Facility

IDENTIFICATION OF HUMAN RESPONSE MODELS IN MANUAL CONTROL SYSTEMS

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Summary

Frequency domain and time domain methods of analysis are reviewed with regard to their application toward identifying pilot models. The models would subsequently be used to study the stability and performance of a man-machine system in which the human controller performs a compensatory tracking task. Sample linear model results are compared and discussed. The inherent requirement constraining the freedom of the form of the pilot model is also discussed. The constraint in the frequency domain consists of smoothing with respect to frequency; whereas, the constraint for the time domain model is more natural and meaningful in that it consists simply of limiting the memory of the pilot model. The linear models determined by both methods were almost identical.

The time domain method of analysis enables the determination of a nonlinear pilot model. The inclusion of a cubic as well as a linear term accounted for only a small additional part of the pilot's remnant and indicated that only a small portion of the total power of the pilot's output is caused by nonlinearities. The power spectral density of an ensemble average of the pilot's outputs is used to determine the upper limit on the amount of power associated with a deterministic response. The indication is that about half the remnant is stochastic and only a small part is due to nonlinear and time-varying response for the example discussed.

Introduction

Analysis of the stability and performance of control systems involving a pilot as an active element have been hampered by the lack of an adequate mathematical model of the pilot's control function. Experiments have been performed to determine such models since 1947.[1] In these experiments the pilot performed a compensatory tracking task in which he tried to minimize the difference between the response of a simple controlled element and a disturbance function.

Data resulting from such experiments have been analyzed, and linear pilot models have been obtained[2,3,4] for a limited set of controlled element dynamics. The methods used to construct the pilot models have been almost exclusively in the frequency domain. Recently, the time domain analysis has been applied to the problem of modeling pilots.

It is the purpose of this paper to assess first the frequency domain method of analysis and then the time domain analysis of Balakrishnan and Hsieh. A comparison of the results of the two forms of analysis applied to a linear model is made and their advantages and disadvantages discussed. Next, the time domain method of analysis is applied to the identification of a nonlinear pilot model. This is the first time that the nonlinear time domain method has been applied to human response data. The power spectral density of an ensemble average of the pilot's output is used to determine an estimate of the amounts of control response that are linear, deterministic, and stochastic.

Description of Experiment

The classical experiment for obtaining data from which pilot models can be identified is illustrated in figure 1. The pilot is asked to minimize the error, e, displayed to him by an oscilloscope, television screen, or meter by manipulating a controller. The controller deflection, c, is sent to an analog computer which computes the response of the controlled element and adds to it the input disturbance function, i, forming an error which, in turn, is sent to the display. Recordings are made of the signals which are later processed to obtain the model of the pilot. Similar experiments have been performed in actual flight in which the pilot maneuvers the airplane.[3,4]

Discussion and Results

Frequency Domain Method

Classically, the model of the pilot is considered to be a linear-describing function plus a remnant signal, r, added to the model output as shown in figure 2. The describing function estimate, $\hat{Y}_p(j\omega)$, is obtained by first computing the cross-spectral density functions $\Phi_{ic}(j\omega)$ and $\Phi_{ie}(j\omega)$. The estimate of $Y_p(j\omega)$ is then the ratio

$$\hat{Y}_p(j\omega) = \frac{\Phi_{ic}(j\omega)}{\Phi_{ie}(j\omega)}$$

Cross-spectral density functions have generally been used instead of Fourier transforms as a means of removing the bias in the estimate of $Y_p(j\omega)$ introduced by the remnant. The use of cross-spectral density functions, however, was shown in reference 5 to have no effect on the bias. The same estimate of $Y_p(j\omega)$, therefore, can be expressed as the ratio of Fourier transforms

$$\hat{Y}_p(j\omega) = \frac{F[c(t)]}{F[e(t)]}$$

and the bias in both cases is

$$\hat{Y}_p(j\omega) - Y_p(j\omega) = \frac{\Phi_{ir}(j\omega)}{\Phi_{ie}(j\omega)} = \frac{F[r(t)]}{F[e(t)]}$$

Figure 3 (from ref. 4) shows an example of the results of a study of human control response on simulators in which describing functions for three different pilots were obtained by using the frequency domain method. The values presented for $\hat{Y}_p(j\omega)$ are the means of 10 runs for pilot A and three runs each for pilots B and C. The vertical lines indicate the range of plus or minus one sigma for each of the points. The lack of a vertical line indicates the range to be less than the height of the symbol. The input disturbance function consisted of the sum of 10 sinusoids. Values of $\hat{Y}_p(j\omega)$ were determined at the input frequencies. The use of sinusoids for the input disturbance function has the advantage of concentrating the power at several frequencies, thereby enhancing the accuracy of the estimate of the pilot-describing function at the input frequencies. If, on the other hand, a random input is used, mathematical difficulties may be encountered when using frequency domain methods. For example, if no constraint is placed on the form of $\hat{Y}_p(j\omega)$, the resulting estimate will account for the entire pilot output, c, erroneously indicating the remnant, r, to be zero. This result follows from applying the relationship

$$\hat{Y}_p(j\omega) = \frac{\Phi_{ic}(j\omega)}{\Phi_{ie}(j\omega)} = \frac{F[c(t)]}{F[e(t)]}$$

at all frequencies. A constraint on $\hat{Y}_p(j\omega)$, therefore, is usually provided by smoothing the values of the cross-spectral density functions or Fourier transforms, or by fairing a curve through the raw estimates of the pilot's describing function, $\hat{Y}_p(j\omega)$, or both. Nevertheless, the required constraint on $\hat{Y}_p(j\omega)$ compromises one of the basic claims made for the analyses in the frequency domain, namely, that the model may be represented with unlimited freedom.

Linear Time Domain Method

Let us now consider a linear analysis in the time domain in which the output of a linear pilot model is expressed in the form (see ref. 6)

$$c(t) = \int_0^{T_M} h_p(\tau)e(t - \tau)d\tau$$

Because the time histories c(t) and e(t) must be sampled for analysis, it is more appropriate to write

$$c(n) = \sum_{m=1}^{M} h_p(m)e(n - m + 1)$$

or in matrix form

$$\underline{c} = E\,\underline{h}_p$$

where

$$E = \begin{bmatrix} e(M) & e(M-1) & \ldots & e(3) & e(2) & e(1) \\ \cdot & & & e(4) & e(3) & e(2) \\ \cdot & & & & e(4) & e(3) \\ \cdot & & & & & e(4) \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ e(N-1) & & & & & e(N-M) \\ e(N) & e(N-1) & \ldots & \ldots & & e(N-M+1) \end{bmatrix}$$

$$\underline{h}_p = \begin{pmatrix} h_p(1) \\ h_p(2) \\ \cdot \\ \cdot \\ h_p(M) \end{pmatrix}$$

$$\underline{c} = \begin{pmatrix} c(M) \\ c(M+1) \\ \cdot \\ \cdot \\ c(N) \end{pmatrix}$$

The sampled impulse response of the pilot model, $h_p(m)$, can be obtained by using the least-squares formulation

$$\hat{\underline{h}}_p = [E^TE]^{-1}E^T\underline{c}$$

Inherent in the time domain representation of the pilot model is the assumption that the output at any one time is a function of only a finite time of the history of the error. This maximum memory is denoted by T_M $(M\Delta\tau)$ in the integral expression or by M in the summation expression of the pilot model output. For the pilot model, T_M was varied (by changing $\Delta\tau$ and keeping M constant) until it was determined that the value of $h_p(\tau)$ was essentially zero beyond about 1 second. The value of T_M, therefore, was selected to be somewhat larger than 1 second. Figure 4 shows a typical result of such an analysis. It can be seen that the model impulse response first peaks at about 0.25 second, then reverses at about 0.45 second to peak in the opposite direction at about 0.6 second, then subsides to zero. The first sample (τ = 0.05 sec) is typically negative but has been faired to correspond to a pure time delay of 0.05 second. One indication of the degree to which a model represents an actual pilot is the ratio outputs of the power of the model in relation to the total power of the pilot's output. Linear pilot models will typically account for 65 to 90 percent of the total for a 4-minute run. The percentage is somewhat higher for shorter runs.

The time domain results can be transformed to the frequency domain for comparison with the frequency domain results through the use of the Fourier transform

$$\hat{Y}_p(j\omega) = F[\hat{h}_p(\tau)]$$

Figure 5 shows such a comparison. The agreement between the two methods is good. One advantage of the time domain method is that $\hat{Y}_p(j\omega)$ can be determined as a continuous function of frequency even when the input consists of sinusoids, as it does for the example shown. The time domain results, by their nature, will always yield $\angle\hat{Y}_p(j\omega) = 0$, or $-180°$, as $\omega \to 0$ and $\frac{d|\hat{Y}_p(j\omega)|}{d\omega} = 0$ as $\omega \to 0$. Had the pilot describing function exhibited gain fluctuations or a phase lag at the lowest frequencies shown, these characteristics would not have been identified by the time domain model unless the value of T_M were increased considerably. This does not mean that the time domain model is limited, but, rather, that T_M should not be unduly limited. Just as it is necessary to constrain the frequency domain model by smoothing, it is also necessary to constrain the time domain model by limiting T_M to a value considerably less than the record length. For pilot models this represents a very natural and meaningful constraint in the time domain, compared with smoothing in the frequency domain. Still another advantage of analysis in the time domain is the capability of constructing nonlinear pilot models.

Nonlinear Time Domain Method

Nonlinear behavior on the part of the pilot accounts for at least part of the remnant of a linear pilot model. It is, therefore, of interest to investigate nonlinear pilot models. The output of a nonlinear time domain pilot model can be expressed by using a Volterra integral series

$$
\begin{aligned}
c(t) = & \underbrace{\int_0^{T_M} h_{p_1}(\tau)e(t-\tau)d\tau}_{\text{(linear)}} \\
& + \underbrace{\int_0^{T_M}\int_0^{T_M} h_{p_2}(\tau_1,\tau_2)e(t-\tau_1)e(t-\tau_2)d\tau_1 d\tau_2}_{\text{(quadratic)}} \\
& + \underbrace{\int_0^{T_M}\int_0^{T_M}\int_0^{T_M} h_{p_3}(\tau_1,\tau_2,\tau_3)e(t-\tau_1)e(t-\tau_2)e(t-\tau_3)d\tau_1 d\tau_2 d\tau_3}_{\text{(cubic)}} \\
& + \underbrace{\ldots\ldots\ldots\ldots}_{\text{(higher order)}}
\end{aligned}
$$

or in the discrete case

$$
\begin{aligned}
c(n) = & \underbrace{\sum_{m=1}^{M} h_1(m)e(n-m+1)}_{\text{(linear)}} \\
& + \underbrace{\sum_{m_1=1}^{M}\sum_{m_2=1}^{M} h_2(m_1,m_2)e(n-m_1+1)e(n-m_2+1)}_{\text{(quadratic)}} \\
& + \underbrace{\sum_{m_1=1}^{M}\sum_{m_2=1}^{M}\sum_{m_3=1}^{M} h_3(m_1,m_2,m_3)e(n-m_1+1)e(n-m_2+1)e(n-m_3+1)}_{\text{(cubic)}} \\
& + \underbrace{\ldots\ldots\ldots\ldots}_{\text{(higher order)}}
\end{aligned}
$$

It was reasoned that the pilot's control response would be symmetrical so that only the first (linear) and third (cubic) terms were used. The algorithm used to perform the analysis was again based on a least-squares solution. [6]

If

$$\underline{c} = E_{1,3}\underline{h}_{1,3}$$

where

$$\underline{c} = \begin{pmatrix} c(M) \\ c(M+1) \\ \cdot \\ \cdot \\ c(N) \end{pmatrix}$$

$$\underline{h}_{1,3} = \begin{pmatrix} h_1(1) \\ h_1(2) \\ \vdots \\ h_1(M) \\ h_3(1,1,1) \\ h_3(1,1,2) \\ \vdots \\ h_3(1,1,M) \\ h_3(1,2,2) \\ \vdots \\ h_3(M,M,M) \end{pmatrix}$$

$$E_{1,3} = \begin{bmatrix} e(M)\ \ldots\ e(1)\ (e(M)e(M)e(M))\ \ldots\ (e(M)e(M)e(1))\ (e(M)e(M-1)e(M-1))\ \ldots \\ e(M+1)\ \ldots\ e(2) \qquad\qquad (e(M)e(M-1)e(1))\ \ldots\ (e(1)e(1)e(1)) \\ e(M+2) \\ e(M+3) \\ \cdot \\ \cdot \qquad\qquad (e(N-M+1)^2e(N-M)) \\ e(N)\ \ldots\ e(N-M+1)\ (e(N)e(N)e(N-1))\ \ldots\ldots\ e(N-M+1)^3 \end{bmatrix}$$

Then

$$\hat{\underline{h}}_{1,3} = \left[E_{1,3}^T E_{1,3}\right]^{-1} E_{1,3}^T \underline{c}$$

It is difficult to present the results of such an analysis in a meaningful form, but it is instructive to look at an example step response. Figure 6 shows the response to (1) a step of very small amplitude so that only the linear term contributes significantly to the response and (2) a large step. The responses have been normalized to the amplitude of the step inputs to facilitate comparison. The response of the pilot model to the larger step is slightly faster, has more overshoot, and has a lower steady-state gain. The inclusion of the cubic term increased the ratio of the power of the model output to the total power of the pilot output by only a few percent. This result would indicate the remnant to be largely stochastic as opposed to nonlinear and deterministic.

If the nonlinear model is expanded to include more samples of the cubic term and higher order terms, dimensionality will become a problem. One means of reducing the total dimension is offered by Balakrishnan and Hsieh. [6,7] Through the employment of the adjoint system of equations, a cubic weighting function of the form

$$h_{p_3}(\tau_1,\tau_2,\tau_3) = \int_0^L f_3(t-\tau)f_3(t-\tau_2)f_3(t-\tau_3)d\tau_1 d\tau_2 d\tau_3$$

is obtained. This equation reduces the weighting function of three variables to a single function of one variable. This technique has not yet been applied to the problem of identifying pilot models, and it is not known if the reduction in dimensionality justifies the added computation required.

Analysis of the Pilot's Output

It would be of interest to know what portion of the pilot's response is deterministic, but not linear, in order to assess the potential of a nonlinear pilot model in describing the pilot's output. It is known that at least part of the pilot's output is stochastic, since results of repeated experiments are never identical. It is possible to estimate the proportioning of the power of the pilot's output by examining the power-spectral density functions of both the pilot's output and its ensemble average. Both functions are shown in figure 7. The cross-hatched peaks in the graph show the amount of power associated with a linear response at the frequencies of the input. The shaded areas show the change in the power as a result of ensemble averaging. Since the deterministic response would be unchanged by averaging, the shaded areas are an indication of the power associated with the stochastic portion of the pilot's output, which will not be accounted for by a deterministic model. The unshaded areas, then, are upper limits on the potential increase in power accounted for by using a nonlinear rather than linear pilot model.

The bar graph at the right of figure 7 shows the proportioning of the power (respective areas) of the pilot's output to be 91.7 percent linear (and time invariant), 4.5 percent stochastic, and 3.8 percent nonlinear and other types of responses for the particular example shown. These results should not be generalized, since changes in the controlled element and input can cause a marked change in the proportioning of the power. It should also be noted that a small amount of power may be attributed to a nonlinearity that is significant in other aspects such as limit cycles.

Concluding Remarks

A review of frequency and time domain methods of analysis shows that both methods require constraints on the freedom of the pilot models. The constraint in the time domain is more natural and straightforward than that of smoothing in the frequency domain. The two methods show good agreement for the linear model when the input disturbance function consists of sinusoids.

The inclusion of a cubic term in the time domain pilot model represents the first time the analysis has been applied to human response data. For the example discussed, only a few percent additional power of remnant was accounted for by the addition of the cubic term. An investigation of the power-spectral density of an ensemble average of pilot output indicates the reason to be the largely stochastic nature of the remnant. The proportioning of the power of the pilot's output appears to be about 92 percent due to linear response, 4 percent due to stochastic response, and 4 percent due to nonlinear and other types of responses for the example discussed.

With this step toward the application of time domain methods of analyzing human control response, the work ahead holds much promise toward the determination of more meaningful and useful pilot models.

Symbols

c	pilot output, control deflection, centimeters
$\overline{c^2}$	mean square or total power of c, centimeters^2
E	error matrix
e	error, radians
$F[\ ]$	Fourier transform
h_p	impulse response of pilot, centimeters/radian
i	input, external disturbance function, radians
L	total record length, seconds
M	maximum value of m, $M = \frac{T_M}{\Delta\tau}$
m	index for the argument of h_p
N	maximum value of n
n	index for time
o	linear output of pilot model (control deflection), centimeters
r	remnant signal of pilot model (control deflection), centimeters
T_M	the maximum memory time of the pilot model, seconds
t	time, seconds
$Y_c(j\omega)$	controlled element transfer function, radians/centimeter
$Y_p(j\omega)$	pilot describing function, centimeters/radian
$\Phi_{xy}(j\omega)$	cross-spectral density of $x(t)$ and $y(t)$
$\Phi_{xx}(\omega)$	power-spectral density of $x(t)$
τ	argument of h_p, seconds
$\Delta\tau$	incremental value of τ, seconds
ω	frequency, radians/second
$\hat{}$	estimate

Matrix notation:

(x), $\underline{x}$	column matrix
$[X]$	rectangular matrix
X^T	transpose
X^{-1}	inverse

Numbers used as subscripts denote the pertinent term, or terms, of the Volterra integral series.

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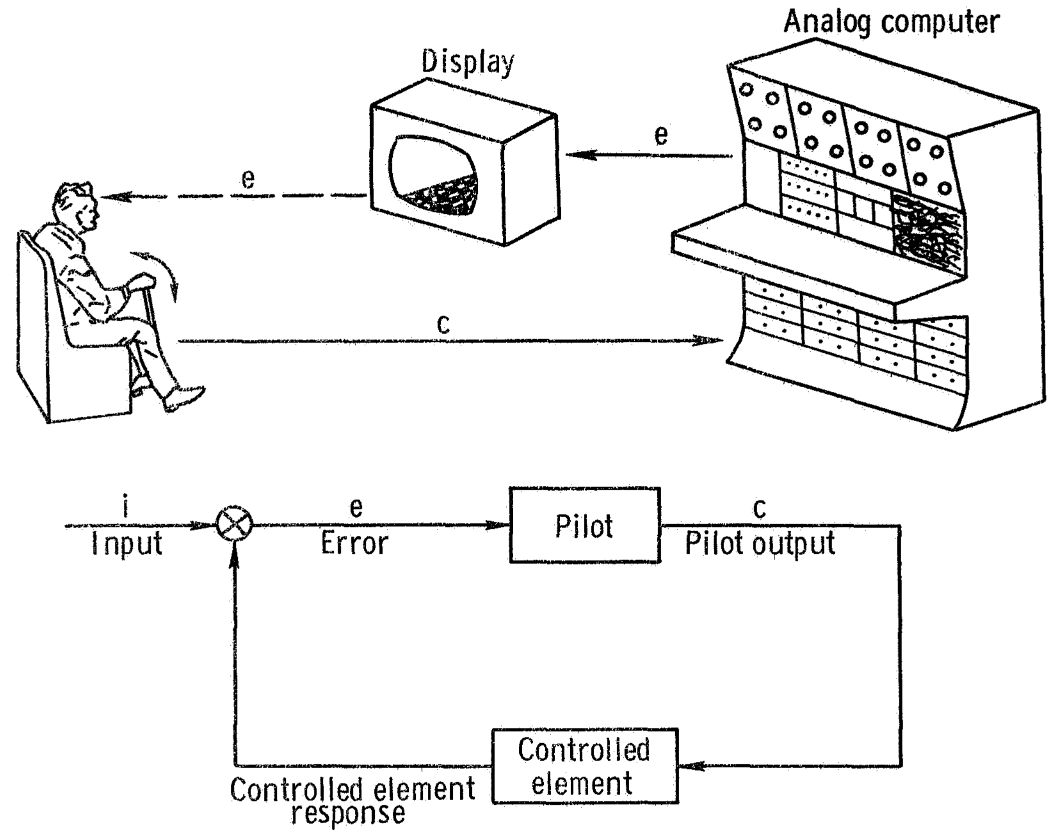

Figure 1.– Block diagram of a pilot in a compensatory tracking task.

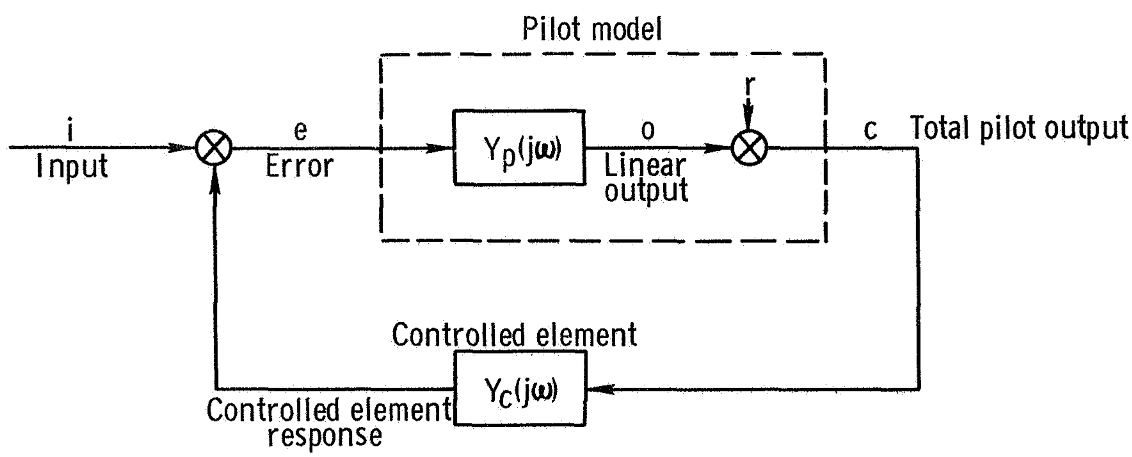

Figure 2.– Frequency domain model of a pilot.

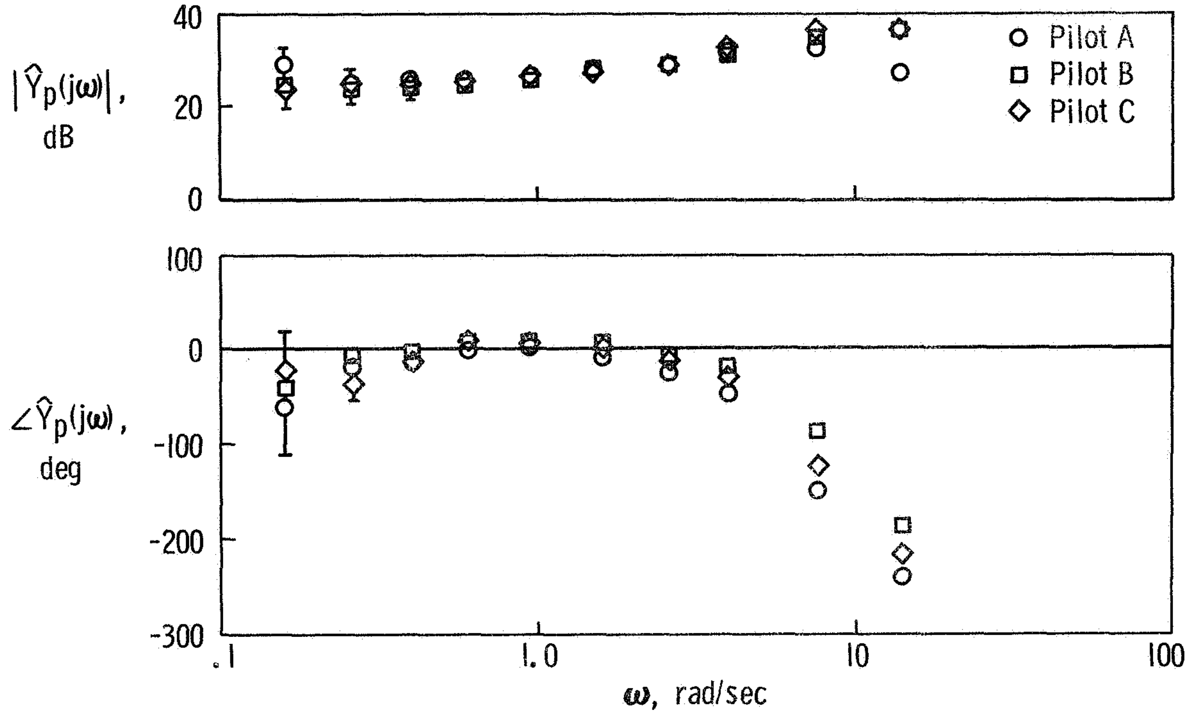

Figure 3.– Example of frequency domain model results.

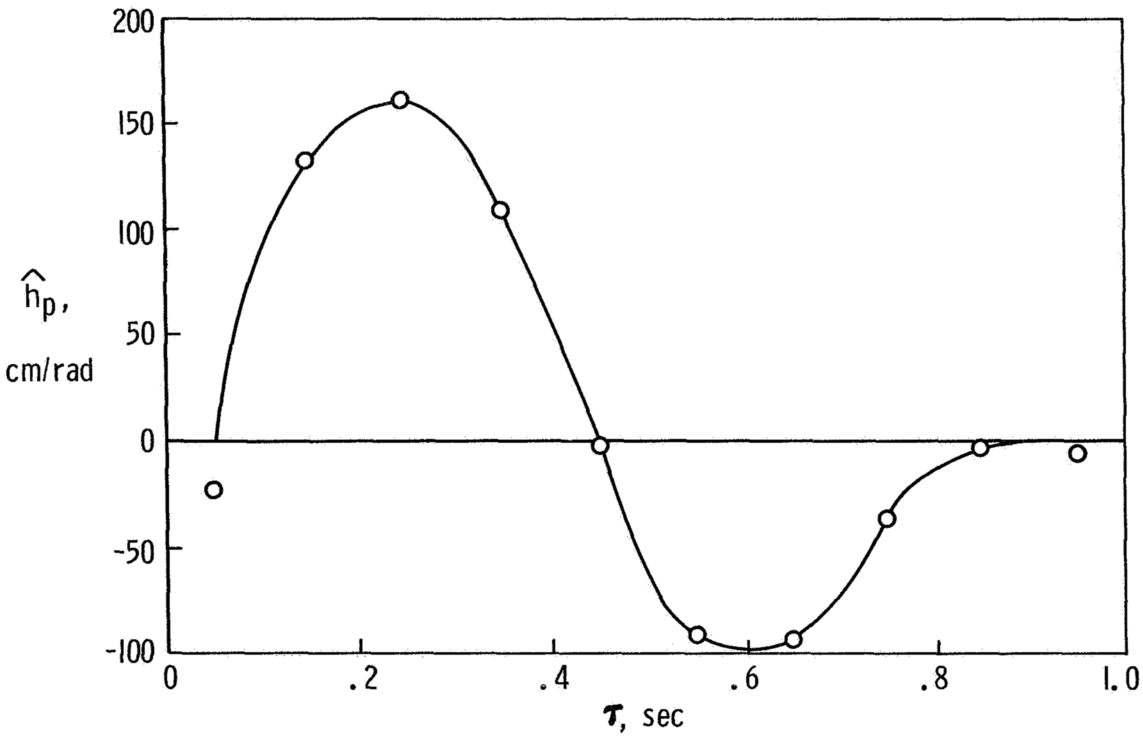

Figure 4.– Linear time domain model of the pilot.

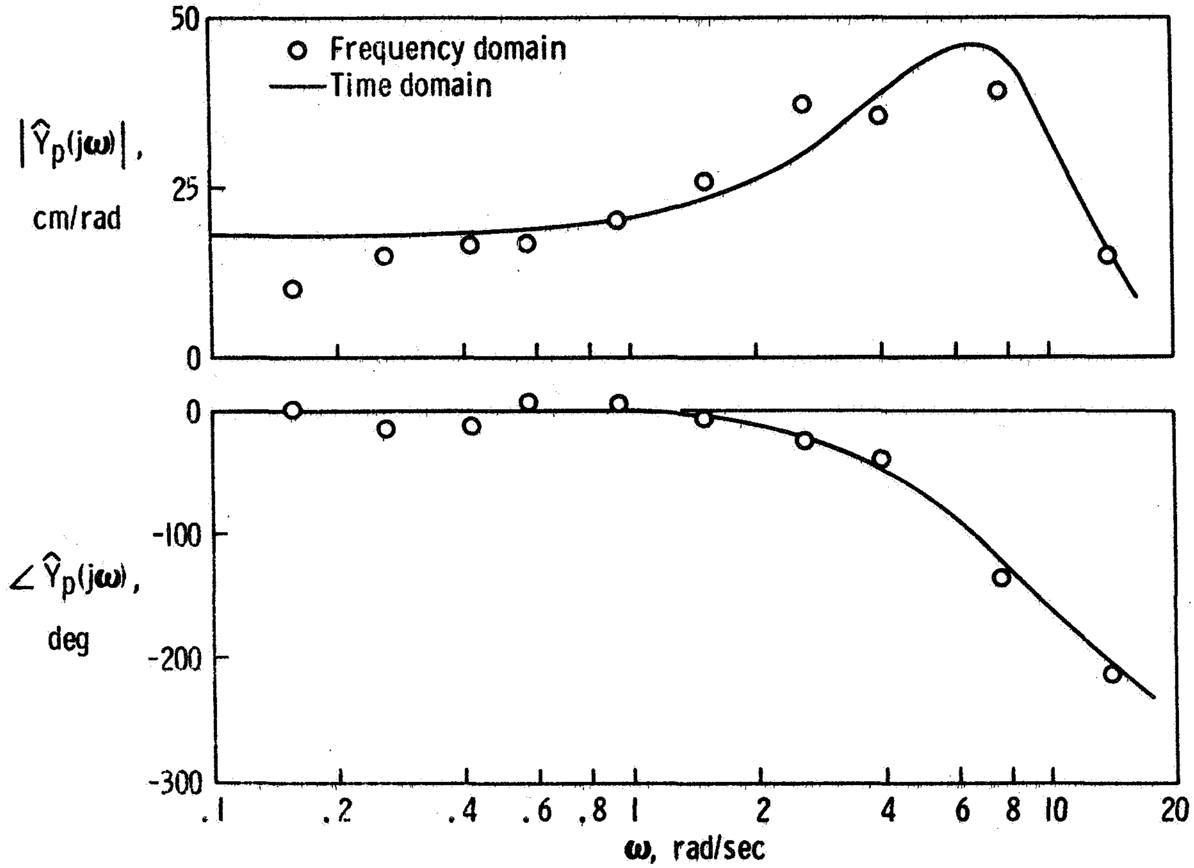

Figure 5.– Comparison of frequency and time domain models.

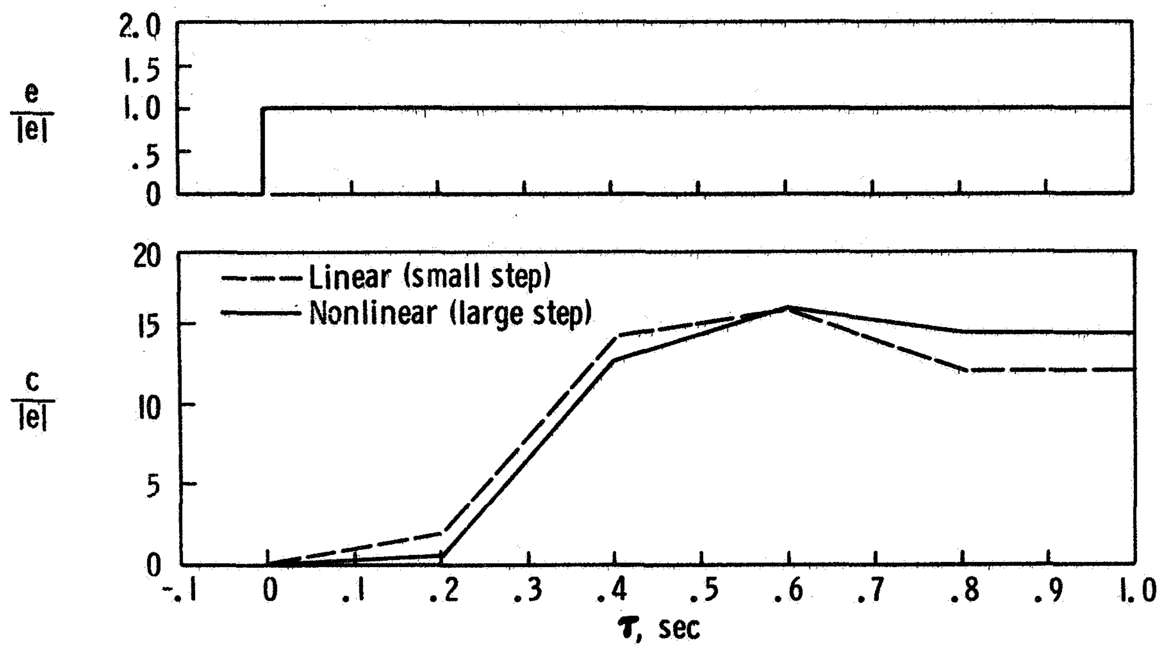

Figure 6.– Step responses of the nonlinear time domain pilot model.

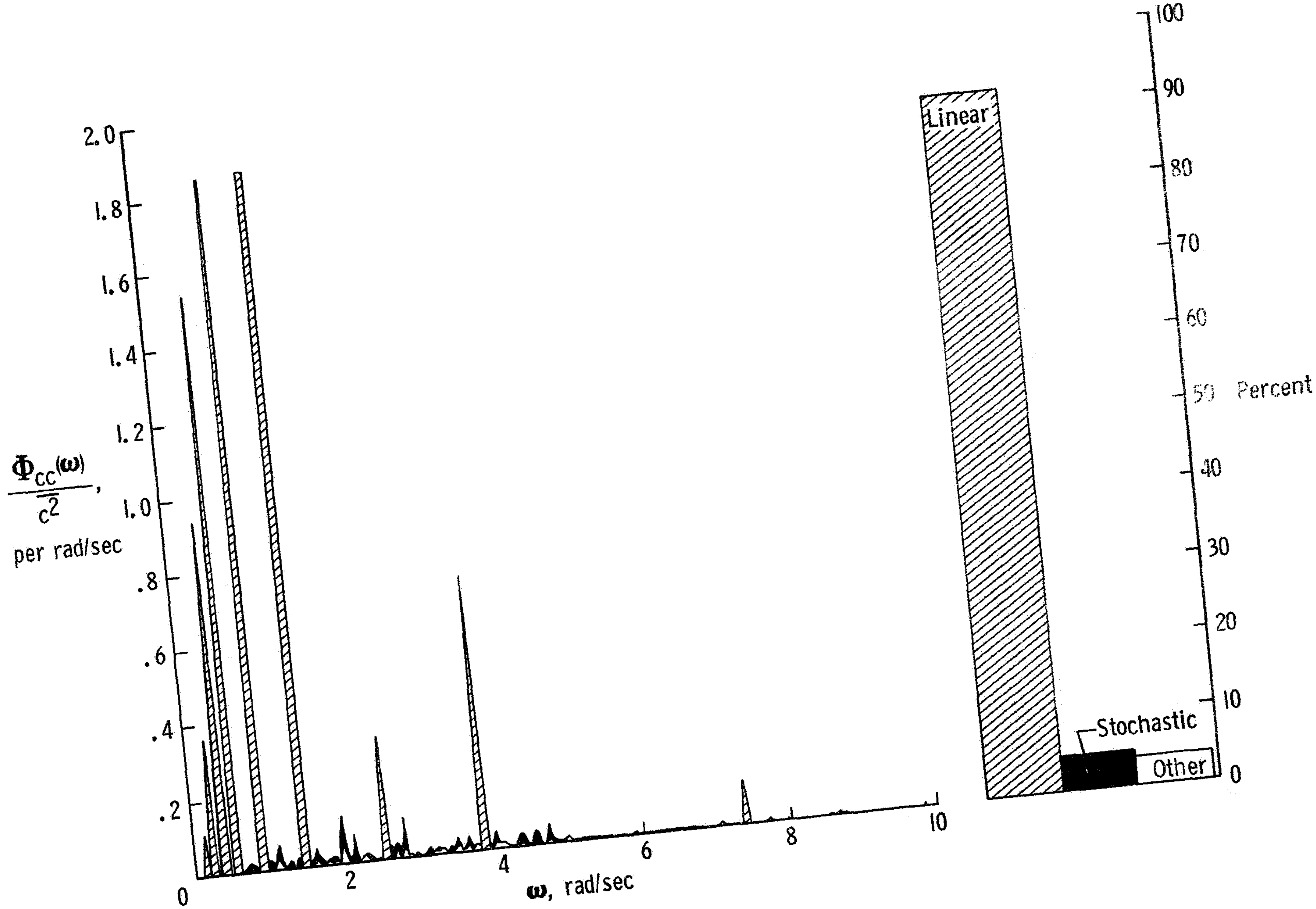

Figure 7.– Spectral analysis of pilot's output.