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An Improved Model for Cosmic Ray Propagation*

by

Peter D. Noerdlinger

Department of Physics and Astronomy
University of Iowa
Iowa City, Iowa 52240

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ABSTRACT

A model for cosmic ray propagation derived by Jokipii is modified to take into account particle mirroring. This removes the unreasonable sensitivity of the original theory to the extreme high frequency end of the interplanetary magnetic field power spectrum P(f). It is then possible to relate the diffusion coefficient for particles of known velocity and rigidity to a limited portion of the field spectrum.

The interplanetary magnetic field, at least up to about 1.2 A.U. from the sun, is known to be fairly ordered, lying principally at the gardenhose angle,[1] but with fluctuations of the order of 30% on quiet days and larger on disturbed days. This paper will deal with cosmic ray propagation on fairly quiet days. At these times, the fluctuations in the field direction are believed[2] to be more important than fluctuations of strength, δB, so we shall assume $\delta B/B_o$ is of the order of 20% to 25%, where $\vec{B}_o$ is the smooth field at the gardenhose angle. Jokipii analyzed cosmic ray propagation in a field of this type in considerable detail,[3] and showed that the motion perpendicular to $\vec{B}_o$ consists of slow diffusion, while the motion along $\vec{B}_o$ is controlled by diffusion in pitch angle $\theta \equiv \cos^{-1} \mu$. Following Jokipii,[3] we denote the direction of $\vec{B}_o$ by z, the particle velocity by V, its charge by Ze, and its energy by $\gamma m_o c^2$. Define ω_o and r_c by $\omega_o = B_o eZ/\gamma m_o c = V/r_c$. The gyro radius is $r_c \sin \theta$. The power spectrum of the field fluctuation $\vec{B}_1 = \vec{B} - \vec{B}_o$ will be a function P(f) of frequency. This spectrum is attributed[4] to a power spectrum $P(kV_w/2\pi)$ of field irregularities of wave number k being carried past the spacecraft at the solar wind velocity V_w. Before indicating the change to be made in the Jokipii theory, we review a few key equations and indicate the problem that arises in the theory when P(f) falls off too steeply at large f. Ignoring the simple, slow diffusion in the XY plane and simplifying to a time-independent diffusion problem, we obtain from (J26)[5]

$$2\mu V(\partial n/\partial z) = (\partial/\partial\mu)[\Delta(\partial n/\partial\mu)] \ , \tag{1}$$

where we denote by Δ the Fokker-Planck coefficient for diffusion in μ that Jokipii denotes by $\langle(\Delta\mu)^2\rangle/\Delta t$. Substituting $n(\mu) = \frac{1}{2} n_o(z) + n_1(\mu)$ in Equation (1), where we assume $\partial n_o/\partial z$ and n_1 are both small, of the same order, and are both independent of z, we obtain

$$n_1(\mu_1) = \frac{1}{2}\int_{\mu_o}^{\mu_1} (\mu^2 - 1)\ \Delta^{-1}(\mu)V(\partial n_o/\partial_z)d\mu \ . \tag{2}$$

In Equation (2), an intermediate constant of integration was evaluated by requiring that $\partial n_1/\partial\mu$ be bounded at $\mu = \pm 1$, and the other constant of integration μ_o will be taken as $\mu_o = -1$ with no loss of generality. The only use to be made of Equation (2) is to evaluate the mean streaming velocity $V\langle\mu\rangle$ along $\vec{B}_o$, viz:

$$\langle\mu\rangle = \int_{-1}^{1} \mu_1\ n_1(\mu_1)d\mu_1 \ . \tag{3}$$

Later, Equation (3) will be modified by deletion of a portion of the range of integration around $\mu_1 = 0$. By the use of Equation (3) and (J27), which is the definition of the diffusion coefficient $D_{zz} = -n_o V\langle\mu\rangle/(\partial n/\partial z)$, we obtain (J28):

$$D_{zz} = \tfrac{1}{2} V^2 \int_{-1}^{1} \mu_1 \left[\int_{-1}^{\mu_1} (1 - \mu^2)\, \Delta^{-1}\, d\mu \right] d\mu_1 \; . \qquad (4)$$

Physically, Equation (4) says that the diffusion depends on the mean drift $\langle\mu\rangle$ in Equation (3), which receives a large contribution from any class of particles that are not scattered much (Δ small), but only a small contribution from particles that are scattered greatly (Δ large), and hence find it difficult to pass freely along $\vec{B}_o$. The <u>form</u> of $\Delta(\mu)$ and D_{zz} must be found from P(f).

Jokipii calculates Δ from P(f) on the basis of two assumptions: (a) immediately after (J17), he assumes that "$V_z\Delta t$ is much greater than the correlation length along z," where Δt is the time interval over which perturbations to the orbit are averaged; (b) he assumes, as is proper, that $r_c \ll L$, where L is the correlation length of the field fluctuations. Whatever value is chosen for Δt, however, assumption (a) must fail for a small percentage of the particles,[6] namely, those with small values of $|\mu|$. Temporarily ignoring this difficulty, however, we pass to Jokipii's result (J37):

$$\Delta = (1 - \mu^2)\,|\,\mu\,|^{-1}\, V^{-1}\, B_o^{-2}\, \omega_o^2\, V_w P(V_w \omega_o / 2\pi\,|\,\mu\,|\,V)\ . \qquad (5)$$

From Equation (5), we see that in this theory all the scattering is resonant; each particle is affected only by fluctuations at wave number k, such that traversing the fluctuations at speed $V_z = \mu V$, it sees them at its gyro frequency ω_o. If $|\,\mu\,|$ is nearly unity, this means $k \approx r_c^{-1}$, but if $|\,\mu\,|$ is very small, fluctuations of large k are the relevant ones. Explicitely, we see that a particle of velocity V and pitch angle θ resonates with the part of the power spectrum P(f) at $f = f_{res}$, where

$$f_{res} = V_w \omega_o / 2\pi\,|\,\mu\,|\,V\ . \qquad (6)$$

Since $0 \leq |\,\mu\,| \leq 1$, f_{res} varies from $V_w \omega_o / 2\pi V$ to ∞ for any class of particles of fixed V. Thus, D_{zz} is sensitive to all frequencies above a limiting frequency, a distressing result.[6]

It is easy to see why this theory includes only resonant scattering. The theory is based on a power series expansion in B_1/B_o, and it is well known[7] that the magnetic moment of a particle $\frac{1}{2} m\gamma^2\, V^2\, B^{-1} \sin^2\theta$ is conserved to all orders in B_1. Since B_o

suffers no secular changes in the present theory, the magnetic moment (and hence θ) can suffer secular changes only through the application of a resonant field fluctuation. In order to explore this problem further, consider applying Equations (1) to (5) to a typical power-law spectrum[2,3,8] $P(f) = \delta/f^n$. A brief calculation gives

$$D_{zz} = \frac{V^{3-n} \; \omega_o^{\,n-2} \; V_w^{\,n-1} \; B_o^{\,2}}{\delta(2\pi)^n \; (2 - n)(4 - n)} \, , \tag{7}$$

for $n < 2$ and $D_{zz} = \infty$ if $n \geq 2$. If $n = 1$, D_{zz} assumes the much-discussed[9] form proportional to $R\beta$, where $\beta = V/c$ and R is the rigidity $R = B_o V/\omega_o$. If $n \to 2$ from below, D_{zz} is unbounded, but approaches in the limit a form independent of R as required by Nathan and Van Allen[8] to fit certain experimental data. Thus, to within experimental accuracy, one could fit the data with the Jokipii theory, provided $n \approx 1.95$, say.[10] This is not satisfactory, both because it seems arbitrary and because we would be in difficulty if further experiment should show that $P(f)$ is as steep[11] as $1/f^2$ at large f. Jokipii has presented in the Addendum[3] a modification of his theory that avoids the divergence problem for these spectra, but the modifications to be given here have advantages that will be pointed out below.

The key to the solution of the foregoing problem lies in careful study of the physical behavior of particles with small $|\mu|$.

These particles undergo the least resonant scattering, according to Equation (5), since $P(f) \rightarrow 0$ as $f \rightarrow \infty$.[12,13] Thus, the magnetic moment of these particles is very well conserved and when they reach a place where δB is sizable, they must mirror. The ones that start at large values of δB will, of course, pick up additional V_z when they reach lower B values, but then their $|\mu|$ value will be large and they will be subject to scattering by the stronger, lower frequency part of P(f). In this theory, the particles follow adiabatically the strong variations of B associated with frequencies well below f_{res}, and this non-resonant contribution to the variation of μ was omitted from the original Jokipii theory by assumption (a). Since $\delta B/B_o$ is of the order of $\leqslant 25\%$ in the solar wind, many particles are subject to mirroring and are thus not free to travel along $\vec{B}_o$ as easily as implied by Equations (1), (2), and (5). One may easily verify that a fluctuation of size δB will reflect all particles with

$$|\mu| < (\delta B/B_o)^{\frac{1}{2}} \leqslant 0.5 \ . \qquad (8)$$

To set up a full theory taking into account mirroring would be an extensive undertaking; one would have to use stochastic theory to estimate the frequency of occurrence of δB as a functional of P(f), and one would have to generalize Equation (1) to allow mirroring. Here, we present a crude theory that is adequate to remove the extreme sensitivity of Jokipii's theory to the asymptotic form of P(f). Regarding the field as static in the rest frame of the solar wind, we recall that isotropic equilibria $n_o(z)$ should still exist,[14] and that locally the Jokipii scattering theory should apply. It is only over large distances, of the order of one correlation length of δB, that we expect to find mirroring. But the effect of this is quite simple: the particle just oscillates back and forth until it is scattered enough to get through the mirror point. This scattering is already accounted for in the Jokipii theory.[5] Therefore, all that we need to do is to say that particles with $|\mu| < \mu_m$ make no average progress along $\vec{B}_o$, and so fail to contribute to the integral in Equation (3), where μ_m, a mean value of μ for mirroring, is established by typical field fluctuations. Thus, we set

$$\langle\mu\rangle = \int_{-1}^{-\mu_m} \mu_1\, n_1(\mu_1)d\mu_1 + \int_{+\mu_m}^{1} \mu_1\, n_1(\mu_1)d\mu_1 \ , \qquad (3')$$

where

$$\mu_m = (\langle\, |\delta B| / B_o \rangle)^{\frac{1}{2}} \ . \qquad (9)$$

If we apply Equations (2), (3′), (5), and (9) to $P(f) = \delta/f^2$, we not get

$$D_{zz} = \frac{V V_w B_o^2}{8 \pi^2 \delta} (\mu_m^2 - 2 \ln\mu_m - 1) \equiv \frac{V V_w B_o^2}{8 \pi^2 \delta} S(\mu_m) , \qquad (10)$$

which defines $S(\mu)$. Figure 1 shows the form of $S(\mu)$; under quiet day conditions, we have $S \approx 0.7$. Under the new theory, particles are sensitive only to frequencies in the range

$$V_w\omega_o/2\pi V < f_{res} < V_w\omega_o/2\pi\mu_m V . \qquad (11)$$

Applying Equations (10) and (11) to the data for 40 keV electrons and 75 MeV protons,[8] we find that P(f) should be of the form $1/f^2$ in the range approximately $5 \times 10^{-4} < f < 1$ c/s. This conclusion still depends, of course, on only two data points, and it is also possible that P(f) could have been anomalously steep on 25-26 May 1965 when the electron data were taken.[15] It is also quite possible that $\vec{B}_1$ contains fluctuations of a kind that present different power spectra to a fixed spacecraft and a diffusing cosmic-ray particle. Specifically, discontinuities between adjacent bundles of field lines may[2] contribute significantly to P(f), and yet particles of low rigidity may stay so much within one bundle that they do not "see" these boundaries.[13] Jokipii has proposed an alternate method to deal with particles of small $|\mu|$, but this method seems less satisfactory because it requires n_1 to be a very smooth function[16] of μ and it fails to bring out the limits on f_{res} shown in Equation (10).

Jokipii also suggests[17] that for particles such that the old theory gives mean scattering length $\lambda \equiv 3D_{zz}/V < L$, the scattering theory must be altered and the simple result $D_{zz} = LV/3$ used. It is difficult to comment on this proposal without a more thorough[13] analysis of what is meant by the "correlation length" L, but it is possible to dispose fairly easily of the assertion[17] that the present scattering theory fails for 40 keV electrons because $\lambda < L \approx 10^6$ km. 40 keV electrons are scattered by frequency components around 0.5 to 1.0 c/s and in this region P(f) is very small; the major contributions to the total power are from much lower frequencies. It is these low frequency components, which the electrons follow adiabatically, that establish the long correlation length quoted by Jokipii. A correlation length of 10^6 km for waves with $f \approx 0.5$ c/s would imply their coherence over $V_w^{-1} f \times 10^6 = 1000$ cycles, which is not likely in a disordered medium like the solar wind. Thus, we believe the correlation length of the field, insofar as it is pertinent to

40 keV electrons, should be taken much smaller. A modification of the type suggested by Jokipii would be more appropriate when the field power density in the range (Equation (11)) is comparable to the total power density.

It would be interesting to apply the results found here to the inward propagation of galactic cosmic rays.

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REFERENCES

1. E. N. Parker, Interplanetary Dynamical Processes (Interscience, New York, 1963), Chap. 10.

2. G. L. Siscoe, L. Davis, Jr., P. J. Coleman, Jr., E. J. Smith, and D. R. Jones, J. Geophys. Res. 73, 61 (1968).

3. J. R. Jokipii, Astrophys. J. 146, 480 (1966); Erratum and Addendum, Astrophys. J. [to be published 1968]. The n^{th} equation of the 1966 paper will be denoted by (Jn).

4. The assumption that B_1 is static in the solar wind reference frame is justified by the smallness of the Alfven velocity $V_A \approx 50$ km/sec. as compared to V_w and V.

5. The modifications to be made here do not affect (J19) and would introduce negligible changes in (J26).

6. This conclusion is true whatever fixed values of V, m_o, or Z are considered. Later it will be shown that for small $|\mu|$, assumption (a) must be replaced by a mirroring condition. Note that $V_z = V_\mu$.

7. T. G. Northrop, The Adiabatic Motion of Charged Particles (Interscience, New York, 1963).

8. K. V. S. K. Nathan and J. A. Van Allen, J. Geophys. Res. 73, 163 (1968).

9. G. Gloeckler and J. R. Jokipii, Phys. Rev. Letters 17, 203 (1966).

10. The absolute magnitude of D_{zz} is very difficult to estimate from the data because of the unknown source function and the unknown dependence P(f) on heliocentric radius r.

11. A careful analysis shows that $D_{zz} \to \infty$ if $\overline{\lim}_{f \to \infty}[f^2 P(f)]$ exists, so that a steep falling off of P(f) leads to a divergence in the Jokipii theory no matter at how high a frequency it occurs. If this were physically correct, diffusive motion would be impossible when $P \sim 1/f^2$ or steeper at large f.

12. In order for the integrated power to be finite, $\overline{\lim}_{f \to \infty}[fP(f)]$ must exist.

13. The actual behavior of P(f) at large f will be discussed in a future communication.

14. E. N. Parker, op. cit., p. 158.

15. J. A. Van Allen and S. M. Krimigis, J. Geophys. Res. 70, 5737 (1965).

16. See Reference 3, Addendum and Erratum.

17. J. R. Jokipii, EFINS preprint, Laboratory for Astrophysics and Space Research, University of Chicago, Chicago, Illinois [unpublished].

FIGURE CAPTION

Figure 1 $S(\mu_m)$ vs. μ_m

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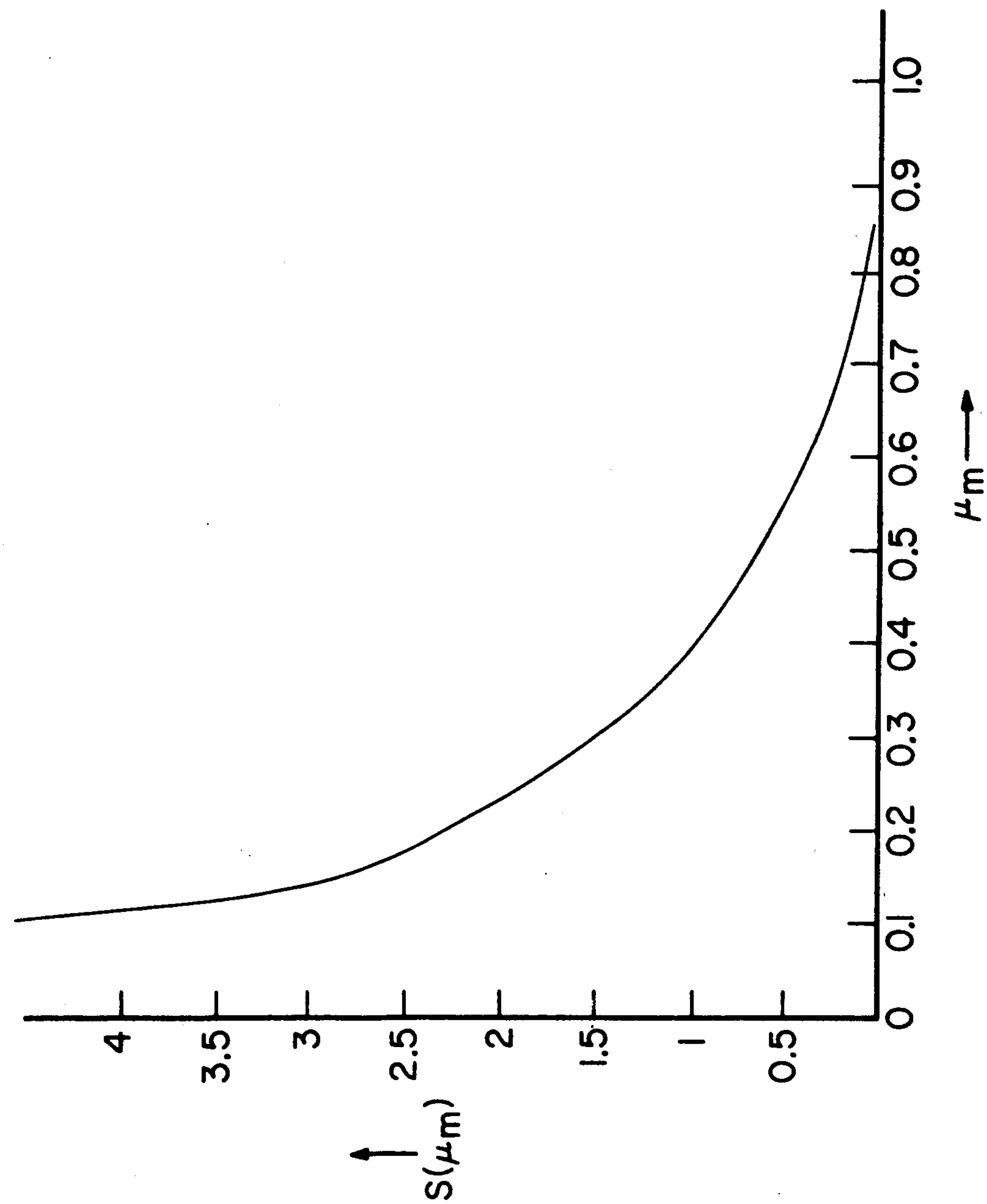

Figure 1

ERRATUM

to

An Improved Model for Cosmic Ray Propagation

After Equation (9) add the following: "Since the mirroring particles have pitch angles that vary from 90° to $\cos^{-1} \mu_m$ as the particles repeatedly mirror, it is reasonable also to set $\Delta = \Delta(\mu_m)$ for $|\mu| < \mu_m$ in calculating n_1. This will be done."

Replace Equation (10) by

$$D_{zz} = \frac{V V_w B_o^2}{8\pi^2 \delta} (1 - \mu_m^2 - 2 \ln \mu_m) \equiv \frac{V V_w B_o^2}{8\pi^2 \delta} S(\mu_m)$$

and note the altered Figure 1 attached.

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