PRELIMINARY RESULTS FROM THE LUNAR ORBITER SELENODESY EXPERIMENT

William H. Michael, Jr.
NASA-Langley Research Center
Langley Station
Hampton, Virginia 23365

[redacted]

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INTRODUCTION

In this presentation, I would like to discuss the progress to date, at the Langley Research Center, on the Selenodesy Experiment being conducted in connection with the Lunar Orbiter Project. The objective of this experiment is to analyze the tracking data from the Lunar Orbiter spacecraft to determine the gravitational field of the moon, and other parameters which can be extracted from the tracking data.

An outline of what I intend to cover in this discussion is shown on the first slide. We start with a definition of the objectives and some description of the data required.

As already indicated, the basic objective of this analysis is the determination of the components of the gravitational field of the moon. The gravitational field results are desired for two main purposes: (1) for orbit prediction for lunar satellites, in general, and (2) for determination of various physical properties of the moon.

The basic data required are tracking data from lunar satellites. The tracking data contain information on the position and velocity of the satellites as a function of time, and from this information one can determine the perturbations produced on the satellite by the gravitational field components. The tracking data used in this analysis are those obtained from the first three Lunar Orbiter spacecraft.

The use of lunar satellites for this purpose has a short history, effectively dating for efforts in this country from last August, when Lunar Orbiter I was successfully injected into orbit about the moon.

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The NASA Lunar Orbiter Project, managed by the Langley Research Center, has the primary objective of obtaining high-quality photographs of the lunar surface, for identification and selection of landing sites for future manned missions, and for other scientific and engineering purposes. A secondary objective is to provide tracking data for use in determination of the gravitational field and other properties of the moon.

Instrumentation required in the spacecraft consists of a power source, an omnidirectional antenna, and a transponder, to receive high frequency radio signals from earth tracking stations and to retransmit the signals back to the stations to provide Doppler frequency measurements. The instrumentation is already available in the spacecraft and is used in determining the coordinates and orbital properties of the spacecraft, for controlling it and its subsystems in accomplishing the primary photographic objective. The additional instrumentation required for providing tracking data for this analysis therefore can be said to require zero weight and zero volume (an unbeatable combination for any experiment). After the photographic phase of a Lunar Orbiter mission is completed, and after the photographic data have been transmitted to earth, the main purpose for maintaining the operation of the spacecraft is for accumulation of tracking data.

To be more specific about what is to be determined by analysis of the tracking data, we refer to slide 2. The equation is the standard representation of the gravitational potential in terms of spherical harmonics. The objective is the determination of a finite number of coefficients $C_{n,m}$ and $S_{n,m}$ in this expansion, resulting in the definition of the gravitational field. In this equation, μ is the product of the gravitational constant and the mass of the moon, R is the radius of the moon, r is the radial distance from the center of mass of the moon, $P_{n,m}$ are associated Legendre polynomials, ϕ is latitude, and λ is longitude.

Other parameters which can be determined are shown on the slide. The spacecraft state at an epoch at the beginning of the data arc is always included in the solution, and the other parameters can be included if desired.

Some discussion of the equation given on the slide is in order here. If all the C's and S's were zero, the acceleration on the spacecraft due to the gravitational potential would be that corresponding to a central gravitational field, and if no other forces were present, the spacecraft would describe an unperturbed elliptical orbit. This is not the case, however, and each of the $C_{n,m}$ and $S_{n,m}$ coefficients introduces a small perturbation in the orbit. The amplitudes and periodicities of the perturbations in the various characteristics of the orbit are more or less attributable to a particular coefficient. These small perturbations in the orbital properties are reflected in the tracking data, and this, of course, is the reason that the coefficients can be defined through analysis of the tracking data.

The polynomials, $P_{n,m}(\sin \phi)$, have some geometrical properties which give an insight into effects which can be expected from various coefficients. The polynomials vanish along $(n - m)$ parallels of latitude, and along $2m$ meridians. From these geometrical properties, one can determine that perturbations in the orbit should be periodic with period of the rotational period of the moon divided by m, or of approximately $\frac{27.3}{m}$ days. The periodic dependence of the perturbations, therefore, allows separation of effects due to coefficients with various values of m. For $m = 0$, and for even n the periods of some of the perturbations are infinite. Thus these coefficients cause secular variations in the node and argument of pericenter, parameters related to the orientation of the orbit in inertial space. For coefficients with $m = 0$ and odd n the perturbations are not secular, but are of very long period, and primarily affect the eccentricity of the orbit. All of these characteristic perturbations are important in separation of effects due to various coefficients and in the determination of the coefficients. However, a basic difficulty remains.

Coefficients with a given value of m, and differing by 2 in n, whether n is odd or even, produce perturbations which are very similar, and therefore from which it is difficult to determine the particular coefficients. This effect is particularly pronounced for low inclination orbits. This is seen physically by observing that low inclination orbits are mainly affected by gravitational forces within the equatorial region, and insufficient sampling of the effects at higher latitudes leads to the problem of not being able to separate effects due to particular coefficients. In an attempt to circumvent this problem, data from orbits with different inclinations, in the range from 12° to 21°, have been used for this analysis. Including data with three different inclinations in the analysis has helped the problem, but has not eliminated it.

Higher inclination orbits, and perhaps a greater variety of other orbital parameters, will be required before the high correlations can be entirely eliminated.

DESCRIPTION OF PROCEDURES FOR THE TRACKING DATA ANALYSIS

The discussion so far has been somewhat of an introduction to the problem. We now proceed to a brief description of the procedures used for the analysis of the tracking data, for determination of the gravitational field parameters.

Although tracking data from lunar satellites have only recently become available, procedures for the analysis of the data have been under development at Langley Research Center for about 3 years. The procedures have some features related to classical methods for orbit determination and some features related to methods employed for analysis of earth satellite tracking data. In general, the procedures make use of the methods of weighted least squares, differential correction, and numerical integration - to determine the values for a set of parameters, with the criterion that the values so determined provide a good fit to the data.

I do not intend to go through a complete description of the procedures. But I do intend to discuss some of the basic formulations, which are rather straightforward, to give some indication of the overall process.

We will discuss the process of differential corrections, outline the procedure for the solution for the parameters, and say a little about the calculations required in the analysis.

Some definitions and equations are shown on slide 3. Here ϕ_i is defined as an observed quantity at same time i, etc. ϕ_i is, in general, a nonlinear function of the set of parameters, p. The set of parameters can include a number of gravitational coefficients, plus the initial conditions defining the spacecraft state at the beginning of the data arc, and other parameters as defined earlier. Initial estimates for the parameters are available. For the state parameters, the initial estimates may be obtained from the nominal conditions for the orbit, or they may be available from previous solutions. The initial estimates for the gravitational coefficients could be zero, or values from previous solutions. The initial estimates are to be corrected with use of the data, to obtain a better estimate for the values of the parameters. The calculated value of the observed quantity is based on the initial estimates for the parameter set.

The first equation on slide 3 is the Taylor's series expansion for $\phi(p_o + \Delta p)$, where Δp represents the corrections to the parameters. The expansion is linearized by neglecting terms involving the second- and higher-order derivatives.

We now obtain an observational equation by forming the difference between the observed quantity ϕ_i and the right side of equation (1). This gives equation (2), which is the basic equation for differential correction of the parameters p.

There is one equation like equation (2) for each observation. In theory, if the number of observations i were equal to the number of parameters in the set p, equations (2) could be solved exactly for the corrections to the parameters Δp. However, this would, in general, not produce a good solution for the parameters for two main reasons: (1) with such a small data sample, even small random errors in the observables would introduce significant errors in the solution, and (2) some of the parameters, such as the gravitational field parameters, produce periodic effects on the orbit and thus on the tracking data, and there must be sufficient long- and short-period sampling of the data to detect these effects in order to obtain a good solution for the parameters. Thus, in general, the number of observations will greatly exceed the number of parameters. In this case, there will be no exact solution to equations (2), but rather the equations will have the form of equations (3) where the ϵ_i are the residuals.

The desired solution to equations (3) is the solution for the parameters for which the residuals are a minimum; specifically, the solution for which the weighted sum of the squares of the residuals is a minimum. The quadratic form to be minimized with respect to p is shown as equation (4), where A is the matrix of partial derivatives. In this formulation, a weighting matrix, W, has been included to provide a means for incorporating weights on the individual observations.

On performing the minimization of Q with respect to p, the weighted least squares condition for the solution is that shown as equation (5). Equations (5) are called the normal equations, and the normal equations constitute j equations for the j parameters to be corrected.

The weighted least squares solution for the corrections to the original estimate of the parameters is thus that shown as equations (6). The new estimates for the parameters are shown as equations (7).

Since a linear approximation was used back up in equations (2), the solution must be iterated by substituting the new estimates of the parameters for the previous estimates and repeating the process until the changes in the parameters become negligible. The resulting values of the parameters are then the best estimates for the parameters in the weighted least squares sense.

We can refer to equation (6) on the slide to get an idea of some of the things involved in the calculations and in the solution. We see that there are three major constituents: the observed quantities ϕ_i, the calculated quantities $\phi_i(p_o)$, and the matrix of partial derivatives, A.

The observed quantity for this analysis is two-way Doppler frequency count. This quantity is proportional to the relative velocity between the tracking station and the spacecraft in orbit about the moon. These quantities are derived from the tracking data. The tracking data are obtained through the facilities of the NASA Deep Space Network, using the tracking stations in California, Australia, and Spain.

The calculated value for the relative velocity must involve three things: (1) the motion of the spacecraft in orbit about the moon, (2) the motion of the moon relative to the earth, and (3) the motion of the tracking station due to the earth's rotation. The motion of the spacecraft is calculated by numerical integration of the equations of motion, taking into account all the perturbations on the spacecraft. The other motions are calculated with use of ephemeris tapes. This information must be interpolated for the time of each observation to be able to compare $\phi_i(p_o)$ with ϕ_i at each observation time.

The partial derivatives are important in the process of differential correction because they contain the information required in arriving at the corrections to the original estimates of the parameters. The partial derivatives are obtained from numerical integration of perturbation equations, along with the integration of the equations of motion.

A number of options are available for the determination of the parameter set. The solution can be obtained with data from a single spacecraft whenever the number of observations exceeds the number of parameters in the solution. Any parameter in the parameter set can be eliminated from the solution, as desired. Data from several spacecraft can be combined to give a solution for the parameters common to the several spacecraft. And, finally, some statistical information is available as a byproduct of the solution, giving an estimate of the standard deviations and correlations for the parameter set.

PRELIMINARY RESULTS ON THE GRAVITATIONAL FIELD OF THE MOON

Tracking data from the first three Lunar Orbiter spacecraft have been used for this analysis. Five separate data arcs have been analyzed, and these data arcs are summarized on slide 4.

The data are generally well distributed throughout the orbits, except during occultations by the moon, which amount to about 45 minutes during the orbital period of about 3-1/2 hours. The 24-day data arc of orbit configuration I-2 and the 21-day data arc of II-1 represent large fractions of complete revolutions of the moon, so the data are well distributed in longitude. However, the orbital elements shown on the slide do not represent a wide variation in these properties, and this is one of the limitations on the data available for analysis at this time. These elements were chosen for photographic purposes, which is the reason for the low inclinations.

Where the early earth satellites had inclinations of 35° or more, most of the data for this analysis are from lunar satellites with inclinations of 12° and $17\text{-}1/2^{\circ}$. Thus the results obtained with these data are strongly influenced by gravitational effects with origin in the equatorial regions.

The lunar gravitational field coefficients obtained in this analysis are presented on slide 5. All the coefficients listed on the slide were included in the solution except for $C_{5,0}$, which is very highly correlated with $C_{3,0}$, and which was set to zero. The standard deviations listed for the coefficients are more an indication of how the solution fits these data arcs than of changes which may be expected in future analyses.

There are a number of fairly high correlations between various coefficients in this set. These high correlations were anticipated from preflight analyses and were mentioned earlier, and they indicate that it is difficult at this stage of the analysis to obtain separation of individual coefficients which produce similar orbital perturbations on the spacecraft. The separations are best obtained through analysis of tracking data from spacecraft with a variety of orbital parameters, and particularly with higher inclination orbits, but such data are not yet available.

Although the results presented on the slide are derived from tracking data which have some inherent limitations, these are nevertheless some of the first results which have become available through direct analysis of the dynamics of lunar satellites. Therefore it is of interest to apply these results to preliminary determinations of various physical properties of the moon, and to compare this new information with that which was previously available.

Comparison of Earth and Moon Gravitational Field Results

On comparing the gravitational field coefficients for the moon and the earth, the oblateness term, $C_{2,0}$, is greater for the earth (-1.08×10^{-3}) than for the moon (-2.22×10^{-4}), as expected from information which was previously available. For comparison of the higher-degree and order terms the quantities shown on the bottom of slide 5 are used as the basis for comparison. For the terms other than $C_{2,0}$, the quantities $\sigma_{n,m}$ for the moon are greater than those for the earth, by factors of from about 10 to about 100. The comparisons indicate that the gravitational field of the moon is somewhat "rougher" than that of the earth, but the higher-order effects decrease in about the same proportion with increasing n for both the earth and the moon.

Kaula has considered respective stress implications in the moon and the earth on the basis of the gravitational coefficients, taking into account the difference in size and mass of the two bodies. He finds that gravitational coefficients for the moon about 36 times those of the earth could be supported by stresses in the moon comparable to those in the earth. As mentioned above, the present results indicate that the lunar gravitational coefficients are 10 to 100 times those of the earth. However, there is no assurance that the coefficients for the earth are necessarily the maximum which could be supported by the strength of the materials in the earth. With the uncertainty in the coefficients for both the earth and the moon, it appears that the factors of 10 to 100 for the ratio of moon and earth coefficients are reasonable. The conclusion from the present results is that the stresses supported by the materials in the moon are probably comparable to those supported by earth materials.

Application of Results to Orbit Prediction

One of the important applications of the gravitational field results is for orbit prediction for lunar satellites, as mentioned earlier. It is of interest to determine how well the present results predict orbital variations over arcs beyond those for which the data were analyzed.

Some typical results are shown on slide 6, which is a plot of the variation of pericenter radius with time for a 2-month period for Lunar Orbiter I. Pericenter radius is the radius of closest approach of the spacecraft to the moon. The shaded boundary on the figure represents the mean radius of the moon, 1738 km. The circles are the values from short-arc orbit determinations. The short-arc orbit determinations are based on fits to one or two orbits of data at each of the times shown. These fits are relatively independent of the gravitational coefficients and serve as a basis for comparison of the predictions. The solid curve represents the predicted values using the coefficients presented here. Whereas the data analyzed in the solution covered a 3-week period near the middle of this time period, the predictions start with initial conditions on August 29 and are projected forward through a 2-month period. The agreement between the predictions and the short-arc results is considered to be good. Similar agreement is obtained in comparisons of the other orbital elements and for the other long-term results (for Lunar Orbiter II) which are available.

The dashed curve on the slide represents the predicted variation in r_p obtained with use of information on two coefficients, $C_{2,0}$ and $C_{2,2}$, which was available prior to the Lunar Orbiter flights. The differences between the dashed curve and the circles thus represent the variations in r_p due to the modified values of $C_{2,0}$ and $C_{2,2}$ and to the other gravitational coefficients. The differences indicate errors which would be obtained if the more complete results were not included.

The variation in pericenter radius represents the change in the eccentricity of the orbit. The other major effects produced by the gravitational field coefficients are on the orientation of the orbit in inertial space, through variations in the ascending node of the orbit, and in the argument of pericenter. For the orbit of Lunar Orbiter I the regression of the node is about $1/2^{\circ}$ per day, and the progression of argument of pericenter is about 1° per day.

The perturbations produced by the gravitational field coefficients become important when precise position and velocity information is required for accomplishment of rendezvous, orbital transfer, or deboost maneuvers. The present gravitational field results, or modified results obtained in future analyses, will be used for orbit calculation and orbit prediction for the Apollo missions and for other future manned and unmanned lunar missions.

Comparison of Lunar Orbiter Results With Luna-10 Results

One additional comparison I would like to make is that of orbit predictions using some recently published Soviet results which they obtained from their Luna-10 spacecraft. The Soviet paper presented a set of 11 coefficients from their analysis of the lunar gravitational field.

For comparison of the Luna-10 and Lunar Orbiter results, these same orbit prediction calculations have been made. The results are shown on slide 7 along with the short-arc data points from Lunar Orbiter I used for the previous comparison. Two curves are shown for the Soviet results because we think some of the signs of their coefficients should be changed to correspond to the coordinate axes we normally use. Whereas the predictions with the coefficients from the present analysis showed good agreement with the circles, neither of the Soviet sets agrees well, as shown on the slide. The conclusion is that the Soviet results do not correspond very well to experience with Lunar Orbiter.

Possible reasons for the disagreements in lunar gravitational field models may involve: misinterpretation of the results presented in the Soviet paper; or real differences in perturbations produced on satellites with highly different orbital parameters, leading to different sets of coefficients. The last possibility is a strong contender in view of the large difference in the inclinations of the satellites, 12° for Lunar Orbiter I and 72° for Luna-10. It is possible that the gravitational field model given here, which fits the Lunar Orbiter data fairly well, would not fit the data from Luna-10. Unfortunately, the data are not available with which to make a comparison. Perhaps at some future date there may be better agreement or a better explanation for the differences.

APPLICATION OF RESULTS TO DETERMINATION OF PHYSICAL PROPERTIES OF THE MOON

The second-degree gravitational field coefficients (that is, those with $n = 2$) can be related to the moments and products of inertia of the moon, to determine what the results of this analysis indicate with respect to the mass distribution in the moon. Definitions of the moments and products of inertia are shown on slide 8.

The equations on the slide show the relations between the gravitational coefficients and the moments and products of inertia of the moon. The quantities defined on the bottom of the slide, L, K, and g, can also be related to the coefficients, particularly $C_{2,0}$ and $C_{2,2}$, and these quantities will be used in later discussion.

It is apparent that there are six quantities involved in the moments and products of inertia, but there are only five second-degree coefficients from which to determine these six quantities. While the products of inertia are defined by particular coefficients, the moments of inertia are defined only in terms of relations involving sums and differences of the moments. Another relation is required to define the individual values of the moments of inertia.

One possibility for the additional relation is to make use of results obtained from previous earth-based analyses of the properties of the moon. But before defining a relation and before showing the application of the present results, we should digress for a few minutes and discuss some of the previously available results. We will also want to use the previous results as a basis for comparison with the results from this analysis.

Previous Results on Mass Distribution in the Moon

Questions concerning the moments of inertia of the moon, the differences in the moments of inertia, and the implications of these results with respect to the dynamics of the rotations of the moon, have occupied astronomers and mathematicians for more than 250 years.

Some indication of the mass distribution in the moon, in terms of quantities related to its moments of inertia, can be obtained through two separate kinds of analyses. These are indicated on slide 9. One approach is based on analyses pertaining to the theory of rotation of the moon, and the other is based on perturbations in the orbit of the moon. Both of these approaches have inherent difficulties. In the rotational theory approach, there is a troublesome problem in not being able to determine whether a particular parameter should be on one or the other side of a resonance value, a situation which affects the results. In the orbit perturbation approach, the differences between observed effects and calculated effects provide extremely small residuals, from which the parameters of the lunar moments of inertia must be determined.

<u>Rotation and physical libration of the moon</u>.- The first concise statements regarding the rotational properties of the moon were put forth in 1693 by J. D. Cassini in three emperical laws. These state that: (1) the moon rotates uniformly about its polar axis with a rotational period equal to the sidereal period of its orbit about the earth, (2) the inclination of the lunar equator to the ecliptic is a small constant angle (approximately 1-1/2°), and (3) that the poles of the lunar equator, of the ecliptic, and of the lunar orbit all lie in a plane in the order given. As a consequence of these laws, and the fact that the moon moves in an elliptical orbit, the moon appears to oscillate in the east-west and north-south directions as seen from the earth. These apparent oscillations are called the optical librations in longitude and latitude. With the diurnal libration, these oscillations allow about 59 percent of the surface of the moon to be viewed from the earth.

Cassini's Laws in effect define a steady state of motion, to a fairly high degree of precision. Only much later was it found that the moon exhibits, in addition to its steady motion, small oscillations about its center of mass, called physical librations, which are at least partially due to the forcing effects of the optical librations. The physical librations are difficult to detect because of their small amplitudes, which have a maximum of about 2 minutes of arc, as measured about the lunar axes, and thus less than 1 second of arc as seen from the earth.

As a consequence of mathematical developments with respect to the optical and physical librations of the moon, the quantities β, f, and γ can be determined. These quantities are defined in terms of the moments of inertia of the moon as shown on the slide.

Motion of the node and perigee of the moon's orbit.- The motions of the node and perigee of the moon's orbit are influenced by the effects of the sun, the earth, the other planets, and by the distribution of mass in the earth and moon. The contributions to these motions due to all known factors can be carefully calculated, and the remaining differences between the calculated and observed quantities can be attributed to the figure of the moon, plus unknown and unaccounted for effects. Results obtained in the past few years from artificial satellites of the earth have helped to remove uncertainties due to the figure of the earth. The differences between the observed and calculated effects are quite small relative to the observed motions, and the calculations accounting for the various effects require the most extreme care and accuracy.

When the differences in observed and calculated values are related to effects produced by the mass distribution in the moon, values can be obtained for the parameters L and K defined on the slide. In these relations M is the mass of the moon and a is its mean radius.

Comparison of results on mass distribution in the moon.- Various typical results from previous analyses are shown on slide 10. For comparison purposes, the results of Jeffreys for β and γ, and the results of Cook and Jeffreys for L and K are taken as a set, and the more recent results of Koziel and Eckert are taken as separate set. The earlier results for β and γ correspond to values of the mechanical ellipticity parameter, f, above the resonance value of 0.662, and the later results correspond to values below the resonance value. Similar remarks apply to the values of f derived from L and K, which are independently obtained, and whose determinations thus are not directly influenced by the discontinuity in f. The close agreement between the values of f obtained by Koziel and by Eckert through independent methods probably indicates that the true value of f is below the resonance value, and that f of about 0.64 is a reasonable value to be compared with values obtained from lunar satellite data.

The quantity g shown on the bottom part of the slide is a quantity related to the polar moment of inertia of the moon. g can be derived in two ways, from L and β, and from K and γ, as indicated on the slide. For homogeneous density distribution in the interior of the moon, g would be 0.6, and for a thin spherical shell, g would be 1.0.

Three of the four values shown for g are considerably larger than the 0.6 corresponding to homogeneous density distribution. In fact, these values for g are close to the value of 1.0 corresponding to a thin spherical shell, a situation which has been called the "hollow moon" phenomenon. Both the fact that the values exceed 0.6, and the amount by which they exceed this value are surprising.

The value of g for the earth is 0.50, and the value for the moon suggested by Jeffreys on the basis of reasonable hypotheses is about 0.6. The implication of values of g greater than 0.6 is of course that the density of the moon is greater in regions near the surface than it is deep in the interior. This is an area in which new data and detailed explanations are especially needed, because of the implications with respect to the internal composition, the origin, and the history of the moon.

Comparison of Present and Previous Results on Mass Distribution in the Moon

The previous results summarized on slide 10 can be used as a basis for comparison with corresponding results obtained from the gravitational field coefficients presented earlier. I have already mentioned that the quantities L and K can be obtained from the coefficients $C_{2,0}$ and $C_{2,2}$, from the present analysis, so these quantities can be compared directly with these previous results. Also, you recall that we need an additional relation to derive the moments of inertia from the gravitational field coefficients - we had only five coefficients to relate to six quantities. The additional relation* can be supplied by either one of the quantities β or γ from the physical libration results shown here.

The present and previous results are compared on slide 11. The information on the previous slide is repeated here and two additional columns have been added. First we look at the next to last column, which shows L, K, and γ derived from the gravitational field results presented in the table shown earlier. The two values of g are derived from the gravitational results with Koziel's values for β and γ.

L is somewhat smaller than that in the previous results, K is between the two values in the table but closer to the more recent value obtained by Eckert, and f is smaller than all the values in the table.

The value for g obtained from $\frac{L}{\beta}(0.68)$, is considered to be the more reliable value and is considerably less than three of the previous values. It is still rather large compared with the result of 0.6 for homogeneous density distribution. The value of g obtained from $\frac{K}{\gamma}$ is considerably larger.

I noted that f is smaller than would be anticipated from the previous results. On looking into the manner in which the gravitational coefficients enter these calculations, the indication of these results is that the value for one of the coefficients, $C_{2,2}$ is probably too

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*For the applications it is assumed that the principal moments of inertia, involved in β and γ, can be taken as the moments of inertia about the X, Y, and Z coordinate axes defined earlier.

large, because of its correlations with other coefficients. Nevertheless, the tentative conclusion is that the moments of inertia of the moon are greater than that obtained for a homogeneous density distribution.

I have noted that the value for the coefficient $C_{2,2}$ appears to be too large. Upon examination of the correlations between the gravitational coefficients, it is found that $C_{2,2}$ is highly correlated with another coefficient, $C_{4,2}$, as expected, and is essentially uncorrelated with any of the other coefficients. This means that these two coefficients are very difficult to separate with the data currently available, and it is possible that various linear combinations of the two would provide solutions which are equally as valid as the solution presented earlier. If so, these other solutions could give additional information on the mass distribution in the moon.

This possibility was investigated using some of the options built into the computational procedures. Starting with the gravitational field results from the next to last iteration, two additional solutions were obtained, one with $C_{2,2}$ fixed and another with $C_{4,2}$ fixed at their next to last values, while solving for all the other coefficients in the set. The results were as anticipated, and it was possible to establish a linear relation between these coefficients.

Now, in applying the linear relation between the coefficients as obtained from the additional solutions, we want the value of g to be consistent. That is to say we want g to be the same value, whether it is obtained from L and β, or from K and γ. I have used Koziel's values for β and γ, and with $\frac{L}{\beta}$ equal to $\frac{K}{\gamma}$ there is obtained a second linear relation to be satisfied.

The results from the additional solutions have been extrapolated to where they intersect this second linear relation. With these manipulations, the results shown in the last column of slide 11, for L, K, and g, are obtained.

These quantities represent a combination of the results obtained from analyses of optical and physical librations and the extrapolated results obtained from the present analysis. These last results are probably more reliable than the values for the set of quantities given in the previous column.

As compared with the other values shown on the slide, the value of 0.647 for g is less than the values in the next to last column and less than three of the values from previous results. As compared with $g = 0.6$, this value still indicates a moment of inertia greater than that corresponding to homogeneous density distribution in the interior of the moon, and this is one of the conclusions of the present analysis. This value for g will be used in some of the following discussions.

Density Distribution in the Moon

As mentioned previously, if g is greater than 0.6, this indicates that the density in the moon is greater near the surface than near the center of the moon. Now I want to consider possible radial density variations which correspond to these present results.

The first possibility which might be considered to account for the density distribution is that of temperature variations.

In a recent paper, a Soviet scientist, B. J. Levin has discussed the influence of thermal effects on the distribution of density in the moon. He indicates that the size of the moon is such that the increase of density toward the center of the moon due to gravitational effects, and the decrease due to thermal expansion effects are almost equal, and that it is difficult to determine which of these two effects is more important. The answer depends on the values for the coefficients of thermal expansion and compressibility of the lunar material, coefficients which are not known very precisely. Levin presents typical values of these coefficients derived from consideration of lunar materials. With his smallest reasonable value of the compressibility coefficient and his largest reasonable value for the thermal expansion coefficient, the increase in density from the center of the moon to the surface is about 6 percent. The maximum value of g obtained from such calculations is $g = 0.604$. While this value is somewhat greater than the value for homogeneous density distribution (0.6), it is considerably smaller than the value of 0.647 shown on the slide. The conclusion from this comparison is that thermal effects in the lunar interior cannot alone account for the density variations indicated by the results obtained in the present analysis.

As an indication of the variation of density required for the values of g obtained in this analysis, calculations have been made with various two-layer models for the moon. A few typical results from these calculations are shown on slide 12. Here σ_1 is the density of the inner layer, $\frac{\rho}{a}$ is the radius of the inner layer, σ_2 is the density required in the outer layer to give the proper mean density for the moon, and g is the parameter related to the moment of inertia. The table on the bottom of the slide gives some typical values of the density for various materials.

The calculations show that various combinations of the inner and outer densities, and radii of the layers, produce values of g around 0.647 or so.

For example, with inner layer density equal to that of the crust of the earth (2.6), an outer layer of 20 percent of the radius of the moon with density of 4.1 gives the desired value for g. In general, the results indicate that outer layer densities between the values for stony meteorites and stony-iron meteorites produce values for g consistent with the results presented here.

It is somewhat premature at this stage of the analysis to advance speculations concerning the internal composition of the moon and its history, but some indications are available from the discussions given here. Consider a model for the moon, with density deep in the interior equal to or somewhat greater than that of the outer regions of the earth, and with density in the outer regions of the moon comparable to a combined value for the densities of the earth's crust and stony and stony-iron meteorites. This model is consistent with the values of g from the present analysis. If the moon was originally composed of material similar to that of the outer regions of the earth, and if in later stages of development it was bombarded with materials with even a small iron content, then the outer regions would have a mixture of the lower and higher density materials. These indications are somewhat speculative, but the evidence from elementary observations of the large maria on the moon, which may be the result of large-scale impacts, is perhaps consistent with this model.

APPLICATION TO THE TOPOGRAPHY OF THE MOON

The final application of the results presented here is for determination of the figure of the moon.

The coefficients of the spherical harmonics in the expansion of the gravitational potential can be related to the coefficients of surface harmonics, representing the figure of the moon, if we make assumptions on the density distribution in the moon.

Contours of the topography of the moon, derived from the gravitational field results, with the assumption of uniform density distribution in the moon, are shown on slide 13. The contours represent differences in radius of the moon from the mean value of 1738 km. The positive contours represent regions of elevated surface, or regions of greater mass than the average and the negative regions represent the opposite effects. If the density distribution in the moon actually corresponded to the assumed density distribution, the contours would represent the actual figure of the moon, but if not, the contours represent deviations from the mean in either mass or elevation, or a combination of the two effects. The plots on the slide show that the maximum deviations from a spherical shape are about 2 km on the portion of the lunar surface facing the earth, and they are about 2.5 km on the far side. The variations in the topography are very much dependent on the values of the higher-degree and order coefficients, and thus are subject to the uncertainties in these coefficients.

For comparison with the topography plots derived with the assumption of homogeneous density distribution, an additional case has been calculated with a density distribution corresponding to the value for g (0.647) obtained from the present analysis. The additional contour plots are shown on slide 14. These plots show less variation from the mean value of 1738 km than the previous plots, as expected with the density distribution assumed. This plot indicates maximum deviations from the mean radius of about 1 km on the near side and they are about 2 km on the far side of the moon. The contours for both sets of figures generally indicate the same kinds of bulges and depressions, but the contours are shifted somewhat.

These topography plots do not have any recognizable correlations with the topographical results obtained from analysis of earth-based photographs, which is not surprising since the results from such analyses do not themselves show much correlation among the several solutions. The present results do not indicate a triaxial figure for the moon or any particularly well defined bulge toward the earth. In general, the results in these two slides are probably best interpreted as an indication of the density or mass distribution over the various regions of the moon.

On comparing either of the last two slides with photographs of the hemisphere of the moon facing the earth, there is a slight indication of correlations between the elevated contours and the maria regions on the moon, and between the depressed contours and the highland regions of the moon. This indication is not wholly consistent and is still somewhat uncertain at this time. However, if it is true that the maria regions represent regions of density greater than the mean density of the moon, then this would be of considerable interest in support of an hypothesis that the maria were formed by impact of higher density materials on the lunar surface, accompanied by large-scale melting of the surface material. And, as mentioned earlier, these higher density materials near the surface of the moon could produce values for g consistent with the values obtained in this analysis.

CONCLUDING REMARKS

The preliminary results for the gravitational field of the moon presented here, and the applications of these results to the determination of various properties on the moon, represent early efforts in a very new field of research. New data are becoming available almost continuously, and these analyses will be updated from time to time as sufficient data are accumulated.

These results and the results from the continuing analyses, provide new information for application to additional studies of the internal and external properties of the moon, and to new theories and hypotheses on the origin and history of the moon. With the research activity currently underway at Langley Research Center and at a number of other organizations throughout the country, there is every reason to expect, within the next few years, a considerable enhancement in knowledge of these fundamental questions about the moon.

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OUTLINE OF PRESENTATION

DESCRIPTION OF THE PROBLEM

- OBJECTIVES
- DATA REQUIRED

BRIEF DESCRIPTION OF PROCEDURES FOR THE ANALYSIS

PRELIMINARY RESULTS ON THE GRAVITATIONAL FIELD OF THE MOON

- DATA ARCS USED IN THE SOLUTION
- DISCUSSION OF THE SOLUTION

APPLICATIONS OF RESULTS

- ORBIT PREDICTION FOR LUNAR SATELLITES
- PHYSICAL PROPERTIES OF THE MOON

COMPARISONS WITH PREVIOUS KNOWLEDGE

CONCLUSIONS AND CONTINUING EFFORT

PARAMETERS TO BE DETERMINED

GRAVITATIONAL COEFFICIENTS, $C_{n,m}$ and $S_{n,m}$ in:

$$U = \frac{\mu}{r}\left[1 + \sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n} P_{n,m}(\sin \Phi)\left\{C_{n,m}\cos \lambda m + S_{n,m}\sin \lambda m\right\}\right]$$

OTHER PARAMETERS:

STATE OF THE SPACECRAFT (6 PARAMETERS)
RADIATION PRESSURE COEFFICIENTS
TRACKING STATION LOCATIONS
CONTROL JET AND GAS LEAK FORCES
VELOCITY OF LIGHT, C
INSTRUMENT AND MEASUREMENT BIASES

DEFINITIONS AND EQUATIONS

Φ_i OBSERVED QUANTITY AT TIME i

$\Phi_i(p_o)$ CALCULATED VALUE OF QUANTITY AT TIME i

p SET OF PARAMETERS TO BE CORRECTED

p_o INITIAL ESTIMATES FOR PARAMETERS

$$\Phi(p_o + \Delta p) = \Phi(p_o) + \frac{\partial \Phi(p_o)}{\partial p_o} \Delta p + \frac{1}{2} \frac{\partial^2 \Phi(p_o)}{\partial p_o^2} \Delta p^2 + \cdots \tag{1}$$

$$\Phi_i - \Phi_i(p_o) = \frac{\partial \Phi_i(p_o)}{\partial p_o} \Delta p \tag{2}$$

$$\Phi_i - \Phi_i(p_o) - \frac{\partial \Phi_i(p_o)}{\partial p_o} \Delta p = \varepsilon_i \tag{3}$$

$$Q = \left(\Phi - \Phi(p_o) - A\Delta p\right)^T W \left(\Phi - \Phi(p_o) - A\Delta p\right) = \varepsilon^T W \varepsilon \tag{4}$$

$$\text{WHERE } A = \frac{\partial \Phi_i(p_o)}{\partial p_o}$$

$$A^T W\left(\Phi - \Phi(p_o) - A\Delta p\right) = 0 \quad \text{OR} \quad A^T WA\, \Delta p = A^T W\left(\Phi - \Phi(p_o)\right) \tag{5}$$

$$\Delta p = \left(A^T WA\right)^{-1} A^T W\left(\Phi_i - \Phi_i(p_o)\right) \tag{6}$$

$$p_1 = p_o + \Delta p \tag{7}$$

DATA ARCS USED FOR THIS ANALYSIS

SPACECRAFT, LUNAR ORBITER	I		II	III	
ORBIT CONFIGURATION	1	2	1	1	2
BEGINNING OF DATA ARC	1966 AUG. 14	1966 SEPT. 17	1966 DEC. 8	1967 FEB. 8	1967 FEB. 12
TOTAL DAYS IN DATA ARC	4	24	21	4	4
APPROXIMATE ORBITAL PARAMETERS					
SEMI-MAJOR AXIS, a, km	2765	2670	2702	2744	2689
ECCENTRICITY, e	0.30	0.33	0.34	0.29	0.33
INCLINATION, deg	≈ 12	≈ 12	≈ 17.5	≈ 21	≈ 21

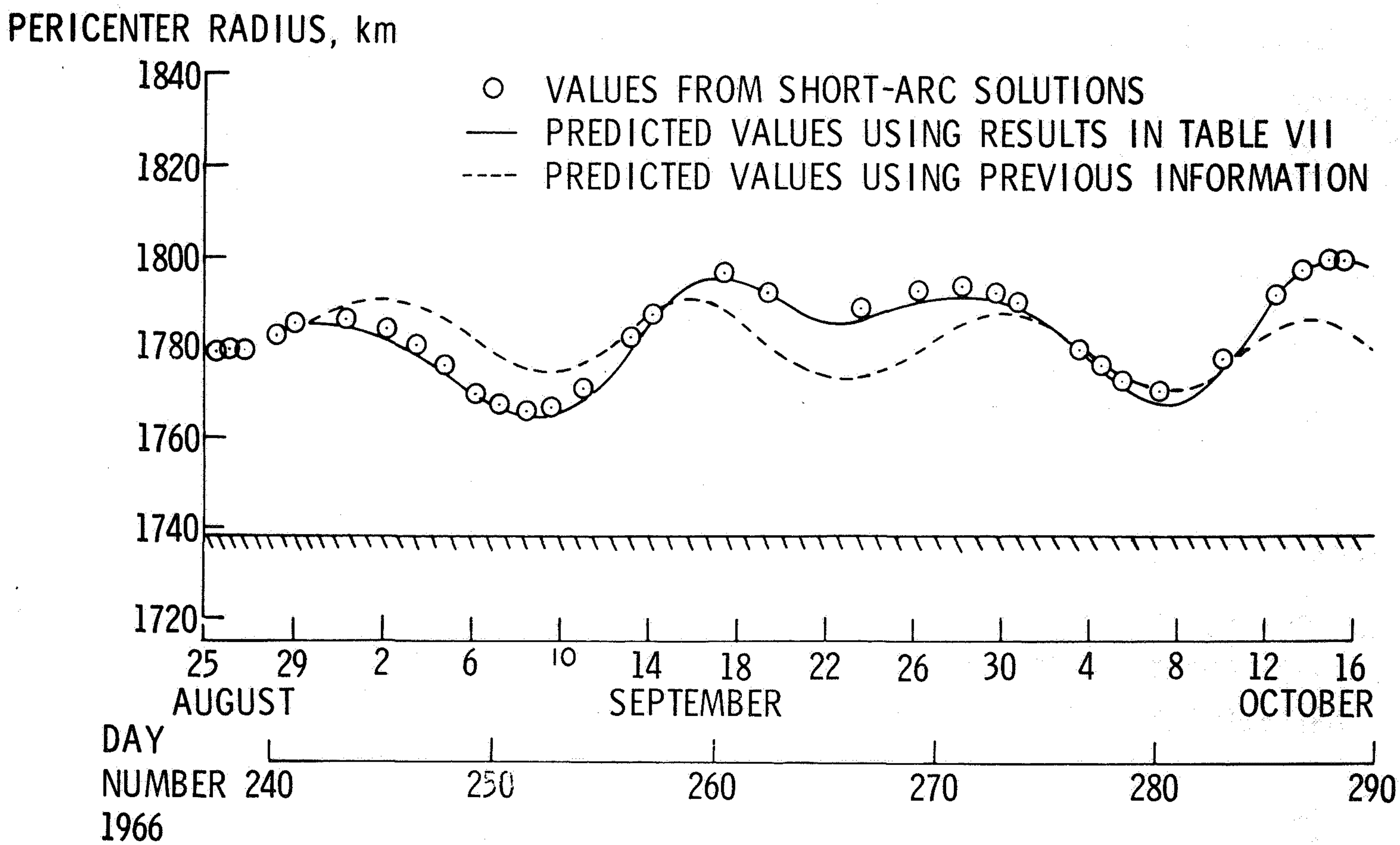
PRELIMINARY LUNAR GRAVITATIONAL FIELD COEFFICIENTS

C			S		
n,m	Coeff. x 10^4	Std. Dev. x 10^4	n,m	Coeff. x 10^4	Std. Dev. x 10^4
2,0	-2.2187	.185	--	--	--
2,1	.1250	.049	2,1	.2145	.053
2,2	.3204	.075	2,2	-.3052	.079
3,0	.2943	.029	--	--	--
3,1	.2350	.049	3,1	.3934	.050
3,2	.1933	.027	3,2	.1440	.025
3,3	-.0317	.028	3,3	.0831	.027
4,0	-.0418	.156	--	--	--
4,1	.0163	.055	4,1	.1908	.037
4,2	-.0276	.014	4,2	-.1035	.016
4,3	.0179	.008	4,3	.0076	.009
4,4	-.0014	.005	4,4	-.0147	.005
5,0	--	--	--	--	--
5,1	-.1274	.037	5,1	.1951	.038
5,2	.0809	.007	5,2	-.0015	.006
5,3	-.0102	.003	5,3	.0058	.003
5,4	.0030	.0005	5,4	-.0044	.0005
5,5	-.0009	.0005	5,5	.0017	.0005

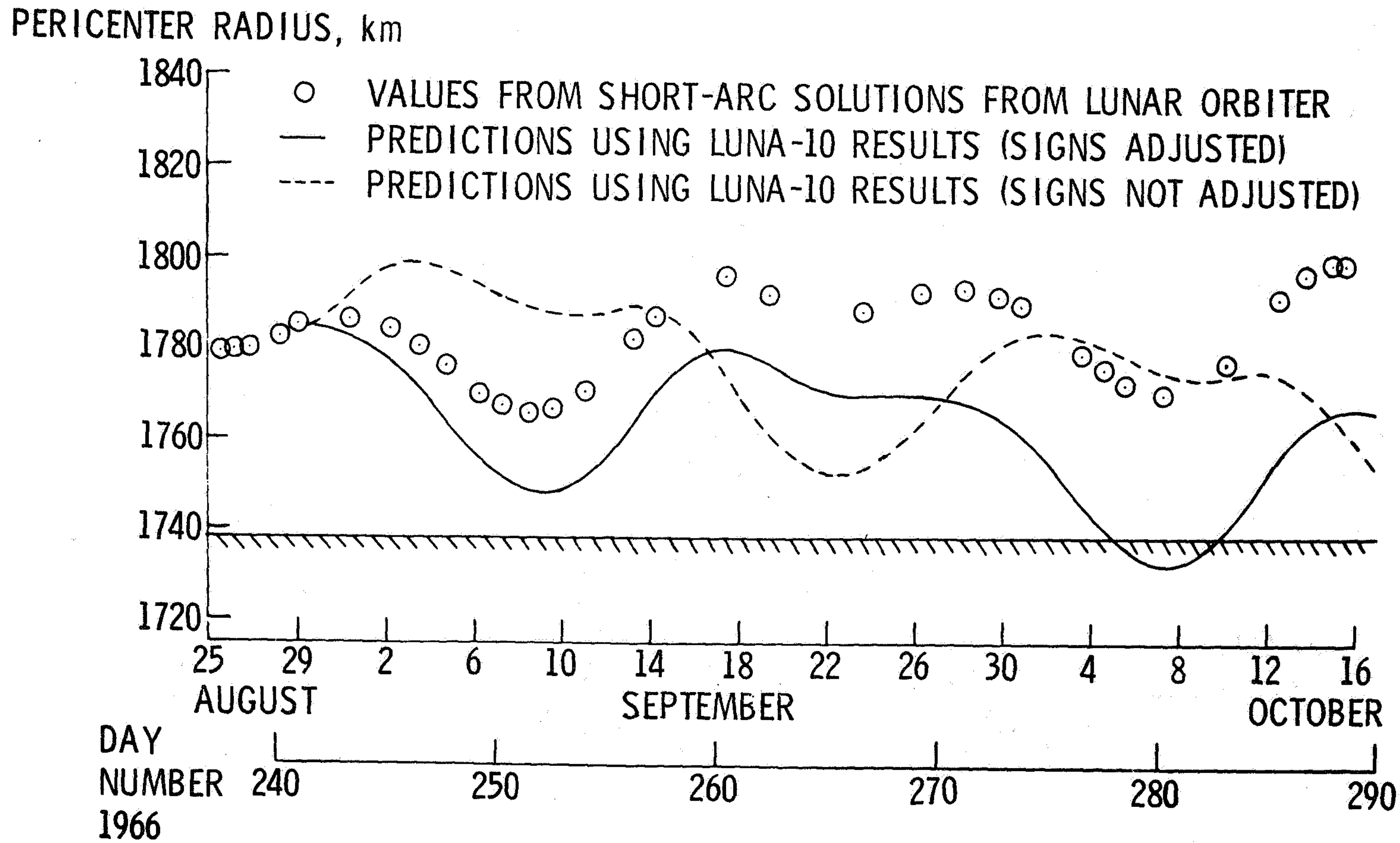
$$\sigma_{n,m} = \sqrt{C_{n,m}^{\;2} + S_{n,m}^{\;2}}$$

Slide 5

PERICENTER VARIATION COMPARISONS FOR LUNAR ORBITER I



PERICENTER VARIATION COMPARISONS FOR LUNAR ORBITER AND LUNA-10



Slide 7

RELATIONS BETWEEN GRAVITATIONAL COEFFICIENTS AND MOMENTS AND PRODUCTS OF INERTIA

$A = I_{xx}$ ABOUT AXIS IN EQ. PLANE POINTING TOWARD EARTH

$B = I_{yy}$ ABOUT $\perp$ AXIS IN LUNAR EQ. PLANE

$C = I_{zz}$ ABOUT LUNAR POLAR AXIS

$D = I_{yz}$ $E = I_{xz}$ $F = I_{xy}$

$$C_{2,1} = \frac{E}{Ma^2} \qquad S_{2,1} = \frac{D}{Ma^2} \qquad S_{2,2} = \frac{F}{2\,Ma^2}$$

$$C_{2,0} = \frac{1}{Ma^2}\left[\frac{A+B}{2} - C\right] \qquad C_{2,2} = \frac{1}{4\,Ma^2}(B - A)$$

$$L = \frac{3}{2}\,\frac{(C - A)}{Ma^2} = 3\,C_{2,2} - \frac{3}{2}\,C_{2,0}$$

$$K = \frac{3}{2}\,\frac{(B - A)}{Ma^2} = 6\,C_{2,2}$$

$$g = \frac{3}{2}\,\frac{C}{Ma^2}$$

CLASSICAL PROCEDURES FOR DETERMINATION OF MASS DISTRIBUTION IN THE MOON

I THEORY OF ROTATION OF THE MOON

CASSINI'S LAWS
OPTICAL LIBRATIONS
PHYSICAL LIBRATIONS

OPTICAL LIBRATIONS $\beta = \frac{C - A}{C}$

PHYSICAL LIBRATIONS $f = \frac{C - B}{C - A}$

$\gamma = \frac{B - A}{C}$

II MOTION OF NODE AND PERIGEE OF MOON'S ORBIT

ACCOUNT FOR ALL KNOWN PERTURBATIONS
ATTRIBUTE RESIDUALS TO LUNAR MOMENTS OF INERTIA

$$L = \frac{3}{2}\,\frac{C - A}{Ma^2}$$

$$K = \frac{3}{2}\,\frac{B - A}{Ma^2}$$

PREVIOUS RESULTS PERTAINING TO MASS DISTRIBUTION IN THE MOON

	JEFFREYS (1961)	COOK (1959) AND JEFFREYS (1961)	KOZIEL (1967)	ECKERT (1965)
$\beta = \frac{C - A}{C}$	6.279×10^{-4}		6.29×10^{-4}	
$\gamma = \frac{B - A}{C}$	2.049×10^{-4}		2.31×10^{-4}	
$L = \frac{3}{2} \frac{C - A}{Ma^2}$		5.46×10^{-4}		6.07×10^{-4}
$K = \frac{3}{2} \frac{B - A}{Ma^2}$		1.07×10^{-4}		2.19×10^{-4}
$f = \frac{\beta - \gamma}{\beta} = 1 - \frac{K}{L}$	0.674	0.804	0.633	0.638
$g = \frac{3}{2} \frac{C}{Ma^2}$ $\frac{L}{\beta}$	0.870		0.965	
$g = \frac{3}{2} \frac{C}{Ma^2}$ $\frac{K}{\gamma}$	0.522		0.949	

Slide 10

COMPARISON OF RESULTS PERTAINING TO MASS DISTRIBUTION IN THE MOON

		JEFFREYS	COOK	KOZIEL	ECKERT	PRESENT RESULTS AS PRESENTED	PRESENT RESULTS EXTRAPOLATED
$\beta = \frac{C - A}{C} \left(\times 10^4\right)$		6.279		6.29			(6.29)
$\gamma = \frac{B - A}{C} \left(\times 10^4\right)$		2.049		2.31			(2.31)
$L = \frac{3}{2} \frac{C - A}{Ma^2} \left(\times 10^4\right)$			5.46		6.07	4.29	4.07
$K = \frac{3}{2} \frac{B - A}{Ma^2} \left(\times 10^4\right)$			1.07		2.19	1.92	1.29
$f = 1 - \frac{K}{L}$		0.674	0.804	0.633	0.638	0.55	0.633
$g = \frac{3}{2} \frac{C}{Ma^2}$	$\frac{L}{\beta}$	0.870		0.965		0.68	0.647
	$\frac{K}{\gamma}$	0.522		0.949		0.83	0.647

RESULTS FROM CALCULATIONS OF TWO-LAYER DENSITY MODELS FOR THE MOON

σ_1	$\frac{\rho}{a}$	σ_2	g
2.4	0.7	3.83	0.645
2.6	.8	4.12	.650
2.8	.9	4.79	.650
3.0	.95	5.38	.636

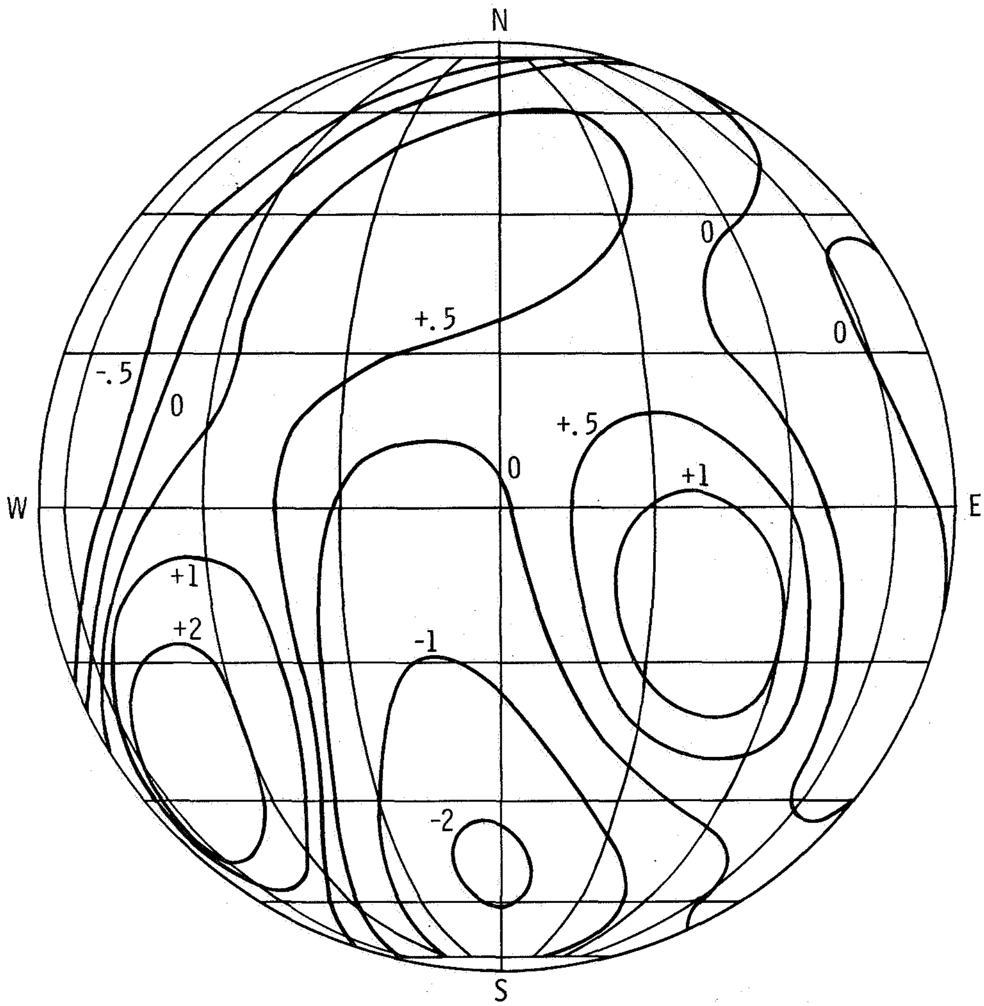
σ_1 = DENSITY OF INNER LAYER (gm/cm^3) $\frac{\rho}{a}$ = RADIUS OF INNER LAYER

σ_2 = DENSITY OF OUTER LAYER

TYPICAL DENSITIES

MEAN DENSITY OF MOON	3.34 gm/cm^3
EARTH CRUST	2.6
STONY METEORITES	3.0 TO 3.5
STONY-IRON METEORITES	5.5 TO 6.0
IRON METEORITES	7.5 TO 8.0

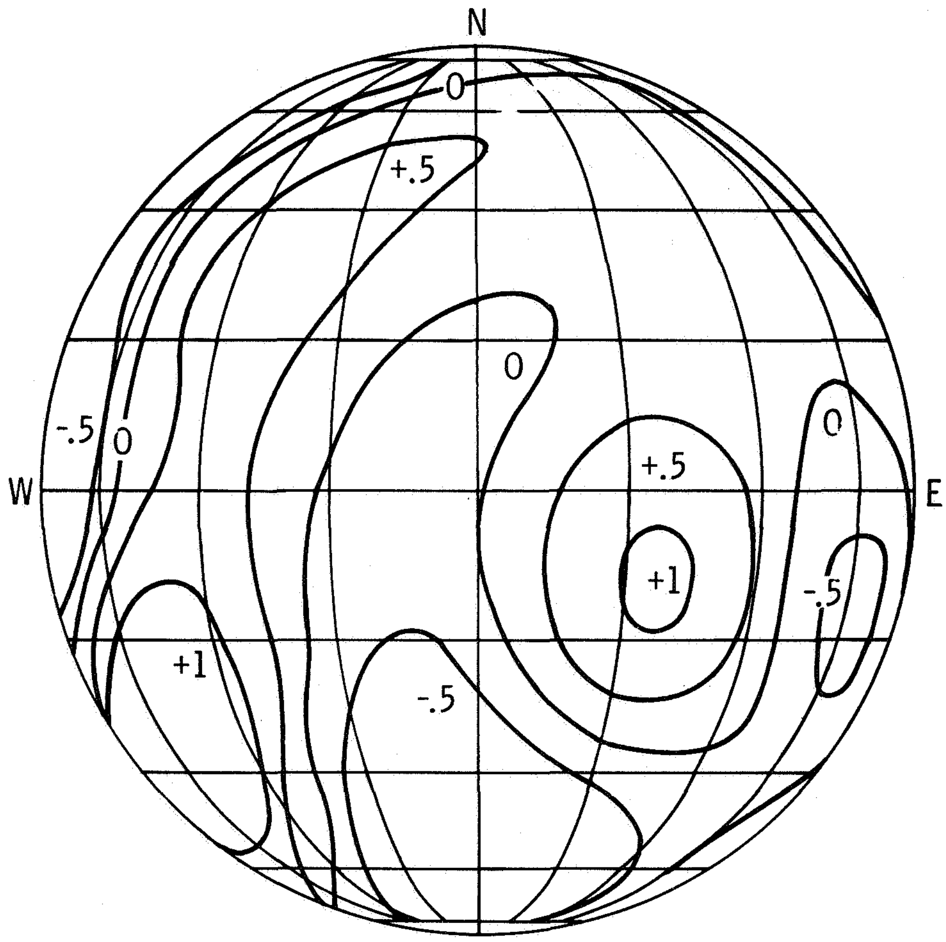
TOPOGRAPHY DERIVED FROM COEFFICIENTS



DEVIATIONS IN km FROM MEAN RADIUS OF 1738 km
HEMISPHERE FACING EARTH

Slide 13

TOPOGRAPHY DERIVED FROM COEFFICIENTS



DEVIATIONS IN km FROM MEAN RADIUS OF 1738 km
HEMISPHERE FACING EARTH
(DENSITY VARYING WITH RADIUS)

Slide 14

NASA-Langley, 1967