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# DETERMINATION OF CONVECTIVE HEAT-TRANSFER COEFFICIENTS ON ADIABATIC WALLS USING A SINUSOIDALLY FORCED FLUID TEMPERATURE



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## ABSTRACT

The response of an insulated wall, over which a heated fluid flows, to a sinusoidally forced fluid temperature was used to calculate the convective heat-transfer coefficients. An exact solution is given which accounts for thermal conductivity and the location of the sensed wall temperature in one-dimensional heat-transfer problems. Charts are included to aid in the calculation. A comparative analysis was made of solutions that do not account for thermal conductivity and the location of the sensed wall temperature and those that do. If the exact solution is not used, errors greater than 23 percent are possible.

STAR Category 33

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## SUMMARY

The response of an insulated wall, over which a heated fluid flows, to a sinusoidally forced fluid temperature was used to calculate the convective heat-transfer coefficients. An exact solution is given which accounts for thermal conductivity and the location of the sensed wall temperature in one-dimensional heat-transfer problems. Charts are included to aid in the calculation. A comparative analysis was made of solutions that do not account for thermal conductivity and the location of the sensed wall temperature and those that do. If the exact solution is not used, errors greater than 23 percent are possible.

## INTRODUCTION

Both transient and steady-state analyses have been used to design calorimeters for use in rocket engines and aerodynamic heat-transfer studies. The steady-state calorimeter makes use of the temperature gradient in a material of known conductivity and geometry; the transient type makes use of the response of a material to a driving temperature, that is, the response of a thin disk to a step change in the surrounding fluid.

The response of a wall to a fluid that flows over it and has a sinusoidally oscillating temperature has been used to calculate the convective heat-transfer coefficient $h$. The solution found by Anderson (ref. 1) for the convective heat-transfer coefficient (herein called slug solution), however, does not account for thermal conductivity or the location of the measured wall temperature. An estimate, given by Anderson, of "the error in the measured time constant $\tau$ caused by heat conduction through the skin" is 1.5 percent. Bell (ref. 2) neglects the effect of thermal conductivity by designing his experiments in such a way as to cause its effect to drop out of his equations, which are in series form.

The objective of this analytical investigation, conducted at NASA Lewis Research Center, was to find a solution for the convective heat-transfer coefficient as a function of the phase lag between the fluid and wall temperatures. Such a solution would take into account the thermoconductivity as well as the location of the measured wall temperature.

The solution is presented along with charts that (for the wall temperature measured at the insulated side of the wall) can be used to determine the heat-transfer coefficient as a function of frequency, wall properties, wall thickness, and phase lag. A comparison is made between this solution and that of Anderson (ref. 1), both of which assume that the back surface of the wall $x = L$ is perfectly insulated.

The slug solution may be substituted for the present analysis when conductivity and temperature-sensor location are not important.

## SYMBOLS

| | |
|---|---|
| $C$ | specific heat of wall material, Btu/(lb)($^{o}$R); J/(kg)(K) |
| $\overline{CON}$ | function defined in eq. (4b) |
| $f$ | frequency of temperature oscillation, cps; Hz |
| $h$ | convective heat-transfer coefficient on surface of wall, Btu/(in.$^2$)(sec)($^{o}$R); W/(m$^2$)(K) |
| $K$ | thermal conductivity, Btu/(in.)(sec)($^{o}$R); J/(m)(sec)(K) |
| $L$ | thickness of wall, in.; m |
| $T$ | wall temperature, $T_W - \overline{T}_G$, $^{o}$R; K |
| $\overline{T}$ | mean temperature, $^{o}$R; K |
| $\Delta T_G$ | amplitude of gas temperature, $T_G - \overline{T}_G$, $^{o}$R; K |
| $x$ | distance measured from fluid-wall interface into wall, in.; m |
| $\alpha$ | thermal diffusivity, $K/\rho C$ |
| $\epsilon_G$ | function defined after eq. (4f) |
| $\eta$ | frequency and diffusivity perimeter, $\sqrt{\omega/2\alpha}$ |
| $\theta$ | time, sec |
| $\pi$ | constant equal to 3.1416 rad |
| $\rho$ | density of wall material, lb/in.$^3$; kg/m$^3$ |
| $\tau$ | time constant, $\rho CL/h$ |

$\varphi$ phase shift between forced fluid temperature $T_G$ and wall temperature $T_w$, $< 0$ for $T_w$ lagging $T_G$, deg

$\omega$ angular velocity of forced fluid temperature $T_G$, $2\pi f$ rad/sec

Subscripts:

G fluid flowing over wall

s values calculated with Anderson's slug-type solution (ref. 1)

w wall over which fluid flows

## DIFFERENTIAL EQUATIONS AND ASSUMPTIONS

The solution for the heat-transfer coefficient as a function of phase lag $\varphi$, frequency of fluid-temperature oscillation f, wall properties, and location of the sensed wall temperature T is now given. Consider an infinite plate on one side of which a fluid flows over the surface (x = 0). The other side is insulated (x = L). The system is illustrated in figure 1.

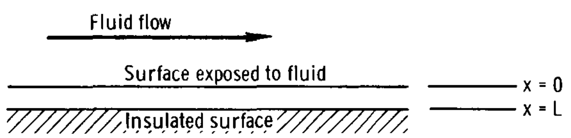



Figure 1. - Basic heat-transfer model.

The applicable differential equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial \theta} \tag{1}$$

where

T difference between wall temperature at any location in wall and average fluid temperature

x distance into wall from fluid side of wall

$\alpha$ thermal diffusivity, $K/\rho C$

$\theta$ time, sec

This solution assumes that the

(1) Thermal conductivity is finite and constant

(2) Convective heat-transfer coefficient h is constant

(3) Density $\rho$ and specific heat C of the wall are constant

(4) Surface of the wall at $x = L$ is insulated

(5) Surface of the wall at $x = 0$ is exposed to a fluid whose temperature is given by

$$T_G - \overline{T}_G = \Delta T_G e^{-i\omega\theta} \tag{2}$$

where

$T_G$ fluid temperature

$\overline{T}_G$ average fluid temperature

$\Delta T_G$ amplitude of gas temperature

$\omega$ angular velocity of temperature oscillation, $2\pi f$ rad/sec

(6) Convective heat transfer at $x = 0$ is

$$-K \frac{\partial T}{\partial x} = h_G(T_G - T) \tag{3}$$

(7) Heat conduction through the wall is one dimensional

For the solution to the differential equation (1), a product solution is assumed and the boundary conditions are applied (i.e., an insulated surface at $x = L$, assumption (4), and convective heat transfer at $x = 0$, assumption (6). The details of the solution are given in the appendix. The convective heat-transfer coefficient is

$$\frac{h_G}{K\eta} = \frac{1 - e^{2\eta L}(\cos 2\eta L + \sin 2\eta L) - \overline{CON}\left[e^{2\eta L}(\cos 2\eta L - \sin 2\eta L) - 1\right]}{e^{2\eta L} \sin 2\eta L + \overline{CON}\left(e^{2\eta L} \cos 2\eta L + 1\right)} \tag{4a}$$

where

$$\overline{CON} = \frac{A(x, L, \eta)\tan \varphi - 1}{A(x, L, \eta) + \tan \varphi} \quad \text{and} \quad \varphi < 0 \tag{4b}$$

for the wall temperature lagging the fluid temperature

$$A(x, L, \eta) = \frac{e^{2\eta(L-x)} \cos \eta(2L - x) + \cos \eta x}{e^{2\eta(L-x)} \sin \eta(2L - x) + \sin \eta x} \tag{4c}$$

where

$$\eta = \sqrt{\frac{\omega}{2\alpha}} \tag{4d}$$

This solution is simplified if the wall temperature is measured at $x = L$, which for many applications is the easiest place to locate a sensor. The $\overline{CON}$ used in equation (4a) reduces to

$$\overline{CON} = \tan(\varphi - \eta L) \qquad \text{at } x = L \tag{4e}$$

The solutions to equation (1) may be used to determine the convective heat-transfer coefficient when thermal conductivity is an important factor.

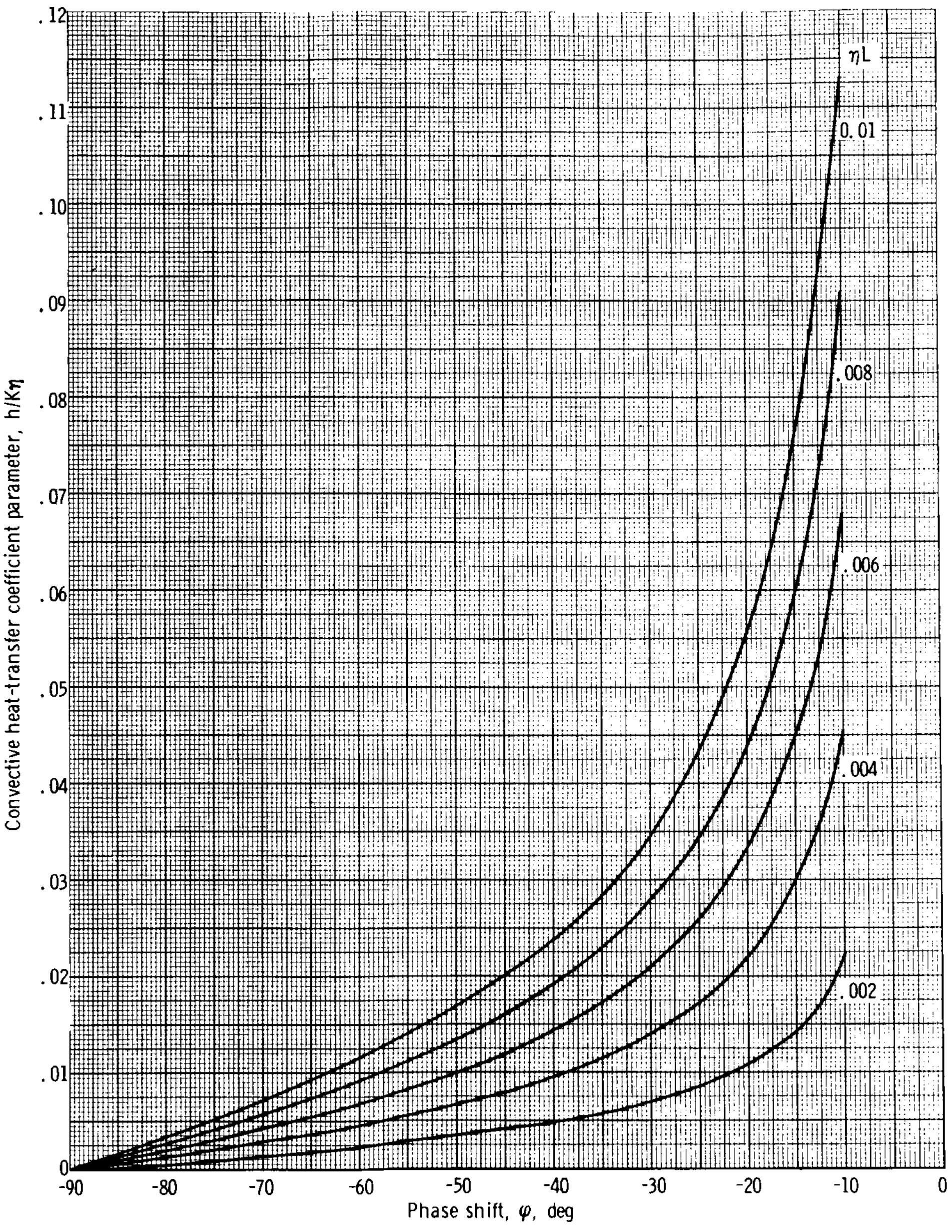
The computations made for the convective heat-transfer coefficient with the present solution (eqs. (4)) are time consuming. The values for $h/K\eta$ were calculated as a function of $\eta L$ and $\varphi$, when $x = L$, and are plotted in figure 2. Using $x = L$ (temperature sensor located at the insulated surface) is reasonable because a temperature sensor is easily installed at this point.

The ratio of the amplitude of the wall- to the fluid-temperature oscillation for $x = L$ can be written by inspection of equation (A15). The ratio is

$$\frac{T_W - \overline{T}_G}{T_G - \overline{T}_G} = \frac{T}{\Delta T_G} = \frac{2e^{2\eta L}}{\sqrt{\left[1 - \frac{K\eta}{h_G} + e^{2\eta L}E \cos(\epsilon_G + 2\eta L)\right]^2 + \left[\frac{K\eta}{h_G} - e^{2\eta L}E \sin(\epsilon_G + 2\eta L)\right]^2}} \tag{4f}$$

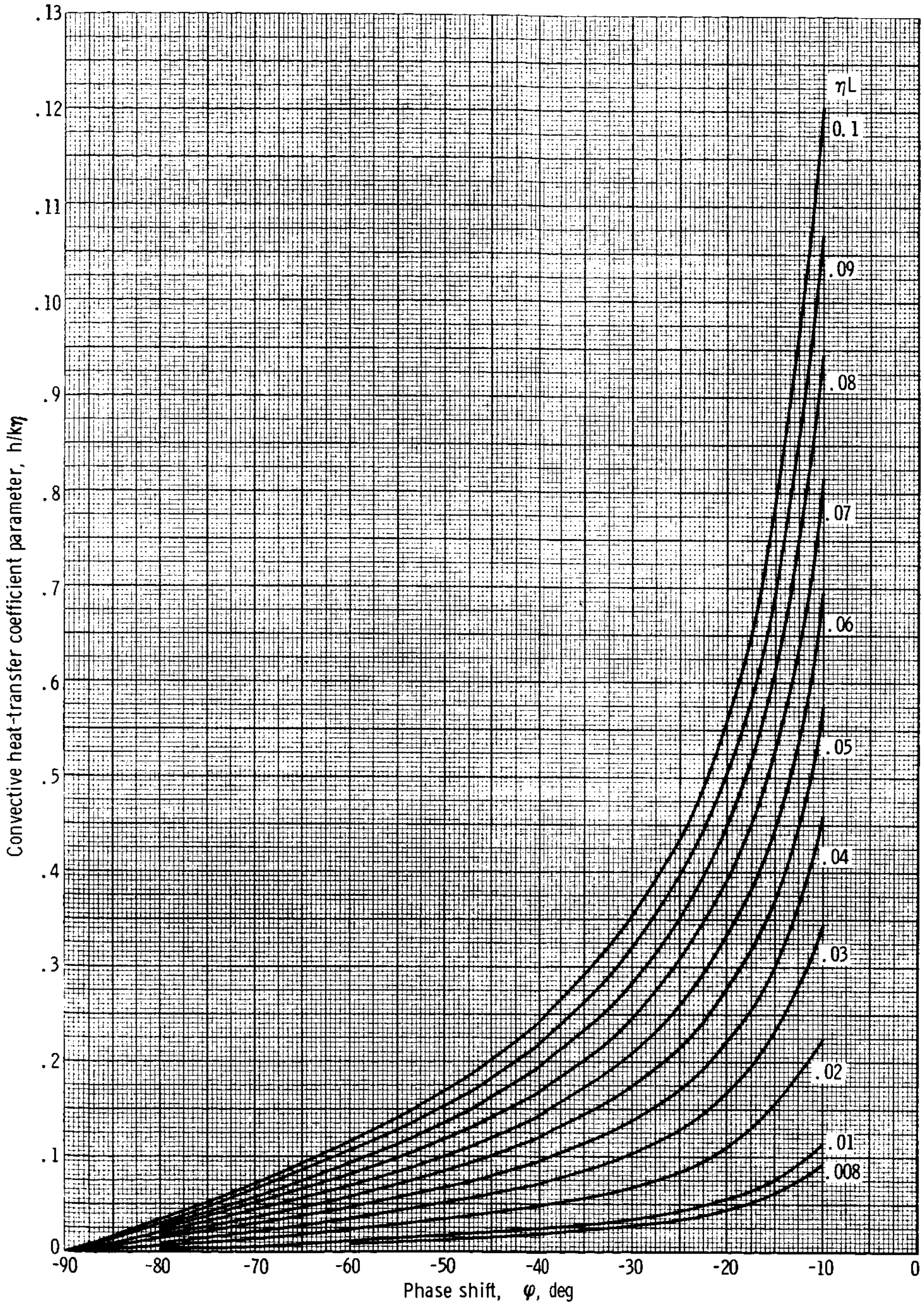
where

$$E = \sqrt{1 + 2\frac{K\eta}{h_G} + 2\left(\frac{K\eta}{h_G}\right)^2}$$



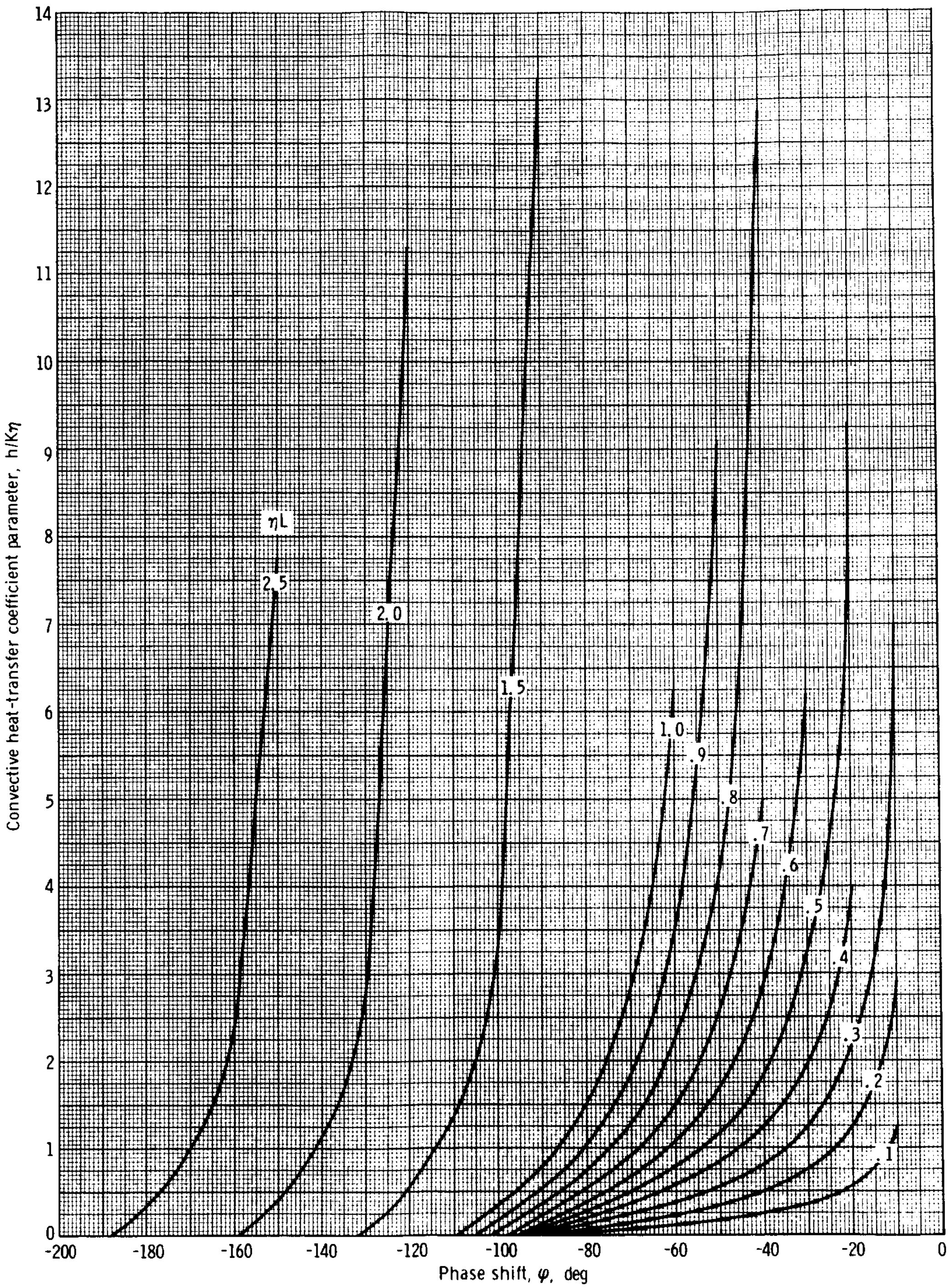

(a) $\eta L$, 0.002 to 0.01.

Figure 2. - Convective heat-transfer coefficient as function of frequency and phase shift with temperature measured at insulated surface.




(b) $\eta L$, 0.008 to 0.1.

Figure 2. - Continued.




(c) $\eta L$, 0.1 to 3.0.

Figure 2. - Concluded.

and

$$\epsilon_G = \tan^{-1} \frac{1}{\frac{h_G}{K\eta} + 1}$$

This ratio can be used to determine the magnitude of the fluid-temperature oscillation $T_G$ that would yield a measurable wall-temperature oscillation at $x = L$.

The determination of the effect of wall properties and plate thickness $L$ on the calculation of $h$ was aided by the expanding of equation (4a) to a series form at $x = L$. The resulting equation for $h_G$ is

$$h_G = \frac{\rho C L \omega}{-\tan \varphi} \left( 1 - \frac{3 + \tan^2 \varphi}{3 \tan \varphi} \eta^2 L^2 + \frac{4}{3} \eta^3 L^3 + \frac{5 + \tan^2 \varphi + 10 \tan^3 \varphi}{5 \tan^2 \varphi} \eta^4 L^4 \dots \right) \tag{4g}$$

In this equation, $\varphi < 0$. Convergence of this series must be checked. However, if $\eta L \ll 1$ and reasonable values of $\varphi$ (e.g., $-45^{\circ}$) are used, the series will converge.

In this report, Anderson's solution is referred to as the slug solution because it equates the rate of change of the temperature of a mass or slug $\rho L$ to the convective heat transfer from a fluid that flows over the slug. The slug solution is derived from the differential equation

$$\tau \frac{\partial T_w}{\partial \theta} + T_w = T_G \tag{5}$$

and assumes that

(1) The thermal conductivity $K$ is infinite (i.e., no temperature gradient in the wall)

(2) The convective heat-transfer coefficient $h$ is constant

(3) The density $\rho$ and specific heat $C$ of the wall are constant

(4) One wall surface is insulated

(5) The other wall surface is exposed to a fluid whose temperature is given by

$$T_G = \overline{T}_G + \Delta T_G \sin \omega\theta \tag{6}$$

The solution to the differential equation (5) is

$$T = \frac{\Delta T_G}{\sqrt{1 + \omega^2\tau^2}}\left\{\sin\left[\omega\theta - \tan^{-1}(\omega\tau)\right]\right\} \tag{7a}$$

where

$$\tau = \frac{\rho C L}{h_G} \tag{7b}$$

is the time constant. The phase lag between the wall and the fluid is $\tan^{-1} \omega\tau = \varphi_s$ from which the convective heat-transfer coefficient can be written as

$$h_{G,s} = \frac{-\rho C L \omega}{\tan \varphi_s} \tag{8}$$

where $\varphi_s$ is $< 0$.

## CRITERION FOR USE OF SLUG SOLUTION

Comparison of the series form of the present solution (eq. (4g)) for $h_G$ with that of the slug solution (eq. (8)) shows that the coefficient of the series solution is simply $h_{G,s}$. If $h_{G,s}$ is substituted in the series solution and all terms having powers greater than second power are neglected, the series solution can be written as

$$\frac{h_G}{h_{G,s}} = 1 - \frac{3 + \tan^2 \varphi}{3 \tan \varphi} (\eta L)^2 \ . \ . \ . \tag{9a}$$

where $\varphi < 0$, and $\eta L$ is assumed to be much smaller than 1. The value of $\varphi$ can and should be approximately $-45^{\circ}$ (as discussed in the section Optimum Phase Angle). The curves in figure 2 can be used to determine the proper frequency that, for a given material, $\eta L$, and $h_G$, will yield the value $\varphi = -45^{\circ}$. The selected value of $\eta L$ is then used in equation (9a) along with $\varphi = -45^{\circ}$ to evaluate the ratio $h_G/h_{G,s}$. For these conditions, equation (9a) reduces to

$$\frac{h_G}{h_{G,s}} = 1 + \frac{4}{3}(\eta L)^2 \ . \ . \ . = 1 + \frac{2}{3}\frac{L}{K} h_{G,s} \tag{9b}$$

TABLE I. - SUMMARY OF CALCULATED VALUES OF CONVECTIVE HEAT-TRANSFER-COEFFICIENT DIFFERENCES AT PHASE LAG OF -45°

(a) U.S. Customary units

| Material | Average temperature, $\overline{T}$, °R | Thermal conductivity, K, $\frac{Btu}{(in.)(sec)(^{o}R)}$ | Specific heat, C, $\frac{Btu}{(lb)(^{o}R)}$ | Density, $\rho$, lb/in.$^3$ | Wall thickness, L, in. | Approximate frequency, f, cps | Heat-transfer coefficient, h, $\frac{Btu}{(in.^2)(sec)(^{o}R)}$ | Convective heat-transfer-coefficient differences | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Exact | | Approximated | |
| | | | | | | | | $\left(\frac{h - h_s}{h}\right)100$, percent | | $\left[\frac{4}{3}(\eta L)^2\right]100$, percent | $\left(\frac{2L}{3K} h_{G,s}\right)100$, percent |
| | | | | | | | | Fluid surface, x = 0 | Insulated surface, x = L | | |
| 347 Stainless steel | 1000 | 0.000253 | 0.128 | 0.286 | 0.040 | 0.11 | 0.001 | 7.0 | 9.6 | 9.5 | 10.4 |
| | | | | | .040 | .24 | .003 | --- | 23.0 | 23.1 | 31.6 |
| | | | | | .010 | .424 | .001 | --- | 2.6 | 2.6 | 2.6 |
| | | | | | .032 | .128 | .001 | --- | 8.0 | 7.9 | 8.4 |
| | | | | | .060 | .065 | .001 | --- | 14.7 | 14.2 | 15.8 |
| | 2000 | 0.000365 | 0.151 | 0.286 | 0.040 | 0.087 | 0.001 | --- | 6.7 | 6.9 | 7.3 |
| Copper | 1000 | 0.004975 | 0.097 | 0.322 | 0.060 | 0.0842 | 0.001 | --- | 0.8 | 0.8 | 0.8 |

(b) SI Units

| Material | Average temperature, $\overline{T}$, K | Thermal conductivity, K, $\frac{J}{(m)(sec)(K)}$ | Specific heat, C, $\frac{J}{(kg)(K)}$ | Density, $\rho$, kg/m$^3$ | Wall thickness, L, m | Approximate frequency, f, Hz | Heat-transfer coefficient, h, $\frac{W}{(m^2)(K)}$ | Convective heat-transfer-coefficient differences | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Exact | | Approximated | |
| | | | | | | | | $\left(\frac{h - h_s}{h}\right)100$, percent | | $\left[\frac{4}{3}(\eta L)^2\right]100$, percent | $\left(\frac{2L}{3K} h_{G,s}\right)100$, percent |
| | | | | | | | | Fluid surface, x = 0 | Insulated surface, x = L | | |
| 347 Stainless steel | 555.5 | 18.92 | $0.536\times10^3$ | $7.92\times10^3$ | $1.016\times10^{-3}$ | 0.11 | $2.942\times10^3$ | 7.0 | 9.6 | 9.5 | 10.4 |
| | | | | | 1.016 | .24 | 8.826 | --- | 23.0 | 23.1 | 31.6 |
| | | | | | .254 | .424 | 2.942 | --- | 2.6 | 2.6 | 2.6 |
| | | | | | .8128 | .128 | 2.942 | --- | 8.0 | 7.9 | 8.4 |
| | | | | | 1.524 | .065 | 2.942 | --- | 14.7 | 14.2 | 15.8 |
| | 1101 | 27.29 | $0.632\times10^3$ | $7.92\times10^3$ | $1.016\times10^{-3}$ | 0.087 | $2.942\times10^3$ | --- | 6.7 | 6.9 | 7.3 |
| Copper | 555.5 | 471.9 | $0.406\times10^3$ | $8.92\times10^3$ | $1.524\times10^{-3}$ | 0.0842 | $2.942\times10^3$ | --- | 0.8 | 0.8 | 0.8 |

where $\varphi = -45^{\circ}$ and $\eta L << 1$. The second term in equation (9b), an approximation to $(h - h_s)/h$, can be used to approximate the error in $h$ if the slug solution, instead of the present solution, is used to calculate $h$. The term may also be used as a first-order correction if the slug solution is used to calculate $h$.

Values for the second term in equation (9b) were calculated for 347 stainless steel and copper and are compared in table I with the values of $(h - h_s)/h \times 100$ percent. The agreement is good, even though the absolute value of the difference is large in several cases. Care must be exercised in using equations (9a) and (9b) because only two terms of the series are used, and if the value of $\eta L$ approaches 1, convergence is not ensured. In addition, values for $\varphi$ that approach either $0^{\circ}$ or $90^{\circ}$ will greatly affect $h_G/h_{G,s}$.

In addition to the aforementioned criteria to be used in the choice of solutions for calculating $h$, a number of calculations were made and are presented to illustrate the practical application of the solutions first given for $h$.

## APPLICATION OF EQUATIONS TO ENGINEERING MATERIALS

The calculation of the relation between phase lag angle $\varphi$ and forcing frequency $f$ necessitates that assumptions be made for values of the convective heat-transfer coefficient and the properties of the wall. The value of the convective heat-transfer coefficient $h_G$ assumed in this calculation is 0.001 Btu per square inch per second per $^{\circ}R$ ($2.942 \times 10^3$ W/(m$^2$)(K)), unless otherwise noted. The wall properties used are those of 347 stainless steel and are given as follows:

| | |
|---|---|
| Average temperature, $\overline{T}$, $^{\circ}R$; K | 1000; 555.5 |
| Density, $\rho$, lb/in.$^3$; kg/m$^3$ | 0.286; $7.92 \times 10^3$ |
| Specific heat, C, Btu/(lb)($^{\circ}R$); J/(kg)(K) | 0.128; $0.536 \times 10^3$ |
| Conductivity, K, Btu/(in.)(sec)($^{\circ}R$); J/(m)(sec)(K) | 0.000253; 18.92 |

These values are approximately those of 347 stainless-steel materials used for simulated rocket-nozzle heat-transfer studies conducted in an air facility. The value of $h_G$ is approximately 5 to 10 times greater than those found by Anderson (ref. 1) in his wind-tunnel tests on a cone.

### Comparison of Phase Lags

The calculated phase lags $\varphi$ as a function of frequency $f$ are shown in figure 3. As would be expected, the slug solution phase lag falls between the values calculated with equation (4a) (present solution) at $x = 0$ and at $x = L$. The decrease in $\varphi$ at $x = 0$ for

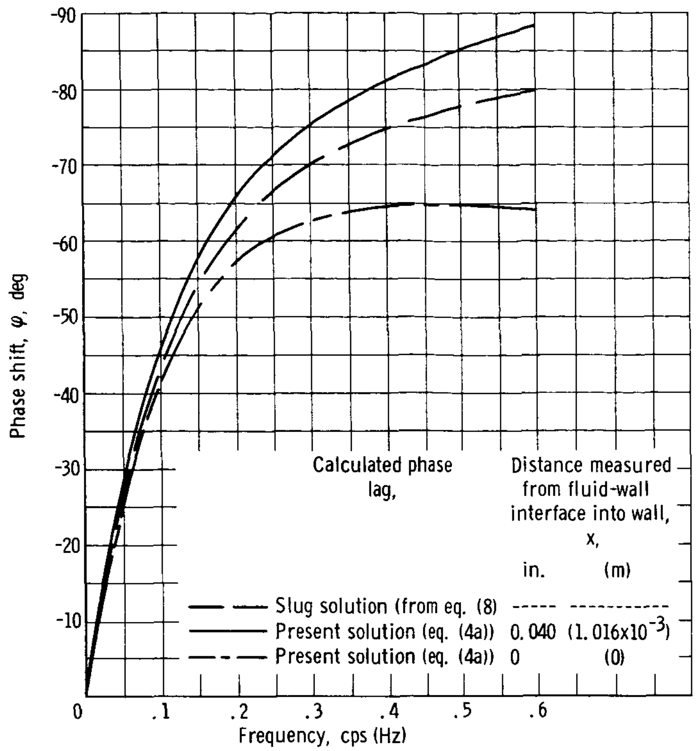



Figure 3. - Phase lag as function of forced fluid-temperature frequency for 347 stainless steel. Fluid temperature, 1000° R (555.5 K); convective heat-transfer coefficient at fluid surface, 0.001 Btu per square inch per second per °R ($2.942\times10^3$ W/(m$^2$)(K)).

$f > 0.45$ hertz can be explained by the reflected wave that counteracts the phase shift of the primary wave.

## Optimum Phase Angle

When the phase lag is measured for use in either of the solutions, an optimum value exists. At this optimum value, a given error in phase lag will produce a minimum error in the heat-transfer coefficient. Figure 4 shows this optimum phase lag to be approximately $-45^{\circ}$, or exactly $-45^{\circ}$ if the slug solution is used. If equation (4a) is used with $x = L = 0.040$ inch ($1.016\times10^{-3}$ m), the optimum $\varphi$ increases slightly, which is the reason for choosing a frequency that will give a value for $\varphi$ of approximately $-45^{\circ}$.

## Phase Lag Differences

The values for the phase lag calculated from both the slug solution and the present

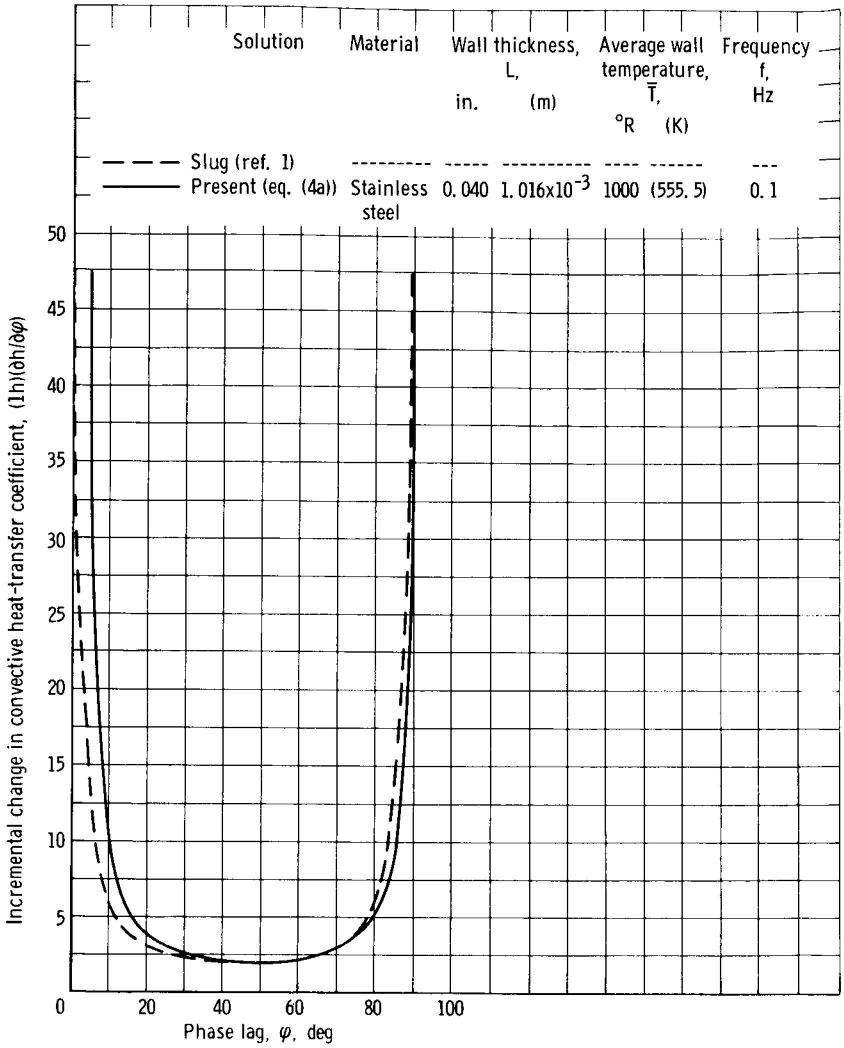



Figure 4. - Change in convective heat-transfer coefficient for incremental change in phase lag as function of phase lag. (For present solution, temperature was measured at insulated surface.

solution do not agree. The percent difference was calculated to determine the magnitude of this disagreement. The percentage is based on the present solution and is shown as a function of frequency in figure 5. For the case where the wall is 0.040 inch ($1.016 \times 10^{-3}$ m) thick and the temperature measurement made is assumed to be at the insulated surface, the minimum difference is 6 percent. The frequency for this point is 0.175 hertz and the phase lag is $-61.5^{\circ}$. If it is possible to measure the temperature on the surface over which the fluid flows ($x = 0$), the difference can be reduced by using a lower frequency. For a phase lag of $-45^{\circ}$, the difference at $x = L$ is 6.3 percent while at $x = 0$ it is 4 percent.

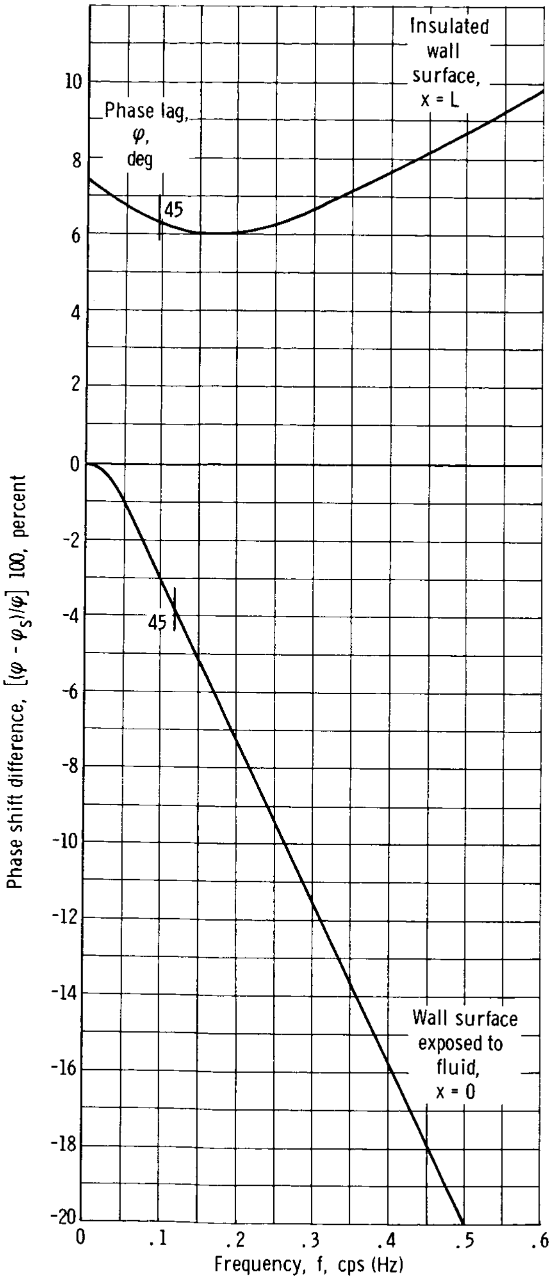



Figure 5. - Difference in phase lag due to location of temperature sensor as function of frequency for 347 stainless steel. Fluid temperature, 1000° R (555.5 K); convective heat-transfer coefficient at fluid surface, 0.001 Btu per square inch per second per °R ($2.942 \times 10^3$ W/(m²)(K)); wall thickness, 0.040 inch ($1.016 \times 10^{-3}$ m).

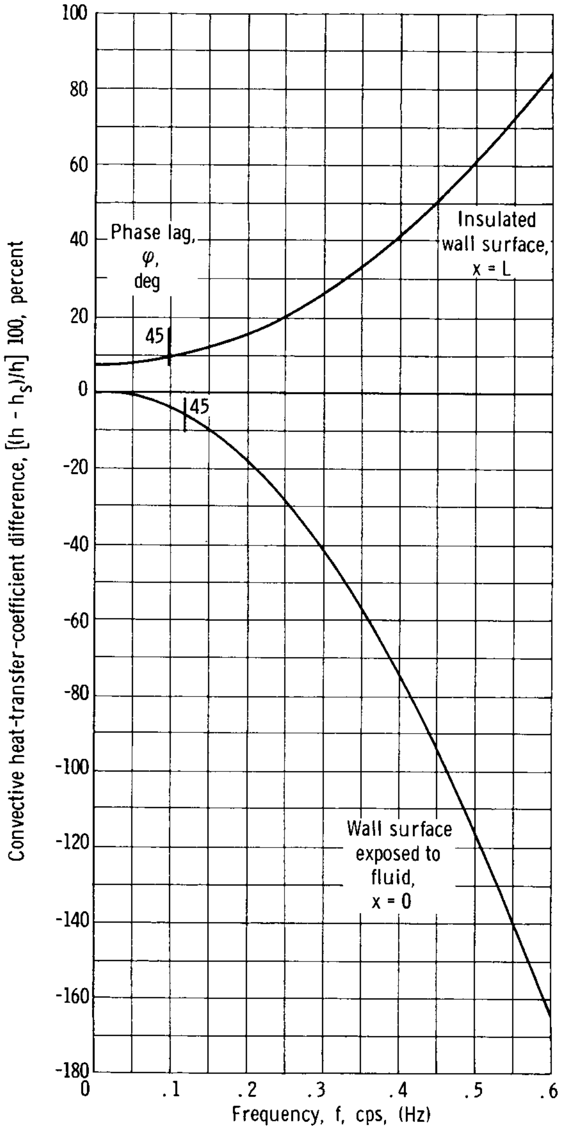



Figure 6. - Difference in convective heat-transfer coefficient due to location of wall temperature sensor as function of frequency for 347 stainless steel. Fluid temperature, 1000° R (555.5 K); convective heat-transfer coefficient at fluid surface, 0.001 Btu per square inch per second per °R ($2.942 \times 10^3$ W/(m²)(K)); wall thickness, 0.040 inch ($1.016 \times 10^{-3}$ m).

## Heat-Transfer-Coefficient Differences

Since a minimum value of the phase lag differences in the case where x = L was observed in figure 5, a similar minimum would be expected to exist when the heat-transfer-coefficient differences are calculated. Figure 6, however, shows that the lowest possible difference calculated for x = L is greater than 7 percent and occurs at a lower frequency than does the minimum phase lag difference shown in figure 5. At phase lags of $-45^{\circ}$, the heat-transfer-coefficient differences are 9.6 percent at x = L and 7 percent at x = 0. The differences will increase with increased frequency or phase lag angle.

## Effect of Heat-Transfer Coefficient

Up to this point in the calculation, the convective heat-transfer coefficient has been

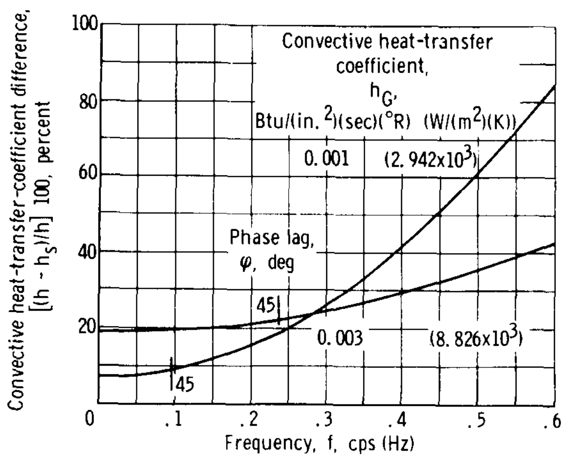



Figure 7. - Difference in convective heat-transfer coefficient for 347 stainless steel. Fluid temperature, 1000° R (555.5 K); wall thickness, 0.040 inch ($1.016\times10^{-3}$ m); temperature measured at insulated surface.

assumed to be 0.001 Btu per square inch per second per $^{\circ}$R ($2.942\times10^{3}$ W/(m$^2$)(K)). Shown in figure 7 are the calculated heat-transfer-coefficient differences at x = L = 0.040 inch ($1.016\times10^{-3}$ m) for h = 0.001 Btu per square inch per second per $^{\circ}$R ($2.942\times10^{3}$ W/(m$^2$)(K)) and for h = 0.003 Btu per square inch per second per $^{\circ}$R ($8.826\times10^{3}$ W/(m$^2$)(K)). Figure 7 shows that, for low frequencies, the differences are greater for the higher h. When a phase lag angle of $-45^{\circ}$ is desired (to minimize the effect of errors in phase lag measurement), the differences are 9.6 percent for h = 0.001 Btu per square inch per second per $^{\circ}$R ($2.942\times10^{3}$ W/(m$^2$)(K)) and 23 percent for h = 0.003 Btu per square inch per second per $^{O}R$ ($8.826 \times 10^3$ W/(m$^2$)(K)). With a phase shift of -45$^{O}$ used as a criterion, it is concluded that increasing the heat-transfer coefficient will increase the heat-transfer-coefficient difference.

## Effects of Temperature

The effect of wall temperature on the convective heat-transfer-coefficient difference is shown in figure 8. Because the temperature affects the wall properties, two tempera-

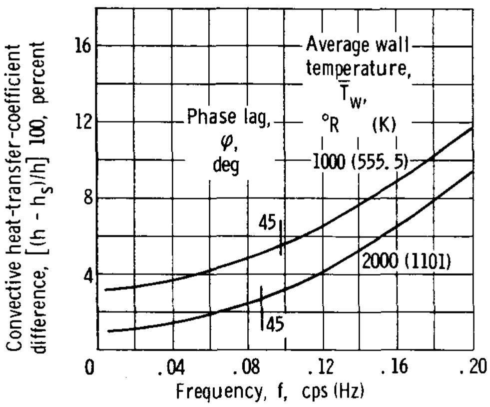



Figure 8. - Difference in convective heat-transfer coefficient due to change in wall temperature as function of frequency for 347 stainless steel. Convective heat-transfer coefficient, 0.001 Btu per square inch per second per °R ($2.942 \times 10^3$ W/(m$^2$)(K)); wall thickness, 0.040 inch ($1.016 \times 10^{-3}$ m); temperature measured at insulated surface.

tures were used, 1000$^{O}$ R (555.5 K) and 2000$^{O}$ R (1101 K). The wall properties for 347 stainless steel at 1000$^{O}$ R (555.5 K) were given in the first part of this section and for 2000$^{O}$ R (1101 K) are as follows:

Density, $\rho$, lb/in.$^3$; kg/m$^3$ . . . . . . . . . . . . . . . . . . . . . . . . . 0.286; $7.92 \times 10^3$
Specific heat, C, Btu/(lb)($^{O}$R); J/(kg)(K) . . . . . . . . . . . . . . . . . . . . 0.151; $0.632 \times 10^3$
Conductivity, K, Btu/(in.)(sec)($^{O}$R); J/(m)(sec)(K) . . . . . . . . . . . . 0.000365; 27.29

The values for the heat-transfer-coefficient differences calculated at $x = L$ show that a 2-percent decrease in the differences exists for a two-to-one change in wall temperature. Property changes due to temperature, in the case of 347 stainless steel, will not significantly affect the convective heat transfer differences.

## Effects of Wall Thickness

Wall thickness can be expected to have an appreciable effect on the convective heat-transfer-coefficient difference, as shown in figure 9. For a frequency that will give a $\varphi$ value of $-45^{\circ}$ and with the use of $x = L$, the following differences can be obtained: for L equal to 0.010 inch ($0.254\times10^{-3}$ m), 2.6 percent; for L equal to 0.060 inch ($1.524\times10^{-3}$ m), 14.7 percent. For thin walls, the differences do not increase rapidly with frequency. The thicker walls cause the differences to increase at lower frequencies than those of the thin wall. Inspection of equation (9a) shows that the differences approach zero as L approaches zero.

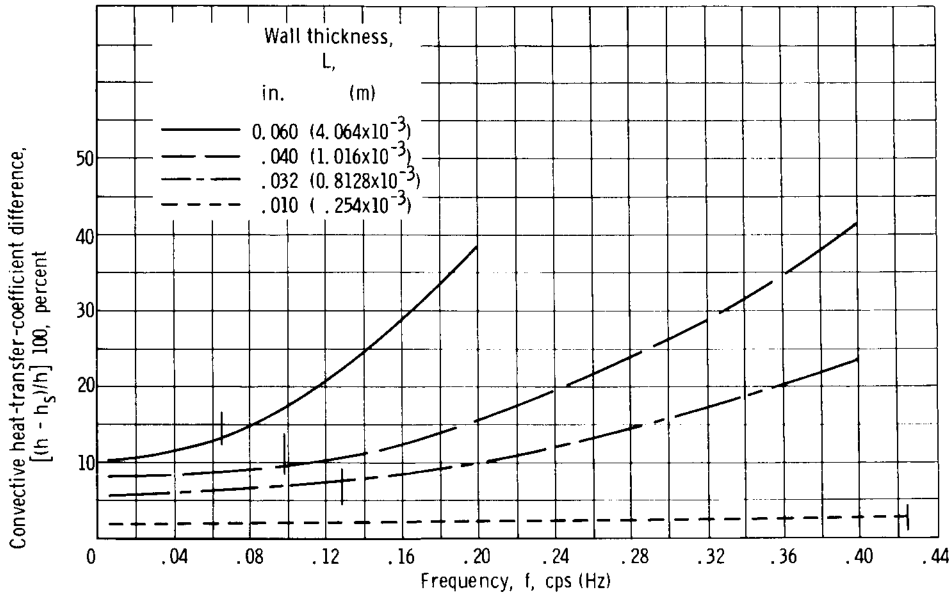



Figure 9. - Difference in convective heat-transfer coefficient due to change in wall thickness as function of frequency for 347 stainless steel. Fluid temperature, 1000° R (555.5 K); convective heat-transfer coefficient, 0.001 Btu per square inch per second per °R ($2.942\times10^{3}$ W/(m$^2$)(K)); temperature measured at insulated surface.

## Effects of Thermal Conductivity

The effect of thermal conductivity on the convective heat-transfer-coefficient difference is shown in table I. A comparison of copper and 347 stainless steel, having wall thicknesses of 0.060 and 0.040 inch ($1.524\times10^{-3}$ and $1.016\times10^{-3}$ m), respectively, shows that the higher conductivity of copper decreases the heat-transfer-coefficient differences by 14.2 percent. Equation (9a) shows that, as K approaches infinity, the differences approach zero.

## COMPARISON OF SOLUTIONS

The slug solution (ref. 1), which neglects the effect of thermal conductivity and temperature-measurement location, may be used in place of the more complicated solution (eqs. (4)) provided that the system is designed properly. Equation (4g) may be used to estimate the error in h when the slug solution is used provided that $\eta L < 1.0$. If a maximum error of 6 percent is to be tolerated, $\eta L$ cannot exceed 0.2 and $\varphi$ must be approximately $-45^{\circ}$. Improper design will result in large errors. For example, a wall made of 347 stainless steel, 0.060 inch ($1.524\times10^{-3}$ m) thick, with the temperature measured at the insulated face will give errors greater than 23 percent if phase lags exceed $-45^{\circ}$. Inspection of equation (4g) shows that if the slug solution is used, thin walls are essential. Although low frequency improves the accuracy of the slug solution, it is well to keep in mind that, at least for $x = L$, the limit of $h_G/h_{G,s} \neq 1$. Also, the accuracy in the measurement of the phase lag angle becomes very poor as $\varphi$ approaches zero (see fig. 4). High thermal conductivity is desirable as are low density and specific heat. If the phase lag is $-45^{\circ}$, an increase in the heat-transfer coefficient will increase the error when the slug solution is used (see fig. 7). An increase in the wall temperature of 347 stainless steel from $1000^{\circ}$ to $2000^{\circ}$ R (555.5 to 1101 K) resulted in only a 2-percent change in the error (fig. 8).

Table I summarizes the results of the comparison made of equations (4a) to (4e) and (8). This table presents calculations made for phase lags of $-45^{\circ}$.

## CONCLUDING REMARKS

The convective heat-transfer coefficient h can be calculated for a fluid flowing over a surface with one insulated side if the fluid temperature is varied sinusoidally. The phase lag between the fluid and wall temperatures, along with the frequency of oscillation and wall material properties, can be used to calculate the convective heat-transfer coefficient h. Two solutions for h are available. Both require a phase lag of approximately $-45^{\circ}$ to minimize the error in h due to errors made in measuring the phase lag angle. Anderson's slug solution (ref. 1) does not account for the wall thermal conductivity or the location of the measured wall temperature, which may result in an error greater than 23 percent in h.

A general one-dimensional solution is given which accounts for a finite thermal conductivity and for the wall-temperature location. This solution is greatly simplified if the wall temperature is measured at the insulated surface. Neither solution is applicable when two- or three-dimensional heat transfer in the wall is important.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, March 25, 1968,
122-29-07-03-22.

## APPENDIX - DERIVATION OF HEAT-TRANSFER COEFFICIENT AS FUNCTION OF PHASE LAG

### Determination of Boundary Conditions

The temperature response of a wall, which has one surface insulated at $x = L$ and the other surface exposed to a fluid with a temperature that varies sinusoidally at $x = 0$, is calculated as follows: First, the boundary conditions are determined with the assumption that

$$T_G = \Delta T_G e^{-i\omega\theta}$$

For $x = 0$,

$$h_G\left[\Delta T_G e^{-i\omega\theta} - T(0,\theta)\right] = -K \frac{\partial T(0,\theta)}{\partial x}$$

For $x = L$

$$-K \frac{\partial T(0,\theta)}{\partial x} = 0$$

The governing differential equation is

$$\frac{\partial^2 T(x,\theta)}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T(x,\theta)}{\partial \theta} \tag{A1}$$

For the solution to the differential equation, assume a product solution

$$T = X(x) \cdot \overline{F}(\theta) \tag{A2}$$

Then

$$\frac{\partial T}{\partial x} = \overline{F}(\theta) \cdot X'(x)$$

$$\frac{\partial^2 \overline{T}}{\partial x} = \overline{F}(\theta) \cdot X''(x)$$

and

$$\frac{\partial T}{\partial \theta} = X(x) \cdot \overline{F}'(\theta)$$

Substituting these expressions in the differential equation (A1) gives

$$\overline{F}(\theta) \cdot X''(x) = \frac{1}{\alpha} X(x) \cdot \overline{F}(\theta)$$

or

$$\frac{X''(x)}{X(x)} = \frac{1}{\alpha} \frac{\overline{F}'(\theta)}{\overline{F}(\theta)} \tag{A3}$$

Since either side of this equation is independent of the other variable, assume that each side must be equal to a constant, $\lambda^2$. Now, $\lambda^2$ can be equal to zero, greater than zero, or less than zero. Then, setting either side of equation (A3) equal to $\lambda^2$ gives

$$\frac{X''(x)}{X(x)} = \lambda^2 \tag{A4}$$

and

$$\frac{1}{\alpha} \frac{\overline{F}'(\theta)}{\overline{F}(\theta)} = \lambda^2$$

$$\frac{\overline{F}'(\theta)}{\overline{F}(\theta)} = \lambda^2 \alpha$$

Then

$$\ln \overline{F} = \lambda^2 \alpha \theta$$

$$\overline{F} = Ce^{\lambda^2 \alpha \theta} \tag{A5}$$

If $\lambda^2 = 0$, $\overline{F} = 1$ and the wall temperature T will not be a function of time. This result cannot be the case physically; therefore, the solution for $\lambda^2 = 0$ is rejected. The choice between $\lambda^2 < 0$ or $\lambda^2 > 0$ is made by attempting to solve the equation by using

$-\lambda^2$ and then $+\lambda^2$, one of which will lead to a solution. With the use of $-\lambda^2$, equation (A4) becomes

$$X''(x) + \lambda^2 \cdot X(x) = 0$$

From Wiley (ref. 3, p. 88), the solution for this equation is

$$X = C_1 e^{i\lambda x} + C_2 e^{-i\lambda x} \tag{A6}$$

Equation (A5) then becomes

$$\overline{F} = Ce^{-\lambda^2 \alpha\theta} \tag{A7}$$

This equation is periodic when $\lambda^2$ is imaginary. The solution requires that the exponent be of the form $\omega\theta$. Therefore, $\lambda^2$ is set equal to $i\omega/\alpha$, and

$$\lambda = \sqrt{i \frac{\omega}{\alpha}}$$

$$\lambda = \pm \sqrt{i} \sqrt{\frac{\omega}{\alpha}}$$

$$\lambda = \pm(1 + i) \sqrt{\frac{\omega}{2\alpha}}$$

Substituting for $\lambda$ and $\lambda^2$ in equations (A6) and (A7) gives

$$X = C_1 e^{\pm i(1+i)\sqrt{(\omega/2\alpha)}x} + C_2 e^{\mp i(1+i)\sqrt{(\omega/2\alpha)}x}$$

and

$$\overline{F} = Ce^{-i\omega\theta}$$

Substituting these solutions into equation (A2) gives a solution to the differential equation (A3) which is periodic.

Set

$$\sqrt{\frac{\omega}{2\alpha}} = \eta$$

The solution thus becomes

$$T = Ae^{\mp\eta x}e^{-i(\omega\theta\mp\eta x)} + Be^{\pm\eta x}e^{-i(\omega\theta\pm\eta x)} \tag{A8}$$

where the constants A and B are $CC_1$ and $CC_2$, respectively, and can be combined with no change in the solution. The choice of sign used in equation (A8) does not matter because the constants are arbitrary. The top sign is used in the following derivation.

Applying the boundary conditions determines the constants A and B, but first some preliminary work is necessary. The boundary conditions require the use of $\partial T/\partial x$. Taking the derivative of equation (A8) with respect to x gives

$$\frac{\partial T}{\partial x} = \eta\left[Ae^{-\eta x}e^{-i(\omega\theta-\eta x)}(i - 1) + Be^{\eta x}e^{-i(\omega\theta+\eta x)}(1 - i)\right] \tag{A9}$$

Using boundary condition 2 and setting equation (A9) equal to 0 at $x = L$ give

$$\frac{A}{B} = \frac{-e^{\eta x}e^{-i(\omega\theta+\eta x)}(1 - i)}{e^{-\eta x}e^{-i(\omega\theta-\eta x)}(i - 1)}$$

$$\frac{A}{B} = e^{2\eta x-i2\eta x}$$

$$\frac{A}{B} = e^{2\eta L(1-i)} \tag{A10}$$

Using boundary condition 1

$$\Delta T_G e^{-i\omega\theta} - Ae^{-i\omega\theta} - Be^{-i\omega\theta} = \frac{-K\eta}{h_G}\left[Ae^{-i\omega\theta}(i - 1) - Be^{-i\omega\theta}(i - 1)\right]$$

and dividing by $e^{-i\omega\theta}$ give

$$\Delta T_G - A - B = \frac{-K\eta}{h_G}\left[(A - B)(i - 1)\right] \tag{A11}$$

Note that, if the fluid temperature is assumed to be $T_G = \Delta T_G e^{+i\omega\theta}$, the constants A and B become functions of $\theta$ because $e^{-i\omega\theta}$ cannot be eliminated by division (see eq. (A11)). Solving equation (A10) for A and substituting in equation (A11) give

$$\Delta T_G - Be^{2\eta L(1-i)} - B = \frac{-K\eta}{h_G}\left\{\left[Be^{2\eta L(1-i)} - B\right](i - 1)\right\}$$

Collecting terms and solving for B give

$$B = \frac{\Delta T_G}{\left[1 - e^{2\eta L(1-i)}\right](i - 1)\frac{K\eta}{h_G} + \left[1 + e^{2\eta L(1-i)}\right]}$$

Rearranging this equation gives

$$B = \frac{\Delta T_G}{\frac{-K\eta}{h_G}\left[1 - e^{2\eta L(1-i)}\right] + \left[1 + e^{2\eta L(1-i)}\right] + i\,\frac{K\eta}{h_G}\left[1 - e^{2\eta L(1-i)}\right]}$$

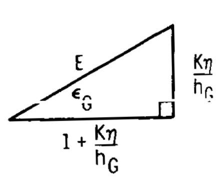
Changing to the polar form gives

$$B = \frac{\Delta T_G}{\frac{-K\eta}{h_G}\left[1 - e^{2\eta L}(\cos 2\eta L - i \sin 2\eta L)\right] + \left[1 + e^{2\eta L}(\cos 2\eta L - i \sin 2\eta L)\right] + i\,\frac{K\eta}{h_G}\left[1 - e^{2\eta L}(\cos 2\eta L - i \sin 2\eta L)\right]}$$

Collecting the terms in the denominator on i gives

$$B = \frac{\Delta T_G}{\left(1 - \frac{K\eta}{h_G}\right) + \left(\frac{K\eta}{h_G} + 1\right) e^{2\eta L} \cos 2\eta L - \frac{K\eta}{h_G} e^{2\eta L} \sin 2\eta L + i\left[\left(\frac{-K\eta}{h_G} - 1\right) e^{2\eta L} \sin 2\eta L + \frac{K\eta}{h_G} - e^{2\eta L}\frac{K\eta}{h_G} \cos 2\eta L\right]}$$

The following trigonometric substitution can be made in the previous equation:



where

$$E = \sqrt{1 + 2\frac{K\eta}{h_G} + 2\left(\frac{K\eta}{h_G}\right)^2}$$

Then

$$B = \frac{\Delta T_G}{1 - \frac{K\eta}{h_G} + Ee^{2\eta L}(\cos\epsilon_G \cos 2\eta L - \sin\epsilon_G \sin 2\eta L) + i\left[\frac{K\eta}{h_G} + e^{2\eta L}E(-\cos\epsilon_G \sin 2\eta L - \sin\epsilon_G \cos 2\eta L)\right]}$$

This equation simplifies to

$$B = \frac{\Delta T_G}{1 - \frac{K\eta}{h_G} + e^{2\eta L}E\cos(\epsilon_G + 2\eta L) + i\left[\frac{K\eta}{h_G} - e^{2\eta L}E\sin(\epsilon_G + 2\eta L)\right]}$$

The complex numbers must be in the numerator so that the phase shift accounts for the resistance of the boundary layer $h_G$. This requirement will become apparent. To put the complex numbers in the numerator, divide the denominator into the numerator in the previous equation for B by multiplying each number by the conjugate of the denominator. The following equation results:

$$B = \frac{\Delta T_G e^{-i\xi}}{\sqrt{\left[1 - \frac{K\eta}{h_G} + e^{2\eta L}E\cos(\epsilon_G + 2\eta L)\right]^2 + \left[\frac{K\eta}{h_G} - e^{2\eta L}E\sin(\epsilon_G + 2\eta L)\right]^2}} \tag{A12}$$

From equation (A10), A is

$$A = \frac{\Delta T_G e^{2\eta L} e^{-i(\xi + 2\eta L)}}{\sqrt{\left[1 - \frac{K\eta}{h_G} + e^{2\eta L}E\cos(\epsilon_G + 2\eta L)\right]^2 + \left[\frac{K\eta}{h_G} - e^{2\eta L}E\sin(\epsilon_G + 2\eta L)\right]^2}} \tag{A13}$$

where

$$\xi = \arc\tan \frac{\frac{K\eta}{h_G} - e^{2\eta L}E \sin(\epsilon_G + 2\eta L)}{1 - \frac{K\eta}{h_G} + e^{2\eta L}E \cos(\epsilon_G + 2\eta L)}$$

From equations (A8), (A12), and (A13), the solution can be written as

$$T = \frac{\Delta T_G\left[e^{\eta(2L-x)}e^{-i(\omega\theta-\eta x+\xi+2\eta L)} + e^{\eta x}e^{-i(\omega\theta+\eta x+\xi)}\right]}{\sqrt{\left[1 - \frac{K\eta}{h_G} + e^{2\eta L}E \cos(\epsilon_G + 2\eta L)\right]^2 + \left[\frac{K\eta}{h_G} - e^{2\eta L}E \sin(\epsilon_G + 2\eta L)\right]^2}} \tag{A14}$$

If the real part of the driving temperature, that is, $T_G = \Delta T_G \cos \omega\theta$, is selected, the imaginary part of the solution can be dropped. Equation (A14) then reduces to

$$T = \frac{\Delta T_G\left[e^{\eta(2L-x)} \cos(\omega\theta - \eta x + \xi + 2\eta L) + e^{\eta x} \cos(\omega\theta + \eta x + \xi)\right]}{\sqrt{\left[1 - \frac{K\eta}{h_G} + e^{2\eta L}E \cos(\epsilon_G + 2\eta L)\right]^2 + \left[\frac{K\eta}{h_G} - e^{2\eta L}E \sin(\epsilon_G + 2\eta L)\right]^2}} \tag{A15a}$$

where, as defined before but restated here,

$$E = \sqrt{1 + 2\frac{K\eta}{h_G} + 2\left(\frac{K\eta}{h_G}\right)^2} \tag{A15b}$$

and

$$\xi = \arc\tan \frac{\frac{K\eta}{h_G} - e^{2\eta L}E \sin(\epsilon_G + 2\eta L)}{1 - \frac{K\eta}{h_G} + e^{2\eta L}E \cos(\epsilon_G + 2\eta L)} \tag{A15c}$$

Equation (A15a) is the required solution. The values for $\lambda^2 > 0$ can be ruled out if the same process using $\lambda^2$ is followed. Boundary condition 1 will then yield a solution for the constants that are functions of time, which cannot be; therefore, $\lambda^2 > 0$ is rejected.

It should be stated that $\lambda^2 > 0$ will work if $T_G = \Delta T_G e^{i\omega\theta}$. The solution for this case will be the same as for equation (A15a).

## Determination of Phase Lag

The phase lag is determined with a value found for $\omega\theta$ such that $T = 0$. This condition will occur when the wall-temperature vector in the complex plane reaches $-\pi/2$ radians. Therefore, the phase lag is

$$\varphi \equiv \frac{\pi}{2} - \omega\theta \tag{A16}$$

This quantity is less than zero for the wall temperature lagging the fluid temperature. Setting $T = 0$ in equation (A15) gives

$$\frac{e^{\eta(2L-x)}}{e^{\eta x}} = -\frac{\cos \omega\theta \cos(\eta x + \xi) - \sin \omega\theta \sin(\eta x + \xi)}{\cos \omega\theta \cos(-\eta x + \xi + 2\eta L) - \sin \omega\theta \sin(-\eta x + \xi + 2\eta L)}$$

Then

$$e^{2\eta(L-x)}\left[\cos(-\eta x + \xi + 2\eta L) - \tan \omega\theta \sin(-\eta x + \xi + 2\eta L)\right] = -\cos(\eta x + \xi) + \tan \omega\theta \sin(\eta x + \xi)$$

from which,

$$\tan \omega\theta = \frac{e^{2\eta(L-x)} \cos(-\eta x + \xi + 2\eta L) + \cos(\eta x + \xi)}{e^{(2\eta(L-x)} \sin(-\eta x + \xi + 2\eta L) + \sin(\eta x + \xi)}$$

and

$$\omega\theta = \arctan \frac{e^{2\eta(L-x)} \cos\left[\xi + \eta(2L - x)\right] + \cos(\xi + \eta x)}{e^{2\eta(L-x)} \sin\left[\xi + \eta(2L - x)\right] + \sin(\xi + \eta x)} \tag{A17a}$$

The heat-transfer coefficient can then be determined from the phase lag. Solving equation (A17) for $\xi$ and then for $h_G$ gives

$$\omega\theta = \text{arc tan}\,\frac{e^{2\eta(L-x)}\left[\cos\xi\cos\eta(2L-x) - \sin\xi\sin\eta(2L-x)\right] + \cos\xi\cos\eta x - \sin\xi\sin\eta x}{e^{2\eta(L-x)}\left[\sin\xi\cos\eta(2L-x) + \cos\xi\sin\eta(2L-x)\right] + \sin\xi\cos\eta x + \cos\xi\sin\eta x}$$

$$\omega\theta = \text{arc tan}\,\frac{e^{2\eta(L-x)}\left[1 - \tan\xi\tan\eta(2L-x)\right] + \dfrac{\cos\eta x}{\cos\eta(2L-x)} - \tan\xi\,\dfrac{\sin\eta x}{\cos\eta(2L-x)}}{e^{2\eta(L-x)}\left[\tan\xi + \tan\eta(2L-x)\right] + \tan\xi\,\dfrac{\cos\eta x}{\cos\eta(2L-x)} + \dfrac{\sin\eta x}{\cos\eta(2L-x)}}$$

$$\omega\theta = \text{arc tan}\,\frac{-\tan\xi\left[e^{2\eta(L-x)}\tan\eta(2L-x) + \dfrac{\sin\eta x}{\cos\eta(2L-x)}\right] + e^{2\eta(L-x)} + \dfrac{\cos\eta x}{\cos\eta(2L-x)}}{\tan\xi\left[e^{2\eta(L-x)} + \dfrac{\cos\eta x}{\cos\eta(2L-x)}\right] + e^{2\eta(L-x)}\tan\eta(2L-x) + \dfrac{\sin\eta x}{\cos\eta(2L-x)}} \tag{A17b}$$

From equation (A16),

$$\omega\theta = \frac{\pi}{2} - \varphi$$

where $\varphi$ is less than zero for the wall temperature lagging the fluid temperature, and

$$\tan\omega\theta = \tan\left(\frac{\pi}{2} - \varphi\right)$$

$$\tan\omega\theta = \cot\varphi$$

The foregoing expression is used in equation (A17a) to write

$$\cot\varphi\left\{\tan\xi\left[e^{2\eta(L-x)} + \frac{\cos\eta x}{\cos\eta(2L-x)}\right] + e^{2\eta(L-x)}\tan\eta(2L-x) + \frac{\sin\eta x}{\cos\eta(2L-x)}\right\}$$
$$+ \tan\xi\left[e^{2\eta(L-x)}\tan\eta(2L-x) + \frac{\sin\eta x}{\cos\eta(2L-x)}\right] - e^{2\eta(L-x)} - \frac{\cos\eta x}{\cos\eta(2L-x)} = 0$$

Solving for $\tan\xi$ gives

$$\tan\xi = \frac{e^{2\eta(L-x)}\left[1 - \tan\eta(2L - x)\cot\varphi\right] + \dfrac{\cos\eta x}{\cos\eta(2L - x)} - \dfrac{\sin\eta x\cot\varphi}{\cos\eta(2L - x)}}{\left[e^{2\eta(L-x)} + \dfrac{\cos\eta x}{\cos\eta(2L - x)}\right]\cot\varphi + e^{2\eta(L-x)}\tan\eta(2L - x) + \dfrac{\sin\eta x}{\cos\eta(2L - x)}} \equiv \overline{CON} \tag{A18}$$

This expression is set equal to $\overline{CON}$ for simplicity and the definition of $\xi$ is used to write

$$\overline{CON} = \tan\xi \equiv \frac{\dfrac{K\eta}{h_G} - e^{2\eta L}E\sin(\epsilon_G + 2\eta L)}{1 - \dfrac{K\eta}{h_G} + e^{2\eta L}E\cos(\epsilon_G + 2\eta L)}$$

The solution for $h_G/K\eta$ is as follows: Keep in mind the trigonometric substitution for E and $\epsilon_G$ and write

$$\frac{K\eta}{h_G} - e^{2\eta L}\left[\frac{K\eta}{h_G}\cos 2\eta L + \left(1 + \frac{K\eta}{h_G}\right)\sin 2\eta L\right] = \overline{CON}\left\{1 - \frac{K\eta}{h_G} + e^{2\eta L}\left[\left(1 + \frac{K\eta}{h_G}\right)\cos 2\eta L - \frac{K\eta}{h_G}\sin 2\eta L\right]\right\}$$

$$\frac{K\eta}{h_G}\left\{1 - e^{2\eta L}(\cos 2\eta L + \sin 2\eta L) - \overline{CON}\left[e^{2\eta L}(\cos 2\eta L - \sin 2\eta L) - 1\right]\right\}$$

$$= e^{2\eta L}\sin 2\eta L + \overline{CON}(e^{2\eta L}\cos 2\eta L + 1)$$

Solving for $h_G/K\eta$ give

$$\frac{h_G}{K\eta} = \frac{1 - e^{2\eta L}(\cos 2\eta L + \sin 2\eta L) - \overline{CON}\left[e^{2\eta L}(\cos 2\eta L - \sin 2\eta L) - 1\right]}{e^{2\eta L}\sin 2\eta L + \overline{CON}(e^{2\eta L}\cos 2\eta L + 1)} \tag{A19}$$

Simplifying equation (A18) by first multiplying the numerator and denominator by $\tan\varphi$, and collecting terms gives

$$\overline{\mathrm{CON}} = \frac{-\left[e^{2\eta(L-x)} \tan \eta(2L - x) + \dfrac{\sin \eta x}{\cos \eta(2L - x)}\right] + \left[e^{2\eta(L-x)} + \dfrac{\cos \eta x}{\cos \eta(2L - x)}\right]\tan \varphi}{+\left[e^{2\eta(L-x)} + \dfrac{\cos \eta x}{\cos \eta(2L - x)}\right] + e^{2\eta(L-x)}\left[\tan \eta(2L - x) + \dfrac{\sin \eta x}{\cos \eta(2L - x)}\right]\tan \varphi}$$

Defining

$$A(x, L, \eta) \equiv \frac{e^{2\eta(L-x)} + \dfrac{\cos \eta x}{\cos \eta(2L - x)}}{e^{2\eta(L-x)} \tan \eta(2L - x) + \dfrac{\sin \eta x}{\cos \eta(2L - x)}}$$

$$A(x, L, \eta) \equiv \frac{e^{2\eta(L-x)} \cos \eta(2L - x) + \cos \eta x}{e^{2\eta(L-x)} \sin \eta(2L - x) + \sin \eta x}$$

Then

$$\overline{\mathrm{CON}} = \frac{A(x, L, \eta)\tan \varphi - 1}{A(x, L, \eta) + \tan \varphi} \tag{A20}$$

Equations (A19) and (A20) are the solutions presented in the text. For $x = L$,

$$A(x, L, \eta) = \cot \eta L$$

$$\overline{\mathrm{CON}} = \frac{\tan \varphi - \tan \eta L}{1 + \tan \varphi \tan \eta L}$$

$$\overline{\mathrm{CON}} = \tan(\varphi - \eta L) \tag{A21}$$

## REFERENCES

1. Anderson, Bernhard H.: Improved Technique for Measuring Heat Transfer Coefficients. Proceedings of the Fourth AFBMD/STL Symposium, Advances in Ballistic Missile and Space Technology. Vol. 2. Charles T. Morrow, ed., Pergamon Press, 1960, p. 352.
2. Bell, J. C.; and Katz, E. F.: A Method for Measuring Surface Heat Transfer Using Cyclic Temperature Variations. Heat Transfer and Fluid Mechanics Institute, ASME, 1949, pp. 243-254.
3. Wylie, Clarence R., Jr.: Advanced Engineering Mathematics. Second ed., Mc-Graw-Hill Book Co., Inc., 1960.



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