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THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
DEPARTMENT OF AEROSPACE ENGINEERING
HIGH ALTITUDE ENGINEERING LABORATORY

Scientific Report

On Solar X-Ray and Radio Emissions from the Sun

S. N. GHOSH

Under contract with:

National Aeronautics and Space Administration
Contract No. NASr-54(05)
Washington, D. C.

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OFFICE OF RESEARCH ADMINISTRATION · ANN ARBOR

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Abstract

Assuming a Maxwell distribution of electron velocities in the solar corona, the X-ray fluxes at the earth for temperatures 7×10^5, 1×10^6, 1×10^7 and 1×10^8 °K are calculated for free-free transitions. These temperatures correspond to typical cases of completely quiet, quiet, active and flare conditions of the sun. The fluxes are then compared with those for free-bound transitions and line emissions obtained by Elwert and Kawabata. It was shown that during flares, the radiations are emitted primarily by free-free transitions, and that coronal temperatures and electron densities during different types of flares can be precisely determined from observations of solar X-ray emissions. For other conditions of the sun, line emissions predominate, and the spectrum becomes harder and the flux increases as the sun passes from the quiet to active condition.

The emission of radio waves by free-free transitions for different conditions of the sun are also calculated and it was shown that the calculated flux for $T = 1.10^6$ °K and $n_e = 10^9 cm^{-3}$ is the mean of the observed value for the quiet sun.

X-Ray Emissions from the Sun

1. Introduction

Rocket observations of solar X-ray flux, its spectrum and penetration through the atmosphere have been obtained. It is observed that with the increase of solar activity, the X-ray spectrum becomes harder and the flux increases. Again, Friedman, (1962) obtained a high correlation of E-layer critical frequencies with solar X-ray fluxes.

Observations through radio telescopes show that the cm and decimetric solar radio flux has a high correlation with solar activity. Also Kundu (1965) reported a remarkable correlation between radio flux and E-layer ionization of the sun (solar flare seems to have no effect on E-layer, but influences D-layer).

The solar X-rays consist of both continuous and line emissions. The probable mechanisms for the emission of solar X-rays, which are supposed to be emanated from the corona, are the following:

(1) Free-free transition or bremsstrahlung of high-velocity electrons in the field of protrons which is the main constituent ion in the corona.

(2) Free-bound transition i. e. recombination of electrons with the heavy ions of the corona e. g. Fe, Ni, Si (ion-electron recombination).

(3) Line emissions (mainly composed of the permitted lines of highly ionized elements e. g. Fe XXIV, Si XIII, Mg X etc.) which are produced by electronic collisions. Since the energy differences between the various levels are great only the first level is excited producing resonance radiations.

The hyperbolic free-free transition of electrons in the field of protons gives continuous emission. The wavelength for maximum emission depends upon the velocity of electrons i. e. on the temperature of the corona.

The amount of maximum emission is proportional to the product of electron and proton densities, or to the square of electron density. Therefore, if the emission is due to free-free transition, knowing the wavelength for maximum emission, the temperature of the corona can be determined and from the amount of maximum emission, the coronal electron density can be obtained. We shall show that when flare occurs, the radiations are emitted primarily by free-free transitions. Hence, coronal temperatures and electron densities during different types of flares can be precisely determined from observations of solar X-ray emissions.

2. Free-free Transition or Bremsstrahlung

The power radiated by free-free transitions of electrons in the field of an atomic nucleus is given by

$$J = n_i n_e \int h\nu \, dq \, v f(v) \, dv$$

where

n_i — ion density

n_e — electron density

dq — cross section for emission of a photon in the energy range $dh\nu$

$f(v)$ — velocity distribution function for electrons.

Assuming the Maxwell distribution of velocities* of electrons and $Z = 1$**, Elwert (1948) showed that the above expression reduces to the following for free-free transition

$$J_{ff} d\nu = C \left(\frac{\chi_H}{KT} \right)^{\frac{1}{2}} e^{-\frac{h\nu}{KT}} \, \bar{g} \, n_i n_e \, d\nu$$

where

$$C = 2^7 \left(\frac{\pi}{3} \right)^{3/2} \alpha^3 \chi_H \, a_o^3 \, f_1$$

$$= 1.7 \times 10^{-40} \, f_1 \quad \text{erg cm}^3$$

*In the solar corona, where the X-rays are supposed to originate, because of the strong gravitational field of the sun, the thermal velocity is less than the escape velocity. Hence, the majority of electrons suffer several collisions and the Maxwell velocity distribution is established (Shklovskii, 1965).

**The corona consists of about 80% H and 20% He and is wholly ionized and hence $Z = 1$.

α - Sommerfield's fine structure constant

$\bar{g}$ - Grant factor*** $= \frac{\sqrt{3}}{\pi} \ln \frac{4KT}{h\nu\xi}$ $\xi =$ Euler's constant

T - absolute temperature of electrons

a_o - radius of the first Bohr orbit

χ_H - ionization energy of the hydrogen atom

f_1 - constant representing the uncertainty of cross section for photo-recombination. It is assumed to be unity.

Therefore

$$J_{ff}d\nu = 7x10^{-41} \left(\frac{10^6}{T}\right)^{\frac{1}{2}} e^{-\frac{h\nu}{KT}} n_i n_e \, d\nu \quad \text{erg cm}^{-3}\text{sec}^{-1}$$

To calculate the flux, F_{ff}, at the earth, $J_{ff}d\nu$ is integrated over the volume of the emitting corona † and divided by $2.4\pi R^2$ where R = sun-earth distance = $1.5x10^{13}$cm (the factor 2 accounts for the fact that one-half of the radiation falls on the earth). Therefore

$$F_{ff}d\nu = \frac{\int J_{ff}d\nu \, dv}{8\pi R^2} = \frac{7x10^{-41}}{8\pi R^2} \left(\frac{10^6}{T}\right)^{\frac{1}{2}} e^{-h\nu/KT} \, d\nu \int n_i n_e \, dv \text{ erg cm}^{-2}\text{sec}^{-1}.$$

Assuming $\int n_i n_e dv = 3x10^{49} \text{cm}^{-3}$ (Elwert, 1954), F_{ff} has been calculated for T = $7x10^5$, $1x10^6$, $1x10^7$ and $1x10^8$ °K which correspond to typical cases of completely quiet, quiet, active and flare conditions of the sun. The fluxes per 1A wavelength interval have been plotted in Figs. 1-4.

***The Grant factor for untraviolet and X-rays is of the order unity (Allen, 1955; Shklovskii, 1965).

† Solar X-rays are believed to be emitted from the inner corona and in the transition zone between chromosphere and corona.

3. Free-bound Transition

The emission by free-bound transition for a frequency interval $d\nu$ is given by (Elwert, 1954)

$$J_{fb}\, d\nu = C\, n_e^2 \left(\frac{\chi_H}{KT}\right)^{3/2} e^{-\frac{h\nu}{KT}} \sum_{h\nu \geq \chi_{n_o} Z_i}^{(Z)} \sum^{(i)} X_{Zi}\, d\nu$$

where

$$C = 1.7 \times 10^{-40} f_1 \text{ erg cm}^3$$

$$X_{Zi} = e^{\chi_{n_o} Z_i / KT} \frac{n_{Z,\,i+1}}{n_e} \frac{\zeta_{n_o},\, Z, i+1}{n_o} \left(\frac{\chi_{n_o} Z_i}{\chi_H}\right)^2$$

Z - same as Z_R

ζ_n - number of unoccupied state in the nth energy level

χ_{n_o} - ionization energy of the atom for the n_o th energy level.

For T = 7×10^5 and 1×10^6 °K, the fluxes at earth calculated by Elwert (1952 and 61) are taken. For T = 1×10^7 and 1×10^8 °K, using Kawabata's data for $J_{fb}/f_1\, n_e^2$ after numerical correction, the fluxes at earth are calculated. These fluxes for a wavelength interval of 1A are plotted in Figs. 1-4.

4. Line Emission

The line emission can be calculated from the formula (Elwert, 1954)

$$J_L = 4\sqrt{\pi}\ f_3 \alpha\ a_o^2\ c \chi_H \left(\frac{\chi_H}{KT}\right)^{\frac{1}{2}} n_e^2 \sum_Z \sum_i Y_{Z_i}$$

$$= 9.5\text{x}10^{-19}\ f_3 \left(\frac{\chi_H}{KT}\right)^{\frac{1}{2}} n_e^2 \sum_Z \sum_i Y_{Z_i}\ \text{erg cm}^{-3}\text{sec}^{-1}$$

where

$$Y_{Z_i} = \frac{n_i}{n_e} \sum_{l_o} \zeta_{n_o l_o}\ C^n_{n_o l_o}\ e^{-h\nu/KT}\ G_3 (h\nu/KT)$$

f_3 - a dimensionless parameter depending on the probable error of the method of calculating cross section.

The values of $\sum_{l_o} \zeta_{n_o l_o} C^n_{n_o l_o}$ and $G_3(h\nu/KT)$ are calculated by Elwert (1954).

For T = $7\text{x}10^5$ and $1\text{x}10^6$ °K, the fluxes for line emissions at the earth are taken from Elwert (1954). For the other two temperatures, T = $1\text{x}10^7$ and $1\text{x}10^8$ °K, they are calculated from the values of $J_L/f_3 n_e^2$ given by Kawabata. These fluxes are also plotted in Figs. 1-4. In these figures, the observed values of X-ray emissions from the sun are also plotted. f_3 is assumed to be unity.

For line emissions, free-free and free-bound transitions, the emission is proportional to $n_i n_e$. Because, in the highly ionized plasma from which the radiations are emitted n_i is proportional to n_e, the emission is proportional to n_e^2.

Analysis of *Figs.* 1 - 4, leads to certain conclusions which are given in Table 1.

Table 1

Solar X-ray Emission by Different Mechanisms

T °K	Mechanisms for Excitation			Agreement of computed curves with the observed values.	Observed Range of Observations*	Remarks
	Free-free transition	Free-bound transition	Line emission			
$7x10^5$ (Completely quiet sun)	Max. emission at 100A; of the three types of emissions, it is least important	Moderately important. Most important below 40A and above 304A.	Most important between 40-304A; strong lines cluster in the region 80-100A.		40-304 A contributed by line emission only.	Free-free transition is unimportant.
$1x10^6$ (quiet sun)	Max emission at 70A; least important	Maximum emission at 45A; moderately important; most important below 18A and above 304A.	Most important for 18-304, around 60-70A lines cluster.	Line intensities and observed wavelength extent of emission agree with Hinteregger's observed values during solar minimum.	22-304 contributed by line emissions only.	-do-
$1x10^7$ (active sun)	Max. emission at 7A; most important emission below 6 and above 25 A	Least important.	Most important for 6-25, strong lines cluster between 10-15 A.	Nicolet's model for flare 2	4-25 contribution by line emission only	Free-bound transition is unimportant.
$1x10^8$ (solar flare)	Max. emission at 7A. Most important for the whole region.	Least important.	Only three unimportant lines.	Nicolet's model for flare 3	.2-13 contribution by free-free emission.	Both free-bound transitions & line emission are unimportant.

*Assuming $6x10^{-4}$ erg $cm^{-2} sec^{-1}$ is the limit of observation.

5. Conclusions

The following conclusions can be drawn from the above theoretical considerations.

1. As the sun passes from the quiet (10^5 - 10^6 °K) to the flare condition (10^8 °K), the emission by the free-free transition increases due to higher temperature. At the sametime, ion-electron recombination which requires slow electrons, decreases and hence emissions by the free-bound transition become less important.

2. The amount of total solar X-ray emission does not increase significantly from the quiet to flare condition of the sun. When the sun is quiet, the velocities of electrons are sufficient to excite resonance radiations of ions, whereas during flares, the low intensities of line emissions are compensated by the higher free-free transitions (the energy differences between the various levels of the strongly ionized ions of the corona are so great that only excitation of the first level of the ions occurs).

3. Hard X-rays are emitted as the sun passes from the quiet to the flare condition.

4. Since at high temperatures, free-free transition predominates, the emission is continuous.

5. The wavelength for maximum emission and the amount of emission by free-free transition depends markedly on temperature (the wavelength for maximum emission shifts to shorter wavelength with increased emission for higher temperatures). Hence, the coronal temperature during different types of flares can be precisely determined by observing solar X-ray emissions.

Radio Emissions from the Sun

The emission of radio waves from the sun can occur by several mechanisms. It is calculated for free-free transition.

The free-free transition (bremsstrahlung) per unit solid angle per unit volume for the frequency range $d\nu$ is given by (Allen, 1955)

$$j_\nu \, d\nu = \frac{16}{3}\left(\frac{\pi}{6}\right)^{\frac{1}{2}} \frac{e^6 Z^2}{c^3 m^2}\left(\frac{m}{KT}\right)^{\frac{1}{2}} g \exp(-h\nu/KT)\, n_e n_i \, d\nu$$

$$= 5.443 \times 10^{-39} Z^2 g \exp(-h\nu/KT) T^{-\frac{1}{2}} n_e n_i \text{ erg cm}^{-3} \text{sec}^{-1} \text{sterad}^{-1}$$

(T in ^oK, n_e and n_i in cm^{-3})

The flux at the earth for the frequency interval $d\nu$

$$F_{ff} = \frac{\int 4\pi j_\nu \, d\nu \, dV}{8\pi R^2}$$

$$= \frac{1}{2R^2} \times 5.443 \times 10^{-39} Z^2 g \exp(-h\nu/KT) T^{-\frac{1}{2}} \int n_i n_e dV \text{ erg cm}^{-2} \text{sec}^{-1}$$

Assuming $\int n_i n_e dV = 3\text{x}10^{49} cm^{-3}$, $\exp(-h\nu/KT) = 1$ in the radio region, $Z = 1$ and the sun-earth distance $R = 1.5\text{x}10^{13}$cm,

$$F_{ff} = 3.628\text{x}10^{-16} g \, T^{-\frac{1}{2}} d\nu \text{ erg cm}^{-2} \text{sec}^{-1}$$

$$= 3.628\text{x}10^{3} g \, T^{-\frac{1}{2}} \text{ RU } \left[\text{IRU} = 10^{-22} \omega \, m^{-2} (c/s)^{-1}\right] \qquad (1)$$

The Grant factor for radio waves is given by (Allen, 1955)

$$g \simeq \frac{\sqrt{3}}{\pi} \ln(4KT/e^2 n_e^{1/3})$$

$$= 1.2695 \, (3.38 + \log_{10} T - \frac{1}{3} \log_{10} n_e) \qquad (2)$$

Using formulas (1) and (2) and assuming the electron density in the corona, $n_e = 10^9 cm^{-3}$, F_{ff} has been calculated for $T = 7x10^5$, $1x10^6$, $1x10^7$ and $1x10^8$ ^oK and is plotted in Fig. 5. In the same figure, observed values of F_{ff} for the quiet sun are also plotted. The curves show the following:

1. The flux decreases with the increase of coronal temperature.
2. For a particular temperature, the flux remains constant with wavelength.
3. For the wavelength range 1-450 cm, the calculated flux for $T=1.10^6$ ^oK and $n_e=10^9 cm^{-3}$ is the mean of the observed value for the quiet sun.

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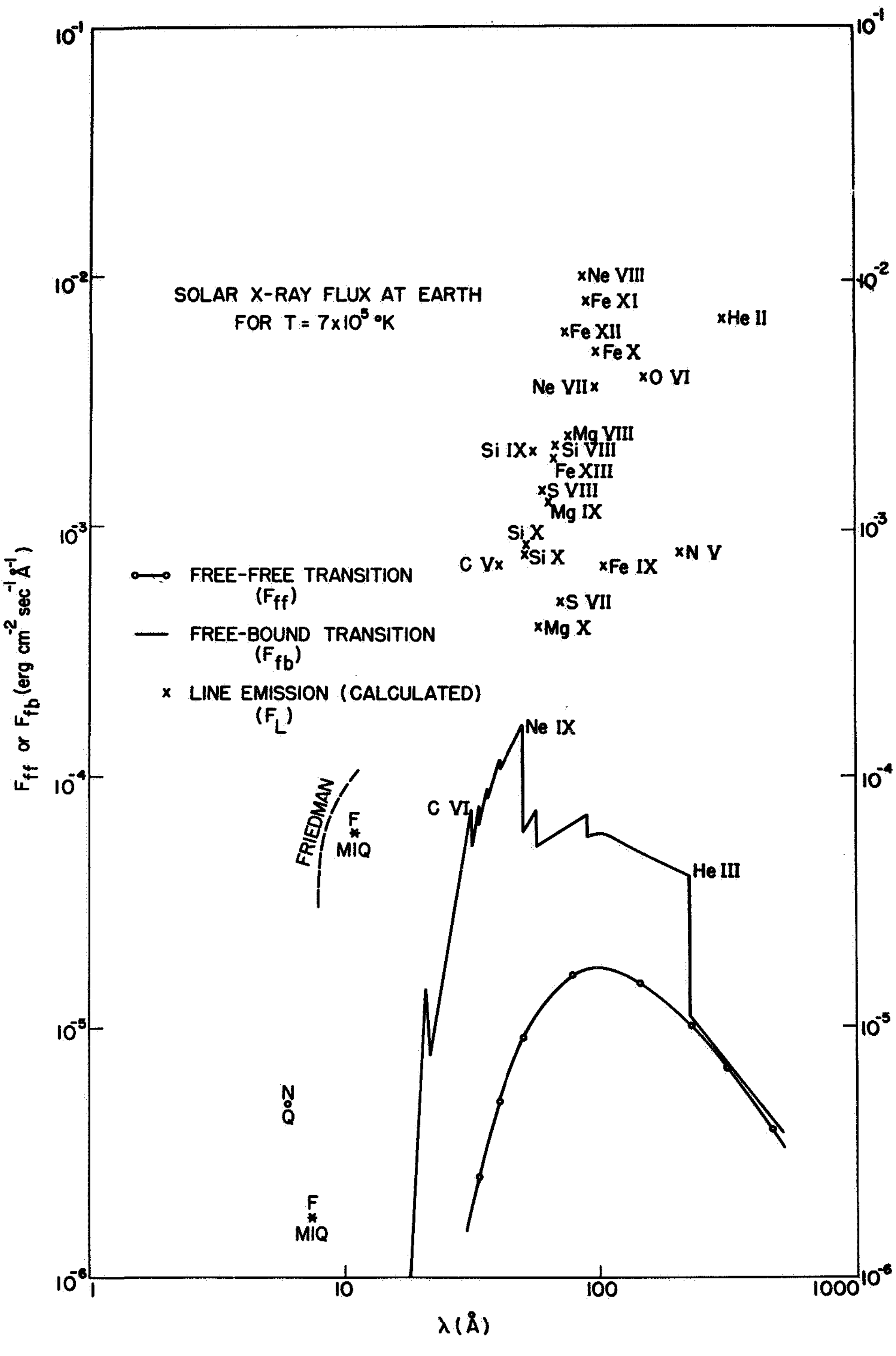

Fig. 1 The variation of the solar X-ray flux at earth with the wavelength for the completely quiet condition of the sun (coronal temperature 7×10^5 °K). Values obtained by Friedman and Nicolet are denoted by F and N. MIQ and Q refer to the minimum quiet and quiet conditions of the sun.

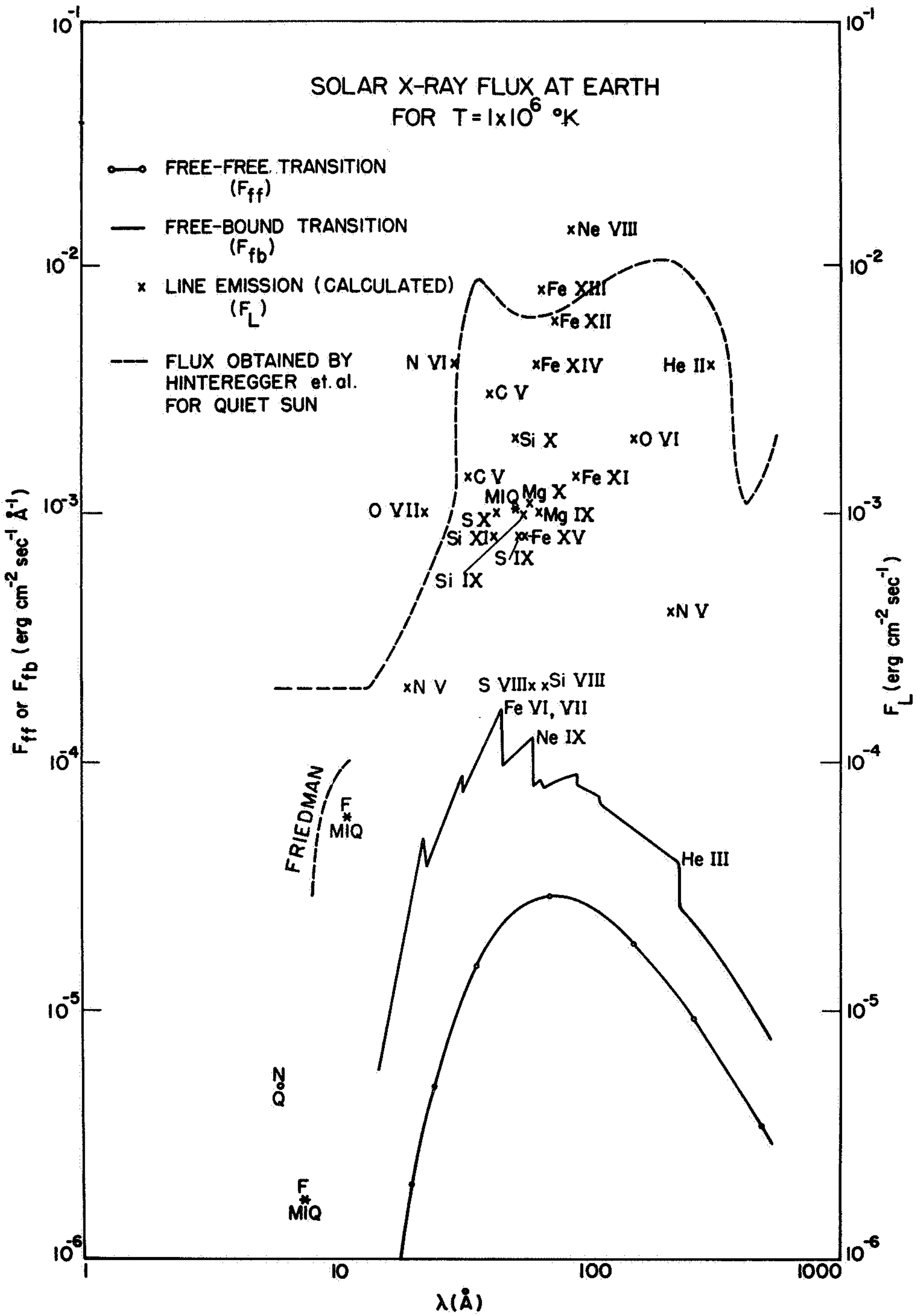

Fig. 2 The variation of the solar X-ray flux at earth with the wavelength for the quiet condition of the sun (coronal temperature $1x10^{6o}K$). Values obtained by Friedman and Nicolet are denoted by F and N. MIQ and Q refer to the minimum quiet and quiet conditions of the sun.

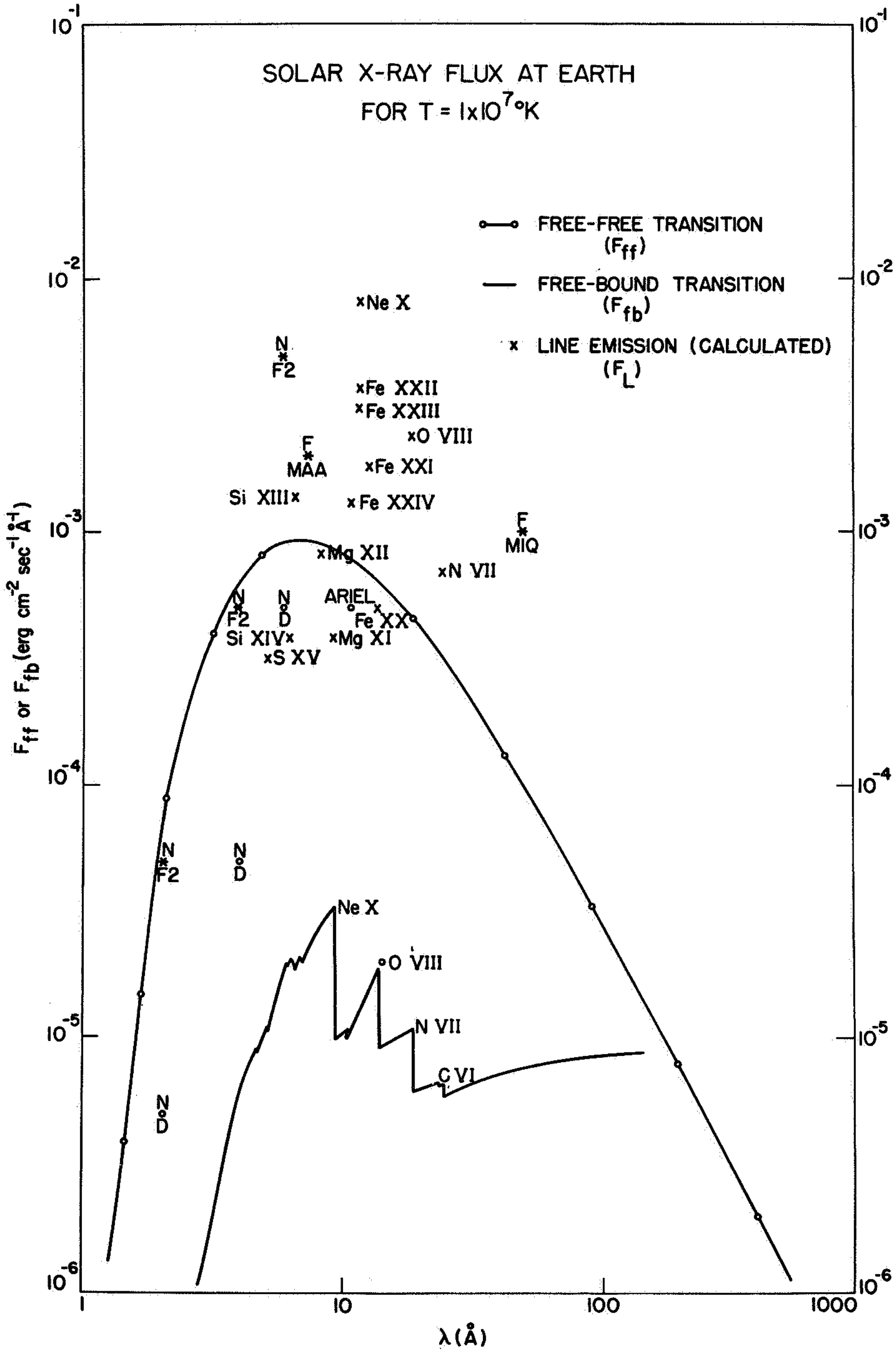

Fig. 3 The variation of the solar X-ray flux at earth with the wave-length for the active condition of the sun (coronal temperature $1x10^7$ °K). Values obtained by Friedman and Nicolet are denoted by F and N. MIQ, MAA, D and F2 refer to the minimum quiet, max. active, disturbed and flare type 2 conditions of the sun.

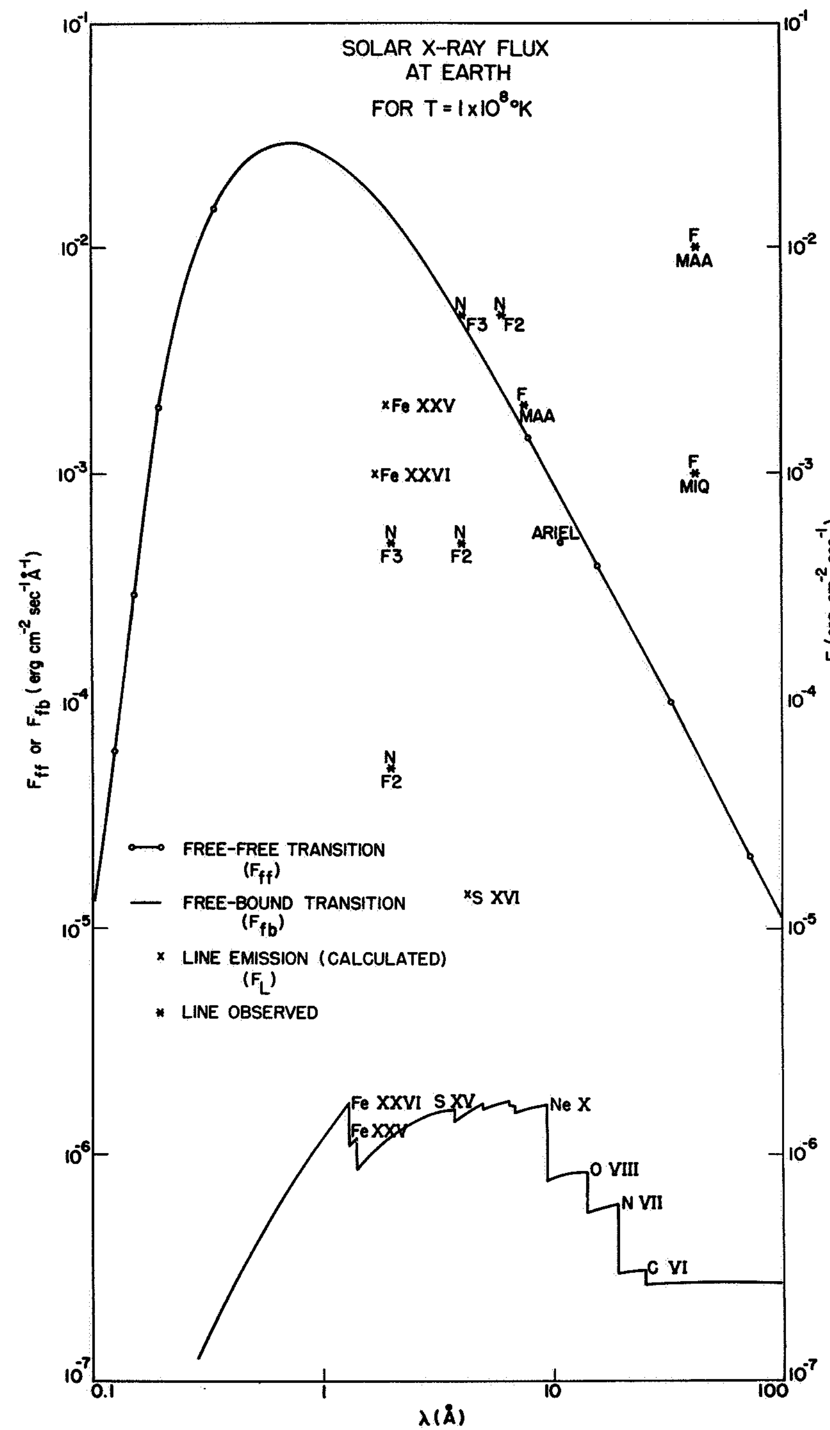

Fig. 4 The variation of the solar X-ray flux at earth with the wavelength for the flare condition of the sun (coronal temperature $1x10^{8}$ °K). Values obtained by Friedman and Nicolet are denoted by F and N. MIQ, MAA, F2 and F3 refer to the minimum quiet, maximum active and flares types 2 and 3 condition of the sun.

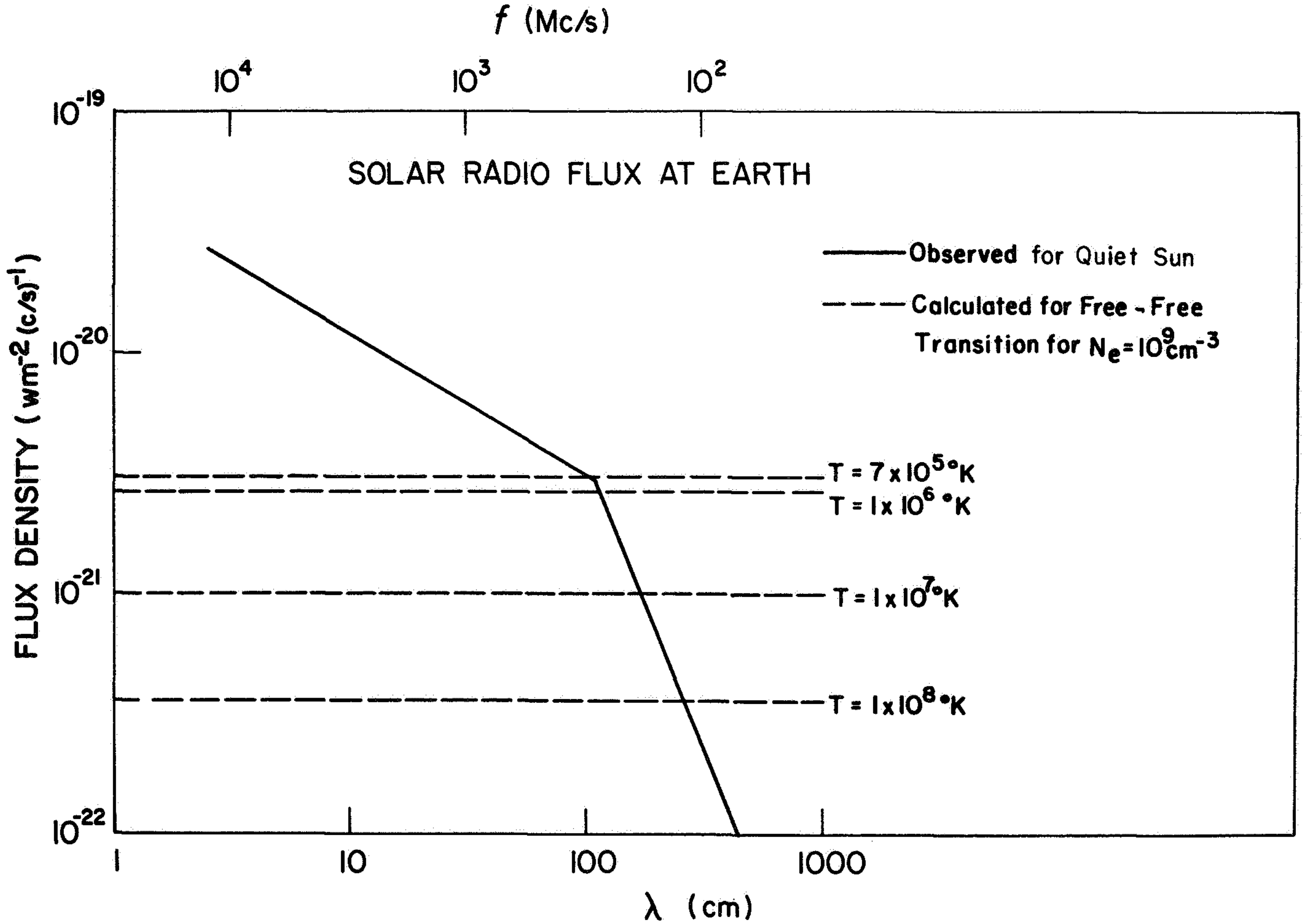

Fig. 5 The comparison of the observed solar radio flux at earth for the quiet sun with those calculated for free-free transitions for $N_e = 10^9 cm^{-3}$ and for different coronal temperatures.