PERFORMANCE OF SELF BIT SYNCHRONIZATION SYSTEMS *

by

E. J. Luecke and P. A. Wintz

ABSTRACT

The optimum (maximum likelihood) estimator for the epoch of a random sequence of anticorrelated signals of known duration in additive gaussian noise was derived and its performance evaluated by Monte Carlo simulation on a digital computer. The mean magnitude of the epoch error vs. signal-to-noise ratio was obtained for three symbol waveforms.

The optimum system is rather complex and, therefore, judged to be impractical. However, the equation defining the optimum system structure can be loosely interpreted as a filtering operation followed by a nonlinearity followed by an averaging operation. A simple (suboptimum) system that performs these operations was considered and its performance computed via the transform method. The suboptimum system performed nearly as well as the optimum system for rounded pulses, but significantly worse for square pulses. A prototype of the suboptimum system was constructed; it performed as predicted.

A second suboptimum system incorporating a hard limiter was constructed and its performance compared to the performance of the first suboptimum system. The first system with rounded pulses performed 10 db • better than the second system with square pulses.

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I. INTRODUCTION

The detection of digital signals requires a time base at the receiver which is in synchronism with the received sequence of digital waveforms. This time base, usually called bit synchronization, may be obtained by transmitting a synchronizing signal on a separate channel, by adding symbol patterns to the data format in order to synchronize local stable clocks, or by properly processing the received data in such a way that the information bearing symbol sequence supplies the synchronization signals. The latter method is called self synchronization.

The current interest in optimum digital detection systems is based on the desire to obtain the most efficient use of the available communication channels. If such optimum overall system results are to be obtained, it is necessary for the synchronizing subsystem to add little or nothing to the total system power and bandwidth requirements. It is clear that self synchronization adds nothing to the total transmitted power requirement. It may, however, increase the required bandwidth if the basic digital symbol waveshapes are chosen such that self synchronization is enhanced at the expense of bandwidth. An extreme example of this occurs with binary anti-correlated signaling using pseudo-random sequences as the basic symbol waveform; the correlation properties of these P-N sequences are excellent for synchronization purposes, but the bandwidth is directly proportional to the length of the sequence. The results presented in this paper are restricted to symbol waveshapes which are chosen to keep the system bandwidth as narrow as possible.

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The synchronization problem is presented graphically in Fig. 1. A random sequence of positive and negative symbols is observed; the problem is to determine the start and stop times of each pulse. If the bit rate T^{-1} is known this is equivalent to learning the epoch ε.

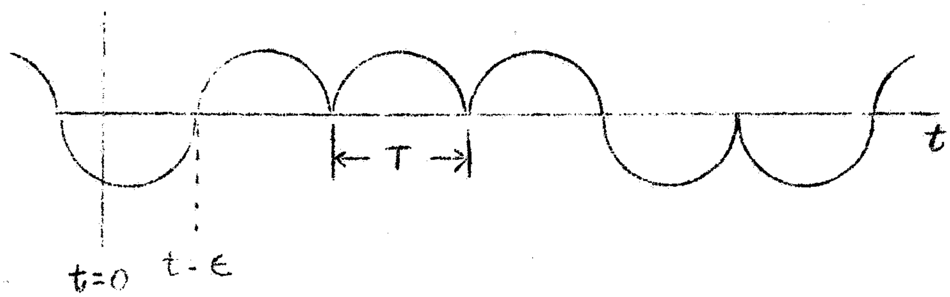

Fig. 1

The process of deriving a synchronizing signal from a digital symbol sequence can also be interpreted as that of detecting the phase of the discrete frequency component of the random signal process at the bit (symbol) frequency. The ease with which this phase can be measured depends on the power in the frequency component corresponding to the bit rate. According to a result by Barnard [1], the discrete components of an m-ary first order Markov digital symbol sequence are given by

$$(1)\quad S_d = \left| \sum_{i=1}^{m} p_i \, G_i \, (j\, 2\pi\, f) \right|^2 \sum_{n=-\infty}^{+\infty} \delta(f - n\, f_B)$$

where S_d is the discrete spectral density, P_i is the probability of occurrence of the i-th symbol, $G_i(s)$ is the Laplace transform of the i-th symbol waveform, f_b is the bit frequency, and $\delta(x)$ is the Dirac delta function.

Eq. (1) indicates that if the spectrum of the individual pulses have a non-zero value at the bit frequency, then the random process will have the desired discrete frequency component in its spectral density. As a rule of thumb, good signal symbol design for self synchronization is obtained when the magnitude of the spectral density of the symbol pulses is large.

For binary anticorrelated symbols Eq. (1) reduces to

$$(2) \quad S_d = |\,(2p-1)\; G\,(j\;2\pi f)\,|^2 \sum_{n=-\infty}^{+\infty} \delta(f - n\,f_B)$$

where p is the probability of a positive pulse. For a well coded system, p will be approximately 1/2. For $p = 1/2$, $S_d = 0$ and there are no discrete components in the spectral density for any choice of symbols. In order to derive a synchronizing signal in this case nonlinear processing is required. Nonlinear processing generates a new random process which has the desired discrete spectral density components. The purpose of this paper is to determine the optimum (nonlinear) synchronizer for binary anticorrelated signals.

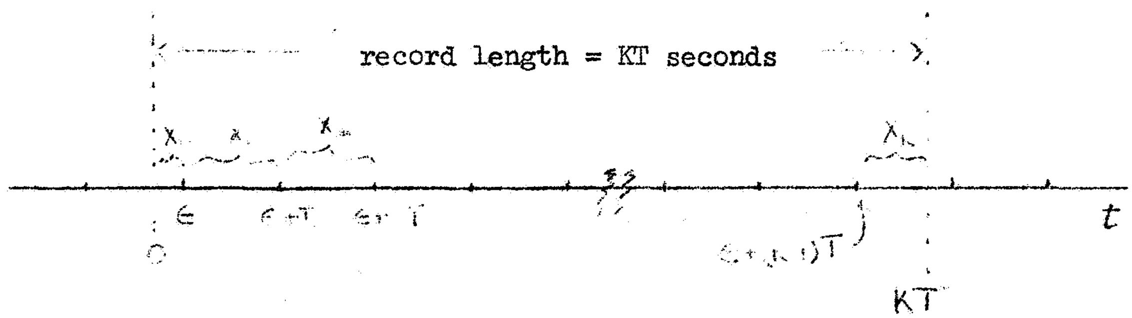

Fig. 2

II. THE OPTIMUM SOLUTION

In this section we compute the optimum (maximum likelihood) self synchronizer for a random sequence of anticorrelated signals in additive gaussian noise. The problem is illustrated by Fig. 1. The assumptions are outlined in Fig. 2. Assume a record of received data precisely KT seconds long where T is the known symbol duration (T^{-1} is the bit rate) and K is an integer. Even though the record is exactly K symbol durations in length it contains parts of K + 1 different symbols since the last ϵ seconds of one symbol is included on the start of the record, and the first $T - \epsilon$ seconds of another symbol at the end of the record. Let the received data (signal plus noise) in the K + 1 time intervals defined by Fig. 2 be represented by the K + 1 column matrices X_i, i = 0, 1, 2, • • •, K+. Hence‡,

$$(3)\quad X_i = \pm S_i + N_i \qquad i = 0, 1, 2, \cdots, K$$

where $\pm S_i$ is a column matrix representing $\pm$ the symbol waveform except that S_o represents the last ϵ seconds and S_K the first $T - \epsilon$ seconds of the symbol waveform; N_j is a column matrix representing the noise waveform in the jth time interval. We also assume that the K + 1 noise covariance matrices in the jth

$$(4)\quad \Phi_i = \overline{N_i\, N_i^T} \qquad i = 0, 1, 2, \cdots, K$$

and that the noises have zero means and are uncorrelated for different time intervals. Finally, we assume that in each time interval the a priori probability of occurrence of a positive symbol and a priori probability of a negative symbol are 1/2.

\+ The representation of waveforms by vectors is discussed in Appendix B of Hancock and Wintz [2]

‡ For notational convenience we assume that all of the column matrices are k X 1. The same result can be obtained if the X_i, S_i, and N_i are assumed to be k_i X 1. (A different number of samples can be used to represent the waveforms in the different time intervals).

According to Baye's rule we can write

$$(5)\quad p(\varepsilon \mid X_o, \cdots, X_K) = \frac{p(\varepsilon)}{p(X_o, \cdots, X_K)}\; p(X_o, \cdots, X_K \mid \varepsilon)$$

Because the signal and noise processes, as well as the individual noise vectors N_i $i = 0, 1, \cdots, K$, are statistically independent, the observations X_i, $i = 0, 1, \cdots, K$ are conditionally (given ε) independent, i.e.,

$$(6)\quad p(X_0, \cdots, X_K \mid \varepsilon) = \prod_{i=1}^{K} p(X_i \mid \varepsilon).$$

If we let the random variable ω take the value ω_{+S} when +S is transmitted, and ω_{-S} is transmitted, we can further write

$$(7)\quad p(X_i \mid \varepsilon) = p(X_i \mid \varepsilon, \omega_{+S})\text{ prob }[\omega = \omega_{+S}] + p(X_i \mid \varepsilon, \omega_{-S})\text{ prob }[\omega = \omega_{-S}]$$

$$= \frac{1}{2}\,[p(X_i \mid \varepsilon, \omega_{+S}) + p(X_i \mid \varepsilon, \omega_{-S})]$$

for each $i = 0, 1, \cdots, K$ since prob $[\omega = \omega_{+S}]$ = prob $[\omega = \omega_{-S}] = \frac{1}{2}$. Furthermore, the two conditional densities in (7) are k-variate gaussian with covariances Φ_i and means $S_i = S_i(\varepsilon)$ and $-S_i = -S_i(\varepsilon)$, respectively:

$$(8)\quad p(X_i \mid \varepsilon, \omega_{\pm S}) = \frac{\exp[-\frac{1}{2}\,(X_i \mp S_i)^T\, \Phi_i^{-1}\, (X_i \mp S_i)]}{(2\pi)^{k/2}\,|\Phi_i|^{1/2}}$$

$$= \frac{\exp\left(-\frac{1}{2}\, X_i^T\, \Phi_i^{-1}\, X_i - \frac{1}{2}\, S_i^T\, \Phi_i^{-1}\, S_i\right)}{(2\pi)^{k/2}\,|\Phi_i|^{1/2}}\, \exp\left(\pm\, X_i^T\, \Phi_i^{-1}\, S_i\right).$$

Hence,

$$(9)\quad p(X_i \mid \varepsilon) = \frac{\exp\left(-\frac{1}{2}\, X_i^T\, \Phi_i^{-1}\, X_i - \frac{1}{2}\, S_i^T\, \Phi_i^{-1}\, S_i\right)}{2\;(2\pi)^{k/2}\;|\Phi_i|^{1/2}}\left[\exp\left(X_i^T\, \Phi_i^{-1}\, S_i\right) + \exp\left(-X_i^T\, \Phi_i^{-1} S_i\right)\right]$$

$$= \frac{\exp\left(-\frac{1}{2}\, X_i^T\, \Phi_i^{-1}\, X_i - \frac{1}{2}\, S_i^T\, \Phi_i^{-1}\, S_i\right)}{(2\pi)^{k/2}\;|\Phi_i|^{1/2}}\,\cosh\left(X_i^T\, \Phi_i^{-1}\, S_i\right).$$

Using (9) in (6) we now have

$$(10)\quad p(X_0,\cdots,X_k|\epsilon) = \frac{\exp\left(-\frac{1}{2}\sum_{i=0}^{K} X_i^T \Phi_i^{-1} X_i\right)}{(2\pi)^{k(K+1)/2}\prod_{i=0}^{K}|\Phi_i|^{1/2}} \exp\left(-\frac{1}{2}\sum_{i=0}^{K} S_i^T\Phi_i^{-1}S_i\right)\prod_{i=1}^{K}\cosh\left(X_i^T\Phi_i^{-1}S_i\right.$$

and using (10) in (5) we now have

$$(11)\quad p(\epsilon|X_0,\cdots,X_K) = \frac{p(\epsilon)}{p(X_0,\cdots,X_K)}\exp\left(-\frac{1}{2}\sum_{i=0}^{K} S_i^T\Phi_i^{-1}S_i\right)\prod_{i=0}^{K}\cosh\left(X_i^T\Phi_i^{-1}S_i(\epsilon)\right)$$

$$= C\, p(\epsilon)\prod_{i=0}^{K}\cosh\left(X_i^T\Phi_i^{-1}S_i(\epsilon)\right)$$

where C is normalizing constant that does not depend on ϵ. The maximum likelihood estimator of the parameter ϵ is the value of ϵ that maximizes $p(\epsilon|X_0, \cdots, X_K)$. It is reasonable to assume that all values of ϵ between 0 and T are equally likely; therefore, we assume a uniform distribution for the a priori density $p(\epsilon)$. Hence, the maximum likelihood estimator of ϵ is the value of ϵ that maximizes

$$(12)\quad g(\epsilon) = \prod_{i=0}^{K}\cosh\left[X_i^T\,\Phi_i^{-1}\,S_i(\epsilon)\right]$$

Or, since $\log g(\epsilon)$ has extrema at the same values of ϵ as $g(\epsilon)$, we can as well maximize

$$(13)\quad \log g(\epsilon) = \sum_{i=0}^{K}\log\cosh\left[X_i^T\,\Phi_i^{-1}\,S_i(\epsilon)\right]$$

Finally, we point out that for unit variance, uncorrelated noise samples Eq. (13) reduces to

$$(14)\quad \log g(\epsilon) = \sum_{i=0}^{K}\log\cosh\left[X_i^T\,S_i(\epsilon)\right]$$

Eq. (14) dictates that the optimum (maximum likelihood) synchronizer operates as follows. To test a record of exactly K durations of received signal for the assumed synchronizing position ε, the first ε seconds of the received data are correlated with the last ε seconds of the basic symbol waveform. The (ln cosh) of the result is computed and stored. The next T seconds of received signal is correlated with the basic symbol and the ln cosh of this result is computed and added to the previously stored result. This process is continued for each of the remaining complete symbol durations. The final partial period is correlated with the first $T-\varepsilon$ seconds of the symbol waveshape, the ln cosh of this result is computed and added to the previous result.

III. SUBOPTIMUM SOLUTION

In the preceding section the maximum likelihood estimator of signal epoch for a random sequence of anticorrelated signals in additive gaussian noise was derived. From Eq. (14) we interpreted the optimum synchronizer structure as consisting of a sequence of three operations: Each received datum is correlated with the known signal waveform, correlator output is passed through a ln cosh nonlinearity, and the K nonlinearity outputs averaged. Now, the correlation operation is equivalent to matched filtering, and an approximation to the matched filter for the pulse type signals assumed is a single pole lowpass filter with the time constant approximately equal to the signal duration. The ln cosh nonlinearity is essentially a square low device for low signal-to-noise ratios and an absolute value device for high signal-to-noise ratios; hence, we follow the lowpass filter with a square low device. Finally, the averaging (summation) operation can be accomplished with a narrowband filter centered at the frequency corresponding to the universe of the bit rate. In this manner we arrive at the rather simple suboptimum structure presented in Fig. 3. The output is a sinuosid whose zero crossings are estimates of the signal start times.

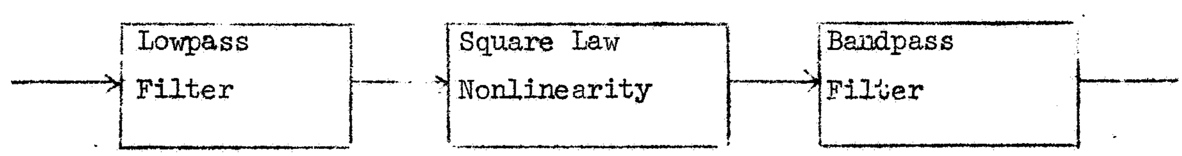

Fig. 3

IV. OPTIMUM AND SUBOPTIMUM SYSTEM PERFORMANCE

The ultimate measure of performance for any digital system is probability of detection error. However, for a subsystem such as the bit synchronizer, such a measure is not possible unless a particular detector is specified. In order to compare the optimum and suboptimum cases without specifing a particular detector, the expectation of the absolute value of the synchronizing error, $\overline{|\epsilon|}$, is chosen as a performance measure. This choice is made for two reasons. First, the particular moment is easily measured for any prototype synchronizer. Secondly, such low order moments generally allow fairly good comparison of random processes which have similar but not identical probability laws. For the density functions dealt with in the systems compared in this paper, the value of $\overline{|\epsilon|}$ is about .75 of its rms value.

The performance of a particular synchronizer also depends on the symbol waveform. Three symbol pulses which have relatively narrow bandwidths have been chosen for comparison. These are the square pulse; a half sine pulse; and a raised cosine. The half sine pulse is defined by

$$(15) \quad S(t) = \sin \frac{\pi}{T} t \quad 0 < t < T$$

and the raised cosine by

$$(16) \quad S(t) = \left(1 - \cos \frac{2\pi}{T} t\right) \quad 0 < t < T$$

The spectral densities for a random sequence $(p = \frac{1}{2})$ of these three pulses are plotted in Fig. 4 so that their bandwidths can be compared. These pulses are chosen because of their ease of generation and because it is felt that they represent fairly well the class of narrowband symbols.

The performance of the optimum synchronizer defined by Eq. (14) was obtained by Monte Carlo simulation on a digital computer. The moment $\overline{|\epsilon|}$ was computed from the histogram of 500 trials.

Fig. 5 shows the optimum system performance obtained with the raised cosine symbol for four different memory times, K. These curves show that $\overline{|\varepsilon|}$ varies inversely with the square root of memory time. Such a variation is also observed for the suboptimum system. It is also evident that $\overline{|\varepsilon|}$ decreases approximately with the square root of 2E/N. Such relationships as these seem to be typical of adaptive systems such as this. A similar relationship is noted in Hancock and Wintz [2] in the chapter on adaptive systems. In Fig. 6 the performance of the optimum system for three different symbols is presented. The roughness of the data makes it hard to conclude that any one symbol gives better performance. It appears, however, that the performance with square pulses may be somewhat better for high signal-to-noise ratios.

The performance of the suboptimum system was computed by the method outlined in the appendix. The performance of the suboptimum system depends not only on memory time and symbol waveshape but also on the bandwidth of the input filter. In Fig. 7 $\overline{|\varepsilon|}$ is plotted against 2E/N for various values of input low pass filter cutoff frequency. Apparently, for realistic values of 2E/N, this bandwidth is not a very significant factor. The best choice is a bandwidth equal to the inverse of the bit rate. For operation at very high values of signal-to-noise ratio, a higher value of cutoff frequency is desirable since a limiting factor on $\overline{|\varepsilon|}$ is the noise produced by the filtering of the signal. This noise is due to the filter memory (intersymbol interference) and occurs even with no additive noise.

As illustrated in Fig. 8, the raised cosine symbol appears best for the suboptimum system. Operation with square pulses is considerably poorer as nearly 8 times as much memory time is required to achieve the same value of $\overline{|\varepsilon|}$.

A prototype of the suboptimum system has been constructed. Fig. 9 presents experimental data similar to Fig. 8 except that the nonlinearity is now an absolute value function instead of a square law function. Experimental evidence indicates that the form of the nonlinearity has no noticeable effect on the performance.

Fig. 10 compares the performance of the optimum and suboptimum systems. For the raised cosine and half sine symbols suboptimum performance is very close to the performance of the optimum system. On the other hand, for the square pulse suboptimum system performance is considered worse than optimum system performance. In order to compare the memory time of the exponential weighting that is characteristic of the bandpass filter in the suboptimum system with the uniform weighting of the optimum system an effective measurement time was defined to be

$$(17)\quad K = \frac{\left(\sum_{i=1}^{\infty} c_i\right)^2}{\sum_{i=1}^{\infty} c_i^2}$$

where c_i is the relative weighting assigned to the i^{th} preceding symbol. (This definition of effective measurement time is due to Price [3]. See also [4]).

The bit synchronizer in most common use is similar to the suboptimum of Fig. 3 but with the square law nonlinearity replaced by a hard limiter. A prototype of this system was also constructed and its performance compared to the performance of the first suboptimum system. As illustrated in Fig. 11 the first system using raised cosine symbols is approximately 10 db superior to the second system using square symbols. A somewhat surprising result, also presented in Fig. 11, is that the performance of the system with raise cosine symbols was significantly better (at least at low and moderate signal-to-noise ratios) than with square symbols.

V. SUMMARY AND CONCLUSIONS

The optimum (maximum likelihood) estimator for the epoch of a random sequence of anticorrelated signals of known duration in additive gaussian noise was derived and its performance evaluated by Monte Carlo simulation on a digital computer. The mean magnitude of the epoch error vs. signal-to-noise ratio was obtained for three symbol waveforms.

The optimum system is rather complex and, therefore, judged to be impractical. However, the equation defining the optimum system structure can be loosely interpreted as a filtering operation followed by a nonlinearity followed by an averaging operation. A simple (suboptimum) system that performs these operations was considered and its performance computed via the transform method. The suboptimum system performed nearly as well as the optimum system for rounded pulses, but significantly worse for square pulses. A prototype of the suboptimum system was constructed; it performed as predicted.

A second suboptimum system incorporating a hard limiter was constructed and its performance compared to the performance of the first suboptimum system. The first system with rounded pulses performed 10 db better than the second system with square pulses.

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APPENDIX

In this Appendix we outline the method used to compute $\overline{|\epsilon|}$ for the suboptimum system of Fig. 3.

The statistics of the output of a square law device when the input is the sum of two independent random processes is well known [5]. The spectral density of the output is

$$(\text{A-1}) \quad S(f) = 2^{-1}\left(R_{s^2}(\tau)\right) + 4\int_{-\infty}^{+\infty} S_s(f^1)\, S_n(f - f^1)\, df^1 + 2\int_{-\infty}^{+\infty} S_n(f)\, S_n(f-f^1)\, df$$

$R_{s^2}(\tau)$ is the autocorrelation function of the square of the input random process, $S_s(f)$ is the spectral density of the signal random process, and $S_n(f)$ is the spectral density of the noise. The second and third terms represent the usual signal-cross-noise and noise-cross-noise terms and can be computed by standard convolution techniques. The first term is more difficult to deal with. It represents the spectral density of the square of the signal random process. It contains both discrete and continuous components.

For sufficiently narrow bandwidth the output of the bandpass filter which is tuned to the inverse of the bit rate can be approximated by a sine wave and additive narrowband gaussian noise. [illegible] This density function, as first derived by Rice [6], has the same general shape and the same lower order moments as the density function usually quoted for phase locked loops [7] when the S/N parameters are properly related. The function for phase, θ, when divided by 2π is the desired density for ϵ.

To evaluate the signal to noise ratio in this density function the magnitude of the sine wave at the bit frequency and the spectral density in the region of the bit frequency must be computed. The convolution integrals of Eq. (A-1) evaluated at one specific frequency, f_b, will provide the signal-cross-noise and noise-cross-noise contributions to the continuous spectrum.

If the input noise is assumed white, the action of the lowpass filter on the noise spectrum is easily computed. The effect of the input lowpass filter on the signal spectral density is computed the same way. The spectral density at the input to the lowpass filter is given by Eq. (2). The components due to the signal-cross-signal term can be computed by assuming that the effective memory time of the lowpass filter is no more than one signal period; then the output of the filter can be considered a new random process which is made up of four symbols. Let $H(t)$ represent the response of the low pass filter to $S(t)$ during the period $(0, T)$, and $T(t)$ the response during $(T, 2T)$. For the anticorrelated signal process the new symbols are $\pm [H(t) \neq T(t)]$ and $\pm [H(t) - T(t)]$. On passing this through the square law nonlinearity, a new random process is formed which has only two symbols: $S_1 = [H(t) + T(t)]^2$ and $S_2 = [H(t) - T(t)]^2$. For the special case of equiprobable symbols, the spectral density is $M(j2\pi f)M(-j2\pi$ where $M(s) = L(H^2(t) + T^2(t))$. For even the most simple symbol wave shapes, the calculations involved are rather tedius. See [8].

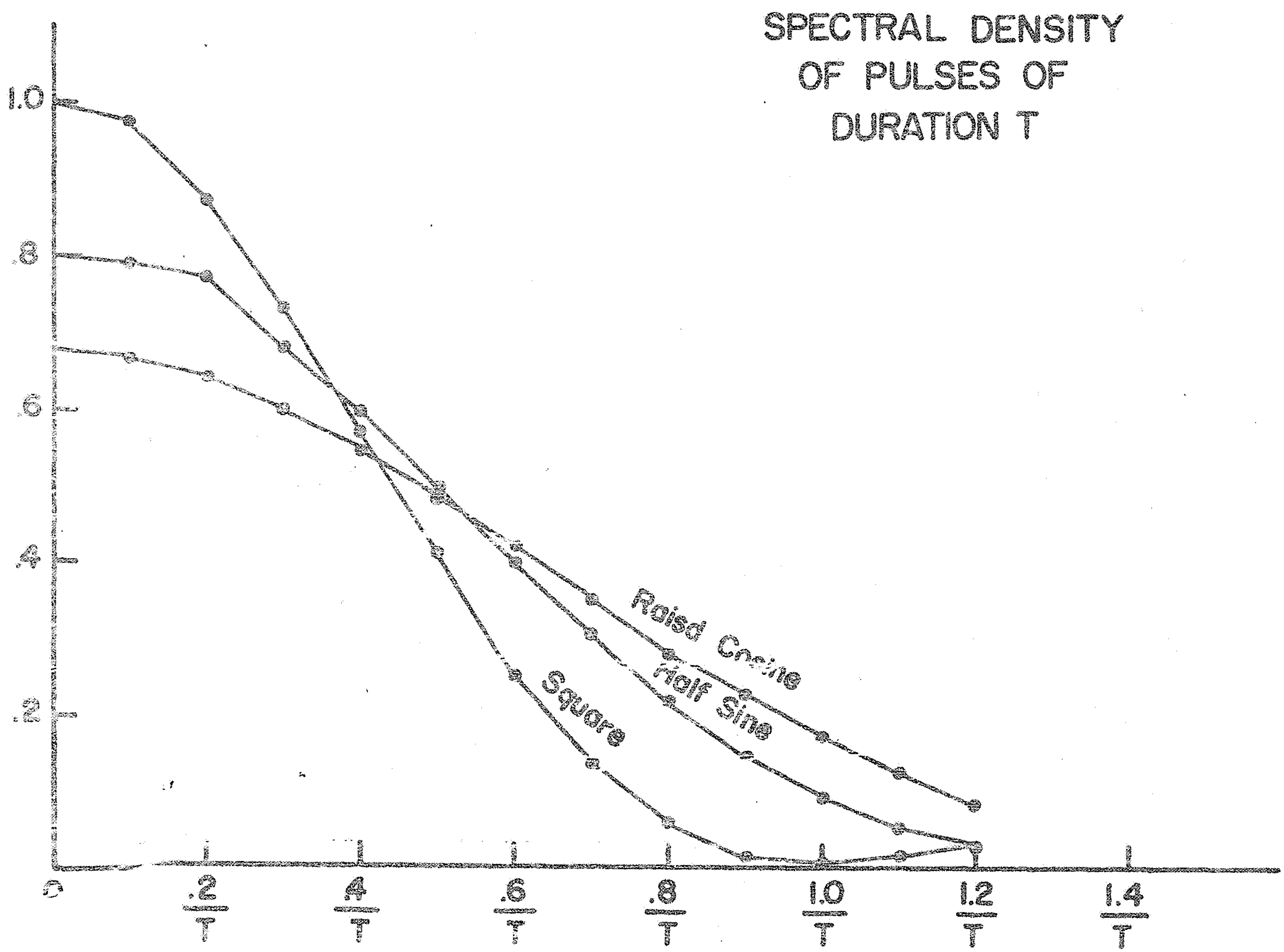

SPECTRAL DENSITY
OF PULSES OF
DURATION T
1.0
.8
.6
.4
.2
0
.2/T
.4/T
.6/T
.8/T
1.0/T
1.2/T
1.4/T
Raisd Cosine
Half Sine
Square

Figure 4

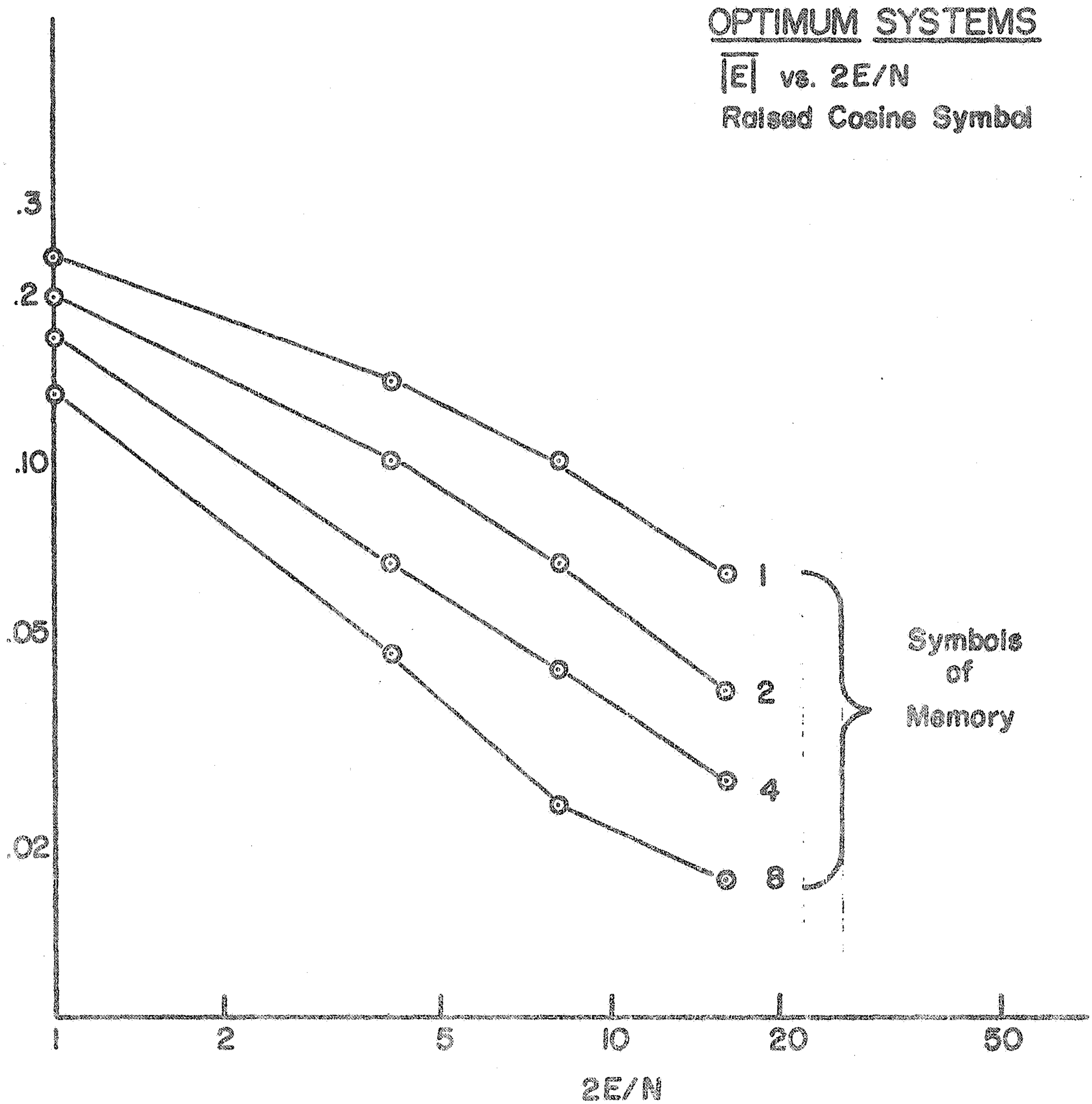

OPTIMUM SYSTEMS
$\overline{|E|}$ vs. 2E/N
Raised Cosine Symbol
.3
.2
.10
.05
.02
1
2
4
8
Symbols of Memory
1
2
5
10
20
50
2E/N

Figure 5

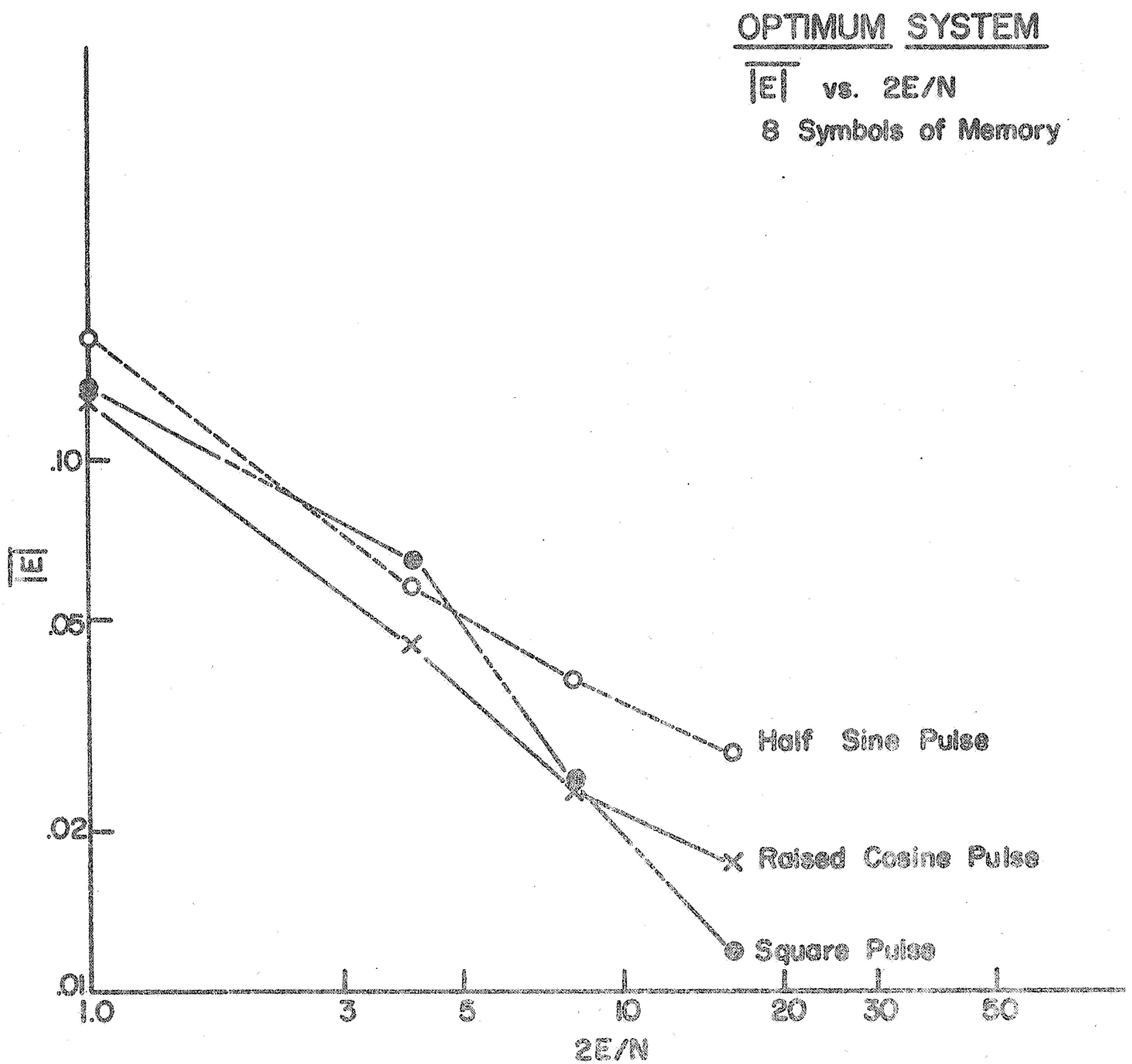
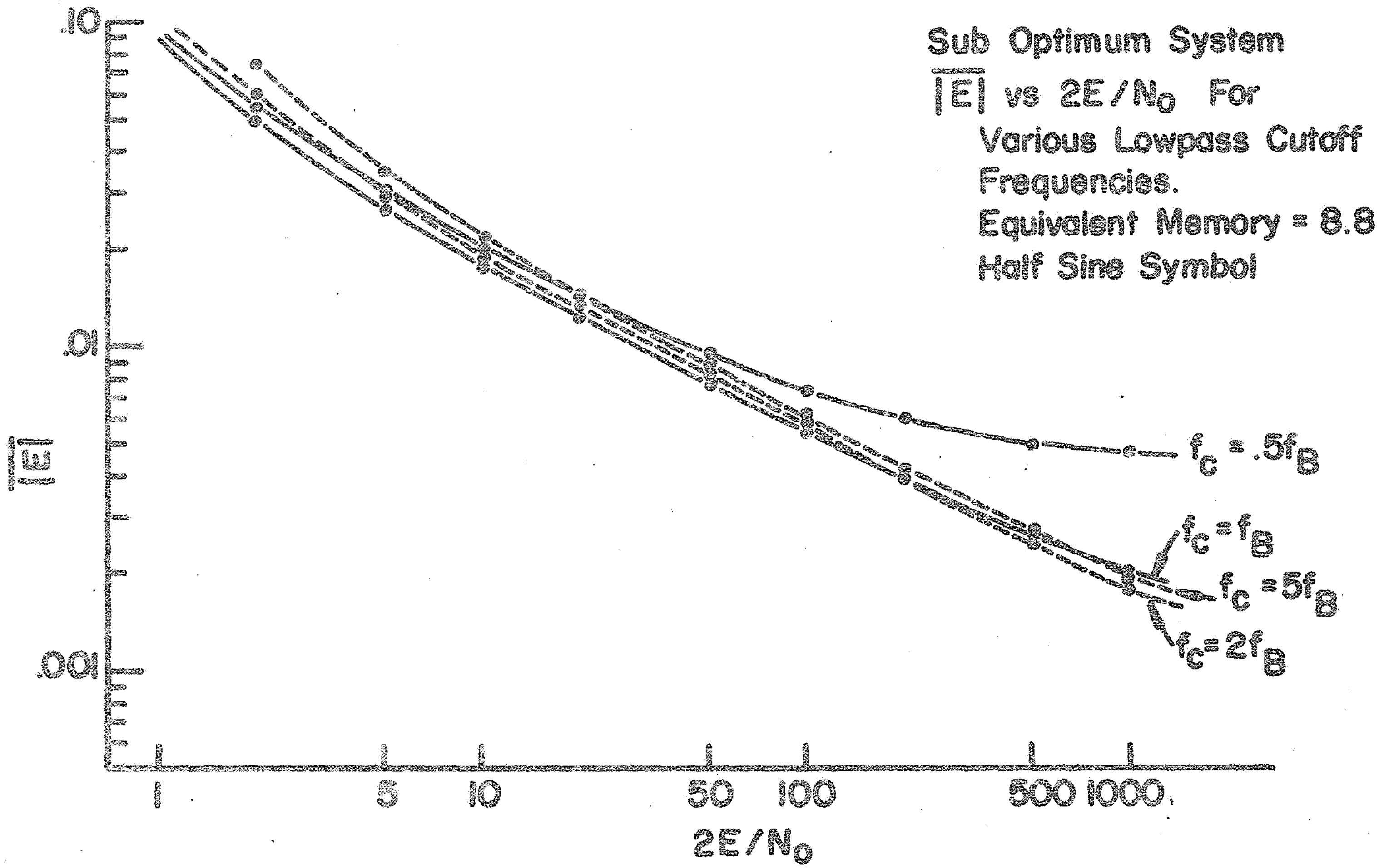

OPTIMUM SYSTEM
$\overline{|E|}$ vs. 2E/N
8 Symbols of Memory
$\overline{|E|}$
.10
.05
.02
.01
1.0
3
5
10
20
30
50
2E/N
Half Sine Pulse
Raised Cosine Pulse
Square Pulse

Figure 6



Sub Optimum System
$\overline{|E|}$ vs $2E/N_0$ For
Various Lowpass Cutoff
Frequencies.
Equivalent Memory = 8.8
Half Sine Symbol
.10
.01
.001
$\overline{|E|}$
1
5
10
50
100
500
1000.
$2E/N_0$
$f_c = .5f_B$
$f_c = f_B$
$f_c = 5f_B$
$f_c = 2f_B$

Figure 7

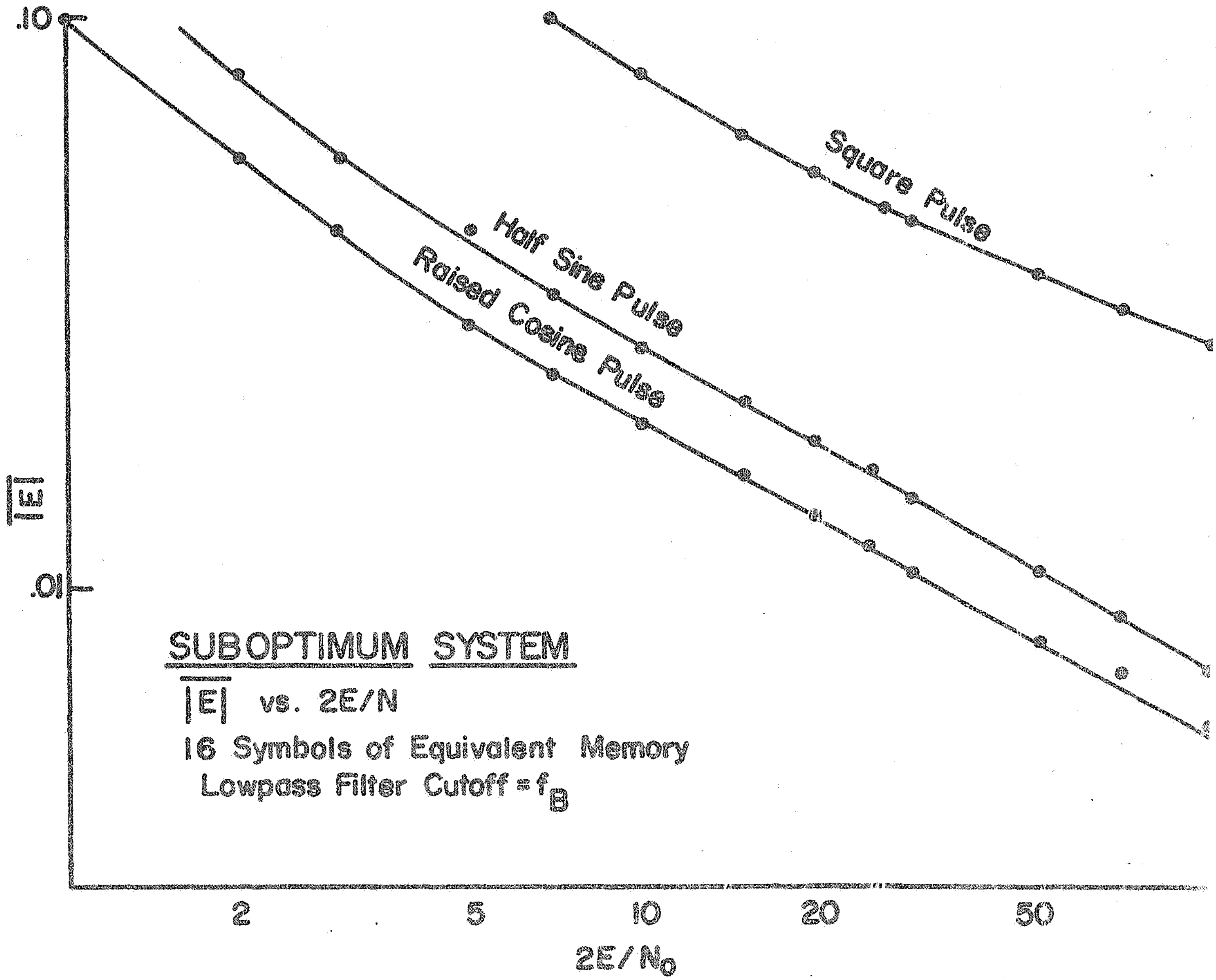
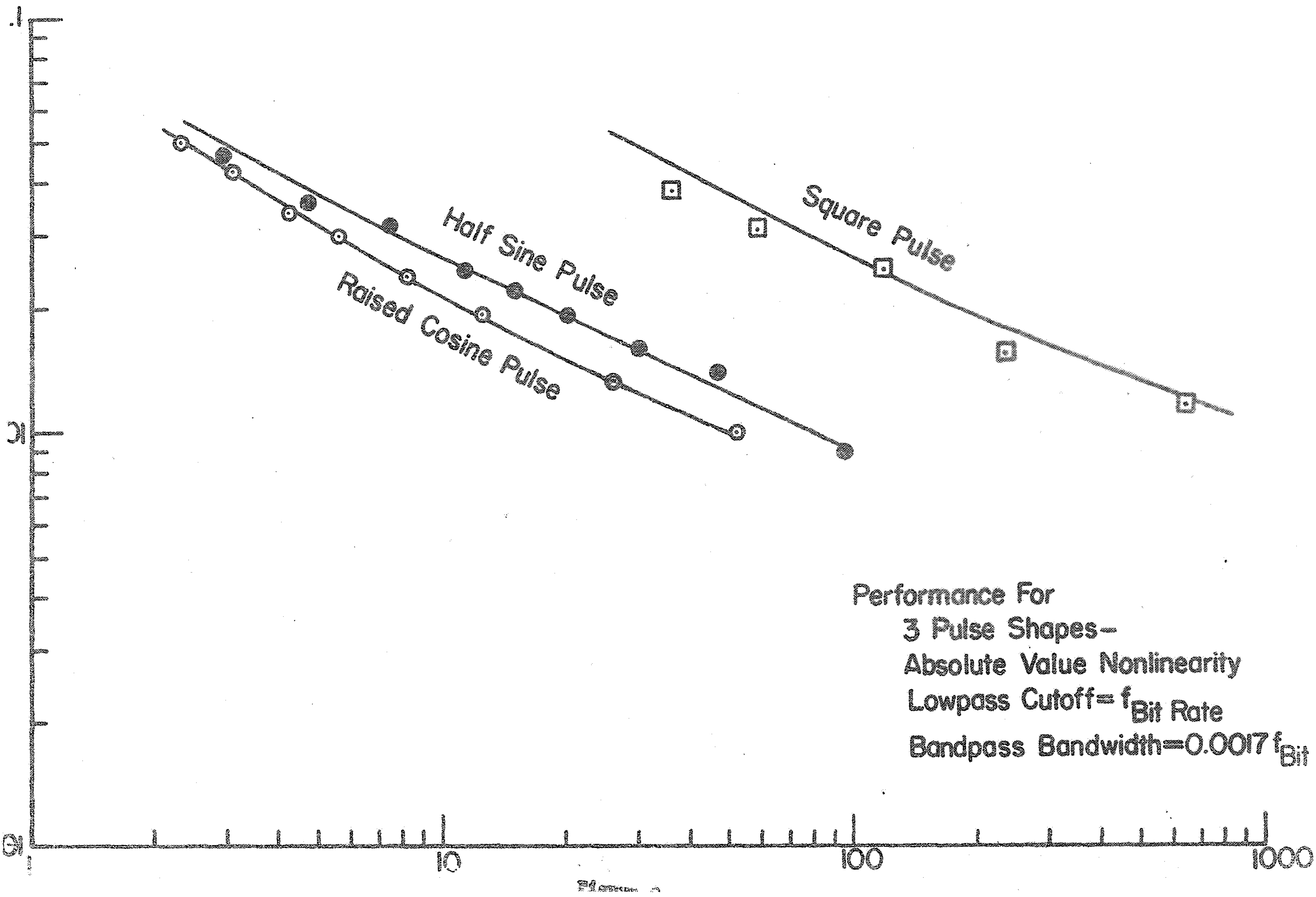

.10
.01
|E|
Square Pulse
Half Sine Pulse
Raised Cosine Pulse
SUBOPTIMUM SYSTEM
|E| vs. 2E/N
16 Symbols of Equivalent Memory
Lowpass Filter Cutoff = f_B
2
5
10
20
50
2E/N_0

Figure 8



Square Pulse
Half Sine Pulse
Raised Cosine Pulse
Performance For
3 Pulse Shapes–
Absolute Value Nonlinearity
Lowpass Cutoff= f_Bit Rate
Bandpass Bandwidth=0.0017 f_Bit
10
100
1000

OPTIMUM AND SUBOPTIMUM SYSTEMS COMPARED

$\overline{|E|}$ vs. 2E/N

8 SYMBOLS OF MEMORY

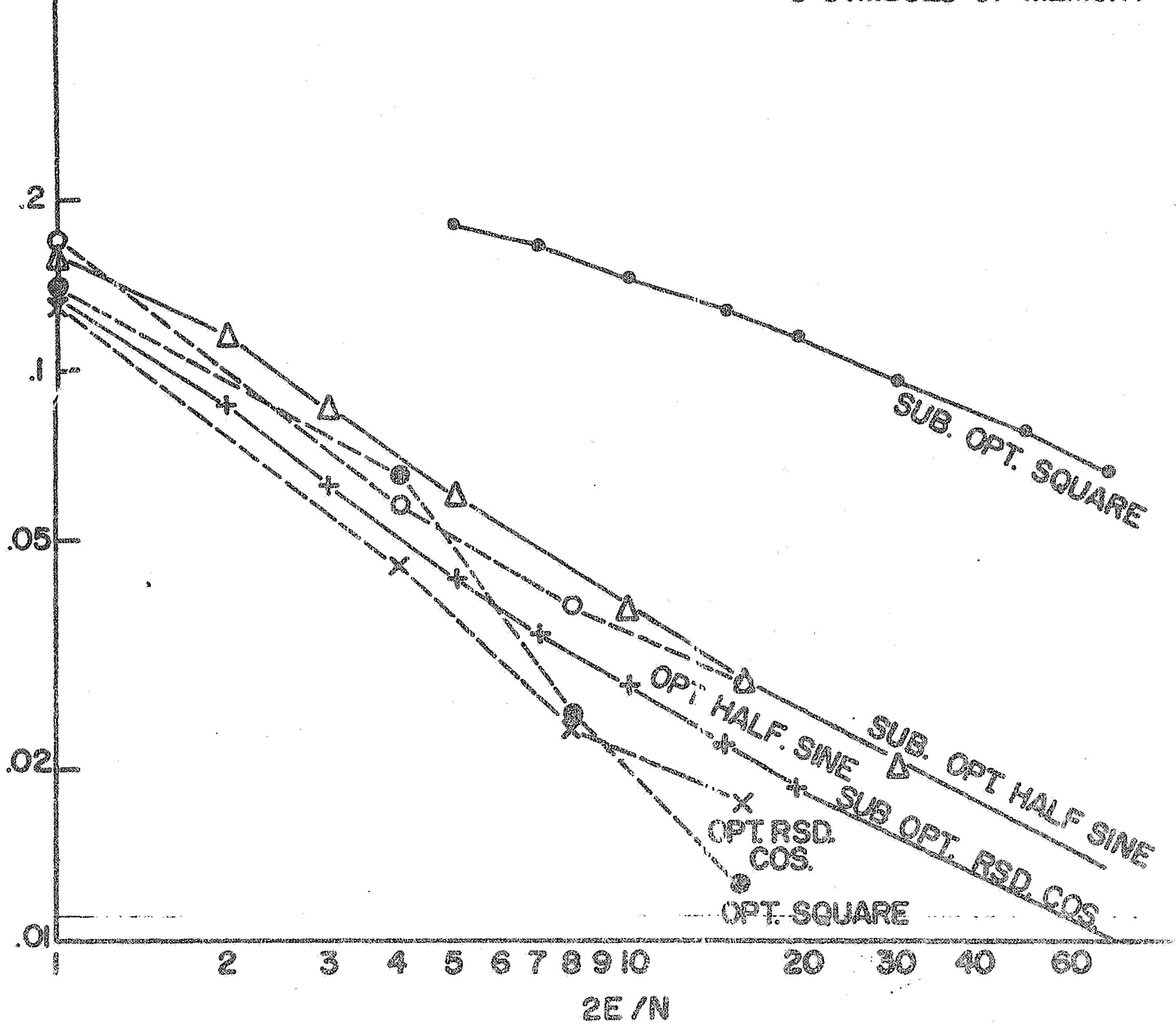
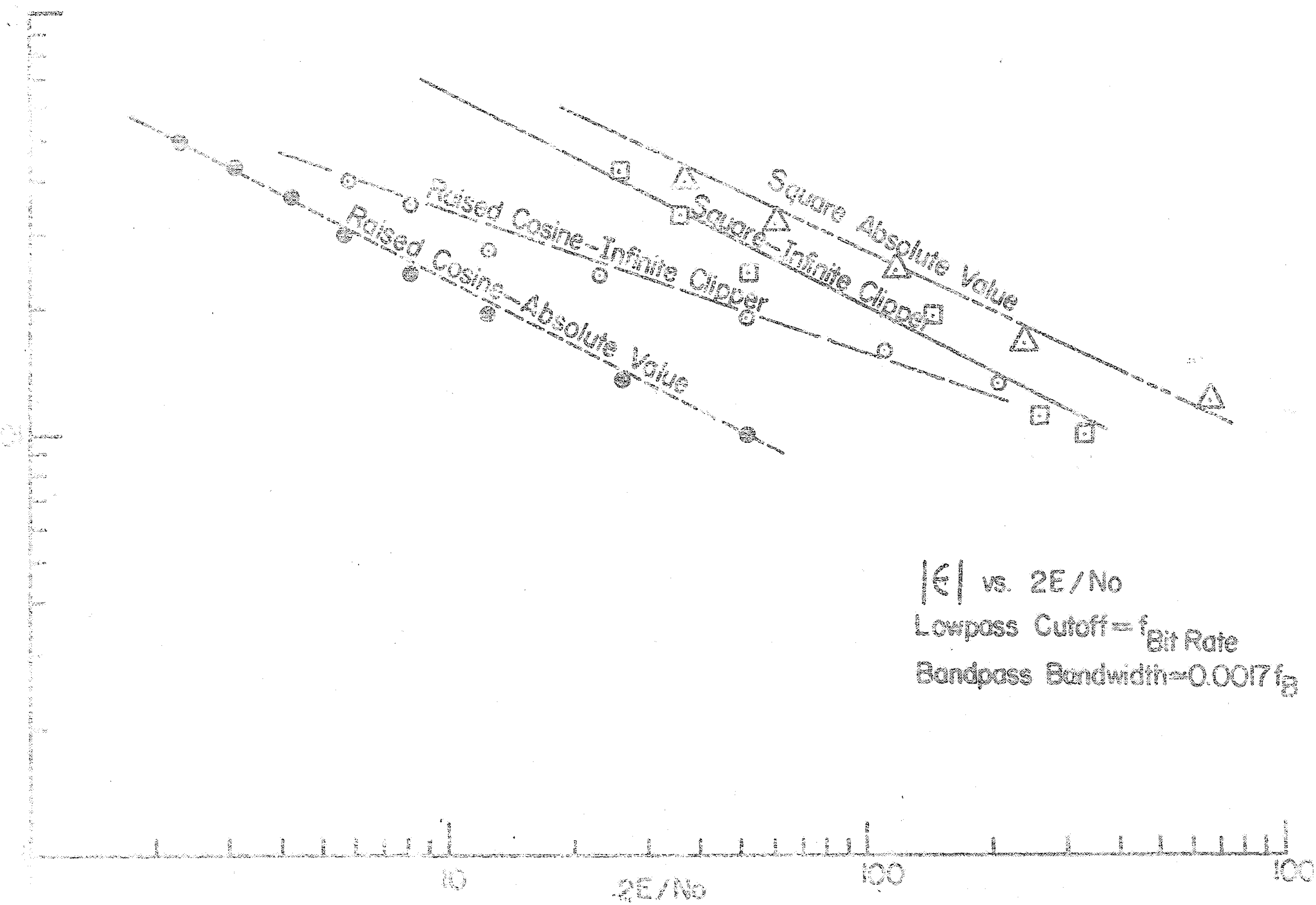


Figure 10


Square Absolute Value
Square-Infinite Clipper
Raised Cosine-Infinite Clipper
Raised Cosine-Absolute Value
|ϵ| vs. 2E/No
Lowpass Cutoff = f Bit Rate
Bandpass Bandwidth = 0.0017 fB
10
100
2E/No