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APPROXIMATION OF THE SPANWISE DISTRIBUTION OF WIND-TUNNEL-BOUNDARY INTERFERENCE ON LIFT OF WINGS IN RECTANGULAR PERFORATED-WALL TEST SECTIONS

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SUMMARY

An approximation method has been developed for calculating the spanwise distribution of wind-tunnel-boundary upwash interference on lift of wings in rectangular perforated-wall test sections. This method is applied to square test sections with an assumed effective permeability constant of 0.6. A problem of considerable difficulty in practical application of the method presented is the estimation of an effective permeability constant. Because of the variation of the upwash interference with Mach number and of the influence of boundary layer on the effective permeability factor, the boundary interference in a perforated-wall wind-tunnel test section at high subsonic Mach numbers is likely to be of the nature of that in an open-throat tunnel.

INTRODUCTION

During the past 15 years a number of wind tunnels have been constructed with test sections having perforated walls. Such walls are of particular advantage in testing through sonic speed and at speeds slightly supersonic because they may reduce the severity of shock-wave disturbances reflected from the walls and impinging on the test models. However these wind tunnels are commonly used also for subsonic testing and, if wing models are of appreciable size relative to the cross section of the tunnel at the test position, it is desirable to correct for the modification of test conditions due to the upwash interference of the tunnel boundaries.

The theory for the upwash interference at a lifting wing of small span mounted at the center of a circular perforated-wall test section has been presented in references 1 and 2, but no comparable theory is known to the authors for the interference in a rectangular perforated-wall test section or for the variation along the wing span of the upwash interference in any perforated-wall test section. It appears possible to extend the theory of reference 2 to cover wings with span not small relative to the tunnel diameter and to permit calculation of the upwash interference along the span. However, for constructional and operational convenience and for avoidance of focusing of reflected shocks in the transonic speed range, perforated-wall test sections are commonly made rectangular in cross section. An approximation method has therefore been developed and is herein reported for estimating the upwash interference along the span of a lifting wing mounted at the center of a rectangular perforated-wall test section. The method has been applied to estimate the upwash interference on wings spanning 0.3 and 0.7, respectively, of the width of a square perforated-wall test section at four different subsonic Mach numbers. A method is also suggested for estimating the interference due to lift at a small-span wing mounted at the center of a square perforated-wall test section by means of comparison with the interference due to lift on a small-span wing in a circular perforated-wall test section. Estimates so obtained are compared with calculated results obtained by the approximation procedure.

SYMBOLS

$A(\omega,g)$ arbitrary function in G

$B(\omega,g)$ arbitrary function in G

$B(\omega,f)$ arbitrary function in F

b semiwidth of test section

C cross-sectional area of test section

C_L lift coefficient

$c_g = \sqrt{\beta^2(\alpha^2 + \omega^2) + g^2}$

$c_f = \sqrt{\beta^2(\alpha^2 + \omega^2) + f^2}$

$D_g = \sqrt{\beta^2\omega^2 + g^2}$

$D_f = \sqrt{\beta^2\omega^2 + f^2}$

F exponential integral Fourier transform of Ω on y

f variable of transformation on y

G	exponential integral Fourier transform of Ω on z
g	variable of transformation on z
H,N	symbols standing for expressions defined by equations (C2) and (C3), respectively
h	semiheight of test section
I_0, I_1	modified Bessel functions of the first kind
K_0, K_1	modified Bessel functions of the second kind
k	quality factor, defined by equation (5)
M	Mach number
n	distance normal to wall, positive outward
p,q,r	variables of integration
Δp	pressure drop through a porous wall
R	permeability factor (defined by eq. (3))
S	area on which lift coefficient is based
s	semispan of horseshoe vortex
V	velocity of tunnel test stream
v	upwash velocity, positive in direction of Z-axis
X,Y,Z	axes of rectangular Cartesian coordinate system
x,y,z	rectangular Cartesian coordinates
α	small positive parameter, $\alpha \to 0$

$\beta = \sqrt{1 - M^2}$

Γ circulation

δ upwash interference factor, $\frac{Cv}{SVC_L}$

η distance along span of horseshoe vortex

ρ density of test medium

ϕ velocity potential

ω variable of transformation on x

Ω exponential integral Fourier transform of ϕ on x

Subscripts:

1 pertaining to direct disturbance caused by lift of wing

2 pertaining to vertical boundaries

3 pertaining to horizontal boundaries

4 pertaining to effect of horizontal boundaries on interference velocity potential inside test section due to vertical boundaries

5 pertaining to effect of horizontal boundaries on interference velocity potential outside test section due to horizontal boundaries

A prime indicates a form containing α; the prime is removed when $\alpha \to 0$, for example, $\lim_{\alpha \to 0} \phi' = \phi$.

ANALYSIS

In this investigation the wing is assumed to be mounted horizontally at the center of a rectangular perforated-wall test section of semiheight h and semiwidth b. The disturbance due to the wing is represented by a horseshoe vortex with span 2s and circulation Γ. The disturbance due to a wing with nonuniform loading can be represented by superposing several horseshoe vortices of differing spans. The total upwash interference velocity is then the sum of the upwash interference velocities corresponding to the several horseshoe vortices considered separately. For convenience in mathematical treatment the vortex system is considered to be made up of elementary horseshoe vortices of constant strength with infinitesimal span $d\eta$. By analogy with equation (40) of reference 2 the free-flow velocity potential ϕ_1 is

$$\phi_1 = \frac{\Gamma}{4\pi}\int_{-s}^{s}\left\{1 + \frac{x}{\sqrt{x^2 + \beta^2\left[(y - \eta)^2 + z^2\right]}}\right\}\frac{z}{(y - \eta)^2 + z^2}\,d\eta \tag{1}$$

where x, y, and z are rectangular Cartesian coordinates with z positive in the direction of lift, y lying along the span, and x positive downstream in the direction of the trailing vortices, and where

$$\beta = \sqrt{1 - M^2}$$

The perforated boundaries are assumed to behave as ideal porous walls. The average velocity normal to the wall is assumed to be proportional to the pressure drop through the wall (a linearized approximation to viscous flow through a porous medium). The pressure outside the wall is assumed to be equal to the free-stream pressure. As in references 1 and 2 these assumptions lead to the boundary condition

$$\frac{\partial\phi}{\partial x} + \frac{1}{R}\frac{\partial\phi}{\partial n} = 0 \tag{2}$$

The permeability factor R is given by

$$R = \frac{\rho V}{\Delta p}\frac{\partial\phi}{\partial n} \tag{3}$$

where

Δp pressure drop through the wall

ρ density of test medium

V stream velocity

n distance normal to wall, positive outward

ϕ velocity potential

The potential ϕ_1 already satisfies the linearized compressible-flow (Laplace) equation (ref. 2)

$$\beta^2\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{4}$$

In order to obtain an approximation to the interference potential, separate potentials ϕ_2, ϕ_3, ϕ_4, ϕ_5, . . ., each of which satisfies equation (4), are added to ϕ_1 such that the sum approximately satisfies the boundary condition (2). This sum necessarily satisfies equation (4) because of its linearity. If the approximation process converges, equation (2) can be satisfied and $(\phi_2 + \phi_3 + \phi_4 + \phi_5 + . . .)$ is the required interference velocity potential. The hypothesis is made that the approximation process next described is convergent.

The potential ϕ_2 is taken to be the interference velocity potential due to plane parallel vertical perforated walls of infinite extent at the y-positions of the side walls of the test section in the presence of the horseshoe vortex. A schematic drawing of the arrangement is shown in figure 1, where V is the test stream velocity. The corresponding upwash interference velocity v_2 is derived in appendix A.

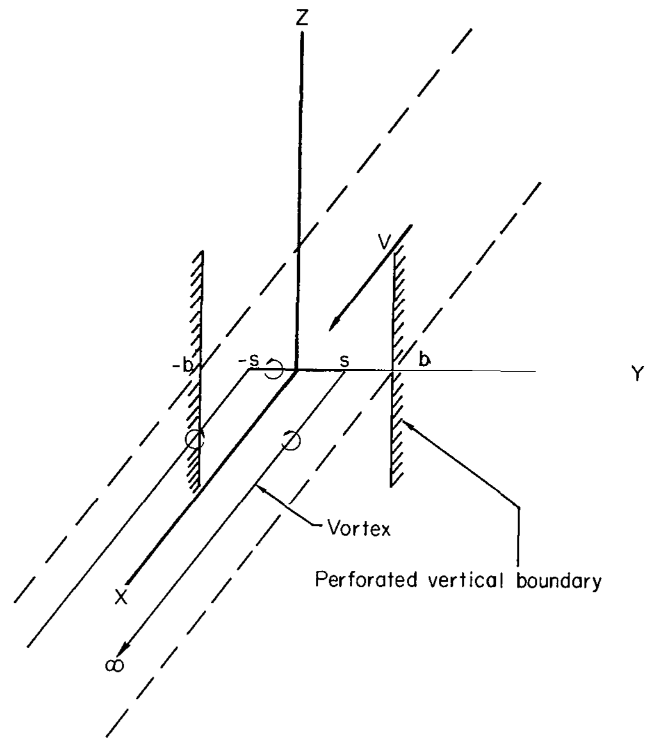

Figure 1.- Schematic drawing showing horseshoe vortex of span 2s in flow between vertical perforated walls of infinite extent parallel to XZ-plane.

The potential ϕ_3 is similarly taken to be the interference velocity potential due to plane parallel horizontal perforated walls of infinite extent at the z-positions of the top and bottom walls of the test section. A schematic drawing of the arrangement is shown in figure 2, and the corresponding boundary interference velocity v_3 is derived in appendix B.

The potential $(\phi_1 + \phi_2)$ satisfies the boundary condition at the vertical boundaries of the test section and $(\phi_1 + \phi_3)$ satisfies the boundary condition at the horizontal boundaries; but in general $(\phi_1 + \phi_2 + \phi_3)$ fails to satisfy the boundary conditions, because ϕ_2 does not in general satisfy the boundary condition at the horizontal top and bottom walls and ϕ_3 does not in general satisfy the boundary condition at the vertical side walls. It should be possible to improve the approximation by adding to ϕ_2 a potential such that the boundary condition at the horizontal boundaries is satisfied. However, the potential ϕ_2 derived in appendix A is valid only inside the vertical boundaries and, as shown in appendix C, another solution must be used outside these boundaries. Correspondingly, two potentials ϕ_4 and ϕ_5 are added to ϕ_2 such that $(\phi_2 + \phi_4 + \phi_5)$ satisfies the boundary condition at the horizontal boundaries. Since $(\phi_1 + \phi_3)$ already satisfies these conditions, then $(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5)$ does also. At this point in the discussion the boundary condition at the vertical boundaries is formally satisfied only up to ϕ_2, but there is reason to hope that the addition of ϕ_4 and ϕ_5 to ϕ_3 may have improved the approximation at the vertical boundaries and that the process has already been carried far enough for practical application. This hope is given substance by consideration of the upwash interference with all-closed or all-open boundaries.

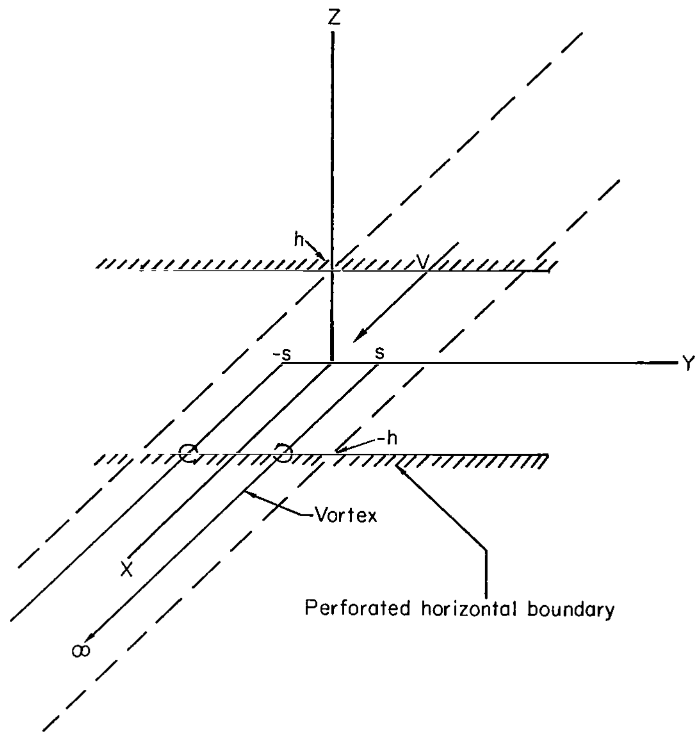

Figure 2.- Schematic drawing showing horseshoe vortex of span 2s in flow between horizontal perforated walls of infinite extent parallel to XY-plane.

Figure 3 indicates the image systems in the far downstream (Trefftz) plane for a lifting wing in a closed and in an open rectangular test section. For the closed test section (fig. 3(a)), with the original lifting wing at $(y = 0,\ z = 0)$ the horizontal row of images at $(z = 0,\ y = \pm 2b, \pm 4b, \ldots)$ yields the potential ϕ_2 inside the test section due to the reaction of the vertical side walls. The vertical row of alternately inverted and erect images at $(y = 0,\ z = \pm 2h, \pm 4h, \ldots)$ yields the potential ϕ_3 inside the test section due to the interaction of top and bottom horizontal walls. The remaining images, those at $(y \neq 0,\ z \neq 0)$, are seen to be exactly the images of the image row at $z = 0$ in the top and bottom horizontal boundaries and therefore yield the potential for the reaction of the top and bottom walls on the interference potential ϕ_2 due to the vertical side walls provided ϕ_2 in the region of the horizontal boundaries outside but near the vertical boundaries is adequately represented by this image row. Thus, subject to this provision, for solid boundaries the addition of the correction $(\phi_4 + \phi_5)$ to $(\phi_2 + \phi_3)$ takes account of all the theoretical first-order boundary interference on the lifting wing. The same conclusion applies to the open test section for which the image system is indicated in figure 3(b). The potentials ϕ_4 and ϕ_5 and the corresponding upwash velocities v_4 and v_5 are derived in appendix C.

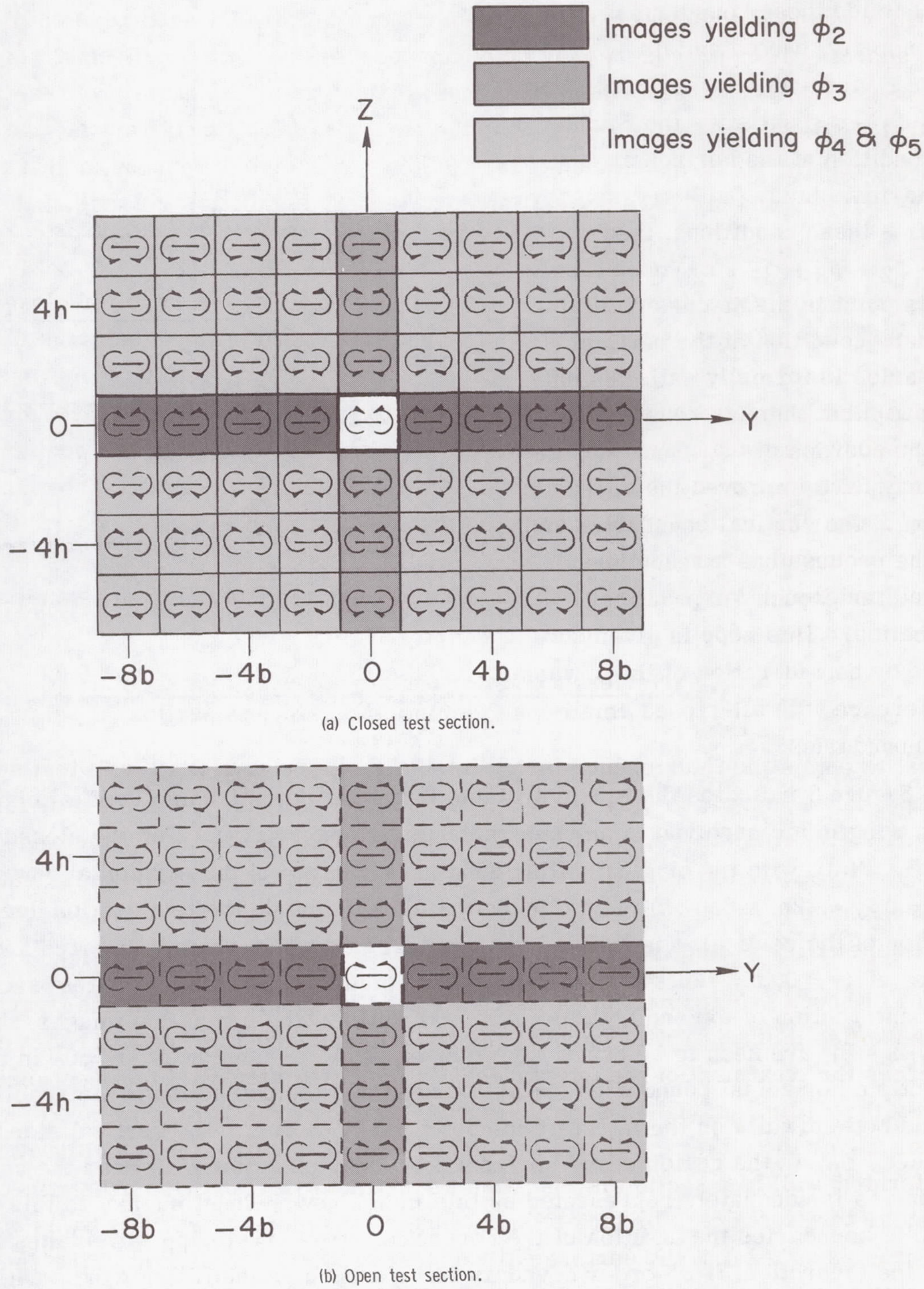

Figure 3.- Image systems in Trefftz plane for lifting wing in closed and in open rectangular test sections.

The upwash interference at a wing of small span (relative to the test-section width) mounted at the center of a square perforated-wall test section as estimated by this approximation procedure may be compared with that at a small-span wing similarly mounted in a circular perforated-wall test section having the same cross-sectional area as that of the square test section. For the same permeability factor R and the same lift, the interferences should be nearly the same since the difference in shape between the square and the circle is not sufficient to be very significant relative to the boundary interference; this is only to say that the upwash interference factor δ $\left(= \frac{Cv}{SVC_L}\right)$ should be approximately the same for the same value of R. For open and for closed boundaries this assertion is borne out by direct comparison between already known values of δ for square test sections and those for circular test sections. A possible improvement in the comparison is suggested by the theory of reference 3 for slotted-wall test sections. In this theory the rectangular test section corresponds to a circular test section through a conformal transformation of the rectangle into the circle. In this transformation the span of the wing is also transformed; however, if the span is small relative to the width of the rectangular test section, the effect of its transformation disappears from the final result and the quality factor k for the rectangular test section is the same as that for the comparable circular test section, where

$$k = \frac{\text{Interference factor for slotted-wall test section}}{\text{Interference factor for closed test section}} \tag{5}$$

Thus, to obtain the interference factor for the rectangular slotted-wall test section with a small lifting wing it is only necessary to multiply the interference factor for the rectangular closed test section of the same cross section by the quality factor for the comparable circular slotted-wall test section. It is seen that the problem is "cut," so that (aside from any effect on the slot distribution for the comparable circular test section) the shape effect is imposed through the closed-test-section interference.

Although proof cannot be obtained through the method of conformal transformation, because this method applies only for the two-dimensional flow in the Trefftz plane whereas the treatment of the interference in the perforated-wall test section is necessarily three-dimensional, it seems reasonable to suppose that a similar relationship exists as for the slotted-wall test section so that the quality factor for the rectangular perforated-wall test section is the same as the factor for an in-some-sense-comparable circular test section. For lack of knowledge of any more reasonable basis of comparability, it is assumed that the perforated-wall test sections correspond in the same way as do the slotted-wall test sections of reference 3, that is, through the conformal transformation of the rectangle into the circle. In this transformation the parts of the periphery of the rectangle farthest from the center become relatively contracted, whereas the parts closest to the center become relatively expanded; but for closely spaced slots or holes the stretching is almost the same for open as for adjacent closed portions of the boundary, so that locally the ratio of open-to-closed boundary area remains unchanged in the transformation. For the slotted-wall test section the distortion of slot distribution produced in the transformation affects the value of k, even though the ratio of open-to-total boundary area may remain unchanged. The effect of the distortion on the perforated boundary is not clear, so that this comparison method is not directly applicable for the general rectangular perforated-wall test section; however, if the effects of distortion were negligible, the effective permeability would remain unchanged and the circular perforated-wall test section with effective permeability factor R would correspond to every rectangular test section with the same effective permeability factor. It seems reasonable to assume that for the square test section this condition is approximately satisfied, so that

Interference factor for square perforated-wall test section

$$= k \text{ (Interference factor for square closed test section)} \tag{6}$$

where k is the quality factor for the circular test section with effective permeability factor R the same as that for the square test section. Examination of equation (49) of reference 2 and of the equation immediately preceding it shows that

$$k = 1 - \frac{\beta}{R}\frac{1}{\pi}\int_0^\infty \frac{\left[K_1(q)\,I_0(q) + K_0(q)\,I_1(q)\right]q^3dq}{q^2I_1{}^2(q) + \left(\frac{\beta}{R}\right)^2\left[I_1(q) - qI_0(q)\right]^2} \tag{7}$$

where I_0, I_1 and K_0, K_1 are modified Bessel functions of the first and second kinds, respectively, and q is a variable of integration. The integrand in equation (7) simplifies to

$$\frac{1}{I_1{}^2(q) + \left(\frac{\beta}{R}\right)^2\left[\frac{I_1(q)}{q} - I_0(q)\right]^2} \qquad \text{or} \qquad \frac{1}{I_1{}^2(q) + \left(\frac{\beta}{R}\right)^2\left[I_1{}'(q)\right]^2}$$

where $I_1{}'(q)$ is the derivative of I_1 with respect to q.

If by this comparison method (for the square test section) or by any other method (for the general rectangular test section) the average upwash interference velocity along the span or that at the center of the span can be estimated, the variation over the span can be estimated from the upwash interference velocity v_2 due to infinite side walls, since the side walls may be expected to be dominant in producing any significant spanwise variation. (See, for instance, ref. 4.)

RESULTS AND DISCUSSION

In order to obtain an indication of the practical convergence of the approximation process herein described, it is applied first to completely closed and to completely open rectangular test sections. The contributions to the upwash interference velocity for a small-span wing, $s/h \rightarrow 0$, at the center of the test section, $y/h = 0$, are computed for the closed test sections from the first terms (the second terms are zero for the closed-throat tunnel) of equations (A14), (B10), (C10), and (C12) and for the open test sections from equations (A15), (B11), (C11), and (C13). The corresponding contributions to the upwash interference factor δ may be put in the form

$$\delta = \left(\frac{b/h}{s/h}\right)\frac{h}{\Gamma} v \tag{8}$$

For the closed-throat tunnel $(R_2 = R_3 \rightarrow 0)$ at $y = 0$ and $s/h \rightarrow 0$, equations (A14) and (B10) may be integrated to give, for $y = 0$, $s \rightarrow 0$, and $R_2 \rightarrow 0$,

$$\frac{h}{\Gamma} v_2 = \frac{s}{b} \frac{h}{b} \frac{\pi}{24} \tag{9}$$

and, for $y = 0$, $s \rightarrow 0$, and $R_3 \rightarrow 0$,

$$\frac{h}{\Gamma} v_3 = \frac{s}{h} \frac{\pi}{48} \tag{10}$$

Similarly, for the open-throat tunnel $(R_2 = R_3 \rightarrow \infty)$ at $y = 0$ and $s/h \rightarrow 0$, equations (A15) and (B11) may be integrated to give, for $y = 0$, $s \rightarrow 0$, and $R_2 \rightarrow \infty$,

$$\frac{h}{\Gamma} v_2 = -\frac{s}{b} \frac{h}{b} \frac{\pi}{48} \tag{11}$$

and, for $y = 0$, $s \rightarrow 0$, and $R_3 \rightarrow \infty$,

$$\frac{h}{\Gamma} v_3 = -\frac{s}{h} \frac{\pi}{24} \tag{12}$$

By using equations (8) to (12) and numerically integrating equations (C10), (C12), (C11), and (C13) for the specified conditions, the velocity contributions v_2, v_3, v_4, and v_5 were computed for small-span lifting wings at the centers of open and of closed test sections with ratio of width to height $2b/2h$ of 0.5, 0.75, 1.00, 1.50, and 2.00. The corresponding contributions δ_2, δ_3, δ_4, and δ_5 and the estimated total upwash interference factors $\delta = \delta_2 + \delta_3 + \delta_4 + \delta_5$ are given in table I. The correct values of δ (computed from formulas of ref. 5) are also shown. It is seen that the approximation is acceptable for all cases shown. The estimated δ is less reliable when the contributions of side walls δ_2 and of top and bottom walls δ_3 are of comparable magnitude and becomes more precise as the contribution of either side walls or top and bottom walls becomes dominant.

This behavior is not unexpected because if the side walls are dominant in producing the interference, then the reaction of the top and bottom walls is small and in the estimation of this reaction the error due to the unsatisfied side-wall boundary condition may also be expected to be small. On the other hand, if the top and bottom walls are dominant, the first neglected term in the approximation procedure – that is, the potential that would be added to $(\phi_4 + \phi_5)$ to satisfy the side-wall boundary condition – should be small.

The approximation procedure herein described has been applied to the calculation of upwash interference factors δ for lifting wings in a square test section with an assumed permeability factor $R = R_2 = R_3$ of 0.6. This value is believed to be representative for perforated material with approximately 20 percent of the surface area open (i.e., occupied by the holes) and with relatively thin boundary layer. It may not be representative of 20 percent open perforated wind-tunnel walls under actual operating conditions. The distributions of the approximated δ and of the contributions δ_2, δ_3, δ_4, and δ_5 along the semispan at Mach numbers 0, 0.6, 0.8, and 0.955 are given for a ratio of wing span to test-section height $2s/2h$ of 0.3 in table II and for $2s/2h$ of 0.7 in table III. Also given in table II are values of δ for small-span wings estimated by the circular-tunnel comparison procedure. In this estimation the quality factors k calculated from equation (7) for permeability factor R of 0.6 and for values of β of 1, 0.8, 0.6, and 0.3 are 0.233, 0.095, -0.087, and -0.477, respectively. The products of these values of k with the value of δ for the square closed test section ($\delta = 0.137$) are the estimated upwash interference factors given in the last column of table II. These estimates are practically the same as the values obtained by the approximation procedure; the two sets of values differ by not more than the maximum error indicated for the approximated values in table I.

In tables II and III is illustrated the already known fact that with porous-wall boundary conditions the boundary lift interference depends on the Mach number (see, for instance, ref. 1). In fact, the equations for v_2, v_3, v_4, and v_5 are easily arranged to show that the interference upwash velocity depends on R_2/β and R_3/β rather than on R_2, R_3, and β separately. Thus, for Mach number approaching unity ($\beta \rightarrow 0$) the boundary condition approaches that for the open tunnel $(R_2 = R_3 \rightarrow \infty)$ with incompressible flow ($\beta \rightarrow 1$).

With the assumed value of the permeability factor of 0.6, the approximated upwash interference factor approaches zero at a Mach number of 0.8, and as seen from table III the variation along the span is very small even for a wing spanning 0.7 of the tunnel width. At all Mach numbers the perforated walls have a beneficial effect on the spanwise variation of δ as compared with the effect of solid side walls. (As shown in ref. 4 the effect of solid side walls can be considerable.)

Large spanwise variations of the upwash interference factor are to be expected only when the wing tips approach the side walls. Since the equations for v_4 and v_5 are somewhat complicated and involve triple integrations, and in consideration of uncertainties in the practical boundary conditions, it is therefore suggested that the upwash interference factor for square perforated-wall test sections should be estimated by obtaining the interference at the center of the span by the circular-tunnel comparison method and superposing the spanwise variation due to infinite side walls. This procedure should yield reasonably accurate spanwise variation of δ provided the interference is not dominated by the top and bottom walls. The estimation is in most cases relatively easy, since equations (7) and (A14) are rapidly convergent for R not either extremely large or very close to zero and can readily be programed for numerical integration by means of high-speed digital computing systems.

Data for such estimations for the square perforated-wall test section with a permeability factor of 0.6 are available in the last column of table II and in the fourth column of table III. Thus at a Mach number of 0,

δ at $y/h = 0$ (estimated by circular-tunnel comparison method)		0.032
δ_2 at $y/h = 0.7$ (from table III)	0.07657	
δ_2 at $y/h = 0$ (from table III)	0.04718	
Difference	0.02939	0.029
δ at $y/h = 0.7$		0.061

Such estimates have been made at Mach numbers of 0 and 0.995 for the spanwise positions for which data are given in table III. The estimated spanwise distributions are shown in figure 4, where also are given for comparison the spanwise distributions (last column of table III) obtained by the approximation procedure. The relatively poorer estimate at Mach number 0.995 is connected with the fact that in this case the contribution of the top and bottom walls to the total interference is several times that of the side walls. This situation might have been detected by noting that δ_2 at $y/h = 0$ is only about a fourth of -0.065, the total upwash interference factor estimated by the circular-tunnel comparison method. However, failure to predict the correct spanwise variation is in this case of little consequence, because the total variation is too small to be of much significance. There is reason to believe that, at least in any practical test-section configuration, any large spanwise variation of the upwash interference would be satisfactorily estimated. There is also reason to believe that in figure 4 the true spanwise variation of δ at $M = 0.995$ lies between that given by the estimation method and that given by the approximation procedure.

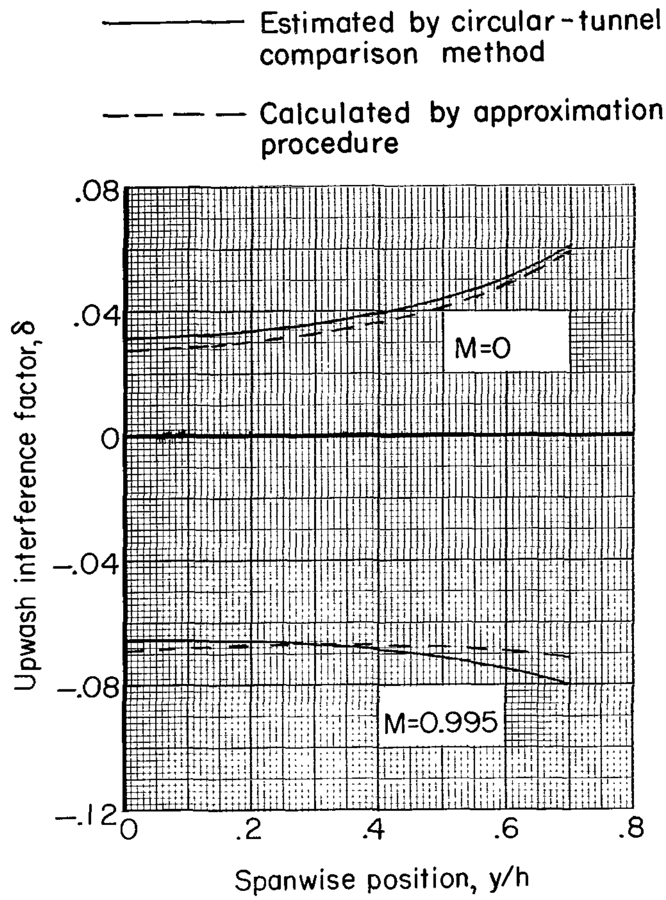

Figure 4.- Comparison of estimated interference factors δ with those calculated by approximation procedure. $b/h = 1$; $s/h = 0.7$.

REMARKS ON PRACTICAL APPLICATION

The reader is cautioned against a naive application of the methods of this paper to the estimation of wind-tunnel-boundary lift interference in a given perforated-wall test section. Although the approximation procedure herein presented and even the suggested estimation method are believed to be quite adequate, the boundary conditions in any given test section are far from certain. It has already been noted that with constant permeability factor R the upwash interference varies strongly with Mach number but there is no reason to suppose that R may be considered constant. In fact, there is reason to believe that with a given test-section-wall configuration R may vary, perhaps separately, with Reynolds number, Mach number, velocity, and density. Moreover, the effective value of R is known to depend on the boundary-layer thickness. It may therefore be expected to vary spatially over the walls and serially with the test model configuration. Partly because of inadequate control of variables and inadequate attention to the problem of scaling, the available investigations of perforated material are believed to be unreliable for determining effective values of R for tests of models in perforated-wall test sections. Even if the required data were available, use of these data would require knowledge and control of test-section conditions such as boundary-layer distribution as related to model configuration (including lift) and removal of boundary layer by suction through the perforated walls. Perhaps the best procedure is to estimate ranges of effective average values of permeability factor R. Some discussion of the behavior of perforated walls is given in reference 6 along with citations of original works. Boundary-layer effects on the behavior of perforated walls in wind tunnels are discussed in reference 7. Because of the variation of the upwash interference with Mach number and of the boundary-layer effect on the permeability constant, the boundary interference in perforated-wall test sections at high Mach numbers is likely to be of the nature of that in open-throat tunnels. Some experimental evidence from North American Aviation, Inc., which is unavailable for reference, supports this assertion.

RÉSUMÉ

An approximation method has been developed for calculating the spanwise distribution of wind-tunnel-boundary upwash interference on lift of wings in rectangular perforated-wall test sections. In this approximation the lifting wing is represented as a horseshoe vortex of finite span. Suggested for the square test section is a method of estimating the upwash interference factor at a small-span wing by a comparison with the circular-test-section interference factor. Close agreement between interference factors obtained by the approximation method and the known values for open and for closed test sections gives confidence in the adequacy of the approximation method. This confidence is strengthened by close agreement between approximated values and those obtained by the circular-tunnel comparison method. A simplified estimation method is suggested by which the spanwise variation, taken as that due to infinite parallel side walls, is superposed on the interference at the center of the span. These methods are applied to square test sections with an assumed effective permeability constant of 0.6. A problem of considerable difficulty in practical application of the methods presented is the estimation of an effective permeability constant. Because of the variation of the upwash interference with Mach number and of the influence of boundary layer on the effective permeability factor, the wind-tunnel-boundary interference in a perforated-wall test section at high subsonic Mach numbers is likely to be of the nature of that in an open-throat tunnel.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., December 12, 1967,
126-13-01-54-23.

APPENDIX A

INTERFERENCE ON HORIZONTAL LIFTING WING CENTERED BETWEEN TWO PARALLEL VERTICAL PERFORATED INFINITE BOUNDARIES

Represent the wing by a horseshoe vortex with bound vortex extending from $-s$ to s along the Y-axis and with trailing vortices extending to $x \rightarrow \infty$ as indicated in figure 1. Let the perforated boundaries with permeability factor R_2 be at $y = b$ and at $y = -b$. Let M be Mach number, Γ be the circulation, and $\beta = \sqrt{1 - M^2}$. The free-air potential of the horseshoe vortex is then

$$\phi_1 = \frac{\Gamma}{4\pi} \int_{-s}^{s} \left\{1 + \frac{x}{\sqrt{x^2 + \beta^2\left[(y - \eta)^2 + z^2\right]}}\right\} \frac{z}{(y - \eta)^2 + z^2} d\eta \tag{A1}$$

An additional potential ϕ_2, such that $\left(\phi_1 + \phi_2\right)$ satisfies the boundary condition for potential ϕ

$$\frac{\partial \phi}{\partial x} \pm \frac{1}{R_2} \frac{\partial \phi}{\partial y} = 0 \tag{A2}$$

at the boundaries $y = +b$ and $y = -b$, is to be found. This potential, like ϕ_1, must satisfy the Laplace equation for potential ϕ

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{A3}$$

To facilitate the determination of ϕ_2, take exponential Fourier transforms on x and z with variables of tranformation ω and g, respectively; thus,

$$\Omega(\omega,y,z) = \int_{-\infty}^{\infty} e^{-i\omega x}\phi(x,y,z)dx \tag{A4}$$

and

$$G(\omega,y,g) = \int_{-\infty}^{\infty} e^{-igz}\Omega(\omega,y,z)dz \tag{A5}$$

By means of formula (11) on page 118 of reference 8, the Laplace equation (A3) transforms to

$$\frac{\partial^2 G(\omega,y,g)}{\partial y^2} = \left(\beta^2\omega^2 + g^2\right) G(\omega,y,g) \tag{A6}$$

A solution of equation (A6) is

$$G(\omega,y,g) = A(\omega,g)\sinh\left(y\sqrt{\beta^2\omega^2 + g^2}\right) + B(\omega,g)\cosh\left(y\sqrt{\beta^2\omega^2 + g^2}\right)$$

APPENDIX A

Because of the symmetry of the physical configuration, the interference velocities as well as the disturbance velocities in the y-direction must be antisymmetrical about $y = 0$. It follows that G_2 corresponding to the interference potential ϕ_2 is of the form

$$G_2(\omega,y,g) = B_2(\omega,g)\cosh\left(y\sqrt{\beta^2\omega^2 + g^2}\right) \tag{A7}$$

The boundary condition (A2) transforms to

$$i\omega G(\omega,y,g) \pm \frac{1}{R_2}\frac{\partial G(\omega,y,g)}{\partial y} = 0 \tag{A8}$$

The sum $\left(G_1 + G_2\right)$ corresponding to $\left(\phi_1 + \phi_2\right)$ must satisfy equation (A8).

Since ϕ_1 does not meet the integrability condition for transformation, use instead (as in ref. 2)

$$\phi_1' = \frac{\Gamma}{4\pi}\int_{-s}^{s}\left(1 - \frac{1}{\alpha}\frac{\partial}{\partial x}\right)\frac{ze^{-\alpha\sqrt{x^2+\beta^2\left[(y-\eta)^2+z^2\right]}}}{(y - \eta)^2 + z^2}\,d\eta \tag{A9}$$

where $\alpha > 0$ and $\phi_1' \rightarrow \phi_1$ as $\alpha \rightarrow 0$. By means of formula (26) on page 16 and formula (11) on page 118 of reference 8, equation (A9) transforms to

$$\Omega_1'(\omega,y,z) = \frac{\Gamma\beta(\alpha - i\omega)}{2\pi\sqrt{\alpha^2 + \omega^2}}\int_{-s}^{s}\frac{zK_1\left\{\beta\sqrt{\left[(y - \eta)^2 + z^2\right]\left(\alpha^2 + \omega^2\right)}\right\}}{\sqrt{(y - \eta)^2 + z^2}}\,d\eta \qquad \begin{pmatrix} \alpha > 0 \\ \beta\sqrt{(y - \eta)^2 + z^2} > 0 \end{pmatrix}$$

(A10)

where K_1 is a modified Bessel function of the second kind.

By using formula (43) on page 112 of reference 8, the transform on z in the vicinity of the boundary $y = \pm b$ where $|y| > |\eta|$ is

$$G_1'(\omega,y,g) = -\frac{i\Gamma}{2}\frac{g(\alpha - i\omega)}{\alpha^2 + \omega^2}\int_{-s}^{s}\frac{e^{-|y-\eta|\sqrt{\beta^2\left(\alpha^2+\omega^2\right)+g^2}}}{\sqrt{\beta^2\left(\alpha^2 + \omega^2\right) + g^2}}\,d\eta \qquad \begin{pmatrix} |y| > s \\ \alpha > 0 \end{pmatrix} \tag{A11}$$

$$G_1'(\omega,y,g) = -\frac{i\Gamma g(\alpha - i\omega)}{c_g^2\left(\alpha^2 + \omega^2\right)}e^{-|y|c_g}\sinh(sc_g) \qquad \begin{pmatrix} |y| > s \\ \alpha > 0 \end{pmatrix} \tag{A12}$$

where

$$c_g = \sqrt{\beta^2\left(\alpha^2 + \omega^2\right) + g^2}$$

Use of $\left(G_1' + G_2\right)$ for G in the boundary condition (A8) at the boundary $y = b$ now gives, with $D_g = \sqrt{\beta^2\omega^2 + g^2}$,

$$i\omega B_2(\omega,g)\cosh\left(D_g b\right) + \frac{D_g}{R_2} B_2(\omega,g)\sinh\left(D_g b\right)$$

$$= -\frac{\omega\Gamma g(\alpha - i\omega)}{c_g{}^2\left(\alpha^2 + \omega^2\right)} e^{-bc_g}\sinh\left(sc_g\right) - \frac{i\Gamma g(\alpha - i\omega)}{R_2 c_g\left(\alpha^2 + \omega^2\right)} e^{-bc_g}\sinh\left(sc_g\right)$$

$$= -\frac{\Gamma g(\alpha - i\omega)}{c_g\left(\alpha^2 + \omega^2\right)}\left(\frac{\omega}{c_g} + \frac{i}{R_2}\right)e^{-bc_g}\sinh\left(sc_g\right)$$

This same equation holds also at the boundary $y = -b$, so that

$$B_2(\omega,g) = -\frac{\Gamma g(\alpha - i\omega)}{c_g\left(\alpha^2 + \omega^2\right)}\frac{\left(\frac{\omega}{c_g} + \frac{i}{R_2}\right)e^{-bc_g}\sinh\left(sc_g\right)}{\frac{D_g}{R_2}\sinh\left(D_g b\right) + i\omega\cosh\left(D_g b\right)}$$

$$= -\frac{\Gamma g\left\{\left(\frac{\omega\alpha D_g}{R_2 c_g} + \frac{\omega D_g}{R_2{}^2}\right)\sinh\left(D_g b\right) + \left(\frac{\omega\alpha}{R_2} - \frac{\omega^3}{c_g}\right)\cosh\left(D_g b\right) + i\left[\left(\frac{\alpha D_g}{R_2{}^2} - \frac{\omega^2 D_g}{R_2 c_g}\right)\sinh\left(D_g b\right) - \left(\frac{\omega^2\alpha}{c_g} + \frac{\omega^2}{R_2}\right)\cosh\left(D_g b\right)\right]\right\} e^{-bc_g}\sinh\left(sc_g\right)}{c_g\left(\alpha^2 + \omega^2\right)\left[\frac{D_g{}^2}{R_2{}^2}\sinh^2\left(D_g b\right) + \omega^2\cosh^2\left(D_g b\right)\right]}$$

Therefore, on inversion of G_2 of equation (A7)

$$\phi_2 = \lim_{\alpha\to 0}\frac{-\Gamma}{4\pi^2}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{i(x\omega+zg)}g\left[\left(\frac{\omega\alpha D_g}{R_2 c_g} + \frac{\omega D_g}{R_2{}^2}\right)\sinh\left(D_g b\right) + \left(\frac{\omega\alpha}{R_2} - \frac{\omega^3}{c_g}\right)\cosh\left(D_g b\right)\right] e^{-bc_g}\sinh\left(sc_g\right)\cosh\left(D_g y\right)}{c_g\left(\alpha^2 + \omega^2\right)\left[\frac{D_g{}^2}{R_2{}^2}\sinh^2\left(D_g b\right) + \omega^2\cosh^2\left(D_g b\right)\right]}\,d\omega\,dg\right.$$

$$\left. + i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{i(x\omega+zg)}g\left[\left(\frac{\alpha D_g}{R_2{}^2} - \frac{\omega^2 D_g}{R_2 c_g}\right)\sinh\left(D_g b\right) - \left(\frac{\omega^2\alpha}{c_g} + \frac{\omega^2}{R_2}\right)\cosh\left(D_g b\right)\right] e^{-bc_g}\sinh\left(sc_g\right)\cosh\left(D_g y\right)}{c_g\left(\alpha^2 + \omega^2\right)\left[\frac{D_g{}^2}{R_2{}^2}\sinh^2\left(D_g b\right) + \omega^2\cosh^2\left(D_g b\right)\right]}\,d\omega\,dg\right\} \qquad \textbf{(A13)}$$

and the upwash velocity $v_2 = \frac{\partial \phi_2}{\partial z}$ at $x = z = 0$ is

$$v_2 = \lim_{\alpha \to 0} \frac{-\Gamma}{4\pi^2} \left\{ i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{g^2 \left[\left(\frac{\omega \alpha D_g}{R_2 c_g} + \frac{\omega D_g}{R_2^2} \right) \sinh(D_g b) + \left(\frac{\omega \alpha}{R_2} - \frac{\omega^3}{c_g} \right) \cosh(D_g b) \right] e^{-bc_g} \sinh(sc_g) \cosh(D_g y)}{c_g(\alpha^2 + \omega^2) \left[\frac{D_g^2}{R_2^2} \sinh^2(D_g b) + \omega^2 \cosh^2(D_g b) \right]} \, d\omega \, dg \right.$$

$$\left. - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{g^2 \left[\left(\frac{\alpha D_g}{R_2^2} - \frac{\omega^2 D_g}{R_2 c_g} \right) \sinh(D_g b) - \left(\frac{\omega^2 \alpha}{c_g} + \frac{\omega^2}{R_2} \right) \cosh(D_g b) \right] e^{-bc_g} \sinh(sc_g) \cosh(D_g y)}{c_g(\alpha^2 + \omega^2) \left[\frac{D_g^2}{R_2^2} \sinh^2(D_g b) + \omega^2 \cosh^2(D_g b) \right]} \, d\omega \, dg \right\}$$

Because c_g and D_g are even in ω, the integrand of the first of these double integrals is odd in ω and the integral over ω is therefore zero. To evaluate the second double integral for $\alpha \to 0$, note that terms of the integrand containing α in the numerator contribute to the integral only for $\omega \to 0$. It follows that

$$v_2 = \lim_{\alpha \to 0} \frac{\Gamma}{4\pi^2} \int_{-\infty}^{\infty} dg \left\{ e^{-|g|b} \frac{\sinh(gs)\cosh(gy)}{\sinh(gb)} \, \alpha \int_{-\infty}^{\infty} \frac{d\omega}{\alpha^2 + \omega^2} \right.$$

$$\left. - \frac{1}{R_2} \int_{-\infty}^{\infty} \frac{g^2 \sinh\left(s\sqrt{\beta^2\omega^2 + g^2}\right) \cosh\left(y\sqrt{\beta^2\omega^2 + g^2}\right)}{\sqrt{\beta^2\omega^2 + g^2} \left[\frac{\beta^2\omega^2 + g^2}{R_2^2} \sinh^2\left(b\sqrt{\beta^2\omega^2 + g^2}\right) + \omega^2 \cosh^2\left(b\sqrt{\beta^2\omega^2 + g^2}\right) \right]} \, d\omega \right\}$$

Evaluating the first integral in braces, multiplying by h/Γ, and writing p for ωh and q for gh gives

$$\frac{h}{\Gamma} v_2 = \frac{1}{2\pi^2} \int_0^\infty dq \left\{ \frac{\pi e^{-\frac{b}{h}q} \sinh\left(\frac{s}{h} q\right) \cosh\left(\frac{y}{h} q\right)}{\sinh\left(\frac{b}{h} q\right)} \right.$$

$$\left. - \frac{2q^2}{R_2} \int_0^\infty \frac{\sinh\left(\frac{s}{h}\sqrt{\beta^2 p^2 + q^2}\right) \cosh\left(\frac{y}{h}\sqrt{\beta^2 p^2 + q^2}\right)}{\sqrt{\beta^2 p^2 + q^2}\left[\frac{\beta^2 p^2 + q^2}{R_2^2} \sinh^2\left(\frac{b}{h}\sqrt{\beta^2 p^2 + q^2}\right) + p^2 \cosh^2\left(\frac{b}{h}\sqrt{\beta^2 p^2 + q^2}\right)\right]} dp \right\}$$

(A14)

This expression may be verified as correct at $y = 0$ and $s \to 0$ for the closed boundary $(R_2 \to 0)$ and for the open boundary $(R_2 \to \infty)$, for which the method of images in the Trefftz plane applies. For the closed boundary the second term within the braces – that is, the term containing the integral – is zero; for the open boundary $(R_2 \to \infty)$ the term must be replaced with

$$-2q \int_0^\infty \frac{\sinh\left(\frac{s}{h} q\right) \cosh\left(\frac{y}{h} q\right)}{q^2 \sinh^2\left(\frac{b}{h} q\right) + p^2 \cosh^2\left(\frac{b}{h} q\right)} dp$$

This expression is obtained by multiplying numerator and denominator by R_2 and replacing p with zero unless it occurs as a product with R_2, in which case $R_2 p$ is replaced with p. This process is equivalent to taking the limit as $(R_2 \to \infty)$. Integration with respect to p gives

$$-\frac{\pi \sinh\left(\frac{s}{h} q\right) \cosh\left(\frac{y}{h} q\right)}{\sinh\left(\frac{b}{h} q\right) \cosh\left(\frac{b}{h} q\right)}$$

which combines with the first term within the braces so that for the open-throat tunnel $(R_2 \to \infty)$ equation (A14) may be written

$$\frac{h}{\Gamma} v_2 = -\frac{1}{2\pi} \int_0^\infty \frac{e^{-\frac{b}{h}q} \sinh\left(\frac{s}{h} q\right) \cosh\left(\frac{y}{h} q\right)}{\cosh\left(\frac{b}{h} q\right)} dq \qquad (R_2 \to \infty) \quad \text{(A15)}$$

APPENDIX B

INTERFERENCE ON HORIZONTAL LIFTING WING CENTERED BETWEEN TWO PARALLEL HORIZONTAL PERFORATED INFINITE BOUNDARIES

Let the boundaries be at $z = h$ and at $z = -h$ as indicated in figure 2. Then, as for vertical boundaries

$$\phi_1 = \frac{\Gamma}{4\pi}\int_{-s}^{s}\left(1 + \frac{x}{\sqrt{x^2 + \beta^2\left[(y - \eta)^2 + z^2\right]}}\right)\frac{z}{(y - \eta)^2 + z^2}\,d\eta \tag{B1}$$

Take exponential integral transforms on x and y with variables of transformation ω and f, where the transform on x is given by equation (A4) and the transform on y is

$$F(\omega,f,z) = \int_{-\infty}^{\infty} e^{-ify}\,\Omega(\omega,y,z)\,dy \tag{B2}$$

The Laplace equation (A3) then transforms to

$$\frac{\partial^2 F(\omega,f,z)}{\partial z^2} = \left(\beta^2\omega^2 + f^2\right)F(\omega,f,z) \tag{B3}$$

The boundary condition for the horizontal perforated boundaries is

$$\frac{\partial\phi}{\partial x} \pm \frac{1}{R_3}\frac{\partial\phi}{\partial z} = 0 \tag{B4}$$

where R_3 is the permeability factor for the horizontal boundaries. The positive sign applies at the boundary $z = h$ and the negative sign applies at $z = -h$. The relation (B4) transforms to

$$i\omega F \pm \frac{1}{R_3}\frac{\partial F}{\partial z} = 0 \tag{B5}$$

An appropriate (antisymmetric) solution of equation (B3) is

$$F_3(\omega,f,z) = B_3(\omega,f)\sinh\left(z\sqrt{\beta^2\omega^2 + f^2}\right) \tag{B6}$$

For the transformation of ϕ_1 use ϕ_1' as for the vertical boundaries. Then, as in appendix A (eq. (A10))

$$\Omega_1'(\omega,y,z) = \frac{\Gamma\beta(\alpha - i\omega)}{2\pi\sqrt{\alpha^2 + \omega^2}}\int_{-s}^{s}\frac{zK_1\left\{\beta\sqrt{\left[(y - \eta)^2 + z^2\right]\left(\alpha^2 + \omega^2\right)}\right\}}{\sqrt{(y - \eta)^2 + z^2}}\,d\eta \qquad \left(\begin{matrix} \alpha > 0 \\ \beta\sqrt{(y - \eta)^2 + z^2} > 0\end{matrix}\right)$$

APPENDIX B

By using formula (44) on page 56 of reference 8, the transform on y in the vicinity of the boundary $z = \pm h$ is

$$F_1'(\omega,f,z) = \frac{\Gamma\beta(\alpha - i\omega)}{2\pi\sqrt{\alpha^2 + \omega^2}} \frac{\pi z e^{-|z|\sqrt{f^2+\beta^2(\alpha^2+\omega^2)}}}{\beta\sqrt{\alpha^2 + \omega^2}|z|} \int_{-s}^{s} e^{-if\eta}\, d\eta \qquad \begin{pmatrix} \alpha > 0 \\ |z| > 0 \end{pmatrix}$$

$$= \frac{\Gamma(\alpha - i\omega)}{\alpha^2 + \omega^2} \frac{z}{|z|} e^{-|z|\sqrt{f^2+\beta^2(\alpha^2+\omega^2)}} \frac{\sin(fs)}{f} \qquad \begin{pmatrix} \alpha > 0 \\ |z| > 0 \end{pmatrix} \qquad \text{(B7)}$$

Now $\left(F_1' + F_3\right)$ must satisfy the boundary condition (B5), and it follows that at the upper boundary $z = h$

$$i\omega B_3 \sinh\left(D_f h\right) + \frac{D_f B_3}{R_3} \cosh\left(D_f h\right) = -\frac{i\omega\Gamma(\alpha - i\omega)}{\alpha^2 + \omega^2} e^{-hc_f} \frac{\sin(fs)}{f} + \frac{c_f}{R_3} \frac{\Gamma(\alpha - i\omega)}{\alpha^2 + \omega^2} e^{-hc_f} \frac{\sin(fs)}{f} \qquad \text{(B8)}$$

where

$$c_f = \sqrt{\beta^2\left(\alpha^2 + \omega^2\right) + f^2}$$

$$D_f = \sqrt{\beta^2\omega^2 + f^2}$$

Equation (B8) holds also at the lower boundary $z = -h$; therefore,

$$B_3(\omega,f) = \frac{\Gamma(\alpha - i\omega)}{\alpha^2 + \omega^2} e^{-hc_f} \frac{\sin(fs)}{f} \frac{\dfrac{c_f}{R_3} - i\omega}{\dfrac{D_f}{R_3} \cosh\left(D_f h\right) + i\omega \sinh\left(D_f h\right)}$$

$$= \frac{\Gamma e^{-hc_f} \sin(fs)}{\left(\alpha^2 + \omega^2\right)\left[\dfrac{D_f^2}{R_3^2} \cosh^2\left(D_f h\right) + \omega^2 \sinh^2\left(D_f h\right)\right] f} \left\{\frac{D_f}{R_3}\left(\frac{\alpha c_f}{R_3} - \omega^2\right)\cosh D_f h\right.$$

$$\left. - \omega^2\left(\frac{c_f}{R_3} + \alpha\right)\sinh\left(D_f h\right) - i\left[\frac{\omega D_f}{R_3}\left(\frac{c_f}{R_3} + \alpha\right)\cosh\left(D_f h\right) + \omega\left(\frac{\alpha c_f}{R_3} - \omega^2\right)\sinh\left(D_f h\right)\right]\right\}$$

Thus, on inversion of F_3 of equation (B6)

$$\phi_3 = \lim_{\alpha \to 0} \frac{\Gamma}{4\pi^2} \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(x\omega+yf)-hc_f}\left[\frac{D_f}{R_3}\left(\frac{\alpha c_f}{R_3} - \omega^2\right)\cosh(D_f h) - \omega^2\left(\frac{c_f}{R_3} + \alpha\right)\sinh(D_f h)\right]}{\left(\alpha^2 + \omega^2\right)\left[\frac{D_f^2}{R_3^2}\cosh^2(D_f h) + \omega^2 \sinh^2(D_f h)\right]} \frac{\sin(fs)}{f} \sinh(D_f z) d\omega \, df \right.$$

$$\left. -i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(x\omega+yf)-hc_f}\left[\frac{\omega D_f}{R_3}\left(\frac{c_f}{R_3} + \alpha\right)\cosh(D_f h) + \omega\left(\frac{\alpha c_f}{R_3} - \omega^2\right)\sinh(D_f h)\right]}{\left(\alpha^2 + \omega^2\right)\left[\frac{D_f^2}{R_3^2}\cosh^2(D_f h) + \omega^2 \sinh^2(D_f h)\right]} \frac{\sin(fs)}{f} \sinh(D_f z) d\omega \, df \right\} \quad \text{(B9)}$$

and the upwash velocity $v_3 = \frac{\partial \phi_3}{\partial z}$ at $x = z = 0$ is (the second double integral is 0 because the integrand is odd in ω)

$$v_3 = \lim_{\alpha \to 0} \frac{\Gamma}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{D_f e^{-hc_f}\left[\frac{D_f}{R_3}\left(\frac{\alpha c_f}{R_3} - \omega^2\right)\cosh(D_f h) - \omega^2\left(\frac{c_f}{R_3} + \alpha\right)\sinh(D_f h)\right]}{\left(\alpha^2 + \omega^2\right)\left[\frac{D_f^2}{R_3^2}\cosh^2(D_f h) + \omega^2 \sinh^2(D_f h)\right]} \frac{\sin(fs)}{f} \cos(fy) d\omega \, df$$

The terms containing α as a factor in the numerator can contribute to the integral only for $\omega \to 0$ and therefore,

$$v_3 = \lim_{\alpha \to 0} \frac{\Gamma}{4\pi^2} \int_{-\infty}^{\infty} df \left[\frac{|f| e^{-h|f|}}{\cosh(hf)} \frac{\sin(fs)}{f} \cos(fy) \int_{-\infty}^{\infty} \frac{\alpha}{\alpha^2 + \omega^2} d\omega \right.$$

$$\left. - \frac{1}{R_3} \frac{\sin(fs)}{f} \cos(fy) \int_{-\infty}^{\infty} \frac{\beta^2\omega^2 + f^2}{\frac{\beta^2\omega^2 + f^2}{R_3^2} \cosh^2\left(h\sqrt{\beta^2\omega^2 + f^2}\right) + \omega^2 \sinh^2\left(h\sqrt{\beta^2\omega^2 + f^2}\right)} d\omega \right]$$

Evaluating the first integral within the brackets, multiplying by h/Γ, and writing p for ωh and r for fh gives

$$\frac{h}{\Gamma} v_3 = \frac{1}{2\pi^2} \int_0^\infty \cos\left(\frac{y}{h} r\right) \left[\frac{\pi e^{-r} \sin\left(\frac{s}{h} r\right)}{\cosh r} \right.$$

$$\left. - \frac{2}{R_3} \frac{\sin\left(\frac{s}{h} r\right)}{r} \int_0^\infty \frac{dp}{\frac{1}{R_3^2} \cosh^2\left(\sqrt{\beta^2 p^2 + r^2}\right) + \frac{p^2}{\beta^2 p^2 + r^2} \sinh^2\left(\sqrt{\beta^2 p^2 + r^2}\right)} \right] dr \qquad \text{(B10)}$$

This expression yields the correct upwash at $y = 0$ and $s \to 0$ for the lifting wing centered in the horizontal plane between two horizontal closed walls $(R_3 \to 0)$ or two horizontal open boundaries $(R_3 \to \infty)$, as may easily be verified by taking the corresponding limits and comparing them with the values obtained by the method of images in the Trefftz plane.

As for equation (A14), the second term within the brackets (the term including the integral) in equation (B10) is zero for the closed-throat tunnel; for the open-throat tunnel $(R_3 \to \infty)$ equation (B10) must be replaced with

$$\frac{h}{\Gamma} v_3 = -\frac{1}{2\pi} \int_0^\infty \frac{e^{-r} \sin\left(\frac{s}{h} r\right) \cos\left(\frac{y}{h} r\right)}{\sinh r} dr \qquad (R_3 \to \infty) \quad \text{(B11)}$$

APPENDIX C

INTERFERENCE OF HORIZONTAL BOUNDARIES ON INTERFERENCE DUE TO VERTICAL BOUNDARIES

From the interference velocity potential ϕ_2 of equation (A13) due to the vertical boundaries, it is easily seen that the corresponding exponential integral transform on x for $\alpha \neq 0$ is

$$\Omega_2'(\omega,y,z) = -\frac{\Gamma}{2\pi}\left[\int_{-\infty}^{\infty} e^{igz}H(\omega,g)\cosh\left(D_g y\right)dg + i\int_{-\infty}^{\infty} e^{igz}N(\omega,g)\cosh\left(D_g y\right)dg\right] \quad \text{(C1)}$$

where

$$H = \frac{g\left[\left(\frac{\omega\alpha D_g}{R_2 c_g} + \frac{\omega D_g}{{R_2}^2}\right)\sinh\left(D_g b\right) + \left(\frac{\omega\alpha}{R_2} - \frac{\omega^3}{c_g}\right)\cosh\left(D_g b\right)\right]e^{-bc_g}\sinh\left(sc_g\right)}{c_g\left(\alpha^2 + \omega^2\right)\left[\frac{{D_g}^2}{{R_2}^2}\sinh^2\left(D_g b\right) + \omega^2\cosh^2\left(D_g b\right)\right]} \quad \text{(C2)}$$

$$N = \frac{g\left[\left(\frac{\alpha D_g}{{R_2}^2} - \frac{\omega^2 D_g}{R_2 c_g}\right)\sinh\left(D_g b\right) - \left(\frac{\omega^2\alpha}{c_g} + \frac{\omega^2}{R_2}\right)\cosh\left(D_g b\right)\right]e^{-bc_g}\sinh\left(sc_g\right)}{c_g\left(\alpha^2 + \omega^2\right)\left[\frac{{D_g}^2}{{R_2}^2}\sinh^2\left(D_g b\right) + \omega^2\cosh^2\left(D_g b\right)\right]} \quad \text{(C3)}$$

and

$$c_g = \sqrt{\beta^2\left(\alpha^2 + \omega^2\right) + g^2}$$

$$D_g = \sqrt{\beta^2\omega^2 + g^2}$$

Let $\Omega_4\left(= \lim_{\alpha\to 0} \Omega_4'\right)$ be the exponential integral transform of ϕ_4, where ϕ_4 is a potential such that $\left(\phi_2 + \phi_4\right)$ satisfies the boundary condition (B4) at the upper and lower perforated boundaries. Equation (B4) transforms on x to

$$i\omega\Omega \pm \frac{1}{R_3}\frac{\partial\Omega}{\partial z} = 0 \quad \text{(C4)}$$

APPENDIX C

Substitution of $\left(\Omega_2' + \Omega_4'\right)$ for Ω in equation (C4) for the boundary condition at $z = h$ gives

$$i\omega\Omega_4' + \frac{1}{R_3}\frac{\partial\Omega_4'}{\partial z} = - i\omega\Omega_2' - \frac{1}{R_3}\frac{\partial\Omega_2'}{\partial z} \qquad (z = h) \quad \text{(C5)}$$

The disturbance represented by equation (C1) is assumed to be effective only on the part of the horizontal boundaries cut off by the side walls (that is, for $-b < y < b$) and is assumed to be zero over the remainder of these boundaries. The exponential integral transform of $\cosh\left(D_g y\right)$ is therefore required for $-b < y < b$. This transform is

$$\int_{-b}^{b} e^{-ify}\cosh\left(D_g y\right)dy = \frac{1}{2}\int_{-b}^{b} e^{-ify}\left(e^{D_g y} + e^{-D_g y}\right)dy = \frac{1}{2}\left[\frac{e^{\left(D_g - if\right)y}}{D_g - if} - \frac{e^{-\left(D_g + if\right)y}}{D_g + if}\right]_{-b}^{b}$$

$$= \frac{1}{2}\left[\frac{e^{\left(D_g - if\right)b} - e^{-\left(D_g - if\right)b}}{D_g - if} - \frac{e^{-\left(D_g - if\right)b} - e^{\left(D_g + if\right)b}}{D_g + if}\right]$$

$$= \frac{\sinh\left[\left(D_g - if\right)b\right]}{D_g - if} + \frac{\sinh\left[\left(D_g + if\right)b\right]}{D_g + if}$$

The transform of Ω_2' on y is, therefore,

$$F_2'(\omega,f,z) = - \frac{\Gamma}{2\pi}\left(\int_{-\infty}^{\infty} e^{igz}H(\omega,g)\left\{\frac{\sinh\left[\left(D_g - if\right)b\right]}{D_g - if} + \frac{\sinh\left[\left(D_g + if\right)b\right]}{D_g + if}\right\}dg\right.$$

$$\left. + i\int_{-\infty}^{\infty} e^{igz}N(\omega,g)\left\{\frac{\sinh\left[\left(D_g - if\right)b\right]}{D_g - if} + \frac{\sinh\left[\left(D_g + if\right)b\right]}{D_g + if}\right\}dg\right) \qquad \text{(C6)}$$

The transform F_4' of Ω_4' on y must satisfy the transformed Laplace equation (B3) of which the appropriate antisymmetric solution is of the form of equation (B6) as follows:

$$F_4'(\omega,f,z) = B_4'(\omega,f)\sinh\left(z\sqrt{\beta^2\omega^2 + f^2}\right) \qquad \text{(C7)}$$

Equation (C5) for the boundary condition transforms to

$$i\omega F_4' + \frac{1}{R_3}\frac{\partial F_4'}{\partial z} = -i\omega F_2' - \frac{1}{R_3}\frac{\partial F_2'}{\partial z}\Bigg]_{z=h} \tag{C8}$$

Substitution of equations (C6) and (C7) into equation (C8) gives

$$i\omega B_4' \sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right) + \frac{B_4'}{R_3}\sqrt{\beta^2\omega^2 + f^2}\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)$$

$$= \frac{i\Gamma}{2\pi}\int_{-\infty}^{\infty} e^{igh}(H + iN)\left(\omega + \frac{g}{R_3}\right)\left\{\frac{\sinh\left[\left(D_g - if\right)b\right]}{D_g - if} + \frac{\sinh\left[\left(D_g + if\right)b\right]}{D_g + if}\right\}dg$$

Solution for B_4' gives

$$B_4' = \frac{i\Gamma}{2\pi}\int_{-\infty}^{\infty}\frac{e^{igh}(H + iN)\left(\omega + \frac{g}{R_3}\right)\left\{\frac{\sinh\left[\left(D_g - if\right)b\right]}{D_g - if} + \frac{\sinh\left[\left(D_g + if\right)b\right]}{D_g + if}\right\}}{i\omega \sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right) + \frac{\sqrt{\beta^2\omega^2 + f^2}}{R_3}\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}dg$$

Use of solution B_4' in equation (C7), inversion on y and on x in order, and passage to the limit as $\alpha \rightarrow 0$ gives the potential

$$\phi_4 = \lim_{\alpha\to 0}\frac{i\Gamma}{8\pi^3}\int_{-\infty}^{\infty} e^{i\omega x}d\omega\int_{-\infty}^{\infty} e^{ify}df\int_{-\infty}^{\infty}\frac{e^{igh}(H + iN)\left(\omega + \frac{g}{R_3}\right)\left\{\frac{\sinh\left[\left(D_g - if\right)b\right]}{D_g - if} + \frac{\sinh\left[\left(D_g + if\right)b\right]}{D_g + if}\right\}\sinh\left(z\sqrt{\beta^2\omega^2 + f^2}\right)}{i\omega \sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right) + \frac{\sqrt{\beta^2\omega^2 + f^2}}{R_3}\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}dg$$

The upwash interference velocity $v_4 = \frac{\partial\phi_4}{\partial z}$ at $x = z = 0$ is, therefore,

$$v_4 = \frac{i\Gamma}{8\pi^3}\int_{-\infty}^{\infty} d\omega\int_{-\infty}^{\infty} e^{ify}df \lim_{\alpha\to 0}\int_{-\infty}^{\infty}\frac{e^{igh}(H + iN)\left(\omega + \frac{g}{R_3}\right)\left\{\frac{\sinh\left[\left(D_g - if\right)b\right]}{D_g - if} + \frac{\sinh\left[\left(D_g + if\right)b\right]}{D_g + if}\right\}}{\frac{i\omega \sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}{\sqrt{\beta^2\omega^2 + f^2}} + \frac{1}{R_3}\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}dg \tag{C9}$$

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As before with $\alpha \to 0$, terms containing α as a factor in the numerator can make a contribution only for $\omega \to 0$. On inserting the values of H and N (eqs. (C2) and (C3)) and with $c_g = \sqrt{\beta^2(\alpha^2 + \omega^2) + g^2}$ and $D_g = \sqrt{\beta^2\omega^2 + g^2}$, equation (C9) can be written

$$
v_4 = \lim_{\alpha \to 0} \frac{i\Gamma}{8\pi^3} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} e^{ify} df \int_{-\infty}^{\infty} \left\{ \frac{g\left[\left(\frac{\omega\alpha D_g}{R_2 c_g} + \frac{\omega D_g}{R_2^2}\right)\sinh(D_g b) + \left(\frac{\omega\alpha}{R_2} - \frac{\omega^3}{c_g}\right)\cosh(D_g b)\right] e^{-bc_g}\sinh(sc_g)}{c_g(\alpha^2 + \omega^2)\left[\frac{D_g^2}{R_2^2}\sinh^2(D_g b) + \omega^2\cosh^2(D_g b)\right]} \right.
$$

$$
\left. + \frac{ig\left[\left(\frac{\alpha D_g}{R_2^2} - \frac{\omega^2 D_g}{R_2 c_g}\right)\sinh(D_g b) - \left(\frac{\omega^2\alpha}{c_g} + \frac{\omega^2}{R_2}\right)\cosh(D_g b)\right] e^{-bc_g}\sinh(sc_g)}{c_g(\alpha^2 + \omega^2)\left[\frac{D_g^2}{R_2^2}\sinh^2(D_g b) + \omega^2\cosh^2(D_g b)\right]} \right\} \frac{e^{igh}\left(\omega + \frac{g}{R_3}\right)\left\{\frac{\sinh\left[(D_g - if)b\right]}{D_g - if} + \frac{\sinh\left[(D_g + if)b\right]}{D_g + if}\right\}}{i\omega \frac{\sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}{\sqrt{\beta^2\omega^2 + f^2}} + \frac{1}{R_3}\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)} dg
$$

$$
= \frac{i\Gamma}{8\pi^3} \int_{-\infty}^{\infty} e^{ify} df \int_{-\infty}^{\infty} e^{igh} dg \left(\frac{\frac{ig|g|}{R_2^2}\sinh(|g|b)e^{-b|g|}\sinh(|g|s)}{\frac{g^2|g|}{R_2^2}\sinh^2(|g|b)} \; \frac{g\left\{\frac{\sinh\left[(|g| - if)b\right]}{|g| - if} + \frac{\sinh\left[(|g| + if)b\right]}{|g| + if}\right\}}{\cosh(h|f|)} \lim_{\alpha \to 0} \int_{-\infty}^{\infty} \frac{\alpha}{\alpha^2 + \omega^2} d\omega \right.
$$

$$
\left. + \int_{-\infty}^{\infty} d\omega \left\{ \frac{\frac{g}{\omega R_2^2}\sinh(D_g b) - \frac{\omega g}{D_g^2}\cosh(D_g b) - i\left[\frac{g}{R_2 D_g}\sinh(D_g b) + \frac{g}{R_2 D_g}\cosh(D_g b)\right]}{\frac{D_g^2}{R_2^2}\sinh^2(D_g b) + \omega^2\cosh^2(D_g b)} \right\} \frac{e^{-bD_g}\sinh(sD_g)\left(\omega + \frac{g}{R_3}\right)\left\{\frac{\sinh\left[(D_g - if)b\right]}{D_g - if} + \frac{\sinh\left[(D_g + if)b\right]}{D_g + if}\right\}}{i\omega \frac{\sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}{\sqrt{\beta^2\omega^2 + f^2}} + \frac{1}{R_3}\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)} \right)
$$

$$
= -\frac{\Gamma}{4\pi^2} \int_{-\infty}^{\infty} e^{ify} df \int_0^{\infty} \cos(gh) dg \left\{ e^{-b|g|}\frac{\sinh(|g|s)}{\sinh(|g|b)} \; \frac{\frac{\sinh\left[(|g| - if)b\right]}{|g| - if} + \frac{\sinh\left[(|g| + if)b\right]}{|g| + if}}{\cosh(h|f|)} \right\}
$$

$$
+ \frac{i\Gamma}{4\pi^3} \int_{-\infty}^{\infty} e^{ify} df \int_{-\infty}^{\infty} d\omega \int_0^{\infty} \left\{ \frac{\frac{\sinh(D_g b)}{\omega R_2^2} - \frac{\omega}{D_g^2}\cosh(D_g b) - i\left[\frac{\sinh(D_g b)}{R_2 D_g} + \frac{\cosh(D_g b)}{R_2 D_g}\right]}{\frac{D_g^2}{R_2^2}\sinh^2(D_g b) + \omega^2\cosh^2(D_g b)} \right\} \frac{e^{-bD_g}\sinh(sD_g)\left\{\frac{\sinh\left[(D_g - if)b\right]}{D_g - if} + \frac{\sinh\left[(D_g + if)b\right]}{D_g + if}\right\}\left[i\omega g\sin(gh) + \frac{g^2}{R_3}\cos(gh)\right]}{i\omega \frac{\sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}{\sqrt{\beta^2\omega^2 + f^2}} + \frac{1}{R_3}\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)} dg
$$

$$
= -\frac{\Gamma}{4\pi^2} \int_{-\infty}^{\infty} e^{ify} df \int_0^{\infty} \cos(gh) dg \left\{ e^{-bg}\frac{\sinh(sg)}{\sinh(bg)} \; \frac{\frac{\sinh\left[(g - if)b\right]}{g - if} + \frac{\sinh\left[(g + if)b\right]}{g + if}}{\cosh(hf)} \right\}
$$

$$
- \frac{\Gamma}{2\pi^3} \int_{-\infty}^{\infty} e^{ify} df \int_0^{\infty} d\omega \int_0^{\infty} dg \left(\frac{e^{-bD_g}\sinh(sD_g)\left\{\frac{\sinh\left[(D_g - if)b\right]}{D_g - if} + \frac{\sinh\left[(D_g + if)b\right]}{D_g + if}\right\}\left\{\left[\frac{\sinh(D_g b)}{R_2^2} - \frac{\omega^2\cosh(D_g b)}{D_g^2}\right]\left[\frac{g}{R_3}\sin(gh)\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right) - \frac{g^2\cos(gh)\sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}{R_3\sqrt{\beta^2\omega^2 + f^2}}\right] - \left[\frac{\sinh(D_g b)}{R_2 D_g} + \frac{\cosh(D_g b)}{R_2 D_g}\right]\left[\omega^2 g\frac{\sin(gh)\sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}{\sqrt{\beta^2\omega^2 + f^2}} + \frac{g^2}{R_3^2}\cos(gh)\cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)\right]\right\}}{\left[\frac{D_g^2}{R_2^2}\sinh^2(D_g b) + \omega^2\cosh^2(D_g b)\right]\left[\omega^2\frac{\sinh^2\left(h\sqrt{\beta^2\omega^2 + f^2}\right)}{\beta^2\omega^2 + f^2} + \frac{1}{R_3^2}\cosh^2\left(h\sqrt{\beta^2\omega^2 + f^2}\right)\right]} \right)
$$

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$$v_4 = -\frac{\Gamma}{\pi^2}\int_0^\infty \cos(fy)\,df \int_0^\infty \cos(gh)\,dg \left\{\frac{e^{-bg}\sinh(sg)\left[g\sinh(bg)\cos(bf) + f\cosh(bg)\sin(bf)\right]}{\left(g^2+f^2\right)\sinh(bg)\cosh(hf)}\right\}$$

$$-\frac{2\Gamma}{\pi^3}\int_0^\infty \cos(fy)\,df \int_0^\infty d\omega \int_0^\infty dg \frac{\left(\begin{aligned}&e^{-bD_g}\sinh(sD_g)\left[D_g\sinh(bD_g)\cos(bf) + f\cosh(bD_g)\sin(bf)\right]\left\{\left[\frac{\sinh(D_g b)}{R_2^2} - \frac{\omega^2\cosh(D_g b)}{D_g^2}\right]\left[\frac{g}{R_3}\sin(gh)\cosh\left(h\sqrt{\beta^2\omega^2+f^2}\right) - \frac{g^2\cos(gh)\sinh\left(h\sqrt{\beta^2\omega^2+f^2}\right)}{R_3\sqrt{\beta^2\omega^2+f^2}}\right]\right.\\ &\left. - \left[\frac{\sinh(D_g b) + \cosh(D_g b)}{R_2 D_g}\right]\left[\frac{\omega^2 g\sin(gh)\sinh\left(h\sqrt{\beta^2\omega^2+f^2}\right)}{\sqrt{\beta^2\omega^2+f^2}} + \frac{g^2}{R_3^2}\cos(gh)\cosh\left(h\sqrt{\beta^2\omega^2+f^2}\right)\right]\right\}\end{aligned}\right)}{\left[\frac{D_g^2}{R_2^2}\sinh^2(D_g b) + \omega^2\cosh^2(D_g b)\right]\left[\frac{\omega^2\sinh^2\left(h\sqrt{\beta^2\omega^2+f^2}\right)}{\beta^2\omega^2+f^2} + \frac{1}{R_3^2}\cosh^2\left(h\sqrt{\beta^2\omega^2+f^2}\right)\right]\left(D_g^2+f^2\right)}$$

With $D_g = \sqrt{\beta^2\omega^2 + g^2}$, $\omega h = p$, $gh = q$, and $fh = r$, this equation may be written

$$\frac{h}{\Gamma}v_4 = -\frac{1}{\pi^2}\int_0^\infty \cos\left(\frac{y}{h}r\right)dr \int_0^\infty \cos(q)\,dq \left\{\frac{e^{-\frac{b}{h}q}\sinh\left(\frac{s}{h}q\right)\left[q\sinh\left(\frac{b}{h}q\right)\cos\left(\frac{b}{h}r\right) + r\cosh\left(\frac{b}{h}q\right)\sin\left(\frac{b}{h}r\right)\right]}{\left(q^2+r^2\right)\sinh\left(\frac{b}{h}q\right)\cosh(r)}\right\}$$

$$+\frac{2}{\pi^3}\int_0^\infty \cos\left(\frac{y}{h}r\right)dr \int_0^\infty dp \int_0^\infty dq \frac{\left(\begin{aligned}&e^{-\frac{b}{h}\sqrt{\beta^2p^2+q^2}}\sinh\left(\frac{s}{h}\sqrt{\beta^2p^2+q^2}\right)\left[\sqrt{\beta^2p^2+q^2}\sinh\left(\frac{b}{h}\sqrt{\beta^2p^2+q^2}\right)\cos\left(\frac{b}{h}r\right) + r\cosh\left(\frac{b}{h}\sqrt{\beta^2p^2+q^2}\right)\sin\left(\frac{b}{h}r\right)\right]\\ &\times\left\{\left[\frac{\sinh\left(\frac{b}{h}\sqrt{\beta^2p^2+q^2}\right) + \cosh\left(\frac{b}{h}\sqrt{\beta^2p^2+q^2}\right)}{R_2\sqrt{\beta^2p^2+q^2}}\right]\left[\frac{p^2 q\sin(q)\sinh\left(\sqrt{\beta^2p^2+r^2}\right)}{\sqrt{\beta^2p^2+r^2}} + \frac{q^2}{R_3^2}\cos(q)\cosh\left(\sqrt{\beta^2p^2+r^2}\right)\right]\right.\\ &\left. - \left[\frac{\sinh\left(\frac{b}{h}\sqrt{\beta^2p^2+q^2}\right)}{R_2^2} - \frac{p^2\cosh\left(\frac{b}{h}\sqrt{\beta^2p^2+q^2}\right)}{\beta^2p^2+q^2}\right]\left[\frac{q}{R_3}\sin(q)\cosh\left(\sqrt{\beta^2p^2+r^2}\right) - \frac{q^2\cos(q)\sinh\left(\sqrt{\beta^2p^2+r^2}\right)}{R_3\sqrt{\beta^2p^2+r^2}}\right]\right\}\end{aligned}\right)}{\left[\frac{\beta^2p^2+q^2}{R_2^2}\sinh^2\left(\frac{b}{h}\sqrt{\beta^2p^2+q^2}\right) + p^2\cosh^2\left(\frac{b}{h}\sqrt{\beta^2p^2+q^2}\right)\right]\left[\frac{p^2\sinh^2\left(\sqrt{\beta^2p^2+r^2}\right)}{\beta^2p^2+r^2} + \frac{1}{R_3^2}\cosh^2\left(\sqrt{\beta^2p^2+r^2}\right)\right]\left(\beta^2p^2+q^2+r^2\right)} \qquad \text{(C10)}$$

For the completely closed test section, $R_2 = R_3 = 0$, the term containing the triple integral in equation (C10) is zero. For the completely open test section, $R_2 = R_3 \to \infty$, the term containing the triple integral must be replaced with

$$+\frac{2}{\pi^3}\int_0^\infty \cos\left(\frac{y}{h}r\right)dr \int_0^\infty dp \int_0^\infty dq \frac{\left(\begin{aligned}&e^{-\frac{b}{h}q}\sinh\left(\frac{s}{h}q\right)\left[q\sinh\left(\frac{b}{h}q\right)\cos\left(\frac{b}{h}r\right) + r\cosh\left(\frac{b}{h}q\right)\sin\left(\frac{b}{h}r\right)\right]\left\{\left[\frac{\sinh\left(\frac{b}{h}q\right) + \cosh\left(\frac{b}{h}q\right)}{q}\right]\left[\frac{p^2 q\sin(q)\sinh(r)}{r} + q^2\cos(q)\cosh(r)\right]\right.\\ &\left. - \left[\sinh\left(\frac{b}{h}q\right) - \frac{p^2\cosh\left(\frac{b}{h}q\right)}{q^2}\right]\left[q\sin(q)\cosh(r) - \frac{q^2\cos(q)\sinh(r)}{r}\right]\right\}\end{aligned}\right)}{\left[q^2\sinh^2\left(\frac{b}{h}q\right) + p^2\cosh^2\left(\frac{b}{h}q\right)\right]\left[\frac{p^2\sinh^2(r)}{r^2} + \cosh^2(r)\right]\left(q^2+r^2\right)}$$

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This expression is obtained by multiplying numerator and denominator by $R_2^2 R_3^2$, replacing $R_2 p$ and $R_3 p$ with p, and otherwise taking p equal to zero. This process is equivalent to taking the limit as $R_2 \rightarrow R_3 \rightarrow \infty$. Integration with respect to p gives

$$+\frac{1}{\pi^2}\int_0^\infty \cos\left(\frac{y}{h} r\right) dr \int_0^\infty dq \frac{e^{-\frac{b}{h}q}\sinh\left(\frac{s}{h} q\right)\left[q \sinh\left(\frac{b}{h} q\right)\cos\left(\frac{b}{h} r\right) + r \cosh\left(\frac{b}{h} q\right)\sin\left(\frac{b}{h} r\right)\right]\left\{\left[\sinh\left(\frac{b}{h} q\right) + \cosh\left(\frac{b}{h} q\right)\right]\left[\frac{\sin q}{\cosh\left(\frac{b}{h} q\right)} + \frac{\cos q}{\sinh\left(\frac{b}{h} q\right)}\right] - \left[\frac{1}{\cosh(r)} - \frac{r}{q \sinh(r)}\right]\left[\sin(q)\cosh(r) - \frac{q}{r}\cos(q)\sinh(r)\right]\right\}}{\left[\cosh\left(\frac{b}{h} q\right)\cosh(r) + \frac{q}{r}\sinh\left(\frac{b}{h} q\right)\sinh(r)\right]\left(q^2 + r^2\right)}$$

and then combination with the first term in equation (C10) yields, for $R_2 = R_3 \rightarrow \infty$,

$$\frac{h}{\Gamma} v_4 = \frac{1}{\pi^2}\int_0^\infty \frac{\sin q}{q} dq \int_0^\infty \frac{r\left[q \sinh\left(\frac{b}{h} q\right)\cos\left(\frac{b}{h} r\right) + r \cosh\left(\frac{b}{h} q\right)\sin\left(\frac{b}{h} r\right)\right]\sinh\left(\frac{s}{h} q\right)\cos\left(\frac{y}{h} r\right)}{\left(q^2 + r^2\right)e^{\frac{b}{h}q}\sinh(r)\cosh\left(\frac{b}{h} q\right)} dr \tag{C11}$$

In the derivation of ϕ_4 the potential ϕ_2 due to side-wall interference has been taken to be zero outside the vertical boundaries. A correction ϕ_5 is therefore to be added. For this purpose a solution ϕ_2 valid in the space outside the vertical boundaries is required. Such a solution, which, as required, is even in y and approaches zero for $y \rightarrow \infty$, is obtained by replacing $\cosh\left(y\sqrt{\beta^2\omega^2 + g^2}\right)$ with $e^{-\sqrt{\beta^2\omega^2+g^2}\,|y|}$ in equation (A7). By examining the processes leading to equation (A13), it is easily seen that the right-hand side of expression (C1) is replaced with

$$-\frac{\Gamma}{2\pi}\int_{-\infty}^{\infty} \frac{e^{igz} g(\alpha - i\omega)\left(\frac{\omega}{c_g} + \frac{i}{R_2}\right)e^{-bc_g}\sinh\left(sc_g\right)e^{-D_g|y|}}{c_g\left(\alpha^2 + \omega^2\right)\left(i\omega - \frac{D_g}{R_2}\right)e^{-D_g b}} dg$$

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Since this part of the potential ϕ_2 is zero between the vertical boundaries, the transform on y requires

$$\int_{-\infty}^{-b} e^{-ify} e^{D_g y} dy + \int_{b}^{\infty} e^{-ify} e^{-D_g y} dy = e^{-D_g b}\left(\frac{e^{ibf}}{D_g - if} + \frac{e^{-ibf}}{D_g + if}\right)$$

$$= 2e^{-D_g b}\left[\frac{D_g \cos(bf) - f \sin(bf)}{D_g^2 + f^2}\right]$$

By analogy with the procedure leading to equation (C9), it is easily seen that

$$\phi_5 = \lim_{\alpha \to 0} \frac{i\Gamma}{4\pi^3} \int_{-\infty}^{\infty} e^{i\omega x} d\omega \int_{-\infty}^{\infty} e^{ify} df \int_{-\infty}^{\infty} \frac{e^{igh}\left(\omega + \frac{g}{R_3}\right) g(\alpha - i\omega)\left(\frac{\omega}{c_g} + \frac{i}{R_2}\right) e^{-bc_g} \sinh(sc_g)\left[D_g \cos(bf) - f \sin(bf)\right] \sinh\left(z\sqrt{\beta^2\omega^2 + f^2}\right)}{c_g\left(\alpha^2 + \omega^2\right)\left(i\omega - \frac{D_g}{R_2}\right)\left(D_g^2 + f^2\right)\left[i\omega \sinh\left(h\sqrt{\beta^2\omega^2 + f^2}\right) + \frac{\sqrt{\beta^2\omega^2 + f^2}}{R_3} \cosh\left(h\sqrt{\beta^2\omega^2 + f^2}\right)\right]} dg$$

and with $\sqrt{\beta^2\omega^2 + f^2} = D_f$ the contribution to the upwash velocity at $x = z = 0$ is

$$v_5 = \lim_{\alpha \to 0} \frac{i\Gamma}{4\pi^3} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} e^{ify} df \int_{-\infty}^{\infty} \frac{e^{igh}\left(\omega + \frac{g}{R_3}\right) g(\alpha - i\omega)\left(\frac{\omega}{c_g} + \frac{i}{R_2}\right) e^{-bc_g} \sinh(sc_g)\left[D_g \cos(bf) - f \sin(bf)\right]}{c_g\left(\alpha^2 + \omega^2\right)\left(i\omega - \frac{D_g}{R_2}\right)\left(D_g^2 + f^2\right)\left[\frac{i\omega \sinh(D_f h)}{D_f} + \frac{1}{R_3} \cosh(D_f h)\right]} dg$$

For $\alpha \to 0$, terms containing α as a factor in the numerator contribute only for $\omega \to 0$, whence

$$\frac{h}{\Gamma} v_5 = \frac{ih}{4\pi^2} \int_{-\infty}^{\infty} e^{ify} df \int_{-\infty}^{\infty} \frac{e^{igh} \frac{g^2}{R_3} \frac{i}{R_2} e^{-b|g|} \sinh(s|g|)\left[|g| \cos(bf) - f \sin(bf)\right]}{|g|\left(\frac{-|g|}{R_2}\right)\left(g^2 + f^2\right)\left[\frac{1}{R_3} \cosh(hf)\right]} dg$$

$$+ \frac{ih}{4\pi^3} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} e^{ify} df \int_{-\infty}^{\infty} \frac{e^{igh}\left(\omega + \frac{g}{R_3}\right) g(-i\omega)\left(\frac{\omega}{D_g} + \frac{i}{R_2}\right) e^{-bD_g} \sinh(sD_g)\left[D_g \cos(bf) - f \sin(bf)\right]}{D_g\omega^2\left(i\omega - \frac{D_g}{R_2}\right)\left(D_g^2 + f^2\right)\left[\frac{i\omega \sinh(D_f h)}{D_f} + \frac{1}{R_3} \cosh(D_f h)\right]} dg$$

$$= \frac{1}{\pi^2} \int_0^{\infty} \cos\left(\frac{y}{h} r\right) dr \int_0^{\infty} \frac{e^{-\left(\frac{bq}{h}\right)} \sinh\left(\frac{s}{h} q\right)\left[q \cos\left(\frac{b}{h} r\right) - r \sin\left(\frac{b}{h} r\right)\right] \cos(q)}{\left(q^2 + r^2\right)\cosh(r)} dq$$

$$+ \frac{2}{R_3\pi^3} \int_0^{\infty} dp \int_0^{\infty} \cos\left(\frac{y}{h} r\right) dr \int_0^{\infty} \frac{q\left[\sin(q)\cosh\left(\sqrt{\beta^2p^2 + r^2}\right) - \frac{q \cos(q)}{\sqrt{\beta^2p^2 + r^2}} \sinh\left(\sqrt{\beta^2p^2 + r^2}\right)\right] e^{-\frac{b}{h}\sqrt{\beta^2p^2 + q^2}} \sinh\left(\frac{s}{h}\sqrt{\beta^2p^2 + q^2}\right)\left[\left(\sqrt{\beta^2p^2 + q^2}\right)\cos\left(\frac{b}{h} r\right) - r \sin\left(\frac{b}{h} r\right)\right]}{\left(\beta^2p^2 + q^2\right)\left(\beta^2p^2 + q^2 + r^2\right)\left[\frac{p^2 \sinh^2\left(\sqrt{\beta^2p^2 + r^2}\right)}{\beta^2p^2 + r^2} + \frac{1}{R_3^2} \cosh^2\left(\sqrt{\beta^2p^2 + r^2}\right)\right]} dq \tag{C12}$$

APPENDIX C

For the closed test section, $R_3 \to 0$, the second term (the term containing the triple integral) in equation (C12) is zero. For the open test section, $R_3 \to \infty$, the second term becomes

$$\frac{2}{\pi^3}\int_0^\infty dp \int_0^\infty \cos\left(\frac{y}{h}r\right)dr \int_0^\infty \frac{q\left[\sin(q)\cosh(r) - \frac{q}{r}\cos(q)\sinh(r)\right]e^{-\frac{b}{h}q}\sinh\left(\frac{s}{h}q\right)\left[q\cos\left(\frac{b}{h}r\right) - r\sin\left(\frac{b}{h}r\right)\right]}{q^2\left(q^2 + r^2\right)\left[\frac{p^2\sinh^2(r)}{r^2} + \cosh^2(r)\right]}dq$$

which on integration with respect to p becomes

$$\frac{1}{\pi^2}\int_0^\infty \cos\left(\frac{y}{h}r\right)dr \int_0^\infty \frac{q\left[\sin(q)\cosh(r) - \frac{q}{r}\cos(q)\sinh(r)\right]e^{-\frac{b}{h}q}\sinh\left(\frac{s}{h}q\right)\left[q\cos\left(\frac{b}{h}r\right) - r\sin\left(\frac{b}{h}r\right)\right]}{q^2\left(q^2 + r^2\right)\frac{\sinh(r)\cosh(r)}{r}}dq$$

This term combines with the first term in equation (C12) to give, for $R \to \infty$,

$$\frac{h}{\Gamma}v_5 = \frac{1}{\pi^2}\int_0^\infty \cos\left(\frac{y}{h}r\right)dr \int_0^\infty \frac{r\left[q\cos\left(\frac{b}{h}r\right) - r\sin\left(\frac{b}{h}r\right)\right]\sin(q)e^{-\left(\frac{b}{h}q\right)}\sinh\left(\frac{s}{h}q\right)}{q\left(q^2 + r^2\right)\sinh(r)}dq \tag{C13}$$

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TABLE I.- UPWASH INTERFERENCE FACTORS FOR SMALL-SPAN WINGS CENTER MOUNTED IN RECTANGULAR OPEN OR CLOSED TEST SECTIONS

Boundary	Width-height ratio, 2b/2h	δ_2	δ_3	δ_4	δ_5	Approximated δ $(\delta_2 + \delta_3 + \delta_4 + \delta_5)$	Correct value of δ
Closed	0.5	0.26180	0.03273	-0.02915	-0.00334	0.262	0.262
Closed	.75	.17453	.04909	-.04820	.00148	.176	.176
Closed	1	.13090	.06545	-.05984	.00364	.140	.137
Closed	1.5	.08727	.09818	-.06395	.00318	.125	.120
Closed	2	.06545	.13090	-.05743	.00169	.141	.137
Open	0.5	-0.13090	-0.06545	0.03920	0.01656	-0.141	-0.137
Open	.75	-.08727	-.09818	.05102	.00998	-.124	-.120
Open	1	-.06545	-.13090	.05131	.00502	-.140	-.137
Open	1.5	-.04363	-.19635	.04151	.00093	-.198	-.197
Open	2	-.03273	-.26180	.03240	.00019	-.262	-.262

TABLE II.- UPWASH INTERFERENCE FACTORS δ IN SQUARE TEST SECTION CALCULATED BY APPROXIMATION PROCEDURE WITH PERMEABILITY FACTOR $R_2 = R_3$ OF 0.6 FOR WING-SPAN—TEST-SECTION-HEIGHT RATIO $\frac{2s}{2h}$ OF 0.3

Mach number	Compressibility factor, β	Spanwise position, y/h	δ_2	δ_3	δ_4	δ_5	Approximated δ $(\delta_2 + \delta_3 + \delta_4 + \delta_5)$	Estimated δ (circular-tunnel comparison method)
0	1	0	0.04302	0.00424	-0.0266	0.0060	0.027	0.032
0	1	.10	.04331	.00400	-.0265	------	.027	
0	1	.20	.04421	.00330	-.0261	.0063	.028	
0	1	.25	.04488	.00279	-.0259	------	.029	
0	1	.30	.04573	.00218	-.0256	.0067	.029	
0.6	0.8	0	0.03180	-0.00752	-0.0222	0.0065	0.009	0.013
.6	.8	.10	.03201	-.00770	-.0221	------	.009	
.6	.8	.20	.03266	-.00823	-.0218	.0068	.009	
.6	.8	.25	.03316	-.00860	-.0216	------	.010	
.6	.8	.30	.03378	-.00906	-.0213	.0073	.011	
0.8	0.6	0	0.01741	-0.02407	-0.0162	0.0071	-0.016	-0.012
.8	.6	.10	.01751	-.02417	-.0161	------	-.016	
.8	.6	.20	.01783	-.02444	-.0159	.0075	-.015	
.8	.6	.25	.01808	-.02463	-.0158	------	-.015	
.8	.6	.30	.01839	-.02486	-.0156	.0080	-.014	
0.955	0.3	0	-0.01318	-0.06375	0	0.0079	-0.069	-0.065
.955	.3	.10	-.01334	-.06361	0	------	-.069	
.955	.3	.20	-.01381	-.06322	0	.0085	-.069	
.955	.3	.25	-.01417	-.06292	0	------	-.068	
.955	.3	.30	-.01463	-.06256	0	.0091	-.068	

TABLE III.- UPWASH INTERFERENCE FACTORS δ IN SQUARE TEST SECTION CALCULATED BY APPROXIMATION PROCEDURE WITH PERMEABILITY FACTOR $R_2 = R_3$ OF 0.6 FOR WING-SPAN—TEST-SECTION-HEIGHT RATIO $\frac{2s}{2h}$ OF 0.7

Mach number	Compressibility factor, β	Spanwise position, y/h	δ_2	δ_3	δ_4	δ_5	Approximated δ $(\delta_2 + \delta_3 + \delta_4 + \delta_5)$
0	1	0	0.04718	0.00130	-0.0256	0.0053	0.028
0	1	.3	.05113	-.00007	-.0245	------	.033
0	1	.5	.05948	-.00231	-.0228	.0070	.042
0	1	.6	.06649	-.00371	-.0216	------	.049
0	1	.7	.07657	-.00524	-.0203	.0086	.060
0.6	0.8	0	0.03485	-0.00971	-0.0215	0.0059	0.010
.6	.8	.3	.03778	-.01070	-.0206	------	.014
.6	.8	.5	.04397	-.01226	-.0192	.0079	.020
.6	.8	.6	.04918	-.01322	-.0183	------	.027
.6	.8	.7	.05670	-.01427	-.0171	.0097	.035
0.8	0.6	0	0.01892	-0.02518	-0.0158	0.0067	-0.015
.8	.6	.3	.02041	-.02557	-.0151	------	-.012
.8	.6	.5	.02356	-.02618	-.0141	.0091	-.008
.8	.6	.6	.02623	-.02652	-.0133	------	-.003
.8	.6	.7	.03011	-.02687	-.0125	.0112	.002
0.955	0.3	0	-0.01539	-0.06200	0.0007	0.0080	-0.069
.955	.3	.3	-.01740	-.06090	.0008	------	-.067
.955	.3	.5	-.02166	-.05903	.0009	.0115	-.068
.955	.3	.6	-.02523	-.05779	.0009	------	-.069
.955	.3	.7	-.03028	-.05636	.0009	.0146	-.071

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