heat exchanger at a temperature above its inlet value, the opposite of the desired effect. Therefore, a negative temperature effectiveness indicates that a fluid intended to be cooled was actually heated, or vice versa. A designer must use θ_1 and θ_3 to determine the effect of the heat exchanger on both fluids.

An auxiliary dependent variable, the overall effectiveness, has been defined to compare the performance of a particular heat exchanger to one of infinite size. The overall effectiveness is $E = \frac{Q_a}{Q_\infty}$ where Q_a is the total heat transferred by an exchanger under fixed operating conditions and Q_∞ is the heat that would be transferred by an infinitely large counterflow heat exchanger operating at the same conditions. It will be shown in a succeeding section that the maximum heat transfer does not necessarily occur for an infinitely large heat exchanger and that with the previous definition, the overall effectiveness can actually be greater than unity.

Method of Presentation of Results

The independent variables θ_1, θ_3, and E are functions of the independent, dimensionless exchanger parameters K_1, K_3, NTU, U, and Δt_i. Since much heat-exchanger design work involves sizing an exchanger for a particular application, the performance factors θ_1, θ_3, and E are presented as functions of NTU for fixed K_1, K_3, and U with Δt_i as a parameter. Results for single-pass exchangers are presented in figures 5 to 31 and results for two-pass exchangers in figures 32 to 37.

Single-Pass Results

A range of values for the independent variables has been chosen to cover a realistic spectrum of operating conditions for the single-pass calculations. The variation of the inlet temperature parameter can be confined to the range 0 to 1 by appropriately numbering the fluids. For example, in the case for which $t_{1,i}$ and $t_{3,i}$ are greater then $t_{2,i}$, Δt_i will always be 1.0 or less if the colder of the two outer fluids is designated as fluid (1). The conductance ratio U may vary from 0 to ∞; however, the selected values of 0.5, 1.0, and 2.0 should be sufficient to cover the range of practical interest. The heat-capacity rate ratios K_1 and K_3 may also vary from 0 to ∞; however, $K_1 + K_3$ must be reasonably close to unity for a balanced heat exchanger. Therefore, the values for K_1 and K_3 of 0.25, 0.5, and 1.0 should adequately cover the range of interest. The variation of NTU_1 from 0 to 7.5 is also sufficient to cover the range of practical sizes. The multiple-run option of the computer program was used to determine performance factors for heat exchangers represented by all possible combinations of the following set of independent parameters.

$$\left.\begin{array}{lll} NTU_1 = 0.25 & NTU_1 = 0.50 & NTU_1 = 1.0 \\ NTU_1 = 2.0 & NTU_1 = 3.0 & NTU_1 = 5.0 \\ & NTU_1 = 7.5 & \end{array}\right\} \tag{22}$$

$$\left.\begin{array}{l} K_1 = 0.25 \\ K_1 = 0.50 \\ K_1 = 1.00 \end{array}\right\} \tag{23}$$

$$\left.\begin{array}{l} K_3 = 0.25 \\ K_3 = 0.50 \\ K_3 = 1.00 \end{array}\right\} \tag{24}$$

$$\left.\begin{aligned} U &= 0.50 \\ U &= 1.00 \\ U &= 2.00 \end{aligned}\right\} \tag{25}$$

and

$$\left.\begin{aligned} \Delta t_i &= 0.25 & \qquad \Delta t_i &= 0.50 \\ \Delta t_i &= 0.75 & \qquad \Delta t_i &= 1.00 \end{aligned}\right\} \tag{26}$$

This set of parameters required 756 separate calculations of the two-dimensional variations of all three-fluid temperatures. The printout option which restricted the output to the overall performance factors θ_1, θ_3, and E was used. Automatic step-size control was used to obtain the required accuracy and to minimize computation time.

In the previous set of independent parameters there are three values each for K_1, K_3, and U; therefore, there are 27 resulting performance charts for single-pass heat exchangers. Even with this many charts, only discrete values of the independent parameters are represented. The succeeding section entitled "Application of Performance Curves for Design" presents an interpolation technique for investigating problems defined by intermediate values of these parameters.

The general trends of the data presented in figures 5 to 31 are similar to most heat-exchanger performance data with some exceptions. In most cases, the effectiveness factors increase with size, rapidly at first, then more slowly tending to some upper limit. This is always true for E and θ_3; however, in some cases θ_1 begins to decrease as size increases and even becomes negative at times (fig. 11, for example). This effect may be explained as follows. Assuming an original intent to cool fluids (1) and (3), fluid (1) will be the coolest of the hot fluids. When θ_1 decreases as size increases, fluid (1) has been actually heated in some portion of the heat exchanger which has been added, thereby reducing the effectiveness of the exchanger with respect to fluid (1). This occurs whenever fluid (2), by virtue of receiving heat from fluid (3), has been heated locally to a temperature greater than that of fluid (1). This effect will be discussed in detail in the succeeding section entitled "Overall Effectiveness." Whenever the overall effect of the heat exchanger has been the heating of fluid (1) and the original intent was to cool fluid (1), the temperature effectiveness θ_1 will be negative.

Other trends can be noted which may be interpreted in terms of physical variables. The patterns of behavior of the performance factors are dependent upon the relative size of the heat exchanger referred to fluids (1) and (3). When the size variable

NTU_1 was previously defined as $\frac{u_{1,2}x_oy_o}{\dot{m}_1c_{p,1}} = A$, it was also noted that a similar variable NTU_3 could be defined as $\frac{u_{2,3}x_oy_o}{\dot{m}_3c_{p,3}} = B$. These parameters are measures of the ability of the heat exchanger to heat or cool fluid (1) or (3) per degree of temperature difference between that of fluid ((1) or fluid (3)) and fluid (2).

The key to interpreting the behavior of the performance factors is the relative magnitude of A and B. Different patterns are noted for $A > B$, $A = B$, and $A < B$. In terms of the variables used in the performance charts, these criteria become $\frac{K_1}{K_3} < U$, $\frac{K_1}{K_3} = U$, and $\frac{K_1}{K_3} > U$. From the definition of the previous parameters, $\frac{K_1}{K_3} = U$ is equivalent to $\left(\frac{\dot{m}_1c_{p,1}}{\dot{m}_3c_{p,3}}\frac{\dot{m}_2c_{p,2}}{\dot{m}_2c_{p,2}}\right) = \frac{u_{1,2}}{u_{2,3}}$ or $\frac{u_{2,3}}{\dot{m}_3c_{p,3}} = \frac{u_{1,2}}{\dot{m}_1c_{p,1}}$. Multiplying both sides by x_oy_o gives $\frac{u_{1,2}x_oy_o}{\dot{m}_1c_{p,1}} = \frac{u_{2,3}x_oy_o}{\dot{m}_3c_{p,3}}$ or $A = B$; similarly, for $A > B$, $\frac{K_1}{K_3} < U$, and for $A < B$, $\frac{K_1}{K_3} > U$.

The performance curves which fall under the above classifications are $\frac{K_1}{K_3} < U$, $A > B$ (figs. 7, 9, 10 to 13, 19, 21, 22, and 31); $\frac{K_1}{K_3} = U$, $A = B$ (figs. 6, 8, 16, 18, 20, 28, and 30); and $\frac{K_1}{K_3} > U$, $A < B$ (figs. 5, 14, 15, 17, 23 to 27, and 29). In cases where $A = B$, the ability of the exchanger to heat or cool fluids (1) and (3) depends only on the temperature differential with the center fluid. When the inlet temperatures $t_{1,i}$ and $t_{3,i}$ are equal ($\Delta t_i = 1$), θ_1 and θ_3 are identical. For smaller values of Δt_i, the temperature differential between fluids (3) and (2) is greater than that between fluids (1) and (3). The result is that heat is more easily transferred to fluid (3), and θ_3 is always greater than θ_1.

When $A < B$, heat is transferred more easily to fluid (3) than to fluid (1). Therefore, in all cases when $A < B$, θ_3 will be higher than θ_1, even for $\Delta t_i = 1$. It should be recalled that fluid (3) has been numbered so that Δt_i is never greater than unity.

There are two effects to consider when A is greater than B, the relative size (NTU) and the temperature differential. The size effect tends to make θ_1 greater than θ_3. When $\Delta t_i = 1$, θ_1 will always be greater than θ_3; however, as Δt_i decreases, the temperature differential between fluid (3) and fluid (2) becomes great enough to overcome the effect of NTU_1 being greater than NTU_3. The net result is that as actual size increases, θ_3 eventually becomes greater than θ_1, when Δt_i is less than 1.0.

Multiple-Pass Results

Numerical results have been obtained for the performance of representative configurations of two-pass, three-fluid, crossflow heat exchangers.

For both the cocurrent and countercurrent cases there are several possibilities for the behavior of the fluids in the elbow sections of multiple-pass exchangers. A fluid may be completely mixed so that it enters one pass at a constant temperature — the average of the exit distribution from the other pass. The fluid may be completely unmixed in the elbow and approach the next pass with the identical temperature distribution with which it left the previous pass. Another possible condition would be no mixing in the elbow with a flow arrangement to invert the fluid prior to entering the next pass. Thus, for each fluid the following possibilities for the behavior in the elbow can be considered.

1. Mixed
2. Unmixed, identical order
3. Unmixed, inverted order

All possible combinations would give 27 different cases to consider for each set of input variables K_1, K_3, and U. To restrict the amount of data presented and still obtain an insight into the performance of three-fluid, multiple-pass, crossflow heat exchangers, numerical results have been obtained for a countercurrent exchanger under the conditions $K_1 = 0.5$, $K_3 = 0.5$, and $U = 0.5$, 1.0, and 2.0 with all fluids mixed in the elbows. To provide comparisons for some of the other possibilities, the following cases have been considered for $K_1 = 0.5$, $K_3 = 0.5$, and $U = 1.0$:

1. Cocurrent mixed
2. Countercurrent, unmixed identical
3. Countercurrent, unmixed inverted

Figures 32 to 37 are the performance curves for the preceding cases.

The size parameter NTU_1 for multiple-pass cases is NA where N is the number of passes and A, as previously defined, is $\frac{u_{1,2}x_o y_o}{\dot{m}_1 c_{p,1}}$. All other parameters are defined in the same manner as in the single-pass analysis.

To compare the performance of two-pass, countercurrent heat exchangers with single-pass exchangers operating at the same conditions, one can compare figures 17, 18, and 19 with figures 32, 33, and 34, respectively. Although all of the effectiveness terms are increased in the two-pass, countercurrent case, the most significant increases occur in θ_1 for the smaller values of Δt_i, particularly 0.25. To obtain a direct comparison, the results for $K_1 = 0.5$, $K_3 = 0.5$, $U = 1.0$, and $\Delta t_i = 0.25$ are presented in figure 38.

It was noted previously in the case of the single-pass heat exchanger that for small Δt_i, the possibility existed for fluid (1) to be heated in some parts of the exchanger, although the original intent was to cool fluids (1) and (3). This effect was the result of fluid (2) being heated by fluid (3) to a level that exceeded the local temperature of fluid (1). This tendency is decreased by the use of multiple passes in a countercurrent arrangement. Fluid (1) may still be heated in pass 1; but in pass 2, fluid (3) has been cooled sufficiently so that it no longer heats fluid (2) above the level of fluid (1). The result is that fluid (1) is well cooled in pass 2; therefore, a significant increase in θ_1 is obtained for small Δt_i.

Figure 35 presents the performance factors for a two-pass cocurrent arrangement for $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$, with mixed flow in the elbows. This configuration is considerably less effective than the single-pass exchanger for similar conditions (fig. 18). The effect of the second pass was to reheat fluid (1) after it had been cooled in pass 1. The effectiveness decreased with an NTU_1 greater than 2.0. These results for a specific case should not categorically condemn the multiple-pass, cocurrent arrangement, but they definitely illustrate the potential problem of reheating (or recooling) associated with this configuration.

Figure 36 represents the performance factors for a two-pass, countercurrent arrangement for $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$ for unmixed, identical-order flow in the elbows. Figure 37 represents a similar case for inverted flow; and figure 33, for mixed flow. Inspection of the curves indicates that all performance factors are highest for inverted order and lowest for identical, with mixed being slightly above identical. The differences are most pronounced for θ_1, when $\Delta t_i = 0.25$. Differences in overall effectiveness are slight; for example, for $NTU_1 = 4.0$, $E_{inv} = 78$ percent, $E_{mix} = 76$ percent, and $E_{ident} = 75$ percent. However, for $\Delta t_i = 0.25$, $\theta_{1,inv} = 45$ percent, $\theta_{1,mix} = 39$ percent, and $\theta_{1,ident} = 37$ percent for $NTU_1 = 4.0$. Again, general conclusions may not be drawn from this specific case. The case does indicate that the differences between the three possibilities for flow in the elbow are worth considering in design and may significantly affect some of the performance factors of the exchanger.

Figure 39 is an illustration of the convergence of $T_{2,i}$ to pass 1 for the case of countercurrent flow with the inverted order $K_1 = 0.5$, $K_3 = 0.5$, $U = 1.0$, $NTU_1 = 3.0$, and $\Delta t_i = 0.25$. Four iterations were required. The initial estimate for

$T_{2,i}$ to pass 1 was automated for the computation of multiple cases. The initial value was assumed to be one-half of the mixing temperature of fluids (1) and (3) for each case.

OVERALL EFFECTIVENESS OF THREE-FLUID, CROSSFLOW HEAT EXCHANGERS

It is useful to define a parameter which compares the performance of a particular heat exchanger to a counterflow heat exchanger of infinite heat-transfer area operating at the same conditions. While the temperature effectivenesses θ_1 and θ_3 are of primary interest to the designer, the overall effectiveness provides additional insight into heat-exchanger performance. The following discussion presents some interesting properties of three-fluid, crossflow heat exchangers that are most easily recognized and explained in terms of overall effectiveness. The overall effectiveness has been previously defined as $E = \frac{Q_a}{Q_\infty}$ where Q_a is the heat transferred by a particular exchanger and Q_∞ is the heat transferred by an infinitely large counterflow exchanger operating at the same conditions. The heat transfer Q_a is obtained from the solution of the basic equations and may be expressed as either

$$Q_a = \dot{m}_2 c_{p,2}\left(t_{2,em} - t_{2,i}\right) \tag{27}$$

or

$$Q_a = \dot{m}_1 c_{p,1}\left(t_{1,i} - t_{1,em}\right) + \dot{m}_3 c_{p,3}\left(t_{3,i} - t_{3,em}\right) \tag{28}$$

In nondimensional form the two previous expressions become

$$\overline{Q}_a = \frac{Q_a}{\dot{m}_2 c_{p,2}\left(t_{1,i} - t_{2,i}\right)} = T_{2,em} \tag{29}$$

or

$$\overline{Q}_a = K_1\left(1 - T_{1,em}\right) + K_3\left(\frac{1}{\Delta t_i} - T_{3,em}\right) \tag{30}$$

Since there is some numerical inaccuracy in the computer program, $\overline{Q}_a$ is equated to the average of these two quantities.

In deriving an expression for Q_∞ for a three-fluid heat exchanger, it is instructive to consider a similar problem for a two-fluid heat exchanger, as illustrated in figure 40. In the two-fluid, counterflow case, the exit temperature of the fluid with the smaller capacity rate will approach the inlet temperature of the fluid with the larger capacity rate as the heat-transfer area becomes infinite. If the hot fluid is denoted by the subscript h and the cold fluid by the subscript c, then for $\dot{m}_h c_{p,h} < \dot{m}_c c_{p,c}$, $t_{h,e} = t_{c,i}$; and for $\dot{m}_h c_{p,h} > \dot{m}_c c_{p,c}$, $t_{c,e} = t_{h,i}$. Therefore, the heat transfer for an exchanger of infinite area is

$$Q_\infty = \left(\dot{m}c_p\right)_{min}\left(t_{h,i} - t_{c,i}\right) \tag{31}$$

Figure 41 depicts the analogous situation for three-fluid, counterflow heat exchangers. In the case for which the capacity rate of the center fluid is greater than the sum of the capacity rates of the outer fluids, the exit temperatures of the outer fluids both approach the inlet temperature of the center fluid.

For $\left(\dot{m}_1 c_{p,1} + \dot{m}_3 c_{p,3}\right) < \dot{m}_2 c_{p,2}$, $t_{1,e,\infty} = t_{3,e,\infty} = t_{2,i}$ and $Q = \dot{m}_1 c_{p,1}\left(t_{1,i} - t_{2,1}\right) + \dot{m}_3 c_{p,3}\left(t_{3,i} - t_{2,i}\right)$. The equivalent nondimensional quantities for $\left(K_1 + K_3\right) < 1$ are

$$T_{1,e,\infty} = T_{3,e,\infty} = T_{2,i} = 0 \tag{32}$$

and

$$\overline{Q}_\infty = \frac{Q_\infty}{\dot{m}_2 c_{p,2}\left(t_{1,i} - t_{2,i}\right)} = K_1 + \frac{K_3}{\Delta t_i} \tag{33}$$

For the situation in which $\left(\dot{m}_1 c_{p,1} + \dot{m}_3 c_{p,3}\right) > \dot{m}_2 c_{p,2}$, $t_{2,e}$ approaches a limiting value $t_{2,e,\infty}$ which lies somewhere between $t_{1,i}$ and $t_{3,i}$. This limiting value should correspond to a single, effective inlet temperature of fluids (1) and (3). This effective inlet temperature is assumed to be the mixing temperature of fluids (1) and (3) defined as

$$t_{2,e,\infty} = t_{1,3,mix} = \frac{\dot{m}_1 c_{p,1} t_{1,i} + \dot{m}_3 c_{p,3} t_{3,i}}{\dot{m}_1 c_{p,i} + \dot{m}_3 c_{p,3}} \tag{34}$$

so that

$$Q_\infty = \dot{m}_2 c_{p,2}\left(t_{2,e,\infty} - t_{2,i}\right) \tag{35}$$

The equivalent nondimensional expressions for $\left(K_1 + K_3\right) > 1$ are

$$T_{2,e,\infty} = \frac{K_1 T_{1,i} + K_3 T_{3,i}}{K_1 + K_3} = \frac{K_1 + \dfrac{K_3}{\Delta t_i}}{K_1 + K_3} \tag{36}$$

and

$$\overline{Q}_\infty = \frac{Q_\infty}{\dot{m}_2 c_{p,2}\left(t_{1,i} - t_{2,i}\right)} = T_{2,e,\infty} = \frac{\dfrac{K_1}{K_3} + \dfrac{1}{\Delta t_i}}{\dfrac{K_1}{K_3} + 1} \tag{37}$$

It should be noted that $T_{2,e}$ depends only on the ratio $\frac{K_1}{K_3}$, and Δt_i and is independent of U.

To understand the variations of the overall effectiveness expression for three-fluid, crossflow heat exchangers, it is necessary to discuss the individual fluid temperature distributions for certain limiting situations. For two-fluid heat exchangers, the effectiveness can never be greater than 100 percent; however, using the above definition, there are isolated cases corresponding to poor design practice for which the overall effectiveness of a three-fluid heat exchanger in crossflow can actually exceed 100 percent. A specific sample will be used to explain the behavior of the fluids when E is greater than 100 percent and to show why this somewhat anomalous result corresponds to poor design practice. The case under consideration will be one of using the center fluid to cool the two outer fluids.

Values of effectiveness greater than 100 percent occur when $\left(K_1 + K_3\right) > 1$ and the configuration is such that $T_{2,em}$ is greater than $T_{2,e,\infty} = T_{1,3,mix}$. Figure 42 represents the performance of a three-fluid, crossflow heat exchanger in terms of θ_1, θ_3, and E for $K_1 = 2.0$, $K_3 = 0.5$, and $U = 2.0$. The value of E reaches a maximum of 101.5 percent at $NTU_1 = 4$ for $\Delta t_i = 0.25$ and begins to decrease toward

100 percent as NTU_1 continues to increase. To understand why it is possible for $T_{2,e}$ to be greater than $T_{2,e,\infty}$, consider the distribution of T_2 along $X = 0$ from $Y = 0$ to $Y = 1.0$. Along this line, T_1 and T_3 are constant at their inlet values $T_{1,i}$ and $T_{3,i}$. Along the narrow strip at $X = 0$, fluids (1) and (3) act as infinite sources between which fluid (2) must flow. If the heat exchanger becomes infinitely large, there is a maximum temperature which fluid (2) may reach along the y-axis. Consider the case for $T_{3,i} > T_{1,i} > T_{2,i}$.

The maximum value of T_2 along the y-axis $T_{2,max}(X=0)$ is reached when the heat flow rate into fluid (2) from fluid (3) is equal to heat flow rate from fluid (2) into fluid (1). This condition is expressed by

$$u_{1,2}\left(T_{2,max} - T_{1,i}\right) = u_{2,3}\left(T_{3,i} - T_{2,max}\right) \tag{38}$$

Solving for $T_{2,max}$

$$T_{2,max}(X=0) = \frac{UT_{1,i} + T_{3,i}}{U + 1} \tag{39}$$

or since $T_{1,i} = 1$ and $T_{3,i} = \frac{1}{\Delta t_i}$

$$T_{2,max}(X=0) = \frac{U + \frac{1}{\Delta t_i}}{U + 1} \tag{40}$$

The value $T_{2,max}$ is a function of Δt_i and U, and is independent of K_1 and K_3; therefore, $T_{2,max}$ is independent of $T_{2,e,\infty}$ for fixed Δt_i. If the value of $T_{2,max}$ is greater than $T_{2,e,\infty}\left(T_{1,3,mix}\right)$, the effectiveness E may be greater than 100 percent for sufficiently large values of NTU_1 (the size parameter).

Figure 43 illustrates the distribution of T_1, T_2, and T_3 along the y-axis for $NTU_1 = 7.5$. T_1 and T_3 are constant, and T_2 asymptotically approaches 2. For

$\Delta t_i = 0.25$ and $U = 2$

$$T_{2,\,max}(X=0) = \frac{U + \frac{1}{\Delta t_i}}{U + 1} = \frac{2 + 4}{3} = 2 \tag{41}$$

Figures 44 to 47 illustrate the temperature distributions as functions of Y for $X = 0.1$, 0.25, 0.5 and 1.0. As X becomes larger, all fluids approach a nondimensional temperature of 1.6 at $Y = 1.0$. This is more clearly illustrated in figure 48 in which all temperatures are plotted versus X at $Y = 1$. For these conditions,

$$T_{2,\,e,\,\infty} = T_{1,\,3,\,mix} = \frac{\frac{K_1}{K_3} + \frac{1}{\Delta t_i}}{\frac{K_1}{K_3} + 1} = \frac{\frac{2}{0.5} + \frac{1}{0.25}}{\frac{2}{0.5} + 1} = 1.6 \tag{42}$$

If the heat-exchanger size were made infinite, the exit temperature of fluid (2) would be $T_{2,\,e,\,\infty}$ as defined previously, since the amount of fluid at a temperature greater than $T_{2,\,e,\,\infty}$ would be negligible. For $T_{2,\,max}(X=0)$ greater than $T_{2,\,e,\,\infty}$, the overall effectiveness E will rise as NTU_1 increases, will reach a maximum value, and will decrease asymptotically to 100 percent as NTU_1 becomes infinite. If $T_{2,\,max}(X=0)$ is less than $T_{2,\,e,\,\infty}$, the average exit temperature of fluid (2) approaches $T_{2,\,e,\,\infty}$ as a maximum, and E increases monotonically to 100 percent as NTU_1 becomes infinite.

Examining the behavior of θ_1, θ_3, T_1, and T_3 for this case, it can be seen that this behavior corresponds to poor design. The average temperature of fluid (1) is not affected and actually exceeds its inlet value at many points in the heat exchanger. Also, it can be seen that very little heat transfer is accomplished after $X = 0.5$, indicating a highly oversized exchanger.

This example should help emphasize the fact that while the overall effectiveness E is a very useful parameter, a designer should direct his attention to θ_1 and θ_3 to analyze effectively the actual performance of the exchanger.

The following tables will assist the designer in quickly identifying the cases for which $T_{2, max}(X=0)$ is greater than $T_{2, e, \infty}$

$$T_{2, max}(X=0) = \frac{U + \frac{1}{\Delta t_i}}{U + 1} \tag{41}$$

U	$T_{2, max}$			
	$\Delta t_i = 0.25$	$\Delta t_i = 0.50$	$\Delta t_i = 0.75$	$\Delta t_i = 1.0$
0.5	3.00	1.667	1.22	1.00
1.0	2.50	1.50	1.167	1.00
2.0	2.00	1.33	1.11	1.00

$$T_{2, e, \infty} = \frac{\frac{K_1}{K_3} + \frac{1}{\Delta t_i}}{\frac{K_1}{K_3} + 1} \tag{42}$$

$\frac{K_1}{K_3}$	$T_{2, e, \infty}$			
	$\Delta t_i = 0.25$	$\Delta t_i = 0.50$	$\Delta t_i = 0.75$	$\Delta t_i = 1.0$
0.25	3.40	1.80	1.267	1.00
1.0	2.50	1.50	1.167	1.00
2.0	2.00	1.33	1.111	1.00
4.0	1.60	1.20	1.067	1.00

Since $T_{2, max}$ depends only on U and Δt_i, and $T_{2, e, \infty}$ depends only on the ratio $\frac{K_1}{K_3}$ and Δt_i, the preceding tables provide an easy means of checking the possibility of E being greater than 100 percent for a given problem. For the case previously discussed, $K_1 = 2.0$ and $K_3 = 0.5$; therefore, $\frac{K_1}{K_3} = 4.0$. For $\Delta t_i = 0.25$, the table indicates $T_{2,e,\infty} = 1.6$. Since $U = 2.0$, the tables indicate $T_{2,max}(X=0) = 2.0$. Therefore, the possibility exists for an overall effectiveness greater than 100 percent, and there will eventually be a decrease in E for an increase in size.

APPLICATION OF PERFORMANCE CURVES FOR DESIGN

A complete graphical presentation of performance data is not practical because of the large number of independent heat-exchanger variables. The approach used in this study is to obtain performance data for selected values of the variables K_1, K_3, U, and Δt_i which bracket the range of practical interest and to develop interpolation techniques for intermediate values. While the data presented by the curves are limited, a fundamental understanding of three-fluid, crossflow heat exchangers may be obtained from them.

Three sample problems are solved below to demonstrate the application of the performance curves for design and to illustrate the physical significance of certain trends in the performance data which contribute to an understanding of the performance of three-fluid, crossflow heat exchangers.

Problem 1

This problem will illustrate the use of the performance curves when no interpolation is required. It is desired to predict the outlet temperatures of three fluids for a heat exchanger operating at the following conditions.

Fluid	$\dot{m}$, lb/hr	c_p, Btu/lb/°F	t_i, °F
1	250	0.5	300
2	500	.5	100
3	250	1.0	500

The surface conductances are $u_{1,2} = 50$ Btu/hr/ft^2/°F and $u_{2,3} = 25$ Btu/hr/ft^2/°F, while the area $x_o y_o$ is 5 ft^2. The resulting nondimensional independent variables are

$$\left.\begin{aligned} K_1 &= \frac{\dot{m}_1 c_{p,1}}{\dot{m}_2 c_{p,2}} = 0.5 \qquad K_3 = \frac{\dot{m}_3 c_{p,3}}{\dot{m}_2 c_{p,2}} = 1.0 \qquad \Delta t_i = \frac{t_{1,i} - t_{2,i}}{t_{3,i} - t_{2,i}} = 0.5 \\ U &= \frac{u_{1,2}}{u_{2,3}} = 2.0 \qquad NTU_1 = \frac{u_{1,2} x_o y_o}{\dot{m}_1 c_{p,1}} = 2.0 \end{aligned}\right\} \tag{43}$$

Referring to figure 22, the resulting nondimensional dependent variables are θ_1 = 48 percent, θ_3 = 30 percent, and E = 51 percent. Average outlet temperatures $t_{1,em}$ and $t_{3,em}$ may be obtained using equations (44) and (45).

$$\theta_1 = \left(\frac{t_{1,i} - t_{1,em}}{t_{1,i} - t_{2,i}}\right) \times 100 \text{ percent} \tag{44}$$

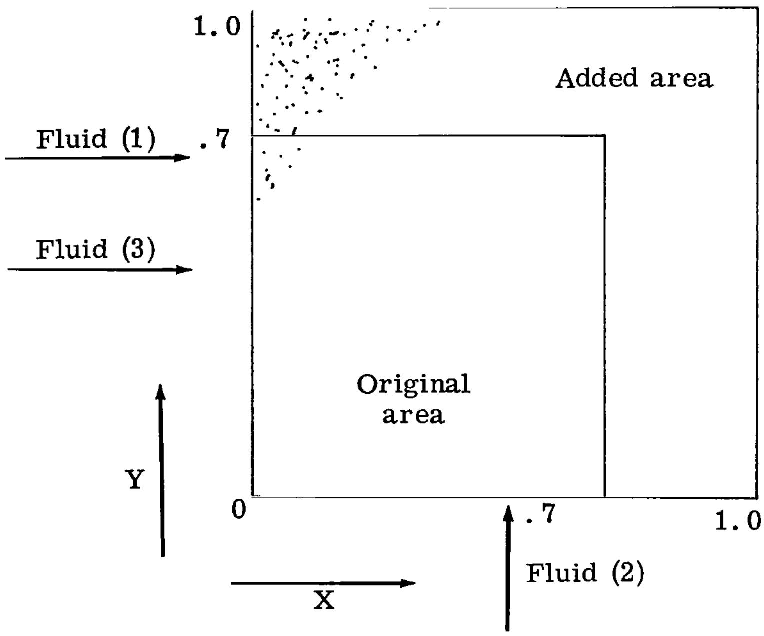
$$\theta_3 = \left(\frac{t_{3,i} - t_{3,em}}{t_{1,i} - t_{2,i}}\right) \times 100 \text{ percent} \tag{45}$$

Thus, for θ_1 = 48 percent, $t_{1,em}$ = 204° F, and for θ_3 = 30 percent, $t_{3,em}$ = 380° F. The average exit value of t_2 may be obtained from an energy balance

$$\left.\begin{aligned} \dot{m}_1 c_{p,1}\left(t_{1,i} - t_{1,em}\right) + \dot{m}_3 c_{p,3}\left(t_{3,i} - t_{3,em}\right) &= \dot{m}_2 c_{p,2}\left(t_{2,em} - t_{2,i}\right) \\ 125(300 - 204) + 250(500 - 380) &= 250\left(t_{2,em} - 100\right) \end{aligned}\right\} \tag{46}$$

thus giving $t_{2,em}$ = 268° F.

It is interesting to note the effect of increasing the size of the heat exchanger to $x_o y_o$ = 10 ft^2 or NTU_1 = 4.0. From figure 22, this change in area results in the following effectiveness factors: θ_1 = 46 percent, θ_3 = 44 percent, and E = 66 percent. Both E and θ_3 have increased; however, θ_1 has decreased by 2 percent. This



decrease occurs because the increased size of the heat exchanger allows fluid (3) (hottest) to heat the center fluid (2) to a temperature greater than that of fluid (1) in some parts of the enlarged heat exchanger.

The dotted region in the sketch indicates the part of the heat exchanger in which fluid (2) is hotter than fluid (1). Since fluid (1) is being heated in the dotted region, an increase in size actually decreases the effectiveness of fluid (1). Fluid (1) is cooled for a smaller value of the coordinate Y. However, as fluid (2) moves through the exchanger, it is heated by both fluids (1) and (3). In the dotted region, heat transferred from the hotter fluid (3) increases the temperature of fluid (2) above that of fluid (1).

The detailed temperature distributions for fluids (1), (2), and (3) are presented in figures 49(a), 49(b), and 49(c) respectively. Figure 49(a) indicates that the temperature of fluid (1) is above its inlet value of 300° F in almost one-third of the heat exchanger. For values of Y less than 0.45, fluid (1) is always cooled as it flows in the X direction. For larger values of Y, fluid (1) is first heated and then cooled as it flows through the exchanger. The regions in which it is being heated correspond to the regions in which fluid (2), the "coolant" fluid, is actually hotter than fluid (1).

Figure 49(b) shows that fluid (2) flowing in the Y direction is heated at a very high rate along the Y-axis where the hot fluids enter the exchanger. The cooling rate decreases for larger values of X.

As indicated in figure 49(c), fluid (3) flowing in the X direction is cooled most rapidly along the X-axis where the coolant fluid (2) enters the exchanger. Since fluid (2) is heated as it passes through the exchanger, its ability to cool the outer fluids is decreased. This is illustrated by the isothermal lines for fluid (3) which indicate a decreasing cooling rate for increasing values of Y.

In figure 50, isothermal contours for all three fluids are superimposed. This figure may be used to determine the part of the heat exchanger in which the temperature of the coolant fluid (2) is actually greater than the temperature of fluid (1). This area corresponds to the dotted region.

Figures 49(a), 49(b), 49(c), and 50 graphically illustrate the cause of the reduction of effectiveness for an increase in size which may occur in some three-fluid, crossflow heat exchangers.

This phenomenon is most pronounced for a small Δt_i $\left(\text{large } t_{3,i}\right)$, a large capacity rate for fluid (3) $\left(\text{large } K_3\right)$, and a small conductance ratio $\left(\text{large } u_{2,3}\right)$. These factors contribute to a high heat-transfer rate from fluid (3) to fluid (2), with the possible result that fluid (1) is reheated in some part of the heat exchanger. In some cases, for example, $K_1 = 1.0$, $K_3 = 1.0$, and $\Delta t_i = 0.25$ (figs. 29 and 31), the effect is so pronounced that fluid (1) is actually heated, and θ_1 becomes negative as NTU increases. The preceding discussion has assumed an original intent to cool fluids (1) and (3) with fluid (2).

Problem 2

The conditions of problem 2 are chosen to illustrate how the performance curves may be used when the independent variables are not equal to those chosen for preparing the curves, namely, the combinations resulting from the values listed in the set of independent parameters presented in the section entitled "Single-Pass Results."

It is desired to determine the temperature effectiveness θ_1 and θ_3 for a heat exchanger operating under the following conditions: $K_1 = 0.40$, $K_3 = 0.75$, $U = 1.3$, $\Delta t_i = 0.85$, and $NTU_1 = 2.0$.

Since these values of the independent variables do not correspond to those for which the performance curves have been prepared, some interpolation scheme must be employed to determine temperature effectiveness for this case. A straightforward, graphical technique has been used. Figure 51 presents the temperature effectiveness $\left(\theta_1, \theta_3\right)$ as a function of the conductance ratio U for $NTU_1 = 2.0$ and $\Delta t_i = 0.85$. Nine curves are required to cover all possible combinations of K_1 and K_3. The data were obtained from the performance charts using visual interpolation for $\Delta t_i = 0.85$. Points were obtained for the three values of U used in the performance curves, namely, 0.5, 1.0, and 2.0. Figure 51 was used to determine θ_1 and θ_3 as functions for K_3 for $U = 1.3$ and $K_1 = 0.25$, 0.50, and 1.0. The results are plotted in parts (a), (b), and (c) of figure 52. These curves were used to determine θ_1 and θ_3 as functions of K_1 for $U = 1.3$ and $K_3 = 0.75$. Figure 52(d) may be used to determine θ_1 and θ_3 for $K_1 = 0.40$, $K_3 = 0.75$, $U = 1.3$, and $\Delta t_i = 0.85$. Entering figure 52(d) at $K_1 = 0.40$ gives $\theta_1 = 58$ percent and $\theta_3 = 41$ percent. The overall effectiveness E could have been obtained in a similar manner; however, it is easier to return to the basic definition and calculate it as

$$\overline{Q}_a = K_1\left(1 - T_{1,em}\right) + K_3\left[\left(\frac{1}{\Delta t_i}\right) - T_{3,em}\right] \tag{47}$$

or

$$\overline{Q}_a = K_1\theta_1 + K_3\ \Delta t_i\theta_3 = 0.40(0.58) + 0.75(0.85)(0.41) = 0.492 \tag{48}$$

Since $K_1 + K_3 > 1$

$$\overline{Q}_\infty = \left(\frac{1}{K_1 + K_3}\right)\left(K_1 + \frac{K_3}{\Delta t_i}\right) = \frac{1.28}{1.15} = 1.11 \tag{49}$$

therefore $E = \frac{\overline{Q}_a}{\overline{Q}_\infty} = 44$ percent. It is evident from examining figures 51 and 52 that linear interpolation is not adequate; hence, at least three points are needed on each curve. An additional point is available on curves which present θ_1 and θ_3 as a function of either K_1 or K_3 (fig. 52). Whenever K_j approaches zero, θ_j approaches unity. Physically, this means that as the flow rate of a fluid becomes infinitely small, it can be cooled or heated very easily. It should also be noted that as U becomes very large, θ_3 will approach zero in figure 51. This trend reflects the physical consequence of $u_{2,3}$ approaching zero. Insulation of fluid (2) from fluid (3) will not alter the temperature. At first it may seem that θ_1 should approach zero as U becomes small; however, if U approaches zero, $NTU_1 = \frac{x_o y_o u_{1,2}}{\dot{m}_1 c_{p,1}}$ could no longer be 2.0 as presumed in the problem. Therefore, it is impossible to consider U approaching zero for a finite, constant value of NTU_1.

When solving a problem where conditions do not correspond to those used in the performance curves, a set of figures similar to figures 51 and 52 must be prepared for every set of values for NTU_1 and Δt_i.

Problem 3

Problems 1 and 2 involved predicting output conditions for a given heat exchanger operating at specific input conditions. Problem 3 is one which is more frequently encountered by a designer: If the inlet conditions and capacity rates of the two outer fluids (1) and (3) are given, determine the size of the exchanger and the mass flow rate of the center fluid (2) that are required to produce specified outlet conditions for fluids (1) and (3).

Consider fluids (1) and (3) entering the exchanger at the following conditions.

$$\left.\begin{aligned} \dot{m}_1 &= 250 \text{ lb/hr} & \dot{m}_3 &= 250 \text{ lb/hr} \\ c_{p,1} &= 1.0 \text{ Btu/lb/}^\circ\text{F} & c_{p,3} &= 0.5 \text{ Btu/lb/}^\circ\text{F} \\ t_{1,i} &= 300^\circ \text{ F} & t_{3,i} &= 500^\circ \text{ F} \end{aligned}\right\} \tag{50}$$

Coolant fluid (2) is available at $T_{2,i} = 100^\circ$ F with $c_{p,2} = 0.5$ Btu/lb/°F. Determine the NTU_1 and $\dot{m}_2$ required to cool both fluids to 220° F. These temperature changes correspond to the following values of effectiveness.

$$\theta_1 = \frac{t_{1,i} - t_{1,e}}{t_{1,i} - t_{2,i}} = \frac{300 - 220}{300 - 100} = 40 \text{ percent} \tag{51}$$

and

$$\theta_3 = \frac{t_{3,i} - t_{3,e}}{t_{3,i} - t_{2,i}} = \frac{500 - 220}{500 - 100} = 70 \text{ percent} \tag{52}$$

Possible solutions may be found by studying the design curves for which $\frac{K_1}{K_3} = 2.0$. This condition is satisfied for two sets of curves: figures 15 to 17 for which $K_1 = 0.5$ and $K_3 = 0.25$ and figures 26 to 28 for which $K_1 = 1.0$ and $K_3 = 0.5$. For each of these sets of curves, a table may be prepared to investigate possible solutions. First, the effectiveness θ_1 is fixed at 40 percent and the number of transfer units NTU_1 is determined for each value of the conductance ratio U. The values of θ_3 at this value of NTU_1 are tabulated. A similar procedure is followed holding θ_3 fixed at 70 percent and determining θ_1. The object is to find a combination of U and NTU_1 for which θ_1 and θ_3 are as close to the desired values as possible. The following table is derived for this problem, for $K_1 = 1.0$ and $K_3 = 0.5$ (figs. 26 to 28).

U	θ_1 = 40 percent		θ_3 = 70 percent	
	NTU_1	θ_3	NTU_1	θ_1
0.5	2.5	79	0.60	21
1.0	2.5	75	1.0	30
2.0	2.5	66	3.5	43

For U = 2.0, a value of NTU_1 between 2.5 and 3.5 should be acceptable. Figure 28 shows that for U = 2.0 and $NTU_1 = 3.0$, $\theta_1 = 42$ percent and $\theta_3 = 68$ percent, which is close enough for design purposes. For this condition the overall effectiveness E is 83 percent. The required value of $\dot{m}_2 c_{p,2}$ is $\frac{\dot{m}_1 c_{p,1}}{K_1} = 250$ Btu/hr/°F. Since $c_{p,2} = 0.5$ Btu/lb/°F, the required $\dot{m}_2$ is 500 lb/hr.

It is still necessary to consider the case of $K_1 = 0.5$ and $K_3 = 0.25$. A similar table is prepared for $K_1 = 0.5$ and $K_3 = 0.25$ (figs. 14 to 16).

U	θ_1 = 40 percent		θ_3 = 70 percent	
	NTU_1	θ_3	NTU_1	θ_1
0.5	0.85	85	0.40	23
1.0	.80	70	.80	40
2.0	.75	45	1.80	57

In this case, the values for $U = 1.0$ give the desired result exactly for $NTU_1 = 0.80$. From figure 15 the overall effectiveness E is 55 percent. The required value of $\dot{m}_2$ is 1000 lb/hr.

The two sets of conditions which satisfy the objectives of the problem are

$$\left.\begin{aligned} NTU_1 &= 3.0 \\ \dot{m}_2 &= 500 \text{ lb/hr} \\ U &= 2.0 \\ E &= 83 \text{ percent} \end{aligned}\right\} \tag{53}$$

and

$$\left.\begin{aligned} NTU_1 &= 0.80 \\ \dot{m}_2 &= 1000 \text{ lb/hr} \\ U &= 1.0 \\ E &= 55 \text{ percent} \end{aligned}\right\} \tag{54}$$

The choice facing the designer is between a large physical size with low flow rate and high effectiveness or small size, larger flow rate, and lower effectiveness. The ultimate choice must be based on factors such as construction, cost, space available, volume of coolant fluid available, and other design factors.

Not all problems which are approached in this manner will have an adequate solution. Consider a case which has the same conditions as the previous problem except that $\frac{K_1}{K_3}$ has the value 0.50 instead of 2.0. That is, assume that the hotter fluid has the higher capacity rate. Performance curves must now be considered for which $K_1 = 0.25$ and $K_3 = 0.50$ (figs. 8 to 10), and $K_1 = 0.50$ and $K_3 = 1.0$ (figs. 20 to 22). A chart is prepared as before for $K_1 = 0.25$ and $K_3 = 0.50$.

U	θ_1 = 40 percent		θ_3 = 70 percent	
	NTU_1	θ_3	NTU_1	θ_1
0.5	0.85	48	2.0	53
1.0	.70	26	4.0	67
[a]2.0	--	--	--	--

[a]θ_1 is always above θ_3 for U = 2.0.

The trends indicate that there will be no satisfactory solution. For $K_1 = 0.5$ and $K_3 = 1.0$, θ_3 never even reaches 70 percent. In this case, the performance curves indicate that the objectives of the problem are impossible under the imposed restrictions.

CONCLUDING REMARKS

Performance characteristics have been determined for a wide range of operating parameters for single-pass, three-fluid, crossflow heat exchangers. The performance of two-pass heat exchangers for both cocurrent and countercurrent flow has been studied for selected operating conditions. The results have been presented in terms of the temperature effectiveness of the two outer fluids as functions of heat-exchanger size for sets of fixed operating conditions.

Selected values have been chosen to bracket the range of practical interest because of the infinite possibilities for combinations of operating conditions. Interpolation techniques have been used to obtain performance data for intermediate values. Sample problems are included to illustrate the use of the performance curves and the interpolation techniques.

An expression for overall effectiveness has been derived which compares the heat transferred by a particular exchanger with that which is transferred by one of infinite size. Isolated cases corresponding to poor design are cited for which the overall effectiveness may be greater than unity. This indicates the importance of using the temperature effectiveness of the two outer fluids as the primary design variables and the overall effectiveness as an auxiliary parameter.

While data are necessarily limited to fixed sets of operating conditions, a fundamental understanding of three-fluid, crossflow heat exchangers may be obtained from the performance curves.

A computer program has been developed for the study of both single- and multiple-pass heat exchangers. Output options are available for detailed studies of temperature distributions within a particular exchanger and for generation of performance data for a large number of heat exchangers.

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APPENDIX A

NUMERICAL PROCEDURE

Basic Logic

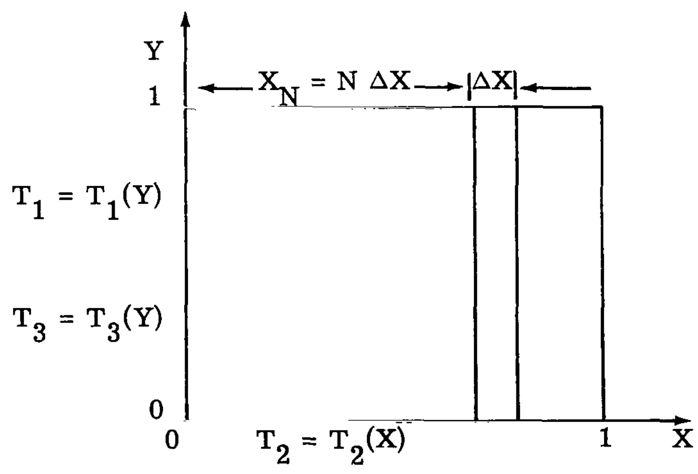
To determine the temperature distributions of each fluid in a three-fluid, cross-flow heat exchanger, the partial differential equations which must be solved simultaneously are

$$\frac{\partial T_1}{\partial X} = A\left(T_2 - T_1\right) \tag{A1}$$

$$\frac{\partial T_2}{\partial Y} = C\left(T_1 - T_2\right) + D\left(T_3 - T_2\right) \tag{A2}$$

$$\frac{\partial T_3}{\partial X} = B\left(T_2 - T_3\right) \tag{A3}$$

The region of solution of these nondimensionalized equations is the portion of the X-Y plane bounded by $X = 0$, $X = 1$, $Y = 0$, and $Y = 1$. The boundary conditions are $T_1 = T_1(Y)$ and $T_3 = T_3(Y)$ at $X = 0$ and $T_2 = T_2(X)$ at $Y = 0$.



The basic logic for solution will be outlined before the details of the integration scheme are discussed.

1. The numerical integration of equations (A1) and (A3) could be initiated in the X direction if T_2 were known along the Y-axis.

2. To obtain $T_2(0, Y)$, equation (A2) is integrated numerically in the Y direction.

3. Using the initial values of T_1 and T_3 and the values of T_2 at $X = 0$ calculated in step 2, T_1 and T_3 can be calculated at $X = \Delta X$ using equations (A1) and (A3).

4. At $X = \Delta X$, the same situation exists as before: T_1 and T_3 are known and T_2 is to be calculated from equation (A2).

5. The above procedure is repeated at each increment ΔX until the solution is obtained over the entire region. The only difference in the initial integration step and all the other steps is that T_1 and T_3 are no longer constant along an $X = \text{constant}$ line.

Integration Scheme

The numerical technique used was devised as a first-order, predictor-corrector integration scheme. The solution of equation (A2) for $T_2(Y)$ at $X = X_N$ will be used to illustrate the procedure. Assume that T_1, T_2, and T_3 are known at $X = X_N$, $Y = Y_N$ and the value of T_2 is desired at $Y = Y_N + \Delta Y$. It will be recalled, from the outline of the basic logic, that T_1 and T_3 are known along the line $X = X_N$ from $Y = 0$ to $Y = 1$. Equation (A2) is used to evaluate $\left(\frac{\partial T_2}{\partial Y}\right)$ at $Y = Y_N$. This derivative will be denoted by $\left(\frac{\partial T_2}{\partial Y}\right)_{Y_N}$. A prediction of the value for T_2 at $Y = Y_N + \Delta Y$ is calculated from

$$T_2^p\left(Y_N + \Delta Y\right) = T_2\left(Y_N\right) + \Delta Y\left(\frac{\partial T_2}{\partial Y}\right)_{Y_N} \tag{A4}$$

This value of T_2^p is then used in equation (A2) along with the known values of T_1 and T_3 at $Y = Y_N + \Delta Y$ to predict $\left(\frac{\partial T_2}{\partial Y}\right)$ at $Y = Y_N + \Delta Y$. This derivative will be denoted by $\left(\frac{\partial T_2}{\partial Y}\right)^p_{Y_N + \Delta Y}$. A corrected value of $T_2 = T_2^c$ is calculated from

$$T_2^c = T_2(Y_N) + \frac{1}{2}\left[\left(\frac{\partial T_2}{\partial Y}\right)_{Y_N} + \left(\frac{\partial T_2}{\partial Y}\right)^p_{Y_N+\Delta Y}\right] \tag{A5}$$

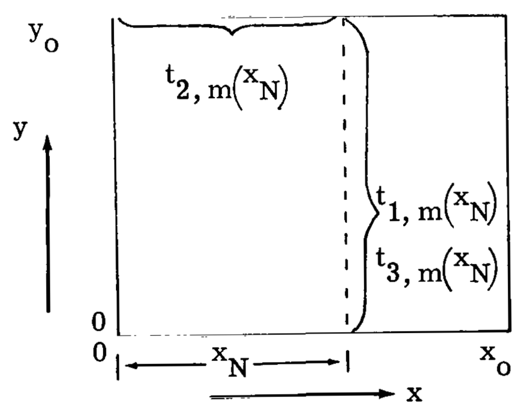
This procedure can be shown to be equivalent to using a second order Taylor series expansion of the function $T_2(Y)$ at the point Y_N with the required first derivatives $\frac{\partial T_1}{\partial Y}$ and $\frac{\partial T_3}{\partial Y}$ approximated by the slopes between Y_N and $Y_N + \Delta Y$.

A similar procedure is used to solve equations (A1) and (A3) in the X direction. In the solution of these equations, T_2 is assumed to be constant over the interval ΔX between X_N and $X_N + \Delta X_N$ at the value $T_2(X_N)$.

Accuracy Check

The accuracy of the computation may be checked at any x coordinate during the integration by comparing the energy gained (lost) by fluid (2) with that which is lost (gained) by fluids (1) and (3).

The calculation of the energy balance proceeds in the following manner:



At any station x_N, t_1, and t_3 are averaged from $y = 0$ to $y = y_o$ while t_2 is averaged from $x = 0$ to $x = x_N$. Conservation of energy requires

$$\left(\dot{m}_1 c_{p,1}\right)\left[t_{1,i} - t_{1,m}\left(x_N\right)\right] + \left(\dot{m}_3 c_{p,3}\right)\left[t_{3,i} - t_{3,m}\left(x_N\right)\right] = \left(\dot{m}_2 c_{p,2}\right)\left(\frac{x_N}{x_o}\right)\left[t_{2,m}\left(x_N\right) - t_{2,i}\right] \tag{A6}$$

The coordinates $\frac{x_N}{x_o}$ may be replaced by X_N. Dividing equation (A6) by $\left(\dot{m}_2 c_{p,2}\right)$, the resulting equation is

$$\left(\frac{\dot{m}_1 c_{p,1}}{\dot{m}_2 c_{p,2}}\right)\left[t_{1,i} - t_{1,m}\left(X_N\right)\right] + \left(\frac{\dot{m}_3 c_{p,3}}{\dot{m}_2 c_{p,2}}\right)\left[t_{3,i} - t_{3,m}\left(X_N\right)\right] = X_N\left[t_{2,m}\left(X_N\right) - t_{2,i}\right] \tag{A7}$$

Since $\frac{\dot{m}_1 c_{p,1}}{\dot{m}_2 c_{p,2}} = K_1$ and $\frac{\dot{m}_3 c_{p,3}}{\dot{m}_2 c_{p,2}} = K_3$, equation (A2) may be written

$$K_1\left[t_{1,i} - t_{1,m}\left(X_N\right)\right] + K_3\left[t_{3,i} - t_{3,m}\left(X_N\right)\right] = X_1\left[t_{2,m}\left(X_N\right) - t_{2,i}\right] \tag{A8}$$

Dividing by $t_{1,i} - t_{2,i}$ gives

$$K_1\left[T_{1,i} - T_{1,m}\left(X_N\right)\right] + K_3\left[T_{3,i} - T_{3,m}\left(X_N\right)\right] = X_N\left[T_{2,m}\left(X_N\right)\right] \tag{A9}$$

Since the boundary conditions are $T_{1,i} = 1$ and $T_{3,i} = \frac{1}{\Delta t_i}$, the accuracy of the overall computation may be checked at any station X_N by comparing the quantities $K_1\left[1 - T_{1,m}\left(X_N\right)\right] + K_3\left[\frac{1}{\Delta t_i} - T_{3,m}\left(X_N\right)\right]$ and $X_N\left[T_{2,m}\left(X_N\right)\right]$. This comparison is used to determine the appropriate step size for the different calculations.

An examination of the basic equations

$$\frac{\partial T_1}{\partial X} = A\left(T_2 - T_1\right) \tag{A1}$$

$$\frac{\partial T_2}{\partial Y} = C\left(T_1 - T_2\right) + D\left(T_3 - T_2\right) \tag{A2}$$

and

$$\frac{\partial T_3}{\partial X} = B\left(T_2 - T_3\right) \tag{A3}$$

would disclose the direct influence of A, B, C, and D on the calculation.

For large A, B, C, or D, the temperature gradients will be large, and a smaller step size will be required to maintain acceptable accuracy. Since any of these constants is a nondimensional representation of the size of the heat exchanger, larger exchangers will require more calculational steps.

It was arbitrarily decided that an acceptable limit for accuracy would be that the two overall energy-balance terms would not differ from each other by more than 2 percent of their average value.

An automatic step-size control was used in the computer program because of the large number of cases which were needed to generate the performance curves. For any calculation, the largest value of the set (A, B, C, D) is denoted as M. The following criteria for step size were established.

M	$\Delta x, \Delta y$
$M > 20$	0.002
$7 \leq M \leq 20$	.005
$4 \leq M < 7$	.01
$2 < M < 4$	.02
$1 \leq M \leq 2$	.05
$M < 1$	.10

In all cases, this set of criteria was sufficient to insure agreement of the energy balance within 2 percent, and the agreement was considerably better in the majority of cases.

APPENDIX B

COMPUTER PROGRAM

A computer program has been developed to solve the basic differential equations using the numerical procedure described in appendix A. The program is capable of handling calculations for both single- and two-pass heat exchangers.

Since the program is written in FORTRAN, it can be run on the IBM 7094 or Univac 1107/1108 computers. The following pages contain a listing of the complete program.

```
      PROGRAM MAIN
C
      DIMENSION T1(1001), T2(1001), T3(1001), XAVE(14),XII(1001),YII(100
     11), BCDX(12), BCDY(12), T2P(1001), T1T(10),T1P(501), T3P(501)
     1 ,THPPP(1001)
      COMMON /AB/ A, B, C, D, DTI
      REAL K1, K3
C
C
      KCOU = 0
C
      READ(5,16)  (XAVE(KJ), KJ=1,14 )
      DO 40 JO=1,14
   40 XAVE(JO) = XAVE(JO) -.001
      IF( XAVE(1) .LT. 0.00001 .AND. XAVE(2) .LT. 0.00001)XAVE(1) = 1.0
      IZERO = 0
      CALL RESET
  100 CONTINUE
      READ(5,995)IPASS, ITYPE, IMIX, IDENT, IPLOT, IPRINT, IALL
 3722 CONTINUE
      READ(5,16)K1, K3, DTI, A, U
      C = A * K1
      B = A * (K1/K3) * (1.0/U)
      D = (K1/U) * A
      WRITE(6,1000)
      IF(ITYPE .EQ. 1)WRITE(6,1001)
      IF(ITYPE .EQ. 2)WRITE(6,1010)IPASS
      IF(ITYPE .EQ. 3)WRITE(6,1011)IPASS
      IF(ITYPE .EQ. 1)GO TO 101
      IF(IMIX .EQ. 0)WRITE(6,1012)
      IF(IMIX .EQ. 1)WRITE(6,1013)
      IF(IDENT .EQ. 0)WRITE(6,1014)
      IF(IDENT .EQ. 1)WRITE(6,1015)
  101 CONTINUE
      ABCD = A
      IF(ABCD .LT. B)ABCD=B
      IF(ABCD .LT. C)ABCD=C
      IF(ABCD .LT. D)ABCD=D
      NX = 500
      IF(ABCD .LE. 20.0 .AND. ABCD .GE. 7.0) NX = 200
      IF(ABCD .LT. 7.0 .AND. ABCD .GE. 4.0) NX = 100
      IF(ABCD .LT. 4.0 .AND. ABCD .GT. 2.0) NX = 50
      IF(ABCD .LE. 2.0 .AND. ABCD .GE. 1.0) NX = 20
      IF(ABCD .LT. 1.0) NX = 10
      NY = NX
      NN = NY + 1
      MP1 = NX + 1
      DELX = 1.0 / FLOAT(NX)
      DELY = 1.0 / FLOAT(NY)
      WRITE(6,1008)A, K1, K3, U, DTI
      WRITE(6,1009)B, C, D
      WRITE(6,1016)DELX, DELY
  306 CONTINUE
      T11 = 1.0
      T22 = 0.0
```

```
      T33 = 1.0 / DTI
      T2GUES = 0.5*( (K1+K3/DTI)/ (K1+K3) )
      WRITE(6,1007)T33
      IF(ITYPE .EQ. 3)WRITE(6,28)T2GUES
      IF(ITYPE .EQ. 3)T22 = T2GUES
      IF(IPLOT .EQ. 0)GO TO 304
      XII(1) = 0.0
      YII(1) = 0.0
      DO 41 IK=1,MP1
      XII(IK+1) = XII(IK) + DELX
      YII(IK+1) = YII(IK) + DELY
   41 CONTINUE
  304 CONTINUE
      IAVE = 1
      IF( IPASS .EQ. 1)IAVE = 0
      WRITE(6,17)
      INV = 0
      DO 42 I66=1,10
      I77 = 1
  203 CONTINUE
      IF(INV .EQ. 1)GO TO 200
      DO 1 I=1,NN
      T1(I) = T11
      T2P(I) = T22
    1 T3(I) = T33
  200 X = 0.0
      INV = 0
      KK = 1
      T2SUM = 0.0
      IMO = MOD(I77,2)
      IF(IPASS .EQ. 1)GO TO 3
      WRITE(6,29)I66
      IF(IMO .NE. 0)WRITE(6,21)
      IF(IMO .EQ. 0)WRITE(6,22)
    3 DO 4 J=1,MP1
      CALL AB1 (T1, T2, T3, DELY, NY , T2P(J) )
      T2P(J) = T2(NN)
      T2SUM = T2SUM + T2(NN)
      KCOU = KCOU + 1
      IF(KCOU .NE. IPRINT)GO TO 202
      KCOU = 0
      WRITE(6,11)X
      WRITE(6,12)
      WRITE(6,10)(T1 (K),K=1,NN)
      WRITE(6,13)
      WRITE(6,10) (T2(K),K=1,NN )
      WRITE(6,14)
      WRITE(6,10) (T3(K),K=1,NN )
  202 CONTINUE
      IF( XAVE(KK) .LT. 0.1)GO TO 371
      IF(XAVE(KK) .LT. 0.1E-03)GO TO 18
      IF(XAVE(KK) .GT. X)GO TO 18
      IF(IALL .EQ. 0)GO TO 310
  371 CONTINUE
      WRITE(6,11)X
```

```
      WRITE(6,12)
      WRITE(6,10)(T1 (K),K=1,NN)
      WRITE(6,13)
      WRITE(6,10) (T2(K),K=1,NN )
      WRITE(6,14)
      WRITE(6,10) (T3(K),K=1,NN )
  310 KK = KK + 1
      T2S = T2SUM / FLOAT(J)
      T1SUM = 0.0
      T3SUM = 0.0
      DO 19 IK=1,NN
      T1SUM = T1SUM + T1(IK)
   19 T3SUM = T3SUM + T3(IK)
      T1SUM = T1SUM / FLOAT(NN)
      T3SUM = T3SUM / FLOAT(NN)
      IF(IPLOT .NE. 1)GO TO 302
      CALL QUIKML (-1, 0.0, 1.0, 0.0,  5.0, 1H1,BCDX,BCDY,NN,YII,T1)
      CALL QUIKML ( 0, 0.0, 1.0, 0.0,  5.0, 1H2,BCDX,BCDY,NN,YII,T2)
      CALL QUIKML ( 0, 0.0, 1.0, 0.0,  5.0, 1H3,BCDX,BCDY,NN,YII,T3)
  302 CONTINUE
      WRITE(6,1021)
      WRITE(6,20)T1SUM, T2S, T3SUM
      COM = K1 * (1.0 - T1SUM) + K3 * (1.0/DTI - T3SUM)
      XX22 = 0.0
      IF(ITYPE .EQ. 2)XX22=T22
      COM1 = X * (T2S-XX22)
      IF( IWO        .NE. 0) COM1 = (T2S - T22) * X
      IF( IWO        .EQ. 0 ) COM = K1 * (T11 - T1SUM) + K3*(T33 -
     1 T3SUM)
      ACC = ABS(COM-COM1) / (COM+COM1)    * 100.0
      IF(ITYPE .LE. 2)WRITE(6,1004) COM, COM1, ACC
   18 CONTINUE
      X = X + DELX
      T1P(J) = T1(NN)
      T3P(J) = T3(NN)
      DO 2 I=1,NN
      DEL = DELX * A * (T2(I) - T1(I) )
      T1P1= T1(I) + DEL
      DEL1 = DELX * A * (T2(I) - T1P1)
      T1(I) = (DEL+DEL1)*0.5      + T1(I)
      DEL = DELX * B * (T2(I) - T3(I) )
      T1P1= T3(I) + DEL
      DEL1 = DELX * B * (T2(I)-T1P1 )
    2 T3(I) = T3(I) + (DEL+DEL1) * 0.5
    4 CONTINUE
      CALL QUIKML(-1,0.0,1.0,0.0,5.0,1H1,BCDX,BCDY,NN,YII,T1P )
      CALL QUIKML( 0,0.0,1.0,0.0,5.0,1H2,BCDX,BCDY,NN,YII,T2P )
      CALL QUIKML( 0,0.0,1.0,0.0,5.0,1H3,BCDX,BCDY,NN,YII,T3P )
      X = 1.0
  311 CONTINUE
      THE1 = 1.0 - T1SUM
      THE2 = DTI * T2S
      THE3 = 1.0 - DTI*T3SUM
      CK = K1 + K3
      QAVE = 0.5 * (COM+COM1)
```

```
      QMAX = (K1 + K3/DTI) * (1.0 / CK)
      IF(CK .LT. 1.0) QMAX = K1 + K3/DTI
      E = QAVE / QMAX
      WRITE(6,1005)
 3000 CONTINUE
      IF(IMO .NE. 0)T2SSS = T2S
      IF(IAVE .EQ. 0)GO TO 100
      I77 = I77 + 1
      IF(I77 .GT. 2)GO TO 44
      IF(IMIX .NE. 0)GO TO 46
      T11 = T1SUM
      T22 = 0.0
      T33 = T3SUM
      IF(ITYPE .EQ. 2)T22=T2S
      GO TO 203
   45 CONTINUE
      T22 = T2S
      T11 = 1.0
      T33 = 1.0 / DTI
      IF(IMIX .EQ. 1)INV = 1
      IF(IDENT .EQ. 0) GO TO 52
      INV = 1
      DO 9 I55=1,NN
      T1(I55) = T11
      T3(I55) = T33
    9 THPPP(I55) = T2P(I55)
      DO 91 I55=1,NN
      I551 = NN - I55 + 1
   91 T2P(I55) = THPPP(I551)
      GO TO 42
   46 CONTINUE
      IF(IDENT .EQ. 0)GO TO 47
      DO 49 I55 = 1,NN
   49 THPPP(I55) = T1(I55)
      DO 5 I55=1,NN
      I551 = NN - I55 + 1
    5 T1(I55) = THPPP(I551)
      DO 6 I55 = 1,NN
    6 THPPP(I55) = T3(I55)
      DO 7 I55=1,NN
      I551 = NN - I55 + 1
    7 T3(I55) = THPPP(I551)
      DO 8 I55=1,NN
    8 T2P(I55) = 0.0
      T11 = T1SUM
      T22 = T2S
      T33 = T3SUM
      IF( ITYPE .EQ. 2)GO TO 56
      GO TO 200
   56 DO 57 I55 = 1,NN
      I551 = NN - I55 + 1
   57 T2P(I55) = T2(I551)
      GO TO 200
   52 DO 53 KKO=1,NN
      T1(KKO) = T11
```

```
   53 T3(KKO) = T33
      IF( INIX .EQ. 1) INV = 1
      GO TO 42
   47 CONTINUE
      IF(ITYPE .EQ. 2)GO TO 58
      DO 48 I55=1,NN
   48 T2P(I55) = 0.0
      GO TO 200
   58 DO 59 KKO=1,NN
   59 T2P(KKO) = T2(KKO)
      GO TO 200
   44 CONTINUE
      IF( ITYPE .LE. 2)GO TO 103
      T1T(I66) = T2S
      IF( I66 .EQ. 1)GO TO 45
      IF( (ABS(T1T(I66-1) - T1T(I66))/T1T(I66-1)) .LT. 0.01 )GO TO 103
      T11 = 1.0
      T22 = T2S
      T33 = 1.0 / DTI
      IF(IDENT .EQ. 0)GO TO 52
      INV = 1
      DO 102 I55=1,NN
      T1(I55) = 1.0
      T3(I55) = T33
  102 THPPP(I55) = T2P(I55)
      DO 92 I55=1,NN
      I551 = NN - I55 + 1
   92 T2P(I55) = THPPP(I551)
   42 CONTINUE
      WRITE(6,23)
  103 CONTINUE
      WRITE(6,1022)
      THE10 = 1.0 - T1SUM
      THE30 = 1.0 - DTI * T3SUM
      Q20 = T2SSS
      IF(ITYPE .EQ. 2) Q20 = T2S
      Q130 = K1 * (1.0-T1SUM) + K3 * (1.0/DTI - T3SUM)
      EO = (Q130 + Q20) * 0.5 / QMAX
      ACC = ABS(Q20-Q130) / (Q130+Q20) * 100.0
      QAVEO = (Q20+Q130) * 0.5
      WRITE(6,1004)Q130, Q20, ACC
      IF(OK .LT. 1.0)WRITE(6,1003)
      IF(OK .GE. 1.0)WRITE(6,1002)
      AA2 = A + A
      WRITE(6,1024)QAVEO, QMAX, AA2, EO, THE10, THE30
      CALL TIME (ITIME)
      IZERO = ITIME - IZERO
      WRITE(6,1029) IZERO, ITIME
      IZERO = ITIME
 1029 FORMAT(1H0,10X,19HTIME FOR THIS CASE ,I5,13H MICROSECONDS, //
     1 11X,11HTOTAL TIME ,I8,13H MICROSECONDS    )
      WRITE(6,1005)
      CALL DMPCUF
      GO TO 100
   10 FORMAT( 6(5X,E14.7) )
```

```
 11 FORMAT(1H0,///,5X,4HX = ,F5.3 )
 12 FORMAT(1H0,///,2X,16HT1(I),    I = 1,N    )
 13 FORMAT(1H0,///,2X,16HT2(I),    I = 1,N    )
 14 FORMAT(1H0,///,2X,16HT3(I),    I = 1,N    )
 15 FORMAT(2I4, 6I1 )
 16 FORMAT(7F10.2)
 17 FORMAT(1H0, //, 55X,14H*** OUTPUT ***    )
 20 FORMAT(1H0,40X,26HTHE AVERAGE VALUE OF T1 = ,F8.3, / 41X,
   1 26HTHE AVERAGE VALUE OF T2 = ,F8.3, / 41X,
   2 26HTHE AVERAGE VALUE OF T3 = ,F8.3    )
 21 FORMAT(1H0,57X, 8HDOMAIN I )
 22 FORMAT (1H0,57X, 9HDOMAIN II )
 23 FORMAT(1H0,20X,45HTHE SYSTEM DID NOT CONVERAGE IN 10 ITERATIONS)
 28 FORMAT(1H0,48X,24HT2 GUESS FOR DOMAIN I = ,F5.2)
 29 FORMAT(1H0,56X,10HITERATION ,I2 )
 55 FORMAT(12A6/12A6)
995 FORMAT( 72I1 )
1000 FORMAT(1H1,     58X,13H*** INPUT *** )
1001 FORMAT(1H0,    59X,11HSINGLE PASS   )
1002 FORMAT(1H0,20X,39HK1 + K3 IS GREATER THAN OR EQUAL TO 1.0  )
1003 FORMAT(1H0,20X,24HK1 + K3 IS LESS THAN 1.0     )
1004 FORMAT(1H , 35X, 44HFOR ACCURACY CHECK ON ENERGY BALANCE COMPARE//
    1 29X, 6HQ13 = ,F8.3,13H  WITH  Q2 = ,F8.3,5X,3HOR ,F7.4        ,
    2  18H PER CENT ACCURACY     )
1005 FORMAT(1H0,128(1H*) )
1007 FORMAT(1H0, 40X, 23HBOUNDARY CONDITIONS            /  43X    ,
    1 9HT1 = 1.00,5X,8HT2 = 0.0,5X,5HT3 = ,F4.2    )
1008 FORMAT(1H0,59X,12HINPUT VALUES / 30X,4HA = ,F4.2,5X,5HK1 = ,
    1 F4.2,5X,5HK3 = ,F4.2,5X,4HU = ,F4.2,5X,11HDELTA TI = ,F4.2    )
1009 FORMAT(1H0, 40X, 39HTHE RESULTING VALUES OF B, C, AND D ARE   /
    1   43X, 4HB = ,F5.2, 5X, 4HC = ,F5.2, 5X, 4HD = ,F5.2    )
1010 FORMAT(1H0,    , 51X, 15HMULTIPLE PASS (,I2. 18H)     PARALLEL FLOW)
1011 FORMAT(1H0,    , 51X, 15HMULTIPLE PASS (,I2, 17H)     COUNTER FLOW )
1012 FORMAT(1H ,        62X, 5HMIXED)
1013 FORMAT(1H , 61X,  7HUNMIXED )
1014 FORMAT(1H , 57X, 14HINVERTED ORDER )
1015 FORMAT(1H , 57X, 15HIDENTICAL ORDER   )
1016 FORMAT(1H0,50X,10HDELTA X = ,F5.3 / 51X,10HDELTA Y = ,F5.3 )
1017 FORMAT(1H0,40X,45HDATA IS PRINTED OUT AT EACH CALCULATED POINT.   )
1018 FORMAT(1H0,40X,48HDATA IS PRINTED OUT AT THE FOLLOWING VALUES OF X
    1)
1019 FORMAT(     51X,2HX(,I2,4H) = , F5.3 )
1020 FORMAT(1H0,40X,7HQAVE = ,F8.3 , 10X,7HQMAX = ,F8.3  // 50X,
    1 4HA = ,F8.3  //30X,4HE = ,F8.3 ,10X,9HTHETA1 = ,F8.3 ,10X
    2, 9HTHETA3 = ,F8.3      )
1021 FORMAT(1H0,34X,33HTHE AVERAGE EXIT TEMPERATURES ARE   )
1022 FORMAT(1H0,41X,32HTHE OVERALL OUTPUT VARIABLES ARE)
1023 FORMAT(1H0,35X,44HFOR ACCURACY CHECK ON ENERGY BALANCE COMPARE //
    1 28X, 7HQ13O = ,F8.3,14H  WITH  Q2O = ,F8.3 ,5X, 3HOR ,F7.4 ,
    2 18H PER CENT ACCURACY    )
1024 FORMAT(1H0,33X,8HQAVEO = ,F8.3, 10X,7HQMAX = ,F8.3 //       48X,
    17HNTU1 = ,F8.3//24X,5HEO = ,F8.3,10X,10HTHETA1O = ,F8.3,10X,
    2 10HTHETA3O = , F8.3    )
     END
```

```
      SUBROUTINE AB1 (T1, T2, T3, DELY, NX , T2G )
      COMMON /AB/ A, B, C, D, DTI
      DIMENSION T1(1001), T2(1001), T3(1001)
      T2(1) = T2G
      DO 1 N=1,NX
      DEL = DELY * (C*T1(N)      + D*T3(N) - (C+D) * T2(N) )
      TEMPT2 = T2(N) + DEL
      DEL1 = DELY * (C*T1(N+1)      + D*T3(N+1) - (C+D) * TEMPT2 )
    1 T2(N+1) = T2(N) + (DEL+DEL1) * 0.5
      RETURN
      END
```

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NASA-S-67-6725

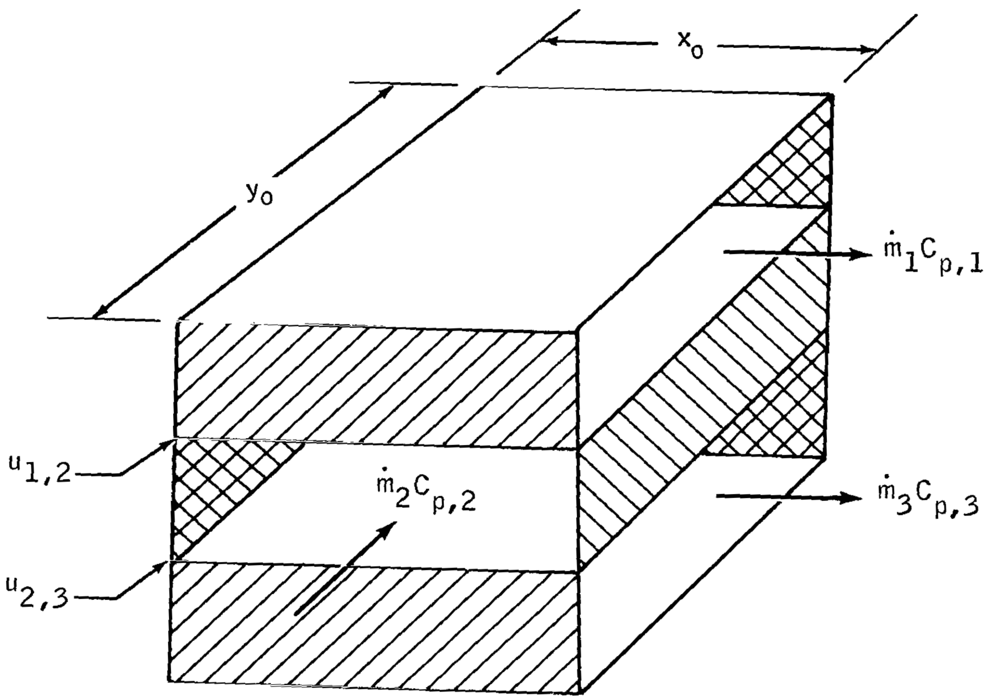


Figure 1. - Schematic representation of a single-pass, three-fluid heat exchanger in crossflow.

NASA-S-67-6726

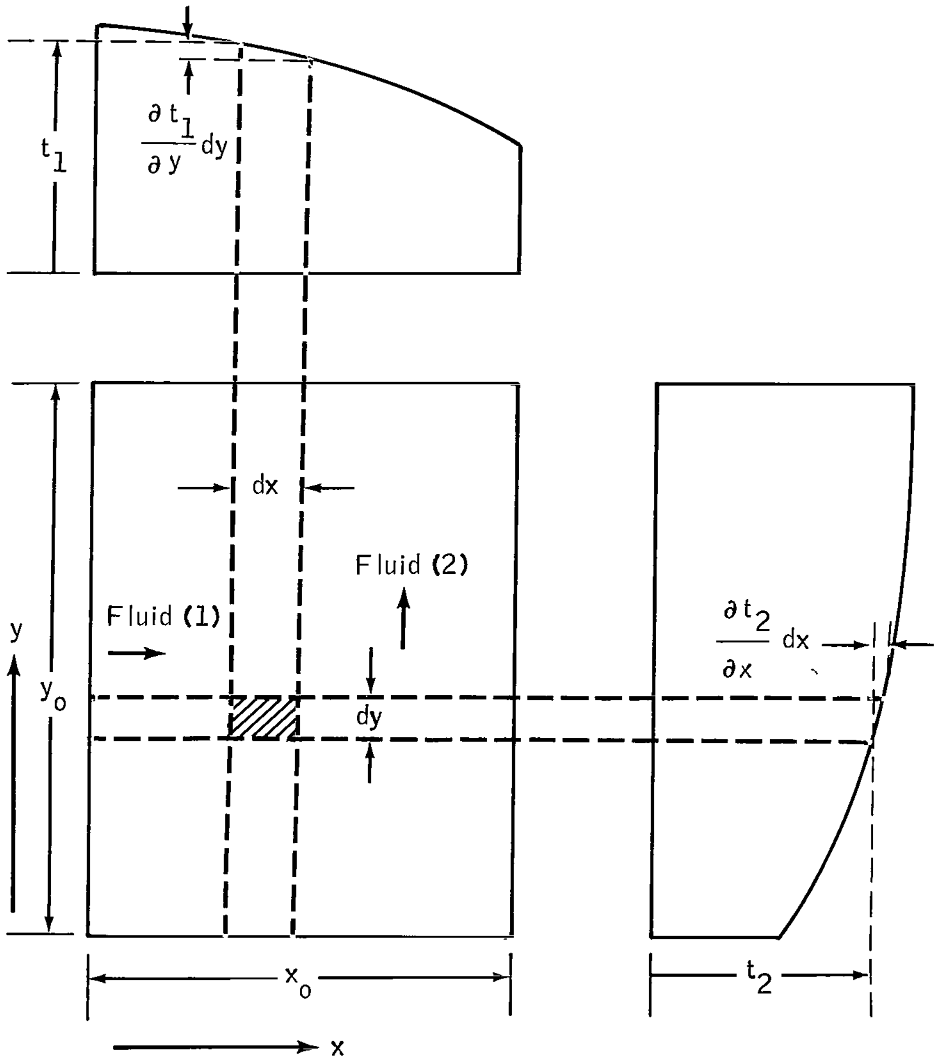


Figure 2. - Schematic representation of the heat-transfer surface between fluids (1) and (2).

NASA-S-67-6727

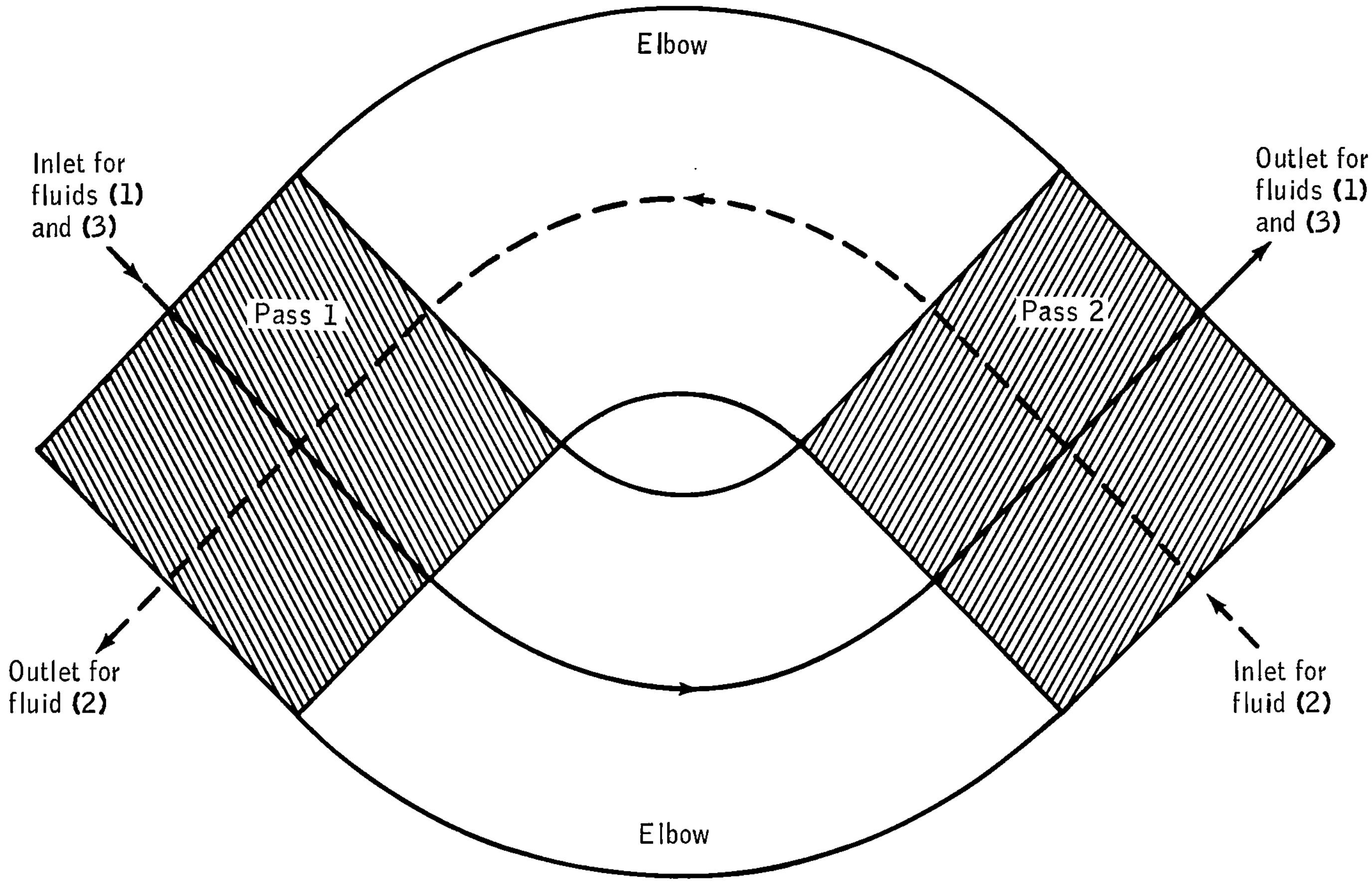


Figure 3. - Schematic representation of a countercurrent two-pass, three-fluid, crossflow heat exchanger.

NASA-S-67-6728

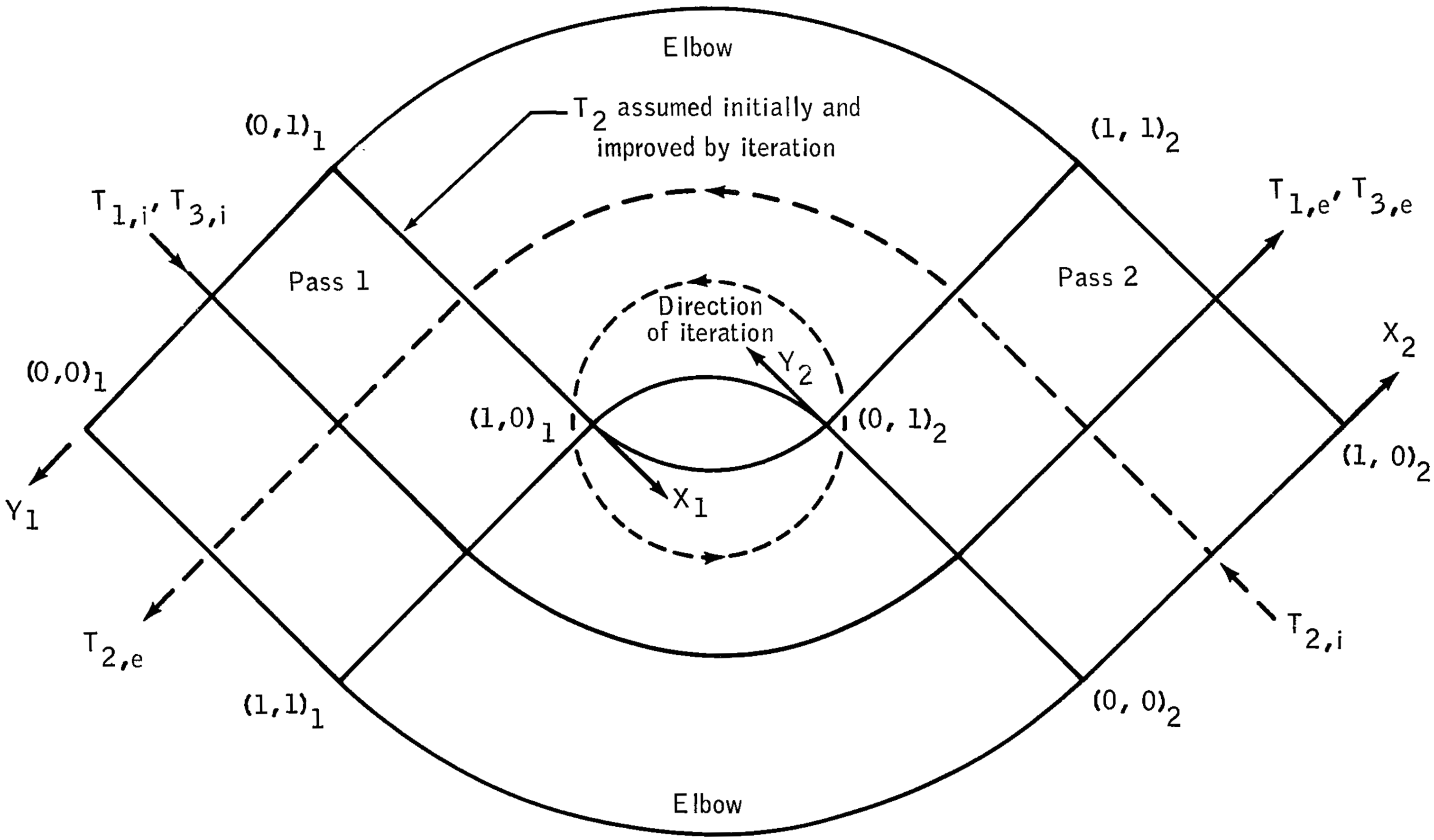


Figure 4. - Mathematical configuration of a countercurrent, two-pass, three-fluid, crossflow heat exchanger.

NASA-S-67-6729

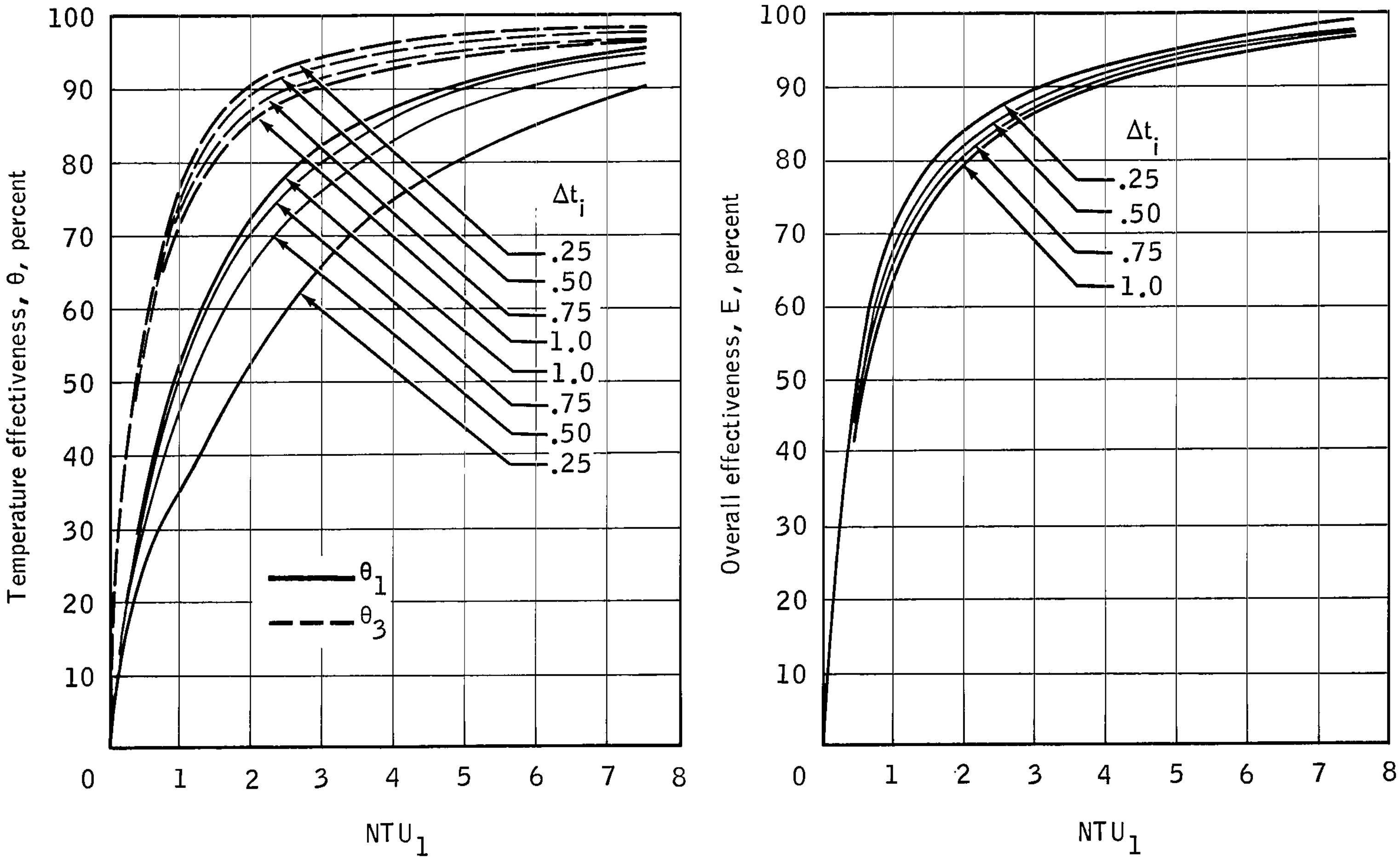


Figure 5. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 0.25$, and $U = 0.50$.

NASA-S-67-6730

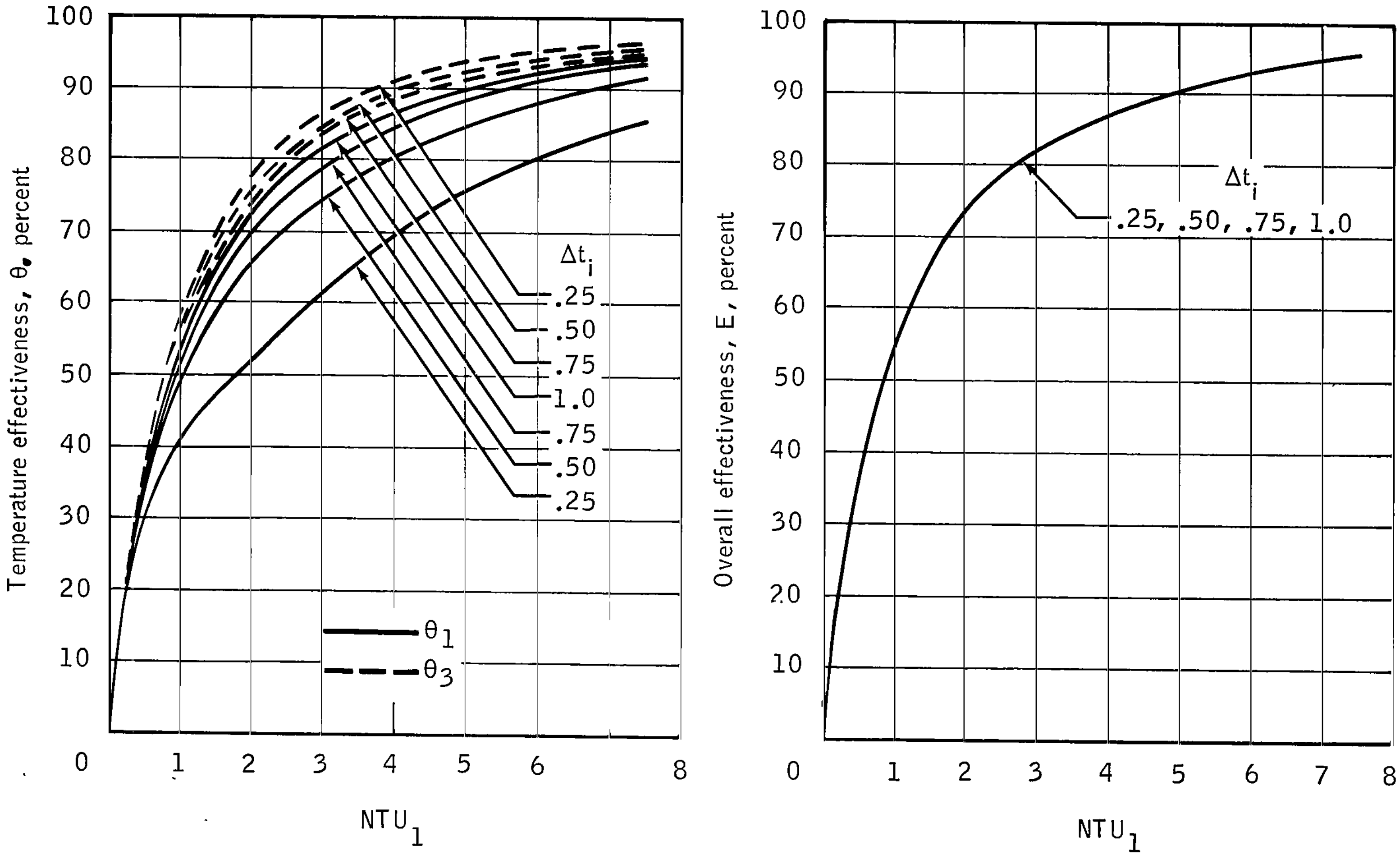


Figure 6. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 0.25$, and $U = 1.0$.

NASA-S-67-6731

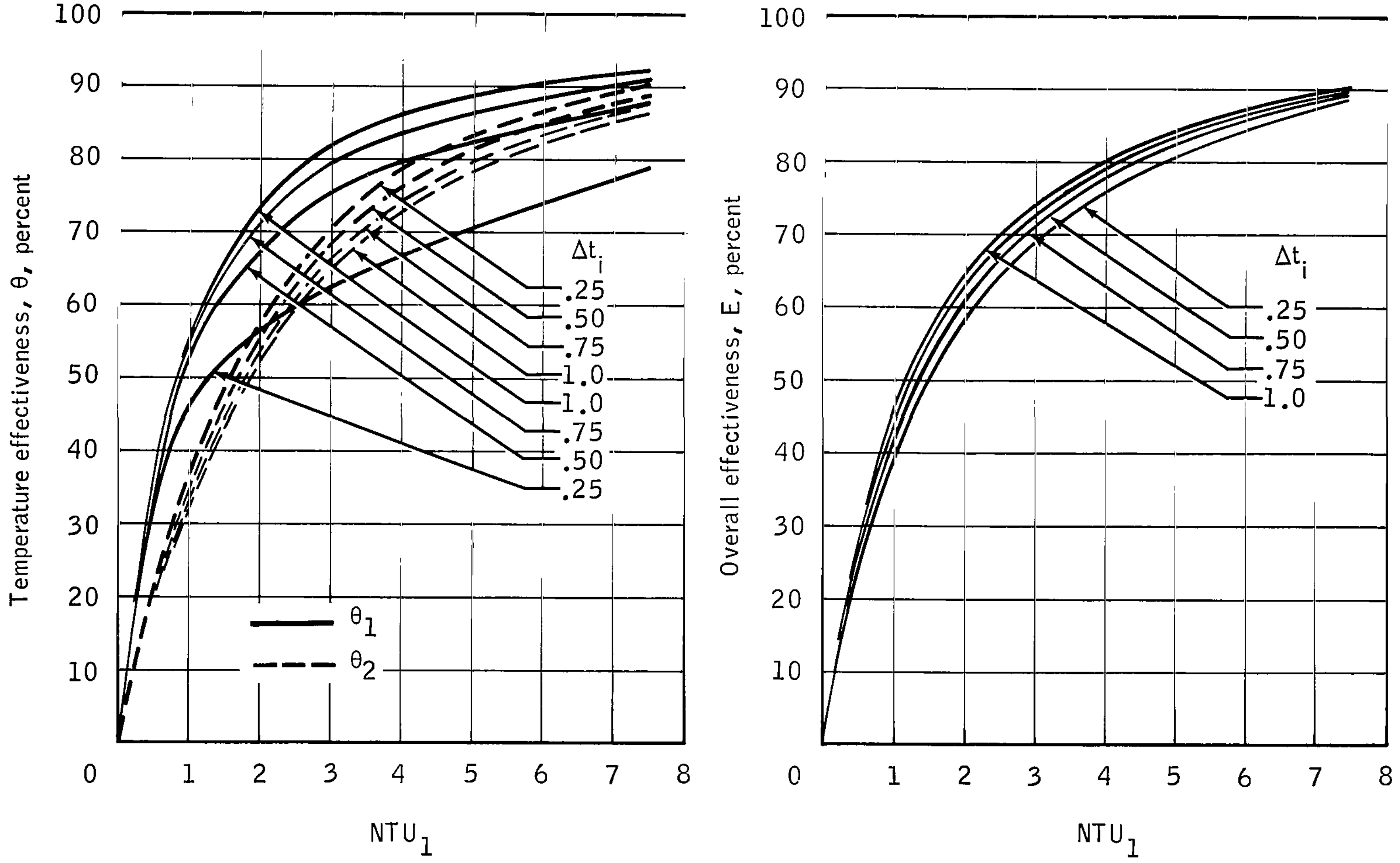


Figure 7. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 0.25$, and $U = 2.0$.

NASA-S-67-6732

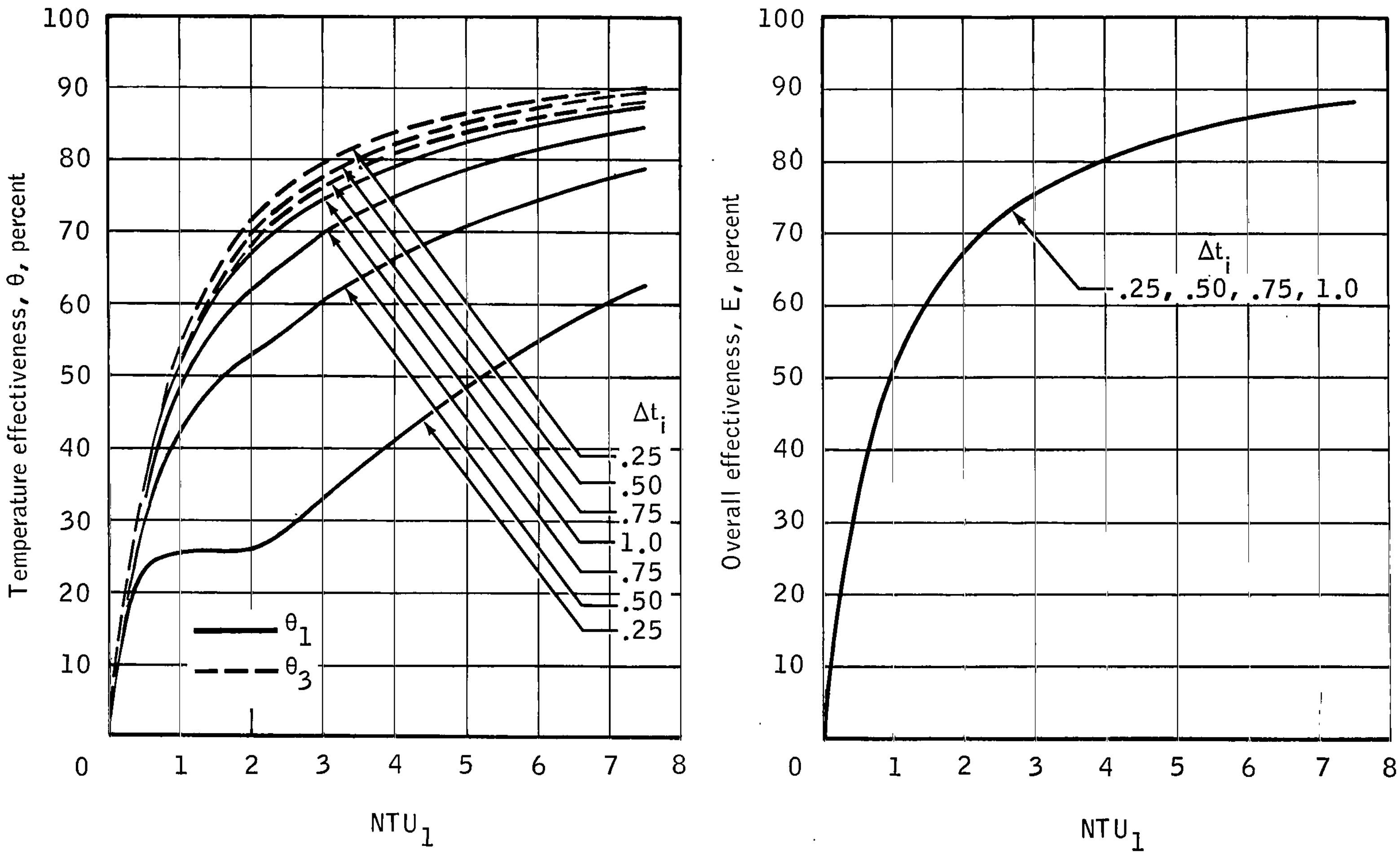


Figure 8. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 0.50$, and $U = 0.50$.

NASA-S-67-6733

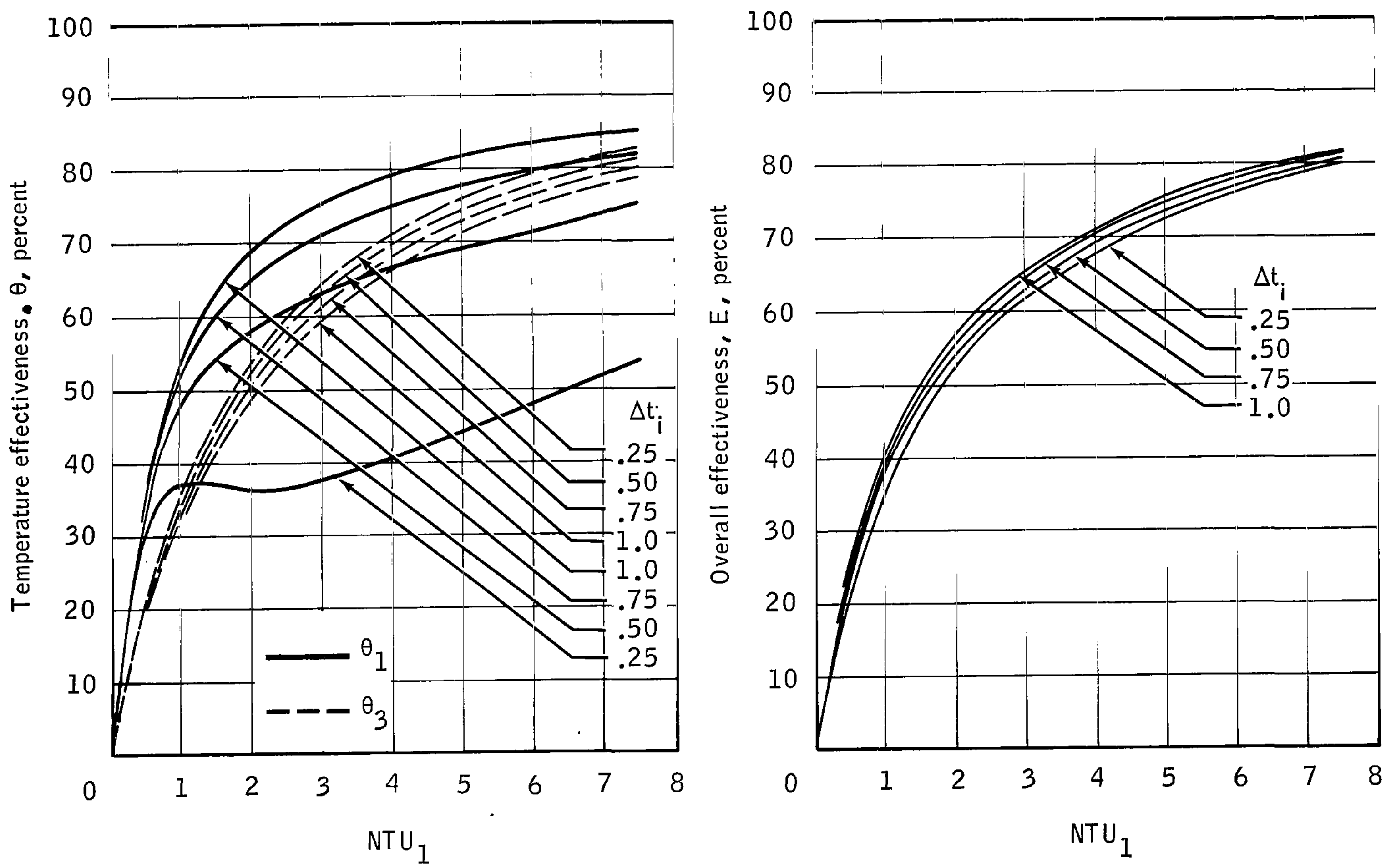


Figure 9. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 0.50$, and $U = 1.0$.

NASA-S-67-6734

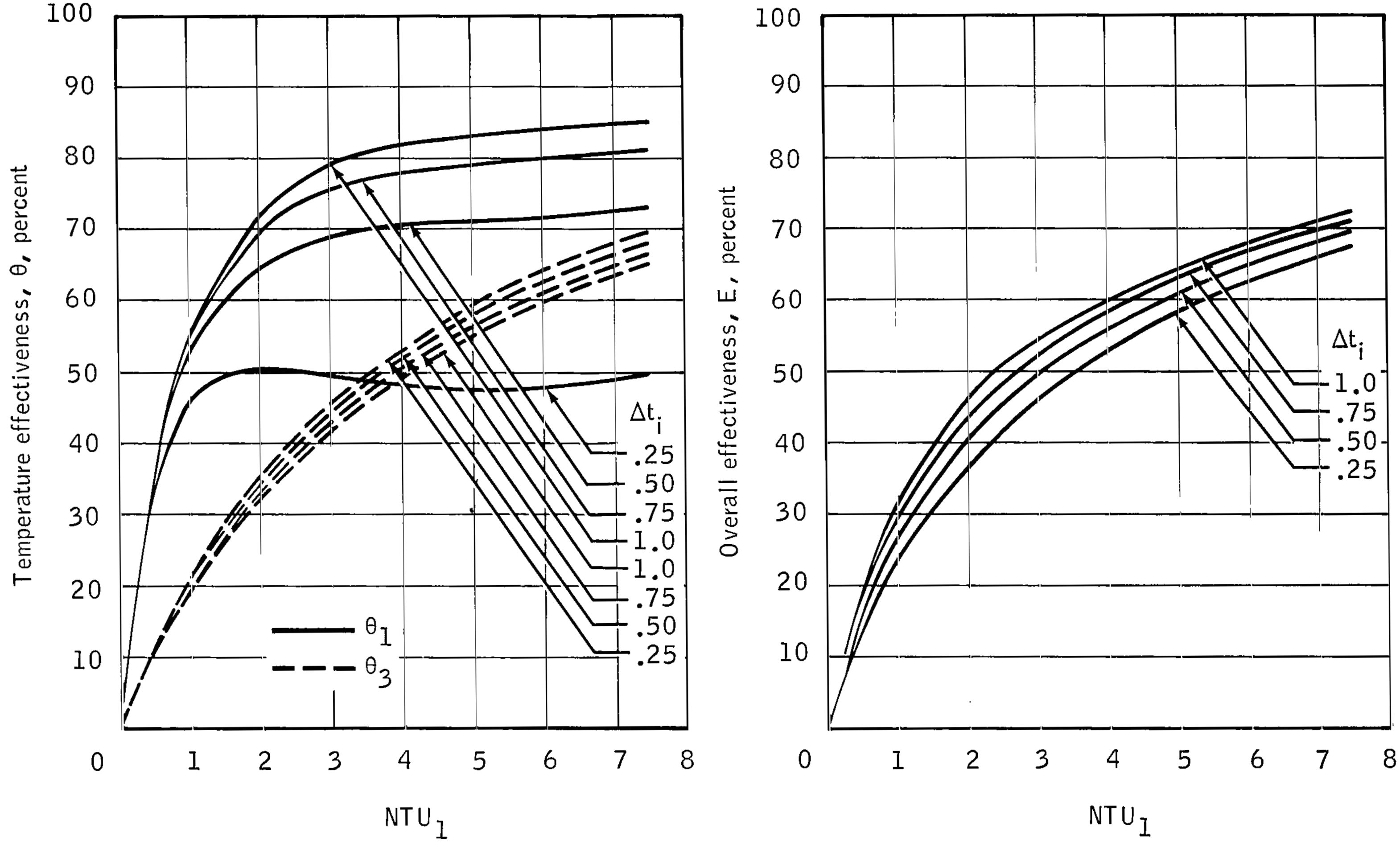


Figure 10. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 0.50$, and $U = 2.0$.

NASA-S-67-6735

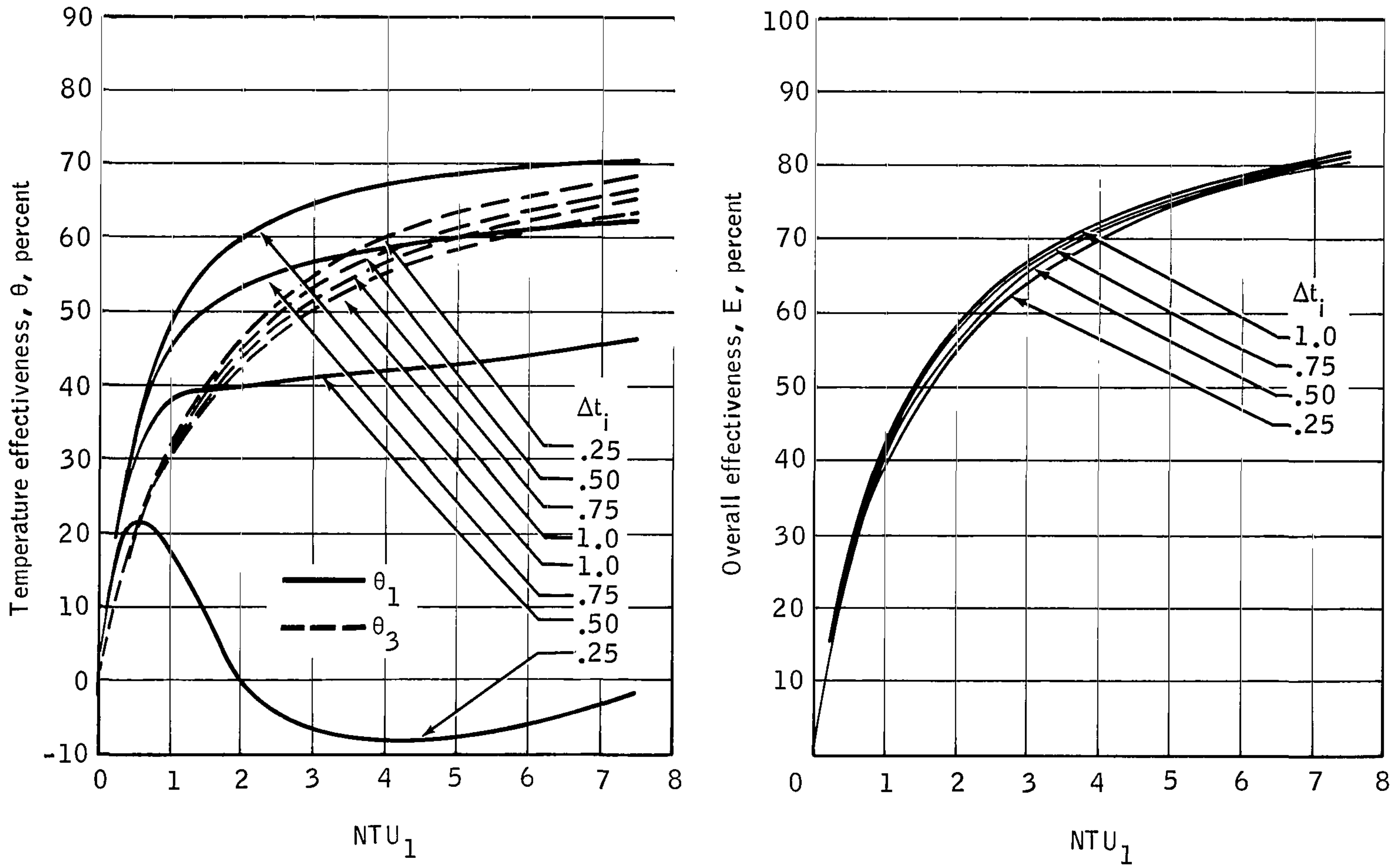


Figure 11. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 1.0$, and $U = 0.50$.

NASA-S-67-6736

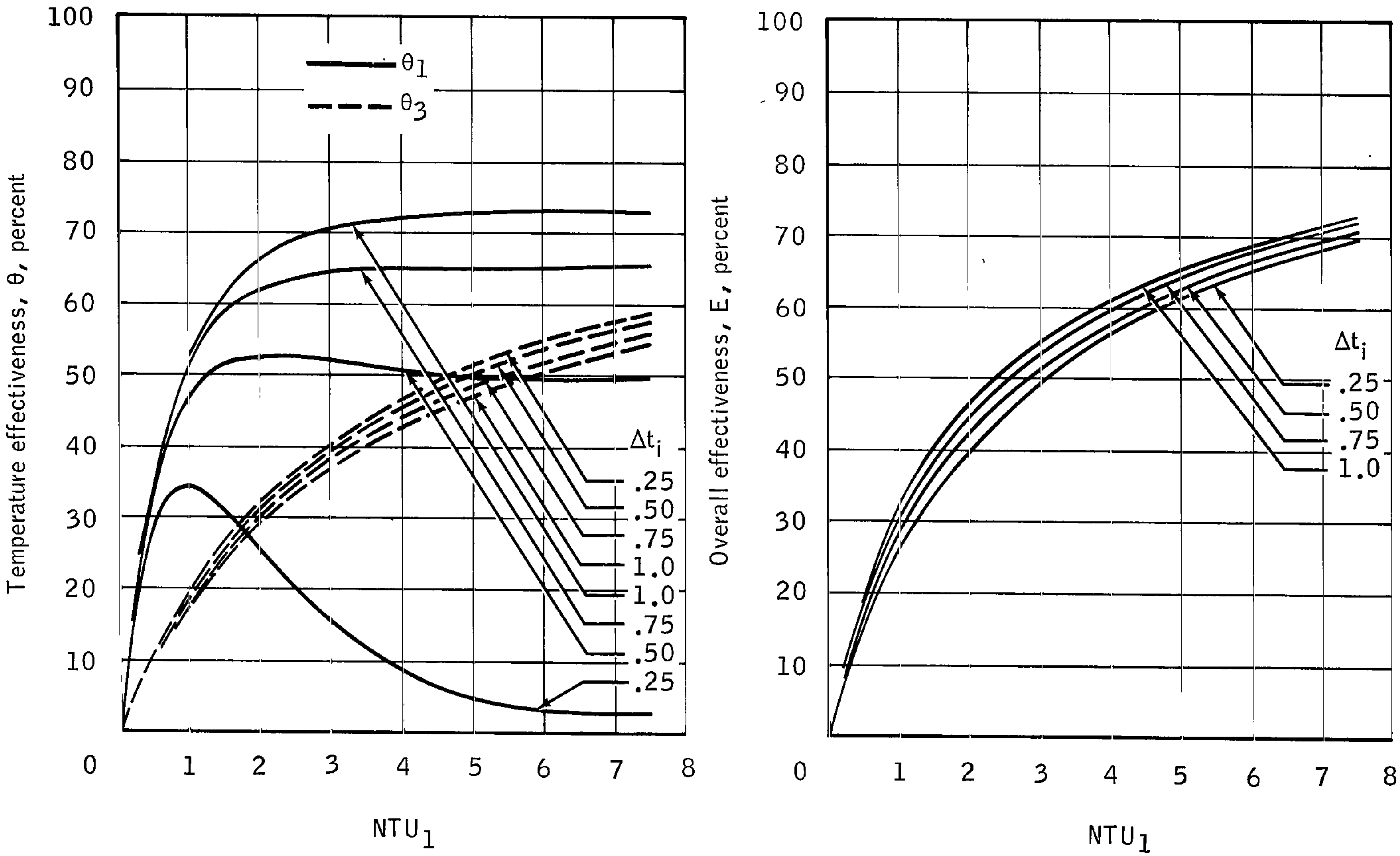


Figure 12. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 1.0$, and $U = 1.0$.

NASA-S-67-6737

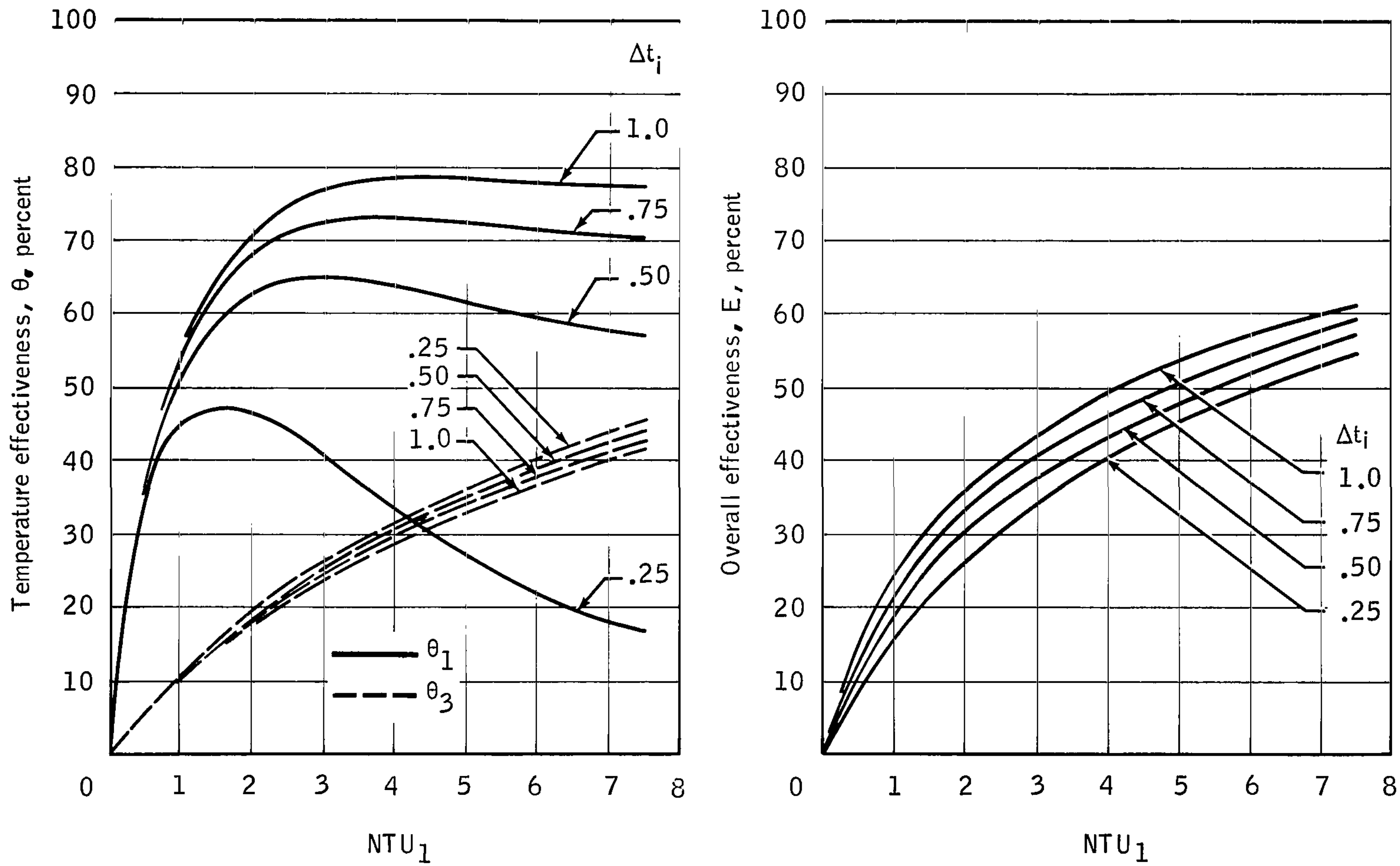


Figure 13. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.25$, $K_3 = 1.0$, and $U = 2.0$.

NASA-S-67-6738

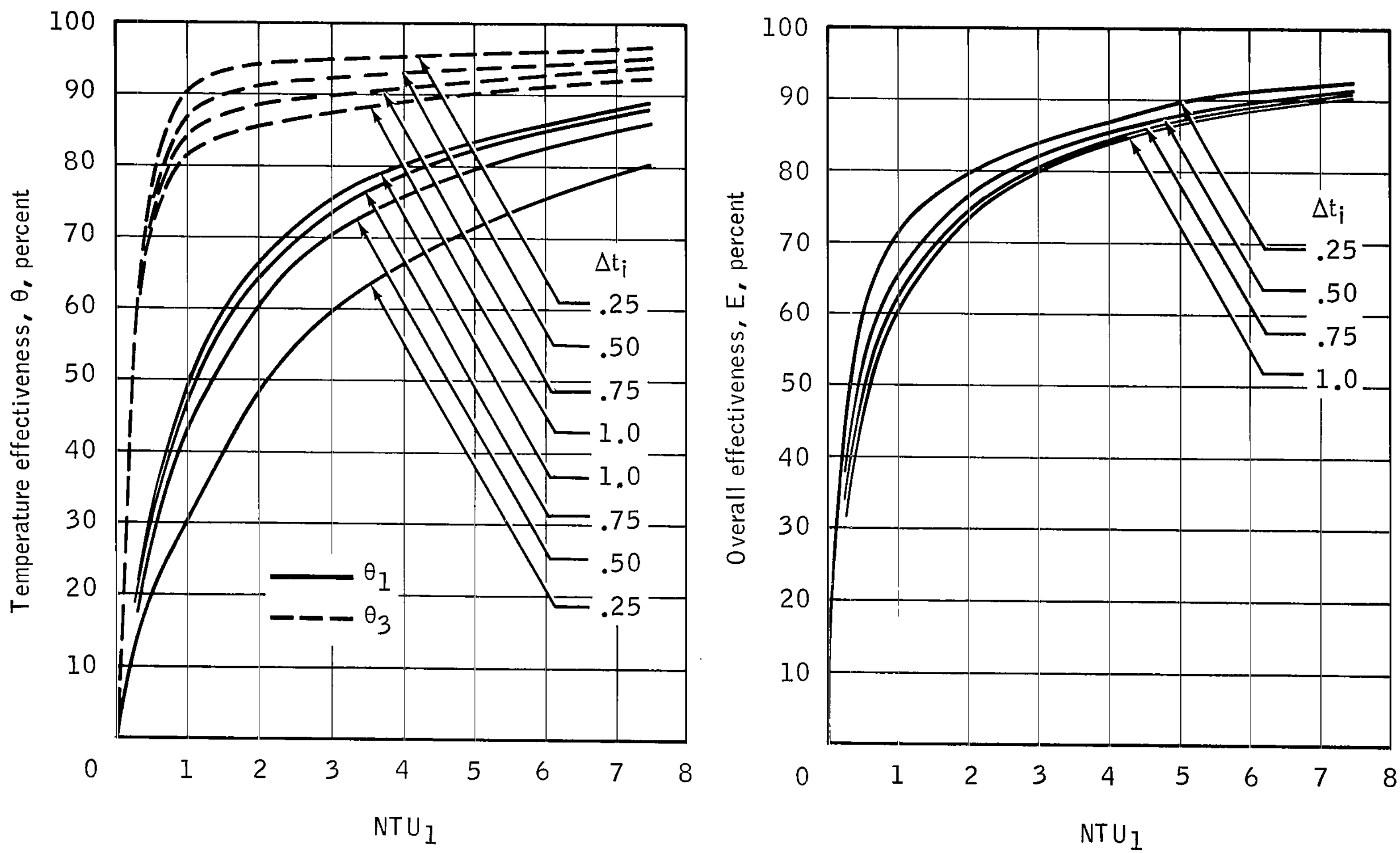


Figure 14. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 0.25$, and $U = 0.50$.

NASA-S-67-6739

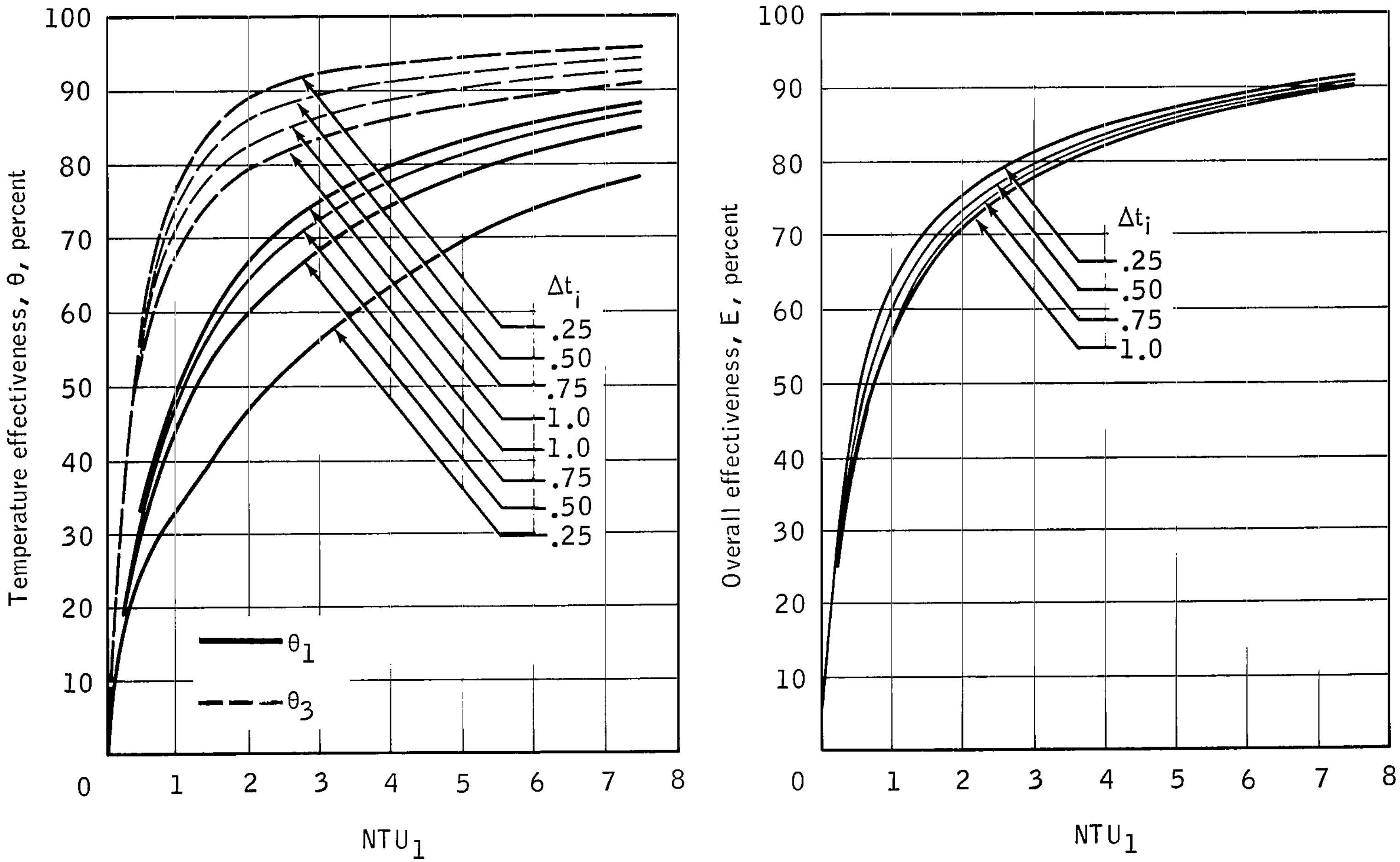


Figure 15. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 0.25$, and $U = 1.0$.

NASA-S-67-6740

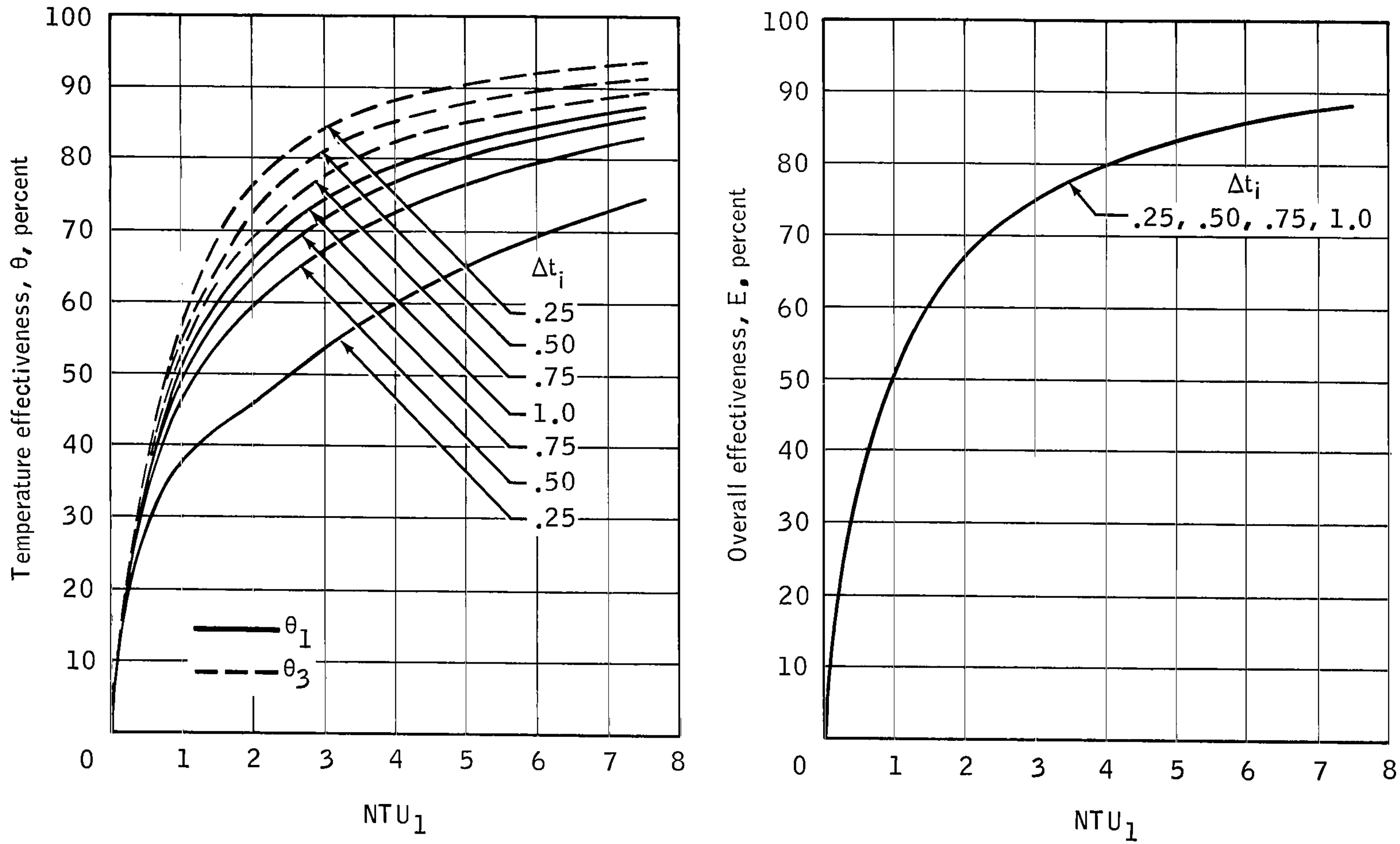


Figure 16. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 0.25$, and $U = 2.0$.

NASA-S-67-6741

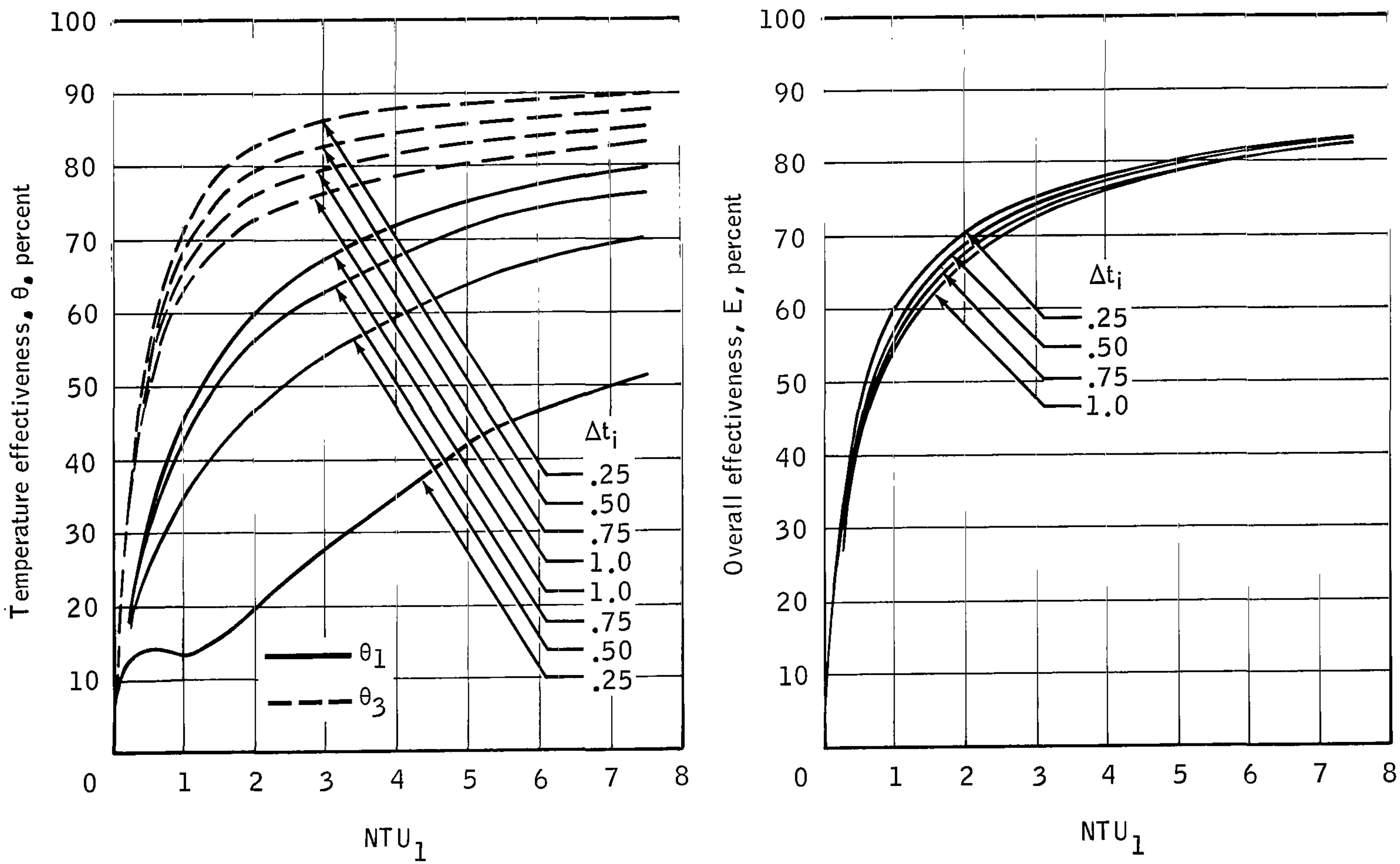


Figure 17. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 0.50$.

NASA-S-67-6742

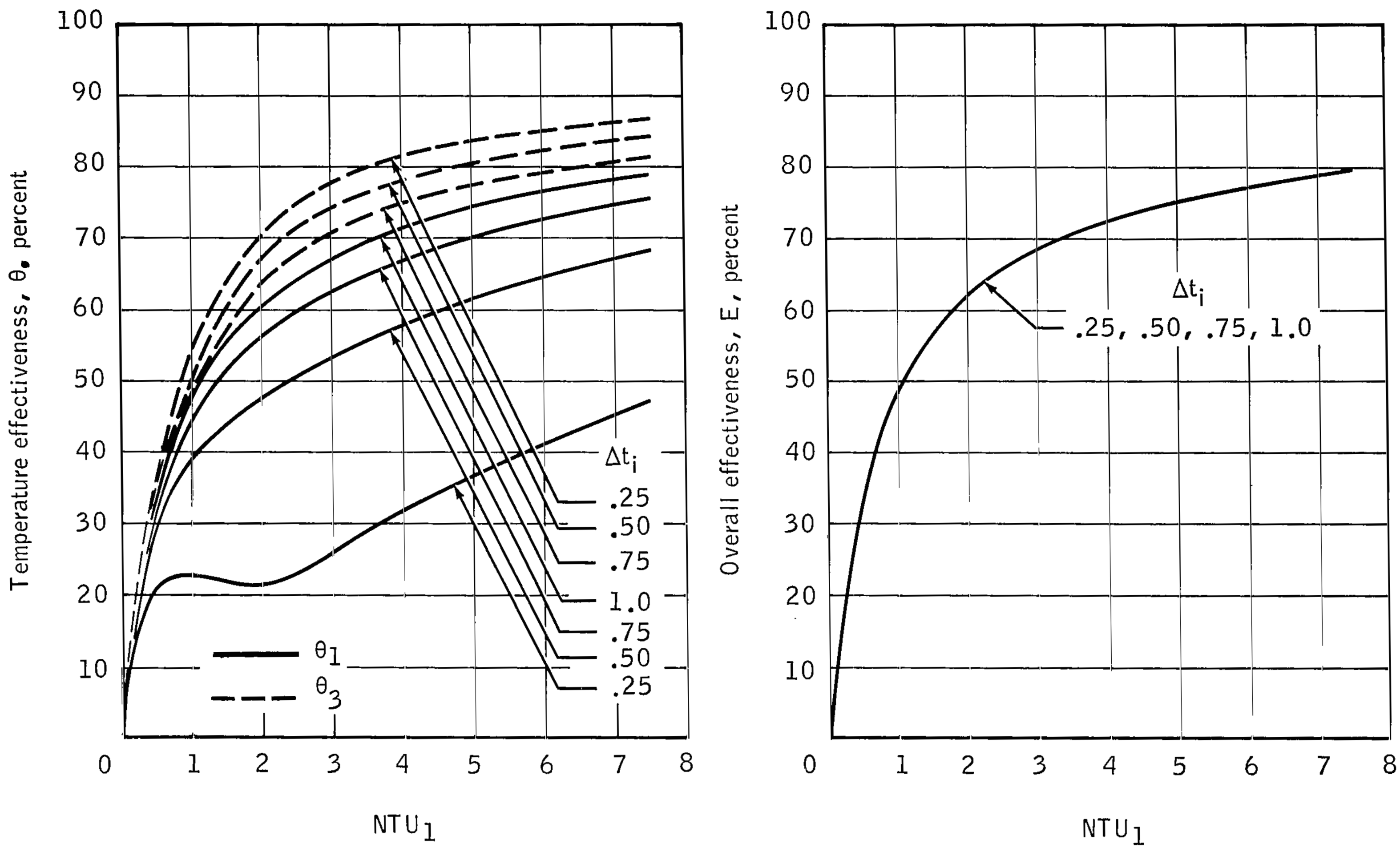


Figure 18. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$.

NASA-S-67-6743

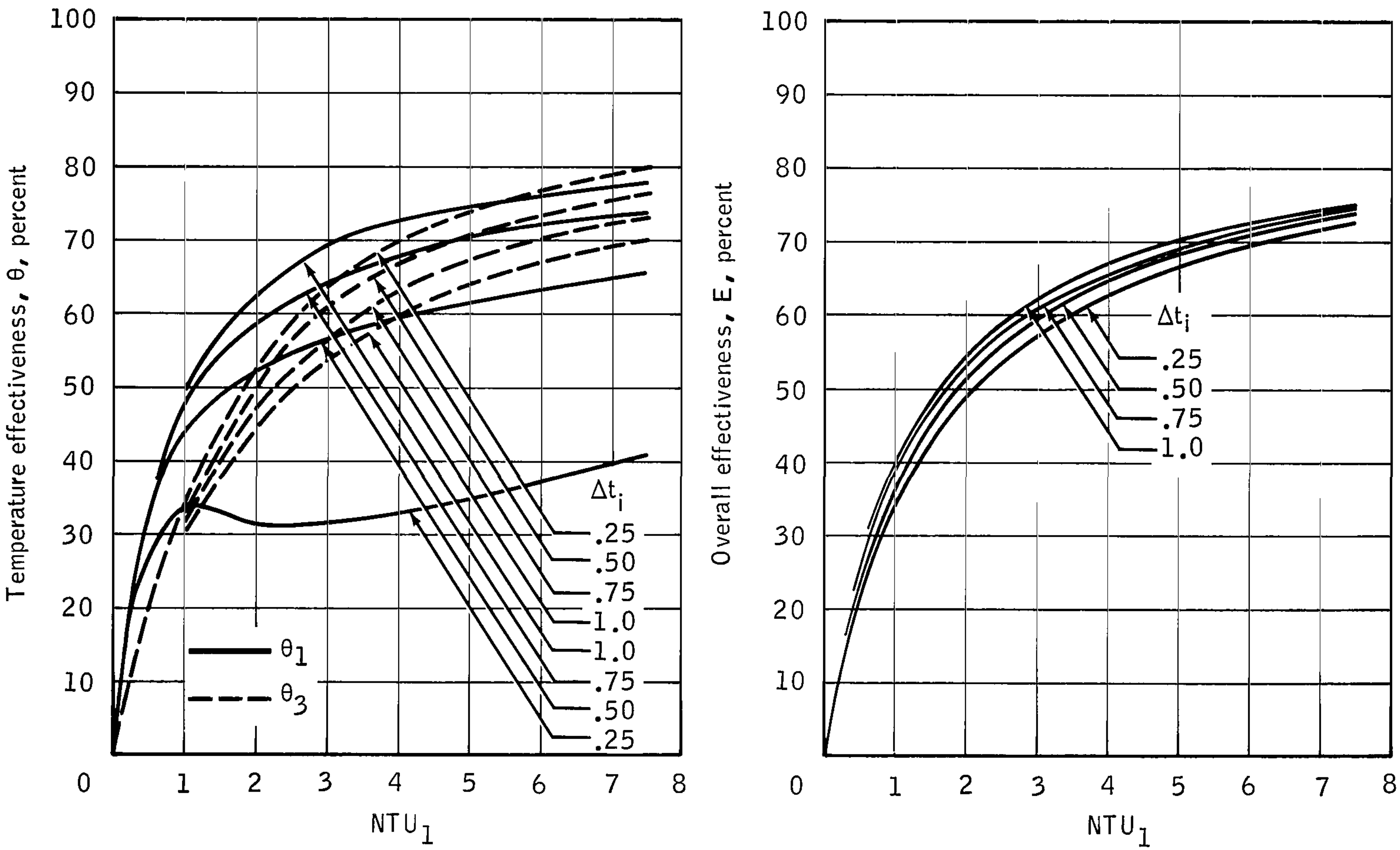


Figure 19. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 2.0$.

NASA-S-67-6744

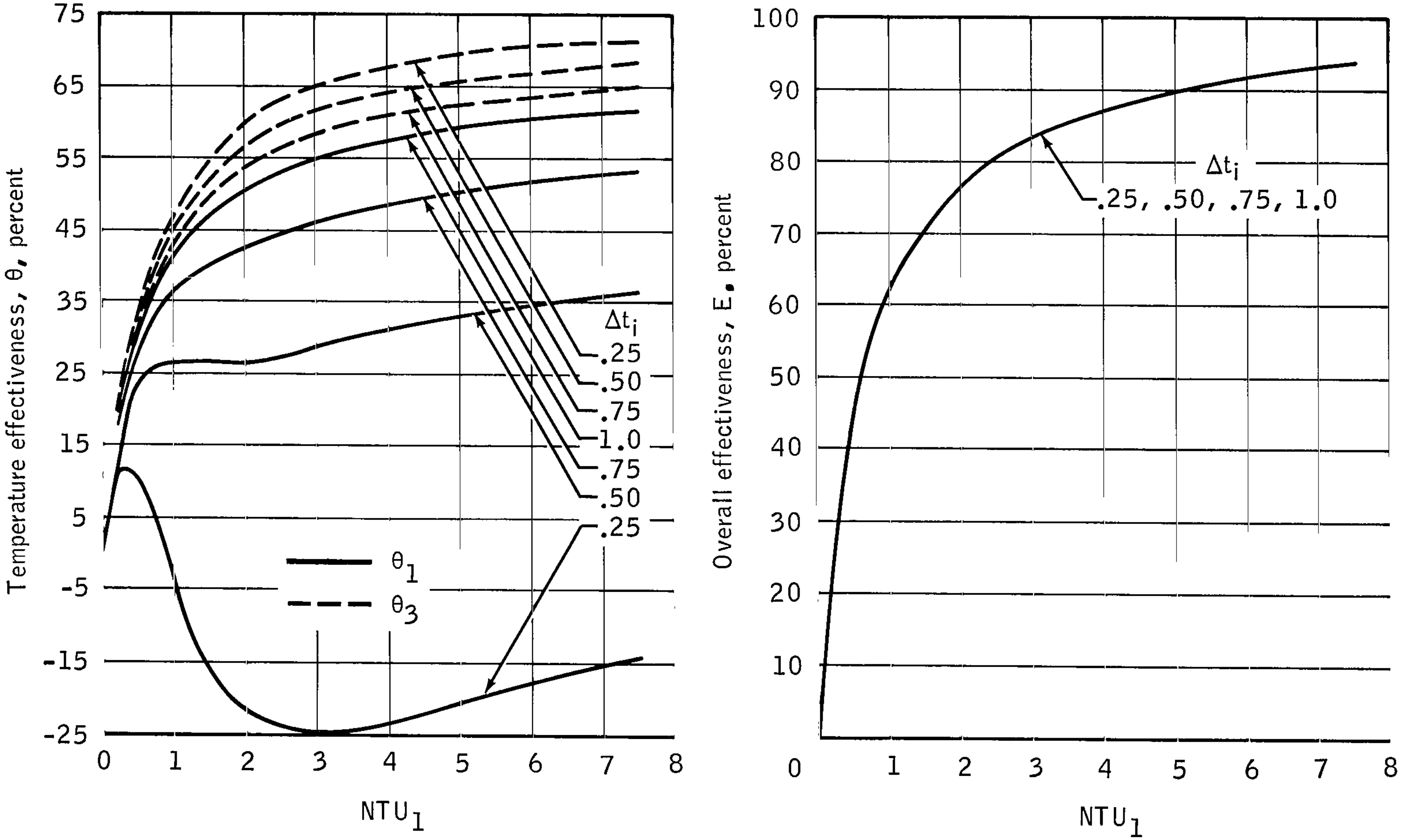


Figure 20. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 1.0$, and $U = 0.50$.

NASA-S-67-6745

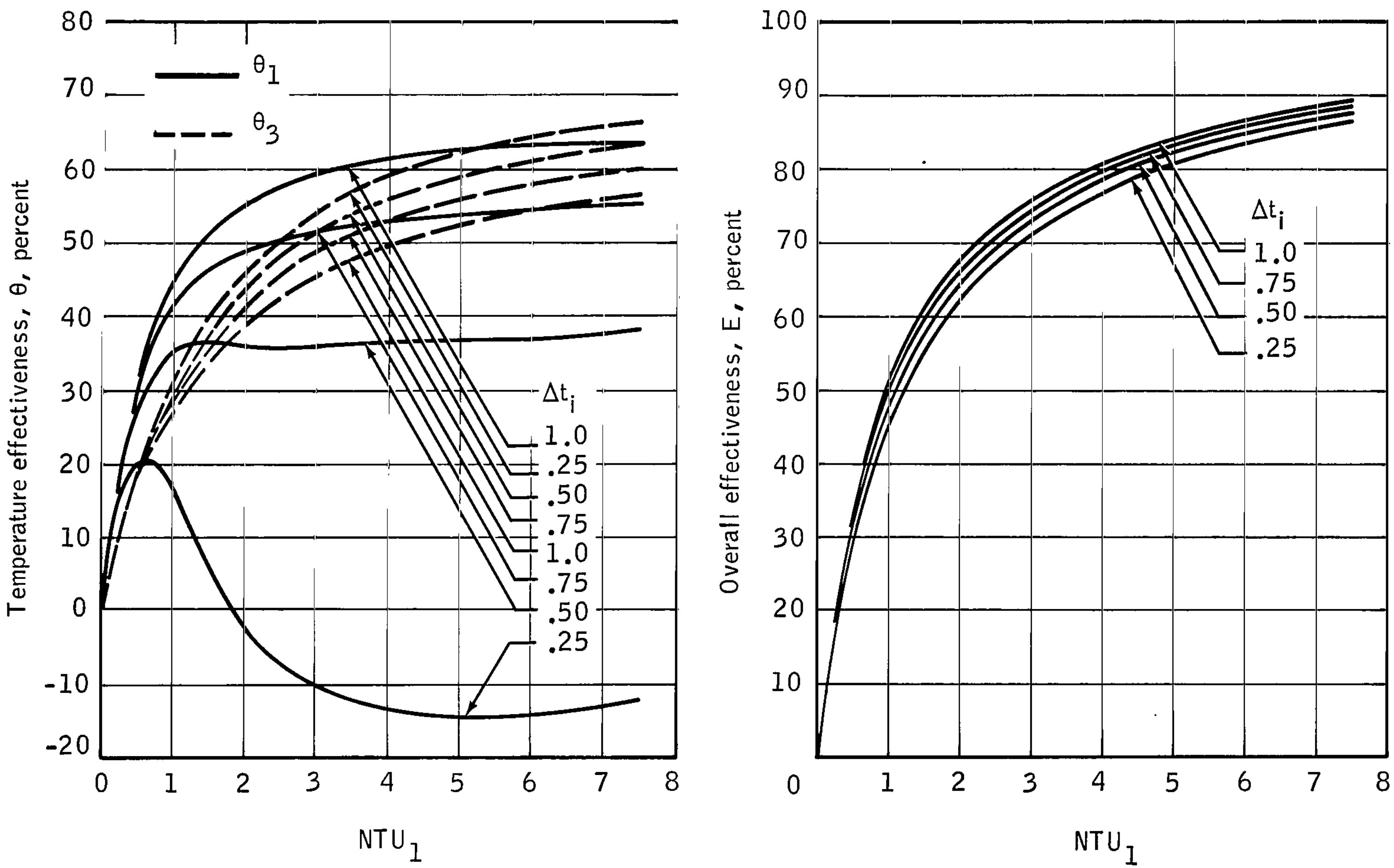


Figure 21. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 1.0$, and $U = 1.0$.

NASA-S-67-6746

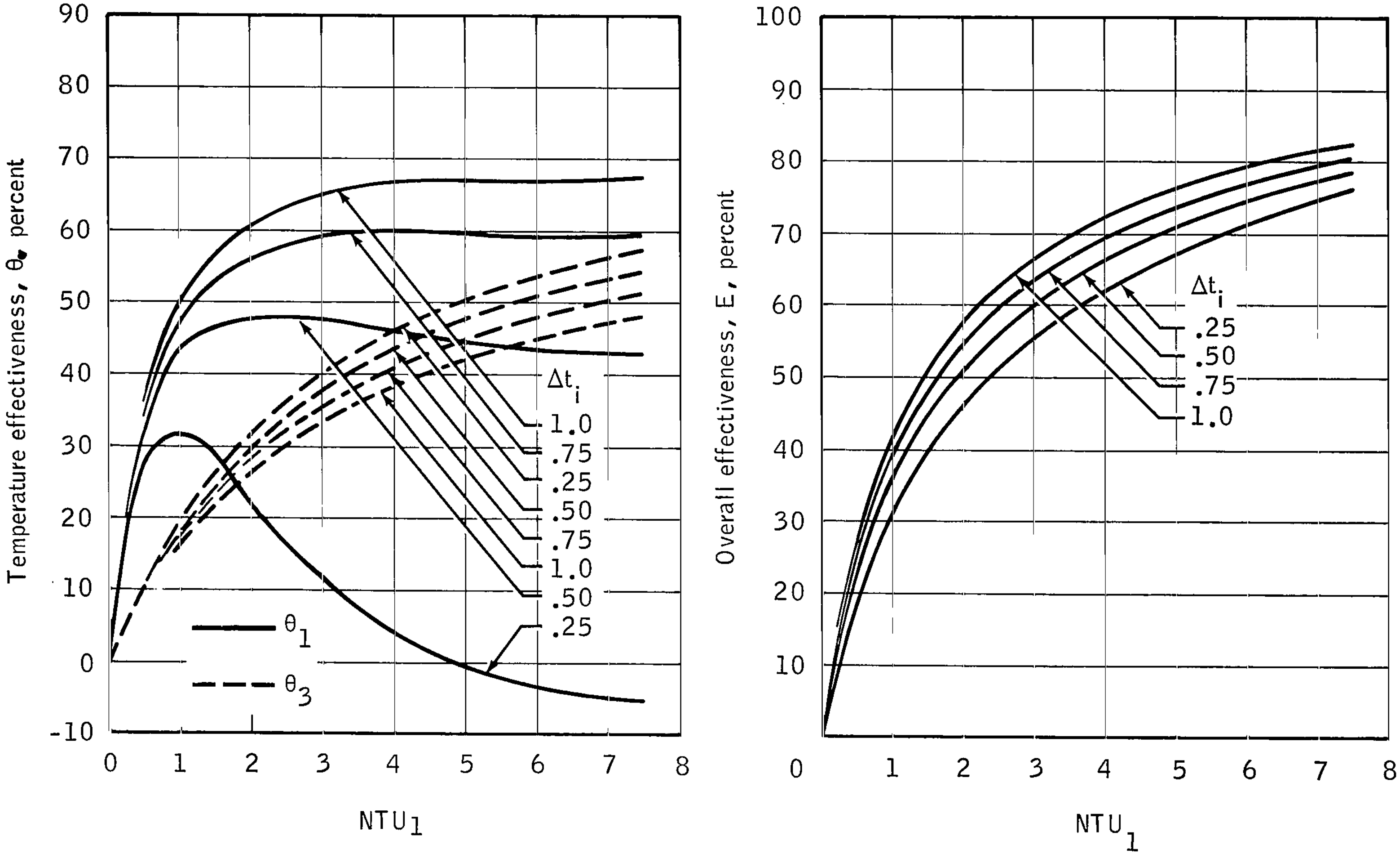


Figure 22. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 0.50$, $K_3 = 1.0$, and $U = 2.0$.

NASA-S-67-6747

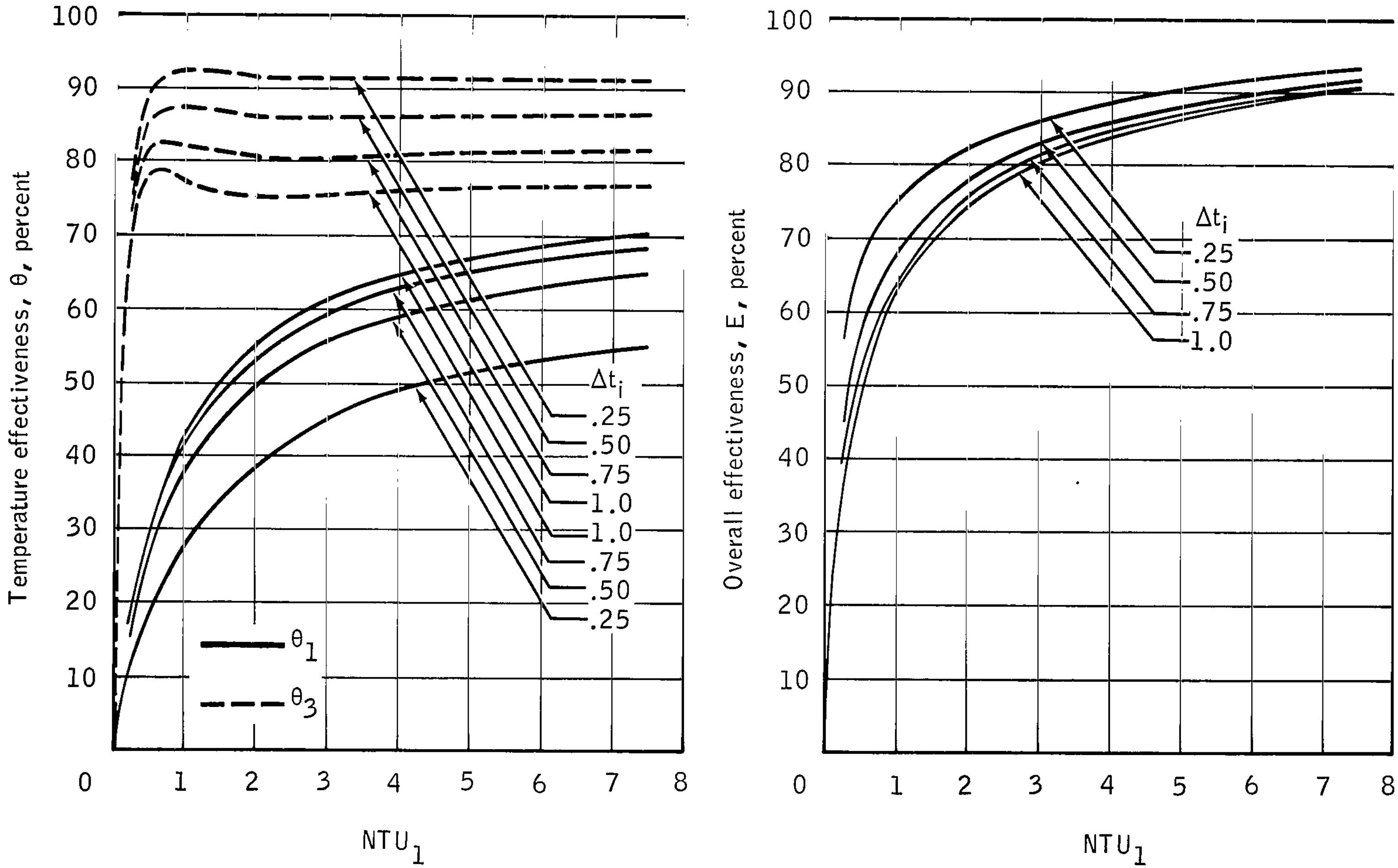


Figure 23. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 0.25$, and $U = 0.50$.

NASA-S-67-6748

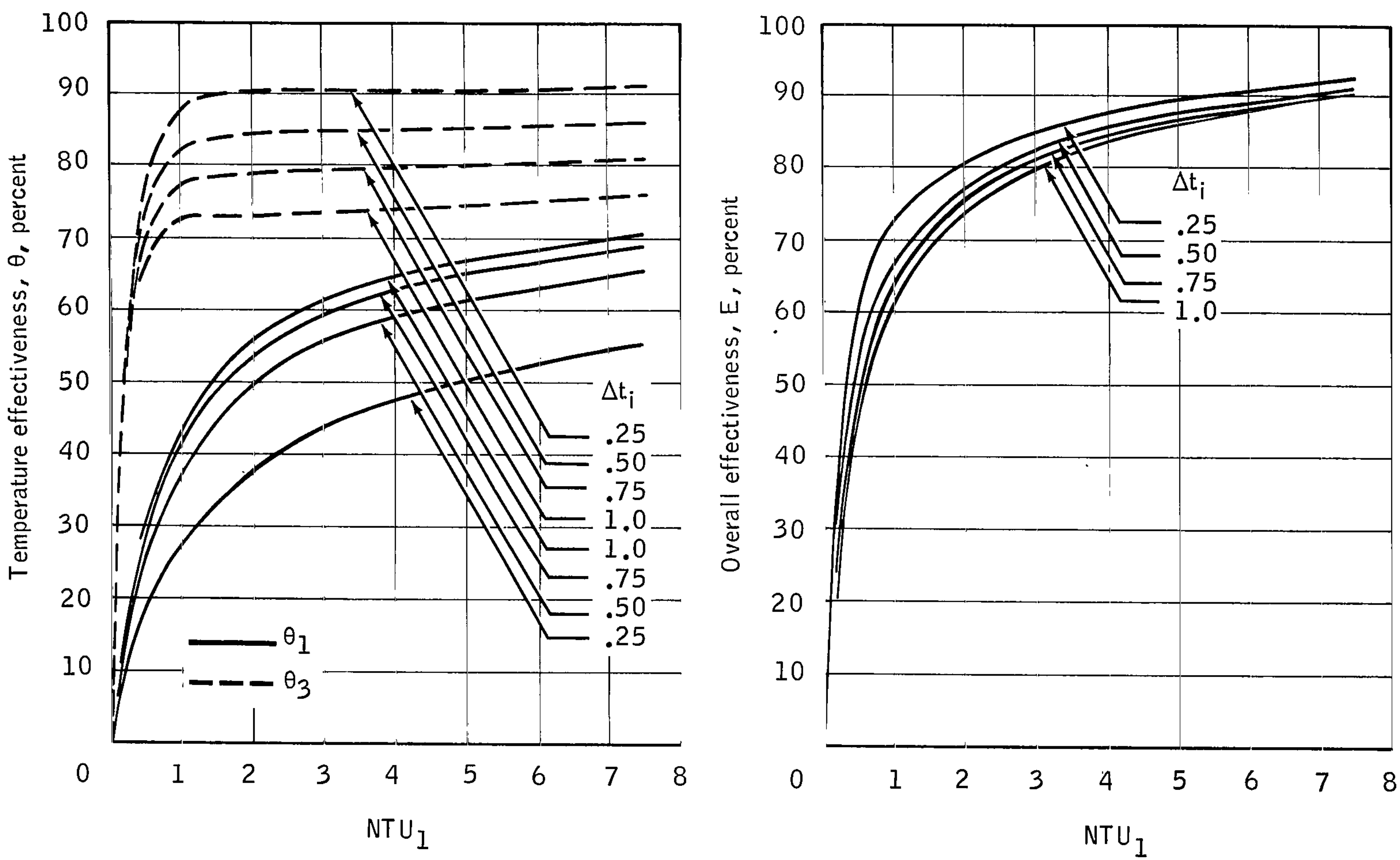


Figure 24. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 0.25$, and $U = 1.0$.

NASA-S-67-6749

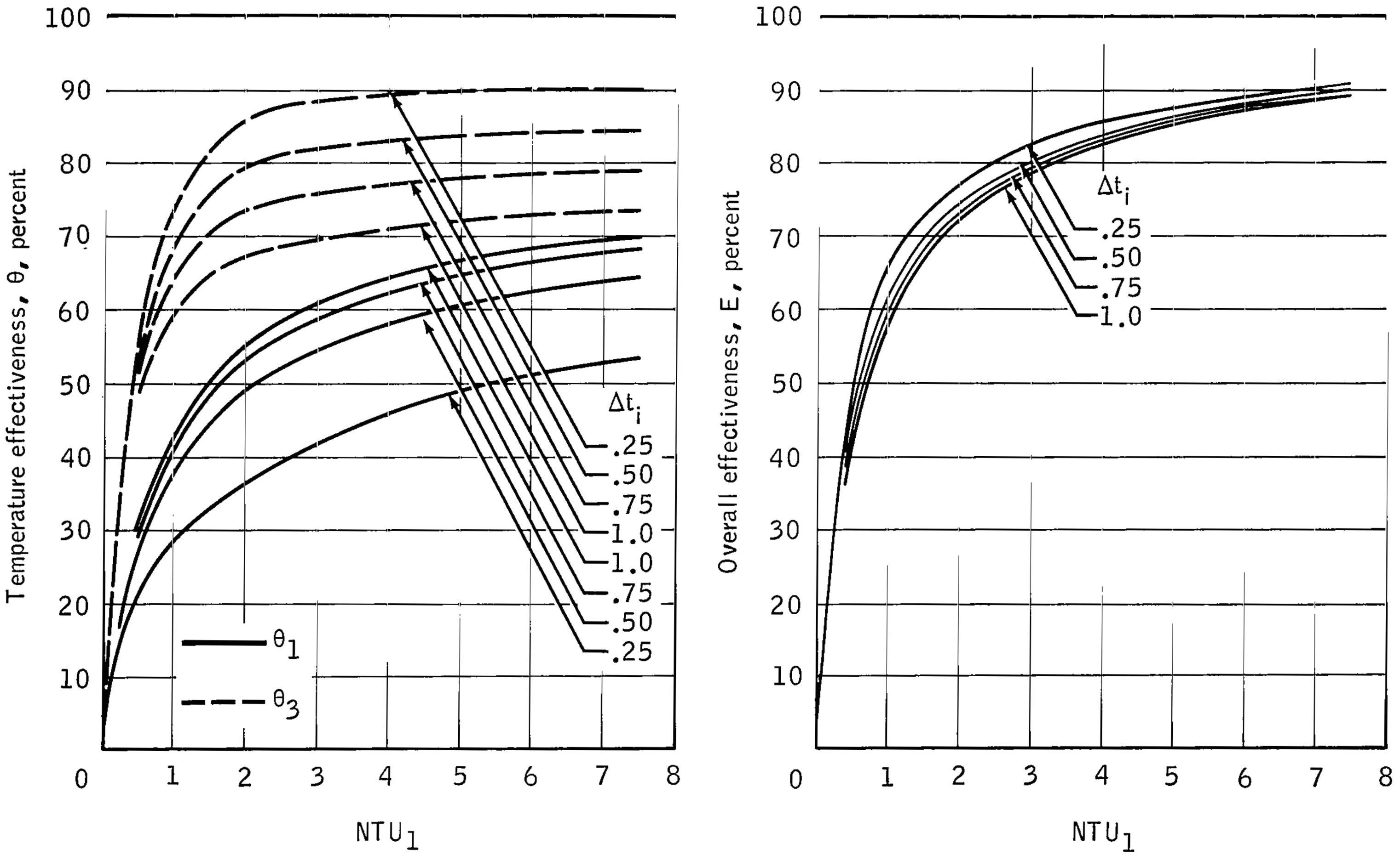


Figure 25. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 0.25$, and $U = 2.0$.

NASA-S-67-6750

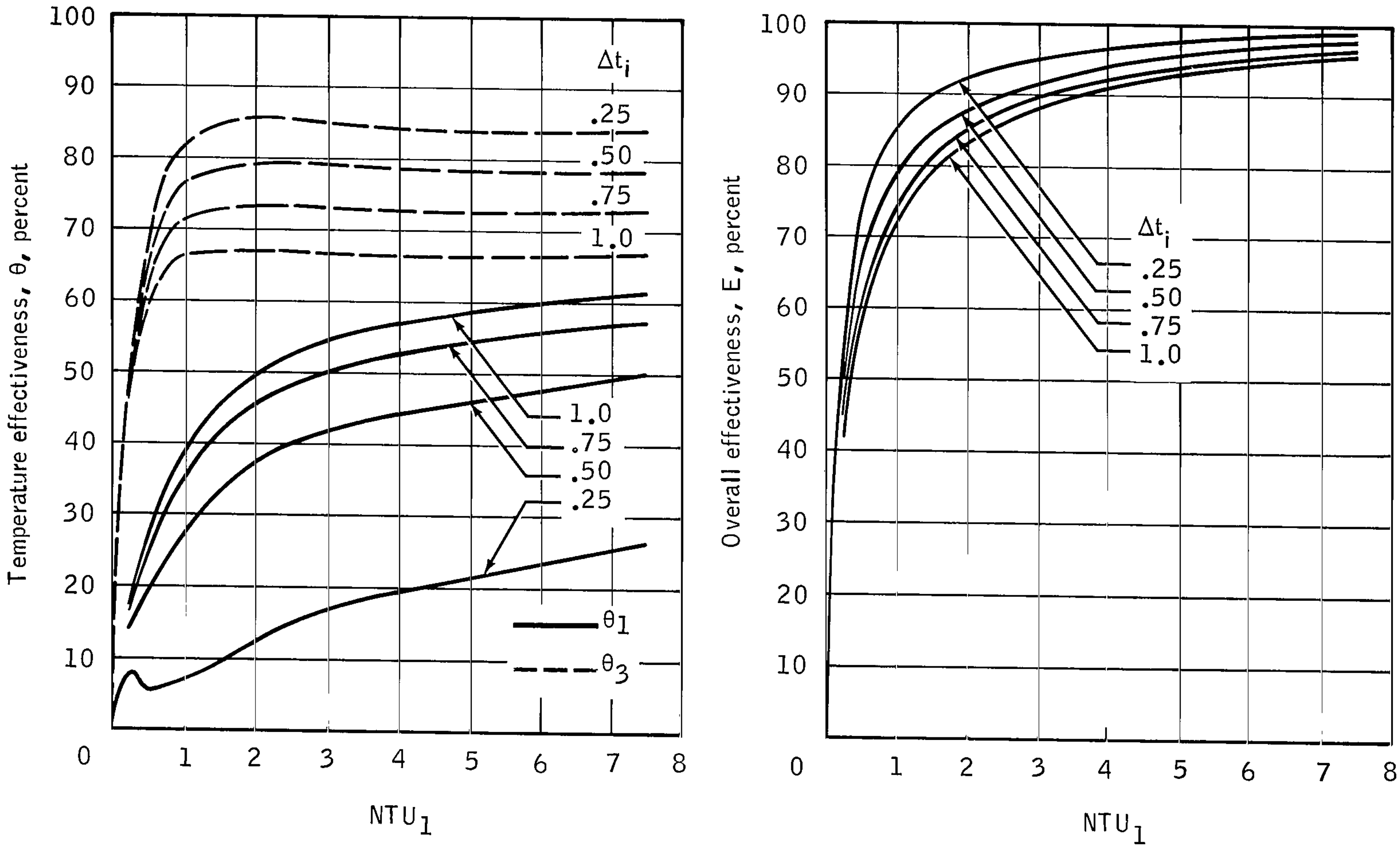


Figure 26. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 0.50$, and $U = 0.50$.

NASA-S-67-6751

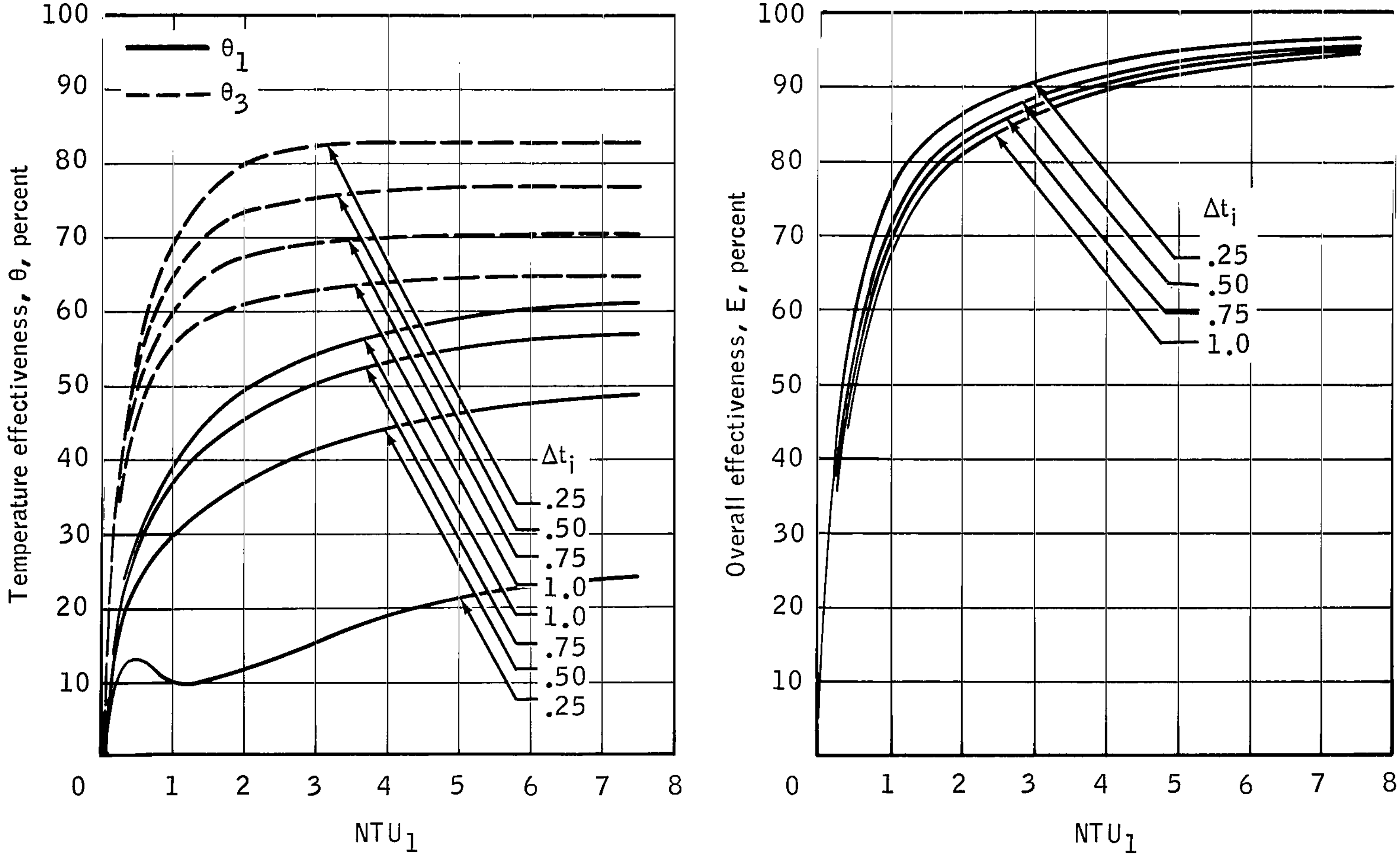


Figure 27. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 0.50$, and $U = 1.0$.

NASA-S-67-6752

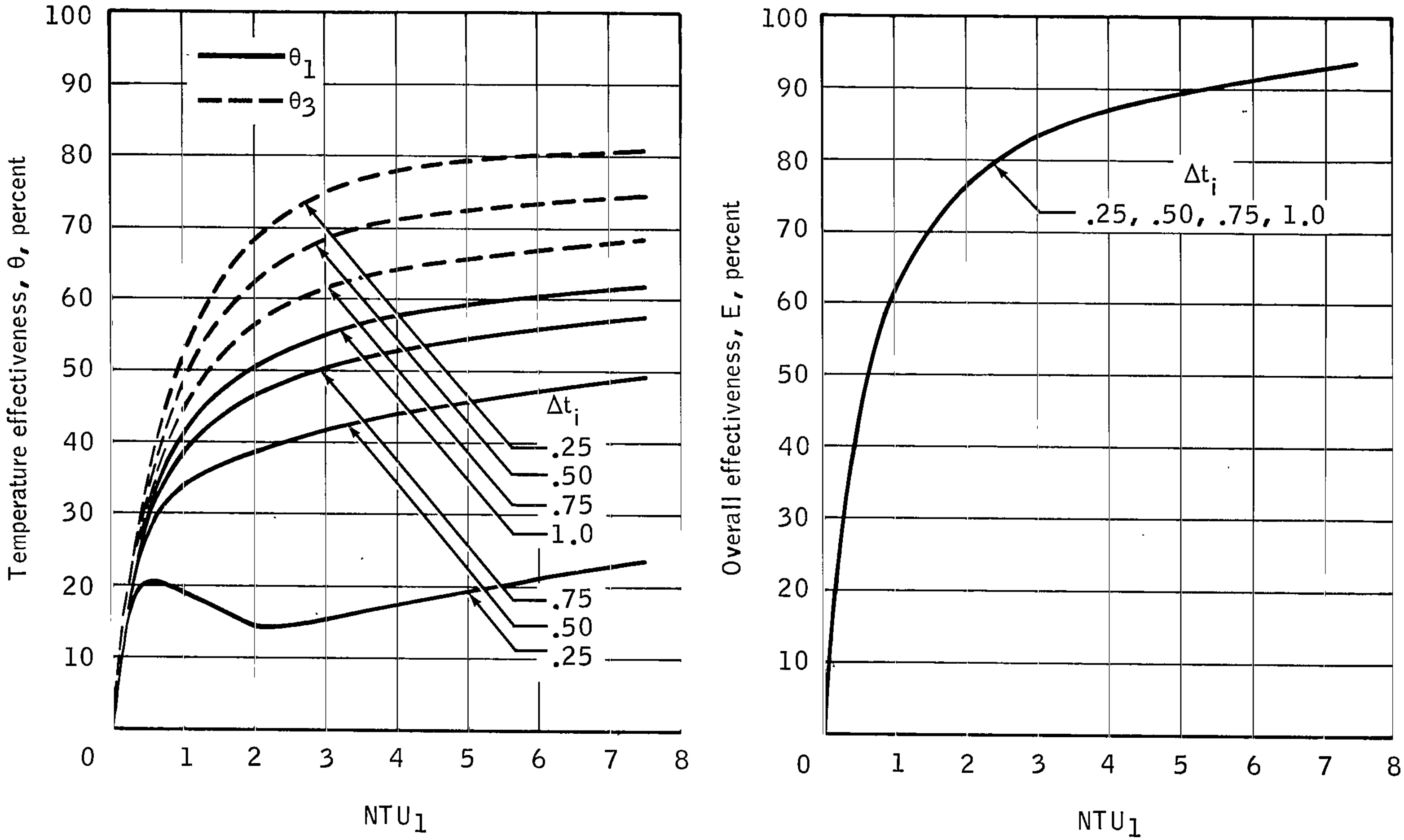


Figure 28. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 0.50$, and $U = 2.0$.

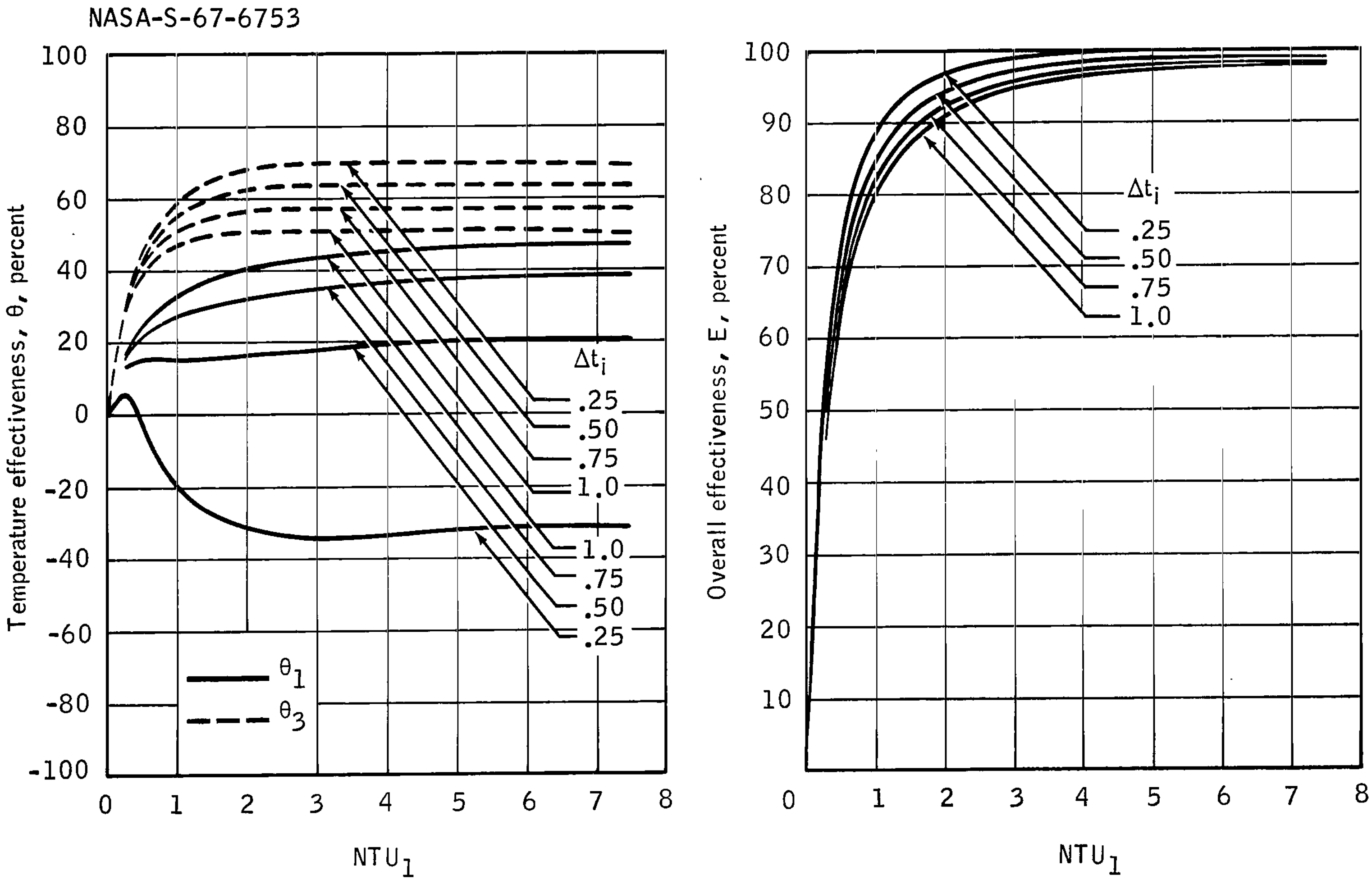


Figure 29. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 1.0$, and $U = 0.50$.

NASA-S-67-6754

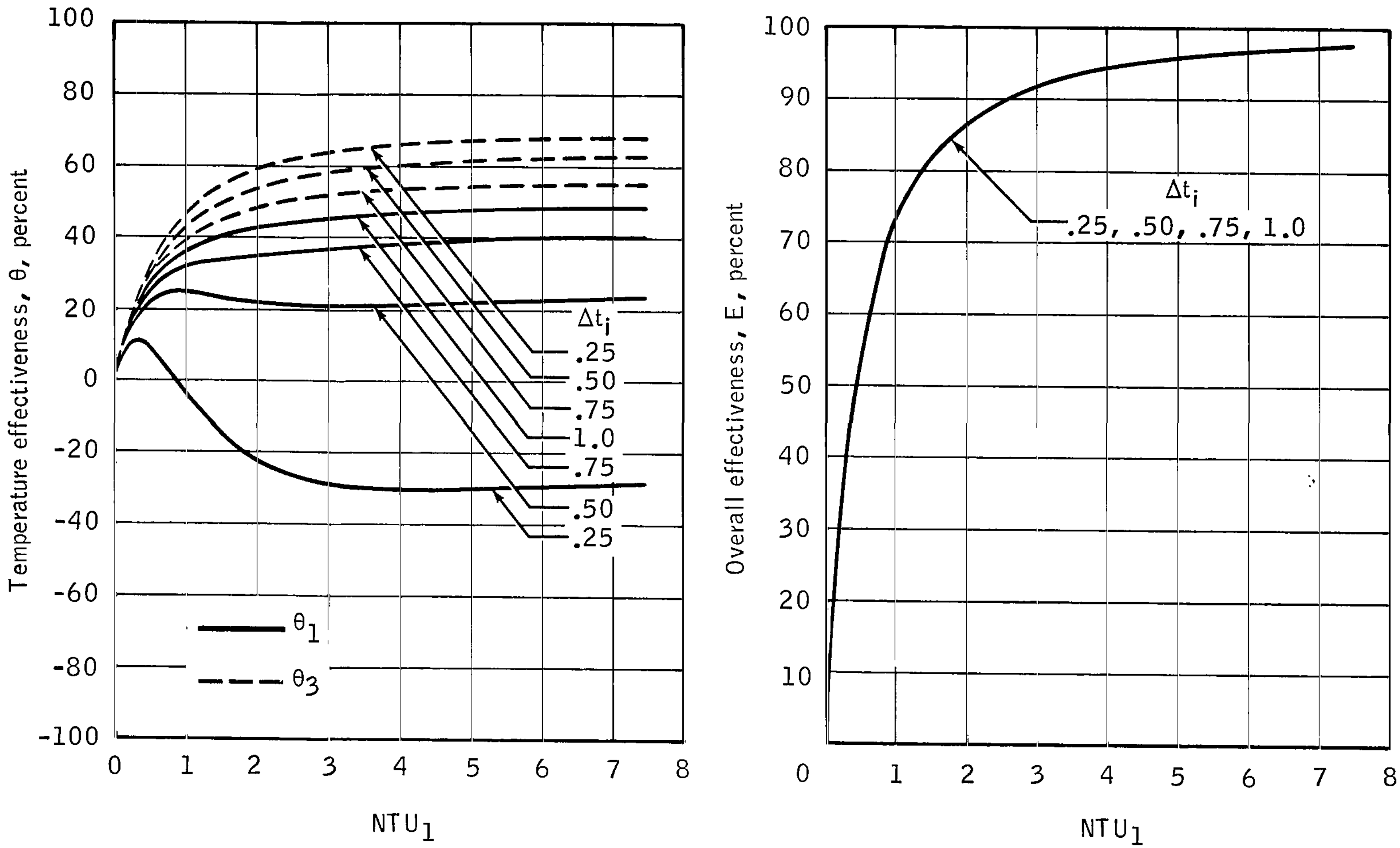


Figure 30. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 1.0$, and $U = 1.0$.

NASA-S-67-6755

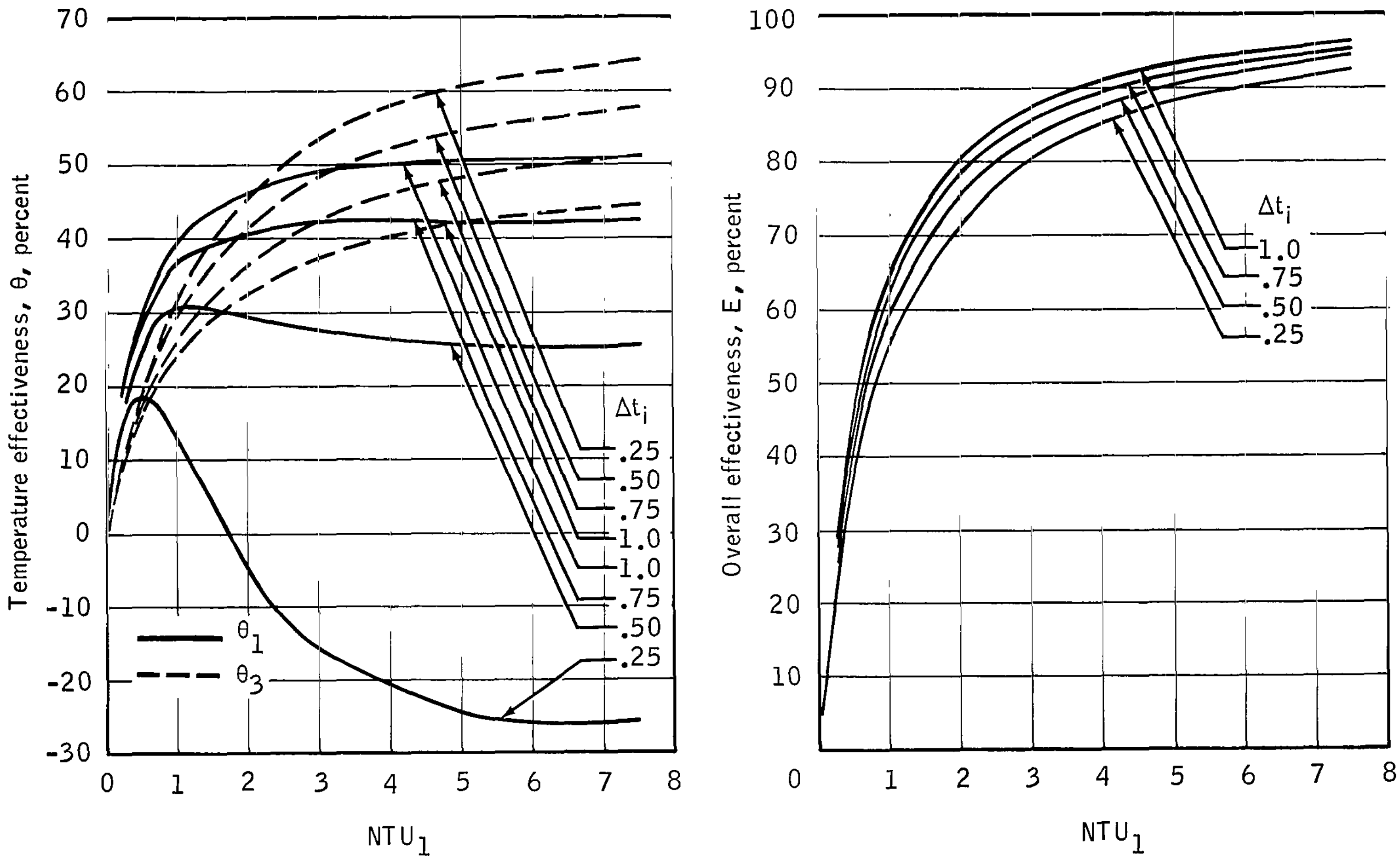


Figure 31. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 1.0$, $K_3 = 1.0$, and $U = 2.0$.

NASA-S-67-6756

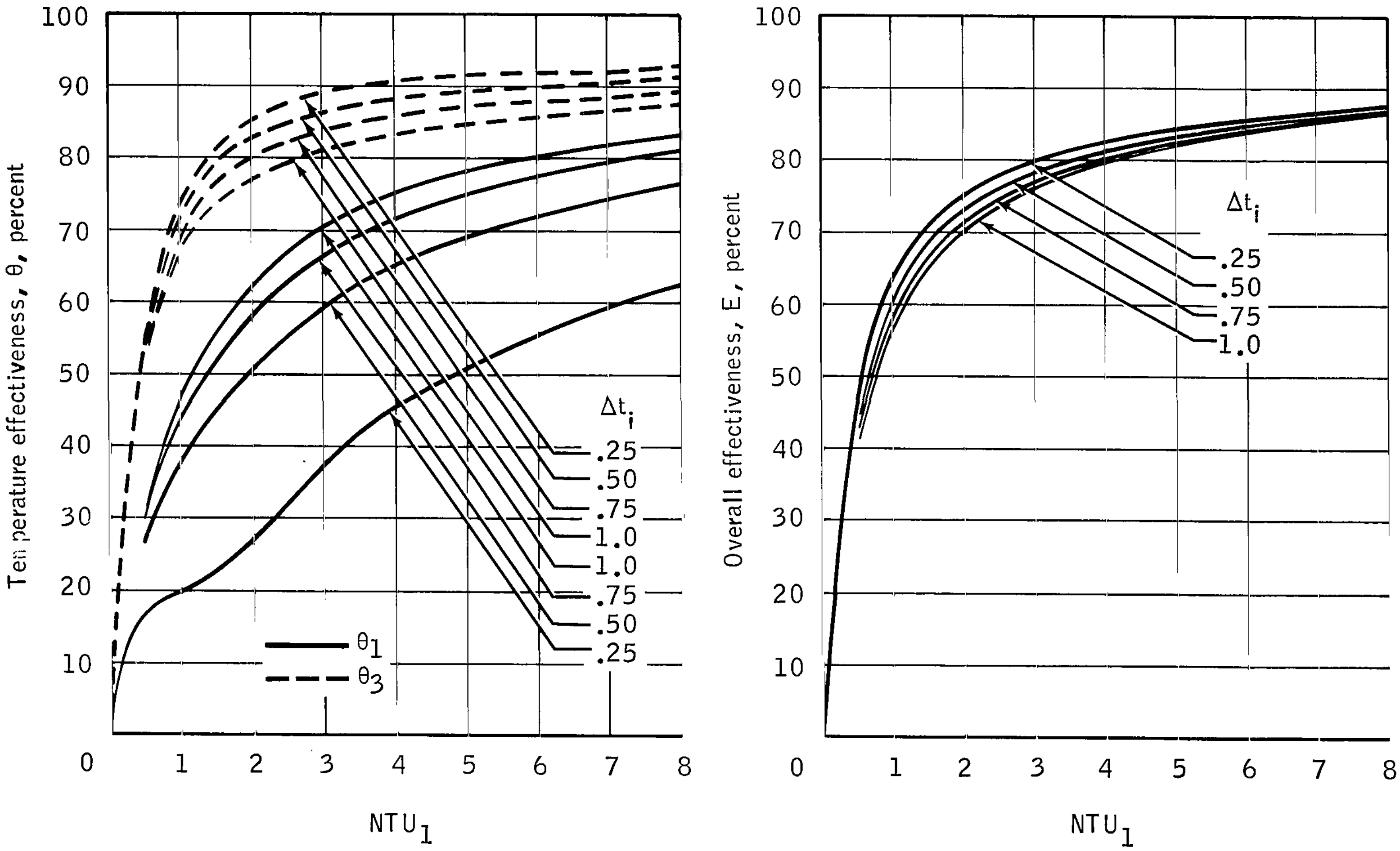


Figure 32. - Effectiveness factors for a two-pass, countercurrent, three-fluid, crossflow heat exchanger with mixed flow in elbows; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 0.50$.

NASA-S-67-6757

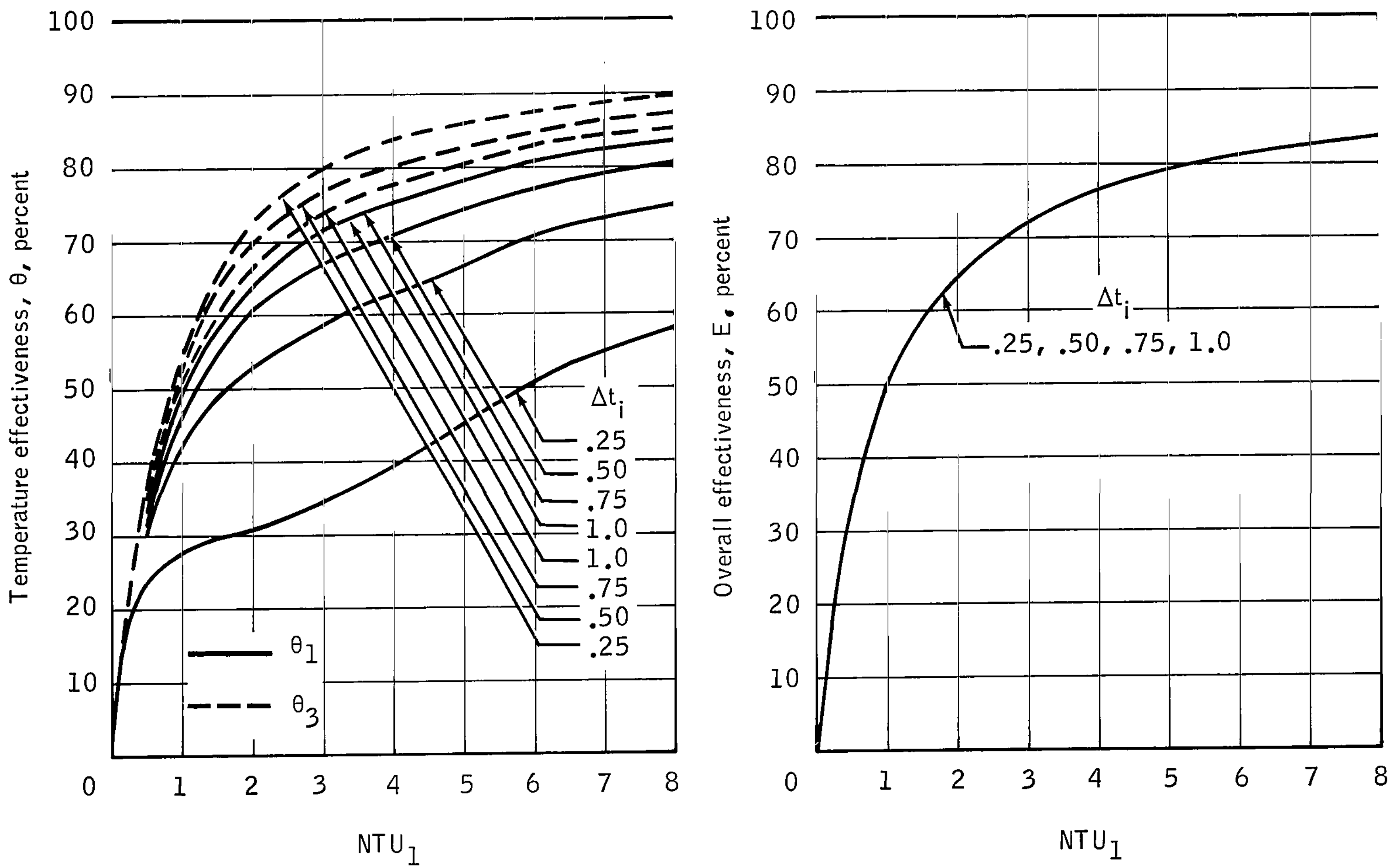


Figure 33. - Effectiveness factors for a two-pass, countercurrent, three-fluid, crossflow heat exchanger with mixed flow in elbows; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$.

NASA-S-67-6758

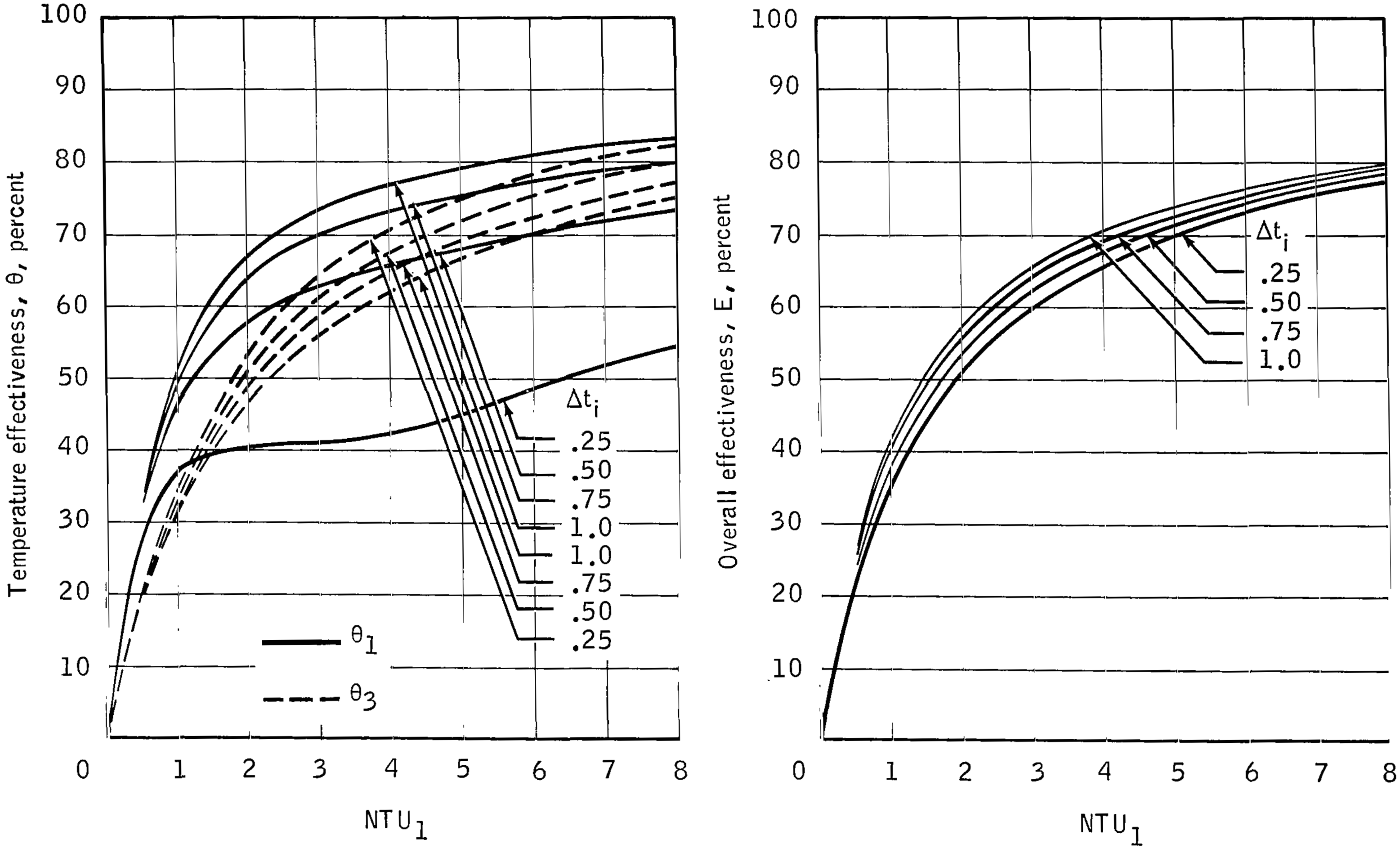


Figure 34. - Effectiveness factors for a two-pass, countercurrent, three-fluid, crossflow heat exchanger with mixed flow in elbows; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 2.0$.

NASA-S-67-6759

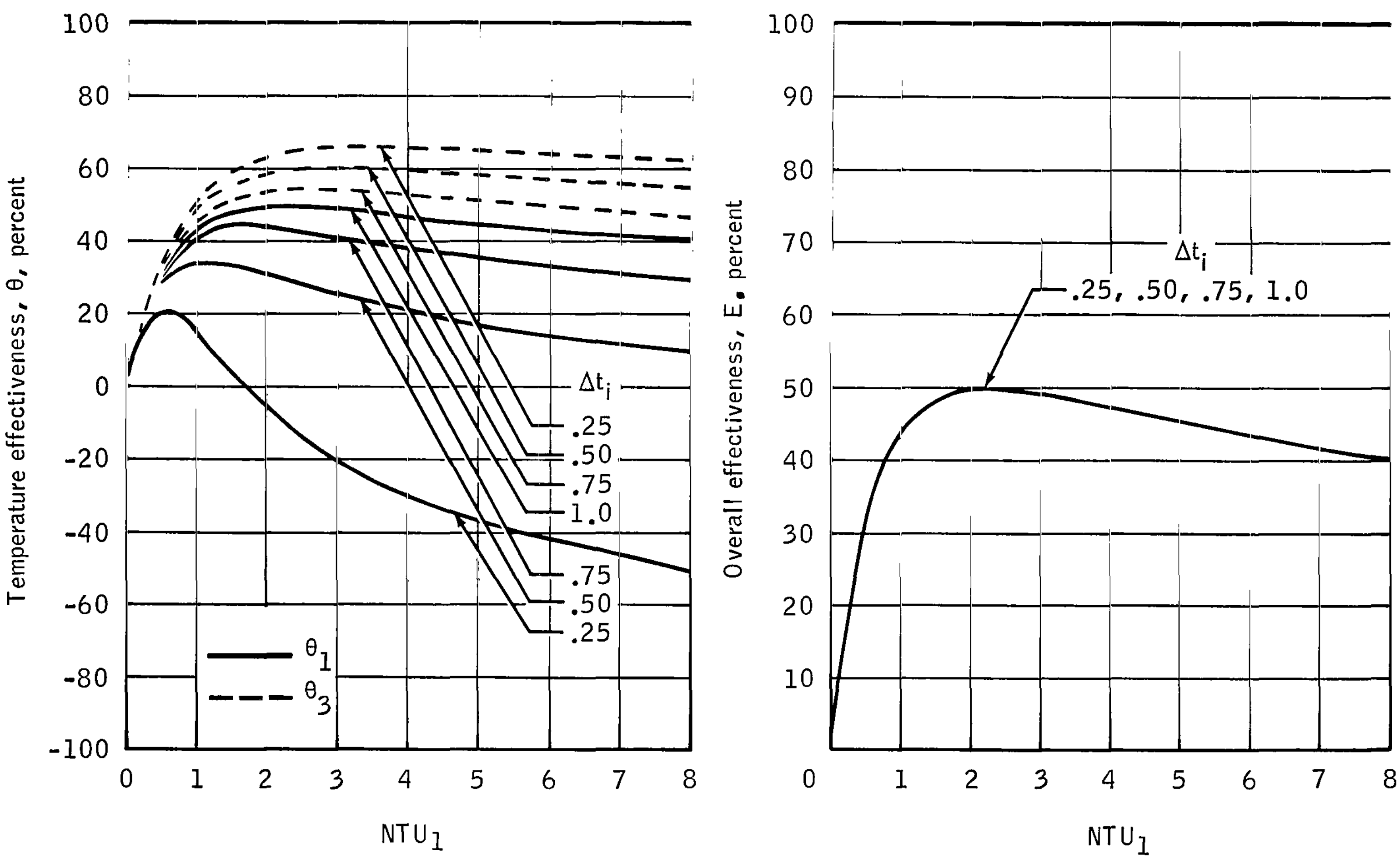


Figure 35. - Effectiveness factors for a two-pass, cocurrent, three-fluid, crossflow heat exchanger with mixed flow in elbows; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$.

NASA-S-67-6760

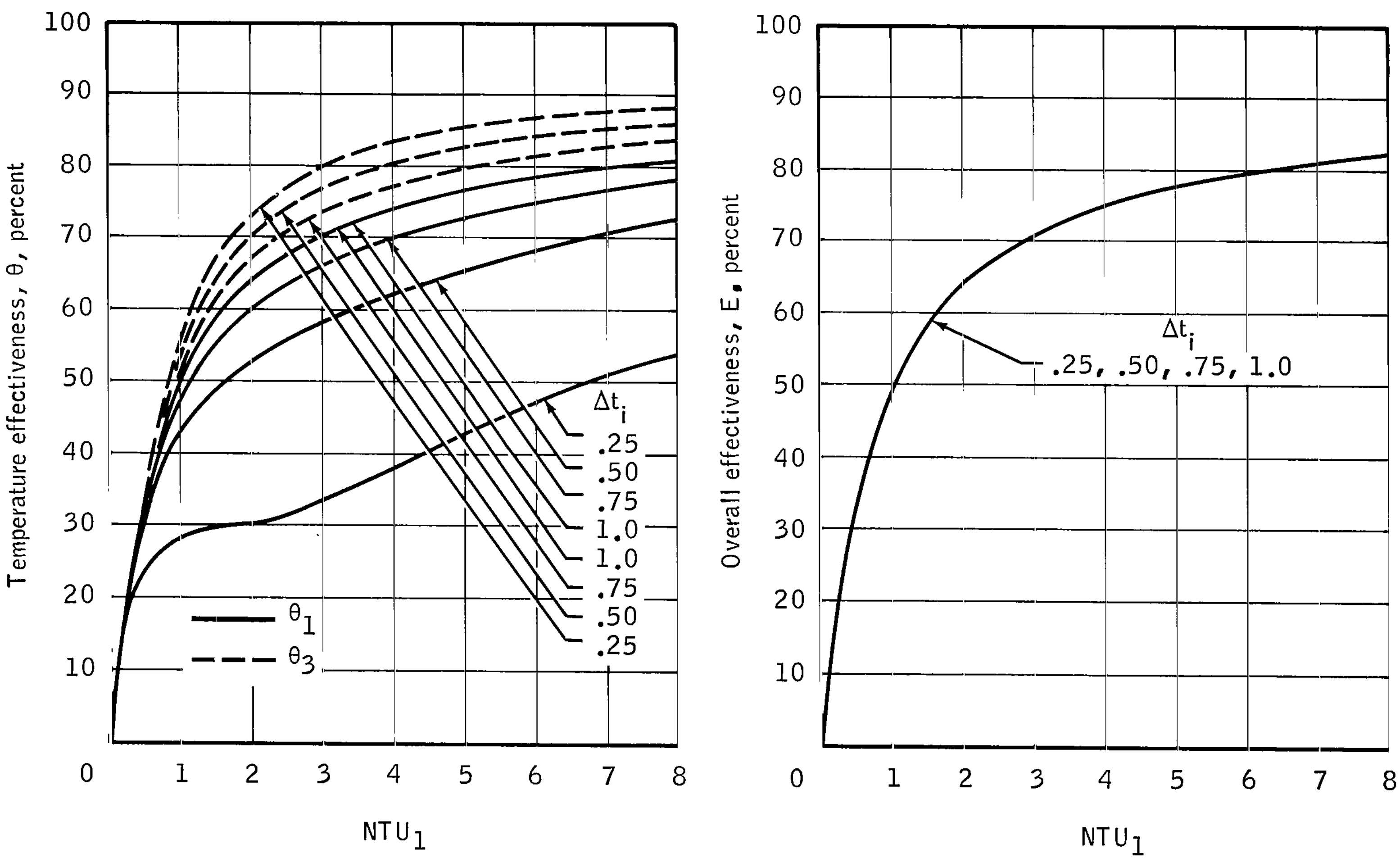


Figure 36. - Effectiveness factors for a two-pass, countercurrent, three-fluid, crossflow heat exchanger with identical flow order in elbows; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$.

NASA-S-67-6761

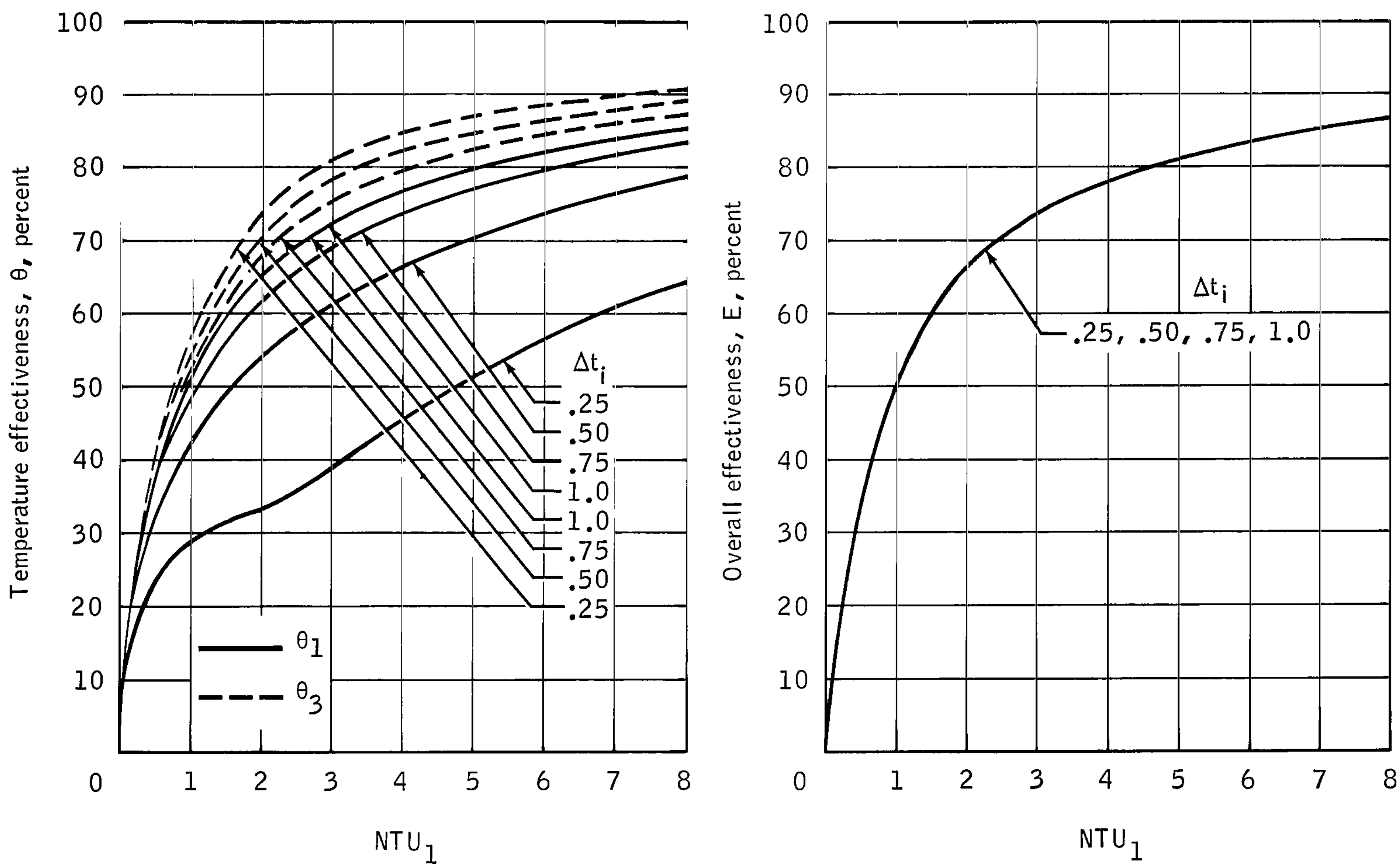


Figure 37. - Effectiveness factors for a two-pass, countercurrent, three-fluid, crossflow heat exchanger with inverted flow order in elbows; $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$.

NASA-S-67-6762

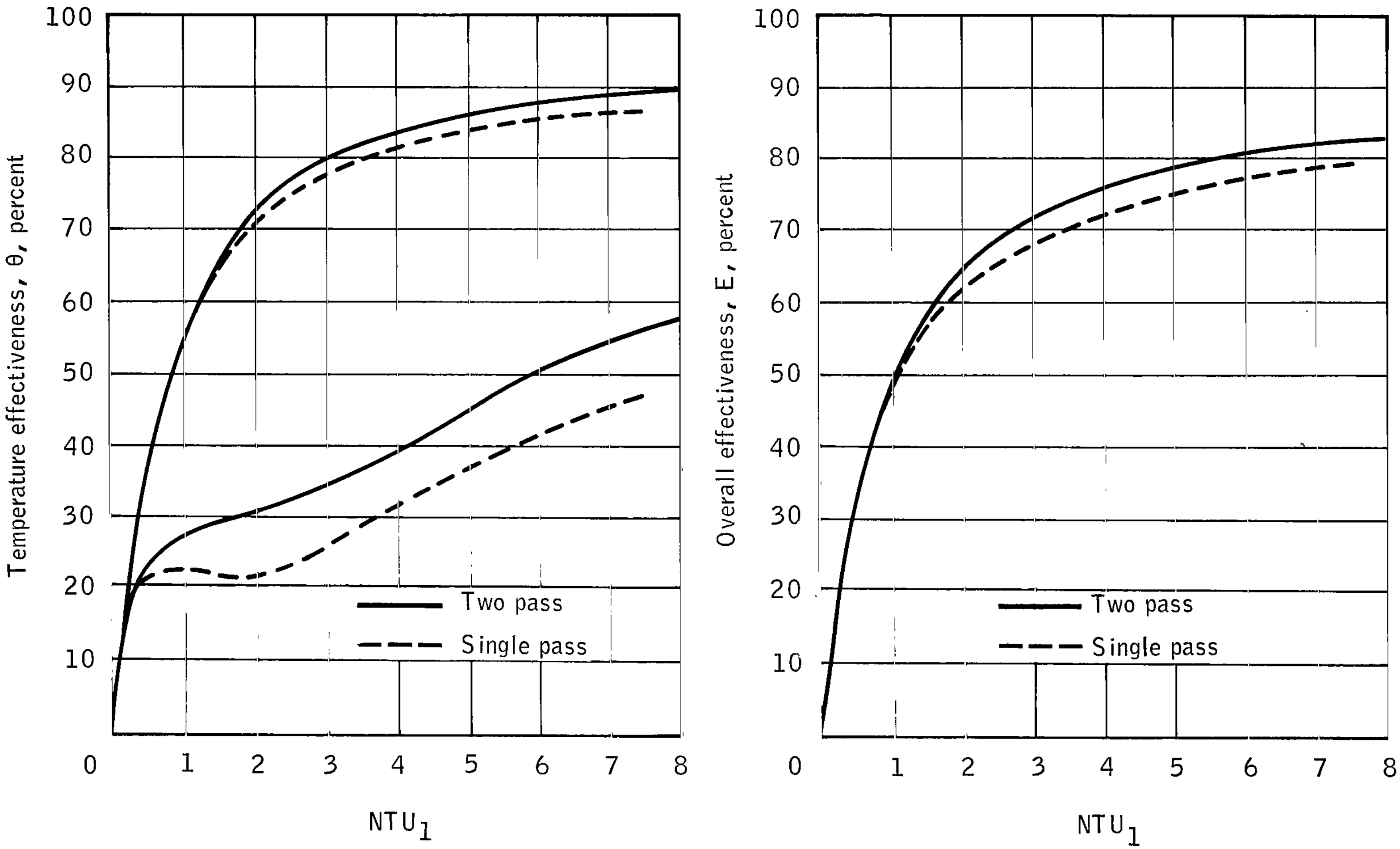


Figure 38. - Comparison of effectiveness factors for single-pass and two-pass heat exchangers; $K_1 = 0.50$, $K_3 = 0.50$, $U = 1.0$, and $\Delta t_i = 0.25$.

NASA-S-67-6763

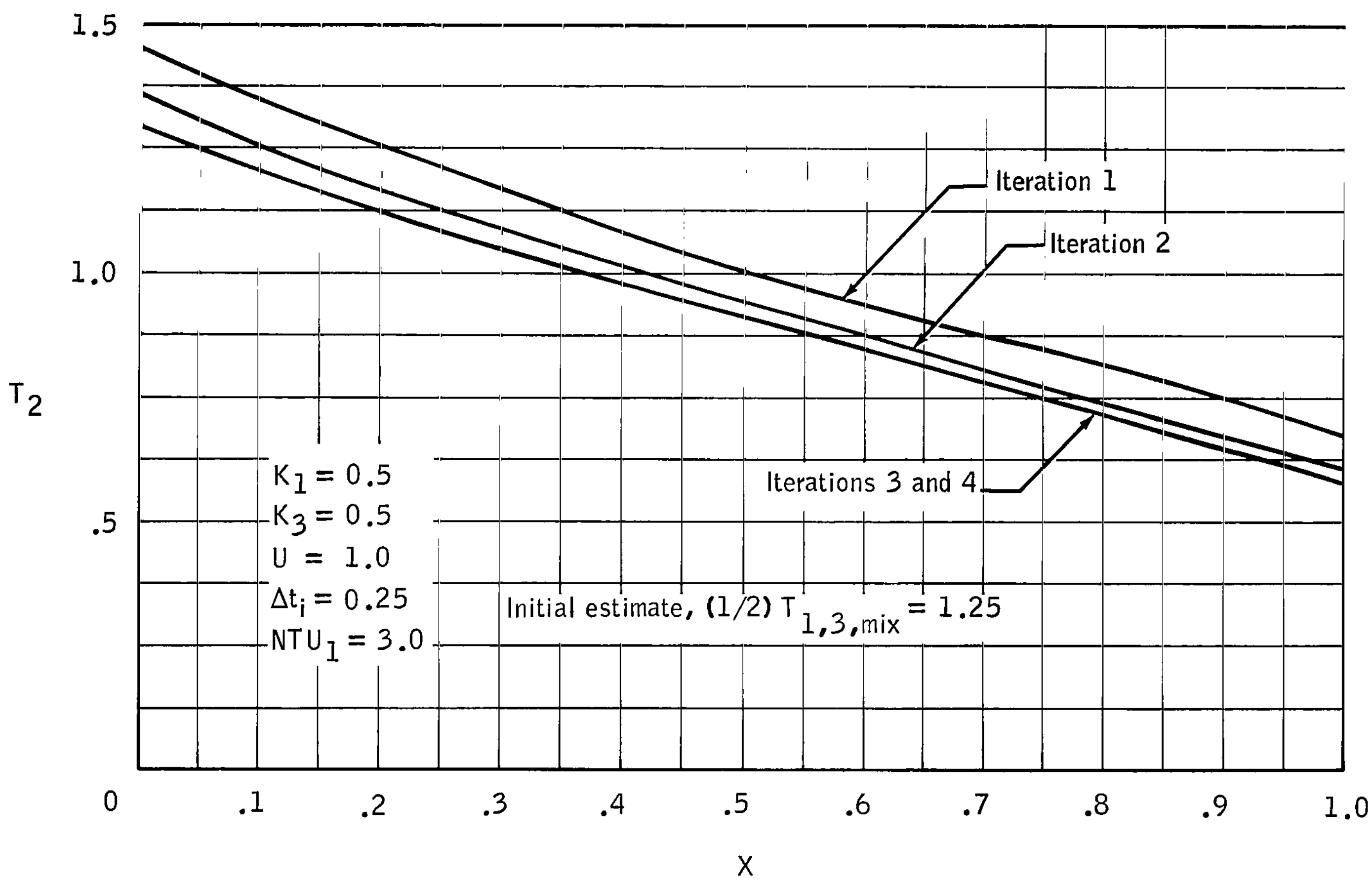


Figure 39. - Convergence of $T_{2,i}$ for pass 1 in a two-pass, countercurrent heat exchanger with inverted flow in the elbows.

NASA-S-67-6764

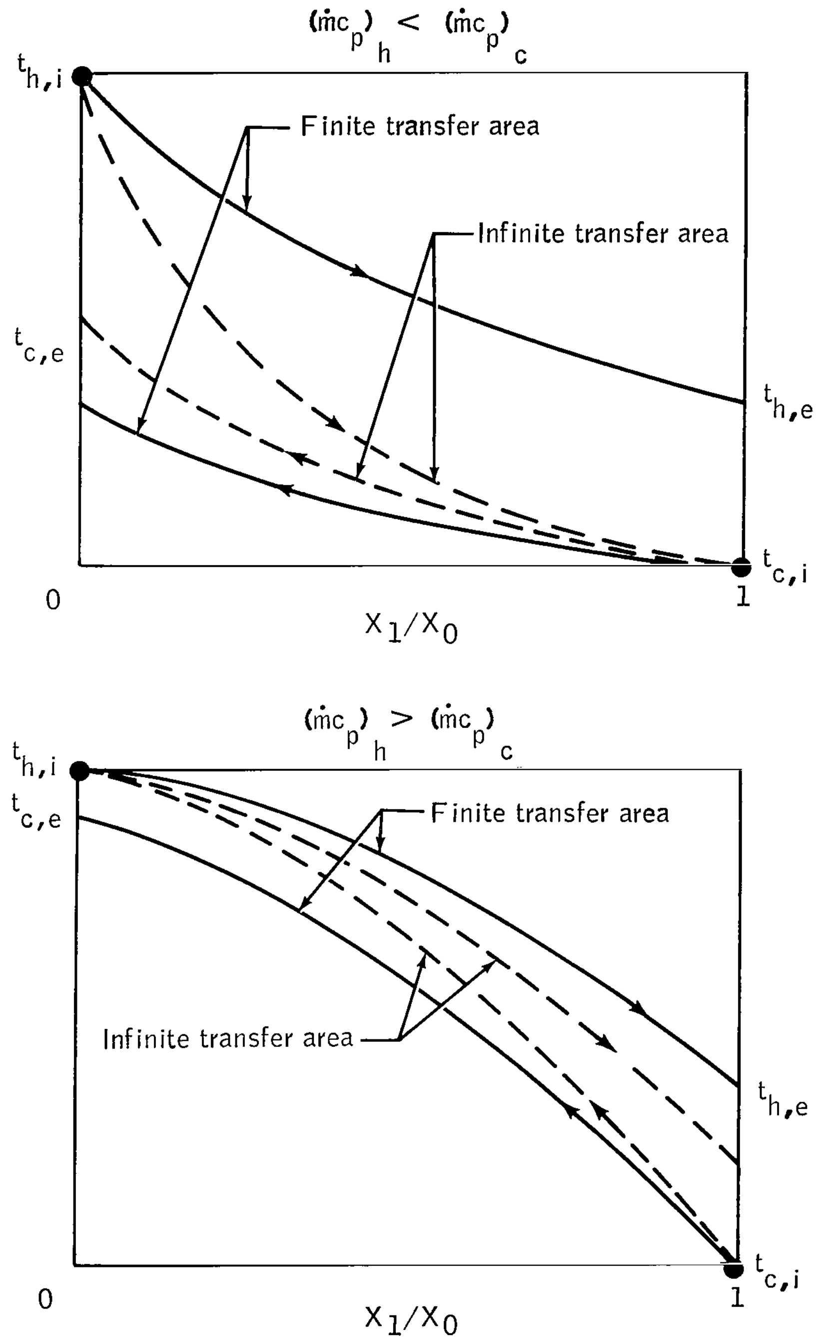


Figure 40. - Temperature distributions in two-fluid, counterflow exchangers with finite and infinite heat-transfer area.

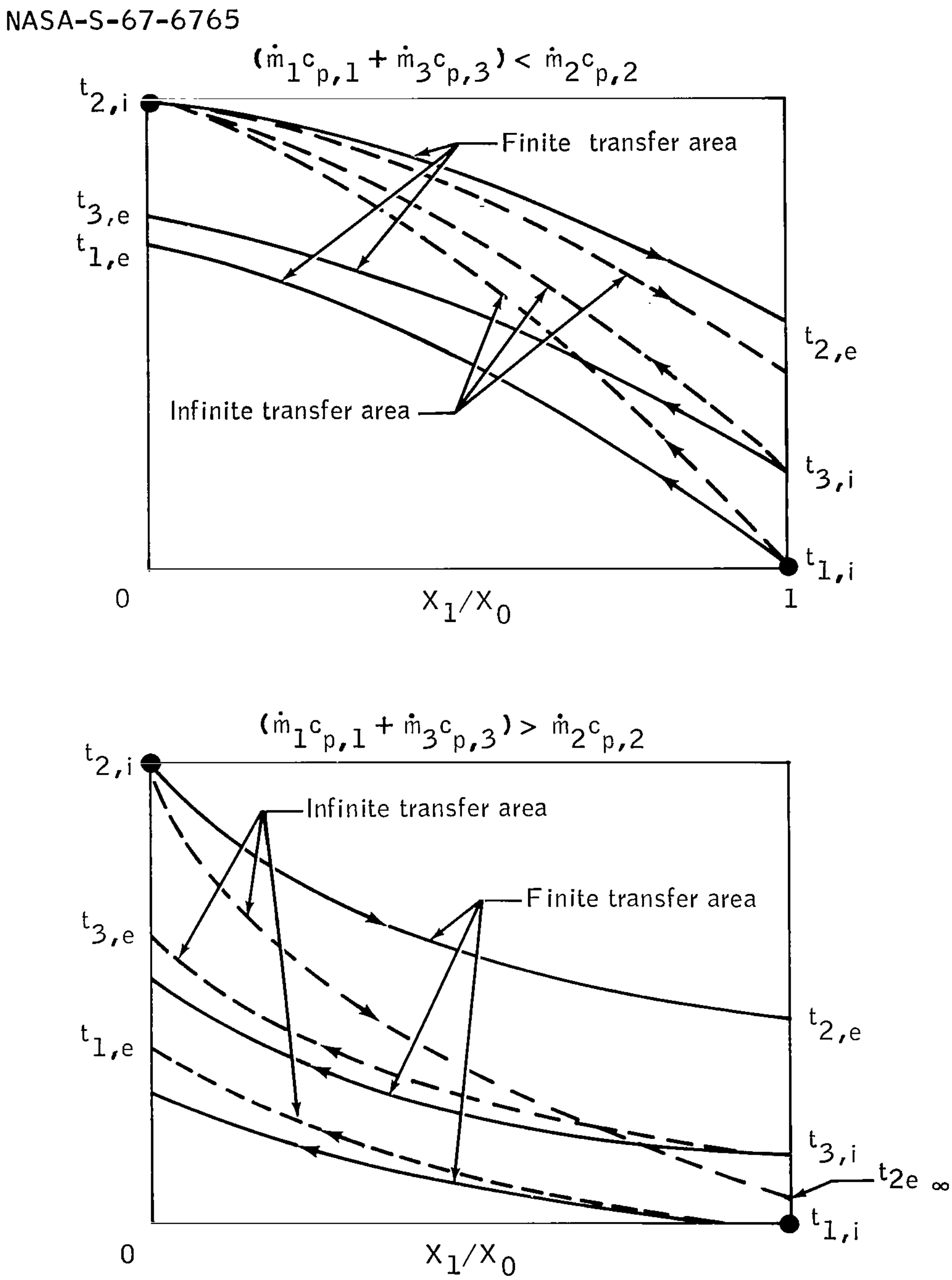


Figure 41. - Temperature distributions in three-fluid, counterflow exchangers with finite and infinite heat-transfer area.

NASA-S-67-6766

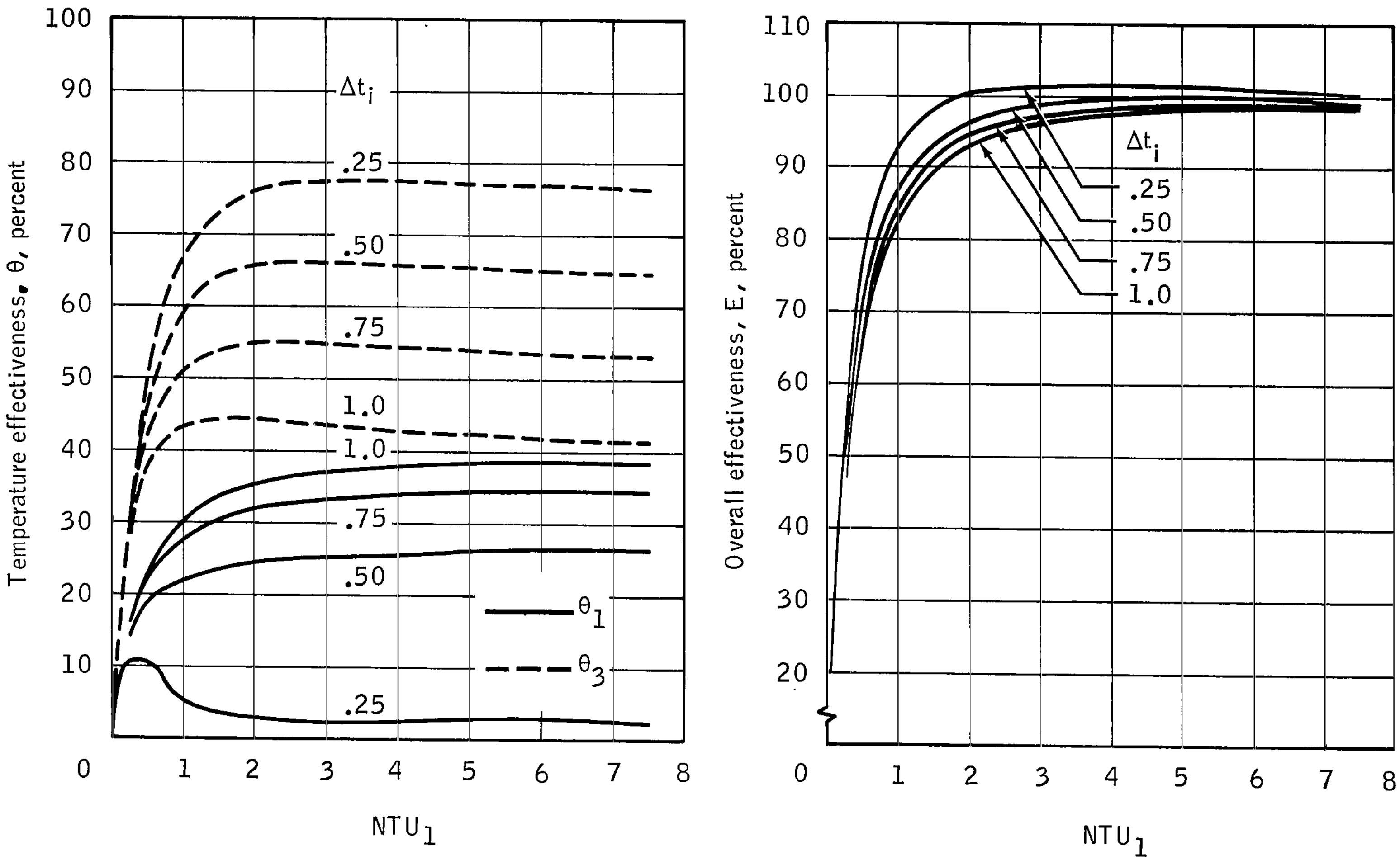


Figure 42. - Effectiveness factors for a single-pass, three-fluid, crossflow heat exchanger; $K_1 = 2.0$, $K_3 = 0.50$, and $U = 2.0$.

NASA-S-67-6767

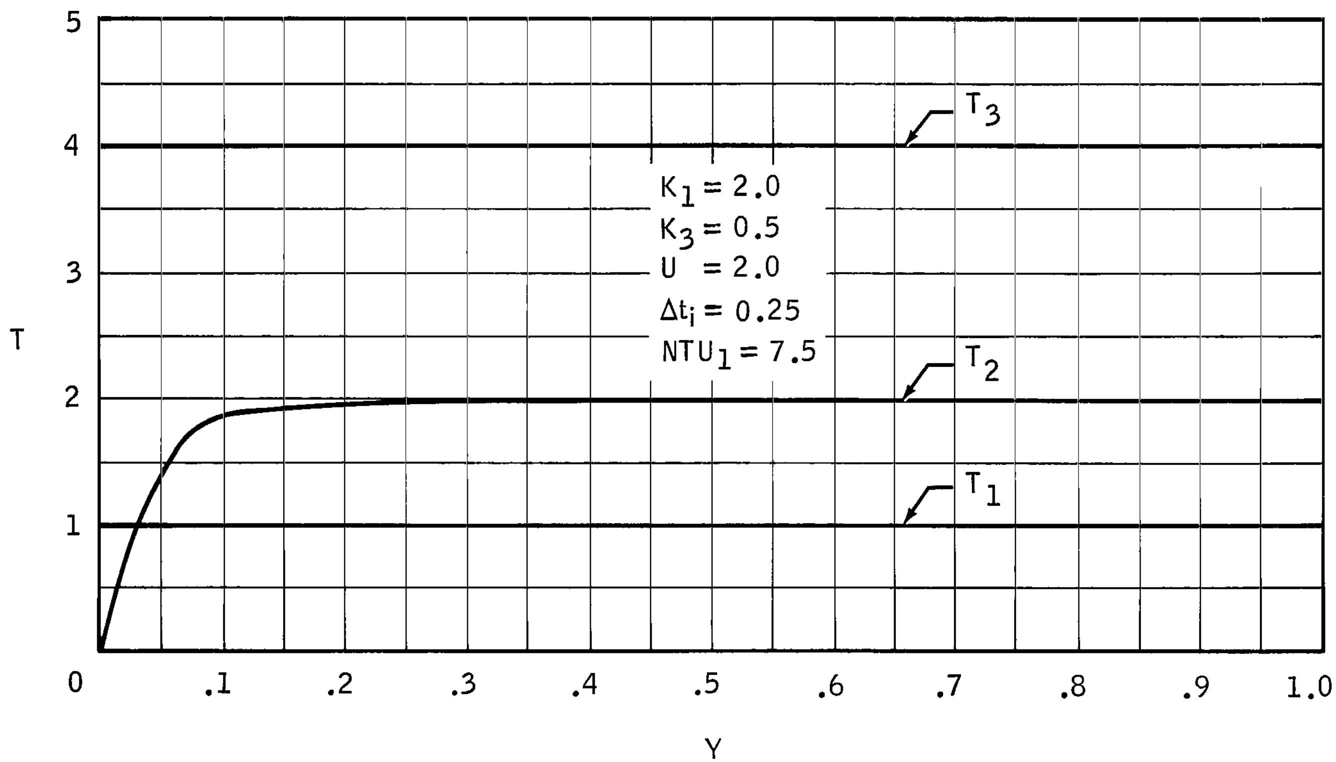


Figure 43. - Temperature distributions as functions of Y for X = 0.

NASA-S-67-6768

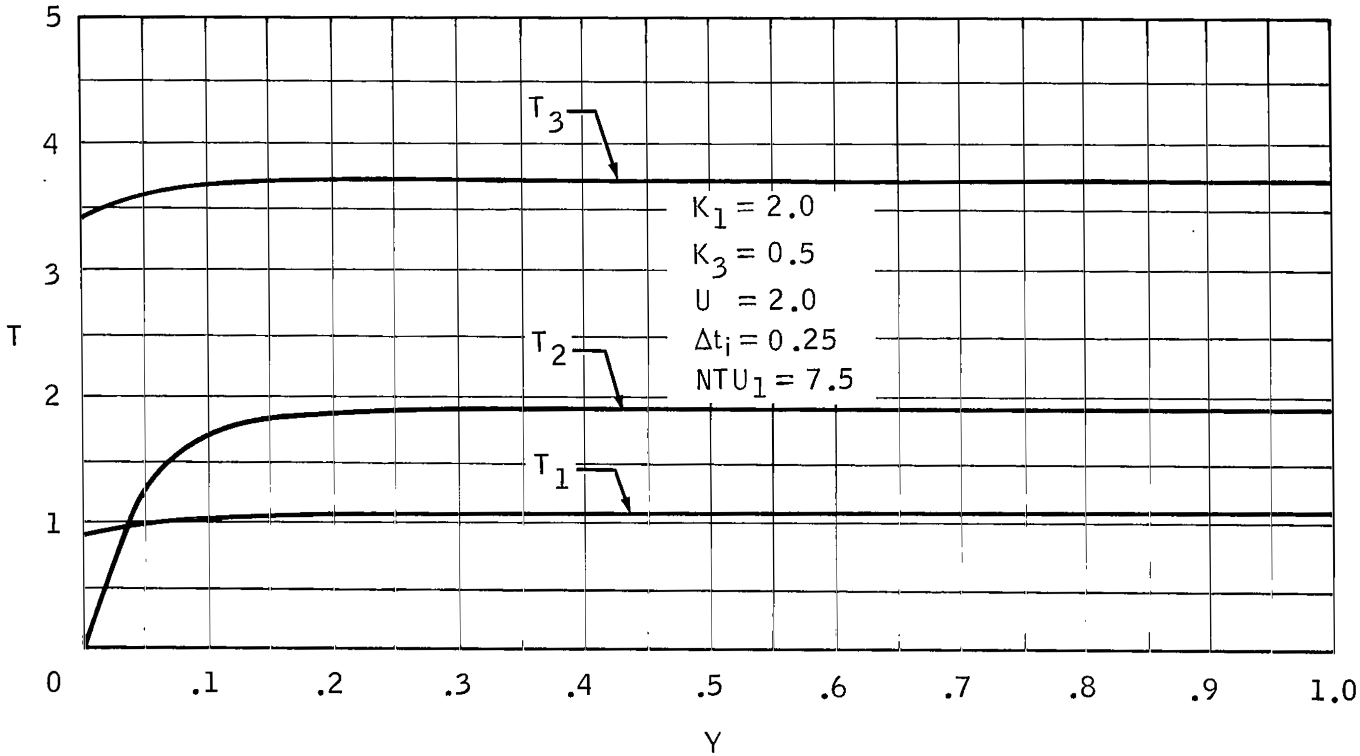


Figure 44. - Temperature distributions as functions of Y for X = 0.10.

NASA-S-67-6769

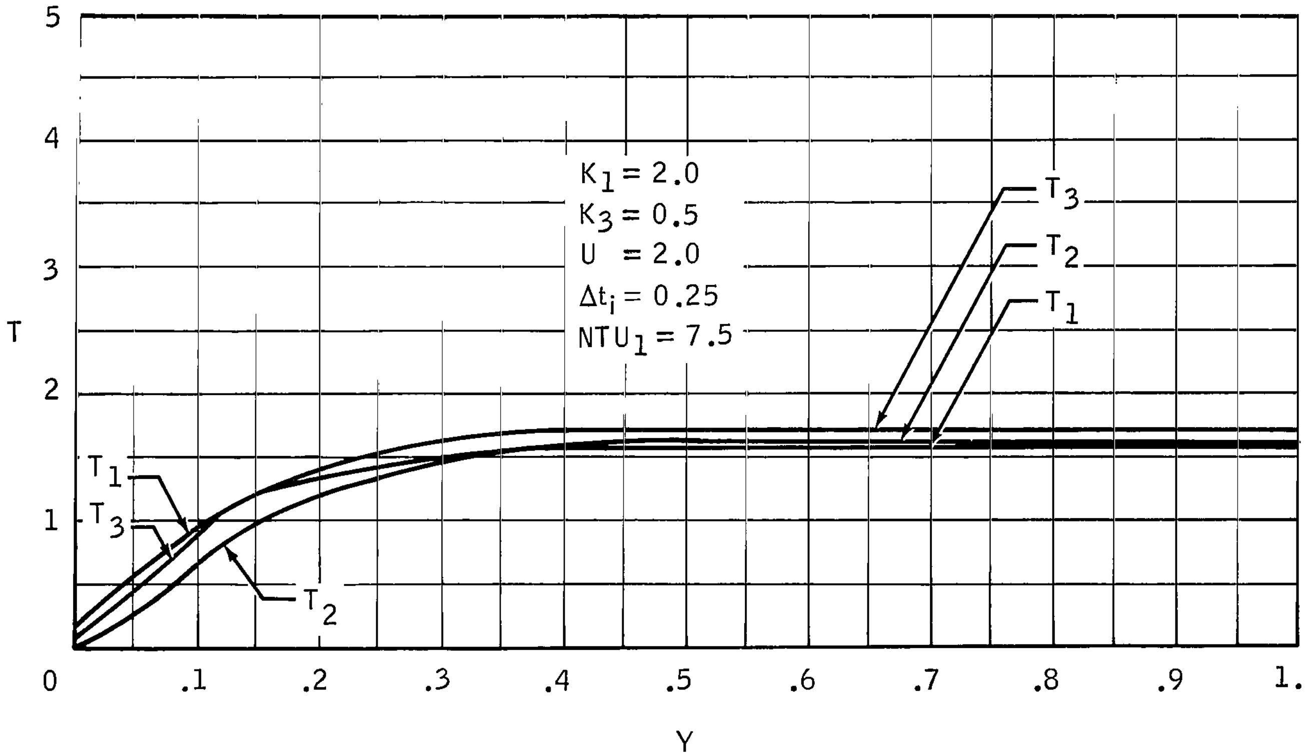


Figure 45. - Temperature distributions as functions of Y for X = 0.25.

NASA-S-67-6770

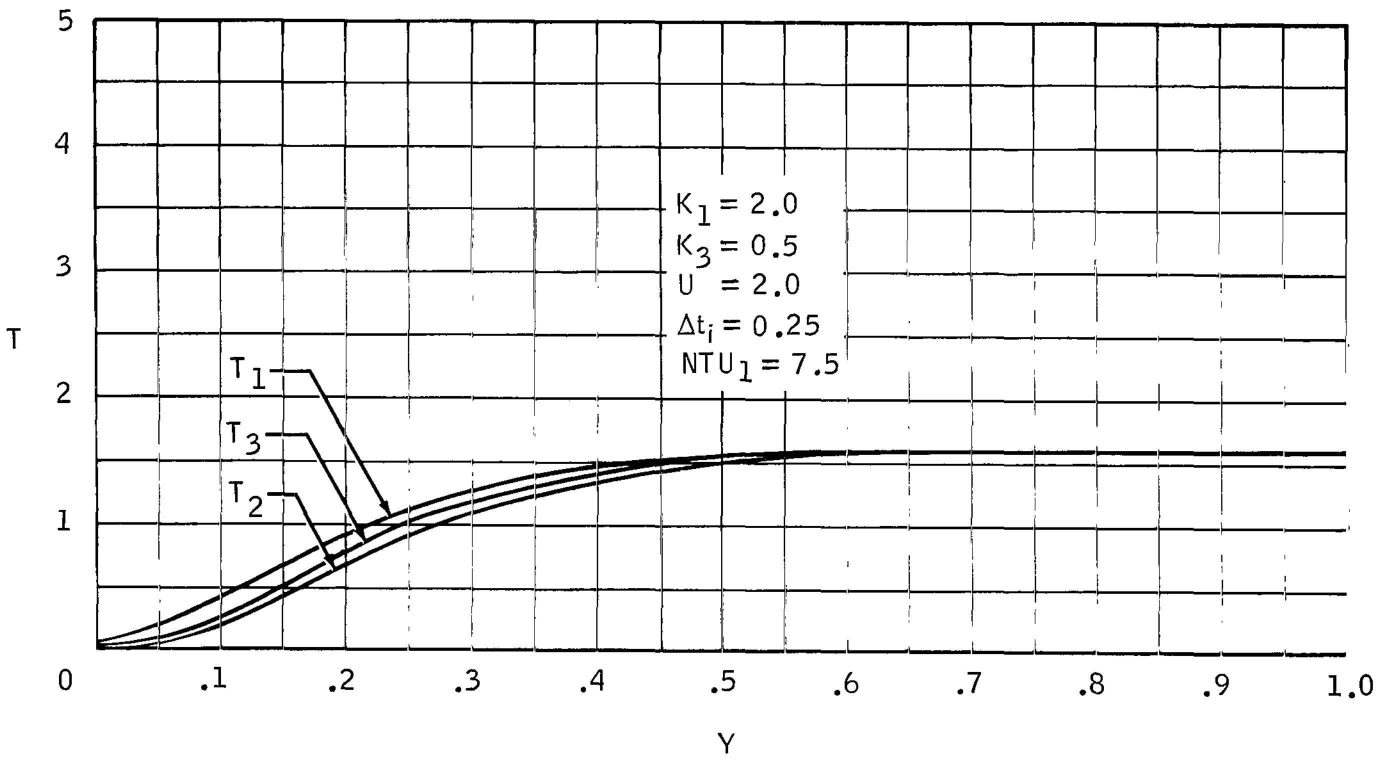


Figure 46. - Temperature distributions as functions of Y for X = 0.50.

NASA-S-67-6771

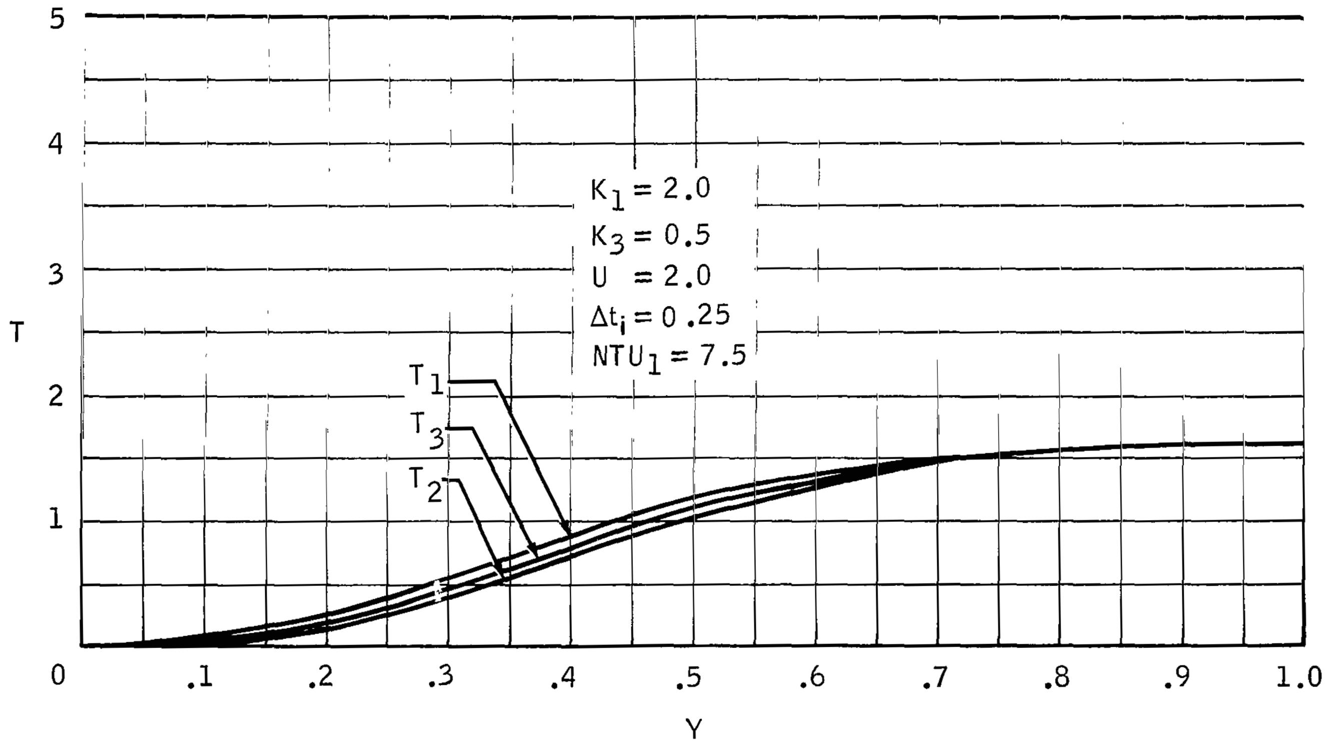


Figure 47. - Temperature distributions as functions of Y for X = 1.0.

NASA-S-67-6772

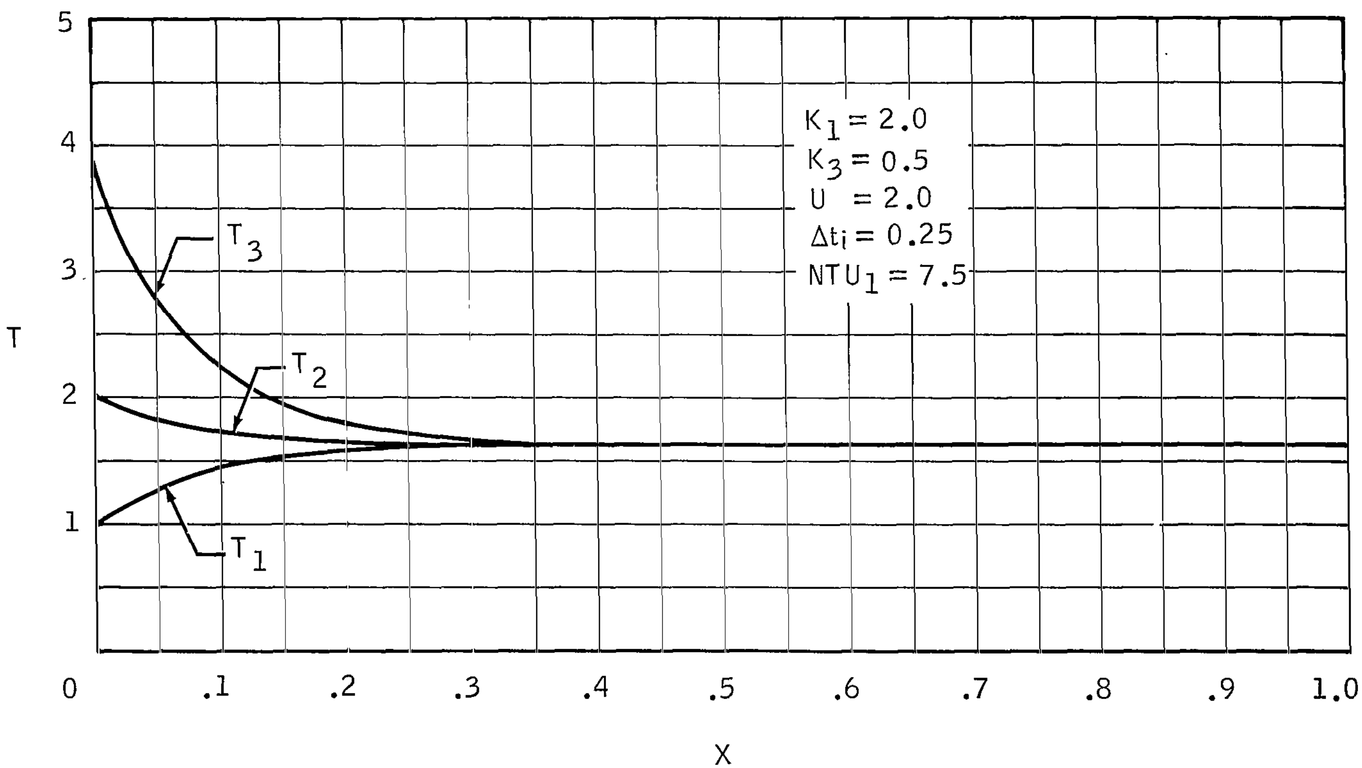
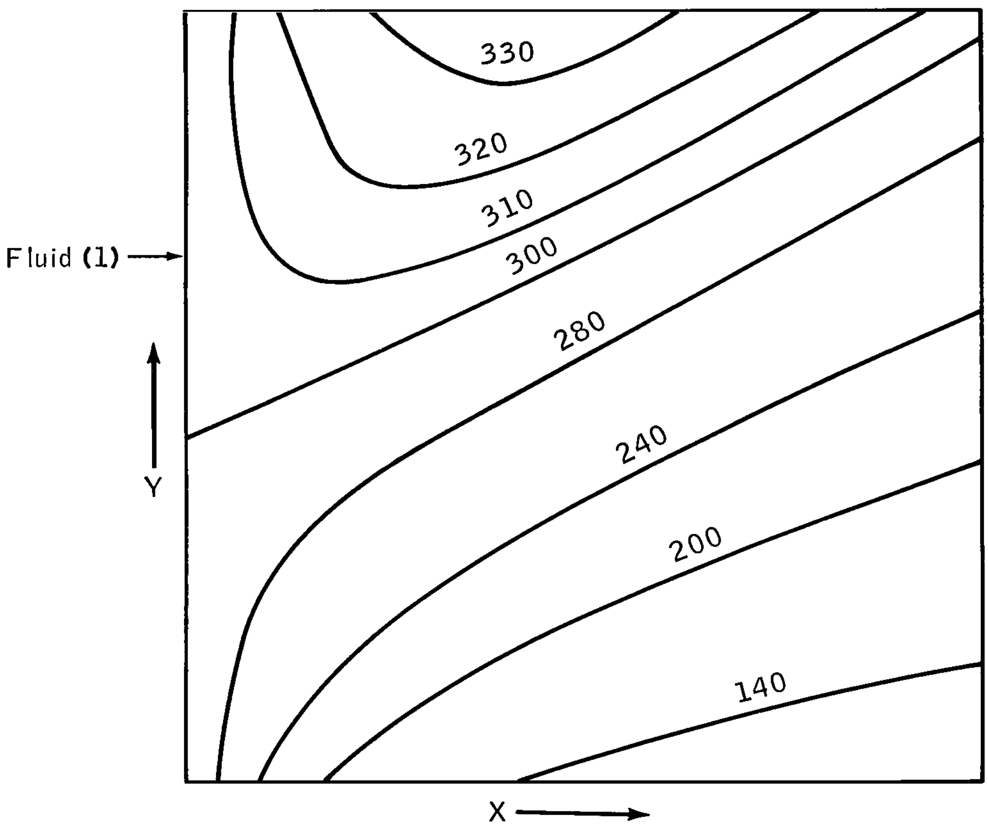


Figure 48. - Temperature distributions as functions of X for Y = 1.0.

NASA-S-67-6773

Fluid	$\dot{m}$, lb/hr	c_p, Btu/lb/ °F	t_i, °F
(1)	250	0.5	300
(2)	500	.5	100
(3)	250	1.0	500

$U_{1,2} = 50$ Btu/hr/ft^2/°F
$U_{2,3} = 25$ Btu/hr/ft^2/°F
$x_o y_o = 10$ ft^2

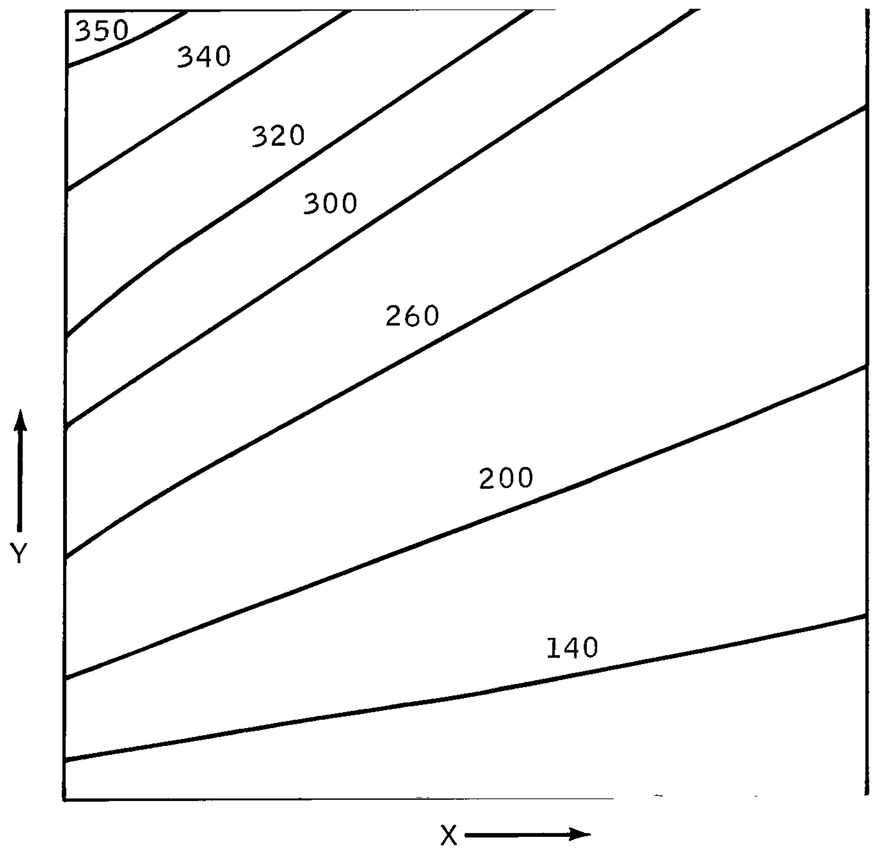


(a) Fluid (1).

Figure 49. - Isothermal contours.

NASA-S-67-6774

Fluid	$\dot{m}$, lb/hr	c_p, Btu/lb/°F	t_i, °F	
(1)	250	0.5	300	$U_{1,2} = 50$ Btu/hr/ft^2/°F
(2)	500	.5	100	$U_{2,3} = 25$ Btu/hr/ft^2/°F
(3)	250	1.0	500	$x_o y_o = 10$ ft^2

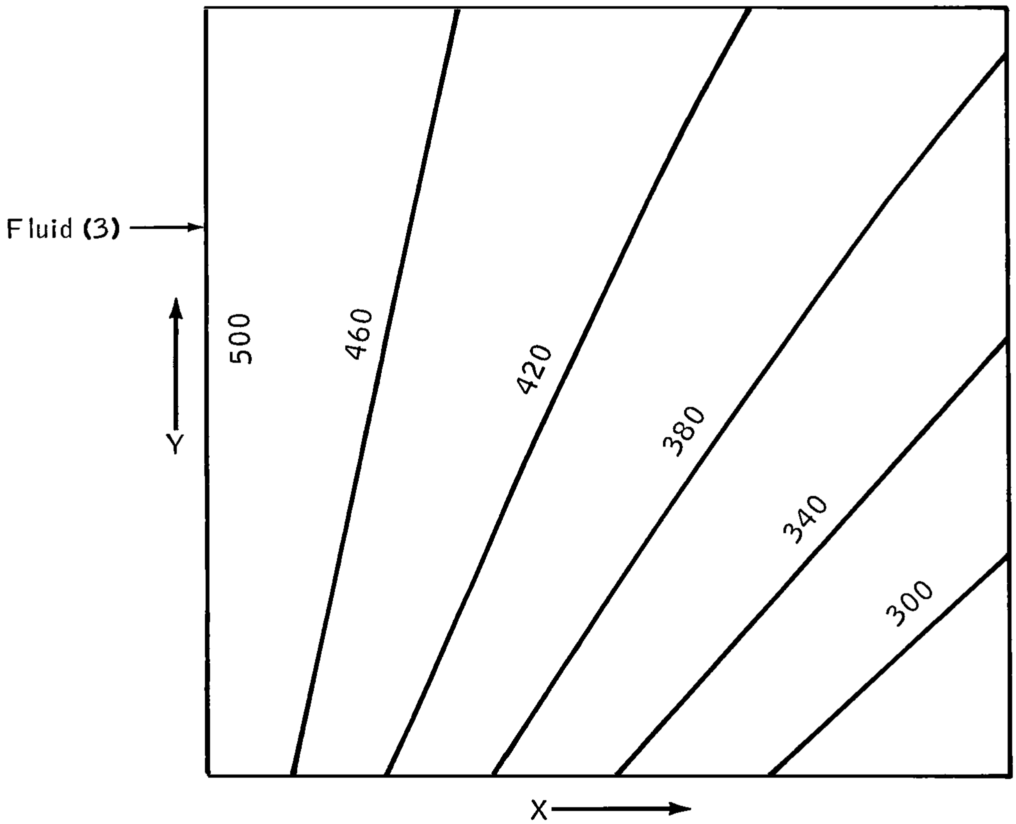


(b) Fluid (2).

Figure 49. - Continued.

Fluid	$\dot{m}$, lb/hr	c_p, Btu/lb/°F	t_i, °F
(1)	250	0.5	300
(2)	500	.5	100
(3)	250	1.0	500

$U_{1,2} = 50$ Btu/hr/ft^2/°F
$U_{2,3} = 25$ Btu/hr/ft^2/°F
$x_o y_o = 10$ ft^2



(c) Fluid (3).

Figure 49. - Concluded.

NASA-S-67-6776

Fluid	$\dot{m}$, lb/hr	c_p, Btu/lb/°F	t_i, °F
(1)	250	0.5	300
(2)	500	.5	100
(3)	250	1.0	500

$U_{1,2} = 50$ Btu/hr/ft^2/°F

$U_{2,3} = 25$ Btu/hr/ft^2/°F

$x_o y_o = 10$ ft^2

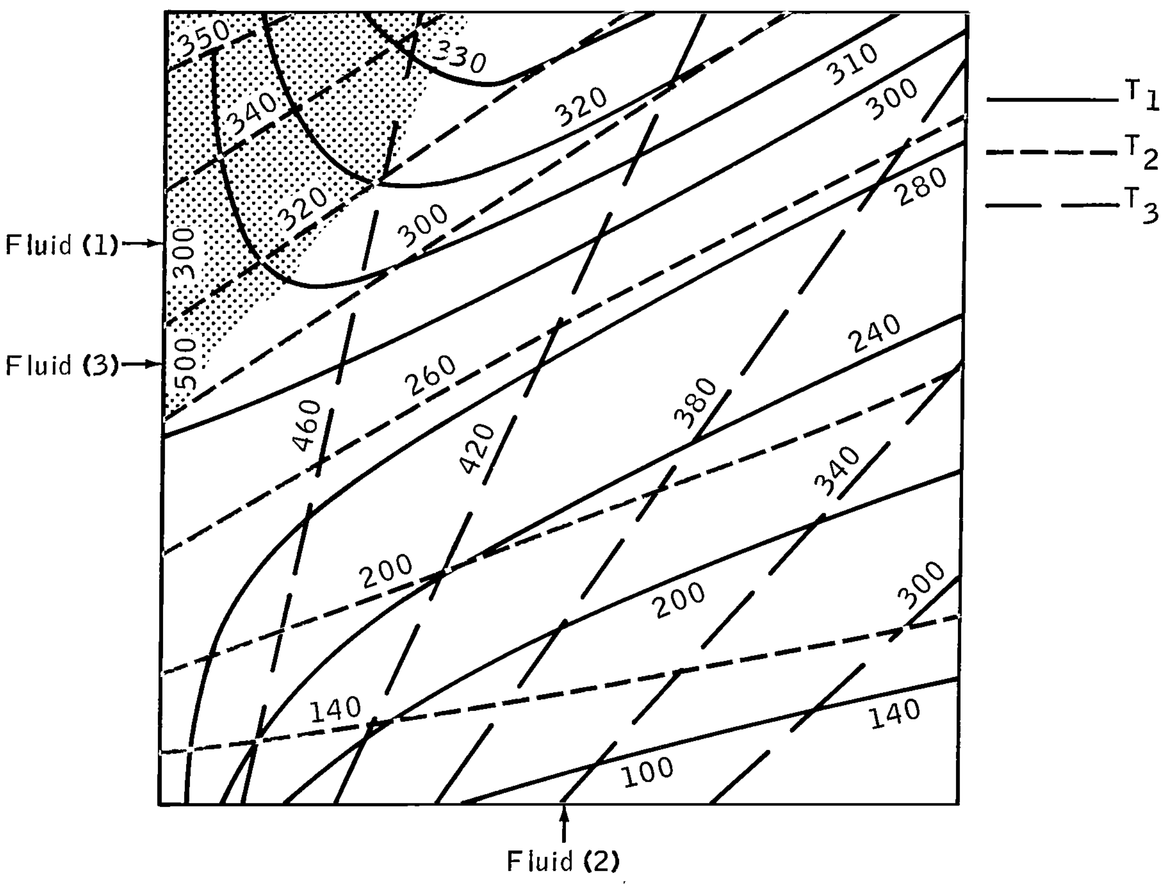


Figure 50. - Isothermal contours for fluids (1), (2), and (3).

NASA-S-67-6777

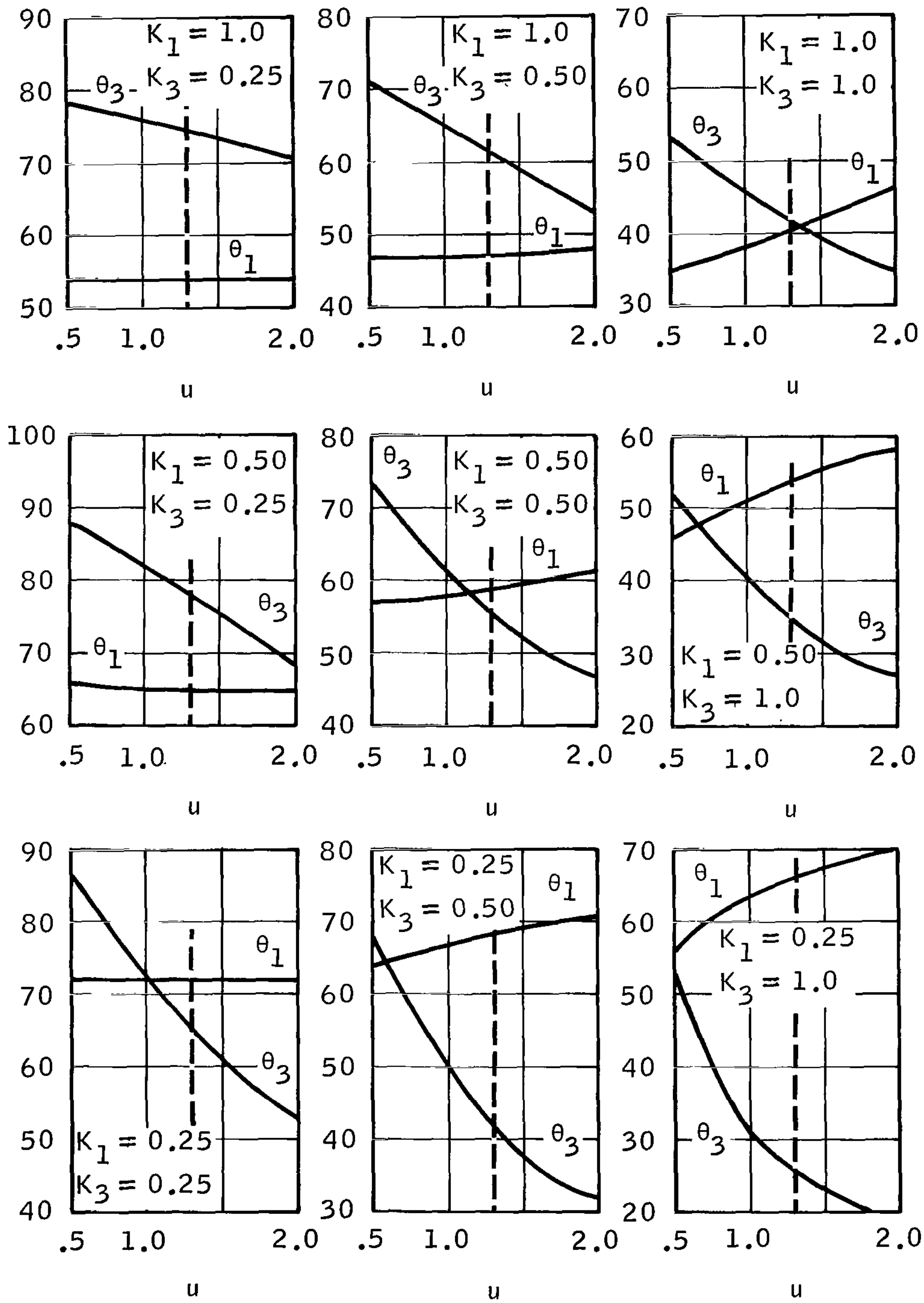


Figure 51. - Temperature effectiveness as a function of conductance ratio for $NTU_1 = 2.0$ and $\Delta t_i = 0.85$.

NASA-S-67-6778

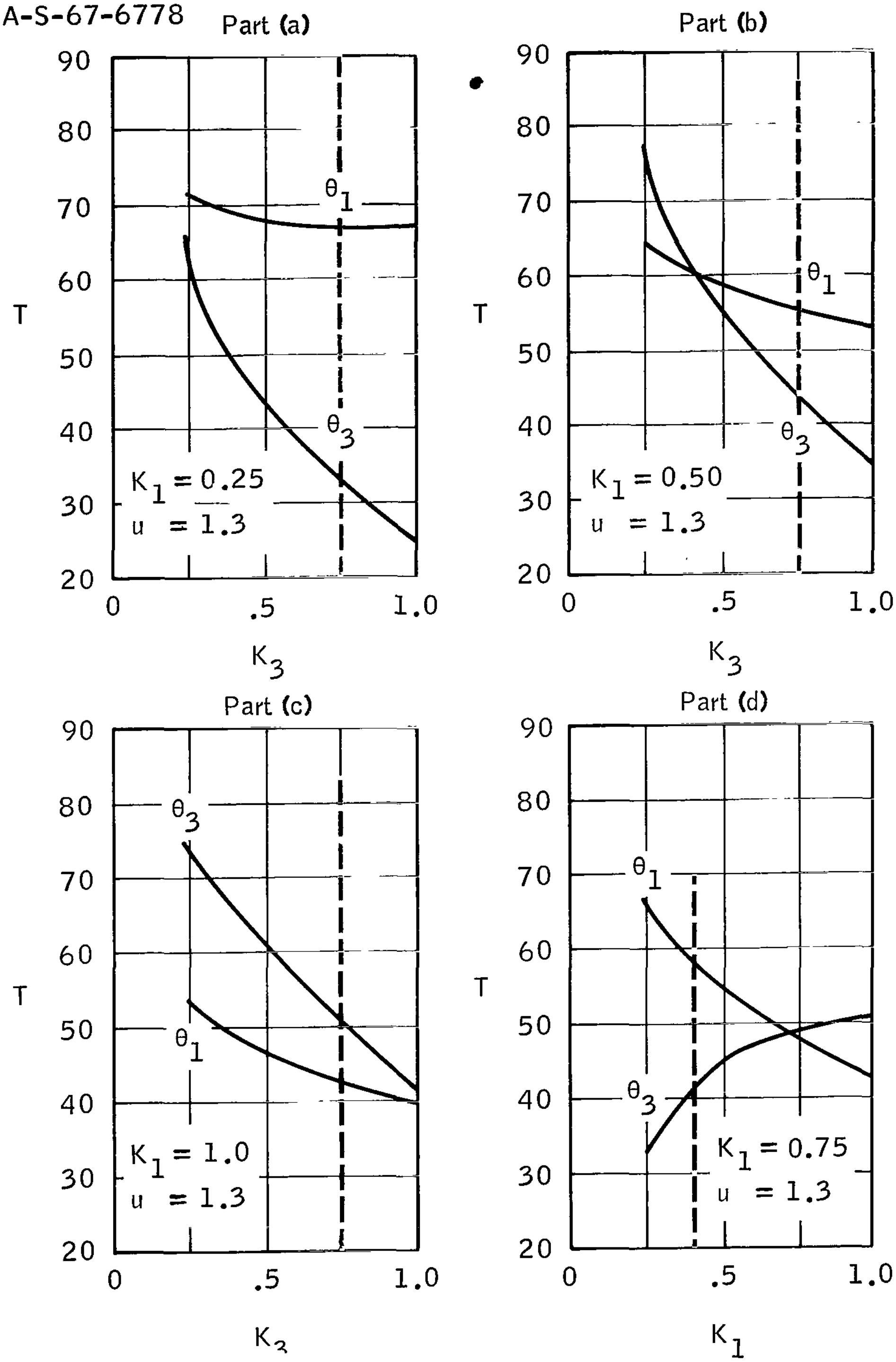


Figure 52. - Temperature effectiveness as a function of capacity-rate ratio for $u = 1.3$, $NTU_1 = 2.0$, and $\Delta t_i = 0.85$.

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