# THE SELECTION OF SIMPLE MODEL ATOMS FOR CALCULATIONS OF ELECTRON DENSITY IN NONEQUILIBRIUM, LOW TEMPERATURE Cs PLASMAS

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# THE SELECTION OF SIMPLE MODEL ATOMS FOR CALCULATIONS OF ELECTRON DENSITY IN NONEQUILIBRIUM, LOW TEMPERATURE Cs PLASMAS

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## ABSTRACT

Free electron number densities $N_e$ are calculated for cesium plasmas with low electron temperatures $T_e$(0.15 - 0.3 eV) using 5-, 3-, and 2-level atomic models. These $N_e$ values are obtained for a Maxwellian distribution of free electrons; this distribution was shown to be a good approximation for $T_e > 2100^\circ$ K in optically thick Cs plasmas. The sensitivity of $N_e$ to the choice of electronic levels, statistical degeneracy and radiative capture coefficients is presented for plasmas either optically thin or thick to resonance radiation. The steady state equation for the electron number density is discussed in the limits of low and high neutral density $N^o_{Cs}$. The ionization fraction $f \equiv N_e/N^o_{Cs}$ was studied as a function of $N^o_{Cs}$ for the range of $T_e$ from 1740° K (0.15 eV) to 3000° K. Results are presented as a function of $T_e$ for $N^o_{Cs} = 3\times10^{22}\ m^{-3}$, (typical values for certain thermionic diodes). Both sets of results are compared with the corresponding Saha curves for plasmas in local thermodynamic equilibrium (LTE). A discussion of rules for choosing reasonably accurate but simple atomic models is included. Those electronic levels which serve as free electron source terms are identified as functions of $N^o_{Cs}$ for fixed $T_e$. Even a 5-level model gives $N_e$ values which are only in fair agreement with optically thick results for a 26-level model under certain conditions. The lumped level degeneracy $g_M$ can be made a function of $N^o_{Cs}$ and $T_e$ to bring the $M = 5$ results into better agreement with the more reliable $M = 26$ results of Norcross and Stone. However, in the absence of such a comparison it does not appear that construction of accurate but simple atomic models is a straightforward process.

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## INTRODUCTION

There is much interest in the accurate calculation of free electron number density $N_e$ in nonequilibrium plasmas of low electron temperature $T_e$ that exist in Cs plasma diodes. The purpose of this study is to arrive at rules for constructing simple but satisfactory model atoms. Such models must ensure reasonably accurate values of $N_e$ for the aforementioned plasmas as well as allow simplified calculations of the free electron distribution function. The accuracy of such models depends upon the number and arrangement of electronic energy levels as well as inelastic cross sections, radiative lifetimes and radiative capture coefficients. For the steady state plasmas of interest, the free electrons are assumed to have a Maxwellian distribution. Solutions of the Boltzmann equation (including all elastic and inelastic collision terms) along the lines described in Ref. 1 indicate that the distribution function $f_e(u)$ is Maxwellian down to $T_e$ values of 2100° K (for Cs plasmas optically thick to resonance radiation). Although the numerical method described in Ref. 1 was convergent only for $T_e \geq 2300^\circ$ K, subsequent calculations down to $T_e = 2100^\circ$ K have been found to yield essentially Maxwellian $f_e(u)$. The high energy tail of $f_e(u)$ may be non-Maxwellian for $T_e$ values $< 2100^\circ$ K but the $N_e$ values can be relatively insensitive to this tail ($u > 3.89$ eV) for optically thick plasmas. This insensitivity is present where ground state ionization becomes a relatively unimportant rate process.

## DETAILS OF THE CALCULATION

A model atom consisting of $M$ levels is studied. The rate of change of the number of bound electrons in any level $L$ is[1]:

$$\dot{N}_L = -N_L(N_e\mathcal{K}_L + \mathcal{A}_L) + N_e \sum_{K\neq L} N_K K_{K\to L} + \sum_{K>L} N_K A_{K\to L} + N_e N_i(K_L^{cap} N_e + \beta_L) \qquad (1)$$

In Eq. (1) $K_{L\to K}$ ($m^3\ sec^{-1}$) is the excitation coefficient for the collision induced transition $L\to K$ and $K_{K\to L}$ ($m^3\ sec^{-1}$) is the corresponding coefficient for de-excitation. The quantities $K_L^{cap}$ ($m^6\ sec^{-1}$) and $\beta_L$ ($m^3\ sec^{-1}$) are the three-body and radiative capture coefficients, respectively. $\mathcal{K}_L$ ($m^3\ sec^{-1}$) is the total collision coefficient for transitions out of state L; i.e. $\equiv \sum_{K\neq L} K_{L\to K} + K_L^{ion}$ where $K_L^{ion}$ ($m^3\ sec^{-1}$) is the ionization coefficient for level L. $A_{K\to L}$ ($sec^{-1}$) is any radiative transition probability for de-excitation into the $L^{th}$ state and $\mathcal{A}_L$ is the sum of the $A_{L\to K}$ ($sec^{-1}$) for radiative transition to all levels below L.

For the steady state treated here $\dot{N}_L = 0$ and the number densities must satisfy the plasma normalization condition:

$$N_i + \sum_{L=1}^{M} N_L = N_{Cs}^o \qquad (2)$$

Equations (1) and (2) together with the condition of charge neutrality ($N_e = N_i$), serve to determine $N_e$ and the $N_L$ values for a specified $f_e(u)$ and initial Cs number density $N_{Cs}^o$. Equation (2) can be rewritten as:

$$f + \sum_{L=1}^{M} N_L' = 1 \qquad (2a)$$

where f is the ionization fraction and $N_L'$ is the normalized (to $N_{Cs}^o$) population of the $L^{th}$ state. In the steady state the total ionization rate from bound levels must balance the total capture rate of free electrons into all such levels. This relation provides a useful check on the numerical calculations. Its application indicated that accurate numerical solutions for f were not possible at low values of $N_e$ without tight convergence criteria.

The condition, $\dot{N}_e = 0$ can be written as:

$$\sum_{L=1}^{M} \left[(K_L^{cap})^* f^2 + \beta_L f\right] = \sum_{L=1}^{M} K_L^{ion} N_L' \qquad (3)$$

where $(K^{cap})^*$ ($m^3\ sec^{-1}$) is an effective two-body capture coefficient $\equiv N_{Cs}^o K_L^{cap}$. Thus the equation for the ionization fraction is essentially quadratic; however, the normalized populations $N_L'$ are strong functions of $N_{Cs}^o$ and optical thickness for fixed $T_e$. The ionization coefficient $K_L^{ion} \equiv \langle Q^{ion}(v_e)v_e\rangle$ is of course a function of $T_e$ alone.

## MODEL ATOM PROPERTIES

The optimization of the model atom depends upon the choices of cross sections for electron impact and coefficients and frequencies for radiative processes involving the $M^{th}$ level. This level must be adjusted so as to simulate the presence of many missing excited levels[1]. The assignments of ionization potential $E_M$ statistical degeneracy $g_M$ and $\mathcal{A}_M$ and $\beta_M$ values thus become critical for accurate calculations of f values. The $N_M'$ for this lumped level is a complicated function of the level structure for variable $N_{Cs}^o$ (see Eq. (1)). The schematic diagram for the three models is shown in Fig. 1 with values of $E_L$, $A_{L\to K}$ and $g_L$.

The early form of the Gryzinski cross sections has been used to compute ionization coefficients and excitation coefficients for optically allowed transitions only.[2] The Gryzinski exchange formulation was used to compute excitation coefficients for optically forbidden transitions.[3] Although the exchange cross sections have relatively small values at maximum, their slopes are steep just above threshold. Since the excitation and ionization thresholds are usually $\gg kT_e$, the behavior of the cross section in this region determines the value of the excitation coefficient. The lumped level was treated as an optically allowed state for all excitation (and de-excitation) collisions. Most allowed and exchange cross sections (monoenergetic) give reasonable agreement with experiment over a range of electron energies from threshold to maximum[4]. The experimental value of the most important excitation cross section $Q^{ex}_{1\to 2}$ has a slope roughly 3 times the Gryzinski value at threshold[4].

The radiative capture coefficients $\beta_L$ were taken from Ref. 5 where they were calculated using an adjusted quantum defect method. These calculations agree well with known oscillator strengths and recombination cross sections.

The radiative transition probabilities were taken from Ref. 6; the agreement of those calculations with cited experimental values is within 50% for all important $A_{LK}$ values (see Ref. 7). The most critical $A_{LK}$ value is for the $2\to 1$ resonance transition; this line is strongly absorbed for all experimental plasmas of interest[8]. For $M = 3$ the $A_{32}$ value was set equal to the sum of $A_{52}$, $A_{42}$, and $A_{32}$. The value of $A_{21}$ was used for all three optically thin cases.

## RESULTS - OPTICALLY THIN PLASMAS

Since the only available $M = 26$ results (optically thin) are several points for $T_e = 3000^\circ$ K, the $M = 2$ and $3$ results will be compared with 5-level results for those plasmas. Since ground state ionization is a significant process at low $N^o_{Cs}$ values, however, a non-Maxwellian tail in $f_e(u)$ could cause large changes in calculated values of $f$ in this region.

Low $N^o_{Cs}$ limit. - In the limit of low $N^o_{Cs}$ values three-body recombination is negligible and the free electron number density is determined by (from Eq. (3))

$$\sum_{L=1}^{M} \beta_L f \cong \sum_{L=1}^{M} K^{ion}_L N'_L \tag{4}$$

In the optically thin case, radiative capture into excited states is the important capture process which balances ground state ionization. Then $f$ is given approximately by:

$$f \cong K^{ion}_1 N'_1 \Big/ \sum_{L=2}^{M} \beta_L \tag{4a}$$

which simplifies for $M = 2$. The product $K^{ion}_1 N'_1 \cong K^{ion}_1$ (for all M), however, since the ground state population is greater than $0.95\ N^o_{Cs}$ for optically thin plasmas in the low $N^o_{Cs}$ regime. The value of $f$ is then simply determined by the ratio of the ground state ionization coefficient to the sum of the largest radiative capture coefficients. For cesium the $\beta_3$ (5D level) value dominates the $\sum^{M} \beta_L$ term; it is three times the maximum $\beta$ value for other excited levels, $\beta$(6P), and 200 times the $\beta_1$ (6S) value[5]. Thus, in choosing simple model atoms, it is important to assign a large value of $\beta_M$ to simulate the missing excited levels at low $N^o_{Cs}$. The $\beta_5$ assigned for $M = 5$ is the sum of $\beta$ values for the six excited states: 7P, 6D, 8S, 4F, 8P, 7D[5].

The results of $f$ versus $N^o_{Cs}$ for 2-, 3-, and 5-level models are shown in Fig. 2 for an electron temperature of $3000^\circ$ K (0.26 eV). The $f$ values for $M = 2$ (6S and 6P levels) with $\beta_2 = \beta(6P) + \beta(5D)$ agree with the $M = 5$ results within 50% at low values of $N^o_{Cs}$ ($\leq 3\times 10^{20}$ m$^{-3}$). This agreement can be improved by adding the sum $(\beta(7S) + \beta_5)$ to the $\beta_2$ value. For a given excitation cross section $Q^{ex}_{L\to K}$ the de-excitation coefficient $K_{K\to L}$ is inversely proportional to the upper state degeneracy $g_K$. The $L^{th}$ level 3-body capture coefficient is directly proportional to the $g_L$ value. The change of degeneracy for the $M^{th}$ state ($M = 2, 3$) does not affect the f value at low $N_{Cs}^{o}$ because the value of $N_1^{\prime}$ is unchanged. However, the degeneracy does affect the maximum ionization fraction calculated for the M = 2 and 3 models. This latter behavior can be described in terms of the free electron source terms although there is no simple limiting expression. The degree of departure of f from LTE conditions is evident by direct comparison with the Saha curve in Fig. 2.

High $N_{Cs}^{o}$ limit. - In the limit of high $N_{Cs}^{o}$ values 3-body capture becomes the dominant loss mechanism for free electrons. Radiative capture is negligible and the value of f can be approximated by:

$$f \cong \left[\sum_{L=1}^{M} K_L^{ion} N_L^{\prime} \Big/ \sum_{L=1}^{M} (K_L^{cap})^*\right]^{1/2} \tag{3}$$

The excited levels have the largest ionization coefficients since the ionization cross section $Q_L^{ion}$ is a strong inverse function of ionization potential $E_L$. These levels are most nearly in equilibrium with the free electrons and f approaches the Saha result. For the simplest model atom, M = 2, one must choose values of $g_M$ and $E_M$ so as to preserve the "correct" ratio of ionization rate to 3-body capture coefficient. In this study the value of $E_2$ was fixed at the 6P value, 2.43 eV, and the value of $g_2$ was varied for best agreement with M = 5 results. However, it will be shown that the $M^{th}$ state for M = 5 is not so simply adjusted to match the M = 26 results.

For a given ratio of ionization and capture coefficients for the lumped level $N_2^{\prime}$ must vary with $N_{Cs}^{o}$ so as to account for the populations of the missing excited levels near the Saha limit. For M = 2, $N_2^{\prime}$ increases so rapidly ($\propto {N_{Cs}^{o}}^{0.95}$) that $K_2^{ion} N_2^{\prime}$ exceeds $(K_2^{cap})^* f^2$ at intermediate values of $N_{Cs}^{o}$. The f is relatively small and only a gradual function of $N_{Cs}^{o}$ so the 3-body rate does not become important for the optically thin case until high $N_{Cs}^{o}$ values. The maxima in the f curves of Fig. 2 occur approximately where the collisional lifetime = $(\mathcal{K}_2 N_e)^{-1}$ equals the radiative lifetime $(A_{21})^{-1}$ for the first excited state (see Ref. 9).

The M = 3 and 5 cases are similar in that net ionization from excited states is a significant process for $N_{Cs}^{o} > 10^{22}\ m^{-3}$. The M = 2 model is unique because the ground state always serves as the free electron source term (i.e. exhibits net ionization). Thus in this case the lumped level necessarily becomes a net capturer since $\dot{N}_e = 0$. The large $\beta_2$ value (60 times $\beta_1$) ensures that this level remain a net capturer at low values of $N_{Cs}^{o}$. The sources of ionization for the M = 3 and 5 models can be determined from the net ionization rates. The rate for the $L^{th}$ state can be written from Eq. (3) as:

$$R_L^{net} = K_L^{ion} N_L^{\prime} - \left[(K_L^{cap})^* f^2 + \beta_L f\right] \tag{6}$$

Only $R_L^{net}$ values which are positive will be considered, i.e., states which are free electron source terms. The variation of such source terms (normalized to the sum of positive terms) $R_L^{\prime}$ is shown for $T_e = 3000^o$ K, optically thin, in Fig. 3 for M = 5 and 3. The L = 4 level (7S) makes the largest contribution to the free electron population at $T_e = 3000^o$ K for M = 5, $N_{Cs}^{o} > 2\times10^{21}\ m^{-3}$. This behavior differs slightly from the $T_e = 0.2$ eV results where the L = 3 state (5D) has the largest $R_L^{\prime}$ value at high $N_{Cs}^{o}$.

The results for M = 5, $g_5 = 50$, $E_5 = 0.6$ eV of Fig. 2 are in good agreement (within a factor of two) with $N_e/(N_e)_{Saha}$ results for the 26-level model at $T_e = 3000^o$ K[8] (see Table I). No optically thin results for M = 26 were available for comparison at lower $T_e$ values. The simpler models should give good agreement with the M = 26 case at low $N_{Cs}^{o}$ ($\leq 10^{19}\ m^{-3}$) since the lumped $\beta_M$ values have been chosen so as to produce that behavior.

## RESULTS - OPTICALLY THICK PLASMAS

Low $N_{Cs}^{o}$ limit. - At low values of $N_{Cs}^{o}$ ( $<10^{19}\ m^{-3}$ in Fig. 4(a)) f is determined by the balance between ionization from excited states and radiative capture. Just as in the optically thin case for high $N^o_{Cs}$, the excited states are free electron source terms, but at low $N^o_{Cs}$. The dominant $R'_L$ is $R'_2$ for $N^o_{Cs} \le 10^{19}\ m^{-3}$ in the 3- and 5-level models. Only the ground state has a positive $R^{net}_L$ value for $M = 2$ through the range of $N^o_{Cs}$ values. The approximate relation for the ionization fraction becomes:

$$f \cong \sum_{L=2}^{M} (K^{ion}_L N'_L) \Big/ \sum_{L=2}^{M} \beta_L \tag{6}$$

For $M = 5$ the above ratio should give reasonably accurate $f$ values when just the $L = 2$ and 3 ionization terms and the $\beta_3$ value are included. However, at intermediate values of $N^o_{Cs}$ 3-body capture is comparable with the radiative process and the lumped level must represent the missing excited states. The excited states are free electron source terms at low $N^o_{Cs}$ because their populations (especially $N'_2$ ) are enhanced in the absence of radiative de-excitation. The ionization rates become large even when populations $N'_L$, $L > 1$ remain $\approx 10^{-2} \rightarrow 10^{-3}$ of $N'_1$ because the ionization coefficients $K^{ion}_L >> K^{ion}_1$. The monoenergetic cross sections $Q^{ion}_L$, $L > 1$, have lower energy thresholds and much higher slopes than $Q^{ion}_1$ in the threshold region. The excitation coefficients for collision-induced transitions between excited levels are correspondingly larger than the ionization coefficients and have low thresholds (some <1 eV).

The $f$ values for the optically thick case are plotted versus $N^o_{Cs}$ for $T_e = 3000^o$ K in Fig. 4(a). The lumped $\beta_M$ value should guarantee accuracy of the simpler models at low $N^o_{Cs}$. This is true if the populations $N'_2$ have nearly the same values as for $M = 26$. Decreasing $\beta_M$ for $M = 2$ gives better agreement with $M = 26$ at $N^o_{Cs} = 10^{20}\ m^{-3}$ but the asymptote is incorrect at $10^{18}\ m^{-3}$. For $M = 3$, the $f$ value is increased at all $N^o_{Cs}$ by making the plasma thick to <u>all</u> line radiation and lowering $g_M$ from 50 to 10. There is no peak in $f$ versus $N^o_{Cs}$ for $M = 2$, $g_2 = 50$ since the $L = 2$ state has a large 3-body capture rate at intermediate $N^o_{Cs}$ due to its large $g_M$ value and relatively high $N_e$. Lowering the degeneracy to $g_2 = 6$ (not shown) does not change the curve shape since 3-body capture continues to dominate excited state ionization. Decreasing the $g_M$ from 50 (curve 4) to 10 (not shown) for $M = 3$ does produce a maximum in $f$ because $3 \rightarrow 2$ de-excitation is increased enough to raise the $N'_2$ value significantly. This $L = 2$ state is the main free electron source term for $M = 3$.

The $f$ values for the optically thick case, $T_e = 2321^o$ K, are plotted versus $N^o_{Cs}$ in Fig. 4(b). Just as for $T_e = 3000^o$ K the $f$ is determined by ionization from the ground level for $M = 2$ and from excited states for $M = 3$ and 5. These ionization processes balance radiative capture into all levels and the curves remain flat where the populations of the ionization sources remain constant, just as for $3000^o$ K. The features of the curves are identical to the $T_e = 3000^o$ K results throughout the range of $N^o_{Cs}$ values. The peak for $M = 5$ is again caused by ionization from the higher excited states $L = 3$ and 4. Four points of $M = 26$ results are also shown in Fig. 4(b). Comparison with the latter results indicate that the simpler atomic models overestimate departures from LTE at values of $N^o_{Cs}$ below $3\times10^{22}\ m^{-3}$. It should be noted, however, that the $N_e$ values from $M = 26$ are considerably lower than Saha values for $N^o_{Cs} \le 10^{21}\ m^{-3}$. The ionization fractions are approximately $10^{-2}$ of the $3000^o$ K values because excited state populations and ionization coefficients are considerably lower. The limiting (low $N^o_{Cs}$) value of $f$ for $M = 3$ is only $10^{-2}$ of the $f$ computed for a 3-level model in Ref. 9. For $M = 2$ the value of $f$ is about 1/15 of that for a 2-level model in Ref. 9. These large differences are due to the present use of more reliable excitation cross sections and $\mathcal{A}_L$ and $\beta_L$ values.

The variation of the free electron source terms for $M = 5$ at $T_e = 2321^o$ K (0.2 eV) and $3000^o$ K are shown in Figs. 5(a) and 5(b), respectively. Qualitatively, the dependences of $R'_L$ are similar but the region of rapidly changing contribution is shifted by a factor of $10^2$ to higher $N^o_{Cs}$ values at the lower value of $T_e$. The $M = 26$ results of Ref. 7 indicate that the maximum $f$ is nearly 5 times the $M = 5$ value for $N^o_{Cs} \cong 10^{21}\ m^{-3}$, $T_e = 2321^o$ K. Figure 5(b) shows that the main contributor of free electrons is the $L = 3$ state for $N^o_{Cs} > 3\times10^{20}\ m^{-3}$ and the ground and first excited states for $N^o_{Cs}$ values $\le 3\times10^{20}\ m^{-3}$. If additional excited states are to become new ionization source terms, their relative populations must fall off slowly enough with binding energy that their higher ionization coefficients can raise f. The sudden drop in the $L = 3$ contribution in the region $N^o_{Cs} \leq 10^{21}\ m^{-3}$ is a result of the large $\beta_3$ value since radiative capture becomes the dominant electron loss process in that region ($N_e \leq 5\times10^{18}\ m^{-3}$).

The agreement between the $M = 5$ and 26 results improve at high $N^o_{Cs}$ ($\cong 10^{22}\ m^{-3}$) for $T_e = 3000^o$ K as can be seen in Fig. 4(a). The f for $M = 5$ is about 90% of the f value for $M = 26$. Also, the disparity near peak ($N^o_{Cs} \cong 10^{20}\ m^{-3}$) is explainable in terms of the $R'_L$ value just as the $T_e = 2321^o$ K results. Figure 5(b) shows that the $L = 4$ level is the chief contributor of free electrons for $N^o_{Cs} > 3\times10^{19}\ m^{-3}$. This occurs because the "ladder" ionization mechanism operates if the excited state populations are large enough. For optically thick plasmas this is the case. It is plausible, therefore, that including additional excited states would raise the value of f in this $T_e$ region if these states serve as large source terms with $R'_L$ values $\cong R'_4$.

High $N^o_{Cs}$ limit. - At high values of $N^o_{Cs}$ the excited levels are still the free electron source terms. Again the balance which determines f is between excited state ionization and three-body capture into these same levels. The f values, optically thick, are uniformly larger at lower $N^o_{Cs}$ values than in the optically thin case. This is because the important $N'_L$ approach their Saha values at lower $N^o_{Cs}$ ($\cong 3\times10^{21}\ m^{-3}$). The value of f can be satisfactorily calculated from Eq. (5) including terms, $L = 2$ to M.

Results for $T_e = 1740^o$ K (0.15 eV). - Curves of f versus $N^o_{Cs}$ were also calculated at $T_e = 1740^o$ K. There is no structure in these curves for $N^o_{Cs} < 10^{25}\ m^{-3}$, optically thin and $N^o_{Cs} < 10^{26}\ m^{-3}$, optically thick. One point was compared with the $M = 26$ results at $N^o_{Cs} = 10^{24}\ m^{-3}$, optically thick. The $M = 26$ model gives an f which is 3 times the value for $M = 5$, $g_5 = 50$. Since the chief free electron source term is $L = 5$ for $M = 5$ the difference between the $M = 5$ and 26 results becomes very sensitive to the lumped state. This level must now simulate the source terms for $N_e$.

The simpler models do not agree well with each other at $T_e = 0.15$ eV, optically thick. This behavior is shown in Fig. 6 where f is plotted as a function of $T_e$ at $N^o_{Cs} = 3\times10^{22}\ m^{-3}$ for $M = 2$, 3, and 5. The largest disparity occurs in the optically thick case where the f for $M = 2$ is only 2% of the value calculated for $M = 5$. This result is consistent with the fact that the former model does not allow for excited state source terms. The $M = 3$ model gives an f value midway between the $M = 2$ and 5 results. The values of f, optically thin, are only $\cong 10^{-3}$ of the optically thick values; this is because the ground state is the sole ionization term in the former case. Thus at this low $T_e$ value the ionization mechanism changes from simple ground state ionization to a "stepwise" ladder process (for $N^o_{Cs} = 10^{18} \rightarrow 10^{25}\ m^{-3}$) as the optical thickness changes. As the electron temperature is increased this disparity between optically thin and thick results decreases markedly. For fixed values of $\beta_M$ and $E_M$, agreement with $M = 26$ results for simpler models can be improved by using variable $g_M = F(N^o_{Cs}, T_e)$ or making the plasma thick to all radiation. However, any scaling laws for construction of atomic models remain to be investigated in the absence of more detailed results for comparison.

## CONCLUDING REMARKS

For certain values of electron temperature and neutral density 2- and 3-level models can be adjusted to give $N_e$ values which agree with results for 5-levels. However, for the experimentally interesting case of optically thick Cs plasmas, results for a 26 level model[7] indicate the "best" $M = 5$ model can seriously underestimate the ionization fraction f. This is especially true for values of electron temperature $<3000^o$ K. This disparity occurs because the sources of free electrons are highly excited levels as a result of a "ladder" ionization mechanism. It does not appear likely that convenient adjustment of simpler models will improve agreement with the $M = 26$ results. For proper choices of $E_M$, $\beta_M$, $\mathcal{A}_L$ for the lumped level a variable $g_M(N^o_{Cs})$ will provide satisfactory agreement for fixed $T_e$. However, it appears that increasing the number of levels is a more desirable approach than this to ensure calculation of accurate $N_e$ values. Further results for $M = 26$ are needed to evaluate the simpler model atoms for the low temperature ($1740^\circ$ - $2321^\circ$ K) optically thin plasmas.

TABLE I. - ($T_e = 3000^\circ$ K; OPTICALLY THIN)

| M | $N_e/(N_e)_{Saha}$ | | | |
|---|---|---|---|---|
| 5 | $2.4\times10^{-2}$ | $6.8\times10^{-2}$ | 0.35 | 0.75 |
| 26* (Ref. 7) | $\cong 1.3\times10^{-2}$ | $\cong 6.5\times10^{-2}$ | $\cong .32$ | $\cong .75$ |

*Points estimated from plots without grid.

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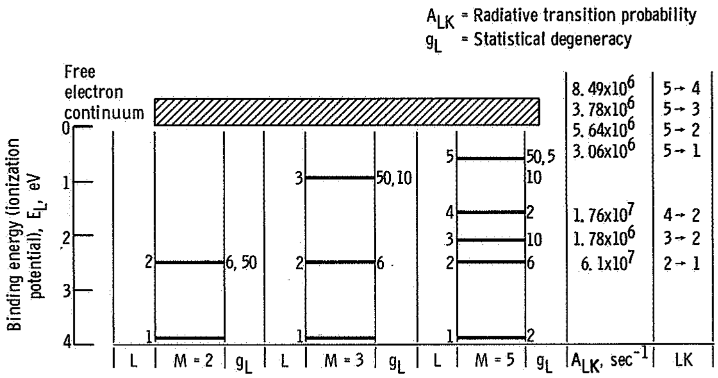



Figure 1. - Schematic diagram of cesium model atoms.

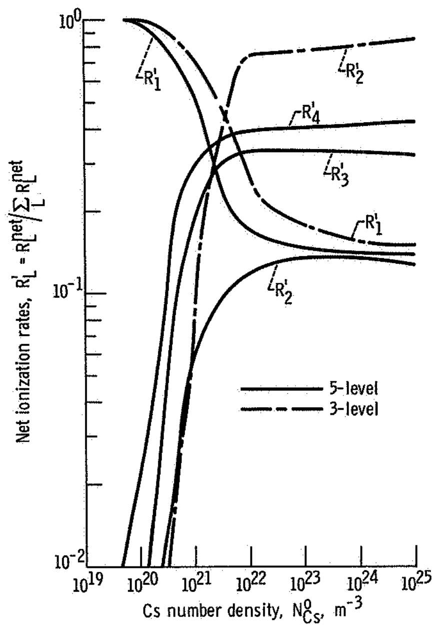



Figure 3. - Plots of normalized net ionization rates as a function of Cs neutral density in plasmas optically thin to resonance radiation for 3- and 5-level atomic models. $T_e$ = 3000° K.

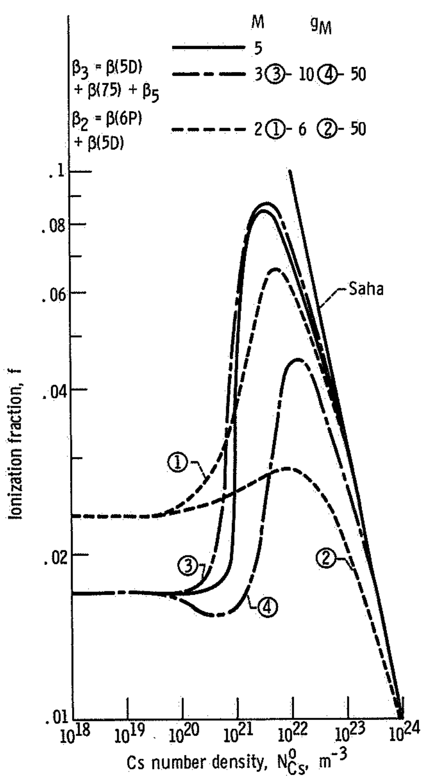
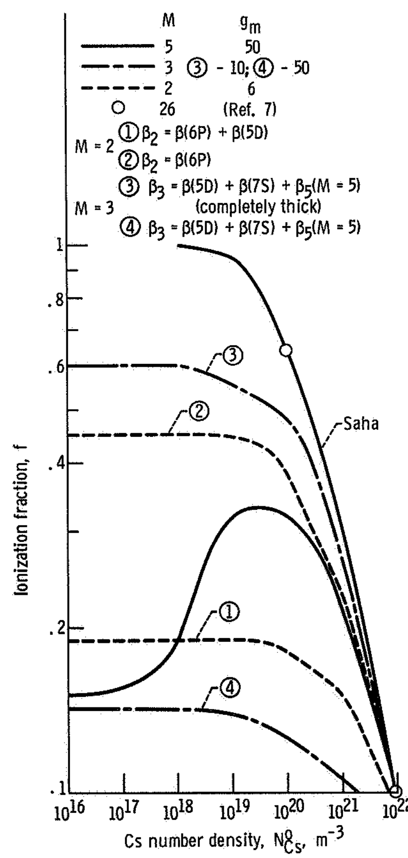



Figure 2. - Plots of ionization fraction as a function of Cs neutral density in plasmas optically thin to resonance radiation for 2-, 3-, and 5-level atomic models. $T_e$ = 3000° K.




(a) $T_e$ = 3000° K. Two points for a 26-level model are shown for comparison.

Figure 4. - Plots of ionization fraction as a function of Cs neutral density in plasmas optically thick to resonance radiation for 2-, 3-, and 5-level atomic models.

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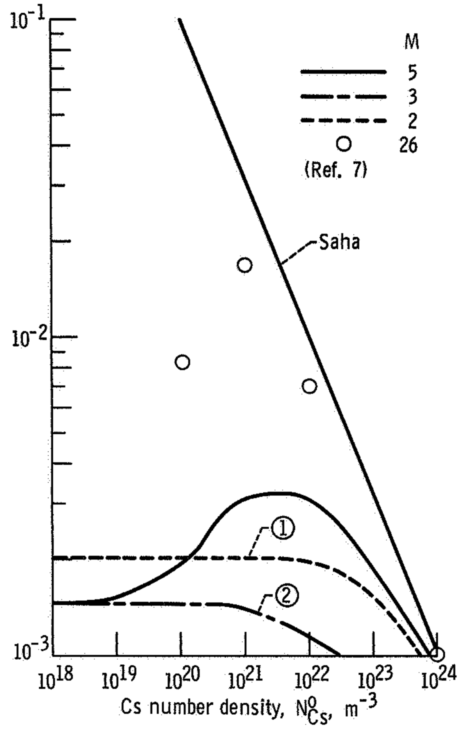
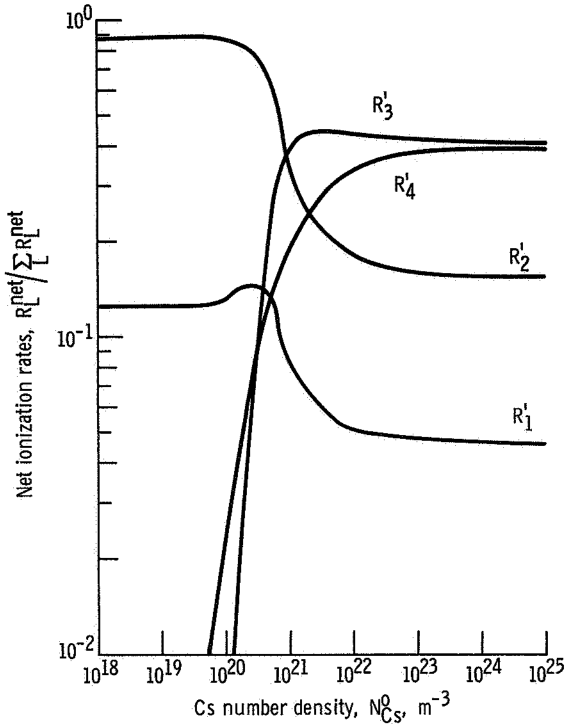


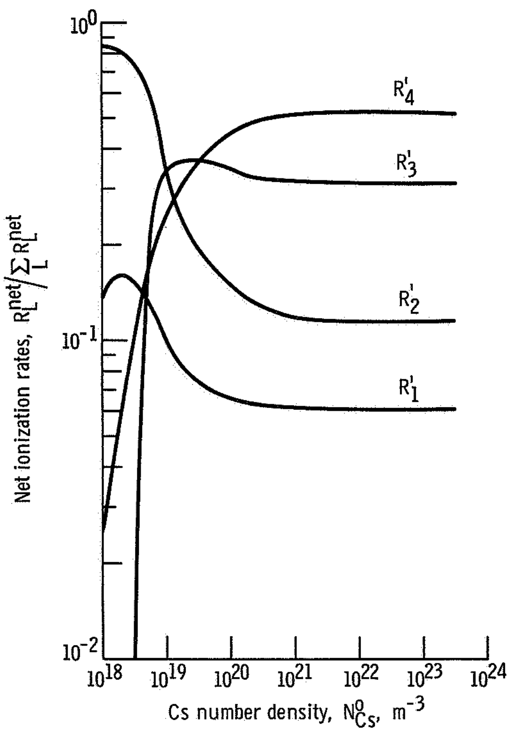

(b) $T_e$ = 2321° K. Four points of 26-level results are included for comparison.

Figure 4. - Concluded.




(b) $T_e$ = 2321° K (0.2 eV).

Figure 5. - Concluded.




(a) $T_e$ = 3000° K.

Figure 5. - Plots of normalized net ionization rates as a function of Cs neutral density in plasmas optically thick to resonance radiation for the 5-level atomic model.

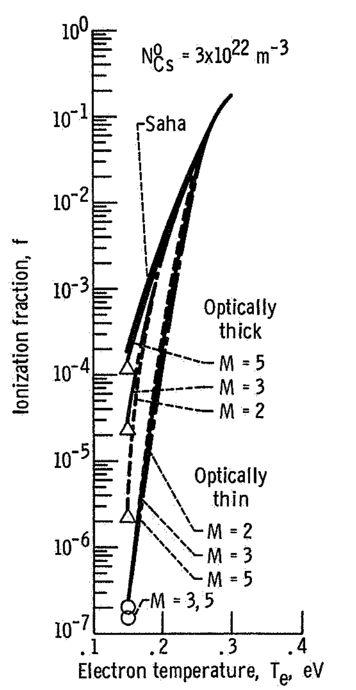



Figure 6. - Plots of ionization fraction as a function of electron temperature in plasmas optically thin and thick to resonance radiation for M = 2-, 3-, and 5-level atomic models.