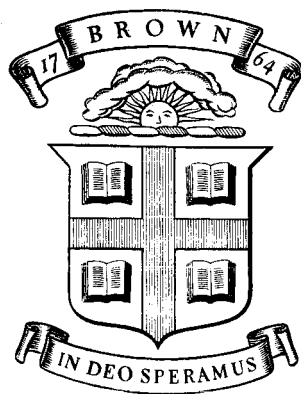




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PART I. EMISSION DOMINATED CASE

J. T. C. LIU



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On Radiative Transfer in the Low Reynolds Number  
Blunt Body Stagnation Region at Hypersonic Speeds.

Part 1. Emission Dominated Case<sup>†</sup>

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Abstract

One objective of this paper is to contribute to an understanding of radiative transfer effects in the low Reynolds number or merged layer regime of hypersonic flow about axisymmetric blunt bodies. The other objective is to illustrate how the concept of thin shock layer theory can be extended to the radiative case, where the shock structure and shock layer are radiatively coupled, through the introduction of a pseudo-jump condition across the shock structure, accompanied by an iteration technique. The radiative transfer is simplified to the emission dominated case. The gas is taken as calorically and thermally perfect, the viscosity taken as a linear function of the temperature, and the absorption coefficient

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of the gas is assumed to be gray. The problem is formulated for the flow about a blunt body but the detailed calculations are carried out for the stagnation region in order to illustrate the techniques used. Radiative cooling decreases the convective heat transfer from the corresponding radiationless case. The additional contributions to the heat transfer due to radiative transfer come from both the shock structure and the shock layer. The relative importance of these contributions are assessed in terms of the rarefaction parameter. Radiative contributions become less important toward the free-molecular range. Radiative cooling decreases the overall shock structure and shock layer thicknesses.

Nomenclature

- $a$  = nose radius  
 $a, b$  = see Eq. (3.3a)  
 $C$  = see Eq. (3.3a)  
 $C_H$  =  $q_w / \rho_\infty U_\infty (H_\infty - H_w)$ , heat transfer coefficient: ( $C_H^C$  - convective,  $C_H^R$  - radiative,  $C_H^{R,SL}$  - radiative, from shock layer,  $C_H^{R,STZ}$  - radiative, from shock transition zone)  
 $c_p, c_v$  = specific heat at constant pressure and at constant volume, respectively  
 $H$  =  $c_p T + \frac{1}{2}(u^2 + v^2)$ , total enthalpy  
 $\mathcal{I}_1, \mathcal{I}_2$  = see Eq. (3.27)  
 $I$  = see Eq. (3.28)  
 $K^2$  =  $\varepsilon \rho_\infty U_\infty a / \mu_0$ , rarefaction parameter  
 $k$  =  $(2Pr/3) \{[(4/K^2) + 1]^{1/2} - 1\}^{-1}$ , also thermal conductivity  
 $Pr$  =  $c_p \mu / k$ , Prandtl number  
 $p$  = pressure  
 $Q$  = radiative energy loss per unit mass  
 $q_w$  = rate of energy transferred to body surface (per unit time and per unit area)  
 $q^R$  = radiative energy flux  
 $R$  = gas constant  
 $Re_b$  =  $\rho_\infty U_\infty a / \mu_0$ , body Reynolds number  
 $T$  = temperature  
 $u, v$  = velocity components in the  $x, y$  directions, respectively

- $x, y$  = orthogonal coordinates along and normal to reference surface  
(see Fig. 2)
- $U_{\infty}$  = free-stream velocity
- $Z$  = distance of reference surface from axis of symmetry
- $\alpha$  = absorption coefficient
- $\beta$  = angle between reference surface and free-stream direction
- $\Gamma$  = radiative energy emission to energy convection ratio  
[see Eq. (3.9)]
- $\gamma$  =  $c_p/c_v$ , heat capacity ratio; also incomplete gamma function  
[see Eq. (3.28)]
- $\epsilon$  =  $(\gamma-1)/2\gamma$
- $\zeta$  =  $[\psi/(\rho_{\infty} U_{\infty} \pi Z^2)]^{1/2}$
- $\bar{n}$  =  $(3/4) Re_b \sin\beta \int_0^y (\mu_o/\mu) d(y/a)$
- $\theta$  =  $(H-H_w)/(H_{\infty}-H_w)$ , dimensionless enthalpy function
- $\theta_o$  = dimensionless enthalpy function for radiationless case
- $\kappa$  = mass absorption coefficient
- $\kappa_c$  =  $d\beta/dx$ , longitudinal curvature of reference surface
- $\mu$  = viscosity coefficient
- $\xi$  =  $x/a$
- $\pi$  = 3.1416
- $\rho$  = density
- $\sigma$  = Stephen-Boltzmann constant
- $\psi$  = stream function

Subscripts

$o$  = stagnation condition in the free stream

$1, \infty$  = conditions in front of the shock and in the free stream,  
respectively

$2, s$  = conditions behind the shock

$w$  = body surface conditions

## 1. Introduction

The study of radiative energy transfer in the hypersonic flow over blunt-nosed bodies is relevant to the design of entry vehicles in a flight regime in which the consideration of such an energy transfer mechanism is important to thermal protection systems. Recent fluid mechanical studies of such radiating flows have, in general, fallen into two categories: the inviscid and the fully viscous radiating shock layers.

Recently Cheng and Vincenti<sup>1</sup> presented solutions for the indirect problem of multidimensional, inviscid radiating flow behind a given paraboloidal shock wave. A review of the various earlier works on inviscid radiating shock layers is also given in this paper.

Howe and Viegas<sup>2</sup> considered a completely viscous, radiating shock layer in the stagnation region of a blunt body, using a local similarity approach. In a series of papers, which contain successive improvements on the radiative transfer model, absorption coefficient and gas properties, Hoshizaki and Wilson<sup>3,4</sup> presented solutions for the direct problem of a viscous, radiating shock layer by an integral technique. Burggraf<sup>5</sup> considered the stagnation region of such a shock layer for the emission-dominated case. He obtained asymptotic expansions for large Reynolds numbers, and clarified the radiative interaction of the viscous boundary layer with the inviscid region. In all these viscous radiating shock layer studies, the shock wave is considered to be infinitesimally thin and the usual Rankine-Hugoniot jump conditions are applied, thereby confining the range of validity up to the viscous-layer regime.<sup>6</sup>

In more recent studies of the viscous hypersonic flow over blunt bodies, Bush<sup>7</sup> and Cheng<sup>8</sup> both rule out the conceptual possibility of a flow regime in which transport effects are important throughout the shock layer but not important immediately behind the shock wave. However, Cheng<sup>8</sup> points out the advantage of using the viscous layer model for numerical computations in the moderately high Reynolds number range corresponding to the nonlinear vorticity interaction regime. Use of the viscous layer model would then eliminate the necessity of matching the inner and outer solutions and is thus particularly advantageous when more realistic gas properties are taken into account.

Numerous recent studies, then, have included the high and moderately-high Reynolds number ranges of radiating flow over blunt bodies. However, the low Reynolds number range of radiating hypersonic flows is virtually unexplored. The ability of the viscous hypersonic thin-shock layer theory to describe the essential features of the transition from continuum to free-molecule flow is well known.<sup>8,9,10</sup> It is thus natural to explore the possibility of extending such an approach to radiating, low Reynolds number hypersonic flow.

In the flow regime in which transport effects are important in the shock layer and in modifying the Rankine-Hugoniot jump conditions, the stagnation enthalpy behind the shock structure or at the outer edge of the shock layer decrease from the free stream stagnation value with decreasing body Reynolds number.<sup>9,11</sup> Any preliminary estimate of the importance of energy transfer by radiation relative to energy transfer by convection must take into account this decrease in stagnation enthalpy.

From the standpoint of radiative heat transfer, it would thus be expected that relatively higher flight speeds could be tolerated in the low Reynolds number regime.

This paper presents discussions of the effect of radiative transfer in the low Reynolds number hypersonic flow about a blunt body. The calculations are specialized to the stagnation region to better illustrate the technique used. The radiation model is simplified to that of the emission-dominated case. The radiation parameter for this case is  $\Gamma_o \alpha_o \epsilon a$ , where  $\Gamma_o$  is the radiative emission to energy convection ratio,  $\alpha_o$  the absorption coefficient and  $\epsilon a$  gives the order of the shock layer thickness. The subscript  $o$  indicates free-stream stagnation conditions. However, as was previously discussed, the free-stream stagnation conditions are not a good indication of the importance of radiative transfer because of the "shock-slip" effects. The appropriate reference condition is one based on the condition at the outer edge of the shock layer,  $\Gamma_s \alpha_s \epsilon a$ . A preliminary estimate of this parameter for various values of the rarefaction parameter,  $K^2$ , can be made through the use of Cheng's<sup>9</sup> solution  $\theta_o(1)$ . The parameter  $\Gamma_s \alpha_s \epsilon a$  is shown in Figure 1, where  $\Gamma_s \alpha_s \epsilon a \approx \theta_o^8(1) \Gamma_o \alpha_o \epsilon a$  is indicated by the dashed lines.\* The  $K^2 \rightarrow \infty$  limit occurs when shock-slip effects are absent and  $\Gamma_s \alpha_s = \Gamma_o \alpha_o$  identically. But for  $K^2 = 0(1)$ , for instance,  $\Gamma_o \alpha_o \epsilon a$  can be as high as  $10^2$  while  $\Gamma_s \alpha_s \epsilon a$  is only  $0(1)$ . The true value of  $\Gamma_s \alpha_s \epsilon a$ , which is not known a priori and is obtained only after the solution has been found, is shown by the solid lines in Figure

\*The term  $\alpha_s T_s^4$  may be replaced by  $\theta_o^8(1) \alpha_o T_o^4$  (see Section 3).

1. Because radiative transfer augments the shock-slip due to transport effects behind the shock wave structure, the dashed lines overestimate this parameter (as is evident from Figure 1).

## 2. Basic Equations

The basic equations of the two-thin-layer formulation for the description of the (non-radiating) low Reynolds number hypersonic flow over blunt bodies are obtained by Cheng.<sup>8,10</sup> The equivalence of this description to Bush's<sup>7</sup> four-layer formulation is pointed out by Cheng.<sup>8</sup>

In this section Cheng's two-layer formulation will be augmented by the inclusion of the effect of radiative transfer in the energy equations. This presupposes that radiative transfer will not alter the order-of-magnitude analyses in the thin-layer theory. Ultimate justification of this assumption must be sought a posteriori from the actual solutions obtained.\*

Throughout this paper, the simplifying assumptions of a perfect gas having constant specific heat and a linear viscosity-temperature law will be retained. It will also be assumed that the gas ahead of the shock structure is cold and neither absorbing nor emitting, that the out going radiation escapes to infinity, that the free stream is uniform, and that the wall is cold. If the smallest local photon mean free path is large compared to the thickness of the shock-transition

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\* In fact, for the radiative effects in the "order-unity" and the "weak" regimes, the thin-layer approximation is made more valid because of the decrease of shock layer and shock-transition zone thicknesses. The strong radiation case requires separate consideration.

zone and of the shock layer, then the effect of radiation can be represented by the emission-dominated case. The gas is assumed to be gray with a frequency-averaged absorption coefficient which depends on the local pressure and temperature. The contribution of radiation to the pressure and internal energy of the gas are neglected.

## 2.1 Shock-layer equations

The orthogonal coordinates used are  $x$  and  $y$ , where  $x$  is the distance along and  $y$  the distance normal to a reference surface. Following Cheng<sup>8</sup>, this reference surface is taken to be the outer edge of the shock layer. (See Figure 2).

The thin-layer approximation then yields the following set of equations for the axially symmetric shock-layer:

$$\frac{\partial}{\partial x}(\rho u Z) + \frac{\partial}{\partial y}(\rho v Z) = 0 \quad (2.1)$$

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \quad (2.2)$$

$$\frac{\partial p}{\partial y} = - \kappa_c \rho u^2 \quad (2.3)$$

$$\rho \left( u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} \right) = \frac{\partial}{\partial y} \left\{ \frac{\mu}{Pr} \frac{\partial}{\partial y} \left[ H - (1-Pr) \frac{u^2}{2} \right] \right\} - \rho Q \quad (2.4)$$

$$p = \rho R T \quad (2.5)$$

The radiative heat loss term,  $-\rho Q$ , is the energy lost per unit volume. For the emission-dominated case

$$-\rho Q = -4\rho\kappa\sigma T^4 \quad (2.6)$$

and  $-\rho Q$  then has the form of local heat sinks.

## 2.2 Shock-transition zone equations

Here  $y$  is measured from and normal to the outer edge of the shock layer and  $x$  is the distance along this layer. The thin-layer approximation then yields the following set of equations for the shock-transition zone:

$$\rho v = \rho_1 v_1 \quad (2.7)$$

$$p + \rho_1 v_1 v - \frac{4}{3} \mu \frac{\partial v}{\partial y} = \rho_1 v_1^2 \quad (2.8)$$

$$\rho_1 v_1 u - \mu \frac{\partial u}{\partial y} = \rho_1 v_1 u_1 \quad (2.9)$$

$$\rho_1 v_1 \frac{\partial H}{\partial y} = \frac{\partial}{\partial y} \left\{ \frac{\mu}{Pr} \frac{\partial}{\partial y} \left[ H - (1-Pr) \frac{u^2}{2} - \left( 1 - \frac{4Pr}{3} \right) \frac{v^2}{2} \right] \right\} - \rho Q \quad (2.10)$$

$$p = \rho RT \quad (2.11)$$

Equations (2.1) through (2.3) are obtained through a simple integration, while (2.4) remains a second-order differential equation. The radiative heat loss,  $-\rho Q$ , is again given by (2.6). The subscript 1 indicates free stream conditions, where  $\rho_1 = \rho_\infty$ ,  $u_1 = U_\infty \cos \beta$ ,  $v_1 = -U_\infty \sin \beta$ , and  $\beta$  is the local inclination of the reference surface.

## 2.3 The unreduced problem

In the nonradiating problem (i.e., for  $-\rho Q = 0$ ) the set of equations (2.7) through (2.10), with Eq. (2.10) integrated once, can be reduced to give a set of shock conservation relations that provide the outer boundary condition for the shock layer.<sup>8,9,10</sup> In the present problem, only Eqs. (2.7) through (2.9) may be reduced; such reduction yields

$$\rho_2 v_2 = \rho_1 v_1 \quad (2.12)$$

$$p_2 = \rho_1 v_1^2 \quad (2.13)$$

$$\rho_1 v_1 (u_2 - u_1) = \left( \mu \frac{\partial u}{\partial y} \right)_2 \quad (2.14)$$

Here the subscript 2 indicates the condition at the outer edge of the shock layer where  $v$  and  $\partial v / \partial y$  are higher-order quantities. Equations (2.12) through (2.14) are the proper jump conditions, independent of the shock structure problem.

Integration of Eq. (2.10) once across the shock-transition zone yields

$$\rho_1 v_1 (H_2 - H_1) = \left\{ \frac{\mu}{Pr} \frac{\partial}{\partial y} \left[ H - (1 - Pr) \frac{u^2}{2} \right] \right\}_2 - \int_1^2 \rho Q dy \quad (2.15)$$

The last term on the right-hand side of Eq. (2.14) represents the accumulated radiative heat loss through the shock-transition zone. Because of this history-dependent term, a jump condition for the total enthalpy is no longer available. That is, the shock layer can no longer be described independently of the details of the shock-transition zone. The two layers are thus radiatively coupled.

### 3. Solution of the Two Radiatively-Coupled Layers

Although the radiative coupling of the shock-transition zone to the shock layer is apparent, the method of solution to be described in this section will adhere to the uncoupled, non-radiating two-layer formulation. Thus, an attempt shall be made to derive a "psuedo-jump" condition from the radiating shock-transition zone energy equation. This would provide the boundary condition for the outer edge of the radiating shock layer.

However, such a condition contains the as yet unknown cumulative radiative heat loss through the shock-transition zone. The solution of the shock layer energy equation, using such a psuedo-jump condition, would then provide the boundary condition for the shock-transition zone at the interface. An overall iterative scheme can then be devised.

For a given boundary condition at the shock-interface, the shock-transition zone solution can be calculated and the cumulative heat loss through this layer obtained. For a given cumulative heat loss at the shock interface, the shock layer solution can be calculated and the boundary condition for the shock-transition zone obtained. The final solution is obtained when consistency is achieved in such a "phase-plane".

The solutions for the individual layers used in the overall iteration scheme can be obtained by actually solving the differential equations appropriate to the respective layers. However, the technique of solution to be described below is, again, through iteration.

### 3.1 Transformation of the shock-transition zone equations

Following Cheng<sup>8,10</sup>, we introduce the following dimensionless variables for the shock-transition zone:

$$\begin{aligned}\bar{u} &= u/u_1, \quad \bar{v} = v/v_1, \quad \bar{p} = p/\rho_1 v_1^2, \quad \bar{T} = T/T_0 \\ \bar{\rho} &= \rho/\rho_1\end{aligned}\tag{3.1}$$

The transformed normal distance measured from the shock interface,  $y_2$ , is defined as

$$\bar{\eta} = \frac{3}{4} \sin\beta \operatorname{Re}_b \int_{y_2=0}^y \left(\frac{u_0}{u}\right) d\left(\frac{y}{a}\right)\tag{3.2}$$

where  $Re_b = \rho_\infty U_\infty a / \mu_o$ .

The gray volumetric absorption coefficient,  $\rho\kappa$ , is assumed to be of the form

$$\alpha = \rho\kappa = C p^a T^b \quad (3.3a)$$

where  $C$ ,  $a$  and  $b$  are constants. For instance, Traugott<sup>12</sup> uses  $a = 1$ ,  $b = 4$  and  $C = 5.37 \times 10^{-25} \text{ sec}^2/\text{gm}^\circ\text{K}$ , where  $p$  is in units of  $\text{dynes/m}^2$  and  $T$  is in  $^\circ\text{K}$ . The dimensionless form of the absorption coefficient  $\bar{\alpha}$  is then written

$$\alpha = \alpha_o(\bar{\alpha}) = \alpha_o(\bar{p}^a \bar{T}^b) \quad (3.3b)$$

where

$$\alpha_o = C(\rho_\infty U_\infty^2)^a T_o^b \quad (3.3c)$$

is based on free stream stagnation conditions.

Substituting Eqs. (3.1) through (3.3) into Eqns. (2.7) through (2.11), one obtains

$$\bar{\rho} \bar{v} = 1 \quad (3.4)$$

$$\bar{p} + \bar{v} + \frac{\partial \bar{v}}{\partial \bar{\eta}} = 1 \quad (3.5)$$

$$\bar{u} + \frac{3}{4} \frac{\partial \bar{u}}{\partial \bar{\eta}} = 1 \quad (3.6)$$

$$\begin{aligned} \frac{\partial \bar{H}}{\partial \bar{\eta}} + \frac{3}{4Pr} \frac{\partial^2}{\partial \bar{\eta}^2} [\bar{H} - (1-Pr) \cos^2 \beta \bar{u}^2 - (1 - \frac{4Pr}{3}) \sin^2 \beta \bar{v}^2] \\ = \left( \frac{\Gamma_o \alpha_o a}{Re_b} \right) \frac{4}{3} \bar{p}^a \bar{T}^{b+5} \end{aligned} \quad (3.7)$$

$$\bar{p} = \epsilon \bar{\rho} \bar{T} \csc^2 \beta \quad (3.8)$$

The dimensionless total enthalpy is defined as

$$\bar{H} = \bar{T} + \cos^2 \beta \bar{u}^2 + \sin^2 \beta \bar{v}^2$$

The linear temperature viscosity law,  $\mu = \mu_o T/T_o$ , is used to arrive at the right-hand side of Eq. (3.7). The radiative-emission to convection ratio is defined as

$$\Gamma_o = \frac{4\sigma T_o^4}{\rho_o U_o c_p T_o} \quad (3.9)$$

where the subscript  $o$  indicates its evaluation at free stream stagnation conditions.

Because of the elimination of the coefficient of viscosity through the transformation given by Eq. (3.2), the two momentum equations, (3.5) and (3.6), are explicitly uncoupled from the energy equation. Thus Cheng's<sup>8</sup> forms of the solutions for  $\bar{v}$  and  $\bar{u}$  are available.

### 3.2 Transformation of the shock layer equations

Again following Cheng<sup>8,9,10</sup>, the shock layer equations are transformed from the  $(x,y)$  coordinates to the  $(x,\psi)$  Von Mises coordinates. With the introduction of the stream function,  $\psi$ , which is related to the velocity components as

$$2\pi Z \rho u = \frac{\partial \psi}{\partial y}, \quad 2\pi Z \rho v = - \frac{\partial \psi}{\partial x}$$

Eq. (2.1) is then identically satisfied. Equations (2.2) through (2.4), with the use of Eq. (2.6), then become

$$\frac{\partial u}{\partial x} = - \frac{1}{\rho u} \frac{\partial p}{\partial x} + (2\pi Z)^2 \frac{\partial}{\partial \psi} (\mu \rho u \frac{\partial u}{\partial \psi}) \quad (3.10)$$

$$\frac{\partial p}{\partial \psi} + \frac{\kappa u}{2\pi Z} = 0 \quad (3.11)$$

$$\frac{\partial H}{\partial x} = (2\pi Z)^2 \frac{\partial}{\partial \psi} \left\{ \frac{\mu \rho u}{Pr} \frac{\partial}{\partial \psi} \left[ H + (Pr-1) \frac{u^2}{2} \right] \right\} - \frac{\alpha 4\sigma T_o^4}{\rho u} \quad (3.12)$$

The no-slip boundary conditions at the wall, a good approximation in the case of cold walls, are

$$\psi = 0: \quad u = 0, \quad H = H_w$$

The boundary conditions at the outer edge of the shock layer are obtained from Section 2.3:

$$\left. \begin{aligned} \psi &= \rho_\infty U_\infty^2 Z^2: & p &= \rho_\infty U_\infty^2 \sin^2 \beta \\ u &= U_\infty \cos \beta - \frac{2\pi Z \mu \rho u}{\rho_\infty U_\infty \sin \beta} \frac{\partial u}{\partial \psi} \\ H &= H_\infty - \frac{2\pi Z \mu \rho u}{\text{Pr} \rho_\infty U_\infty \sin \beta} \frac{\partial}{\partial \psi} \left[ H - (1 - \text{Pr}) \frac{u^2}{2} \right] + \frac{q_2^R}{\rho_\infty U_\infty \sin \beta} \end{aligned} \right\} \quad (3.13)$$

The outer boundary condition for  $H$  is the "psuedo-jump" condition, where

$$q_2^R = \int_{\infty}^{y_2} \rho \kappa 4 \sigma T^4 dy \quad (3.14)$$

is an integral across the shock-transition zone.

With the introduction of the dimensionless dependent variables

$$\left. \begin{aligned} \bar{p} &= p / (\rho_\infty U_\infty^2 \sin^2 \beta) \\ \bar{u} &= u / (U_\infty \cos \beta) \\ \theta &= (H - H_w) / (H_\infty - H_w) \end{aligned} \right\} \quad (3.15)$$

and the independent variables

$$\xi = x/a, \quad \zeta = [\psi / (\rho_\infty U_\infty^2 Z^2)]^{1/2} \quad (3.16)$$

Eqs. (3.10) through (3.13) can then be transformed into a form suitable for the calculation of flow over (smooth) blunt bodies.<sup>10</sup> However, the stagnation region, which is the zeroth order term in a series expansion in

$\xi$ , must be considered first, in order to provide initial information for calculations around the body.

In this paper, only the stagnation region will be discussed; in this region the application of the "psuedo-jump" condition and the overall iteration scheme can be illustrated simply.

### 3.3 The stagnation region

In the stagnation region,  $\sin\beta \approx 1$ ,  $\cos\beta \approx \xi$  and  $Z/a \approx \xi$ . Then, in the leading approximation the following ordinary differential equations are obtained:

$$\bar{u}^2 - \zeta \bar{u} \bar{u}' = [\bar{u}/K^2 \zeta] [(\bar{u}/\zeta) \bar{u}']' \quad (3.16)$$

$$\bar{u}(0) = 0, \quad \bar{u}(1) = 1 - \bar{u}(1) \bar{u}'(1)/K^2 \quad (3.17)$$

$$-\zeta \bar{u} \theta' = [\bar{u}/(\text{Pr} K^2 \zeta)] [(\bar{u}/\zeta) \theta']' - \Gamma_0 \alpha_0 \epsilon a \theta^9 \quad (3.18)$$

$$\theta(0) = 0, \quad \theta(1) = 1 - \bar{u}(1) \theta'(1)/(\text{Pr} K^2) + \bar{q}_2^R \quad (3.19)$$

where  $\bar{q}_2^R = q_2^R/(\rho_\infty U_\infty^3/2)$ , and primes indicate differentiation with respect to  $\zeta$ .

The momentum equation (3.16) is uncoupled from the energy equation (3.18) through the use of a linear viscosity-temperature law and the neglect of the tangential pressure gradient term. This term is of order  $K^2$  and is not of importance in the  $K^2 = O(1)$  or merged layer regime of flow. (For further discussion of this point, see Cheng.<sup>8</sup>) The solution obtained by Cheng<sup>9</sup> is

$$\bar{u} = \text{Pr} K^2 \zeta / (3K) \quad (3.20)$$

Hence, with the use of (3.20), the final form of the shock layer energy equation and its boundary conditions becomes

$$\left. \begin{aligned} \theta'' + 3k\zeta^2\theta' &= (\Gamma_o \alpha_o \epsilon a) [3k/(\text{Pr}K^2)] \theta^9 \\ \theta(0) &= 0, \quad \theta(1) = 1 - \theta'(1)/3k + \frac{R}{q_2} \end{aligned} \right\} \quad (3.21)$$

The shock-transition zone energy equation, when specialized to the stagnation region, becomes, with primes indicating differentiation with respect to  $\bar{\eta}$

$$\left. \begin{aligned} \bar{H}'' + \bar{H}' &= (4/3)(\Gamma_o \alpha_o \epsilon a/K^2) \epsilon(\bar{H} - \bar{v}^2)^{10} / \bar{v} \\ \bar{H}(\infty) &= 1, \quad \bar{H}(0) = \theta(1) \end{aligned} \right\} \quad (3.22)$$

where the shock-interface is taken as located at  $\bar{\eta} = 0$ . Cheng's solution<sup>8,10</sup> for the shock-transition zone momentum equations are

$$\bar{u} = 1 - (1 - \bar{u}_2) e^{-4\bar{\eta}/3} \quad (3.23)$$

and

$$(1 - 2\bar{v}_*)\bar{\eta} = \ln[(\bar{v} - \bar{v}^*)^{\bar{v}^*}/(1 - \bar{v})] + \text{constant} \quad (3.24)^\dagger$$

where  $\bar{v}^* = \bar{v}_*/(1 - \bar{v}_*)$  and  $\bar{v}_* = \epsilon \bar{H}_2$ . See Cheng<sup>8</sup> for details of the approximate solution for  $\bar{v}$ .

In terms of the shock-transition zone variables,  $\frac{R}{q_2}$  in Eq. (3.21) is

$$\frac{R}{q_2} = (4/3) (\Gamma_o \alpha_o \epsilon a/K^2) \int_{\infty}^0 [\epsilon(\bar{H} - \bar{v}^2)^{10} / \bar{v}] d\bar{\eta} \quad (3.25)$$

<sup>†</sup>The exact value of the constant of integration is not important. The constant could be fixed by matching  $\partial\bar{v}/\partial\bar{\eta}$  with that of the shock layer solution at  $\bar{\eta}=0$  or it could be entirely omitted. In both cases, the normal velocity and temperature profiles are expected to be discontinuous at the interface to order  $\epsilon^2$ . (See Reference 8). In this paper, for simplicity, the constant is omitted.

Since the integrand in Eq. (3.25) is positive and the integration is taken from  $\infty$  to 0,  $\bar{q}_2^R$  is a negative number, representing a further mode of heat loss besides conduction at the shock interface.

The overall iteration scheme is as follows: The parameters  $\Gamma_o \alpha_o \epsilon a$ ,  $\epsilon$ ,  $K^2$  and  $Pr$  are first specified. We decouple Eq. (3.21) from Eq. (3.22) by assuming various values of  $\bar{q}_2^R$  to obtain the solution for  $\theta(\zeta)$ . From these solutions,  $\theta(1)$  (and hence  $\theta'(1)$ ) is obtained as a function of the assumed values of  $\bar{q}_2^R$ .

On the other hand, Eq. (3.22) may be decoupled from Eq. (3.21) by assuming various values of  $\theta(1)$  in the solutions for  $\bar{H}$ . From these solutions  $\bar{q}_2^R$  can be calculated from Eq. (3.25) and obtained as a function of the assumed values of  $\theta(1)$ . The intersection of the two "independent" solution curves in the  $\bar{q}_2^R - \theta(1)$  phase plane then gives the final consistent overall solution.

Because of the nonlinearity of Eqs. (3.21) and (3.22), analytical solutions are not expected. The technique of solving the individual decoupled Eqs. (3.21) and (3.22) is a matter of choice. They could be numerically integrated, subject to their respective two-point boundary conditions. However, here their solutions will be obtained by iteration.

To proceed, it is assumed that the right-hand sides of the differential equations in Eqs. (3.21) and (3.22) can be treated as known nonhomogenous terms. Upon applying the respective boundary conditions, we obtain for the shock layer

$$\begin{aligned} \theta(z) = & \Gamma_o \alpha_o \epsilon a (3k^{1/3}/PrK^2) \{ \mathcal{J}_2(z) - \theta_o(z) [\mathcal{J}_2(k^{1/3}) + e^{-k} \mathcal{J}_1(k^{1/3})/3k^{2/3}] \} \\ & + (1 + \bar{q}_2^R) \theta_o(z) \end{aligned} \quad (3.26)$$

where  $z = k^{1/3} \zeta$

$$\left. \begin{aligned} \mathcal{J}_1(z) &= \int_0^z e^{-\bar{z}^3} \theta^9(\bar{z}) d\bar{z} \\ \mathcal{J}_2(z) &= \int_0^z e^{-\bar{z}^3} \mathcal{J}_1(\bar{z}) d\bar{z} \end{aligned} \right\} \quad (3.27)$$

and

$$\theta_0(z) = I(z) / [I(k^{1/3}) + e^{-k} / (3k^{2/3})]$$

is the solution for the nonradiating stagnation region shock layer.<sup>9</sup> The integral  $I(z)$  is defined by

$$I(z) = \int_0^z e^{-z^3} dz = \frac{1}{3} \gamma\left(\frac{1}{3}, z^3\right), \quad (3.28)$$

which is related to the incomplete gamma function of order  $1/3$  and argument  $z^3$ .

For the shock-transition zone, we obtain

$$\bar{H} = 1 - [1 - \theta(1) - \int_0^{\bar{n}} e^{\eta' \bar{q}^R(\eta')} d\eta'] e^{-\bar{n}} \quad (3.29)$$

where

$$\bar{q}_2^R(\bar{n}) = (\Gamma_0 \alpha_0 \varepsilon a / K^2) (4/3) \int_{\infty}^{\bar{n}} [\varepsilon (\bar{H} - \bar{v}^2)^{10} / \bar{v}] d\eta \quad (3.30)$$

In Eq. (3.29),  $\theta(1)$  is the value of the dimensionless shock layer enthalpy function at the interface  $\zeta = 1$  (or  $z = k^{1/3}$ ), which is assumed known. The shock-transition zone velocity,  $\bar{v}$ , is in the form given by Eq. (3.24) and is consistent with the assumed  $\theta(1)$ .

An alternate form of Eq. (3.29), in terms of  $\bar{T}$ , can be obtained:

$$\bar{T} = 1 - \bar{v}^2 - [1 - \theta(1) - \int_0^{\bar{n}} e^{\eta' \bar{q}^R(\eta')} d\eta'] e^{-\bar{n}} \quad (3.31)$$

with

$$\bar{q}^R(\bar{\eta}) = (\Gamma_o \alpha_o \epsilon a / K^2) (4/3) \int_{\infty}^{\bar{\eta}} (\epsilon \bar{T}^{10} / \bar{v}) d\bar{\eta} \quad (3.32)$$

#### 4. Results and Discussion

As anticipated in Section 3.3, the intersection of the shock layer and shock-transition zone solution curves in the  $\theta(1) - \bar{q}_2^R$  phase plane would give the final consistent solution. Each point on the respective solution curves is here obtained by iteration, using the energy equations in the form given by Eq. (3.26) and either (3.29) or (3.31). The calculations begin as follows: first, the parameters  $\Gamma_o \alpha_o \epsilon a$ ,  $K^2$ ,  $\epsilon$  and  $Pr$  are fixed. Then the radiationless shock layer solution<sup>9</sup> (i.e. fixing  $\theta(1)$ ) corresponding to these values of  $K^2$ ,  $\epsilon$ , and  $Pr$ , is used to obtain  $\bar{v}$  in the shock-transition zone. Then  $\bar{T}$  in the shock-transition zone is obtained from Eq. (3.31) as accurately as desired through iteration, the zeroth approximation used being the radiationless solution<sup>8,10</sup> for  $\bar{T}$ . Thus  $\bar{q}_2^R$  can be obtained from Eq. (3.25) for the particular value of  $\theta(1)$ ; this corresponds to the first circle in Figure 3. The next step is to use this value of  $\bar{q}_2^R$  in Eq. (3.16). The iteration begins by using the radiationless solution,  $\theta_o$ , as the zeroth approximation in the integrals  $\mathcal{I}_1$  and  $\mathcal{I}_2$ . Again,  $\theta$  is obtained to the desired accuracy for the particular value of  $\bar{q}_2^R$  used, and this yields a new  $\theta(1)$  which corresponds to the first square in Figure 3. In this figure the horizontal paths indicate fixed  $\theta(1)$  and the vertical paths indicate fixed  $\bar{q}_2^R$ . Such a procedure produces a spiral path in approaching the final consistent solution, as

is dictated by physical reasoning from the implications of such a procedure.

The particular values of parameters for the trajectory shown in the phase plane of Figure 3 are  $\Gamma_0 \alpha_0 \epsilon a = 100$ ,  $K^2 = 1$ ,  $\epsilon = 0.1428$  ( $\gamma = 1.40$ ) and  $Pr = 0.75$ . From Figure 1 we see that this combination of  $\Gamma_0 \alpha_0 \epsilon a$  and  $K^2$  falls in the regime where radiative emission effects are of "order one". When calculations are performed in the "weak radiation" regime, only a single vertical path in the  $\theta(1) - \bar{q}_2^R$  plane is necessary, which corresponds to a small perturbation away from the radiationless case. Since for the "strong radiation" regime, radiation becomes the dominant mode of energy transfer, this regime must be considered separately. The techniques described here then encompass both the order-one and weak radiation cases, but not the strong radiation case.

Of particular interest are the contributions of the various modes of heat transfer to the body surface. In this discussion the local surface heat transfer rates will be made dimensionless with respect to  $\rho_\infty U_\infty (H_\infty - H_w)$  ( $= \rho_\infty U_\infty^3 / 2$  for a highly cooled wall and a hypersonic free stream) to give the coefficients of heat transfer  $C_H$ . The contributions from the radiative heat flux to the wall are obtained by the following integrals:

$$q_w^R = q_w^{R,SL} + q_w^{R,STZ} = \frac{1}{2} \int_0^{y_s} \rho \kappa 4\sigma T^4 dy_{SL} + \frac{1}{2} \int_{y_s}^{\infty} \rho \kappa 4\sigma T^4 dy_{STZ} \quad (4.1)$$

The radiative heat flux to the wall is taken as half of the total energy radiated from the shock layer and the shock-transition zone. The thin

radiating slab approximation is implicit in this consideration. This would be consistent with the thin-layer approximations already used in treating the shock layer and shock-transition zone. In terms of the heat transfer coefficient and of the respective variables used for the two layers, then:

$$C_H^{R,SL} = [(\Gamma_o \alpha_o \epsilon a) 3k^{2/3} / (2PrK^2)] \int_0^{k^{1/3}} \theta^9(\bar{z}) d\bar{z} \quad (4.2)$$

$$C_H^{R,STZ} = [(\Gamma_o \alpha_o \epsilon a) 4 / (6K^2)] \int_0^\infty (\epsilon \bar{T}^{10} / \bar{v}) d\bar{\eta} = -\bar{q}_2^R / 2 \quad (4.3)$$

The form of the absorption coefficient given by Eq. (3.3) shows its dependence on pressure. In the shock layer  $\bar{p} = 1$  in the stagnation region. In the shock-transition zone,  $\bar{p} = \epsilon \bar{T} / \bar{v}$ . As with the  $\bar{p}$  term in the momentum equation (3.5), the contribution of  $\epsilon \bar{T} / \bar{v}$  does not become important until the region near the shock interface is approached, where  $\bar{v} = 0(\epsilon)$ . In general, because of this pressure dependence, it is expected that the contribution to  $C_H^R$  from the shock layer,  $C_H^{R,SL}$ , is more important than that from the shock transition zone,  $C_H^{R,STZ}$ . However, for decreasing  $K^2$ , the shock layer thickness,  $y_s / \epsilon a$ , decreases (for the case of a cold wall) while the shock transition zone stretches out. In this case,  $C_H^{R,STZ}$  overtakes  $C_H^{R,SL}$ . This is shown in Figure 4 for  $\Gamma_o \alpha_o \epsilon a = 10$ ,  $\epsilon = 0.1428$ ,  $Pr = 0.75$ , where  $C_H^R$  is broken down into the contributions from the two regions. Since radiative emission decreases the shock layer temperature, and hence its gradient, the convective heat transfer coefficient,  $C_H^C$ , becomes lower than in the

radiationless case  $C_{H_0}$ . The total heat transfer coefficient,  $C_H^{R+C}$ , is asymptotic to  $C_H^R$  at the higher end of  $K^2$  and asymptotic to  $C_H^C$  at the lower end of  $K^2$ . Radiative effects become negligible for small  $K^2$  for a fixed  $\Gamma_0 \alpha_0 \epsilon a$ , since the lower temperature level in this range of  $K^2$  puts the radiative energy transfer in the "weak" range, (see Figure 1). The effect of changes in  $\Gamma_0 \alpha_0 \epsilon a$  on  $C_H^{R+C}$  is shown in Figure 5.

The dimensionless enthalpy function at the shock interface,  $\theta(1)$ , is shown as a function of  $K^2$  in Figure 6, and the shock detachment distance, or the thickness of the shock layer, is shown in Figure 7. Because the overall temperature level in the shock layer decreases for increasing  $\Gamma_0 \alpha_0 \epsilon a$ , the shock layer thickness decreases correspondingly.

Some typical stagnation region profiles are shown in Figure 8 for the case of  $K^2 = 1$ . The discontinuity in the temperature profile at the shock interface is as expected in the thin shock layer treatment [see Footnote to Eq. (3.24)]. The explicit form of the tangential and normal velocity profiles are identical to the radiationless case in their respective transformed plane. In particular, the tangential velocity differs from the corresponding radiationless case in the physical plane because of coordinate stretching only. The normal velocity differs from the corresponding radiationless case because of the modifications upon  $\bar{H}_2$  in addition to coordinate stretching. These differences become more pronounced with increasing  $\Gamma_0 \alpha_0 \epsilon a$ . The effects of radiative cooling on the temperature profile is illustrated. The temperature profiles become less steep near the wall and the entire

shock layer and shock-transition zone decrease in thickness with increasing importance of radiation.

Throughout this paper, comparisons of the present solutions have been made only with the corresponding radiationless solutions. Comparisons with other radiative solutions at the higher Reynolds number range have not been made. Because of the various differing assumptions on the gas properties, the equation of state and the radiation model, such comparisons are not found to be meaningful.

Since the gas near the cold wall and in the outer portion of the shock-transition zone is relatively cool, reabsorption effects would become important. An extension to the corresponding low Reynolds number hypersonic flow, including reabsorption effects, is under way.

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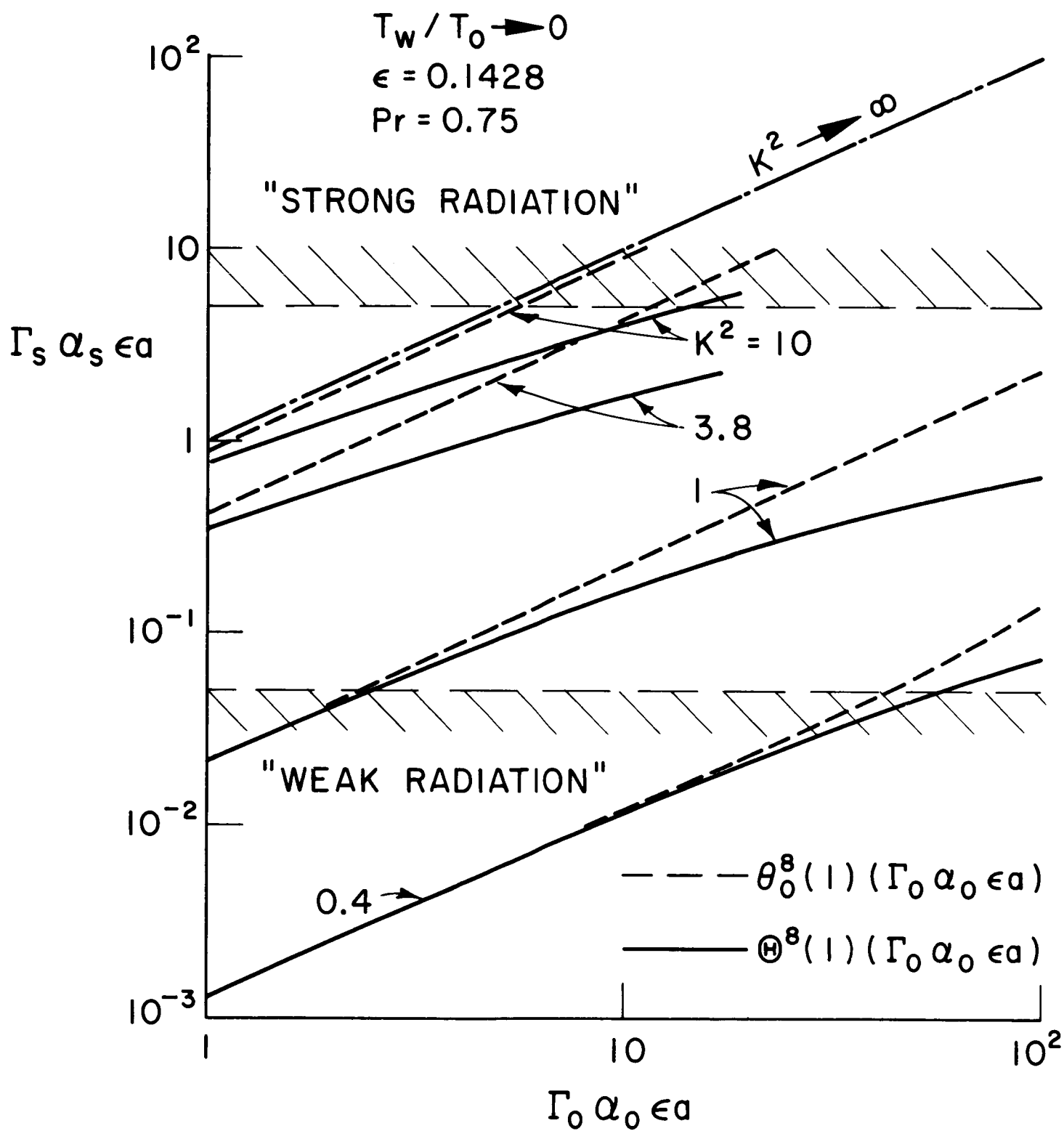


Figure 1. The radiation parameter.

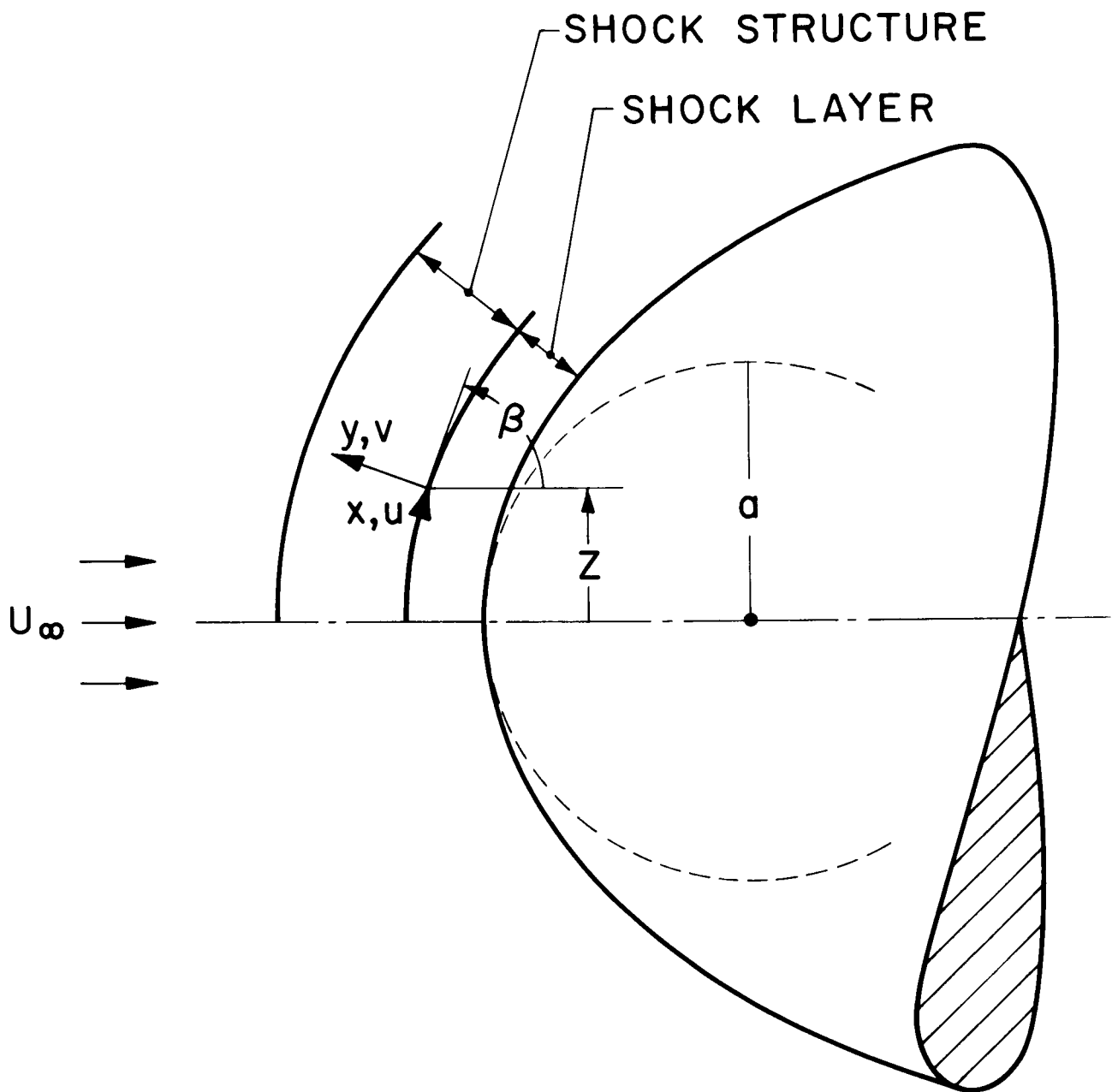


Figure 2. Thin shock structure and shock layer notation. Schematic.

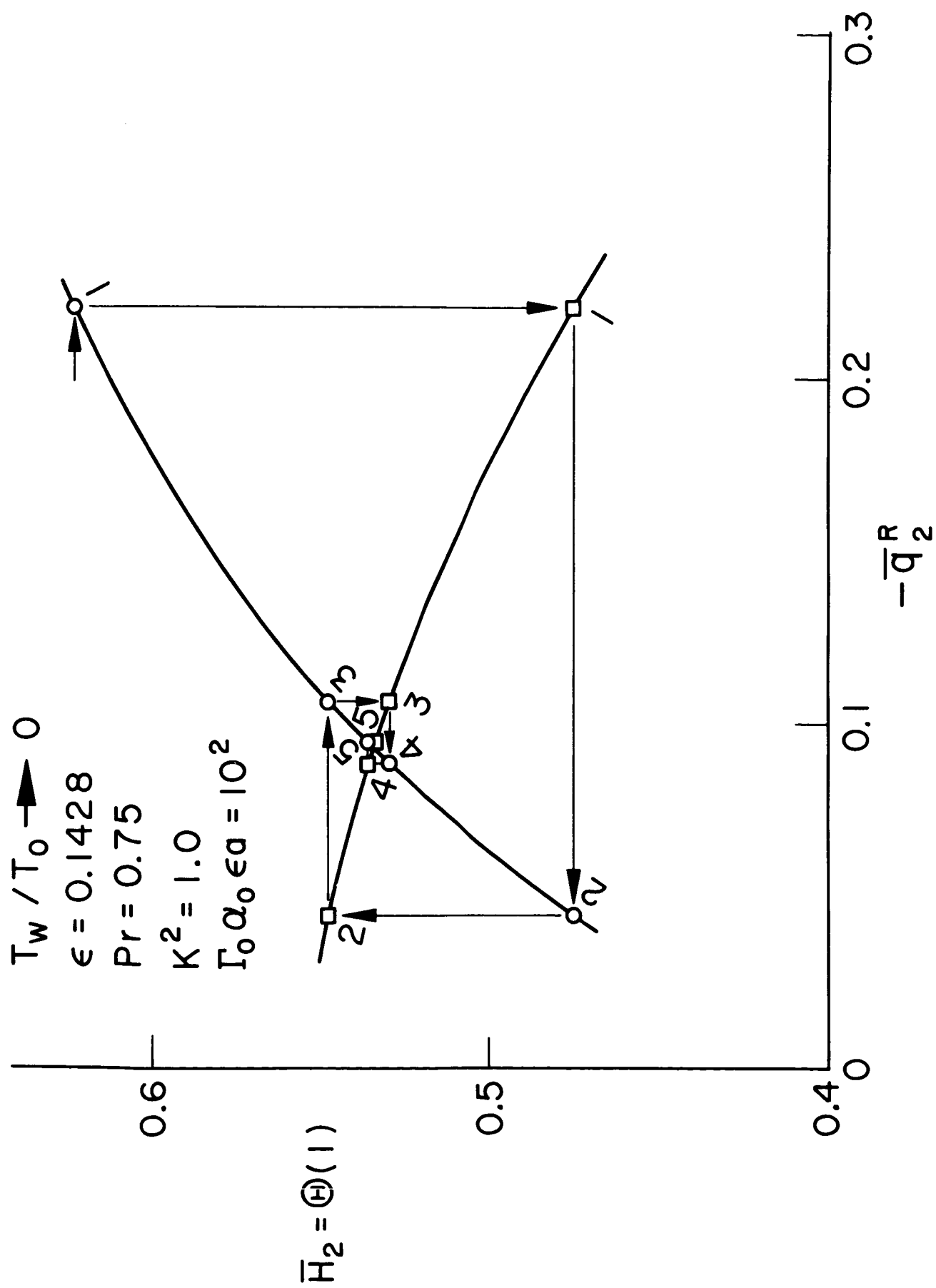


Figure 3. The  $\theta(1) - \bar{q}_2^R$  phase plane.

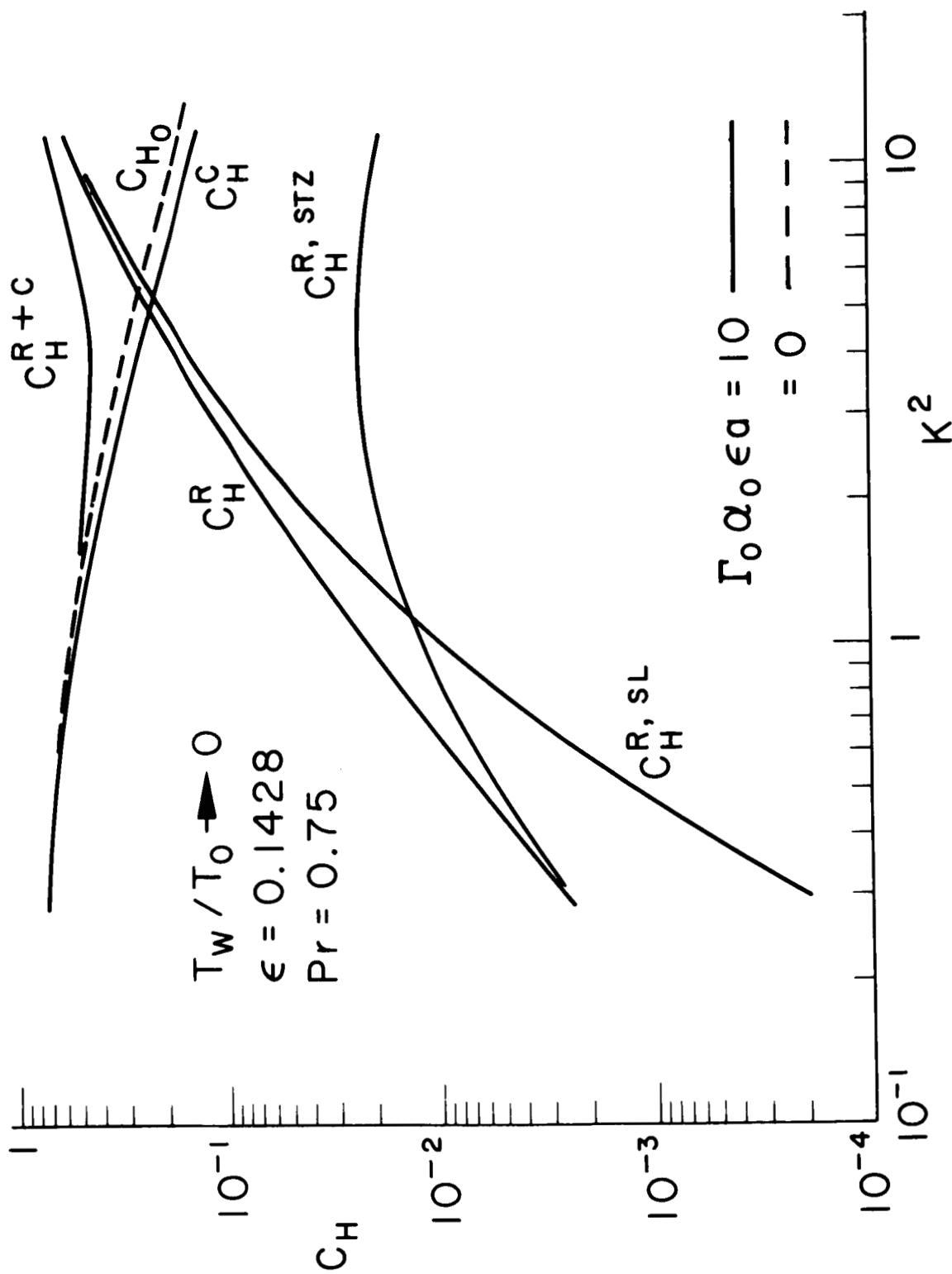


Figure 4. Contributions to the heat transfer coefficient: convective heat transfer and radiative heat transfer from shock layer and shock-transition zone as functions of the rarefaction parameter.

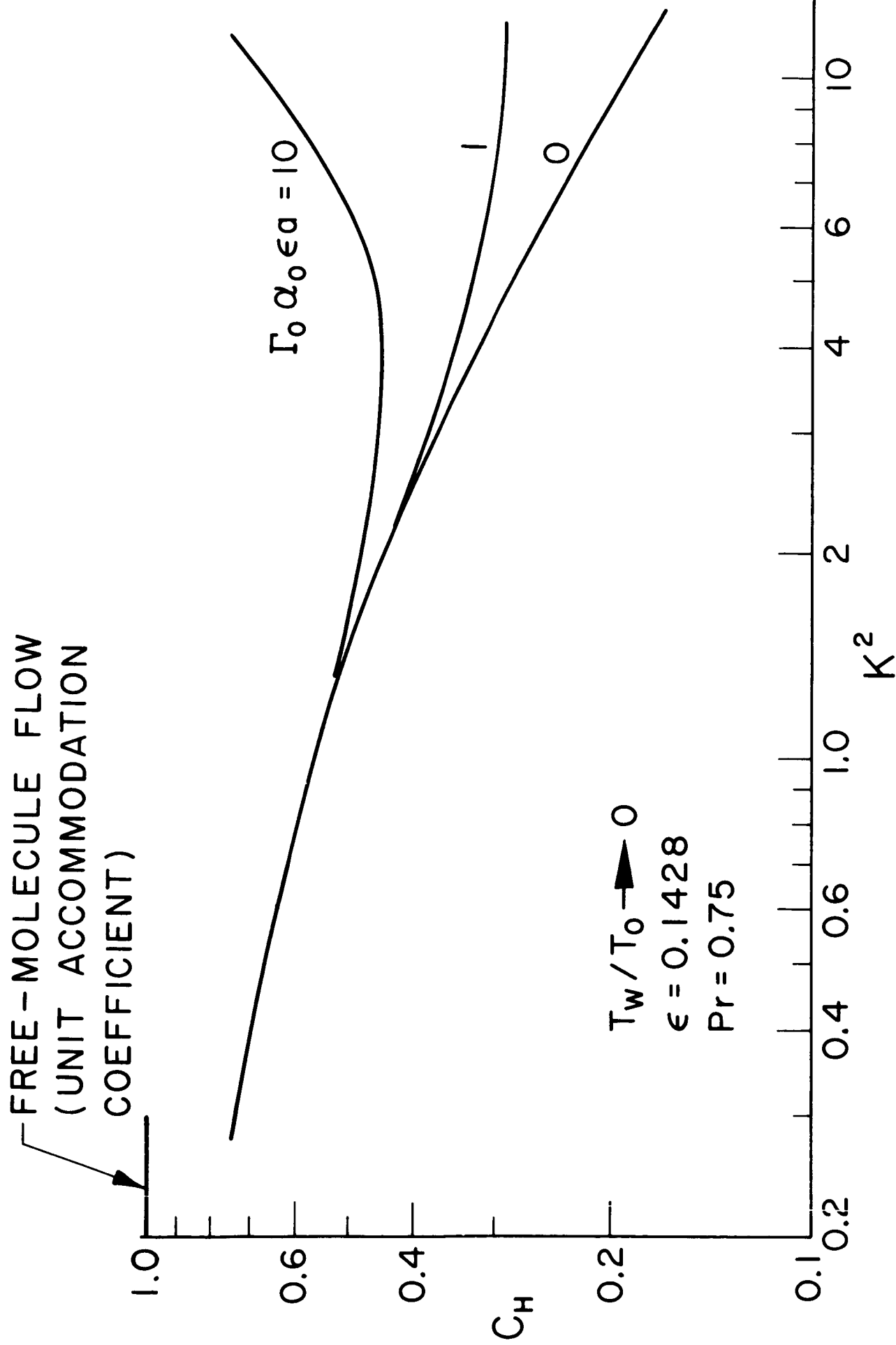


Figure 5. Overall heat transfer coefficient as a function of the rarefaction parameter.

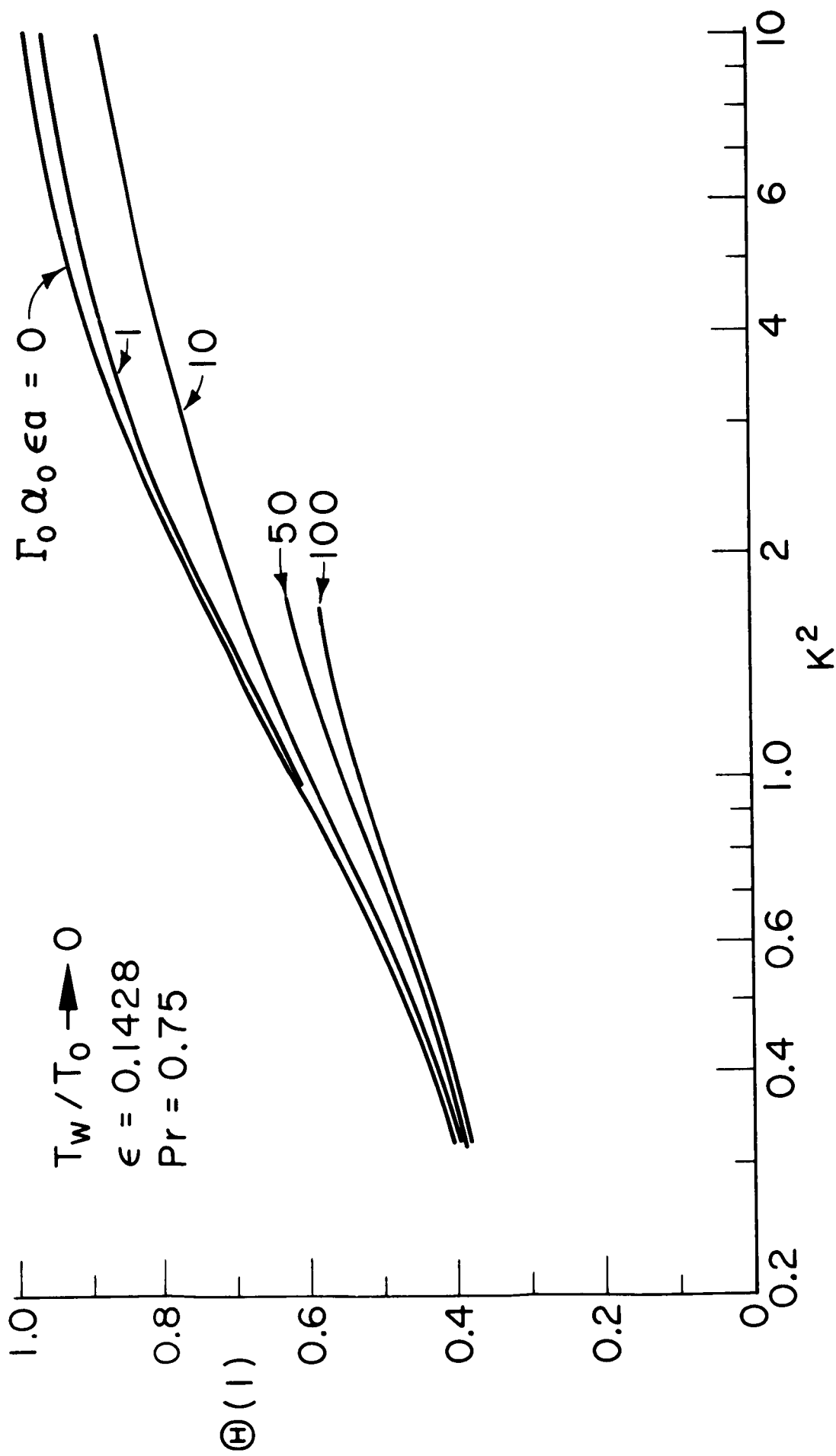


Figure 6. Dimensionless enthalpy at the outer-edge of the shock layer as a function of the rarefaction parameter.

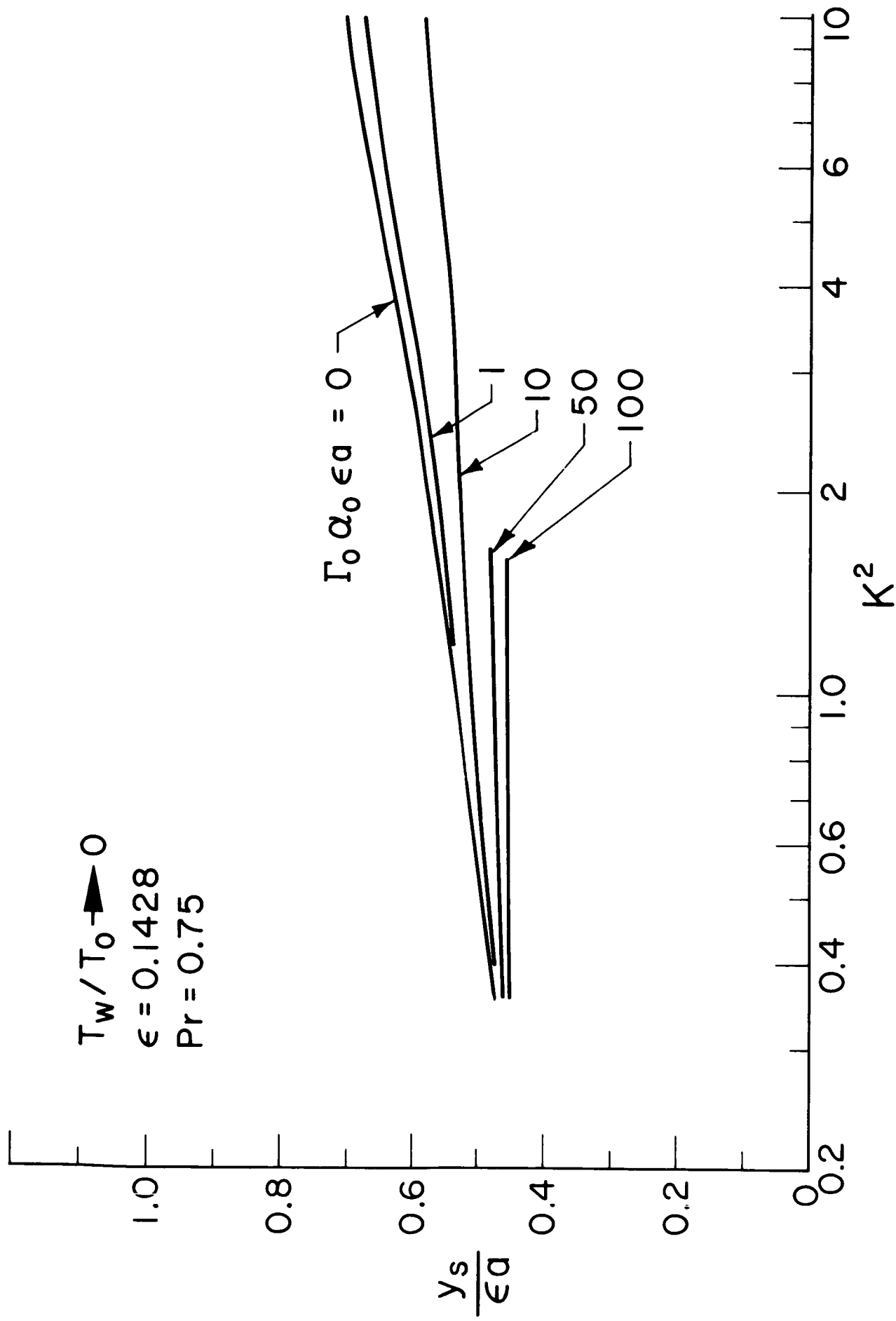


Figure 7. Shock layer thickness as a function of the rarefaction parameter.

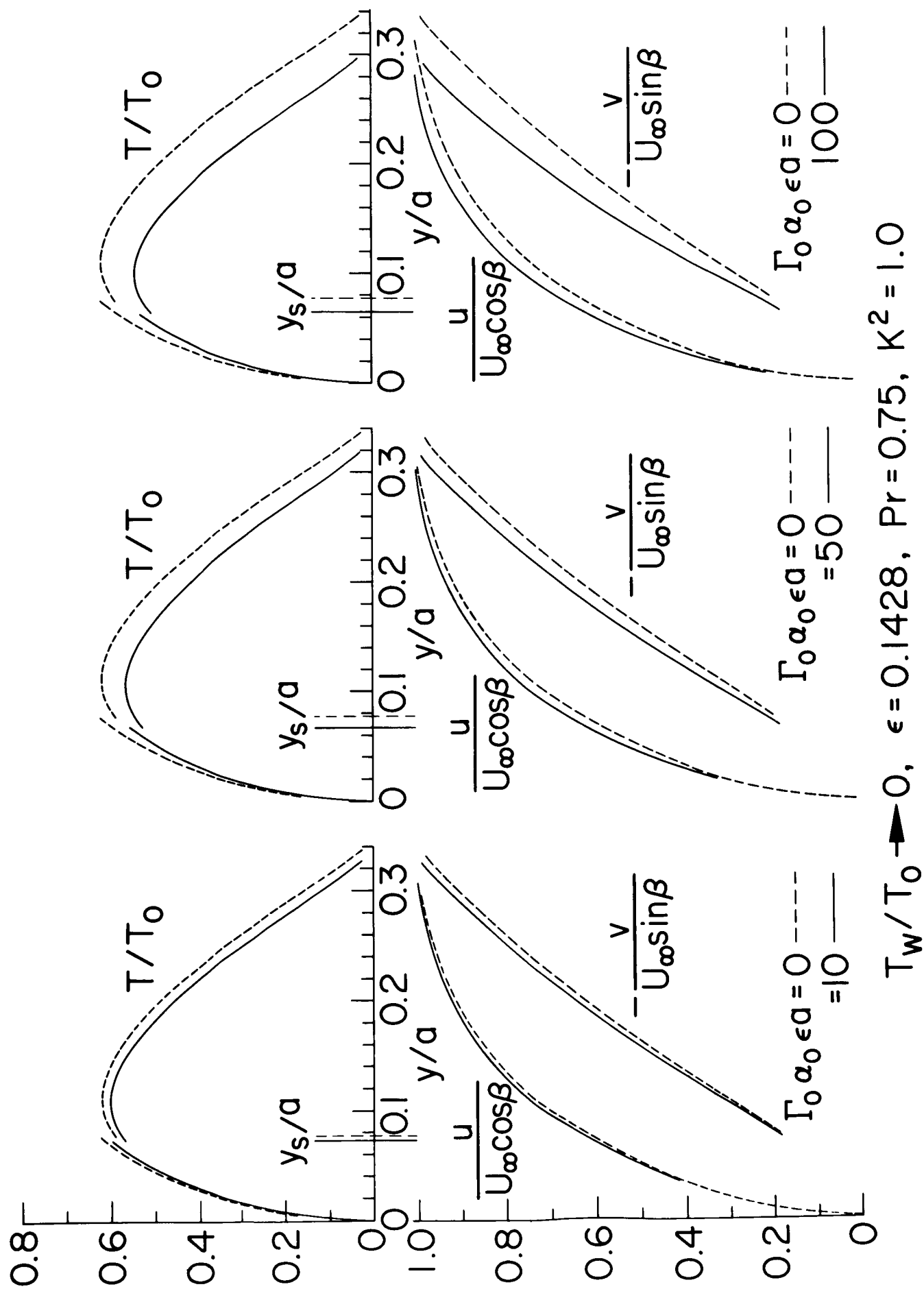


Figure 8. Dimensionless temperature, tangential velocity and normal velocity profiles for various values of the radiation parameter.

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13. ABSTRACT

One objective of this paper is to contribute to an understanding of radiative transfer effects in the low Reynolds number or merged layer regime of hypersonic flow about axisymmetric blunt bodies. The other objective is to illustrate how the concept of thin shock layer theory can be extended to the radiative case, where the shock structure and shock layer are radiatively coupled, through the introduction of a psuedo-jump condition across the shock structure, accompanied by an iteration technique. The radiative transfer is simplified to the emission dominated case. The gas is taken as calorically and thermally perfect, the viscosity taken as a linear function of the temperature, and the absorption coefficient of the gas is assumed to be gray. The problem is formulated for the flow about a blunt body but the detailed calculations are carried out for the stagnation region in order to illustrate the techniques used. Radiative cooling decreases the convective heat transfer from the corresponding radiationless case. The additional contributions to the heat transfer due to radiative transfer come from both the shock structure and the shock layer. The relative importance of these contributions are assessed in terms of the rarefaction parameter. Radiative contributions become less important toward the free-molecular range. Radiative cooling decreases the overall shock structure and shock layer thicknesses.

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|     | Radiative Transfer<br><br>Low Reynolds Number Hypersonic Flow<br><br>Blunt-Body Flow |        |    |        |    |        |    |

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