"Maximum-Likelihood Receiver for Digital Data Transmission"

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ABSTRACT

(This paper is intended for the Communication Technology Group, Session 8: Advances in Data Communications, Telegraph Systems, and Facsimile.)

A novel receiver structure for binary data with limited intersymbol interference uses non-linear elements and combines features of the optimum linear and "tail cancellation" receivers. The receiver is maximum-likelihood so it is optimum in that it minimizes per-bit probability of error, P_e. Bounds on P_e and certain extensions are presented

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SYNOPSIS

High speed data communication via pulse amplitude modulation (PAM) is reliable only if one can simultaneously minimize the effects of intersymbol-interference (ISI) and random noise. This problem has been examined recently by Tufts[1], who describes the joint optimization of transmitter and receiver, and by Aaron and Tufts.[2] Their receiver, constrained to be linear, has the form of a matched filter followed by a tapped delay line (transversal equalizer). Tufts[1] also discusses, briefly, the historical and some of the more recent methods of controlling ISI and noise. These methods include linear equalization, which has been given renewed interest by Lucky[3], and "tail cancellation".

In this paper we describe the maximum-likelihood (ML) receiver for data transmission via PAM and show that it employs elements of the optimum linear receiver and the "tail cancellation" receiver. The ML receiver is known to be optimum in the sense that it minimizes the per-bit probability of error P_e. Thus its performance can be used as a basis of comparison for other receivers. The structure is quite simple so it merits consideration in an actual binary transmission system with the limited ISI of assumption 2 below.

We make the following assumptions:

1. The received signal is

$$x(t) = \sum_{k=-\infty}^{\infty} \mu_k \, s(t-kT), \qquad (1)$$

   where μ_k is 1 or -1 and represents the k^{th} information symbol. The μ_k's are independent and $s(t)$ is known.

2. The signal $s(t)$ is smeared only into one adjacent baud giving rise to limited intersymbol-interference (ISI).

3. The noise is stationary, white, Gaussian and additive. Colored noise can be handled by "pre-whitening", but the output of the whitening filter, $s(t)$, is subject to 2 above.

4. The transmitter and receiver are in synchronism.

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In references [4] and [5] we show that the ML receiver first computes the correlation statistic

$$A_k = \frac{4}{N_o} \int_{kT}^{(k+2)T} y(t)\, s(t-kT)\, dt, \tag{2}$$

where y(t) is made up of x(t) as given by (1) plus random noise of double-sided power spectral density $N_o/2$ watts/cps. The receiver then bases its decision about the polarity of μ_k on the statistic

$$\begin{aligned} \Lambda_k = A_k &+ Z\{A_{k-1} + Z\{A_{k-2} + \cdots\}\} \\ &+ Z\{A_{k+1} + Z\{A_{k+2} + \cdots\}\}, \end{aligned} \tag{3}$$

where

$$Z\{x\} \equiv \log_e \frac{e^x + e^R}{1 + e^{x+R}} \tag{4}$$

and

$$R \equiv \frac{4}{N_o} \int_0^{2T} s(t)\, s(t+T)\, dt. \tag{5}$$

R is a measure of the ISI. If, for example, the μ_k's are equally likely to be 1 or -1 and the costs associated with each type of error are equal, then the receiver decides

$$\mu_k = \begin{cases} 1 & \text{if } \Lambda_k \geq 0 \\ -1 & \text{if } \Lambda_k < 0 . \end{cases} \tag{6}$$

Equations (2) and (3) define the ML receiver and the corresponding structure is shown in Fig. 1. Note the similarity between this structure and the optimum linear receiver[1,2], that is, a matched filter followed by a tapped delay line.

In this structure, however, the useful output is not merely a weighted sum of the tap outputs. Here each output is added to its neighbor after the neighbor is passed through a non-linear amplifier (the box labelled Z and defined by Equation (4)). The M taps to the right of the center tap, the useful output, indicate that M bauds of the past data have been optimally processed to aid in the decision on μ_k. The N taps to the left indicate that N future bauds have also been considered. In theory one should let $M \to \infty$ and $N \to \infty$ to achieve the optimum detector structure. We can, in fact, achieve $M \to \infty$, that is, we can consider all past data, by a modification of Fig. 1. This modification, employing a feedback loop, is shown within the dotted lines of Fig. 1 and replaces the M taps to the right. Obviously, a consideration of all future data requires infinite delay (for an infinite binary sequence) so a compromise, finite N must be chosen.

In Fig. 2 we show the Z-box amplifier input-output characteristics for several values of R. Note that these curves saturate at $\pm R$.

To provide insight into the detector operation let us assume that the detector of Fig. 1 uses only the feedback structure. Hence the decision statistic becomes

$$\Lambda_k \equiv A_k + Z\{A_{k-1} + Z\{A_{k-2} + \cdots\}\}. \tag{7}$$

We see that the k^{th} decision statistic contains, first of all, the correlation of $s(t-kT)$ with the received signal, namely A_k. The other additive term is bounded by $\pm R$. In fact, assuming high SNR, A_{k-1} will be either large positive for $\mu_{k-1} = 1$ or large negative for $\mu_{k-1} = -1$. Thus, referring to Fig. 2, we will subtract R from A_k if μ_{k-1} is positive and we will add R to A_k if μ_{k-1} is negative. This addition or subtraction of R is mathematically equivalent to subtracting out the channel memory and is, therefore, a tail cancellation scheme. The novelty here is that the tail cancellation occurs on a probabilistic basis. This is, we no not simply allow R or -R but we choose an intermediate value based on $A_{k-1} + Z\{A_{k-2} + \cdots\}$, which is a measure of our certainty concerning μ_{k-1}.

The non-linearity of the ML receiver discourages an exact analytical determination of P_e. We have found reasonably tight upper and lower bounds, however. The lower bound is easily established by considering the favorable $R = 0$ case with the resulting

$$P_e(\text{lower}) = \text{Erfc}\,\sqrt{\rho}\,, \tag{8}$$

where ρ is the signal-to-noise ratio

$$\rho \equiv \frac{2}{N_o} \int_0^{2T} s^2(t)\, dt \tag{9}$$

and

$$\text{Erfc } x \equiv \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt. \tag{10}$$

The upper bound is found by considering two tail cancellation receivers, one operating in positive time and the other in negative time. This constitutes a sub-optimum detector and gives

$$P_e(\text{upper}) = Q^2 \text{ Erfc } \sqrt{\rho} + QP \left[\text{Erfc } \sqrt{\rho}\ (1+2r) + \text{Erfc } \sqrt{\rho}\ (1-2r) \right]$$

$$+ P^2 \left[\frac{1}{2} \text{ Erfc } \sqrt{\rho} + \frac{1}{4} \left(\text{Erfc} \sqrt{\rho}\ (1+4r) + \text{Erfc } \sqrt{\rho}\ (1-4r) \right) \right], \tag{11}$$

where

$$P \equiv \frac{\text{Erfc} \sqrt{\frac{\rho}{2}}}{1 + \text{Erfc} \sqrt{\frac{\rho}{2}} - \frac{1}{2} \text{ Erfc} \sqrt{\frac{\rho}{2}}\ (1-4r) - \frac{1}{2} \text{ Erfc} \sqrt{\frac{\rho}{2}}\ (1+4r)}, \tag{12}$$

$$Q \equiv 1-P, \tag{13}$$

and

$$r \equiv \frac{\int_0^{2T} s(t)\ s(t+T)\ dt}{\int_0^{2T} s^2(t)\ dt} . \tag{14}$$

To simplify presentation of these results, we have assumed that the energy in $s(t)$ is equally divided between the intervals 0 to T and T to 2T.

In Fig. 3 we show the upper and lower bounds on the ML receiver versus ρ for $r = .25$. We also show the tail cancellation receiver and the optimum linear receiver (for $r = .2$, as appears in [2]). Note that this optimum linear receiver passes in and out of the ML receiver's upper bound so that it could have been used as a tighter upper bound over some ranges of ρ.

The receiver's mathematical specification for multi-level data is straightforward[4], however, an efficient implementation and calculation of P_e appears to be difficult. The major limitation of this approach, the restriction to limited ISI may be removed by a heuristic argument suggesting a receiver structure similar to that of Fig. 1.

References

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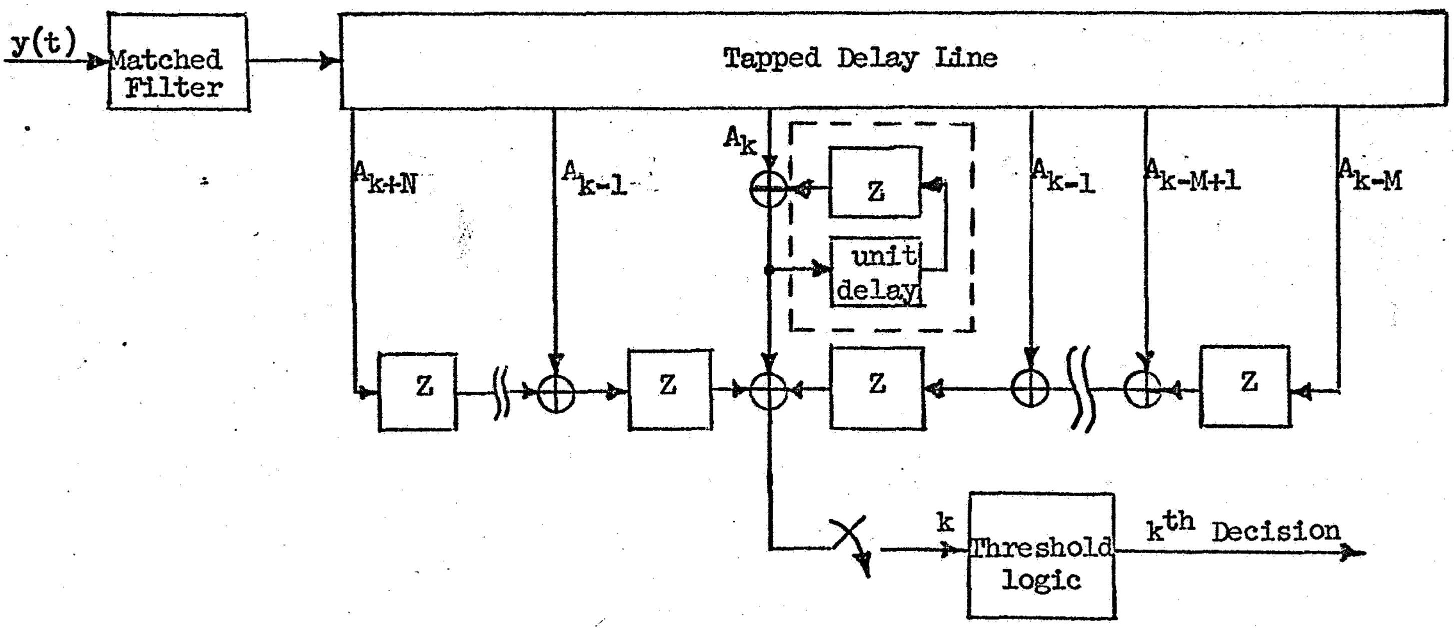

Fig. 1 ML Receiver for Limited ISI

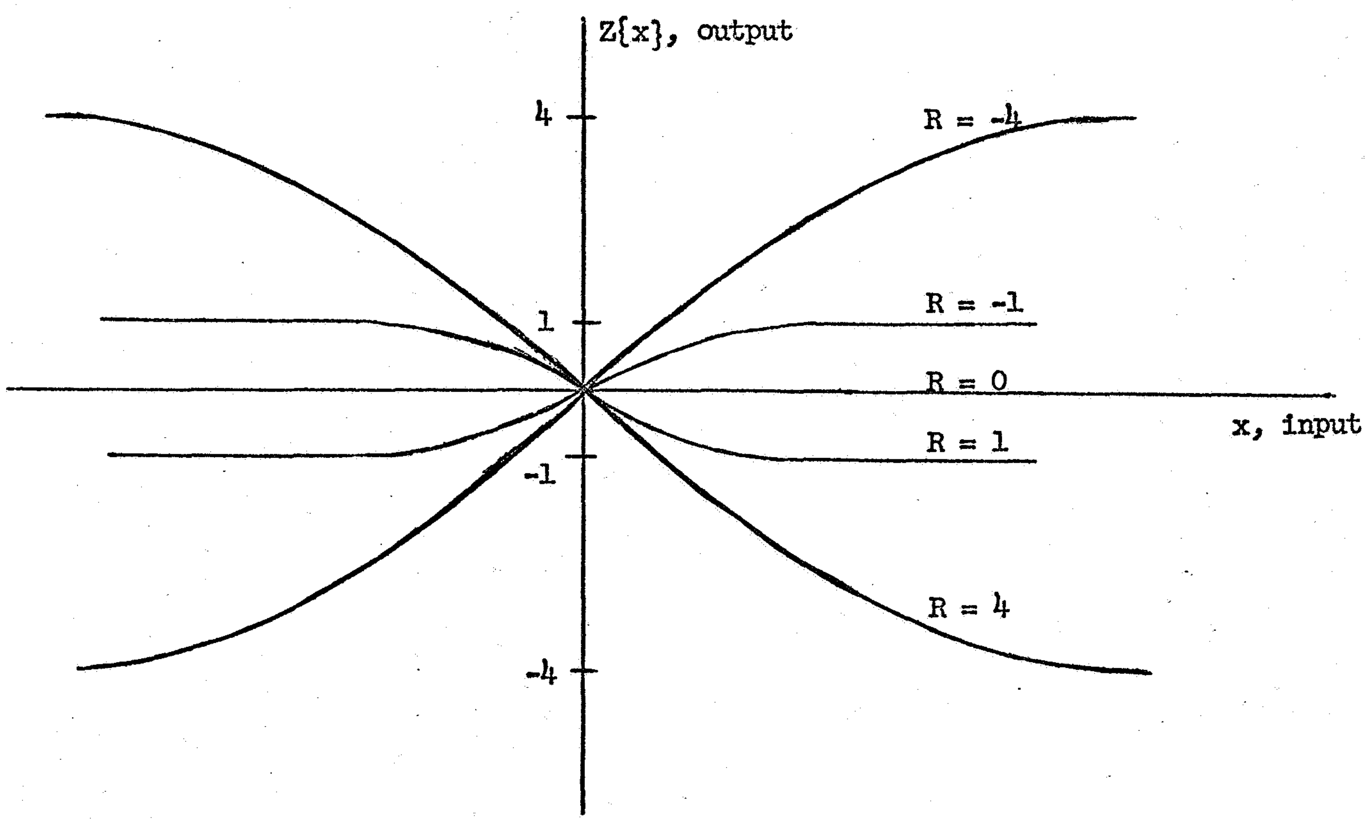

Fig. 2 The Non-Linear Amplifier, Z{x}

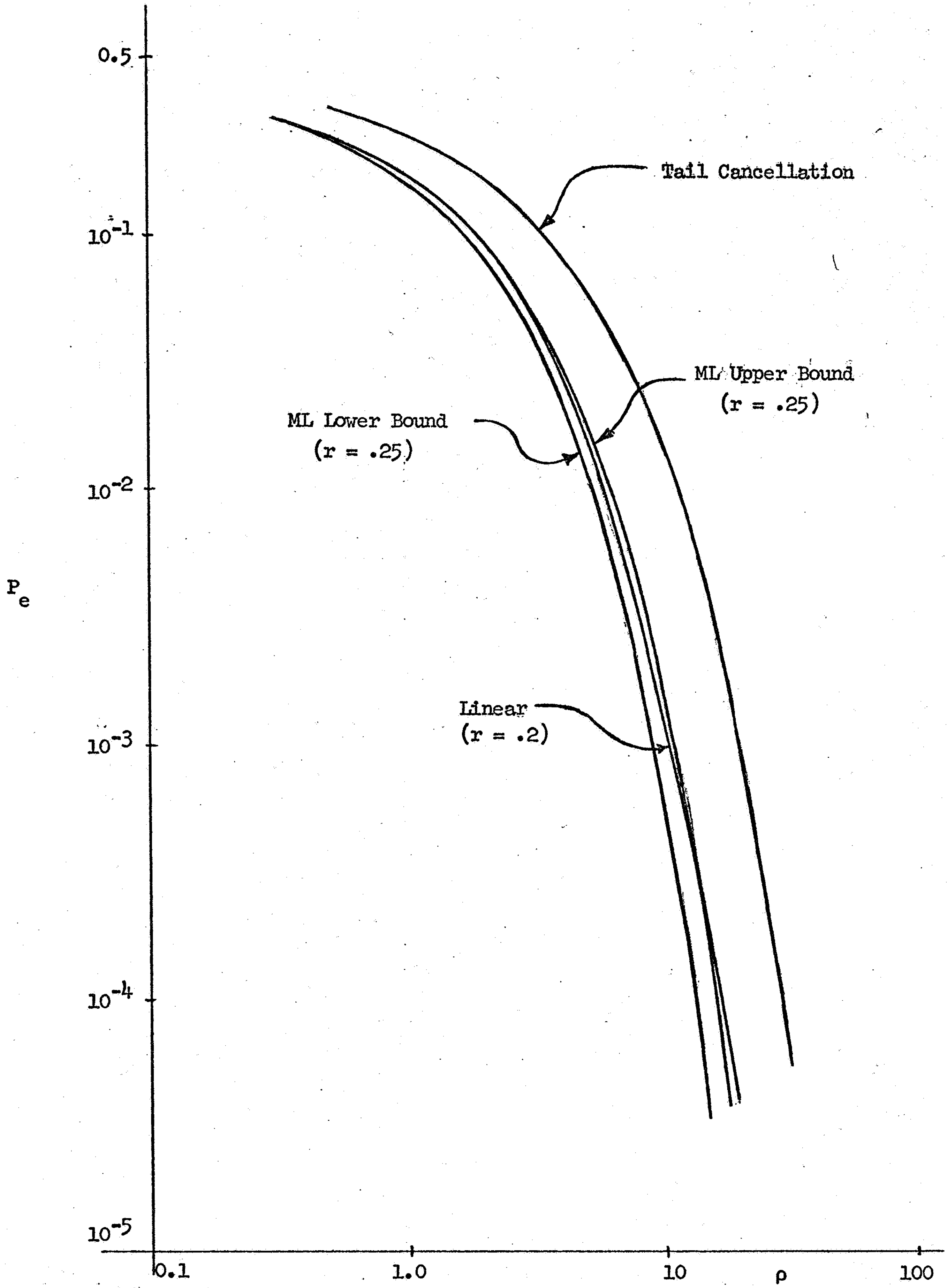

Fig. 3 Comparison of the ML, Tail Cancellation and Linear Receivers