NASA CONTRACTOR REPORT

NASA CR-61223

May 1968

NASA CR-61223

N 68-25812 (ACCESSION NUMBER) (THRU)

109 (PAGES) 1 (CODE)

CR-61223 (NASA CR OR TMX OR AD NUMBER) 30 (CATEGORY)

FACILITY FORM 602

ROBOT

APOLLO AND AAP PRELIMINARY MISSION PROFILE OPTIMIZATION PROGRAM

Part I- Mathematical Formulation

Prepared under Contract No. NAS 8-20082 by
Robert G. Gottlieb

APPLIED ANALYSIS, INC.

GPO PRICE $ ________

CFSTI PRICE(S) $ ________

Hard copy (HC) 3.00

Microfiche (MF) .65

ff 653 July 65

For

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

May 1968 NASA CR-61223

ROBOT

APOLLO AND AAP PRELIMINARY MISSION PROFILE OPTIMIZATION PROGRAM

Part I- Mathematical Formulation

(Contractor's report dated Dec. 25, 1967)

By

Robert G. Gottlieb

Prepared under Contract No. NAS 8-20082 by

APPLIED ANALYSIS, INC.
Holiday Office Center
Huntsville, Alabama

For

Aero-Astrodynamics Laboratory

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

FOREWORD

This report presents the results of development and implementation of steepest-ascent optimization theory as applied to trajectory computation. This program depicts the results of the developments performed by efforts contracted by the Advanced Studies Office and the Mission Analysis Branch of Aero-Astrodynamics Laboratory at MSFC. Questions and requests pertaining to this program should be addressed to the Mission Analysis Branch, Aero-Astrodynamics Laboratory, MSFC.

ABSTRACT

ROBOT is a minimum Hamiltonian-steepest ascent multistage booster trajectory optimization program. It can simulate up to 15 thrust or coast events, provide rigorous Saturn V and Saturn-IB ground launch simulation and can also be started in orbit.

The payoff and terminal constraints can be selected from a library of eleven functions. In addition intermediate point constraints, selected from the same library, may be imposed on the trajectory following any one of the thrust events.

Through the use of input switches, a variety of vehicle parameters can be optimized in conjunction with the control variables χ-pitch and χ-yaw. Tank limits of stages being optimized can be held and performance reserves, as a function of ΔV, can be calculated. The impact point of any stage can be calculated and publishable tables can be printed. The working coordinate system and the environmental simulation conform to Apollo standards.

This document contains the ROBOT input description and an example problem.

TABLE OF CONTENTS

		PAGE
1.	Introduction	1-1
2.	Coordinate Systems	2-1
3.	Geophysical Properties	3-1
	3.1 Gravitational Accelerations	3-1
	3.2 Geometric Form	3-3
	3.3 Atmospheric Properties	3-5
4.	Control Variables	4-1
5.	Aerodynamic Forces	5-1
6.	Booster Configuration	6-1
	6.1 Thrust and Flow Rate	6-4
7.	The Forward Trajectory	7-1
	7.1 The Equations of Motion	7-1
	7.2 Ground-Launch Trajectory Flight Phases	7-3
	7.2.1 GRR, Δt_o, t_o	7-4
	7.2.2 Ground-Launch Initial Conditions	7-4
	7.2.3 Lift-Off -- Phases 1 and 2	7-5
	7.2.4 Tilt-Over -- Phase 3	7-7
	7.2.5 Pitch-Plane Gravity Turn -- Phase 4	7-7
	7.2.6 Chi-Freeze -- Phase 5	7-8
	7.2.7 Exo-Atmospheric Flight	7-9
	7.3 Intermediate and/or Terminal Functions	7-10
	7.4 Control Parameters, Propellant Tank Limits and Flight Performance Reserves	7-11
	7.4.1 Control Parameters	7-17
	7.4.2 Propellant Tank Limits	7-18
	7.4.3 Flight Performance Reserves	7-20
	7.5 Jump Start	7-23
	7.6 10 km, Q max, 14 km	7-24
	7.7 Impact Point	7-25
	7.8 Chi-Yaw Options	7-26
	7.8.1 Lift-Off Yaw	7-26
	7.8.2 First Stage Yaw	7-26
	7.9 Aeroheat Constraint	7-26
	7.10 Output Tables	7-27

TABLE OF CONTENTS (Cont'd)

		PAGE
8.	The Backward Trajectory	8-1
8.1	Boundary Conditions	8-2
8.2	The Adjoint Differential Equations	8-3
8.3	Impulse Response Functions and I Integrals	8-4
8.4	Influence Coefficients	8-5
8.4.1	Influence Coefficients for Lift-Off Weight and Tilt-Over $\dot{\chi}$	8-5
8.4.2	Influence Coefficients for the τ_{1i}	8-6
8.4.3	Influence Coefficients with Tank Limits and FPR	8-7
8.4.4	Parameter I Matrices	8-8
8.5	Steepest Ascent Formulae	8-9
8.6	The Automatic Convergence Scheme	8-10
References		8-17
Appendix I	First Partial Derivatives of Spherical-Plumbline Transformations	I-1
Appendix II	First Partial Derivatives of Gravitational Acceleration With Respect to Plumbline Position Coordinates	II-1
Appendix III	Input Description and Example Problem	III-1
Table 1	Function Library	7-11
Figure 1	Reference Coordinate System $\hat{X}\hat{Y}\hat{Z}$	2-1
Figure 2	Plumbline Coordinate System $\hat{x}\hat{y}\hat{z}$	2-2
Figure 3	Geocentric Spherical Coordinate System $\hat{\phi}\hat{r}\hat{\theta}$	2-4
Figure 4	The Earth - An Ellipsoid	3-4
Figure 5	χ_p, χ_y and $\hat{p}\hat{c}\hat{\varphi}$ Coordinate System	4-1
Figure 6	Aerodynamic Force Quantities	5-1
Figure 7	Thrust Profile - A Sequence of Thrust Events	6-1
Figure 8	Ground-Launch Flight Phases	7-3
Figure 9	Saturn V Thrust Profile	III-1

SYMBOL INDEX

This index gives an alphabetical list of symbols and page no.(s) where definition(s) given. More than one page number implies a multiple use of a symbol.

Symbol(s)	Page(s)	Symbol(s)	Page(s)	Symbol(s)	Page(s)
A	2-3	F_{AA}, F_{AN}	5-1	h	3-5
Ae_i	6-2	F_{AX}, F_{AY}, F_{AZ}	5-3	I	7-3,8-4,II-1
A_z	2-3	F_i	6-2	$I_{zz}^a, I_{\phi\phi}^a$	8-4
a	3-5, 8-9	F_x, F_y, F_z	7-2	$I_{\phi\psi}^a, I_{\psi\psi}^a$	8-4
a_{ij}	2-5	FPR	7-20	$I_{zz}^b, I_{\phi\phi}^b$	8-9
B	2-6	f	3-4	$I_{\phi\psi}^b, I_{\psi\psi}^b$	8-9
b	8-9	G_{11}, G_{TO}	3-2	$I_{\phi\psi}, I_{\psi\psi}$	8-9
C	4-2	G_{22}, G_{23}	II-2	$I_{\phi\phi}$	8-10
C_A, C_N'	5-3	G_{32}, G_{33}	II-2	i	7-15
CJ	3-1	G_{zp}, G_{zy}	8-4	J	II-1
C_1	7-13	$G_z, G_{\phi p}, G_{\phi y}$	8-4	k	8-9
C_3	7-12	$G_{\psi p}, G_{\psi y}$	8-4	k_1, k_2, k_3, k_4	7-21
$c\dot{m}_i$	7-18	GRR	2-2	$\underline{L}^i$	8-14
D	2-6	g_o	7-2	L_{zi}	8-6
DJ	3-1	g_x, g_y, g_z	3-2	L_{zm}	8-5
d_{ij}	2-6	g_{xx}, g_{xy}, g_{xz}	II-1	L_{zx}	8-6
db	8-9	g_{yy}, g_{yz}, g_{zz}	II-1	L_z, L_ϕ, L_ψ	8-8
$d\phi$	8-10	H	3-1,8-12	M	5-3
E	8-9	H_a	8-12	m	7-2,8-1
F	8-3	H_{aa}	8-13	m_a	7-2

SYMBOL INDEX (Cont'd)

Symbol(s)	Page(s)	Symbol(s)	Page(s)	Symbol(s)	Page(s)
m_c	7-21	r	2-4	w, u, v	2-3
m^d	7-22	S	5-3,I-1	w_s, u_s, v_s	2-6
m_i	7-22	S_i	6-2	$\underline{w}, \underline{u}, \underline{v}$	5-2
m_L	7-22	T	6-4	X, Y, Z	2-5
m_o	7-5	t_D	7-5	x, y, z	2-3
m_x	7-18	t_L	7-5	$\overline{x}, \overline{y}, \overline{z}$	7-6
$\dot{m}$	6-5	t_Q	7-9	Y_{zj}	8-6
$\dot{m}_i$	6-2	t_T	7-7		
N	I-1	t_f	7-3	α	5-1
n_i^w	6-3	t_o	7-4	β	7-14
n_{ij}	I-1	t_χ	7-8	γ	7-11
np	8-11	t_{1i}	6-2	Δt_o	7-4
nv	8-7	t_{2i}	6-2	$\Delta V_g, \Delta V_p$	7-20
nw	8-7	$U(r, \theta)$	3-1	$\Delta \phi_o$	7-4
P	I-1	V_I	7-11	δa	8-9
P_i	8-11	V_R	5-1	η	7-1
p_a	3-5	V_o	7-4	θ	2-4
p_s	6-4	Vex	7-21	θ_g	3-5
$\dot{p}_i, i=1\ldots 8$	7-1	W_i	8-11	θ_L	7-6
QY	8-10	W_a^{-1}	8-5	θ_o	2-3
q	5-2	W_b^{-1}	8-9	θ_1	2-3
R_e	3-1	W_i^d	6-3	θ'	7-23
$R(\theta)$	3-5	W_i^J	6-2	λ	8-2

SYMBOL INDEX (Cont'd)

Symbol(s)	Page(s)
λ_z	8-3
$\lambda_\phi, \lambda_\psi$	8-2
μ	7-1
μ_e	3-1
ν	8-1
ν_i	6-4
ν_{ij}	6-2
ρ	3-5
σ	5-1
τ_i	6-2
τ_i^w	6-3
τ_p	7-21
Φ	8-2
ϕ	2-4,8-1
ϕ_L	7-6
χ_p, χ_y	4-1
$\dot{\chi}$	7-7
ψ	8-1
Ω_e	3-3
ω	7-16
$\underline{\omega}$	7-23

Symbol(s)	Page(s)
Coordinates	
$\hat{X}\hat{Y}\hat{Z}$	2-1
$\hat{x}\hat{y}\hat{z}$	2-2
$\hat{\phi}\hat{r}\hat{\theta}$	2-4
$\hat{p}\hat{c}\hat{\varphi}$	4-1

1. INTRODUCTION

The ROBOT (ROBert's Optimum Trajectory) program described in this report is designed to optimize a large variety of multistage booster trajectories. This objective is achieved through the use of the Min-H* steepest-ascent trajectory optimization technique described in Reference 1. Briefly, the steepest-ascent technique requires that a reasonable, but nevertheless arbitrary choice of the controls be used to calculate a nominal trajectory. In general, neither the desired terminal state will result, nor will the performance index be optimum. Next, by solving the adjoint differential equations associated with the linearized perturbation equations about the nominal trajectory, impulse response functions may be determined for arbitrary small variations in the control variables, and influence coefficients may be determined for arbitrary small variations in the control parameters. The choice of small changes in these controls, which simultaneously moves the terminal state closer to the desired terminal state and improves the performance index, is calculated. This change in the controls is added to the nominal control history and the process is repeated until the optimum is reached.

The ROBOT program can simulate a multistage booster having up to 15 thrust events. The program can be used for both ground-launch and orbital-start trajectories. Internal logic is available which will adhere rigorously to the atmospheric flight profile of the Saturn-IB and the Saturn V. The working coordinate system and the environmental simulation conform to Apollo standards.

---

*minimum Hamiltonian

The payoff and terminal constraints can be selected from a library of eleven functions. In addition intermediate point constraints, selected from the same library, may be imposed on the trajectory following any one of the thrust events.

Through the use of input switches, a variety of vehicle parameters may be optimized in conjunction with the control variables χ-pitch and χ-yaw. Tank limits of stages being optimized can be held and performance reserves, as a function of Δ V, can be calculated. Also, the impact point of any stage can be calculated and publishable tables can be printed.

The ROBOT program has a new simple straightforward automatic convergence logic and a new dynamic updating scheme for the control parameter weighting matrix making convergence much more reliable and sure than previous programs [2] using Min-H.

For the most part, this report is devoted to a description of the mathematical model used in formulating ROBOT and to such a limited discussion of the logic structure as affords a complete description of the program flexibility. A companion report, "RØBØT - Apollo and AAP Preliminary Mission Profile Optimization Program, Part II - Program Description," contains a detailed description of the computer program.

2. COORDINATE SYSTEMS

The basic reference coordinate system in the ROBOT program is the inertial geocentric cartesian coordinate system $\hat{X}\hat{Y}\hat{Z}$ shown in Fig. 1. This coordinate system has the $\hat{Y}$ axis pointing north, the $\hat{X}$ and $\hat{Z}$ axes in the equatorial plane, and the $\hat{Z}$ axis in the meridian plane that

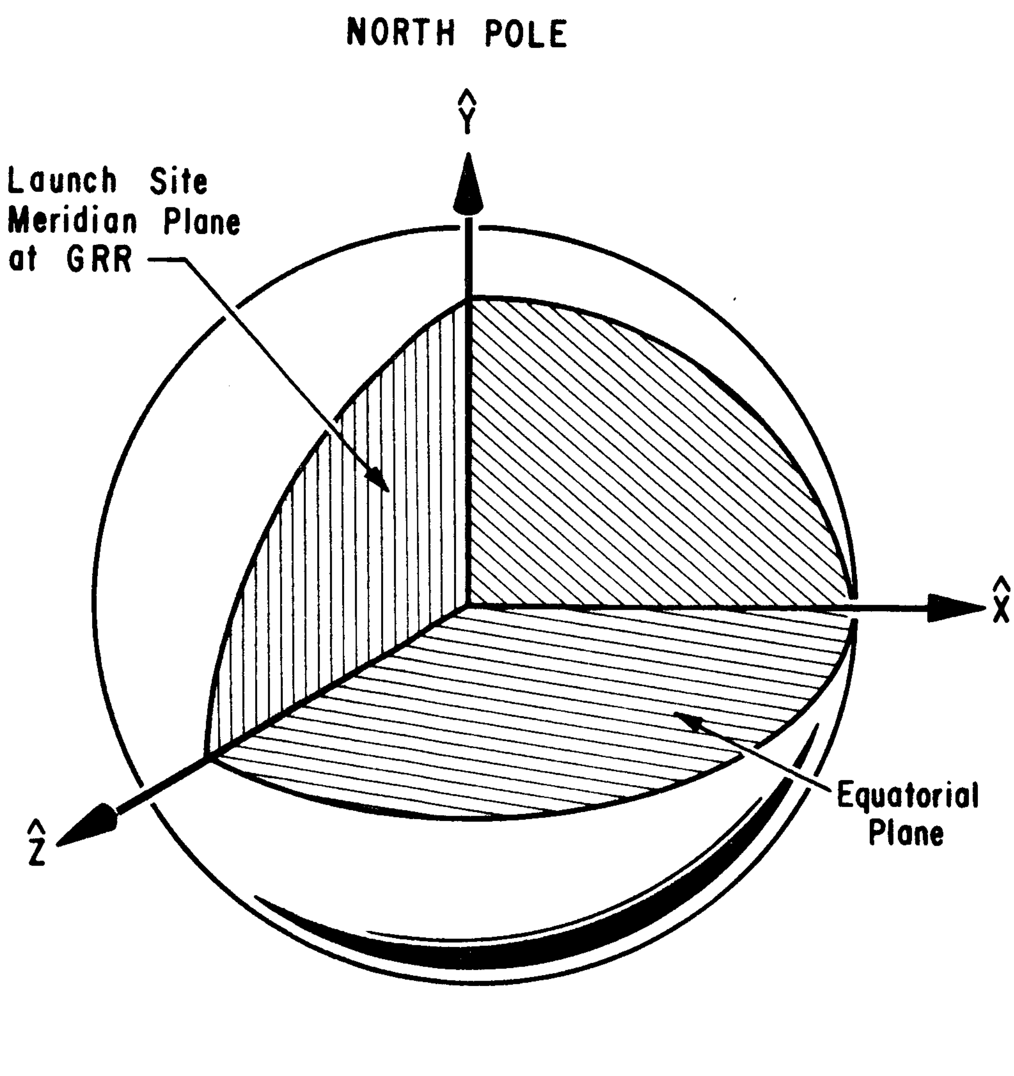

FIG. 1. REFERENCE COORDINATE SYSTEM $\hat{X}\ \hat{Y}\ \hat{Z}$

contains the launch site at gyro release time. In the ROBOT program gyro release time or guidance reference release (GRR) is a reference time occurring either prior to or at liftoff.

Next described is the inertial cartesian plumbline coordinate system $\hat{x}\hat{y}\hat{z}$, in which the equations of motion are written.

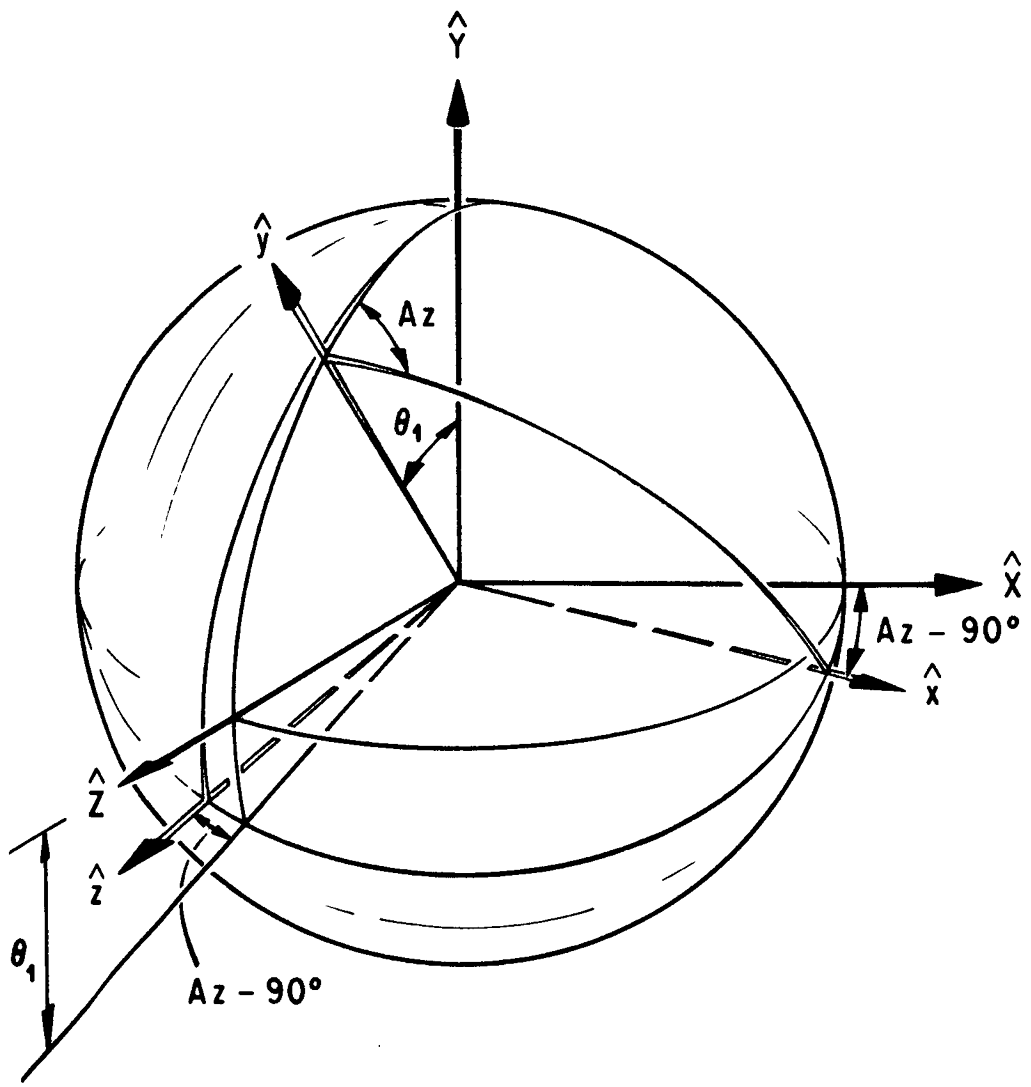

FIG. 2. PLUMBLINE COORDINATE SYSTEM $\hat{x}\ \hat{y}\ \hat{z}$

The plumbline coordinate system $\hat{x}\hat{y}\hat{z}$, shown in Fig. 2, is formed from $\hat{X}\hat{Y}\hat{Z}$ by first rotating counterclockwise about $\hat{X}$ through θ_1 and then clockwise about $\hat{y}$ through $A_z - 90$. A_z is the launch azimuth angle and $\theta_1 = \pi/2 - \theta_o$ where θ_o is the geodetic latitude of the launch site. Both A_z and θ_o are input quantities.

The equations for transforming a vector from the $\hat{X}\hat{Y}\hat{Z}$ system to the $\hat{x}\hat{y}\hat{z}$ system are

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = A \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}$$

where

$$A = \begin{bmatrix} \sin A_z & \cos A_z \sin\theta_1 & -\cos A_z \cos\theta_1 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ \cos A_z & -\sin A_z \sin\theta_1 & \sin A_z \cos\theta_1 \end{bmatrix}$$

Since A_z is an input constant and θ_1 is the complement of an input constant, the matrix A is also constant.

In the plumbline system the position coordinates x, y, z and the velocity components w, u, v, are measured in the $\hat{x}$, $\hat{y}$, $\hat{z}$ directions, respectively.

The plumbline system in ROBOT differs from the Apollo 13 coordinate system [3] only in the names of the axes, i.e.,

$$\begin{bmatrix} \hat{z} \\ \hat{x} \\ \hat{y} \end{bmatrix}_{\text{Apollo 13}} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}_{\text{ROBOT}}$$

The third coordinate system used in RC'BOT is the geocentric spherical polar coordinate system $\hat{\phi}\hat{r}\hat{\theta}$ with coordinates ϕ, r, and θ. The $\hat{\phi}\hat{r}\hat{\theta}$ axes, shown in Fig. 3, point in the direction of increasing ϕ, r, and θ, respectively, and may be formed by first rotating counterclockwise about $\hat{Y}$ through ϕ and then rotating counterclockwise about $\hat{\phi}$ through θ.

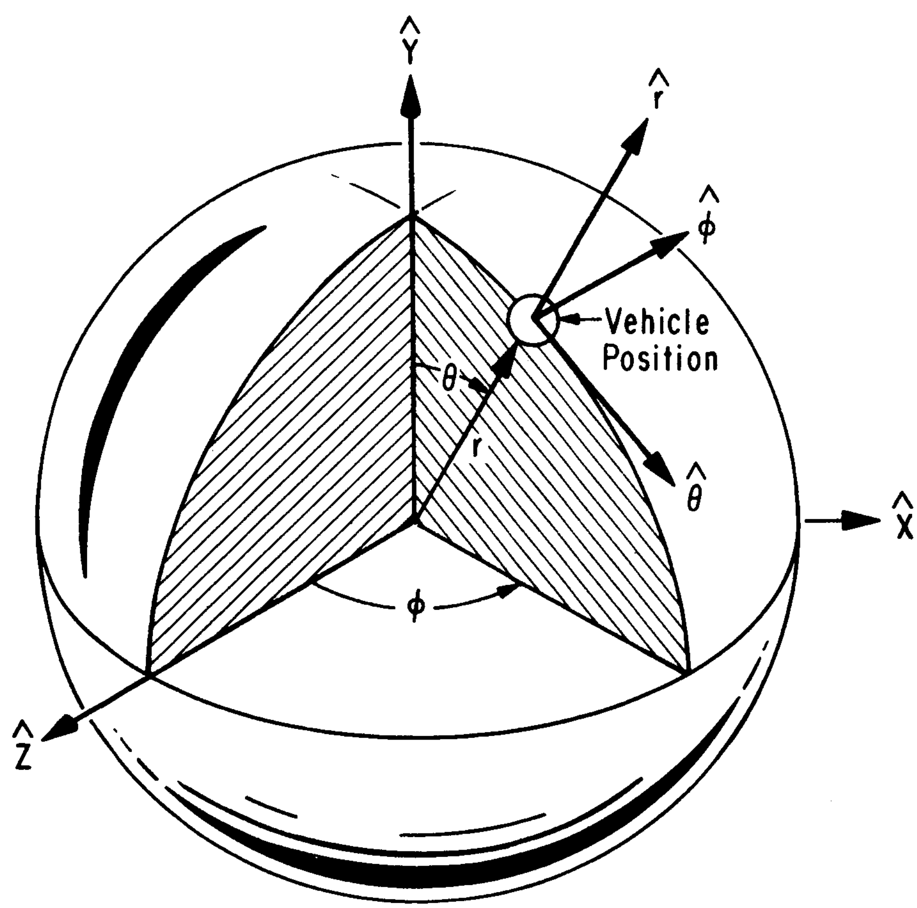

FIG. 3. GEOCENTRIC SPHERICAL COORDINATE SYSTEM $\hat{\phi}\hat{r}\hat{\theta}$

The projections of r on $\hat{X}\hat{Y}\hat{Z}$ are

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = r \begin{bmatrix} \sin\theta \sin\phi \\ \cos\theta \\ \sin\theta \cos\phi \end{bmatrix}$$

and therefore the projections of r on $\hat{x}\hat{y}\hat{z}$ are

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = r \; A \begin{bmatrix} \sin\theta \sin\phi \\ \cos\theta \\ \sin\theta \cos\phi \end{bmatrix}$$

The transformation from x, y, z to ϕ, r, θ is therefore

$$\phi = \tan^{-1}\left(\frac{a_{11} x + a_{21} y + a_{31} z}{a_{13} x + a_{23} y + a_{33} z} \right)$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\left((a_{12} x + a_{22} y + a_{32} z) / r \right)$$

where the a_{ij} are elements of the A matrix described previously.

The equations for transforming a vector from the $\hat{\phi}\hat{r}\hat{\theta}$ system to the $\hat{X}\hat{Y}\hat{Z}$ system are

$$\begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix} = B \begin{bmatrix} \hat{\phi} \\ \hat{r} \\ \hat{\theta} \end{bmatrix}$$

where

$$B = \begin{bmatrix} \cos\phi & \sin\phi\sin\theta & \sin\phi\cos\theta \\ 0 & \cos\theta & -\sin\theta \\ -\sin\phi & \cos\phi\sin\theta & \cos\phi\cos\theta \end{bmatrix}$$

and therefore the equations for transforming a vector from the $\hat{\phi}\,\hat{r}\,\hat{\theta}$ system to the $\hat{x}\,\hat{y}\,\hat{z}$ system may be written

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = D \begin{bmatrix} \hat{\phi} \\ \hat{r} \\ \hat{\theta} \end{bmatrix}$$

where $D = A \cdot B$

Also, the inertial velocity components in the $\hat{\phi}$, $\hat{r}$, $\hat{\theta}$ directions, w_s, u_s, v_s respectively, may be written

$$\begin{bmatrix} w_s \\ u_s \\ v_s \end{bmatrix} = D^T \begin{bmatrix} w \\ u \\ v \end{bmatrix}^*$$

If the multiplication $A \cdot B$ is performed and substitutions for ϕ and θ are made in terms of x, y and z, the elements of the D matrix become

$$d_{11} = (a_{22}z - a_{32}y) / r\sin\theta$$

$$d_{21} = (a_{32}x - a_{12}z) / r\sin\theta$$

$$d_{31} = (a_{12}y - a_{22}x) / r\sin\theta$$

$*(\ )^T$ Denotes matrix transpose.

$$d_{12} = x / r$$

$$d_{22} = y / r$$

$$d_{32} = z / r$$

$$d_{13} = (d_{12} \cos\theta - a_{12}) / \sin\theta$$

$$d_{23} = (d_{22} \cos\theta - a_{22}) / \sin\theta$$

$$d_{33} = (d_{32} \cos\theta - a_{32}) / \sin\theta$$

3. GEOPHYSICAL PROPERTIES

Described in this section are three geophysical properties of the earth which affect a rocket trajectory: gravitational acceleration, geometric form, atmospheric properties.

3.1 Gravitational Accelerations

The gravitational potential function, $U(r, \theta)$, used in ROBOT is [4]

$$U(r,\theta) = \frac{\mu_e}{r}\left[1 + \frac{CJ}{3}\left(\frac{R_e}{r}\right)^2 (1 - 3\cos^2\theta) + \frac{H}{5}\left(\frac{R_e}{r}\right)^3 (3 - 5\cos^2\theta)\cos\theta + \frac{DJ}{35}\left(\frac{R_e}{r}\right)^4 (3 - 30\cos^2\theta + 35\cos^4\theta)\right]$$

where CJ, H, DJ, R_e, μ_e are input parameters which are, however, pre-set to

$$CJ = 1.62345 \times 10^{-3}$$

$$H = -0.575 \times 10^{-5}$$

$$DJ = 0.7875 \times 10^{-5}$$

R_e = Earth equatorial radius
= 6378165. m

μ_e = Product of universal gravity constant and earth mass
= $3.986032 \times 10^{14}\ m^3/sec^2$

The components of the gravitational acceleration vector in the plumbline system are calculated as the first partial derivatives of

$U(r, \theta)$ with respect to the plumbline position coordinates, i.e., g_x, g_y and g_z are calculated as

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \begin{bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \\ \frac{\partial U}{\partial z} \end{bmatrix} = \frac{\partial U}{\partial r} \begin{bmatrix} \frac{\partial r}{\partial x} \\ \frac{\partial r}{\partial y} \\ \frac{\partial r}{\partial z} \end{bmatrix}^{*} + \frac{\partial U}{\partial \theta} \begin{bmatrix} \frac{\partial \theta}{\partial x} \\ \frac{\partial \theta}{\partial y} \\ \frac{\partial \theta}{\partial z} \end{bmatrix}^{*}$$

These equations may be rearranged into the form

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = G_{11} \begin{bmatrix} x \\ y \\ z \end{bmatrix} - G_{TO} \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$$

where

$$G_{11} = -\frac{\mu_e}{r^3}\left[1 + CJ\left(\frac{R_e}{r}\right)^2 (1 - 5\cos^2\theta) + H\left(\frac{R_e}{r}\right)^3 (3 - 7\cos^2\theta)\cos\theta + DJ\left(\frac{R_e}{r}\right)^4 \left(\frac{3}{7} - (6 - 9\cos^2\theta)\cos^2\theta\right)\right]$$

$$G_{TO} = \frac{\mu_e}{r^2}\left[2CJ\left(\frac{R_e}{r}\right)^2 \cos\theta - H\left(\frac{R_e}{r}\right)^3 \left(\frac{3}{5} - 3\cos^2\theta\right) + DJ\left(\frac{R_e}{r}\right)^4 \left(\frac{12}{7} - 4\cos^2\theta\right)\cos\theta\right]$$

Equations of the same general form are used in the Saturn V flight computer.[5]

*These partials are given in Appendix I.

In the event that a spherical earth is to be simulated these equations become

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = G_{11} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where

$$G_{11} = -\mu_e / r^3$$

and, of course,

$$G_{TO} = 0$$

3.2 Geometric Form

The earth is taken to be an ellipsoid, [6] as shown in Fig. 4, which rotates about its polar axis with an angular velocity Ω_e.

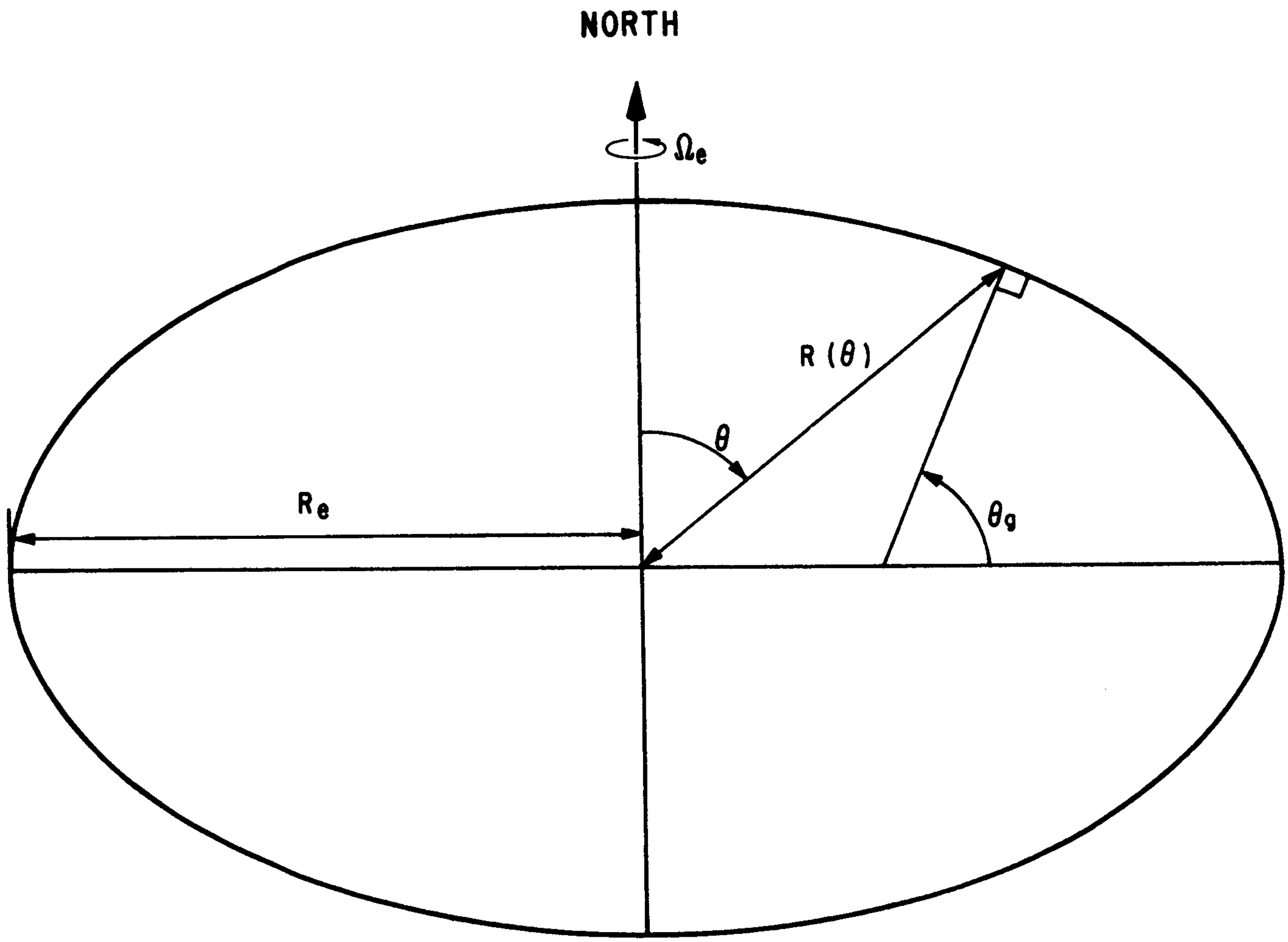

FIG. 4. THE EARTH — AN ELLIPSOID

The angular velocity, Ω_e, and the flattening, f, are input constants that are preset to

$$\Omega_e = 7.2921158 \times 10^{-5} \text{ rad / sec}$$

$$f = 1 / 298.3$$

The relationship between geocentric colatitude, θ, and geodetic latitude, θ_g, is expressed by

$$\text{ctn}\theta = (1-f)^2 \tan\theta_g$$

The radius of the earth as a function of colatitude, $R(\theta)$, is

$$R(\theta) = (1-f) R_e / \sqrt{(1-f)^2 \sin^2\theta + \cos^2\theta}$$

The derivative of $R(\theta)$ with respect to θ, which is needed in order to calculate the time at which maximum dynamic pressure occurs, is given by

$$\frac{dR(\theta)}{d\theta} = \frac{R(\theta)^3 f(2-f) \sin\theta \cos\theta}{(R_e)^2 (1-f)^2}$$

3.3 Atmospheric Properties

The earth is assumed to have an atmosphere which rotates with it at the same angular velocity, so that there is no wind over the earth's rotating surface.

The PRA63 model atmosphere [7] routine on the MSFC system tape is presently used to calculate density, ρ, pressure, p_a and speed of sound, a, as a function of the altitude, h, where h is calculated from

$$h = r - R(\theta)$$

4. CONTROL VARIABLES

The time history of the orientation in space of the centerline, $\hat{c}$, of the boost vehicle is determined by the control variable attitude angles χ_p (chi-pitch) and χ_y (chi-yaw) shown in Fig. 5.

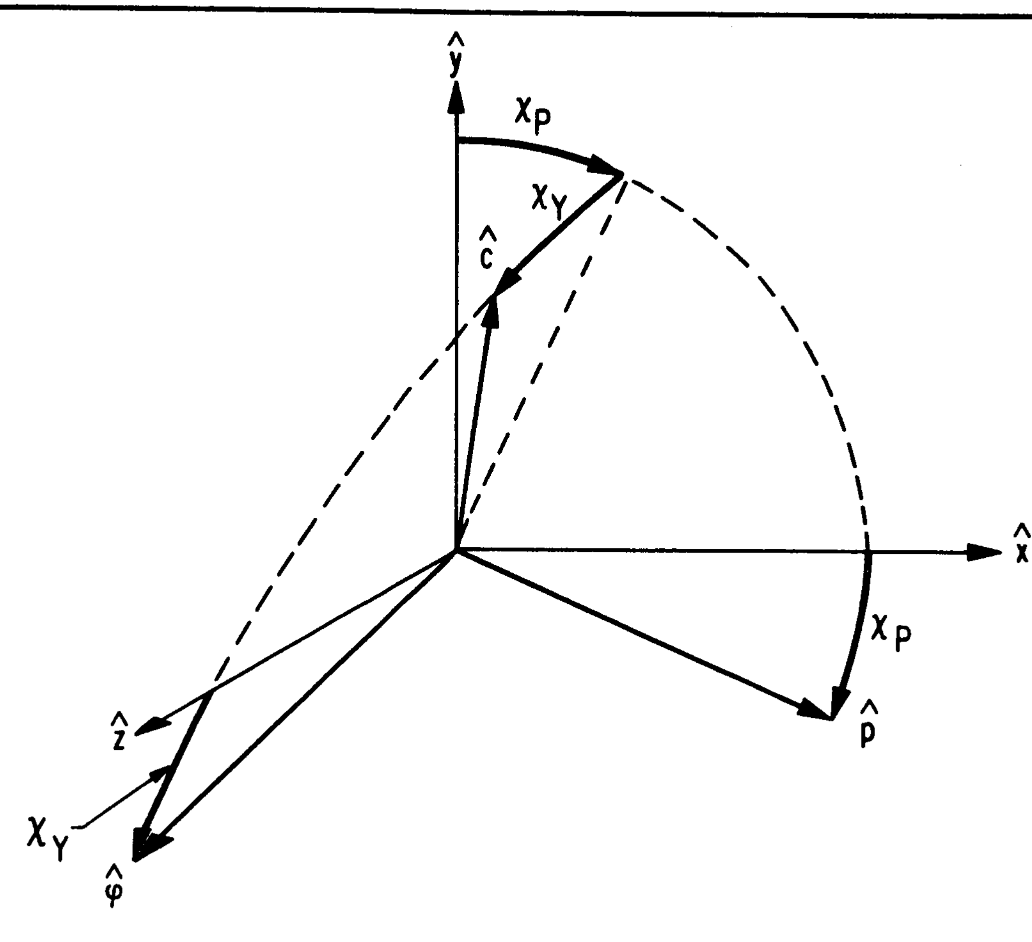

FIG. 5. χ_P, χ_Y AND $\hat{p}\hat{c}\hat{\varphi}$ COORDINATE SYSTEM

In addition to defining the position of the centerline, $\hat{c}$, χ_p and χ_y may be thought of as defining the auxiliary coordinate axes $\hat{p}\,\hat{c}\,\hat{\varphi}$ shown in Fig. 5. This auxiliary coordinate system is formed by rotating clockwise about $\hat{z}$ through χ_p and then counterclockwise about $\hat{p}$ through χ_y.

The equations for transforming a vector from the $\hat{p}\,\hat{c}\,\hat{\varphi}$ system into the $\hat{x}\,\hat{y}\,\hat{z}$ system are

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = C \begin{bmatrix} \hat{p} \\ \hat{c} \\ \hat{\varphi} \end{bmatrix}$$

where

$$C = \begin{bmatrix} \cos\chi_p & \sin\chi_p \cos\chi_y & -\sin\chi_p \sin\chi_y \\ -\sin\chi_p & \cos\chi_p \cos\chi_y & -\cos\chi_p \sin\chi_y \\ 0 & \sin\chi_y & \cos\chi_y \end{bmatrix}$$

5. AERODYNAMIC FORCES

The passage of the vehicle through the atmosphere gives rise to aerodynamic forces defined to act coincident with and normal to the vehicle body axis $\hat{c}$. In ROBOT, the relative velocity, V_R, is considered to be the velocity of the vehicle relative to the atmosphere. The orientation of V_R in the auxiliary coordinate system $\hat{p}\,\hat{c}\,\hat{\varphi}$ is accomplished using the angle of attack, α, and the relative velocity heading angle, σ. These items along with the aerodynamic axial force, F_{AA}, and the aerodynamic normal force, F_{AN}, are shown in Fig. 6.

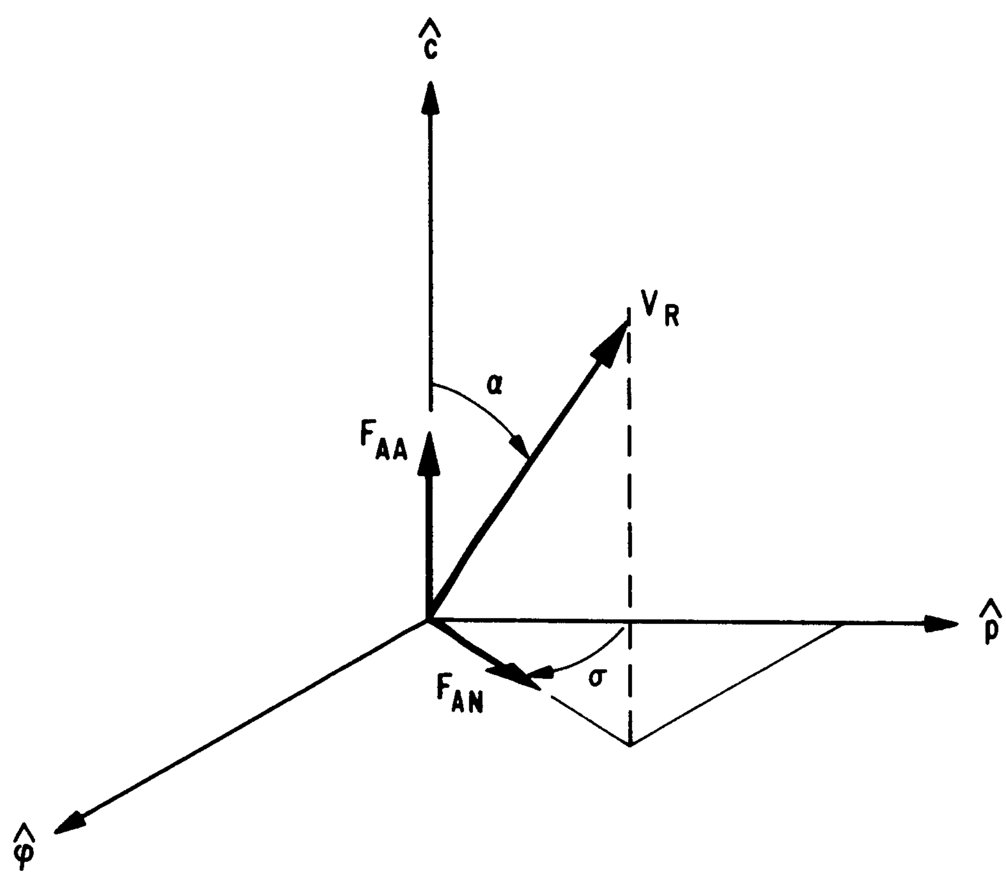

FIG. 6. AERODYNAMIC FORCE QUANTITIES

The equations for transforming V_R into the plumbline system are

$$\begin{bmatrix} \underline{w} \\ \underline{u} \\ \underline{v} \end{bmatrix} = V_R C \begin{bmatrix} \sin\alpha\cos\sigma \\ \cos\alpha \\ \sin\alpha\sin\sigma \end{bmatrix}$$

The relative velocity in the plumbline system may also be calculated as the difference between the plumbline inertial velocity and the transformed velocity required by an object in order to remain over a given point on the rotating earth. This results in

$$\begin{bmatrix} \underline{w} \\ \underline{u} \\ \underline{v} \end{bmatrix} = \begin{bmatrix} w \\ u \\ v \end{bmatrix} - D \begin{bmatrix} r\,\Omega_e \sin\theta \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} w - (a_{22} z - a_{32} y)\,\Omega_e \\ u - (a_{32} x - a_{12} z)\,\Omega_e \\ v - (a_{12} y - a_{22} x)\,\Omega_e \end{bmatrix}$$

Therefore, the relative velocity, V_R, the angle of attack, α, and the relative velocity heading angle, σ, are

$$V_R = \sqrt{\underline{w}^2 + \underline{u}^2 + \underline{v}^2}$$

$$\alpha = \cos^{-1}\left(\frac{\underline{w}}{V_R} \sin\chi_p \cos\chi_y + \frac{\underline{u}}{V_R} \cos\chi_p \cos\chi_y + \frac{\underline{v}}{V_R} \sin\chi_y\right)$$

$$\sigma = \tan^{-1}\left(\frac{-\underline{w}\sin\chi_p \sin\chi_y - \underline{u}\cos\chi_p \sin\chi_y + \underline{v}\cos\chi_y}{\underline{w}\cos\chi_p - \underline{u}\sin\chi_p}\right)$$

The dynamic pressure, q, is calculated as

$$q = \frac{1}{2}\rho V_R^{\,2}$$

The Mach number, M, is calculated as

$$M = \frac{V_R}{a}$$

and is used to calculate

$$C_A = C_A(M)$$

$$C_N' = C_N'(M)$$

The axial force, F_{AA}, and the normal force, F_{AN}, are now calculated as

$$F_{AA} = q \cdot S \cdot C_A$$

$$F_{AN} = q \cdot S \cdot C_N' \cdot \alpha$$

where S is the reference area.

Two sets of tables of $C_A(M)$ and $C_N'(M)$ are provided in ROBOT, one set for use while thrusting, the other set for use while coasting. The aerodynamic forces F_{AX}, F_{AY} and F_{AZ} in the $\hat{x}$, $\hat{y}$, $\hat{z}$ directions, respectively, are calculated as

$$\begin{bmatrix} F_{AX} \\ F_{AY} \\ F_{AZ} \end{bmatrix} = -C \begin{bmatrix} F_{AN} \cos\sigma \\ F_{AA} \\ F_{AN} \sin\sigma \end{bmatrix}$$

The minus sign is used since the velocity of the atmosphere relative to the vehicle is the negative of V_R.

6. BOOSTER CONFIGURATION

In ROBOT, the simulation of the thrust profile of a multistage booster is accomplished by synthesizing the profile from a sequence of up to 15 thrust events. By allowing the grouping of these thrust events into stages to be specified by input rather than by fixed internal logic, a great deal of generality is obtained. Fig. 7 depicts five thrust events as an example of such a sequence.

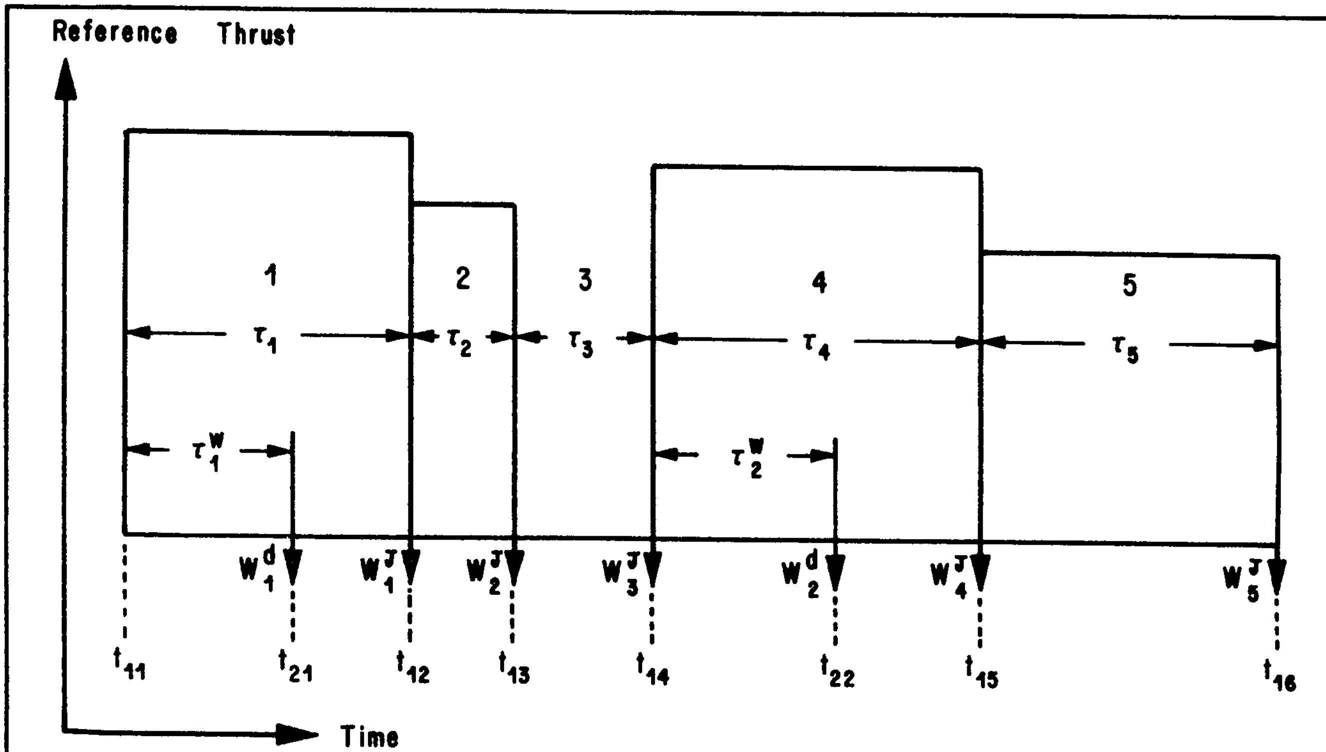

FIG. 7. THRUST PROFILE - A SEQUENCE OF THRUST EVENTS

The ith thrust event is characterized by five (seven in the atmosphere) items:

1)	F_i	Reference thrust per engine
2)	$\dot{m}_i$	Flow rate per engine
3)	ν_{i1}	Number of inboard engines
	ν_{i2}	Cant angle of inboard engines
	ν_{i3}	Number of outboard engines
	ν_{i4}	Cant angle of outboard engines
4)	τ_i	Thrust event duration
5)	W_i^J	Weight jettisoned at the end of each thrust event

and in the atmosphere.....

6)	Ae_i	Engine exit area
7)	S_i	Aerodynamic reference area

The convention used in ROBOT for labeling thrust event and miscellaneous weight drop event times is also depicted in Fig. 7. Thrust event times are labeled t_{1i} and miscellaneous weight drop event times are labeled t_{2i}.

Note that there are six t_{1i} but only five thrust events of duration τ_i. The same is true of a picket fence, in that there is always one more picket than there are spaces. The τ_i may therefore be thought of as "spaces", and the i subscript of t_{1i} as the "picket" number, with $i = 1$ at the beginning of the first thrust event.

From the figure it is apparent that the t_{1i} are calculated as

$$t_{1i+1} = t_{1i} + \tau_i$$

with t_{11} being defined as some input initial time.

In addition to thrust event items, Fig. 7 also depicts two miscellaneous weight drops. A miscellaneous drop weight, as distinguished from a jettison weight, can be dropped at any time. The ith miscellaneous weight drop is characterized by three items:

1) W_i^d Miscellaneous weight dropped

2) τ_i^w Time interval between beginning of n_i^wth thrust event and miscellaneous weight drop.

3) n_i^w Weight drop time is calculated from the beginning of this thrust event. Can also be thought of as "picket" number of the thrust event time to which τ_i^w is added to get miscellaneous weight drop time.

The ith miscellaneous weight drop occurs at t_{2i}. The t_{2i} are calculated as

$$t_{2i} = t_{1j} + \tau_i^w$$

where

$$j = n_i^w$$

Note that with this definition, none, one or many miscellaneous weight drop events may be defined relative to any given thrust event, and may occur during that or any other thrust event. The only restriction being that t_{2i+1} must be greater than t_{2i}.

In ROBOT the thrust events are grouped into stages through the use of the input array NØEVNT. The first member of the NØEVNT array contains the number of thrust events in the first stage, the second member contains the number in the second stage, etc. However they are grouped, all thrust events must be accounted for!

6.1 Thrust and Flow Rate

The use of the four numbers ν_{i1}, ν_{i2}, ν_{i3}, ν_{i4}, to describe the effective number of engines leads to a rather cumbersome notation if they are used in each equation where the number of engines is required. Consequently, an effective number of engines operator, ν_i, is defined to be:

$$\nu_i = \begin{cases} \nu_{i1}\cos\nu_{i2} + \nu_{i3}\cos\nu_{i4} & \text{if } \nu_i \text{ multiplies } F_i \text{ or } Ae_i \\ \nu_{i1} + \nu_{i3} & \text{if } \nu_i \text{ multiplies } \dot{m}_i \text{ or } c\dot{m}_i{}^{*} \end{cases}$$

The input thrust levels for first stage component rockets are considered to be nominal sea level thrusts. The total thrust, T, for all thrust events considered to be in the first stage is calculated from

$$T = \nu_i(F_i + Ae_i(p_s - p_a))$$

where p_s is the sea level atmospheric pressure.

---

*$c\dot{m}_i$ is defined in Section 7.4.2.

The input thrust levels for all thrust events other than those in the first stage are considered to be vacuum thrust levels, and the total thrust is calculated from

$$T = \nu_i (F_i - Ae_i p_a)$$

while still in the atmosphere and

$$T = \nu_i F_i$$

once the atmosphere is dropped.

The total flow rate, $\dot{m}$, in any thrust event is calculated from

$$\dot{m} = \nu_i \dot{m}_i$$

7. THE FORWARD TRAJECTORY

This section contains the equations of motion integrated in ROBOT, a description of the forward trajectory flight phases, the terminal functions which may be selected to define an optimization problem, and various users' options associated with the forward trajectory.

7.1 The Equations of Motion

In general form the equations of motion integrated in ROBOT are written

$$\dot{p}_1 = \dot{w} = F_x / m + g_x$$

$$\dot{p}_2 = \dot{u} = F_y / m + g_y$$

$$\dot{p}_3 = \dot{v} = F_z / m + g_z$$

$$\dot{p}_4 = \dot{x} = w$$

$$\dot{p}_5 = \dot{y} = u$$

$$\dot{p}_6 = \dot{z} = v$$

$$\dot{p}_7 = \dot{\mu} = -\dot{m}$$

$$\dot{p}_8 = \dot{\eta} = \begin{cases} q V_R / (\frac{\pi}{2} - \alpha) & \text{in the atmosphere} \\ \text{not integrated} & \text{outside the atmosphere} \end{cases}$$

The forcing functions F_x, F_y and F_z are

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} T \sin\chi_p \cos\chi_y - F_{Ax} \\ T \cos\chi_p \cos\chi_y - F_{Ay} \\ T \sin\chi_y \quad - F_{Az} \end{bmatrix} = C \begin{bmatrix} - F_{AN} \cos\sigma \\ T - F_{AA} \\ - F_{AN} \sin\sigma \end{bmatrix}$$

with, of course, F_{AA} and F_{AN} set to zero when the atmosphere is dropped.

The mass, m, is calculated from

$$m = \mu + m_a$$

where μ is continuous and consists of the total propellants to be burned plus the payload, and

$$m_a = (\sum_i W_i^d + \sum_j W_j^J) / g_o$$

The constant g_o relates mass to weight and is taken to be

$$g_o = 9.80665 \text{ m / sec}^2$$

Since m_a is constant from one weight drop to the next, $\dot{\mu} = - \dot{m}$ for all $t \neq t_{1i}$ or t_{2i}.

By this artifice, the seventh state variable, μ, is made to be continuous at all times, including those times at which mass is discontinuous. The primary advantage of integrating this particular choice of state variable is the ease with which mass can be reconstructed during the adjoint integration. Since μ is continuous, it may be stored as a function of time on the forward trajectory, and therefore, even if $\dot{m}$ is a time varying function obtained from a thrust tape, the mass can be calculated during the adjoint integration by looking μ up, updating m_a at the t_{1i} and t_{2i} and adding the two together.

The eighth state variable, η, is an aerodynamic heating indicator.

7.2 Ground-Launch Trajectory Flight Phases

The flight profile of a ground-launch trajectory is separated into a number of phases. These phases are depicted graphically below for a booster having n-1 thrust events. The symbols in Fig. 8 are discussed below.

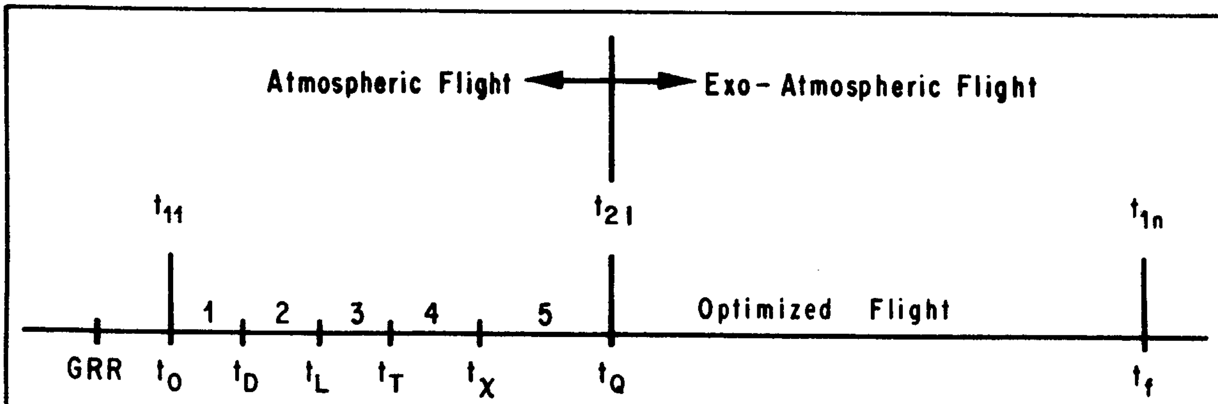

FIG. 8. GROUND-LAUNCH FLIGHT PHASES

7.2.1 GRR, Δt_o, t_o

The input quantity Δt_o is the time interval between the time the coordinate systems are defined, GRR, and the lift-off time, t_o. t_o is an input constant which is generally taken to be zero. t_{11}, the time the first thrust event begins, is set to t_o. If a non-zero value of Δt_o is used, the boost vehicle, which is fixed to the earth, will not be in the $\hat{Y}\hat{Z}$ plane at lift-off.

7.2.2 Ground-Launch Initial Conditions

The calculation of the initial, t_o, conditions for a ground-launch trajectory proceeds directly from Δt_o and the input value of the geodetic latitude of the launch site, θ_o. The geocentric colatitude of the launch site is

$$\theta = \pi/2 - \tan^{-1}\left((1-f)^2 \tan\theta_o\right)$$

The radius of the launch site is $R(\theta)$ and the initial velocity of the launch site is

$$V_o = R(\theta)\,\Omega_e \sin\theta$$

The longitude angle subtended by the launch site during the time interval Δt_o is

$$\Delta\phi_o = \Omega_e \Delta t_o$$

The initial plumbline velocity components are

$$\begin{bmatrix} w_o \\ u_o \\ v_o \end{bmatrix} = V_o A \begin{bmatrix} \cos\Delta\phi_o \\ 0 \\ -\sin\Delta\phi_o \end{bmatrix}$$

The initial plumbline position coordinates are

$$\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} = R(\theta) A \begin{bmatrix} \sin\Delta\phi_o \sin\theta \\ \cos\theta \\ \cos\Delta\phi_o \sin\theta \end{bmatrix}$$

The initial value of the seventh state variable is calculated from m_a, and the input value of initial mass, m_o, as

$$\mu_o = m_o - m_a$$

The initial value of the eighth state variable is, of course

$$\eta_o = 0$$

7.2.3 Lift-Off -- Phases 1 and 2

The interval $t_o \rightarrow t_L$, Phases 1 and 2 of Fig. 8, is the lift-off portion of the trajectory. During this interval the control variables χ_p and χ_y are chosen so that the launch vehicle will clear the launch tower.

During the interval $t_o \rightarrow t_D$, Phase 1 of Fig. 8, F_{AA} is augmented so as to be

$$F_{AA} = qSC_A + \text{DRAG1} \Big/ \left(1 + \frac{.55(\sinh t)^{2.3}}{\sqrt{1+t^2}}\right)$$

This DRAG1 term is included to account for the effect of the launch tower on the axial force. For the Saturn V, t_D is taken to be four secounds greater than t_o. This augmentation of F_{AA} can be omitted by inputting t_D equal to t_o. Also, to avoid numerical problems, α is defined to be zero during Phase 1.

During Phase 2, however, the full three dimensional forms for α and σ are used. Since the launch tower is constructed normal to the reference ellipsoid, the angular separation of the launch tower and north is θ_L, where

$$\theta_L = \theta_1$$

The longitude of the launch site is ϕ_L, where

$$\phi_L = \Delta\phi_o + \Omega_e (t - t_o)$$

A unit vector in the launch tower direction can be transformed into the plumbline system as

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = A \begin{bmatrix} \sin\phi_L \sin\theta_L \\ \cos\theta_L \\ \cos\phi_L \sin\theta_L \end{bmatrix} = \begin{bmatrix} \sin\chi_p \cos\chi_y \\ \cos\chi_p \cos\chi_y \\ \sin\chi_y \end{bmatrix}$$

Therefore, in the interval $t_o \rightarrow t_L$, χ_p and χ_y are calculated to be

$$\chi_p = \tan^{-1}\left(\frac{\bar{x}}{\bar{y}}\right)$$

$$\chi_y = \sin^{-1} \bar{z}$$

Since the A matrix and θ_L are constant and ϕ_L depends only on t, χ_p and χ_y during lift-off are functions of time only. Both t_D and t_L are input constants.

7.2.4 Tilt-Over -- Phase 3

During the interval $t_L \rightarrow t_T$, Phase 3 of Fig. 8, the vehicle is caused to tilt over in the $\hat{x}\hat{y}$ plane by calculating χ_p and χ_y as

$$\chi_p = \dot{\chi} \cdot (t - t_L)$$

$$\chi_y = 0$$

where $\dot{\chi}$ is a trajectory parameter and t_T is an input constant.

During $\chi_y = 0$ flight, the equations given previously for α and σ reduce to

$$\alpha = \cos^{-1}\left(\frac{\underline{w}}{V_R}\sin\chi_p + \frac{\underline{u}}{V_R}\cos\chi_p\right)$$

$$\sigma = \tan^{-1}\left(\frac{\underline{v}}{\underline{w}\cos\chi_p - \underline{u}\sin\chi_p}\right)$$

7.2.5 Pitch-Plane Gravity Turn -- Phase 4

Following the tilt-over, a pitch-plane gravity turn is flown in which

$$\chi_y = 0$$

and χ_p is chosen so that the angle of attack in the pitch ($\hat{x}\hat{y}$) plane is zero. This requires that during Phase 4

$$\chi_p = \tan^{-1}\left(\frac{\underline{w}}{\underline{u}}\right)$$

and therefore,

$$\alpha = \cos^{-1}\left(\frac{\sqrt{\underline{w}^2 + \underline{u}^2}}{V_R}\right)$$

and

$$\sigma = \mathrm{sgn}(\underline{v})\frac{\pi}{2}$$

7.2.6 Chi-Freeze -- Phase 5

The pitch-plane gravity turn terminates at t_χ, an input constant marking the beginning of Phase 5. During Phase 5 neither χ_p nor χ_y is allowed to vary and hence they may be thought of as being "frozen" with values

$$\chi_y = 0$$

and

$$\chi_p = \tan^{-1}\left(\frac{\underline{w}}{\underline{u}}\right)_{t=t_\chi}$$

On the Saturn V, χ-freeze is initiated towards the end of the first stage and is held until the launch escape system is jettisoned. Consequently, the internal logic of ROBOT is arranged so that t_Q is a miscellaneous weight drop event time, i.e.,

$$t_Q = t_{2I}$$

with I being the number of the miscellaneous weight drop event which terminates χ-freeze. The end of χ-freeze marks the end of atmospheric flight and hence t_Q must be defined on every ground launch trajectory. Note that this implies that there must always be at least one miscellaneous weight drop event. If none is actually desired, then a zero weight must be dropped.

7.2.7 Exo-Atmospheric Flight

At t_Q the atmosphere is dropped and the eighth state variable is no longer integrated. The ROBOT program shifts to a different set of derivative routines at this point in order to avoid bypassing terms that have to do with the atmosphere and also because, logically, the control variables are handled differently.

Prior to t_Q the thrust vector control angles χ_p and χ_y are obtained as a direct consequence of a sequence of internal logic phases. After t_Q, χ_p and χ_y are considered to be tabular functions of time. Time, χ_p and χ_y can be specified at a maximum of 196 tabular points. These are broken up into four sets of control tables with a limit of 49 points each. Through input it is possible to specify the thrust event "picket" number at which control tables start and stop and the number of points in a table. Control tables should not continue across a coast or an intermediate point constraint.* Since Simpson's rule is used to integrate products of impulse response functions during the adjoint solution, there should always be an odd number of points in a control table.

The steepest ascent process converges on the optimal χ_p, χ_y time histories by updating the tabular control programs of χ_p and χ_y (if specified by input) at each iteration. If the input quantity KWTA is set to 3, both χ_p and χ_y are varied. If KWTA is input as 2, χ_y is held at zero and χ_p is varied.

7.3 Intermediate and/or Terminal Functions

In order to define an optimization problem it is necessary to specify the trajectory constraints as well as the quantity to be maximized or minimized. Table 1 consists of a library of eleven (at present) intermediate and/or terminal functions and their non-zero partial derivatives. Any one of these functions may be selected as the payoff and be maximized or minimized at the terminal time. Any physically realizable set of the remaining functions may be selected as trajectory constraints and imposed at the terminal time. In addition, any physically realizable set of these functions may be imposed as constraints at an intermediate time by inputting the number of the thrust event following which the constraints are imposed as NVRST.

---

*Described in Section 7.3.

Table 1. Function Library

Code No.	Function Name	Symbol	Formula	Non-zero Partial Derivatives
1	Mass (Payload if payoff)	m		$\frac{\partial m}{\partial \mu} = 1$
2	Inertial Velocity	V_I	$V_I = \sqrt{w^2 + u^2 + v^2}$	$\frac{\partial V_I}{\partial w} = \frac{w}{V_I}$, $\frac{\partial V_I}{\partial u} = \frac{u}{V_I}$, $\frac{\partial V_I}{\partial v} = \frac{v}{V_I}$
3	Inertial Flight Path Angle	γ	$\gamma = \sin^{-1}\left(\frac{u_s}{V_I}\right)$	$\frac{\partial \gamma}{\partial w} = \frac{1}{V_I \cos\gamma}\left(n_{21}^{*} - \frac{w \sin\gamma}{V_I}\right)$ $\frac{\partial \gamma}{\partial u} = \frac{1}{V_I \cos\gamma}\left(n_{22} - \frac{u \sin\gamma}{V_I}\right)$ $\frac{\partial \gamma}{\partial v} = \frac{1}{V_I \cos\gamma}\left(n_{23} - \frac{v \sin\gamma}{V_I}\right)$ $\frac{\partial \gamma}{\partial x} = \frac{n_{24}}{V_I \cos\gamma}$ $\frac{\partial \gamma}{\partial y} = \frac{n_{25}}{V_I \cos\gamma}$ $\frac{\partial \gamma}{\partial z} = \frac{n_{26}}{V_I \cos\gamma}$

*The n_{ij} are defined in Appendix I

Table 1. (Cont'd)

Code No.	Function Name	Symbol	Formula	Non-zero Partial Derivatives
4	Radius	r	$r = \sqrt{x^2 + y^2 + z^2}$	$\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$
5	Energy	C_3	$C_3 = V_I^2 - \frac{2\mu_e}{r}$	$\frac{\partial C_3}{\partial w} = 2w$ $\frac{\partial C_3}{\partial u} = 2u$ $\frac{\partial C_3}{\partial v} = 2v$ $\frac{\partial C_3}{\partial x} = \frac{2\mu_e x}{r^3}$ $\frac{\partial C_3}{\partial y} = \frac{2\mu_e y}{r^3}$ $\frac{\partial C_3}{\partial z} = \frac{2\mu_e z}{r^3}$

Table 1. (Cont'd)

Code No.	Function Name	Symbol	Formula	Non-zero Partial Derivatives
6	Angular Momentum	C_1	Defining: $A = yv - uz$ $B = zw - vx$ $C = xu - wy$ $C_1 = \sqrt{A^2 + B^2 + C^2}$	$\frac{\partial C_1}{\partial w} = \frac{zB - yC}{C_1}$ $\frac{\partial C_1}{\partial u} = \frac{xC - zA}{C_1}$ $\frac{\partial C_1}{\partial v} = \frac{yA - xB}{C_1}$ $\frac{\partial C_1}{\partial x} = \frac{uC - vB}{C_1}$ $\frac{\partial C_1}{\partial y} = \frac{vA - wC}{C_1}$ $\frac{\partial C_1}{\partial z} = \frac{wB - uA}{C_1}$
7	Inertial Longitude	ϕ	$\phi = \tan^{-1}\left(\frac{a_{11}x + a_{21}y + a_{31}z}{a_{13}x + a_{23}y + a_{33}z}\right)$	$\frac{\partial \phi}{\partial x} = n_{44}$ $\frac{\partial \phi}{\partial y} = n_{45}$ $\frac{\partial \phi}{\partial z} = n_{46}$

Table 1. (Cont'd)

Code No.	Function Name	Symbol	Formula	Non-zero Partial Derivatives
8	Inertial Heading Angle	β	$\beta = \tan^{-1}\left(\frac{w_s}{v_s}\right)$	$\frac{\partial\beta}{\partial w} = \frac{v_s n_{11} - w_s n_{31}}{w_s^2 + v_s^2}$ $\frac{\partial\beta}{\partial u} = \frac{v_s n_{12} - w_s n_{32}}{w_s^2 + v_s^2}$ $\frac{\partial\beta}{\partial v} = \frac{v_s n_{13} - w_s n_{33}}{w_s^2 + v_s^2}$ $\frac{\partial\beta}{\partial x} = \frac{v_s n_{14} - w_s n_{34}}{w_s^2 + v_s^2}$ $\frac{\partial\beta}{\partial y} = \frac{v_s n_{15} - w_s n_{35}}{w_s^2 + v_s^2}$ $\frac{\partial\beta}{\partial z} = \frac{v_s n_{16} - w_s n_{36}}{w_s^2 + v_s^2}$
9	Colatitude	θ	$\theta = \cos^{-1}(a_{12}x + a_{22}y + a_{32}z)$	$\frac{\partial\theta}{\partial x} = n_{64}$ $\frac{\partial\theta}{\partial y} = n_{65}$ $\frac{\partial\theta}{\partial z} = n_{66}$

Table 1. (Cont'd)

Code No.	Function Name	Symbol	Formula	Non-zero Partial Derivatives
10	Inclination	i	$i = \cos^{-1}(\sin\theta \sin\beta)$	$\frac{\partial i}{\partial w} = -\frac{\sin\beta\cos\theta}{\sin i} n_{61} - \frac{\cos\beta\sin\theta}{\sin i}\frac{\partial\beta}{\partial w}$ $\frac{\partial i}{\partial u} = -\frac{\sin\beta\cos\theta}{\sin i} n_{62} - \frac{\cos\beta\sin\theta}{\sin i}\frac{\partial\beta}{\partial u}$ $\frac{\partial i}{\partial v} = -\frac{\sin\beta\cos\theta}{\sin i} n_{63} - \frac{\cos\beta\sin\theta}{\sin i}\frac{\partial\beta}{\partial v}$ $\frac{\partial i}{\partial x} = -\frac{\sin\beta\cos\theta}{\sin i} n_{64} - \frac{\cos\beta\sin\theta}{\sin i}\frac{\partial\beta}{\partial x}$ $\frac{\partial i}{\partial y} = -\frac{\sin\beta\cos\theta}{\sin i} n_{65} - \frac{\cos\beta\sin\theta}{\sin i}\frac{\partial\beta}{\partial y}$ $\frac{\partial i}{\partial z} = -\frac{\sin\beta\cos\theta}{\sin i} n_{66} - \frac{\cos\beta\sin\theta}{\sin i}\frac{\partial\beta}{\partial z}$

Table 1. (Cont'd)

Code No.	Function Name	Symbol	Formula	Non-zero Partial Derivatives
11	Line of Nodes	ω	$\omega = \phi + \tan^{-1}(\cos\theta \tan\beta)$	Defining: $A \quad \frac{w_s v_s \sin\theta}{v_s^2 + w_s^2 \cos^2\theta}$ $B = \frac{(v_s^2 + w_s^2)\cos\theta}{v_s^2 + w_s^2 \cos^2\theta}$ $\frac{\partial\omega}{\partial w} = B \frac{\partial\beta}{\partial w}$ $\frac{\partial\omega}{\partial u} = B \frac{\partial\beta}{\partial u}$ $\frac{\partial\omega}{\partial v} = B \frac{\partial\beta}{\partial v}$ $\frac{\partial\omega}{\partial x} = (n_{44} - A n_{64}) + B \frac{\partial\beta}{\partial x}$ $\frac{\partial\omega}{\partial y} = (n_{45} - A n_{65}) + B \frac{\partial\beta}{\partial y}$ $\frac{\partial\omega}{\partial z} = (n_{46} - A n_{66}) + B \frac{\partial\beta}{\partial z}$

7.4 Control Parameters, Propellant Tank Limits and Flight Performance Reserves

In addition to optimizing the χ_p and χ_y time histories during exo-atmospheric flight, the ROBOT program can simultaneously optimize control parameters selected by input from the control parameter library.

7.4.1 Control Parameters

Table 2 contains the members of the control parameter library. The maximum value of n in Table 2 is 15.

Table 2. Control Parameter Library

Library No.	Parameter Name	Symbol
1	Launch Weight	m_o
2	Tilt-over Chi-dot	$\dot{\chi}$
3	1 st Thrust event duration	τ_{11}
4	2 nd Thrust event duration	τ_{12}
⋮	⋮	⋮
n + 2	n th Thrust Event Duration	τ_{1n}

The library number of each parameter to be optimized is specified by putting a non-zero value into the equivalently numbered element of the input array KDB. Thus it is the position of non-zero elements in KDB which indicates an active parameter. Although all τ_{1i} are provided a library number, only those τ_{1i} terminating outside the atmosphere may be selected for optimization.

7.4.2 Propellant Tank Limits

In a great number of real problems the total propellant in a given stage is fixed, albeit allocated among a number of different thrust events. Also, since the available fuel and oxidizer will not, in general, be exhausted simultaneously when mixture-ratio shifts are considered, tank limits in ROBOT are based upon "critical" propellant rather than actual propellant.

The τ_{1i} can be connected by logic so as to maintain the relationship

$$m_x = \sum_i \nu_i c\dot{m}_i \tau_{1i}$$

where m_x, when tank limits alone are considered, is defined by the input values of the critical flow rate $c\dot{m}_i$ and the τ_{1i}. Since m_x cannot vary when the τ_{1i} are being varied by the steepest-ascent process

$$\sum_i \nu_i c\dot{m}_i d\tau_{1i} = 0$$

Therefore, all the connected thrust events cannot be optimized independently. One of the τ_{1i}, the jth, must be dependent and result from a choice of the others, i.e.,

$$d\tau_{1j} = -\sum_{i \neq j} \frac{\nu_i c\dot{m}_i}{\nu_j c\dot{m}_j} d\tau_{1i}$$

The procedure used in ROBOT for specifying that the jth thrust event is connected to the ith with the ith being independent, i.e., KDB(i+2) $\neq$ 0, is to put the difference between j and i into the same element of the input array KDT, i.e., KDT(i+2) = j - i. One restriction on this procedure is that j must be greater than i. If no connection is desired, the appropriate element of KDT is set to zero. Note that if KDT(i+2) = j - i > 0, then KDB(j+2) must be zero since the same parameter cannot be specified as both dependent and independent. (If this requirement is not met, the program will print a warning, run a forward trajectory and go to the next case.) If, for example, KDB(6) ≠ 0 indicating that τ_{14} is to be optimized and if KDT(6) = 1, then τ_{15} is altered to keep m_x constant and KDB(7) must be zero. If, on the other hand, KDT(6) = 2, then τ_{16} is altered to keep m_x constant and KDB(8) must be zero. If, however, KDT(6) = 0, then τ_{14} is optimized without regard to limits.

As is implied by the equation for $d\tau_{1j}$, the same thrust event can be specified as dependent by more than one independent parameter. For example, the input arrays

KDT = 0, 0, 0, 0, 0, 4, 3, 2, 0, 0, 0

KDB = 1, 1, 0, 0, 0, 1, 1, 1, 0, 0, 0

indicate that m_o, $\dot{X}$, τ_{14}, τ_{15}, and τ_{16} are to be optimized and that

$$d\tau_{18} = -\frac{\nu_4 c\dot{m}_4}{\nu_8 c\dot{m}_8} d\tau_{14} - \frac{\nu_5 c\dot{m}_5}{\nu_8 c\dot{m}_8} d\tau_{15} - \frac{\nu_6 c\dot{m}_6}{\nu_8 c\dot{m}_8} d\tau_{16}$$

It should be noted that although the rationale for the development of the connection logic comes from the necessity of holding stage tank limits, the connection logic is independent of stage specification.

7.4.3 Flight Performance Reserves

Flight performance reserves (hereinafter called FPR) is a name given to the propellants held in reserve on a design flight to provide an increment of velocity over and above the design velocity in the event it should be necessary on an actual flight. As such, FPR are jettisoned along with the jettison weight of the last thrust event and do not appear as part of the payload.

The input quantity IPR is the number of the thrust event from which the FPR are withheld. If IPR = 0, FPR are not calculated. There are several accompanying requirements if IPR is not to be zero. First of all the IPR th thrust event must be in the last stage. Secondly, the maximum amount of critical propellant in the last stage, m_x, must be input as WPMX. Thirdly, although the IPR th does not have to be the last thrust event, no thrust event which follows the IPR th may be optimized.

The ROBOT program calculates FPR on the basis of two input ΔV requirements. These are, ΔV_g to account for geometry perturbations and ΔV_p to account for performance perturbations. FPR are related to ΔV_g and ΔV_p through the equations

$$GPR = m_c\left(1 - e^{-\Delta V_g / Vex}\right)$$

$$PPR = \left(m_c - GPR\right)\left(1 - e^{-\Delta V_p / Vex}\right)$$

$$FPR = GPR + PPR$$

where m_c is the mass at cutoff of the IPR th thrust event, and $Vex = g_o I_{sp}$ of the IPR th thrust event. Defining

$$k_1 = 1 - e^{-\Delta V_g / Vex}$$

$$k_2 = 1 - e^{-\Delta V_p / Vex}$$

The FPR can be calculated as

$$FPR = m_c k_4$$

where

$$k_4 = k_1 + k_3$$

$$k_3 = (1 - k_1) k_2$$

Denoting IPR by j, and the mass at the beginning of the IPR th thrust event by m_j, the cutoff mass, m_c, can be written as

$$m_c = m_j - \nu_j \dot{m}_j \tau_{1j}$$

The problem of course is to find τ_{1j} such that the sum of the critical propellant contained in the FPR and that consumed during the remainder of the last stage is equal to m_x. This may be written

$$m_x = \sum_{i \neq j} \nu_i c \dot{m}_i \tau_{1i} + \nu_j c \dot{m}_j (\tau_{1j} + \tau_p)$$

where the summation by i is over the thrust events in the last stage, and τ_p is defined by

$$\tau_p = \frac{FPR}{\nu_j \dot{m}_j} = k_4 \left(\frac{m_j}{\nu_j \dot{m}_j} - \tau_{1j} \right)$$

This leads to

$$\tau_{1j} = \frac{1}{\nu_j \dot{cm}_j (1 - k_4)} \left(m_x - k_4 m_j \frac{\dot{cm}_j}{\dot{m}_j} - \sum_{i \neq j} \nu_i \dot{cm}_i \tau_{1i} \right)$$

If in addition to FPR, τ_{1i} are optimized in the last stage, a different form of the equation for τ_{1j} is useful in the calculation of the steepest ascent influence coefficients. Denoting the mass at the beginning of the first thrust event in the last stage by m_L and noting that

$$m_j = m_L - \sum_{i<j} \nu_i \dot{m}_i \tau_{1i} - m^d$$

where m^d is the sum of the weights dropped (if any) in the interval between m_L and m_j, the equation for τ_{1j} may be written

$$\tau_{1j} = \frac{1}{\nu_j \dot{cm}_j (1 - k_4)} \left(m_x - k_4 \frac{\dot{cm}_j}{\dot{m}_j} (m_L - m^d) + \sum_{i<j} (k_4 \frac{\dot{cm}_j}{\dot{m}_j} \dot{m}_i - \dot{cm}_i) \nu_i \tau_{1i} \right.$$

$$\left. - \sum_{i>j} \nu_i \dot{cm}_i \tau_{1i} \right)$$

The situation that exists when τ_{1i} in the last stage are optimized and FPR are calculated, and when there is KDT connection between the ith and IPR th thrust events is essentially the same, since m_x is constant in either case. The difference is that in straight KDT connection the input thrust event durations define an m_x, whereas with FPR, m_x = WPMX defines τ_{1j}. The similarity between IPR and KDT connection can readily be seen for the case where $\Delta V_g = \Delta V_p = 0$, in which case $k_4 = 0$, and for both IPR and KDT connection

$$d\tau_{1j} = - \sum_{i \neq j} \frac{\nu_i \dot{m}_i}{\nu_j \dot{m}_j} d\tau_{1i}$$

The situations are in fact so similar logically that the ROBOT program sets up and uses KDT connection logic whenever τ_{1i} are optimized in a stage that has FPR.

7.5 Jump Start

The input variable JUMP is the thrust event "picket" number at which a trajectory begins. If JUMP = 1, the trajectory will progress through the ground-launch logic. If JUMP ≠ 1, the trajectory will begin out of the atmosphere at that thrust event "picket" number. The starting state is specified through the input array VIV, the starting time by TZERØ and the starting weight by WZERØ. When there is a jump start, t_Q is set to TZERØ and all KDB and KDT below the jump start point are set to zero. If VIV(7) = 0, the plumbline state w, u, v, x, y, z must be read into VIV(1) → VIV(6). If VIV(7) = 2, V_I, γ, r, azimuth (A_z), latitude (θ') and ω must be read into VIV(1) → VIV(6).

Setting

$$a = 180 - A_z$$

and

$$\theta = 90 - \theta'$$

$$\underline{\omega} = \tan^{-1}(\cos\theta \tan a)$$

$$\phi = \omega - \underline{\omega}$$

Then, constructing a B matrix using θ and ϕ above and using the launch site A matrix, a D matrix can be constructed and used to calculate the initial plumbline state as

$$\begin{bmatrix} w_o \\ u_o \\ v_o \end{bmatrix} = V_I D \begin{bmatrix} \cos\gamma \cos a \\ \sin\gamma \\ \cos\gamma \sin a \end{bmatrix}$$

$$\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} = r D \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

7.6 10 km, Qmax, 14 km

The program prints out as it crosses 10 km altitude, 14 km altitude and the point of maximum dynamic pressure. In order to find the latter, the time derivative of dynamic pressure, $\dot{q}$, is used. $\dot{q}$ is calculated by forming the dot product of the partials of q wrt the plumbline state, $\frac{\partial q}{\partial p}$, and the time derivatives of the plumbline state, $\dot{p}$. That is,

$$\dot{q} = \frac{\partial q}{\partial p} \dot{p}$$

where

$$\left[\frac{\partial q}{\partial p}\right]^T = \begin{bmatrix} \rho \underline{w} \\ \rho \underline{u} \\ \rho \underline{v} \\ \frac{q}{\rho}\frac{d\rho}{dh}\left(d_{12} - \frac{dR(\theta)}{d\theta} d_{13}\right) - \rho(a_{32}\underline{u} - a_{22}\underline{v})\Omega_e \\ \frac{q}{\rho}\frac{d\rho}{dh}\left(d_{22} - \frac{dR(\theta)}{d\theta} d_{23}\right) - \rho(a_{12}\underline{v} - a_{32}\underline{w})\Omega_e \\ \frac{q}{\rho}\frac{d\rho}{dh}\left(d_{32} - \frac{dR(\theta)}{d\theta} d_{33}\right) - \rho(a_{22}\underline{w} - a_{12}\underline{u})\Omega_e \end{bmatrix}$$

and $\frac{d\rho}{dh}$ is calculated numerically using the PRA63 atmosphere routine.

7.7 Impact Point

The ROBOT program integrates the trajectory of the jettison weight of the IMPth thrust event (W^J_{IMP}) to impact ($h = 0$) if the input constant IMP is > 0. The forcing functions F_x, F_y and F_z on the impact trajectory are

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -\rho V_R \underline{w} \\ -\rho V_R \underline{u} \\ -\rho V_R \underline{v} \end{bmatrix}$$

where $\rho = 0$ for altitudes greater than 690 km and ρ calculated from PRA63 as a function of altitude for $h < 690$ km. The equations for μ and η are not integrated on the impact trajectory.

7.8 Chi-Yaw Options

Two different χ_y options may be activated during the atmospheric portion of flight.

7.8.1 Lift-Off Yaw

In order to provide positive launch tower clearance, it is possible to activate a supplemental trapezoidal χ_y history during Phase 2. The trapezoid is defined by the input times TCY1, TCY2, TCY3, TCY4 and the plateau value of χ_y, CYTM. A non-zero value of the input constant NCYT will activate this χ_y profile. Since this option must occur inside Phase 2, TCY1 must be $\geq t_D$ and TCY4 must be $\leq t_L$.

7.8.2 First Stage Yaw

A non-zero χ_y during Phase 4 can be obtained by inputting a non-zero value of NFSCY. A χ_y rate, FSCYD, and a time to initiate the rate, TFSCY, are also inputs. Since this χ_y logic can only be initiated during Phase 4, TFSCY must be $t_L \leq$ TFSCY $\leq t_\chi$. The value of χ_y at t_χ is retained during chi-freeze (Phase 5).

7.9 Aeroheat Constraint

By inputting a non-zero value of NAHI, the aerodynamic heating indicator, η, can be constrained to the input value AHIMAX at t_Q. Each time the launch weight is changed, the ROBOT program does a linear search on the tilt rate, $\dot{\chi}$, until $\eta(t_Q)$ = AHIMAX. If NAHI $\neq$ 0, the program sets KDB(2) = 0 since $\dot{\chi}$ cannot be optimized and used to satisfy the aeroheat constraint at the same time.

7.10 Output Tables

By inputting a non-zero value of NTABLE output tables suitable for publication can be obtained. The output tables are printed only for converged trajectories. The format of the tables is discussed in Part 2 of this report.

If tables are desired, additional input described in Appendix III is required.

8. THE BACKWARD TRAJECTORY

Since the steepest ascent method converges on the optimum set of controls by adding beneficial changes to the nominal set, the effect of small changes in the controls on the terminal and intermediate functions must be calculated. This is accomplished through the use of the adjoint differential equations. One solution of the adjoint differential equations is required for each terminal or intermediate function being either optimized or constrained. The adjoint solutions proceed backward in time from the final time for the payoff and terminal constraints, and from the intermediate constraint time if there are intermediate constraints.

The adjoint variables are used to form impulse response functions which give the effect of changes in χ_p and χ_y and influence coefficients which give the effect of changes in the parameters. These impulse response functions and influence coefficients are then used in the steepest ascent formulae to calculate beneficial changes in the controls.

Notation traditionally used to describe the adjoint solution is introduced below.

- ϕ The scalar payoff function.
- ψ An m x 1 matrix of constraints. Includes both terminal and intermediate constraints. (Constraints satisfied when $\psi = 0$)
- ν An m x 1 matrix of constant Lagrange multipliers associated with the constraints. (This ν should not be confused with the effective number of engines ν_i defined in Section 7.)

Φ The augmented scalar payoff function $\phi + \nu^T \psi$.

λ_ϕ A 7 x 1 matrix of particular adjoint solutions associated with the payoff function.

λ_ψ A 7 x m matrix of particular adjoint solutions associated with the constraints.

λ A 7 x 1 matrix of adjoint solutions associated with the function Φ. When appearing without a subscript, λ is the equivalent of the Euler-Lagrange variables used in the calculus of variations (c. o. v. λ's) and are formed as

$$\lambda = \lambda_\phi + \lambda_\psi \nu$$

8.1 Boundary Conditions

The boundary condition on the Euler-Lagrange variables λ are known to be

$$\lambda^T = \frac{\partial \Phi}{\partial p}$$

Consequently, the boundary conditions on λ_ϕ and λ_ψ are chosen to be

$$\lambda_\phi^T = \left.\frac{\partial \phi}{\partial p}\right|_{t = t_f}$$

and

$$\lambda_\psi^T = \left.\frac{\partial \psi}{\partial p}\right|_{t = \begin{cases} t_f \text{ for terminal constraints} \\ t_{1j},\ j = \text{NVRST} + 1 \text{ for intermediate constraints} \end{cases}}$$

8.2 The Adjoint Differential Equations

Defining the 7 x m + 1 matrix λ_z to be $\lambda_z = \left[\lambda_\phi \,\vdots\, \lambda_\psi\right]$, the Euler-Lagrange or adjoint differential equations become

$$\dot{\lambda}_z = + F^T \lambda_z \quad \text{(for backwards integration)}$$

The 7 x 7 matrix F is the matrix of partial derivatives

$$F^T = \left[\frac{\partial \dot{p}}{\partial p}\right]^T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ g_{xx}^* & g_{yx} & g_{zx} & 0 & 0 & 0 & 0 \\ g_{xy} & g_{yy} & g_{zy} & 0 & 0 & 0 & 0 \\ g_{xz} & g_{yz} & g_{zz} & 0 & 0 & 0 & 0 \\ -\frac{F_x}{m^2} & -\frac{F_y}{m^2} & -\frac{F_z}{m^2} & 0 & 0 & 0 & 0 \end{bmatrix}$$

m + 1 sets of adjoint equations are integrated backwards to t_Q (one set for ϕ and one set for each of the m ψ's). The reconstruction of the plumb-line state, needed to calculate F during the adjoint run, is accomplished by looking up stored values of the state as a function of time.

*The gravity partials are given in Appendix II.

8.3 Impulse Response Functions and I Integrals

The impulse response functions for χ_p and χ_y are defined by the equations

$$G^T_{zp} = \frac{\partial(\lambda^T_z \dot{p})}{\partial \chi_p} = \frac{T}{m}(\lambda^w_z \cos\chi_y \cos\chi_p - \lambda^u_z \cos\chi_y \sin\chi_p)$$

$$G^T_{zy} = \frac{\partial(\lambda^T_z \dot{p})}{\partial \chi_y} = \frac{T}{m}(-\lambda^w_z \sin\chi_y \sin\chi_p - \lambda^u_z \sin\chi_y \cos\chi_p + \lambda^v_z \cos\chi_y)$$

In the scalar product $\lambda^T_z \dot{p}$, λ^w_z, λ^u_z and λ^v_z are coefficients of $\dot{w}\,\dot{u}\,\dot{v}$ respectively. Impulse response functions are calculated at every tabular point in use in the χ_p-χ_y control tables.

Denoting G_z by

$$G^T_z = \begin{bmatrix} G_{\phi p} \\ --- \\ G_{\psi p} \end{bmatrix} \quad \text{if KWTA = 2}$$

$$G^T_z = \begin{bmatrix} G_{\phi p} & G_{\phi y} \\ --- & --- \\ G_{\psi p} & G_{\psi y} \end{bmatrix} \quad \text{if KWTA = 3}$$

The m + 1 x m + 1 matrix of control variable "I" integrals is calculated during the backward trajectory as

$$I^a_{zz} = \begin{bmatrix} I^a_{\phi\phi} & \vdots & I^a_{\phi\psi} \\ -- & \vdots & -- \\ I^a_{\psi\phi} & \vdots & I^a_{\psi\psi} \end{bmatrix} = \int_{t_Q}^{t_f} G^T_z W^{-1}_a G_z \, dt, \quad I^a_{zz}(t_f) = 0$$

where W_a^{-1} is a time varying weighting matrix defined in Section 8.6.

If there are intermediate constraints, the above definition of I_{zz}^a may be used provided that after the intermediate constraint time the elements of G_z corresponding to the intermediate constraints are taken to be zero.

8.4 Influence Coefficients

The influence coefficients for a parameter give the changes in trajectory functions resulting from a unit change in that parameter and hence may be considered trajectory to trajectory partial derivatives. The influence coefficients for lift-off weight and tilt-over $\dot{\chi}$ are calculated using numerical derivatives; whereas, those for the τ_{1i} are calculated using analytic partials.

8.4.1 Influence Coefficients for Lift-Off Weight and Tilt-Over $\dot{\chi}$

If the launch weight is to be optimized, two trajectories are run from $t_o \rightarrow t_Q$ with m_o changed by $\pm \Delta m_o$. The influence coefficients for the first parameter are then calculated as

$$L_{zm} = \left[\frac{p^+(t_Q) - p^-(t_Q)}{2\,\Delta m_o}\right]^T \lambda_z(t_Q)$$

where p^+ and p^- refer to the plumbline state from positive and negative variations of Δm_o respectively.

If the tilt-over $\dot{\chi}$ is to be optimized, two trajectories are run from $t_o \rightarrow t_Q$ with $\dot{\chi}$ changed by $\pm \Delta \dot{\chi}$. The influence coefficients for the second parameter are then calculated as

$$L_{z\chi} = \left[\frac{p^+(t_Q) - p^-(t_Q)}{2 \Delta \dot{\chi}} \right]^T \lambda_z (t_Q).$$

8.4.2 Influence Coefficients for the τ_{1i}

The calculation of the influence coefficients for the τ_{1i} proceeds through three phases. In the first phase the influence coefficients for the effect of shifting the time at which a discontinuity occurs are calculated as

$$L_{zi} = \Delta \dot{p}^T \lambda_z$$

for the effect of shifting each t_{1i} and as

$$Y_{zj} = \Delta \dot{p}^T \lambda_z$$

for the effect of shifting each t_{2j} where $\Delta \dot{p} = \left[\dot{p}^- - \dot{p}^+ \right]$ is the discontinuity in the plumbline state derivatives resulting from a discontinuous change in either thrust or mass or both. The $\Delta \dot{p}$ are calculated and stored during the forward trajectory; and L_{zi} and Y_{zj} are calculated and stored during the backward trajectory.

When t_{1i} is the final time, $\dot{p}^+$ is set to zero. When t_{1i} is the intermediate orbit time, $\dot{p}^+$ is set to zero for the multiplication of those columns of λ_z corresponding to the intermediate constraints.

In the second phase cognizance is taken of the fact that the t_{2j} are pinned to the t_{1i} via τ^w_j. Since the τ^w_j are constant,

$$dt_{2j} = dt_{1i}, \quad i = N\emptyset WD(j)$$

and therefore the following additions are performed in sequence with j running from $1 \rightarrow nw$

$$L_{zi} = L_{zi} + Y_{zj}, \quad i = N\emptyset WD(j)$$

where nw is the total number of miscellaneous weight drop events.

In the third phase cognizance is taken of the fact that for the τ_{1i} to be parameters

$$\frac{\partial t_{1j}}{\partial \tau_i} = \frac{\partial t_{1i}}{\partial \tau_i} = 1, \quad j > i$$

and therefore the following additions are performed in sequence with i running from $nv - 1 \rightarrow 1$

$$L_{zi} = L_{zi} + L_{zj}, \quad j = i + 1$$

where nv is the total number of thrust events.

8.4.3 Influence Coefficients with Tank Limits and FPR

If flight performance reserves are withheld from the jth thrust event, the influence coefficients for launch weight and τ_{1i} become

$$L_{zm} = L_{zm} - \nu_j \dot{m}_j \frac{k_4}{(1 - k_4)} L_{zj}$$

$$L_{zi} = L_{zi} + \frac{\nu_i c\dot{m}_i k_4}{\nu_j \dot{m}_j (1 - k_4)} L_{zj} \quad (i < i_L)$$

where i_L is the first thrust event in the last stage, and

$$L_{zi} = L_{zi} - \left(1 - k_4 \frac{c\dot{m}_j \dot{m}_i}{c\dot{m}_i \dot{m}_j}\right) \frac{\nu_i c\dot{m}_i}{\nu_i c\dot{m}_j (1 - k_4)} L_{zj} (i_L \leq i < j)$$

In the above equations, when and if the element corresponding to payload is augmented, k_4 is set to zero.

The proper augmentation of L_{zi} when tank limits alone are considered and the jth thrust event is connected to the ith via KDT, is

$$L_{zi} = L_{zi} - \frac{\nu_i c\dot{m}_i}{\nu_j c\dot{m}_j} L_{zj} \quad (i < j)$$

Note that this result can be obtained from the last equation given above for FPR if k_4 and i_L are taken to be zero.

8.4.4 Parameter I Matrices

Grouping the active parameters into a matrix $L_z = \left[L_\phi \; \vdots \; L_\psi\right]$ the m + 1 x m + 1 parameter "I" matrix I_{zz}^b can be formed as

$$I_{zz}^{b} = \begin{bmatrix} I_{\phi\phi}^{b} & I_{\phi\psi}^{b} \\ \hline I_{\psi\phi}^{b} & I_{\psi\psi}^{b} \end{bmatrix} = L_z^T W_b^{-1} L_z$$

where W_b^{-1} is a weighting matrix defined in Section 8.6.

8.5 Steepest Ascent Formulae

Denoting the vector of active control parameters by b, and the vector of active control variables by a, (if KWTA = 2, a is a scalar equal to χ_p) the steepest ascent formulae for the changes in the controls are

$$\delta a = \pm W_a^{-1} (G_\phi - G_\psi I_{\psi\psi}^{-1} I_{\psi\phi}) E - W_a^{-1} G_\psi I_{\psi\psi}^{-1} k\psi$$

$$db = \pm W_b^{-1} (L_\phi - L_\psi I_{\psi\psi}^{-1} I_{\psi\phi}) E - W_b^{-1} L_\psi I_{\psi\psi}^{-1} k\psi$$

where

$$I_{\psi\psi} = I_{\psi\psi}^{a} + I_{\psi\psi}^{b}$$

$$I_{\psi\phi} = I_{\psi\phi}^{a} + I_{\psi\phi}^{b}$$

In the control equations above the plus sign is used when ϕ is to be maximized, the minus sign is used when ϕ is to be minimized, $0 \leq E \leq 1$ is a constant chosen to aid convergence, ψ is the vector of terminal constraints violations, and k is the decimal fraction of the constraint violation to remove.

If there are connected thrust events involving tank limits only, the $d\tau_{1i}$ for optimized thrust events will appear as elements of the db vector and the corresponding $d\tau_{1j}$ must be calculated as indicated in Section 7.4.2.

The changes in the controls calculated using the above equations are then added to the nominal set to get the controls for the next iteration.

The change in the payoff function ϕ resulting from the control changes is

$$d\phi = \pm(I_{\phi\phi} - I_{\phi\psi} I_{\psi\psi}^{-1} I_{\psi\phi}) E - I_{\phi\psi} I_{\psi\psi}^{-1} k\psi$$

where the sign is chosen as before and

$$I_{\phi\phi} = I_{\phi\phi}^{a} + I_{\phi\phi}^{b}$$

8.6 The Automatic Convergence Scheme

It is the function of the automatic scheme to pick k, E, W_a^{-1} and W_b^{-1} in order to speed convergence, and to terminate a run when it does converge. The logic for picking k and E is quite straightforward and is directly related to iteration number. On the first iteration, E is set to zero and k is chosen such that $.5 \leq k \leq 1$. A starting value of k can be input as DP2, however the program will ignore $k < .5$ or $k > 1$. If k is input ≥ 1 the iteration number is advanced to 2. On the second iteration $k = 1$ and $E = 0$. On the third iteration $k = 1$ and $E = QY/2$, where QY is an input constant which should be chosen $0 < QY \leq 1$. On the fourth and subsequent iterations $k = 1$ and $E = QY$.

The choice of the weighting matrices W_a^{-1} and W_b^{-1} is also dependent on iteration number. On the first iteration W_a^{-1} is chosen to be

$$W_a^{-1} = \begin{cases} \dfrac{m}{T}^{*} & \text{if KWTA} = 2 \\[2ex] \begin{bmatrix} \dfrac{m}{T} & 0 \\ 0 & \dfrac{m}{T} \end{bmatrix} & \text{if KWTA} = 3 \end{cases}$$

and W_b^{-1} is chosen to be

$$W_b^{-1} = \begin{bmatrix} W_1 P_1 & 0 & \cdots & 0 \\ 0 & W_2 P_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & W_n P_n \end{bmatrix}$$

where the W_i are an input set of weighting numbers for the np active parameters. The W_i are input as WIBT and should generally be left at their preset value of 1 unless experience dictates otherwise. On the first and subsequent iterations the P_i are chosen automatically so that the largest contribution of the ith parameter to the diagonal of $I_{\psi\psi}^{a}$ is equal to one. Denoting the influence coefficients of the ith parameter on the constraints by L_{ψ}^{i}, the P_ith scale factor is

---

*m/T is mass/thrust.

$$P_i = \frac{1}{\underset{j=1,m}{\text{Max}} \left(\frac{\left(L^{i}_{\psi j}\right)^2}{I^a_{\psi_j \psi_j}} \right)}$$

For the second and third iterations the constant Lagrange multipliers on the constraints, ν, are

$$\nu = - I^{-1}_{\psi\psi} I_{\psi\phi}$$

thereafter, ν is formed as

$$\nu = - I^{-1}_{\psi\psi} I_{\psi\phi} \mp I^{-1}_{\psi\psi} \psi$$

where the minus sign is used if maximizing and the plus sign if minimizing.

Once ν has been calculated, min-H on the control variables can begin since the Euler-Lagrange multipliers λ can be formed as

$$\lambda = \lambda_\phi + \lambda_\psi \nu$$

the variational Hamiltonian H as

$$H = \lambda^T \dot{p}$$

the first partial of H w r t χ_p and χ_y as

$$H_a = \begin{bmatrix} \frac{\partial H}{\partial \chi_p} & \frac{\partial H}{\partial \chi_y} \end{bmatrix}$$

and the second partial of H w r t χ_p and χ_y as

$$H_{aa} = \begin{bmatrix} \frac{\partial^2 H}{\partial \chi_p^2} & \frac{\partial^2 H}{\partial \chi_p \partial \chi_y} \\ \frac{\partial^2 H}{\partial \chi_p \partial \chi_y} & \frac{\partial^2 H}{\partial \chi_y^2} \end{bmatrix}$$

Therefore on the second and subsequent iterations W_a^{-1} is taken to be

$$W_a^{-1} = \mp H_{aa}^{-1}$$

where the minus sign is used if maximizing and the plus sign is used if minimizing.

The elements of H_{aa} are

$$\frac{\partial^2 H}{\partial \chi_p^2} = -\frac{T}{m} (\lambda_1 \cos\chi_y \sin\chi_p + \lambda_2 \cos\chi_y \cos\chi_p)$$

$$\frac{\partial^2 H}{\partial \chi_p \partial \chi_y} = -\frac{T}{m} (\lambda_1 \sin\chi_y \cos\chi_p - \lambda_2 \sin\chi_y \sin\chi_p)$$

$$\frac{\partial^2 H}{\partial \chi_y^2} = -\frac{T}{m} (\lambda_1 \cos\chi_y \sin\chi_p + \lambda_2 \cos\chi_y \cos\chi_p + \lambda_3 \sin\chi_y)$$

where λ_1, λ_2, λ_3 are the coefficients of $\dot{w}\,\dot{u}\,\dot{v}$ respectively in the calculation of H.

If H_{aa} is ill conditioned, χ_p and χ_y satisfying $H_a = 0$ are used to calculate a backup H_{aa} having elements

$$\frac{\partial^2 H}{\partial \chi_p^2} = \mp \frac{T}{m} (\lambda_1^2 + \lambda_2^2) / \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$$

$$\frac{\partial^2 H}{\partial \chi_p \partial \chi_y} = 0$$

$$\frac{\partial^2 H}{\partial \chi_y^2} = \mp \frac{T}{m} \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$$

where the minus sign is used if maximizing and the plus sign is used if minimizing.

If KWTA = 2, H_{aa} is

$$H_{aa} = -\frac{T}{m} (\lambda_1 \sin\chi_p + \lambda_2 \cos\chi_p)$$

with backup

$$H_{aa} = \mp \frac{T}{m} \sqrt{\lambda_1^2 + \lambda_2^2}$$

If a backup H_{aa} matrix is used, the output quantity KAT will be 1; otherwise KAT = 0. KAT should never be 1 on a converged run.

On each iteration a normalized total influence coefficient for each parameter, $\underline{L}^i$ is formed as

$$\underline{L}^i = (L_\phi^i + L_\psi^i \nu) \Big/ \left(\max_{j=1,m} \left(\left|L_\phi^i\right|, \left|\nu_j L_{\psi_j}^i\right| \right) \right)$$

Prior to the fourth iteration the input values of W_i are used in the construction of W_b^{-1}. For the fourth and subsequent iterations each W_i is altered according to the following logic:

$$W_i \text{ unchanged if } \left| \underline{L}^i \right| < .005$$

$$W_i \text{ unchanged if } \left| \underline{L}^i \right|_{present} < \left(1 - \frac{E}{2}\right) \left| \underline{L}^i \right|_{last}$$

otherwise

$$W_i = 2\,W_i \text{ if } \left| \underline{L}^i_{present} - \underline{L}^i_{last} \right| < \left| \underline{L}^i \right|_{present}$$

$$W_i = W_i / 2 \text{ if } \left| \underline{L}^i_{present} - \underline{L}^i_{last} \right| \geq \left| \underline{L}^i \right|_{present}$$

The W_i are printed out as WIBT between iterations.

This dynamic updating of W_i will generally insure smooth convergence of the parameters. The relative magnitude of the W_i on a converged run can be used as a guide in picking input WIBT.

If there have been at least 3 iterations, if $|dm_o| < 100$ kg, if $|d\dot{\chi}| < .00002$ radians, if all $|d\tau_{1i}| < .5$ seconds and if in addition all $|\underline{L}^i| < .005$, the parameters are considered to be converged and the output quantity BETCON will be T; otherwise BETCON will be F.

The convergence test for the control variables χ_p and χ_y is

$$\begin{bmatrix} \left| d\chi_p \right|_{max} \\ \left| d\chi_y \right|_{max} \end{bmatrix} = \begin{matrix} \text{Max} \\ \text{overall} \\ \text{points in} \\ \text{chi-tables} \end{matrix} \left| H_{aa}^{-1} H_a \right| < .005$$

This implies that the max deviation of either χ_p or χ_y from the optimum anywhere along the trajectory is less than .005 radians. The max deviation in χ_p from the optimum is labeled DEL CHIP MAX in the output and the max deviation in χ_y is labeled DEL CHIY MAX.

As soon as $\left| d\chi_p \right|_{max} < .005$, $\left| d\chi_y \right|_{max} < .005$ and BETCON is T, a run is considered converged. A final forward trajectory is then run at the input print interval, integrated impact (if any) and output tables (if any) are run from this trajectory and then ROBOT looks for input for the next case.

REFERENCES

1.) Gottlieb, R.G., "Rapid Convergence to Optimum Solutions Using A Min-H Strategy," AIAA Journal, Vol 5, No. 2, pp 322-329 (February 1967)

2.) Battle, C. T., and Gottlieb, R.G., "Optimization of Multistage Three Dimensional Boost Trajectories, Pt. 1 - Program Formulation," Raytheon Co., Space and Information Systems Div., Rept. BR-2758-1 (1 May 1964)

3.) "Project Apollo Coordinate System Standards" NASA SE 008-001-1 (June 1965)

4.) Clarke, V.C., "Constants and Related Data for Use in Trajectory Calculations as Adopted by the ad hoc NASA Standard Constants Committee" JPL TR 32-064 (March 6, 1964)

5.) Martin, D. T., Sievers, R.F., O'Brian, R.M., Rice, A.F., "Saturn V Guidance, Navigation, and Targeting," AIAA J. of Spacecraft and Rockets, Vol 4, No. 7, pp 891-898, (July 1967)

6.) Fischer, I., "An Astrogeodetic World Datum from Geoidal Heights Based on the Flattening f = 1 / 298.3," J. of Geophysical Research, Vol 65, No. 7, (July 1960)

7.) Smith, O. E. and Weidner, D.K., "A Reference Atmosphere for Patrick AFB, Florida, Annual (1963 Revision," NASA TM X-53139, (September 23, 1964)

APPENDIX I

FIRST PARTIAL DERIVATIVES OF SPHERICAL - PLUMBLINE TRANSFORMATIONS

The matrix N is defined to be the matrix of first partial derivatives

$$N = \frac{\partial S}{\partial P}$$

where S here is the 6 x 1 vector of spherical state components

$$S = \begin{bmatrix} w_s \\ u_s \\ v_s \\ \phi \\ r \\ \theta \end{bmatrix}$$

and P is the 6 x 1 vector of plumbline components

$$P = \begin{bmatrix} w \\ u \\ v \\ x \\ y \\ z \end{bmatrix}$$

The matrix N may be partitioned into four 3 x 3 submatrices, i.e.,

$$N = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$$

These submatrices are defined by the equations:

$$N_{11} = \begin{bmatrix} \frac{\partial w_s}{\partial w} & \frac{\partial w_s}{\partial u} & \frac{\partial w_s}{\partial v} \\ \frac{\partial u_s}{\partial w} & \frac{\partial u_s}{\partial u} & \frac{\partial u_s}{\partial v} \\ \frac{\partial v_s}{\partial w} & \frac{\partial v_s}{\partial u} & \frac{\partial v_s}{\partial v} \end{bmatrix} = D^T$$

$$N_{12} = \begin{bmatrix} \frac{\partial w_s}{\partial x} & \frac{\partial w_s}{\partial y} & \frac{\partial w_s}{\partial z} \\ \frac{\partial u_s}{\partial x} & \frac{\partial u_s}{\partial y} & \frac{\partial u_s}{\partial z} \\ \frac{\partial v_s}{\partial x} & \frac{\partial v_s}{\partial y} & \frac{\partial v_s}{\partial z} \end{bmatrix}$$

where,

$$\frac{\partial w_s}{\partial x} = \left(a_{32} u - a_{22} v - w_s (d_{13} \cos\theta + d_{12} \sin\theta)\right) / (r \sin\theta)$$

$$\frac{\partial w_s}{\partial y} = \left(a_{12} v - a_{32} w - w_s (d_{23} \cos\theta + d_{22} \sin\theta)\right) / (r \sin\theta)$$

$$\frac{\partial w_s}{\partial z} = \left(a_{22} w - a_{12} u - w_s (d_{33} \cos\theta + d_{32} \sin\theta)\right) / (r \sin\theta)$$

$$\frac{\partial u_s}{\partial z} = (w - d_{12} u_s) / r$$

$$\frac{\partial u_s}{\partial x} = (u - d_{22} u_s) / r$$

$$\frac{\partial u_s}{\partial z} = (v - d_{32} u_s) / r$$

$$\frac{\partial v_s}{\partial x} = (d_{11} w_s \operatorname{ctn}\theta - d_{13} u_s) / r$$

$$\frac{\partial v_s}{\partial y} = (d_{21} w_s \operatorname{ctn}\theta - d_{23} u_s) / r$$

$$\frac{\partial v_s}{\partial z} = (d_{31} w_s \operatorname{ctn}\theta - d_{33} u_s) / r$$

$$N_{21} = \begin{bmatrix} \frac{\partial \phi}{\partial w} & \frac{\partial \phi}{\partial u} & \frac{\partial \phi}{\partial v} \\ \frac{\partial r}{\partial w} & \frac{\partial r}{\partial u} & \frac{\partial r}{\partial v} \\ \frac{\partial \theta}{\partial w} & \frac{\partial \theta}{\partial u} & \frac{\partial \theta}{\partial v} \end{bmatrix} = 0$$

$$N_{22} = \begin{bmatrix} \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \\ \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} & \frac{\partial r}{\partial z} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} & \frac{\partial \theta}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{d_{11}}{r \sin\theta} & \frac{d_{21}}{r \sin\theta} & \frac{d_{31}}{r \sin\theta} \\ d_{12} & d_{22} & d_{32} \\ \frac{d_{13}}{r} & \frac{d_{23}}{r} & \frac{d_{33}}{r} \end{bmatrix}$$

APPENDIX II

FIRST PARTIAL DERIVATIVES OF GRAVITATIONAL ACCELERATION WITH RESPECT TO PLUMBLINE POSITION COORDINATES

The matrix J is defined to be the matrix of first partial derivatives of the gravitational acceleration vector in the plumbline system with respect to the plumbline position coordinates. This matrix is used in the gravity related terms of the adjoint (Euler-Lagrange) equations.

$$J = \begin{bmatrix} \frac{\partial g_x}{\partial x} & \frac{\partial g_x}{\partial y} & \frac{\partial g_x}{\partial z} \\ \frac{\partial g_y}{\partial x} & \frac{\partial g_y}{\partial y} & \frac{\partial g_y}{\partial z} \\ \frac{\partial g_z}{\partial x} & \frac{\partial g_z}{\partial y} & \frac{\partial g_z}{\partial z} \end{bmatrix} = \begin{bmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{yx} & g_{yy} & g_{yz} \\ g_{zx} & g_{zy} & g_{zz} \end{bmatrix}$$

$$J = G_{11} I + \begin{bmatrix} x & a_{12} \\ y & a_{22} \\ z & a_{32} \end{bmatrix} \begin{bmatrix} G_{22} & G_{23} \\ G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} x & y & z \\ a_{12} & a_{22} & a_{32} \end{bmatrix}$$

G_{11} is defined in Section 3.1, I is a 3 x 3 identity matrix, the a_{ij} are elements of the A matrix, and

$$G_{22} = \frac{1}{r}\left(\frac{\partial G_{11}}{\partial r} + \frac{\text{ctn}\,\theta}{r}\frac{\partial G_{11}}{\partial \theta}\right)$$

$$= -\frac{3}{r^2}\left[G_{11} - \frac{\mu_e}{r^3}\left(CJ\left(\frac{R_e}{r}\right)^2\left(\frac{2}{3} - \frac{20}{3}\cos^2\theta\right) + H\left(\frac{R_e}{r}\right)^3 (4 - 14\cos^2\theta)\cos\theta\right.\right.$$

$$\left.\left. + DJ\left(\frac{R_e}{r}\right)^4\left(\frac{4}{7} - (9 - 18\cos^2\theta)\cos^2\theta\right)\right)\right]$$

$$G_{23} = G_{32} = -\frac{1}{r\sin\theta}\frac{\partial G_{11}}{\partial\theta} = -\frac{1}{r}\left(\frac{\partial G_{TO}}{\partial r} + \frac{\text{ctn}\theta}{r}\frac{\partial G_{TO}}{\partial\theta}\right)$$

$$= \frac{\mu_e}{r^4}\left[10\,CJ\left(\frac{R_e}{r}\right)^2\cos\theta - H\left(\frac{R_e}{r}\right)^3(3 - 21\cos^2\theta) + DJ\left(\frac{R_e}{r}\right)^4(12 - 36\cos^2\theta)\cos\theta\right]$$

$$G_{33} = \frac{1}{r\sin\theta}\frac{\partial G_{TO}}{\partial\theta}$$

$$= -\frac{\mu_e}{r^3}\left[2\,CJ\left(\frac{R_e}{r}\right)^2 + 6H\left(\frac{R_e}{r}\right)^3\cos\theta + \frac{12}{7}DJ\left(\frac{R_e}{r}\right)^4(1 - 7\cos^2\theta)\right]$$

The fact that J is symmetric can be anticipated, since J is also the matrix of second partial derivatives of the gravitational potential function $U(r, \theta)$ with respect to the plumbline position coordinates.

In the event that a spherical earth is being simulated J reduces to

$$J = G_{11}I + \begin{bmatrix} x \\ y \\ z \end{bmatrix} G_{22}\,[x \quad y \quad z]$$

with

$$G_{22} = -\frac{3}{r^2}G_{11}$$

since $G_{23} = G_{32} = G_{33} = 0$.

APPENDIX III

INPUT DESCRIPTION AND EXAMPLE PROBLEM

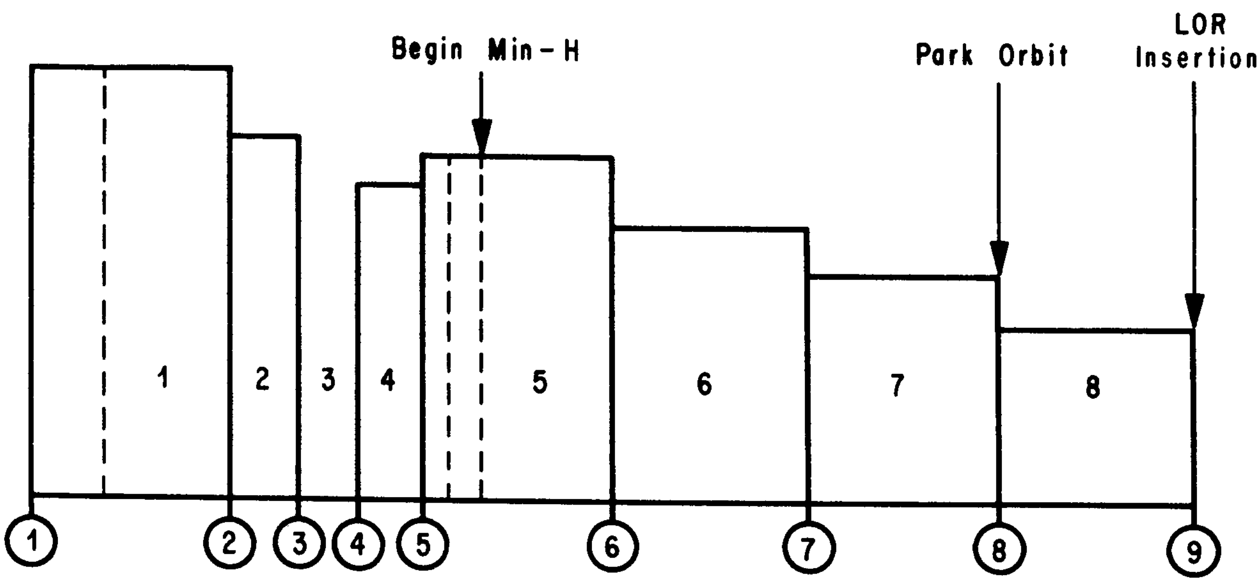

FIG. 9. SATURN V THRUST PROFILE

The user of the ROBOT program will find it helpful to sketch a thrust profile before setting up input for the problem he wishes simulated. Sketched above is an 8 thrust event representation of a three stage Saturn V thrust history. Vertical lines and horizontal lines will be referred to as "pickets" and "spaces", respectively. The "picket" numbers in Fig. 9 are circled. Note, there is always one more picket than spaces. A thrust event must be defined every time there is a discontinuity in the total thrust. Dashed vertical lines represent miscellaneous weight drops. Spaces are thrust duration times and are denoted TAUT. The elapsed time between the Jth miscellaneous weight drop event (dashed vertical lines) and some thrust event picket is denoted TAUW(J). The particular thrust event picket to use is denoted NØWD(J). ROBOT drops the atmosphere and begins optimizing χ_p and χ_y at the IWDCHIth miscellaneous weight drop event. Therefore, a miscellaneous weight must be dropped where Min-H is to begin even if it is a zero (0) weight drop.

The ROBOT program controls exo-atmospheric flight by looking up χ_p and χ_y as a function of time out of control tables. The Min-H steepest ascent process adjust these tabular points until they take on optimal values. A "control table" consists therefore of three tabular arrays: time, χ_p, χ_y. ROBOT contains four control tables, each containing a maximum of 49 points. In order to provide generality for the user the Jth control table begins at the NBGCT(J)th picket, ends at the NENDCT(J)th picket and has a maximum of NP(J) $\leq$ 49 points. NP should be odd for all tables in use and zero for all others. Control tables should not extend over coasts or over an intermediate constraint point. If Min-H is to begin in the middle of a thrust event, NBGCT(1) should be set to the picket at the beginning of the thrust event.

MAVRIK INPUT DESCRIPTION

INPUT SYMBOL	INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
HEAD		(15)	Identification for print out (60 characters)		
TZERØ	= TZERØ		Initial time		sec
TZERØ+1	= TDRAG		Time at which DRAG1 (launch tower induced drag) is dropped		sec
TZERØ+2	= TLIFT		End of lift-off; beginning of tilt		sec
TZERØ+3	= TTILT		End of Tilt		sec
TZERØ+4	= TCHFRZ		Begin chi freeze		sec
TZERØ+5	= DTZ		Time from GRR to lift-off		sec
F		(15)	Thrust per engine/thrust event	0.	lbs
TNE		(4, 15)	Number of engines/thrust event. Four numbers for each thrust event: the number of inboard engines, their cant angle (deg), the number of outboard engines, their cant angle (deg).	0.	
XMD		(15)	Flow rate per engine/thrust event	0.	lbs/sec
XMD+15	= CFR	(15)	Critical flow rate per engine/thrust event		lbs/sec
WD		(15)	Weight dropped during a weight drop event	0.	lbs
WD+15	= WJET	(15)	Jettison weight/thrust event	0.	lbs
AE		(15)	Engine exit area/thrust event	0.	m^2

INPUT SYMBOL	INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
S		(15)	Aerodynamic reference area/thrust event	0.	m^2
TAUT		(15)	Thrust event duration time/thrust event	0.	sec
TAUW		(15)	Elapsed time between a thrust event and a weight drop event	0.	sec
NØWD		(15)	Denotes picket number from which TAUW is defined	0	
NØEVNT		(5)	The total number of thrust events which comprise a stage	0	
PRINT		(15)	Print time increment/thrust event	10.	sec
STEP		(15)	Integration step-size increment for forward run/thrust event	8.	sec
BSTEP		(15)	Integration step-size increment for backward run/thrust event	16.	sec
AAETC			Due to a limitation of the UNIVAC 1107 MAVRIK call, the vector AAETC was selected to read in miscellaneous floating point data.		
AAETC	= AA		Launch azimuth	90.	deg
AAETC+1	= THZ		Initial geodetic latitude	28.531855	deg
AAETC+2	= DRAG1		Initial launch tower induced drag	12697.2	lbs
AAETC+3	= XJEXT		= 1. if maximizing payoff =-1. if minimizing payoff	1.	
AAETC+4	= CASE		Case number	1.	

INPUT SYMBOL		INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
AAETC+5	=	DP2		Decimal fraction of constraint error to remove on first iteration	.5	
AAETC+6	=	QY		Decimal fraction of H_a to remove per iteration	.8	
AAETC+7	=	CHIDØT		$\dot{\chi}$ for tilt-over during first stage pitch	.1	deg/sec
AAETC+8	=	WZERØ		Lift-off weight at TZERØ		lbs
AAETC+9	=	DELVG		Δ V for geometry reserves	0.	m/sec
AAETC+10	=	DELVP		Δ V for performance reserves	0.	m/sec
AAETC+11	=	WPMX		Maximum critical propellant in stage from which performance reserves are taken	0.	lbs
AAETC+12	=	TCHIR		Time of chi roll initiation (for report tables)		sec
AAETC+13	=	CHRDØT		Roll rate (for report tables)		deg/sec
AAETC+14	=	FAZ		Azimuth at which Fin 1 points (for report tables)		deg
AAETC+15	=	ALØNGØ		Longitude of the launch site (measured positive west)	80.5649528	deg
AAETC+16	=	AHIMAX		Max value of aerodynamic heating indicator	1.E20	$n \cdot m/m^2$
AAETC+17	=	TCY1		Time to initiate lift-off χ_y trapezoid	0.	sec
AAETC+18	=	TCY2		Lift-off χ_y begin plateau time	0.	sec

INPUT SYMBOL		INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
AAETC+19	=	TCY3		Lift-off χ_y end plateau time	0.	sec
AAETC+20	=	TCY4		End of lift-off of χ_y trapezoid	0.	sec
AAETC+21	=	CYTM		Lift-off χ_y trapezoid plateau value	0.	deg
AAETC+22	=	FSCYD		First stage $\dot{\chi}_y$	0.	deg/sec
AAETC+23	=	TFSCY		Time to initiate first stage $\dot{\chi}_y$	0.	sec
AAETC+29	=	EU		Upper error bound in forward integration	1. E-5	
AAETC+30	=	BEU		Upper error bound in backward integration	2. E-5	
AAETC+31	=	AYL		Used for error check in forward integration	2. E-3	
AAETC+32	=	BYL		Used for error check in backward integration	4. E-5	
AAETC+33	=	HMN		Minimum step-size for forward integration	.25	sec
AAETC+34	=	BHMN		Minimum step-size for backward integration	.50	sec
AAETC+42	=	CMUE		Gravitational constant	3. 986032E14	m^3/sec^2
AAETC+43	=	ØMEGA		Angular rotational velocity of earth	7. 2921158E-5	rad/sec
AAETC+44	=	CJ		First coefficient in gravitational expansion	1. 62345E-3	
AAETC+45	=	H		Second coefficient in gravitational expansion	-5. 75E-06	

INPUT SYMBOL		INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
AAETC+46	=	DJ		Third coefficient in gravitational expansion	7.875E-06	
AAETC+47	=	FLAT		Flattening of Fischer ellipsoid	1/298.3	
AAETC+48	=	RE		Equatorial radius	6378165.0	m
AAETC+49	=	GZERØ		Relates mass to weight	9.80665	m/sec^2
		JRBETC		Due to a limitation of the UNIVAC 1107 MAVRIK call the vector JRBETC was selected to read in miscellaneous fixed point numbers.		
JRBETC	=	JØRB		=1 if spherical earth =0 if oblate earth	0	
JRBETC+1	=	JUMP		Jump start at this picket number if JUMP>1	1	
JRBETC+2	=	IWDCHI		The number of the weight drop event where Min-H begins	1	
JRBETC+3	=	KIND		Type of integration used: =1 for variable step size Adams-Moulton =2 for Runge-Kutta =3 for fixed step Adams	3	
JRBETC+4	=	KWTA		=2 if χ_p only optimized =3 if χ_p and χ_y optimized	2	
JRBETC+5	=	NMAX		Total number of iterations	0	

INPUT SYMBOL		INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
JRBETC+6	=	NØTRAC		Thrust and flow rate are looked up in thrust tables for all thrust event numbers ≤ NOTRAC. This overrides whatever was input in F and XMD. The tables are data blocks in subroutine ATTRAC.	0	
JRBETC+7	=	NTABLE		=1 if output tables are wanted for publication	0	
JRBETC+8	=	NVRST		Intermediate constraints imposed at termination of this thrust event. Must be zero if no intermediate constraints wanted.	0	
JRBETC+9	=	IPR		Thrust event from which performance reserves are taken. (IPR must be zero if no performance reserves are wanted). If IPR≠0, WPMX and XMD+15 must be input.	0	
JRBETC+10	=	LAST		=0 if only one case is run; =1 if more cases are run	0	
JRBETC+11	=	KRDER		Order of differences in integration package for forward run.	3	
JRBETC+12	=	KINDB		Type of integration used in backward run (See JRBETC+3).	3	
JRBETC+13	=	KRDERB		Order of differences in integration package used for backward run.	3	
JRBETC+14	=	IMP		Jettison weight of this thrust event will be integrated to impact. (Can not be the last thrust event).	0	

INPUT SYMBOL	INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
JRBETC+15	= NAHI		=1 if aeroheat constrained =0 if aeroheat not constrained	0	
JRBETC+16	= NCYT		=1 if lift-off χ_y =0 if no lift-off χ_y	0	
JRBETC+17	= NFSCY		=1 if first stage $\dot{\chi}_y$ =0 if no first stage $\dot{\chi}_y$	0	
KCDPHI		(10)	Terminal function codes. (Code in KCDPHI(1) is payoff)		
PSIREQ		(10)	Constraint values desired at terminal point. (Value in PSIREQ(1) is constraint for code in KCDPHI(2), etc.)		
KCDRES		(6)	Intermediate constraint function codes		
PSIRST		(6)	Constraint values desired at restart point.		
	KDB		Control parameter switches		
KDB			INSERT 1 TO OPTIMIZE WZERØ	0	
KDB+1			" 1 " " CHIDØT	0	
KDB+2			" 1 " " TAUT (1)	0	
.			. . . . .	.	
.			. . . . .	.	
.			. . . . .	.	
KDB+16			INSERT 1 TO OPTIMIZE TAUT (15)	0	

INPUT SYMBOL	INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
KDT		(17)	Companion vector to KDB. Contains in corresponding locations the number of the thrust event from the present one which is to be altered in order to hold tank limit.	0	
WIBT		(17)	Used to speed up or slow down convergence of one parameter relative to another. 1 st element of WIBT goes with 1 st active parameter, 2 nd with 2 nd active parameter, etc.	1.	
NBGCT		(4)	J th control table begins at NBGCT(J) th picket		
NENDCT		(4)	J th control table ends at NENDCT(J) th picket		
NP		(4)	The number of points in a control table (Must be an odd number of points.)	0 pts	

CONTROL TABLES

INPUT SYMBOL	INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
TTBL TTBL+50 TTBL+100 TTBL+150		(49 pts.)	1 st time table (real time from TZERØ) 2 nd time table (real time from TZERØ) 3 rd time table (real time from TZERØ) 4 th time table (real time from TZERØ)		sec

INPUT SYMBOL	INTERNAL SYMBOL	SIZE	EXPLANATION	PRESET VALUE	UNITS
CPTBL		(49 pts.)	1 st χ_p table		rad
CPTBL+50		"	2 nd χ_p table		
CPTBL+100		"	3 rd χ_p table		
CPTBL+150		"	4 th χ_p table		
CYTBL		(49 pts.)	1 st χ_y table		rad
CYTBL+50		"	2 nd χ_y table		
CYTBL+100		"	3 rd χ_y table		
CYTBL+150		"	4 th χ_y table		
VIV		(8)	Vector of initial conditions for a jump start. If VIV(7)=0., input: $\dot{x}, \dot{y}, \dot{z}, x, y, z$ ($\dot{z}, \dot{x}, \dot{y}, z, x, y$ Apollo 13) If VIV(7)=2., input: V_I, γ, r, A_z, Lat, Node		

To avoid program failure due to input negligence the AAETC and JRBETC vectors have been preset in the main subroutine (with the exception of WZERØ). Any of the preset values may be changed by input. Note the discontinuity in the AAETC vector, i.e., between AAETC+23 and AAETC+29 no variables have been defined. Space was left available for the time more input variables are desired. The same is true of the JRBETC vector which does not fill its dimension.

For multiple-case runs, care must be taken in reinitializing any values which may have been changed during the computation of previous cases.

When not designated, the internal symbol is the same as the input symbol.

Aerodynamic Coefficients -- Input as follows on the 7094 only:

PØWER ØN (up to 44 points)

MACH NØ'S	CA TABLE	CN-PRIME TABLE
PNM =	CAN =	CNN =
PNM+1 =	CAN+1 =	CNN+1 =
. . .	. . .	. . .
PNM+43=	CAN+43=	CNN+43=

PØWER ØFF (up to 25 points)

MACH NØ'S	CA TABLE	CN-PRIME TABLE
PFM =	CAF =	CNF =
PFM+1 =	CAF+1 =	CNF+1 =
. . .	. . .	. . .
PFM+24=	CAF+24=	CNF+24=

On other machines this data must be included in a block data routine.

EXTRA MAVRIK CALL FOR OUTPUT TABLES

If JRBETC+7=1 (NTABLE) output tables will be printed and data for a second call of MAVRIK must be provided. Also, the user should be sure to set AAETC+12(TCHIR), AAETC+13(CHRDØT) and AAETC+14(FAZ). Data for the second MAVRIK call are:

TITLE	-	48 columns of BCD information
ØFFICE	-	12 columns of BCD information
DATE	-	12 columns of BCD information
NCASE	-	Fixed point case number; should be < 1000

Both KCDPHI and KCDRES can select any of the following:

1 = Payload (MASS) kg

2 = Inertial Velocity (VEL) m/sec

3 = Inertial Flight Path Angle (GAM) deg

4 = Radius (R) m

5 = Energy (C3) m^2/sec^2

6 = Angular momentum (C1) m^2/sec

7 = Inertial Longitude (LONG) deg

8 = Inertial Heading angle measured east from south (BETA) deg

9 = Colatitude (CO-LAT) deg

10 = Inclination (INCL) deg

11 = Line of nodes (NODES) deg

The alignment of the codes and constraints is

KCDPHI	=	payoff code	1 st constraint code	2 nd constraint code	etc.
PSIREQ	=		1 st constraint value	2 nd constraint value	

KCDRES	=	1 st constraint code	2 nd constraint code	etc.
PSIRST	=	1 st constraint value	2 nd constraint value	

EXAMPLE PROBLEM

Maximize payload into a given inclination LOR* conic and pass thru a 185.2 km circular parking orbit on the way up. Launch due east from Cape Kennedy over an oblate earth using the three stage Saturn V sketched in Fig. 9. Withhold performance reserves from the third stage and hold critical fuel limits on both the 2nd and 3rd stages. Controls to be optimized are: lift-off weight, tilt-over $\dot{\chi}$, mixture ratio shift time in 2nd stage, parking orbit insertion time, and χ_p and χ_y outside the atmosphere.

Data for this problem are given below:

Thrust Event	1	2	3	4	5	6	7	8
Thrust/engine (lb)	1.5E6**	1.5E6	0	2.E5	2.3E5	1.92E5	2.3E5	2.0E5
Flow rate/engine (lb/sec)	5754	5754	0	480	542	446	542	446
Critical flow rate/ engine (lb/sec)	-	-	-	240	271	223	271	223
Jettison weight (lb)	0	357000	0	0	0	100000	0	30000
Number of engines	5	4	0	5	5	5	1	1
Engine exit area (m^2)	9.93	9.93	0	3	3	-	-	-
Aerodynamic Ref. area (m^2)	79.4	79.4	79.4	0	0	-	-	-
Burn times*** (sec)	156	4	3.5	2.5	260	110	100	350
Integration step (sec)	2	4	8	8	8	8	8	8
Print interval (sec)	10	10	20	20	50	50	50	50

*Apollo Lunar Orbit Rendezvous earth-moon transfer ellipse
**Note: 1.5E6 means 1.5×10^6
***Starting values

Lift-off time = 0

Drop DRAG1 at 4 secs

Begin $\dot{\chi}$ tilt-over at 12 secs

End $\dot{\chi}$ tilt-over at 35 secs

Begin χ freeze at 150 secs

Gyro Release 17 secs prior to lift-off

Group thrust events as follows:

> 1 st stage - 3 thrust events; 2 nd stage - next 3 thrust events; and 3 rd stage - next 2 thrust events

Drop 1100 lbs 75 secs after lift-off

Drop 9500 lbs 25 secs after 2 nd stage ignition

Drop 8500 lbs 30 secs after 2 nd stage ignition, and begin Min-H after this weight drop.

Start 1 st control table at picket 5 and end it at picket 8

Start 2 nd control table at picket 8 and end it at picket 9

Have 41 points in 1 st table and 31 in 2 nd

Estimate χ_p at start of 2 nd stage to be about 1 rad and at parking orbit insertion to be about 2 rad. Estimate χ_p goes from 2. to 2.2 rad between parking orbit and LOR. Estimate χ_y zero all the time.

Estimate starting tilt-over $\dot{\chi}$ to be .135 deg/sec

Estimate starting lift-off weight 6340000 lbs

Withhold 20 m/sec ΔV_g and 10 m/sec ΔV_p from third stage (8 th thrust event). Max critical fuel in third stage 100000 lbs

Conditions at Intermediate Orbit (termination of 7 th thrust event)

> Vel = 7794. m/sec
>
> Gam = 0
>
> R = 6563365. m

Conditions at LOR Insertion:

$C_3 = -1.4986E6\ m^2/sec^2$

$INCL = 28^o$

Since mixture ratio shifts are notoriously sensitive choose WIBT = 2., .5, .2, .5

Also publish report tables

This problem converges in 7 iterations. All constraints are met to within a small tolerance and the max payload is 109253. lbs

THE MAVRIK AND DRAG DATA FOR THIS PROBLEM ARE GIVEN BELOW

```
HEAD=(ROBOT EXAMPLE PROBLEM(
TZERO=0.,4.,12.,35.,150.,17.,
F=1.5E6,1.5E6,0.,2.25,2.3E5,1.92E5,2.3E5,2.0E5,
XMD=5754.,5754.,0.,480.,542.,446.,542.,446.,
XMD+18=240.,271.,223.,271.,223.,
WD+16=357000.,
WD+20=100000.,
WD+22=30000.,
TME=5.,0.,0.,0.,4.,0.,0.,0.,0.,0.,0.,0.,5.,0.,0.,0.,5.,0.,0.,0.,
TME+20=5.,0.,0.,0.,1.,0.,0.,0.,1.,
AE=9.93,9.93,0.,3.,3.,
S=79.4,79.4,79.4,
TAUT=156.,4.,3.5,2.5,260.,110.,100.,350.,
STEP=2.,4.,8.,8.,8.,8.,8.,8.,
PRINT=10.,10.,20.,20.,50.,50.,50.,50.,
TAUW=75.,25.,30.,
NOWD=1,4,4,
WD=1100.,9500.,8500.,
NOEVNT=3,3,2,
NBGCT=5,8,
NENDCT=8,9,
NP=41,31,0,0,
TTBL=193.,610.,
CPTBL=1.,2.,
TTBL+50=610.,950.,
CPTBL+50=2.,2.2,
KCDPHI=1,5,10,
PSIREG=-1.4986E6,28.,
KCDRES=2,3,4,
PSIRST=7794.,0.,6563365.,
ORBETC+2=3,
ORBETC+4=3,20,
ORBETC+7=1,7,8,
AAETC+5=1.,
AAETC+7=.135,6340000.,20.,10.,100000.,10.,1.,120.,
KDT=0,0,0,0,0,0,1,
KDB=1,1,0,0,0,0,1,0,1,
WIBT=2.,.5,.2,.5,
/
TITLE=(ROBOT EXAMPLE PROBLEM(
OFFICE=(APPLIED ANAL(
DATE=(DEC 25,1967(
NCASE=1,
/
```

```
 DATA (PNM(I),I=1,24)/0.,.07,.15,.2,.25,.5,.6,.7,.82,.86,1.,1.05,
A1.1,1.5,2.,2.5,3.,3.5,4.,5.,6.,7.,10.,1000./
 DATA (CAN(I),I=1,24)/2.,.91,.8,.63,.57,.415,.388,.385,.4,.45,.73,
A.77,.77,.595,.445,.36,.315,.277,.227,.107,-.035,-.035,-.035,-.035,
B/
 DATA (CNN(I),I=1,24)/5.75,5.75,5.75,5.75,5.75,5.8,5.86,6.,6.2,6.3,
A6.21,6.18,6.15,5.95,5.18,4.98,5.06,5.10,4.99,4.65,4.25,3.85,
B3.3,3.3/
```

DISTRIBUTION

DIR
- Dr. von Braun
- Dr. Rees

R-DIR
- Mr. Weidner

DEP-T

R-AS
- Mr. Williams
- Mr. Page
- Mr. Davies
- Mr. Neighbors

R-P&VE
- Dr. Lucas

R-ASTR
- Dr. Haeussermann

R-RP
- Dr. Stuhlinger
- Dr. Shelton
- Mr. Wood

R-AERO
- Dr. Geissler
- Mr. Jean
- Mr. Horn
- Mr. Thomae
- Mr. Scott
- Mr. T. Deaton
- Mr. Cremin
- Mr. von Puttkamer
- Mr. Goldsby (10)
- Mr. Toelle (20)
- Mr. Young
- Mr. Ellison
- Mr. Jump
- Mr. Sumrall
- Mr. Wheeler
- Mr. Leonard
- Mr. Smart
- Mr. Perkinson
- Mr. Fulks
- Mrs. Chandler
- Mr. W. Deaton

R-AERO (Cont'd)
- Mr. Lindberg
- Mr. Baker
- Mr. Causey
- Mr. W. Vaughan
- Dr. H. Krause
- Dr. Teuber
- Mr. Walker
- Mr. Murphee
- Mr. Loftin
- Mr. Wittenstein
- Mr. Dickerson
- Mr. Gillis
- Mr. Perrine
- Mr. Mullins
- Mr. Little
- Mr. Telfer
- Mrs. Bauer
- Mr. Mabry
- Mr. Phillips

MS-IP
MS-IL (8)
CC-P
I-RM-M
MS-H
MS-T (6)

Tech. & Sci. Info. Facility (25)
Box 33
College Park, Md.
Attn: NASA Rep. (S-AK/RKT)

MSC
Houston, Texas 77058
Attn: Library (5)

Applied Analysis Inc.
Holiday Office Center
Huntsville, Ala. 35805
Attn: Dr. Nesline
Mr. Gottlieb (25)
Mr. Hall

Distribution (Continued)

Northrop Space Laboratories
Technology Drive
Huntsville, Ala. 35805
Attn: Dr. Hu
Mr. G. Smith
Mr. R. Keith
Mr. J. Morehead
Library (5)

Lockheed Missiles and Space Co.
Technology Drive
Huntsville, Ala.
Attn: Mr. W. B. Tucker
Mr. Peete
Library (5)

Mr. J. Winch (5)
Mail Stop AC-50
Boeing Co.
P. O. Box 1680
Huntsville, Ala. 35807

Mr. John E. Canady
NASA - Langley Research Center
Langley Sta., Va. 23365
Mail Stop 215

NASA Headquarters
Attn: Col. Burke, MTV (5)
Library (5)
RMP (5)

NASA - Ames Research Center
Mountain View, Calif. 94035
Attn: Library (5)

NASA - Langley Research Center
Hampton, Va. 23365
Attn: Library (5)

NASA - Electronics Research Center
Cambridge, Mass.
Attn: Mr. W. Miner
Library (5)

Jet Propulsion Laboratory
Pasadena, Calif.
Attn: Library (5)

J. J. Modorelli (5)
Tech. Info. Div.
NASA - Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135