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METHOD FOR CALCULATING ALLOWABLE CREEP STRESS IN LINEARLY INCREASING STRESS ENVIRONMENT

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ERRATA

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Page 5: Equation (10) should read

$$\left(1 - \frac{1}{mT}\right)^{-mT}$$

Page 6, figure 1: The values of the slope of the Larson-Miller correlation should be -1.9×10^{-4}, -1.8, -1.3, -1.2, and -1.1.

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SUMMARY

An analytical technique was developed to calculate the allowable design stress for a specified creep limit of a material under conditions of linear stress buildup and constant temperature. The analysis utilizes constant-stress creep data in the integration of a stress-dependent creep rate over the material history. Allowable design stress limits for 1-percent creep of refractory metals in the temperature range 800° to 1600° K are increased by 25 to 57 percent when compared with calculations based on the constant end-of-life stress condition.

INTRODUCTION

An accurate assessment of the allowable stresses that cause high-temperature creep in materials is important for space power systems currently under study. A great body of experimental creep data has been obtained for materials of interest under conditions of constant stress and temperature. The Larson-Miller (ref. 1) correlation, for example, which plots allowable constant stress against a combined time and temperature parameter for a fixed value of creep, is widely used for interpolation among the data. However, conditions of variable stress and/or temperature may be encountered in practice, and few creep data for such conditions exist. The situation that prompted the present work was the calculation of cladding thickness for a cylindrical nuclear-reactor fuel element designed to contain fission gases being generated at a constant rate. Internal pressure and, therefore, stresses are not constant (although temperature can be reasonably assumed to be), but increase linearly from some initial low value. Assuming that the end-of-life stress is constant over the entire reactor lifetime is unduly conservative, and the calculational results of cladding thicknesses and core volumes are greater than necessary.

This assumption may severely penalize high-temperature-reactor concepts intended for space power.

If the creep rate of a material under stress is assumed to be independent of prior creep sustained at other conditions, then total creep can be accumulated. This idea is not new (ref. 2) and has been experimentally verified for several materials (refs. 3 to 5). Recently, this technique has been used for the design of radioisotope capsules where helium, a decay product, builds up internal pressure (ref. 6).

The purpose of this analysis is to apply the concept of accumulation of total creep for variable-stress situations and to use as the basis for the analysis experimental data obtained under constant-stress conditions.

ANALYSIS

General Case

The basic assumption used in the following analysis is that the creep rate of a material under stress is independent of prior creep sustained at other conditions and that total creep, therefore, can be accumulated, according to the relation

$$\int_0^{\epsilon_f} d\epsilon = \int_0^{\tau_f} \dot{\epsilon}(\sigma, T)d\tau \tag{1}$$

where ϵ is the creep of a sample; $\dot{\epsilon}(\sigma, T)$ is the creep rate of a sample and is in general a function of the stress σ and the temperature T to which the sample is subjected; and ϵ_f and τ_f represent the final fractional elongation and final time, respectively.

If the creep rate is now assumed to be constant for the time period of an experiment at constant stress and temperature (second-stage creep only), then the creep rate $\dot{\epsilon}$ can be written as the ratio of some fractional elongation ϵ_0 to the time $t_0(\sigma, T)$ over which this elongation occurred. Thus

$$\dot{\epsilon}(\sigma, T) = \frac{\epsilon_0}{t_0(\sigma, T)} \tag{2}$$

To facilitate the translation of these data to other values of ϵ and t, the time $t_f(\sigma, T)$ required for the design creep ϵ_f to occur can be defined as

$$t_f(\sigma, T) \equiv \frac{\epsilon_f}{\dot{\epsilon}(\sigma, T)} \tag{3}$$

where the subscript f is used to differentiate between design conditions and the experimental test conditions in equation (2). Substitution of this relation into equation (1) and integrating the left side yield

$$\epsilon_f = \epsilon_f \int_0^{\tau_f} \frac{d\tau}{t_f(\sigma, T)}$$

$$1 = \int_0^{\tau_f} \frac{d\tau}{t_f(\sigma, T)} \tag{4}$$

To carry out the integration, $t_f(\sigma, T)$ is written as an explicit function of stress and temperature. A useful relation to accomplish this is the Larson-Miller (ref. 1) correlation. Under those conditions where the Larson-Miller correlation can be represented by a straight line on a semilog graph, it can be written as

$$\log \sigma = mP + \log a = mT(K + \log t_0) + \log a \tag{5}$$

where

m slope of equation

P Larson-Miller parameter

a ordinate σ intercept

T absolute temperature

K constant dependent on material

t_0 time of exposure

Rearranging equation (5) into an explicit relation of time t_0 yields

$$t_0 = \left(\frac{\sigma}{a}\right)^{1/mT} 10^{-K} \tag{6}$$

and combining equations (2), (3), (4), and (6) yields

$$\frac{\epsilon_f}{\epsilon_0} = 10^K \int_0^{\tau_f} \left(\frac{a}{\sigma}\right)^{1/mT} d\tau \tag{7}$$

Implicit in the derivation of equation (7) is the use of constant-stress - constant-temperature experimental data to describe the creep occurring under conditions of variable stress and temperature. However, this procedure is proper only if the basic assumption holds that creep rate is independent of prior creep. It should be noted also that, in the derivation of equation (7), no assumptions are required on the variation of stress and temperature over the operating lifetime of the sample. These variables often can be described as functions of the operating history of the sample; for example, $\sigma = \sigma(\tau)$ and $T = T(\tau)$, which reduce the integrand to a function of time τ. Depending on the circumstances, equation (7) can then be integrated, either numerically or in closed form, to determine the allowable end-of-life stress for a fixed core life or vice versa.

Linearly Varying Stress and Constant Temperature

A closed-form integration of equation (7) is obtainable for the relatively simple and interesting case for which the operating temperature remains constant over the sample history, while the stress varies linearly with time. This case is applied to the calculation of creep in the cladding of a pin-type nuclear-reactor fuel element operating at constant power and constant temperature. Since fission-product gases are generated at a nearly constant rate, the pressure and, hence, stresses within the fuel element can be considered to build up linearly with time. Under this condition, the rate of increase of stress is constant and can be set equal to the stress at the end of the operating life divided by the time period over which this rate occurred. Thus,

$$\frac{d\sigma}{d\tau} = \frac{\sigma_f}{\tau_f} \tag{8}$$

where the implicit assumption has been made that the stress is initially zero.

Equation (8) can now be used to effect a change of variable of integration in equation (7) from operating time τ to stress σ. Equation (7) then becomes

$$\sigma_f = \frac{\epsilon_0}{\epsilon_f} 10^K \tau_f a^{1/mT} \int_0^{\sigma_f} \sigma^{-1/mT} d\sigma$$

Integration yields

$$\sigma_f = \frac{\epsilon_0}{\epsilon_f} a\left(10^K \tau_f\right)^{mT} \left(\frac{1}{1 - \frac{1}{mT}}\right)^{mT} \tag{9}$$

At this point, examination of equation (6) reveals that the quantity $a\left(10^K \tau_f\right)^{mT}$ is equal to the stress value predicted by the Larson-Miller correlation for the condition $\epsilon_0 = \epsilon_f$. This point is denoted as σ_{LM} and a stress modifying factor ψ is defined as

$$\left(1 - \frac{1}{mT}\right)^{-1/mT} \tag{10}$$

Thus, equation (9) becomes

$$\sigma_f = \sigma_{LM}\psi \tag{11}$$

and states that, when the stress is linearly increasing and the temperature is constant over the operating history, the allowable end-of-life stress σ_f to achieve a desired creep value is equal to the product of the stress σ_{LM} (obtained from the Larson-Miller correlation for an equivalent value of creep) and a factor ψ that depends on the material and the operating temperature.

The ψ factor was evaluated for several high-temperature refractory alloys of interest and is shown plotted as a function of temperature in figure 1. Data for these materials were taken from references 7 and 8 and were assumed to have a constant Larson-Miller slope over the temperature range plotted. For the temperature range 800° to 1600° K, the least sensitivity is exhibited by NAS-36 with ψ values of 1.25 to 1.4, respectively, and the greatest sensitivity is exhibited by FS-85 with ψ values of 1.37 to 1.57, respectively. From figure 1 it can be seen that ψ increases with operating temperature and is nearly linear over a broad range.

To illustrate the use of the ψ factor, imagine a hypothetical application requiring 10 000 hours' service at a constant temperature of 2200° F (1480° K) for which the stress on the sample is linearly increasing. If 1-percent creep and the material T-111 are specified, for example, use of the Larson-Miller data given in reference 7 indicates a design stress limit σ_{LM} of about 5000 psi (0.18 MN/m^2). From figure 1, ψ for T-111 at 2200° F (1480° K) is about 1.42. Hence, a stress σ_f of about 7100 psi (0.26 MN/m^2) at the end of life could be allowed.

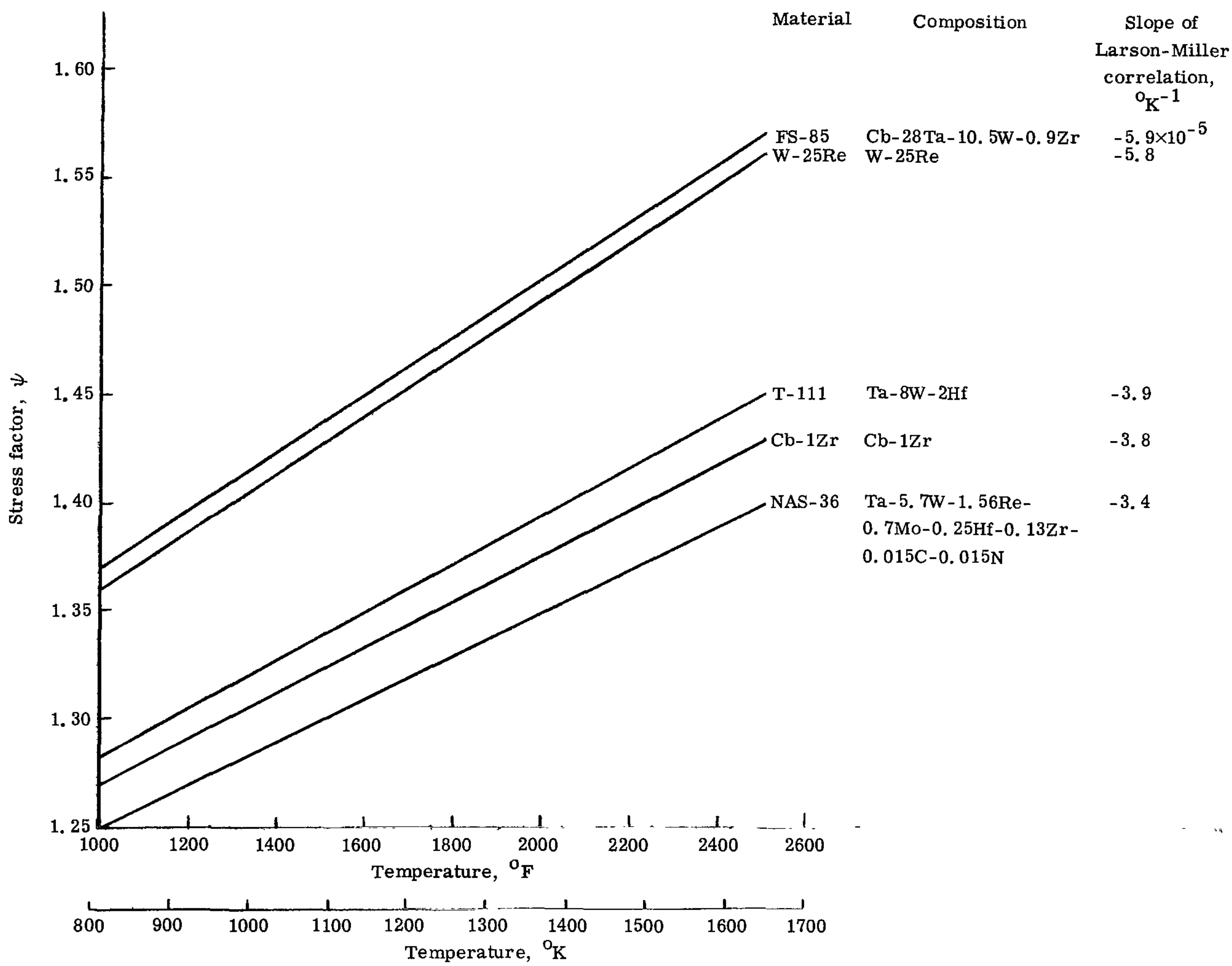

Figure 1. - Calculational factor to increase allowable design stress for 1-percent creep in several refractory-metal alloys.

Application to Reactor-Fuel-Clad Design

In a study of a small, fast, gas-cooled, high-temperature reactor employing pin-type fuel elements, the cladding thickness was calculated for the constant stress σ_{LM} as well as for the linearly varying stress. The allowable design stress and the pressure on the fuel cladding were related by means of the tangential stress equation for thick wall cylinders (ref. 9) evaluated at the inside surface of the tube wall

$$\sigma = \frac{\left(r_o^2 + r_i^2\right)P_i - 2r_o^2 P_o}{\left(r_o^2 - r_i^2\right)} \tag{12}$$

where r is the tube radius, P is the pressure, and the subscripts i and o refer to inside and outside conditions, respectively. Equation (11) was used to evaluate the allowable stress from which the cladding thickness was determined by equation (12). The cladding material was tungsten - 25 percent rhenium, and the operating conditions were 25 000 hours at 1800° K with 1 percent creep. The factor ψ was 1.6 under these conditions, with a 60-percent increase allowed in design stress over the constant-stress assumption. A saving in cladding thickness was effected, which allowed an increased fuel loading that resulted in a core volume reduction of 18 percent with the use of the technique reported herein as compared with that of the constant-stress assumption.

The foregoing analysis utilized the following approximation to actual reactor conditions. Because the initial fission-gas pressure was zero and the gaseous coolant exerts a constant pressure on the outside of the element, the cladding is initially in compression. As the fission gases build up throughout the reactor life, this compression decreases and changes to tension when the fission-gas pressure exceeds the coolant pressure. At the end of life, the net pressure on the cladding equals the fission-gas pressure minus the coolant pressure. These pressure histories are presented in figure 2. The upper dashed line shows the fission-gas pressure alone, and the lower dashed line shows the net pressure (fission gas minus coolant) on the cladding. The solid line starting at zero

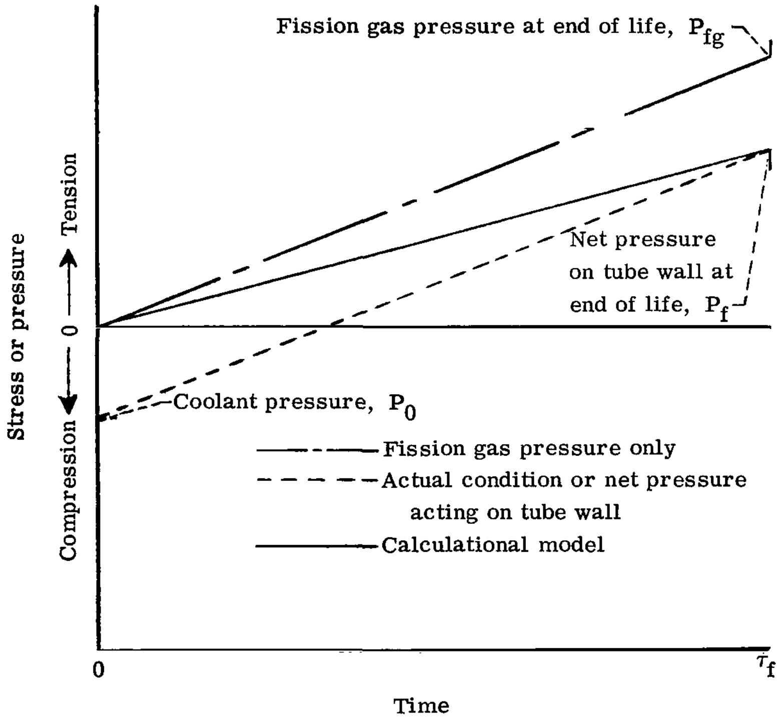

Figure 2. - Pressure history of a nuclear reactor fuel element with respect to containment of fission gases.

and ending at the end-of-life net pressure P_f was the assumed pressure history used for the stress calculations. This assumption is considered conservative, since the assumed tensile stress was always equal to or greater than the actual.

DISCUSSION

The analytical method presented herein appears mathematically suitable for any total creep limit within the constant-creep-rate assumption and is not restricted to the creep limit of the experimental test data. To assure such application, however, its use should be limited to low creep limits. Also, uncertainties exist in the use of the Larson-Miller creep rate correlation where longtime test data for materials of interest are usually nonexistent, and thus extended extrapolations are required. Fortunately, the stress-increase factor ψ plotted in figure 1 is relatively insensitive to changes in both the value of the slope of the Larson-Miller correlation and the temperature. For example, a 100-percent increase in either m or T causes the factor to increase about 15 percent.

In theory, any stress-temperature-creep correlation and/or any stress history could be used if a closed analytical solution were not required. Thus, equation (4) could be solved numerically by use of a digital computer if sufficient test data and/or problem history data were available.

Method application to nuclear-reactor design is restricted to those situations in which the fuel cladding thickness is determined by creep as opposed to those situations in which limitations are related to material elastic properties or strength. In general, this application will include high-power-density - high-temperature systems that are required to retain fission gases in the core. Of course, fission gas pressure is only one of the factors upon which cladding thickness depends, and in a detailed reactor design, additional items, such as thermal cycling, heat-transfer effects, and fabrication, must be considered.

SUMMARY OF RESULTS

An analytical method was developed to calculate allowable end-of-life stress for a design creep limit of a material under conditions of linearly increasing stress and constant temperature. For a given material and creep limit, a temperature dependent factor was calculated which represents the ratio of the allowable end-of-life stress to the constant stress obtained from Larson-Miller data.

Calculations were performed to show that the allowable end-of-life stress for refractory metals, for the conditions of 1-percent creep, and linearly increasing stress, and a temperature range of 800° to 1600° K, was increased by 25 to 57 percent when compared with constant-stress calculations.

Also, the core volume of a small, fast-spectrum, high-temperature, gas-cooled reactor was reduced by 18 percent when the method presented herein was used for the determination of the required fuel cladding thickness.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 7, 1967,
120-27-06-05-22.

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