NGR-23-005-166

AEROELASTIC STABILITY OF PLATES AND CYLINDERS

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ABSTRACT

Linear stability criteria are presented for the panel flutter of thin plates and thin-walled cylinders. These structures are exposed to fluid flow passing parallel to an outer surface. The expression for fluid pressure is simplified in order to emphasize the dynamic properties of the systems. The pressures are derived from steady flow relations (frequency effects are ignored). An arbitrary spatial phase angle is included in the pressure expression. As this phase angle is varied in a continuous manner, the fluid flow passes from "subsonic" character to supersonic character. The results are useful in classifying several types of instability and discussing several pathological cases which are usually treated separately.

The analysis is intended to serve as an aid to understanding the mechanism of panel flutter; however, it can be applied directly to several problems. It is accurate for the static divergence and "coupled mode" flutter of flat panels in supersonic flow, and also for divergence problems wherever experimental measurements can supply the values for the necessary aerodynamic parameters. One result is to point out the

importance of static instability for flat panels in a transonic viscous flow. A second result is to illustrate that the asymmetric divergence of cylindrical shells is very sensitive to small changes in the pressure distribution.

A, $\tilde{A}$	Aerodynamic pressure parameter; $\frac{\rho U^2 L^3}{\sqrt{\lvert M^2 - 1 \rvert} D}$, $\frac{\rho U^2 R^3}{\sqrt{\lvert M^2 - 1 \rvert} D}$
D	$Eh^3/12(1 - \nu^2)$
F	Airy stress function
h	Panel thickness
K_m	Aerodynamic pressure constant, Eq. (5)
L	Length of panel
M	Mach number
m	Axial wave number
N	Number of modes
N_x	Axial stress resultant due to initial load
N_Θ	Circumferential stress resultant due to initial load
$p(x,t)$	Aerodynamic load
q	Integer
R	Radius of cylinder
t	Time
U	Flow velocity
w	Panel displacement in transverse direction
x	Spatial coordinate, flow direction
z	Spatial coordinate
δ_{qm}	Kronecker delta
ϵ	Amplitude constant
Θ	Angular coordinate
λ	Eigenvalue
$\tilde{\lambda}$	Eigenvalue

ρ	Fluid density
ρ_s	Panel density
σ_x	Axial stress in cylindrical shell due to shell motion
ψ_m	Spatial phase shift
ω	Frequency, rad/sec

LIST OF FIGURES

1. INTRODUCTION

The elastic instability of thin panels exposed to fluid flow is under intensive study at the present time. Typical problems involve thin-walled structural elements with one surface exposed to fluid flow essentially parallel to the surface. Figure 1 illustrates the flow situation for a flat plate and a cylinder. The usual question of interest is whether the elastic panels incur divergence (static instability) or flutter (dynamic instability) at some value of flow velocity.

The fluid pressures exerted on oscillating panels are difficult to derive in many cases. The role of fluid viscosity, frequency of oscillation, and panel geometry have complicated panel flutter studies to the point where the results are often difficult to understand.

The present study is based on an intuitive simplification of the pressure distribution on the panel. It illustrates the effect of the spatial distribution of pressures. The pressures are taken from steady flow results and are hence independent of the frequency of oscillation. The results are valid only for instabilities occurring at relatively low frequencies.

An approximate solution is required because of the nature of the assumptions on the pressures. These assumptions are equivalent to a specification of the generalized forces on a discrete system. Galerkin's method is used to pose the eigenvalue problem in matrix form.

2. FLUID PRESSURES

The pressure expression used in this study is motivated by the solution for flow over an infinitely long, two-dimensional stationary wavy wall (Fig. 2). For the case of inviscid, isentropic flow, one finds that a deflection

$$w(x) = \epsilon \sin \frac{2\pi x}{\ell} \tag{1}$$

yields a pressure of the form

$$p(x) = \epsilon \frac{\rho U^2}{\sqrt{|M^2 - 1|}} \frac{2\pi}{\ell} \cos\left(\frac{2\pi x}{\ell} + \psi\right) \tag{2}$$

where ψ takes the value 0 for a supersonic flow and $\pi/2$ for subsonic flow. The solution is not valid near Mach 1.

The pressure expression given in Eq. (2) is "exact" within the framework of linearized potential flow for the stationary wall under consideration. We will view this expression, however, as an approximation which has been provided to describe a given physical situation: a panel of finite length with viscous flow effects, real gas effects, etc. As an example, for transonic flow, McClure[1] measured pressures of the form

$$p(x) = \frac{\epsilon \rho U^2}{\sqrt{|M^2 - 1|}} \frac{2\pi}{\ell} K \cos\left(\frac{2\pi x}{\ell} + \psi\right) \tag{3}$$

for a stationary wavy wall. The constants K and ψ are functions of Mach number, fluid properties and wavelength. McClure found the amplitude constant K to be near unity. His measured values of ψ ranged from

20 to 45°. We hence see that values of ψ lying between 0 and 90° have physical significance in practical cases.

Let us consider the pressure expression, Eq. (3) as sufficient for our purposes. We will generalize this expression slightly by using subscripts to show the dependence of the constants K and ψ upon the wavelength. For a given deflection of a wall

$$w(x,t) = e^{i\omega t} \sum_{m=1}^{N} a_m \sin \frac{m\pi x}{L} \tag{4}$$

one then has a pressure expression of the form

$$p(x,t) = e^{i\omega t} \frac{\rho U^2}{\sqrt{|M^2 - 1|}} \sum_{m=1}^{N} a_m \frac{m\pi}{L} K_m \cos\left(\frac{m\pi x}{L} + \psi_m\right) \tag{5}$$

Note that each term in Eq. (4) represents a wave with length $\frac{2L}{m}$.

In the following examples, it will be assumed that the constants K_m and ψ_m are known. (This is equivalent to assuming that the generalized forces are known for the discrete system.) For example, if slender wing (Ackeret) theory were used for supersonic flow over a finite panel, Eq. (5) would result with $K_m = 1$ and $\psi_m = 0$ for all m.

3. FLAT PANEL OF FINITE LENGTH

Consider the case of a two-dimensional flat panel exposed to fluid flow over one surface Fig. 3. The plate is of uniform thickness, length L and simply supported at both ends. The aerodynamic expression of Eq. (5) will be used to provide fluid pressures above the panel.

The fluid below the panel is at rest and at the same static pressure as the upper flow.

The equation of motion for small deflections of the plate is

$$D \frac{\partial^4 w}{\partial x^4} - N_x \frac{\partial^2 w}{\partial x^2} + \rho_s h \frac{\partial^2 w}{\partial t^2} + p(x,t) = 0 \qquad (6)$$

and the boundary conditions are

$$w(0,t) = w(L,t) = \frac{\partial^2 w}{\partial x^2}(0,t) = \frac{\partial^2 w}{\partial x^2}(L,t) = 0$$

The solution is assumed to be of the form

$$w(x,t) = e^{i\omega t} \sum_{m=1}^{N} a_m \sin \frac{m\pi x}{L}$$

Galerkin's method yields a set of coupled, linear algebraic equations of motion

$$\sum_{m=1}^{N} \left\{ \left[(m\pi)^4 + \frac{N_x \ell^2}{D} (m\pi)^2 - m\pi A K_m \sin\psi_m - \lambda \right] \delta_{mq} + A K_m \cos\psi_m \, \eta_{mq} \right\} a_m = 0 \qquad (q = 1,2,\ldots N) \qquad (7)$$

where

$$\lambda = \frac{\rho_s h \omega^2 L^4}{D} \qquad \eta_{mq} = \begin{cases} \dfrac{4mq}{m^2 - q^2} & \text{if } m + q \text{ is odd} \\ 0 & \text{if } m + q \text{ is even} \end{cases}$$

$$A = \frac{\rho U^2 L^3}{\sqrt{|M^2 - 1|}\, D}$$

and δ_{mq} is the Kronecker delta.

This is a linear eigenvalue problem in the eigenvalue λ. It is non-Hermitian and hence in general we may have complex eigenvalues.

The characteristic polynomial is solved for the eigenvalue as a function of A and $\frac{N_x L^2}{D}$.

To interpret the stability of the system, we must remember that the frequency of oscillation varies as the square root of the eigenvalue:

$$\omega \propto \lambda^{1/2}$$

and hence

$$w(x,t) \propto e^{i\lambda^{1/2}t}$$

The square root must be considered a multivalued function of the complex variable λ. If all eigenvalues λ are real and positive, then neutral stability results. If λ is real and negative, static divergence occurs. If λ is complex, then flutter occurs.

Results have been calculated for the stability of a panel with no membrane tension ($N_x = 0$). Extensive experience with Galerkin's method as applied to fourth order differential equations has shown excellent convergence when four modes are used. Two-mode, four-mode, and eight-mode calculations were used here; the results were found to converge adequately.

The stability boundaries shown in Fig. 4 are from a four-mode analysis. For this special case, the amplitude constants and the spatial phase shift have been set equal for all modes:

$$K_1 = K_2 = K_3 = K_4 = K$$

$$\psi_1 = \psi_2 = \psi_3 = \psi_4 = \psi$$

As a result, the amplitude constant is easily incorporated into the ordinate. The figure hence emphasizes the role played by ψ.

The panel is stable for sufficiently low values of A, regardless of the value for ψ. As A increases, however, the panel becomes unstable at some critical value of A. This can be either divergence or flutter, depending on the value of ψ.

It is interesting that for $\psi = 0$ ("supersonic" flow) only flutter is possible. (Experimental evidence indicates that this theoretical solution is correct for $\psi = 0$.) Also, for $\psi = 90°$ ("subsonic" flow) only divergence is possible. These limiting cases are well known. On the other hand, for phase angles ψ between 25° and 90°, one encounters divergence first and then flutter.

The results for small values of ψ, say from 0° to 40° are important. In transonic flow, for instance, ψ depends upon boundary layer thickness, fluid viscosity, etc. If a given test were carried out for varying boundary layer properties, the type of instability might well change from a dynamic type to a static type because of this spatial phase shift. (It must be remembered that the present analysis cannot predict the single-degree-of-freedom type of flutter which often typifies transonic flow. On the other hand, this analysis is "exact" for simply supported plates which diverge and hence is sufficient to predict static instability.)

For phase angles ψ near 90°, one finds that increasing dynamic pressure causes first a static divergence, followed by dynamic instability and finally a static divergence. This might be a confusing factor in some subsonic experimental work, where spatial phase angles might be near, but not exactly, 90°.

4. ASYMMETRIC FLUTTER OF A CYLINDER OF FINITE LENGTH

The stability of a finite elastic cylinder Fig. 1 will be investigated in the same spirit as the flat panel. The shell is of uniform thickness and unstiffened. Conventional cylindrical coordinates x, r, Θ will be used. Donnell's cylinder equations are adequate to describe the deflections of interest here:

$$D\nabla^4 w - N_x \frac{\partial^2 w}{\partial x^2} - \frac{N_\Theta}{R^2}\frac{\partial^2 w}{\partial \Theta^2} + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \rho_s h \frac{\partial^2 w}{\partial t^2} + p(x,t) = 0 \quad (8)$$

$$\nabla^4 F - \frac{Eh}{R}\frac{\partial^2 w}{\partial x^2} = 0 \quad (9)$$

The boundary conditions are taken to be the freely-supported case:

$$v = w = \frac{\partial^2 w}{\partial x^2} = \sigma_x = 0 \quad (\text{at } x = 0, L)$$

Again, for a deflection of the form

$$w(x,\Theta,t) = e^{i\omega t}\cos n\Theta \sin\frac{m\pi x}{L}$$

the fluid forces will be taken as

$$p(x,\Theta,t) = \frac{\rho U^2}{\sqrt{|M^2 - 1|}} e^{i\omega t}(\cos n\Theta)\, K_m \frac{m\pi}{L} \cos\left(\frac{m\pi x}{L} + \psi_m\right)$$

If one again applies Galerkin's method to the equations of motion (8) and (9), one obtains a system of linear algebraic equations

$$
\begin{aligned}
\sum_{m=1}^{N} a_m \Bigg[\Bigg\{ & \left[\left(\frac{m\pi R}{L}\right)^2 + n^2\right]^2 + 12(1-\nu^2)\left(\frac{R}{h}\right)^2 \left(\frac{m\pi R}{L}\right)^2 \left[\left(\frac{m\pi R}{L}\right)^2 + n^2\right]^{-2} + \frac{N_x R^2}{D}\left(\frac{m\pi R}{L}\right)^2 \\
& + \frac{N_\theta R^2}{D}\, n^2 - \tilde{\lambda} - \tilde{A} K_m \frac{m\pi R}{L} \sin \psi_m \Bigg\} \delta_{qm} \\
& + \left\{ \tilde{A} K_m \frac{m\pi R}{L} \cos \psi_m \right\} \eta_{qm} \Bigg] = 0 \qquad (q = 1,2,\ldots N)
\end{aligned}
$$

where

$$\tilde{\lambda} = \frac{\rho_s h \omega^2 R^4}{D}$$

$$\tilde{A} = \frac{\rho U^2}{\sqrt{|M^2 - 1|}\, D}$$

and η_{qm} is defined as for the plate.

These equations can be solved for the eigenvalues $\tilde{\lambda}$ as a function of the fluid dynamic pressure ratio $\tilde{A}$ and the phase shift ψ. We will consider numerical results for a case corresponding to wind tunnel tests carried out by Olson [2].

$N_x = 0$

$N_\theta = 0$

$R = 8.00$ inch

$h = 0.004$ inch

$\ell = 15.4$ inch

$\nu = 0.35$

$n = 28$

We will again choose

$$\psi_1 = \psi_2 = \ldots \psi_n = \psi$$
$$K_1 = K_2 = \ldots K_n = K$$

The results for a four mode solution are given in Fig. 5. Here it is seen, as for a flat plate, that for $\psi = 0$ only flutter can occur. For values of ψ between 60° and 120°, there is an unexpected result. The case of static divergence does indeed occur, but at relatively large values of $\tilde{\Lambda}$. In this case, if ψ is not exactly 90°, then flutter can occur at much lower values of $\tilde{\Lambda}$.

This analysis shows the danger inherent in using an aerodynamic theory which predicts that $\psi = 90°$ exactly. Resulting calculations might not reveal a flutter situation which occur at a much lower dynamic pressure ratio.

Note that the flutter boundary is very insensitive to changes in ψ from -30° to 60°. This means that the details of the pressure distribution on the cylinder are not of much importance in the stability analysis. This explains why one of the simplest aerodynamic theories, Ackeret theory, can be used with success to predict cylinder flutter which occurs at low frequencies [3].

5. CONCLUSIONS

The appearance of a spatial phase shift as a free parameter in the fluid pressure expression results in some new observations. It illustrates the change, in a continuous manner, from subsonic (or slender body) flow character to supersonic character. The intermediate values of the phase angle have physical application to the cases of viscous transonic flow over flat plates and supersonic flow over cylindrical shells.

The analysis is limited to two types of elastic instability: coupled mode flutter and divergence. The study cannot predict single degree-of-freedom flutter because of the use of steady flow relations for the fluid forces.

Several examples were studied in which the pressure amplitudes K_m were identical in all modes and the phase angles ψ_m were identical in all modes. This case was chosen because of its simplicity. Conclusions for the flat plate and the cylinder will be discussed separately.

The flat plate exhibits both divergence and flutter. For one range of the spatial phase angle ψ (-90° to -60°), the plate is stable for all dynamic pressures. For a second range of ψ (-60° to 25°), only flutter is possible. Finally, for a third range of ψ (25° to 90°), divergence is the critical form of instability, occurring at a much lower dynamic pressure than flutter. The stability diagram indicates that experiments carried out for certain phase angles might be confusing in the sense that different regions of stability and instability could be observed in turn as the dynamic pressure is raised.

Divergence occurs for flat plates at a relatively low value of dynamic pressure ratio. As a result, divergence may be a distinct problem for the case of viscous transonic flow, where previous pressure measurements indicate that the necessary phase shift does occur [1].

The cylinder example studied was for a particular cylinder geometry, chosen to match the only successful experiments to date. The cylinder exhibits coupled mode flutter over the entire phase angle range of physical interest. This flutter boundary is surprisingly insensitive to the value of ψ. This is fortunate from a practical standpoint. It means that coupled mode flutter calculations can be carried out for such a shell with less attention paid to the details of the spatial pressure distribution.

The occurrence of divergence for the cylinder is not a simple phenomenon. In the past, divergence has been predicted for some types of cylinders in supersonic flow (where axial wavelengths are long compared to circumferential wavelengths). For the cylinder studied here the divergence would be of little practical interest. Very small phase shifts from $\psi = 90°$ cause flutter to occur at much lower dynamic pressures than divergence.

It is not prudent to extend the results of this simple analysis too far. On the other hand, it can serve as a qualitative aid to investigators in panel flutter. There are times when the methods of analysis are so cumbersome that one restricts his techniques (or his interest) to only divergence or to flutter. It is apparent that one must be careful to not overlook one of the possible instabilities.

6. ACKNOWLEDGEMENT

This investigation was supported by NASA Research Grant NGR-23-005-166.

7. REFERENCES

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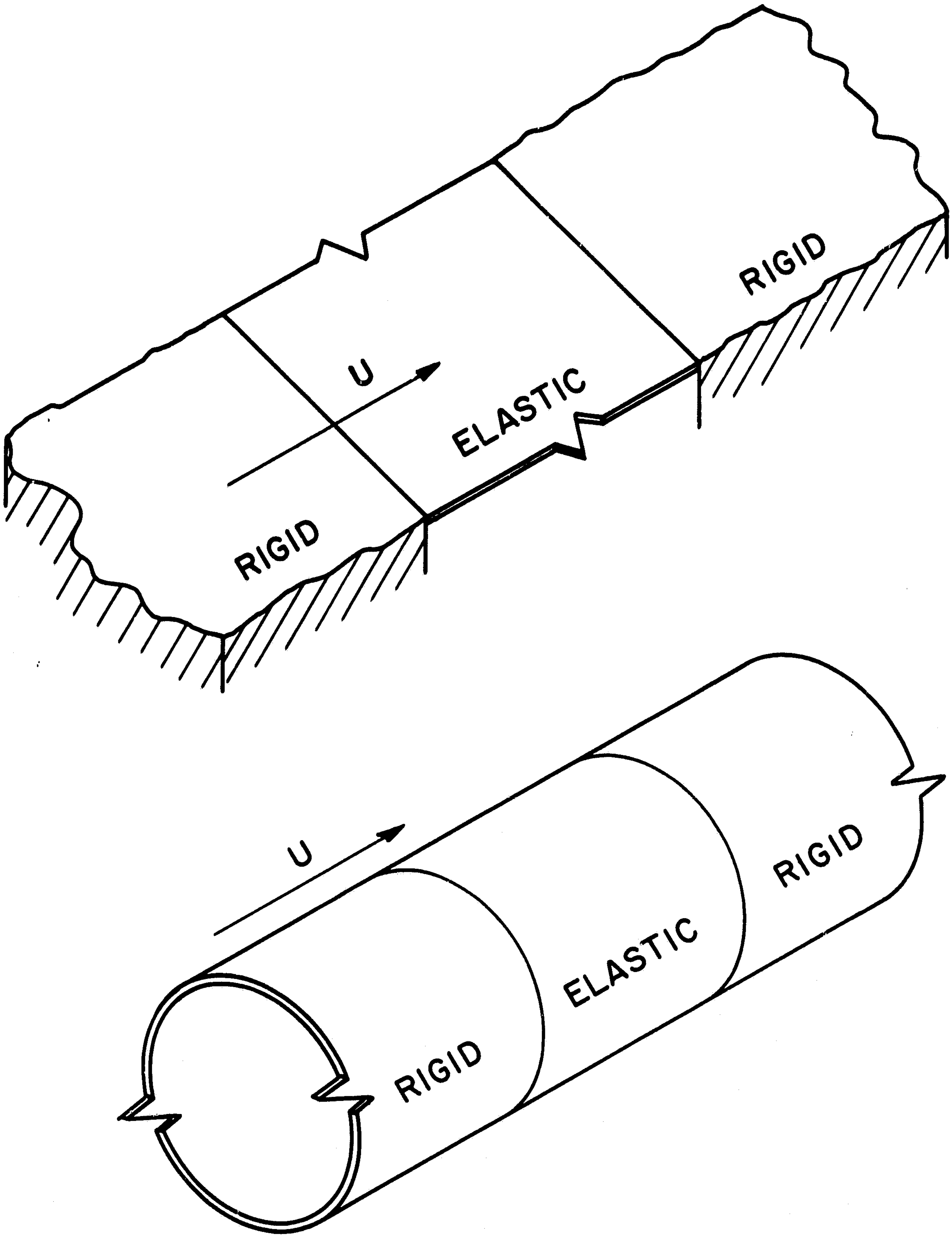

Fig. 1. Typical panel flutter problems.

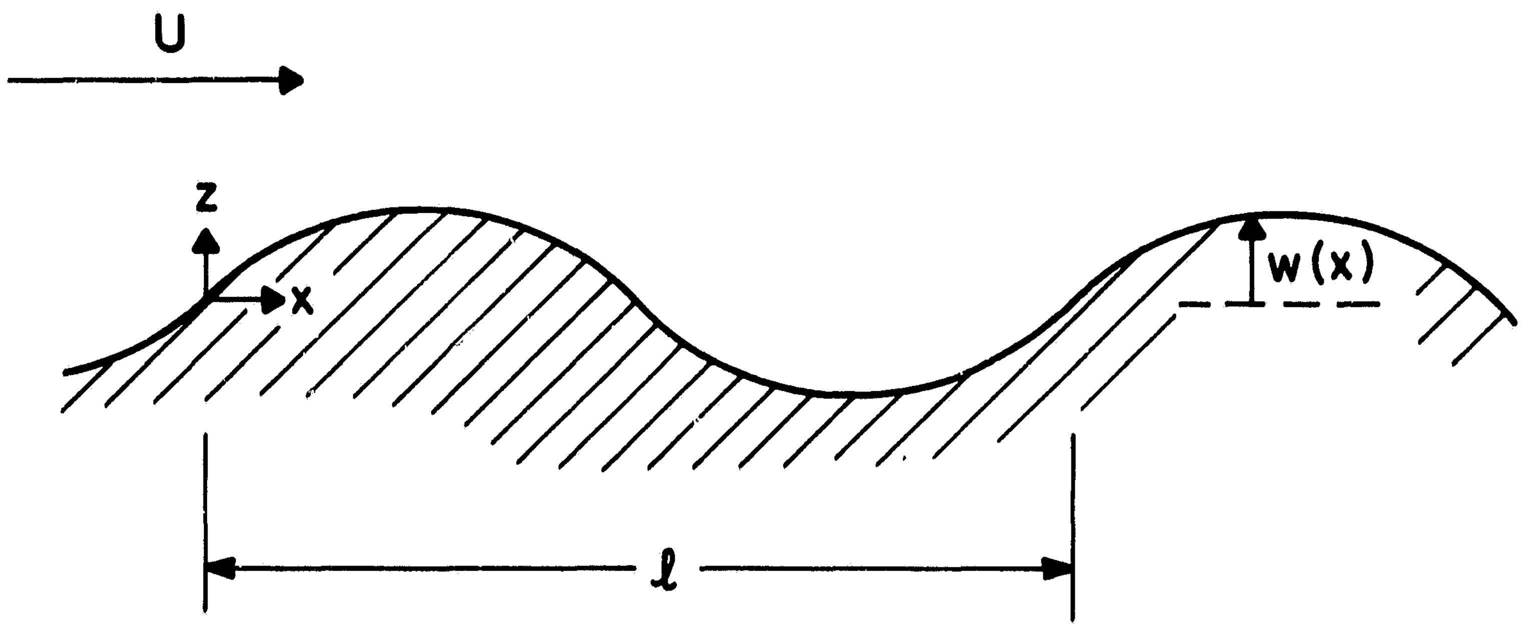

Fig. 2. Flow over an infinitely long, stationary, two-dimensional wavy wall.

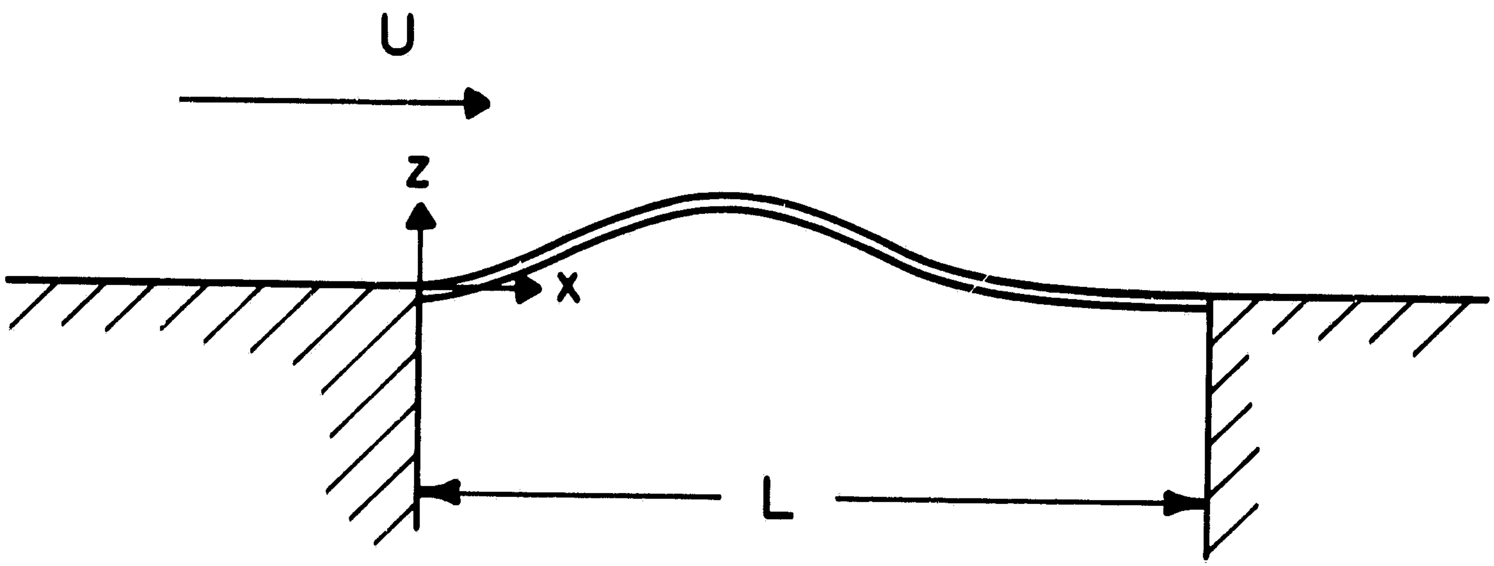

Fig. 3. Flow over a two-dimensional flat panel.

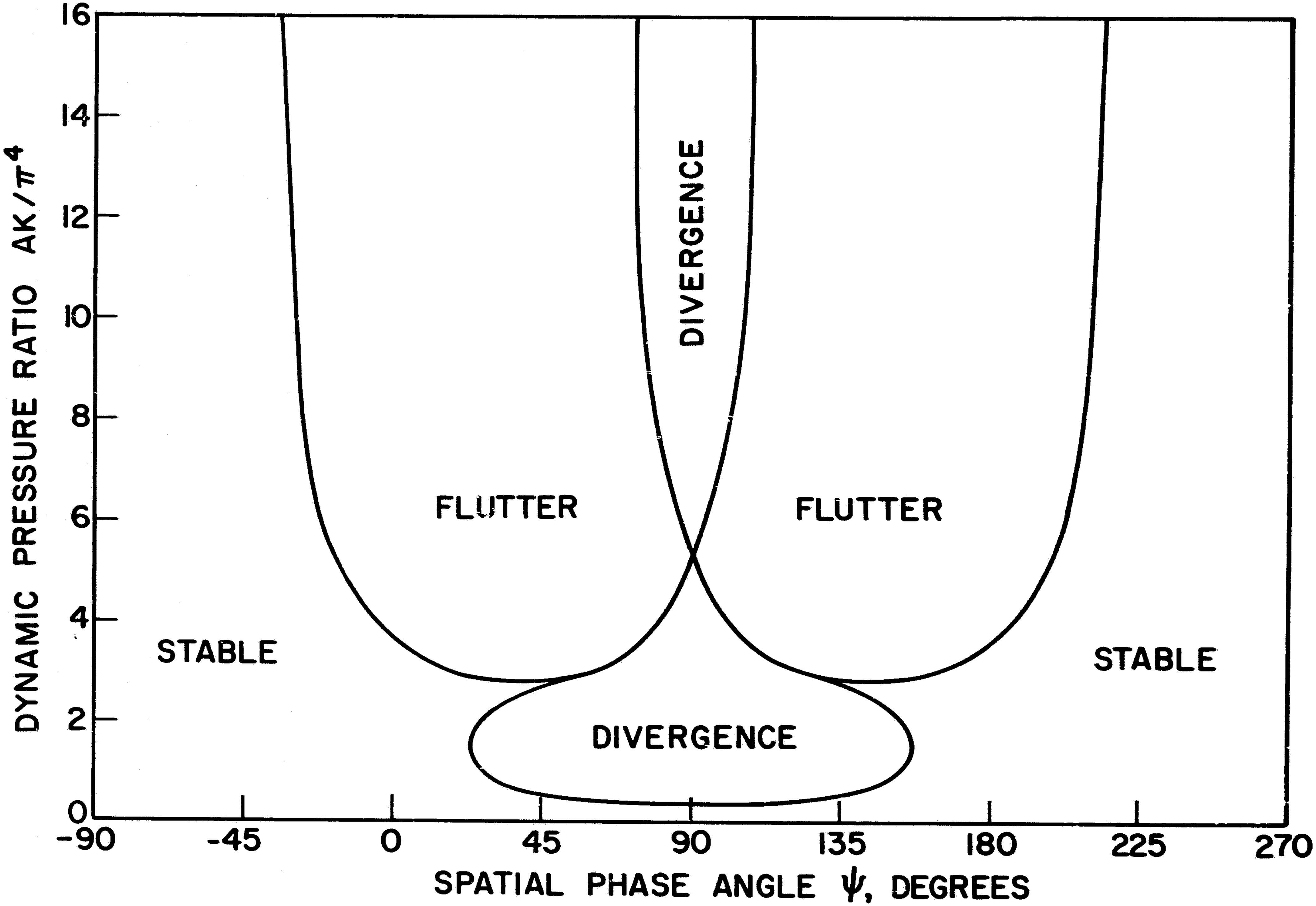

Fig. 4. Stability boundaries for a flat plate.

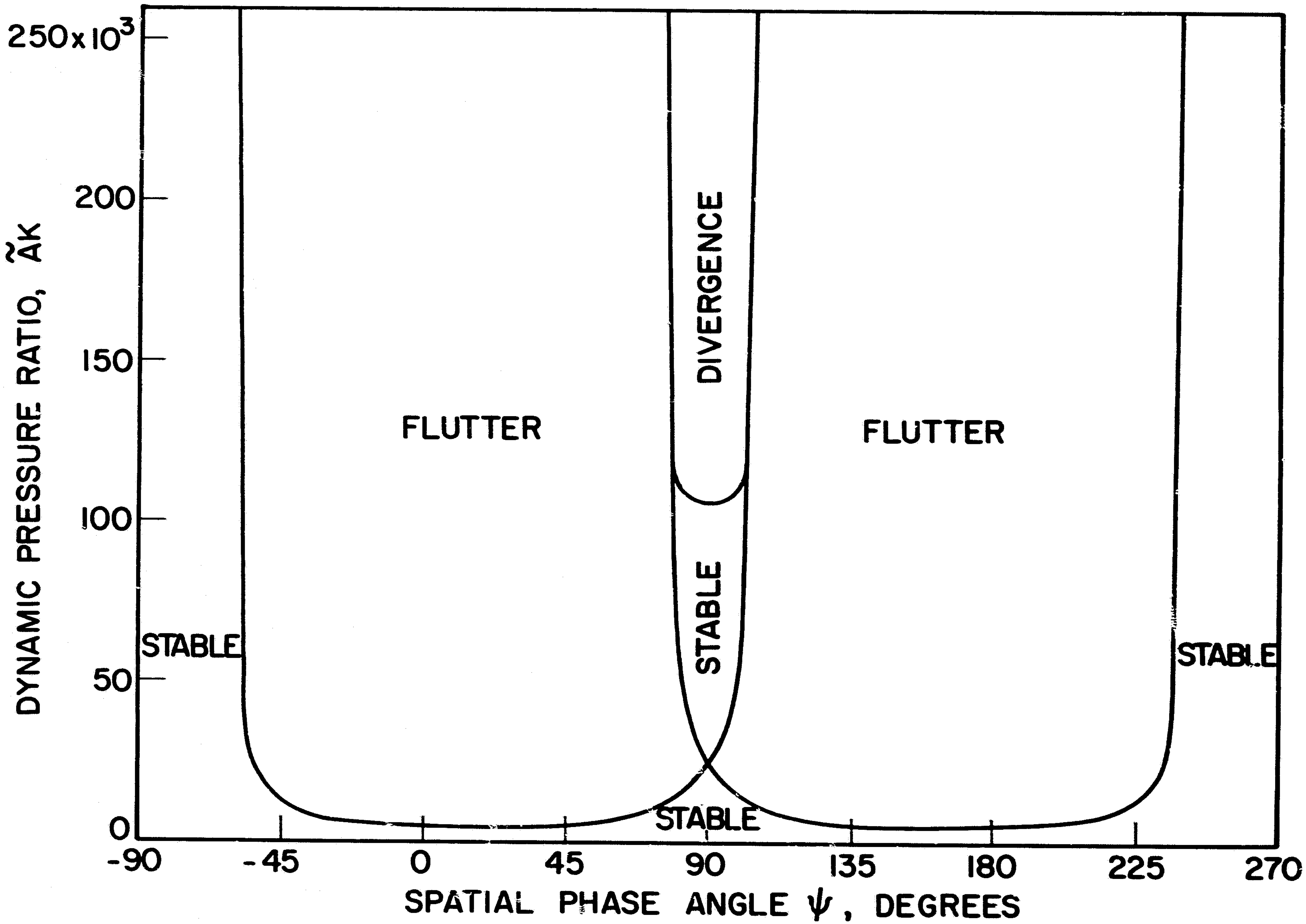

Fig. 5. Stability boundaries for a cylinder.