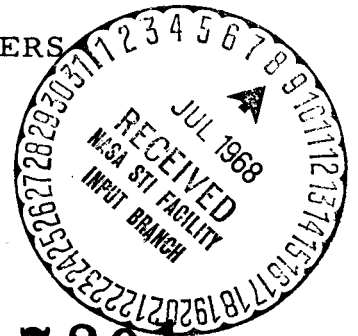# USE OF ANALOG COMPUTER FOR THE

EQUALIZATION OF ELECTROMAGNETIC SHAKERS

IN TRANSIENT TESTING*

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SUMMARY

A method which uses an analog computer to perform an open-loop equalization for multi-shaker environmental vibration testing is presented. The computer is programmed to simulate the equations of motion of the electromechanical system. The input of the computer is an electrical signal representing the acceleration time histories to be reproduced at the base of the spacecraft. An example of multi-shaker equalization is shown for transient testing on a beam excited at two points. The method is valid for any type of vibration testing.

INTRODUCTION

Environmental transient vibration testing is a recent type of testing which has been performed on some of the latest spacecraft at the Jet Propulsion Laboratory (JPL). In this paper, transient vibration testing implies transient signal of low-frequency content, say below 200 cps, in contrast to shock testing, for which the transient signal has a frequency content much above this limit. The basic problem is to reproduce a given acceleration transient time history at the points of attachment of the spacecraft on the carrying vehicle.

For the spacecraft tested at JPL, the transient time history was derived from flight data (refs. 1 and 2), but transient requirements of other origins could also be specified. In any case, in order to use the method of testing described later, it is necessary to have available an electrical signal representing the time history of the transient acceleration to be reproduced on the strp ̄e.

The method presented here for the simulation of transients is restricted to linear structures. It makes use of electrodynamic shakers, which are well suited to reproduce given time histories in the low-frequency range, due to

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the ease of converting an electrical signal into a force. However, when the shakers are hooked on the structure, their motional impedance is strongly influenced by the structure in such a way that it is necessary to place an equalizer between the input signal to be reproduced and the power amplifier that drives the shaker.

The methods of equalization currently used in environmental vibration testing for single-shaker excitation (servo system for sine sweep testing and automatic equalizer for random testing) fail for transient testing since those methods are specifically designed for steady- or quasi-steady-state excitation. The peak and notch filter bank used before the advent of the automatic equalizer, although not restricted to steady-state excitation, is nevertheless not entirely satisfactory, since phase coherence is lost through the filters. Therefore an analog equalization method that is valid for non-steady-state excitation and assures phase coherence has been sought in this paper.

This analog equalization is an open-loop method which simulates the differential equations of motion of the electromechanical system on an analog computer to shape the signal to be presented to the shaker. The analog equalization was first tested for the classical single-shaker environmental excitation (ref. 1); here it is extended to the case of multi-shaker excitation. The multi-shaker environmental vibration testing has been gaining favor in recent years because of the increasing size of the structures to be tested and because of the possibility of more realistic testing (ref. 3).

However, the problem of equalization of multi-shaker excitation is a difficult one even for steady-state operation. The open-loop analog technique presented here gives a solution to that problem for both random and transient excitations. The proper equations of motion will be derived in this paper and the corresponding analog schematic indicated. An example of a structure excited at two points by a transient acceleration will be considered.

EQUATIONS OF MOTION FOR ANALOG EQUALIZATION

Consider a structure (fig. 1) on which we want to run a transient environmental vibration test with a multi-shaker system. The structure has p degrees of freedom and it is excited by N shakers. Each shaker has to produce a prescribed acceleration $\ddot{u}_s$ ($s = 1, 2, \ldots, N$) in a prescribed direction. We note that up to three shakers could be attached to the same point. The equalization problem consists in determining the time histories of the shaker voltages e_1, $e_2 \ldots e_N$ required to reproduce the given accelerations $\ddot{u}_1$, $\ddot{u}_2$, $\ldots$, $\ddot{u}_N$. The equations of motion of the system have been derived in reference 3, in which each shaker has been approximated by a one-degree-of-freedom system. This assumption is valid for low enough frequencies and will be kept here. From reference 3, these equations are

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = [A]^T\{F\} \tag{1}$$

$$\{u\} = [A]\{y\} \tag{2}$$

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$$\{F\} = [A]\{i\} - [M']\{\ddot{u}\} - [C']\{\dot{u}\} - [K']\{u\} \tag{3}$$

$$\{e\} = [R]\{i\} + [L]\left\{\frac{di}{dt}\right\} + [A]\{\dot{u}\} \tag{4}$$

where

$$\{y\} = \begin{Bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_p \end{Bmatrix} = \text{a system of } p \text{ independent coordinates}$$

$$\{u\} = \begin{Bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_N \end{Bmatrix} = \text{a column of the shaker displacements}$$

$[M]$ = the mass matrix

$[C]$ = the damping matrix

$[K]$ = the stiffness matrix

$[A]$ = a transformation matrix that relates $\{u\}$ to $\{y\}$

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ \cdot \\ F_N \end{Bmatrix} = \text{a column of the shaker forces}$$

$[M'] = \begin{bmatrix} \ddots & & \\ & m'_s & \\ & & \ddots \end{bmatrix}$ = the mass matrix for the moving elements of the shakers

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$$[C'] = \begin{bmatrix} \ddots & \\ & c'_s \\ & & \ddots \end{bmatrix} = \text{the damping matrix of the shakers}$$

$$[K'] = \begin{bmatrix} \ddots & \\ & k'_s \\ & & \ddots \end{bmatrix} = \text{the stiffness matrix of the shakers}$$

$$[R] = \begin{bmatrix} \ddots & \\ & R_s \\ & & \ddots \end{bmatrix} = \text{the resistance matrix of the shakers}$$

$$[L] = \begin{bmatrix} \ddots & \\ & L_s \\ & & \ddots \end{bmatrix} = \text{the inductance matrix of the shakers}$$

$$[\Lambda] = \begin{bmatrix} \ddots & \\ & \lambda_s \\ & & \ddots \end{bmatrix} = \text{the force-current coefficient matrix of the shakers}$$

$$\{e\} = \begin{Bmatrix} e_1 \\ e_2 \\ \vdots \\ \vdots \\ e_N \end{Bmatrix} = \text{the column of the shaker voltages}$$

In the system of Eqs. (1) through (4), the acceleration column $\{\ddot{u}\}$ is known and the voltage column $\{e\}$ has to be determined. The objective of the analog equalization is to simulate these equations with an analog computer in order to produce $\{e(t)\}$ for the given $\{\ddot{u}(t)\}$ (fig. 2).

We note that a fast analog computer made of high-speed operational amplifiers with a flat frequency response up to at least 1000 cps must be used in order to take advantage of an on-line hook-up with the power amplifiers, permitting real-time operation.

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Before programming Eqs. (1) through (4) on the computer, a few remarks are in order. At least two considerations limit the use of an analog computer:

(1) The size of the computer, i.e., the number of operational amplifiers that are available.

(2) The susceptibility to feedback instability, a situation that can occur whenever an even number of operational amplifiers is used in the feedback loops of coupled equations.

Therefore one must choose the proper set of coordinates $y_1, y_2, \ldots, y_p$ that will minimize the size of the equations and further eliminate the coupling if possible. We first note that Eq. (1) does not represent a forced vibration problem since $F_1, F_2, \ldots, F_N$ are unknown forces. Instead, the $\ddot{u}_1, \ddot{u}_2, \ldots, \ddot{u}_N$ are the forcing functions for the equations. Consequently, we seek to introduce $\ddot{u}_1, \ddot{u}_2, \ldots, \ddot{u}_N$ in Eq. (1). To this end, we choose for $y_1, y_2, \ldots, y_p$ a system of coordinates which is formed by all the shaker displacements $u_1, u_2, \ldots, u_N$ and the normal coordinates $q_1, q_2, \ldots, q_r$ ($r = p-N$) of the structure in a configuration tied down at the points of excitation, i.e., for $u_1 = u_2 = \ldots = u_N = 0$. This means that $q_1, q_2, \ldots, q_r$ represent the normal coordinates in a relative motion with respect to the shaker armatures. We have

$$\{y\} = \left\{ \begin{array}{c} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_N \\ \hline q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_r \end{array} \right\} = \left\{ \frac{u}{q} \right\} \tag{5}$$

The transformation matrix $[A]$ becomes

$$[A] = [I \mid O] \tag{6}$$

where

$[I]$ = the identity matrix.

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The matrices $[M]$, $[C]$, and $[K]$ are partitioned accordingly, and Eqs. (1) and (2) become

$$\begin{bmatrix} M^{uu} & M^{un} \\ \hline M^{nu} & M^{nn} \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} C^{uu} & C^{un} \\ \hline C^{nu} & C^{nn} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{q} \end{Bmatrix}$$

$$+ \begin{bmatrix} K^{uu} & K^{un} \\ \hline K^{nu} & K^{nn} \end{bmatrix} \begin{Bmatrix} u \\ q \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} \tag{7}$$

where the mass, damping, and stiffness matrices are symmetric.

The matrix $[K^{un}]$, which represents the stiffness coupling between the shaker motion and the normal mode motion, is a null matrix. Indeed, since the variables q_1, q_2, ..., q_r are relative to the shaker motion, there should be no modal response q_1, q_2, ..., q_r for static forces applied through the shaker. The matrix $[K^{nu}]$, which is the transpose of $[K^{un}]$, is also null:

$$[K^{un}] = [K^{nu}] = 0 \tag{8}$$

Nothing of the sort can be said for the damping coupling matrices $[C^{un}]$ and $[C^{nu}] = [C^{un}]^T$; in general, those matrices are not null. However, if the damping is limited to internal structural damping, each term of the damping matrix is proportional to the stiffness. On this basis, it is usually assumed that if a term of the stiffness matrix is zero, then the corresponding term of the damping matrix is also zero. Therefore, in this case we can write

$$[C^{un}] = [C^{nu}] = 0 \tag{9}$$

Since, in general, the matrix $[C^{un}]$ will be very difficult, if not impossible, to determine, one would have to make the assumption of Eq. (9) to be able to proceed. These matrices, together with the stiffness matrix $[K^{uu}]$ and the damping matrix $[C^{uu}]$, are effectively null for a statically determinate excitation. In this case the displacements u_1, u_2 ..., u_N become rigid-body displacements and we have (ref. 4)

$$[K^{uu}] = 0 \tag{10}$$

$$[C^{uu}] = [C^{un}] = [C^{nu}] = 0 \tag{11}$$

The matrix $[M^{uu}]$ becomes the mass matrix for the rigid-body modes, and the matrices $[M^{un}] = [M^{nu}]^T$ are the rigid -- elastic couplings (ref. 4). The statically determinate motion will occur for $N \leq 6$ for a three-dimensional motion and $N \leq 2$ for a plane motion.

The matrices $[M^{nn}]$ and $[K^{nn}]$ are diagonal matrices, and we will assume that the damping matrix $[C^{nn}]$ is also diagonal. Expanding Eqs. (3), (4), and (7), we obtain

$$m_j^{nn} \ddot{q}_j + c_j^{nn} \dot{q}_j + k_j^{nn} q_j = - \sum_{s=1}^{n} m_{js}^{nu} \ddot{u}_s - \sum_{s=1}^{n} c_{js}^{nu} \dot{u}_s \tag{12}$$

$$j = 1, 2, \cdots, r$$

$$F_s = \sum_{i=1}^{N} m_{si}^{uu} \ddot{u}_i + \sum_{i=1}^{N} c_{si}^{uu} \dot{u}_i + \sum_{i=1}^{N} k_{si}^{uu} u_i$$

$$+ \sum_{j=1}^{r} m_{sj}^{un} \ddot{q}_j + \sum_{j=1}^{r} c_{sj}^{un} \dot{q}_j \tag{13}$$

$$s = 1, 2, \cdots, N$$

$$i_s = \frac{1}{\lambda_s} \left(F_s + m'_s \ddot{u}_s + c'_s \dot{u}_s + k'_s u_s \right) \tag{14}$$

$$e_s = R_s i_s + L_s \frac{di_s}{dt} + \lambda_s \dot{u}_s \tag{15}$$

This is the system of equations that can be programmed with operational amplifiers on an analog computer. The system is electrically stable if we proceed as follows. We remark that the only differential equations that we have to solve are the equations of the subsystem (12) which represent the forced oscillation of r-independent, one-degree-of-freedom systems due to the linear combination of the known $\ddot{u}$'s and $\dot{u}$'s. These equations are obviously uncoupled in terms of the variables q_1, q_2, $\cdots$, q_r, since these variables are normal coordinates, and hence they lead to a stable electrical network.

Further, each equation in (12) gives responses $\ddot{q}_j$ and $\dot{q}_j$, which are easily combined linearly together and with the u's to give the N forces F_s according to Eq. (13). The shaker currents i_s and then the shaker voltages e_s are obtained by linear combination of the known quantities F_s and u_s on operational summers with no stability problem.

EXAMPLE OF A BEAM EXCITED AT TWO POINTS

Structure

The technique of analog equalization for multi-shaker excitation was tested on a Plexiglas beam of rectangular cross section 3/4 x 2 x 51 in., excited at two points, P_1 and P_2 (fig. 3). The first five natural modes in the tied-down configuration at P_1 and P_2 were taken to represent the structure, spanning a frequency range from 0 to 250 cps. Since it is a plane motion and only two shakers are used, the excitation is statically determinate and the matrices $[C^{uu}]$, $[C^{un}]$, and $[K^{uu}]$ are null. Equations (12) through (15) become

$$m_j^{nn}\ddot{q}_j + c_j^{nn}\dot{q}_j + k_j^{nn}q_j = -m_{j1}^{nu}\ddot{u}_1 - m_{j2}^{nu}\ddot{u}_2 \qquad (16)$$

$$j = 1, 2, 3, 4, 5$$

$$\lambda_1 i_1 = m_{11}^{uu}\ddot{u}_1 + m_{12}^{uu}\ddot{u}_2 + \sum_{j=1}^{5} m_{1j}^{un}\ddot{q}_j + m_1'\ddot{u}_1 + c_1'\dot{u}_1 + k_1'u_1 \qquad (17)$$

$$\lambda_2 i_2 = m_{21}^{uu}\ddot{u}_1 + m_{22}^{uu}\ddot{u}_2 + \sum_{j=1}^{5} m_{2j}^{un}\ddot{q}_j + m_2'\ddot{u}_2 + c_2'\dot{u}_2 + k_2'u_2 \qquad (18)$$

$$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + \lambda_1 \dot{u}_1 \qquad (19)$$

$$e_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + \lambda_2 \dot{u}_2 \qquad (20)$$

These equations were programmed with operational amplifiers (44 amplifiers were needed, see Appendix A). The numerical coefficients for the equations are indicated in Appendix A, together with the methods used to obtain them and the electrical network. The block diagram for the test is shown in figure 4.

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Transient Testing

Once the analog equalizer is set for a given structure, the setting is valid for any kind of specified excitation. The only requirement is that the desired time history of the accelerations $\ddot{u}_1(t)$ and $\ddot{u}_2(t)$ be available in the form of an electrical signal. A flight-type transient was available on magnetic tape and the time history of the transient is shown in figure 5a. Most of the frequency content of this transient is between approximately 10 cps and 250 cps. It was postulated that the test would be the reproduction of this same transient at both locations P_1 and P_2. The two inputs of the analog computer were driven by the same transient from the tape, and the responses $\ddot{u}_1$ and $\ddot{u}_2$ at points P_1 and P_2 were monitored and recorded on magnetic tape. Figures 5b and 5c show the two time histories that were obtained. A perfect equalization would give three identical figures. Comparing the three figures, it can be seen that as a whole the two transients were very well reproduced with the correct levels. It is only in the detail that differences exist, and those differences remain small. Therefore, we will say that the system was properly equalized.

CONCLUSION

It has been shown in this paper that an analog equalization is possible for multi-shaker environmental vibration testing of linear structures. The proper equations have been written and were applied successfully for the equalization of a simple beam excited at two points. Nevertheless it is realized that obtaining the coefficients for the equations and setting the corresponding potentiometers of the analog result in a complex operation. Therefore, a mechanization of the operation would have to be developed for any application to practical environmental vibration testing.

Finally, we recall that the analog equalization is limited to linear structure and is an open-loop method with the built-in disadvantage that any change in the structure during the test would not be corrected by the equalizer. In this respect a closed-loop system would not have this disadvantage; however, a closed-loop system does not seem possible in the present state of the art for transient vibration testing, since the control would have to be done on instantaneous values rather than on average values.

Before closing, we note that the analog equalizer is also valid for multi-shaker random vibration testing with the advantage of reproducing the proper correlation between shakers. Tests have been made with correlated and uncorrelated band-limited random noise inputs at P_1 and P_2, and the responses $\ddot{u}_1$ and $\ddot{u}_2$ have been satisfactorily reproduced.

REFERENCES

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2. Trubert, M. R., A Fourier Transform Technique for the Prediction of Torsional Transients for a Spacecraft from the Flight Data of Another Spacecraft Using the Same Booster, Technical Memorandum 33-350. Jet Propulsion Laboratory, Pasadena, Calif., Oct. 1967.

3. Trubert, M. R., "Structural and Electromechanical Interaction in the Multiple Exciter Technique for Random Vibration Testing," J. Acoust. Soc. Am., Vol. 41, No. 5, May 1967.

4. Hurty, W. C., Dynamic Analysis of Structural Systems by Component Mode Synthesis, Technical Report 32-530, Jet Propulsion Laboratory, Pasadena, Calif., Jan. 1964.
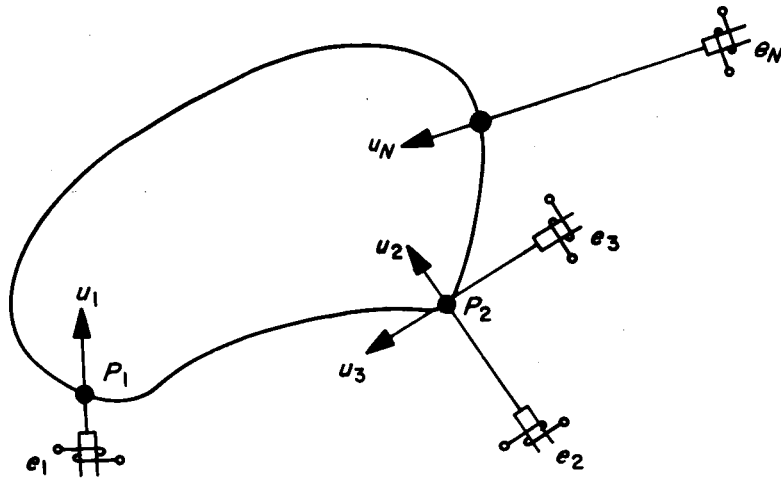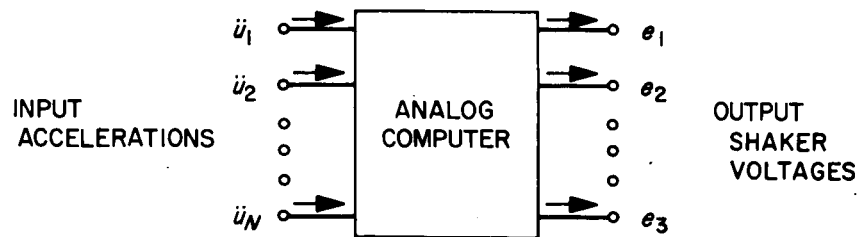
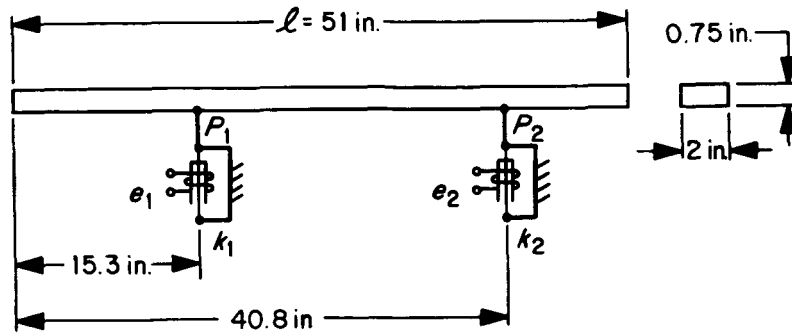Figure 1.- Multi-shaker excitation.

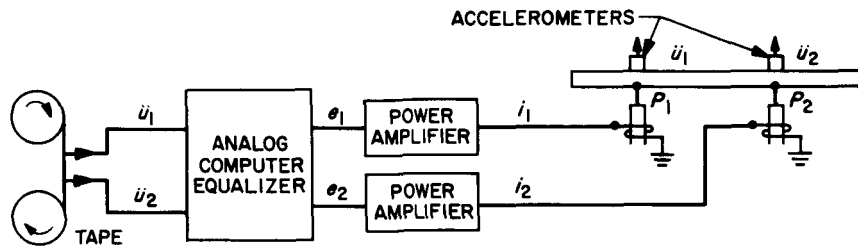Figure 2.- Analog equalizer.

Figure 3. - Beam excited by two shakers.

Figure 4. - Beam test setup.

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(a)
SPECIFIED
ACCELERATION
(INPUT TO COMPUTER)

(b)
BEAM ACCELERATION
POINT P_1

(c)
BEAM ACCELERATION
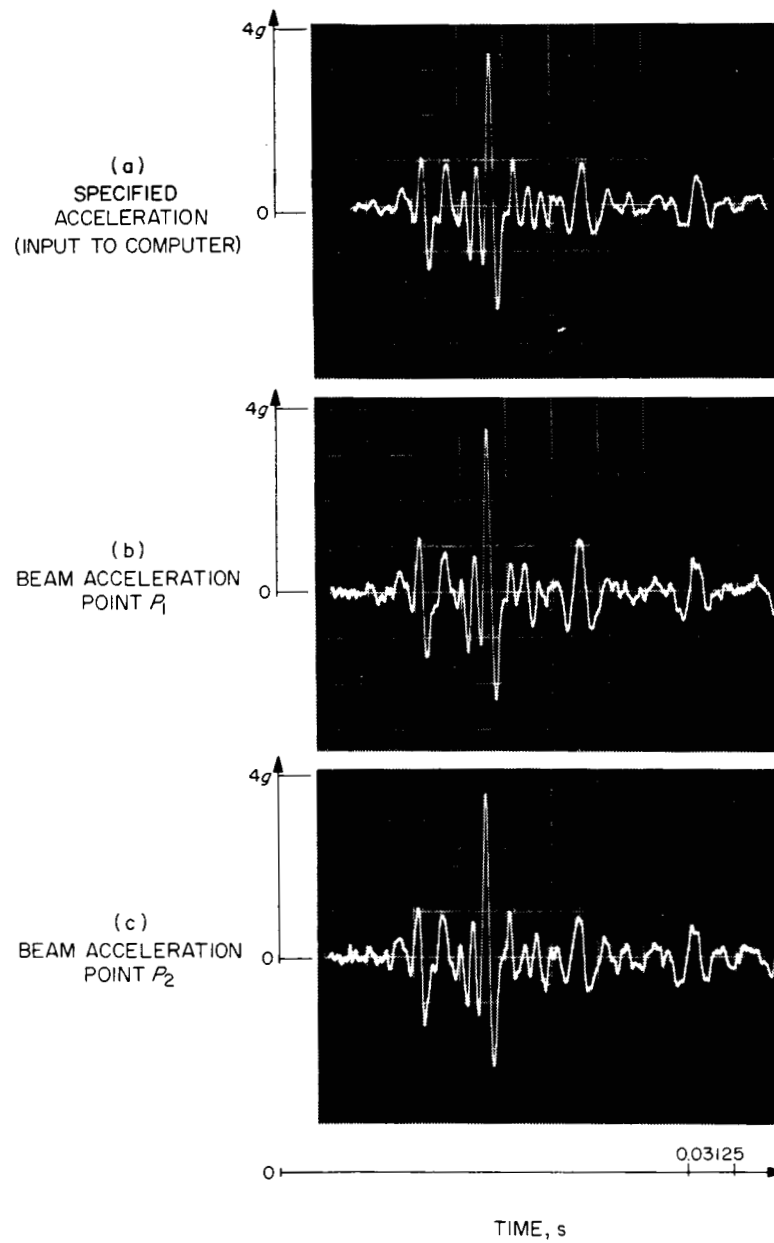POINT P_2

0.03125

TIME, s

Figure 5. - Transient testing.

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APPENDIX A

DETERMINATION OF THE EQUALIZER COEFFICIENTS

SHAKER CHARACTERISTICS

The exciters are two 20-lb shakers, the characteristics of which were determined experimentally before the test. Table A-1 shows the values of these characteristics.

Table A-1. Shaker Characteristics

Designation	Shaker No. 1	Shaker No. 2
Resistance R, ohms	3.85	3.53
Inductance L, mH	1	1
Force-current coef λ, newton/A	12.99	12.86
Mass of moving assembly m', kg	0.377	0.377
Flexure stiffness k', newton/m	2140	2080
Damping c', newton/m/s	0.860	0.846

BEAM CHARACTERISTICS

The rigid-body mass matrix $[M^{uu}]$ was computed from the geometry and the mass properties of the beam and its attachments, including accelerometer pick-ups and armature masses m'_1 and m'_2 previously computed.

The matrix is

$$[M^{uu}] = \begin{bmatrix} 1.16 & -0.152 \\ -0.152 & 1.48 \end{bmatrix} kg$$

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Several different techniques can be used to obtain the dynamic characteristics of the elastic beam. We first note that the generalized masses m_j^{nn} can be set to unity by proper normalization, and the remaining coefficients that need be determined are c_j^{nn}, k_j^{nn}, and m_{js}^{nu} for $j = 1, 2, 3, 4, 5$ and $s = 1, 2$.

Method 1

The first method is a modal approach for which the natural modes of the beam in the configuration tied down at P_1 and P_2 are either computed or measured experimentally in a separate test. This method gives the natural frequencies f_j and the mode shapes $\{\phi_j^n\}$ from which one can get k_j^{nn} and m_{js}^{nu}:

$$k_j^{nn} = 4\pi^2 f_j^2 m_j^{nn} \qquad \left(m_j^{nn} = 1\right) \tag{A-1}$$

$$m_{js}^{nu} = \left\{\phi_j^n\right\}^T \left[\diagdown m \diagdown\right] \left\{\phi_s^u\right\} \tag{A-2}$$

where $\left[\diagdown m \diagdown\right]$ is the mass distribution of the structure and $\left\{\phi_s^u\right\}$ are the mode shapes of the rigid-body modes. The coefficients C_j^{nn} are obtained by introducing a modal reduced damping ξ_j (percent of critical damping), which can be measured from decay curves made at each natural frequency

$$c_j^{nn} = 4\pi f_j \xi_j m_j^{nn} \qquad \left(m_j^{nn} = 1\right) \tag{A-3}$$

Method 2

The second method is an impedance test made on the electromechanical system itself, therefore not necessitating any separate setup. This is the method that was used for this example. The method consists in locking one shaker at a time, for example shaker No. 2, such that $\ddot{u}_2(t) = 0$. A low-level sine sweep in the frequency range of interest is then made by driving the other shaker with a sweep oscillator which is controlled to maintain the acceleration $\ddot{u}_1(t)$ at point P_1 at a constant amplitude level A_1:

$$\ddot{u}_1(t) = A_1 \sin \omega t \tag{A-4}$$

The amplitude E_1 of the required voltage $e_1(t)$ (or current $i_1(t)$) is plotted versus the sweep frequency. The process is repeated, switching the role of the shakers. We obtain two curves, E_1 and E_2, as shown in figures A-1 and A-2.

The curves E_1 and E_2 exhibit a series of maxima that correspond to the natural frequencies of the structure in the tied-down configuration at P_1 and P_2, since one of the shakers is locked and the other one is controlled for an acceleration of constant amplitude. The locations of the maxima along the frequency axis give the numerical values of the natural frequencies f_j from which the stiffnesses k_j^{nn} can be obtained according to Eq. (A-1). Those stiffnesses are indicated in Table A-2.

The maxima of the curves E_1 and E_2 are followed by troughs that correspond to the natural frequencies of the beam tied down at only one point, P_1 or P_2. Those modes were chosen to determine the modal damping since it was very convenient to obtain the decay curve from this configuration. The modal damping ξ_j of the modes tied down at P_1 and P_2 was assumed to be the same. The damping coefficients c_j^{nn} were computed according to Eq. (A-3) and are indicated in Table A-2.

We now turn to the determination of the coefficients $m_{sj}^{un} = m_{js}^{nu}$. The method used to obtain those coefficients was the following. We first program Eqs. (16) through (20) on the analog computer. A schematic of the electrical network is shown in figures A-3 — A-5. Then the computer is placed in line with the power amplifiers (fig. 4) and all the previously determined coefficients are set on the corresponding potentiometers. All the coefficients m_{sj}^{un} are first set to zero. Next the computer is driven by a sine wave of constant amplitude corresponding to a low level acceleration. The beam is still in the one-shaker-locked configuration and the other shaker is driven by the output of the computer. Then the double potentiometer corresponding to $m_{sj}^{un} = m_{js}^{nu}$ is adjusted in order to obtain the desired level at the first natural frequency f_1. This is repeated in turn for all natural frequencies and for both shakers to obtain all the ten coefficients m_{sj}^{un}. There remains a sign ambiguity which is resolved by unlocking the two shakers. The values of the coefficients obtained by this method are indicated in Table A-2.

Figures A-6 and A-7 are relative to a sine sweep made on the beam with the computer in line to show the effectiveness of the equalization. The computer was driven on its two inputs with a sine wave of constant amplitude and the amplitude of the responses of the beam $\ddot{u}_1$ and $\ddot{u}_2$ was monitored. A perfect equalization would be achieved if the two responses $\ddot{u}_1$ and $\ddot{u}_2$ were flat in terms of frequency. Those curves have to be compared with figures A-1 and A-2 to show the magnitude of the necessary equalization. The curves of figures A-6 and A-7 show that the equalization is effective within ±1 dB from 10 cps to 100 cps, with more divergence in the higher frequency range.

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Before concluding this section, let us note that a random input was also used in place of the sine wave to determine the coupling coefficients m_{sj}^{un}, and the potentiometers were adjusted so that the power spectral density of the responses of the beam $\ddot{u}_1$ and $\ddot{u}_2$ was as flat as possible. The random method turned out to be just as convenient as the sine wave method.

Table A-2. Beam Characteristics

Mode j	Natural frequency f_j, cps	Modal damping ξ_j	Stiffness k_j^{nn}, newton/m	Damping c_j^{nn}, newton/m/s	m_{j1}^{un}, kg	m_{j2}^{un}, kg
1	19.6	0.0340	0.151×10^5	0.084×10^2	-0.210	0.570
2	40.6	0.0315	0.650×10^5	0.160×10^2	0.351	-0.620
3	80.1	0.0253	2.53×10^5	0.254×10^2	0.576	0.370
4	182	0.0220	13.1×10^5	0.500×10^2	0.208	0.084
5	239	0.0206	22.5×10^5	0.620×10^2	-0.171	0.380
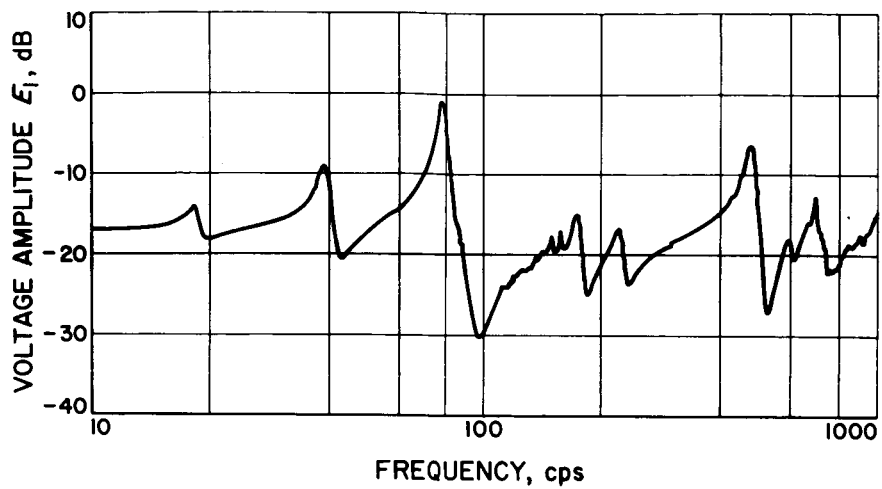
Figure A-1. - Shaker No. 1 voltage vs frequency
for shaker No. 2, locked.
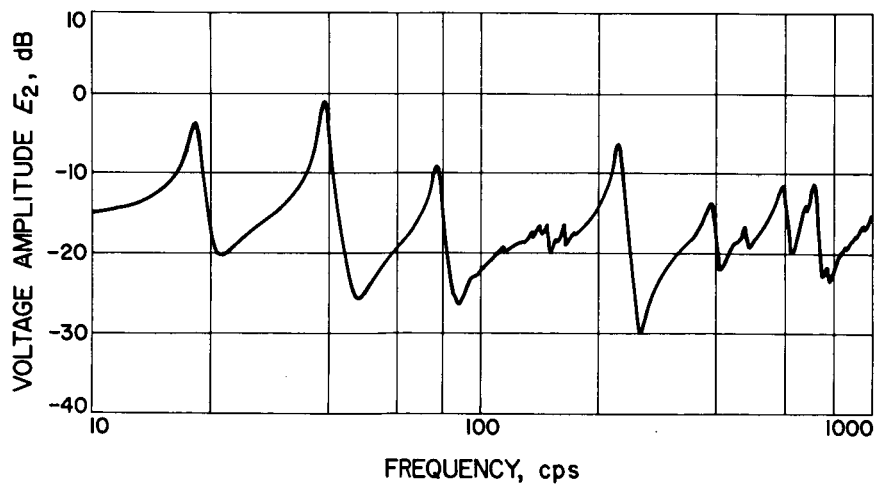
Figure A-2. - Shaker No. 2 voltage vs frequency
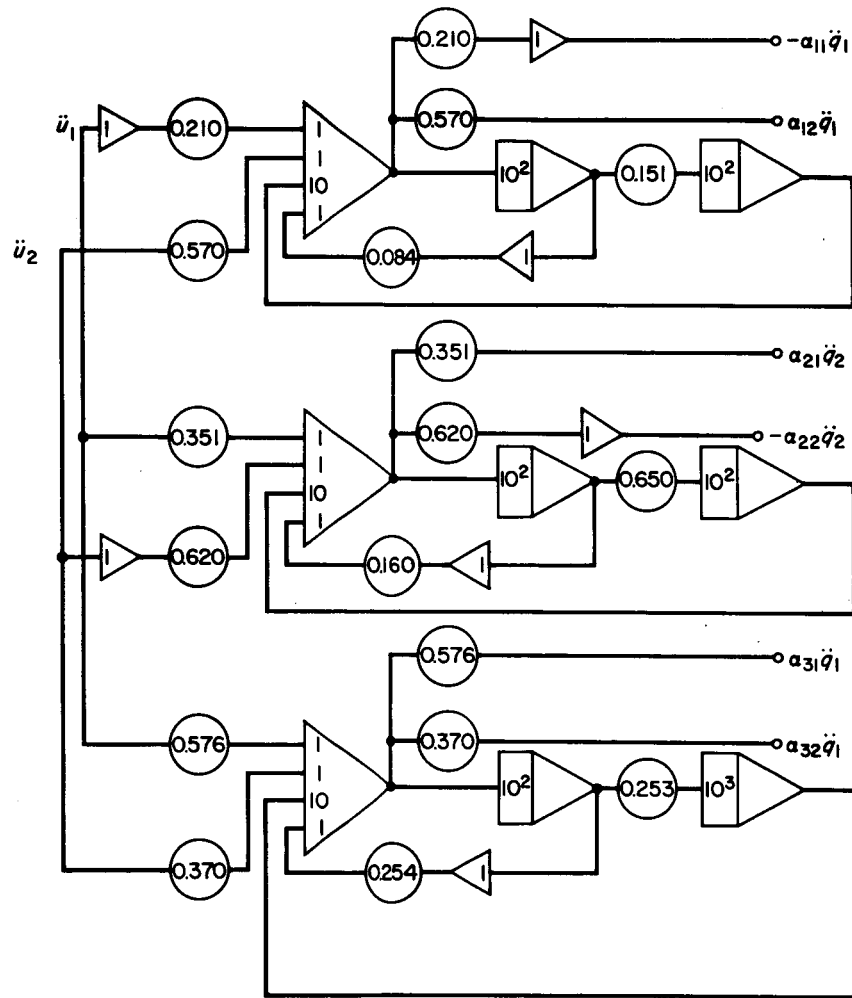for shaker No. 1, locked.

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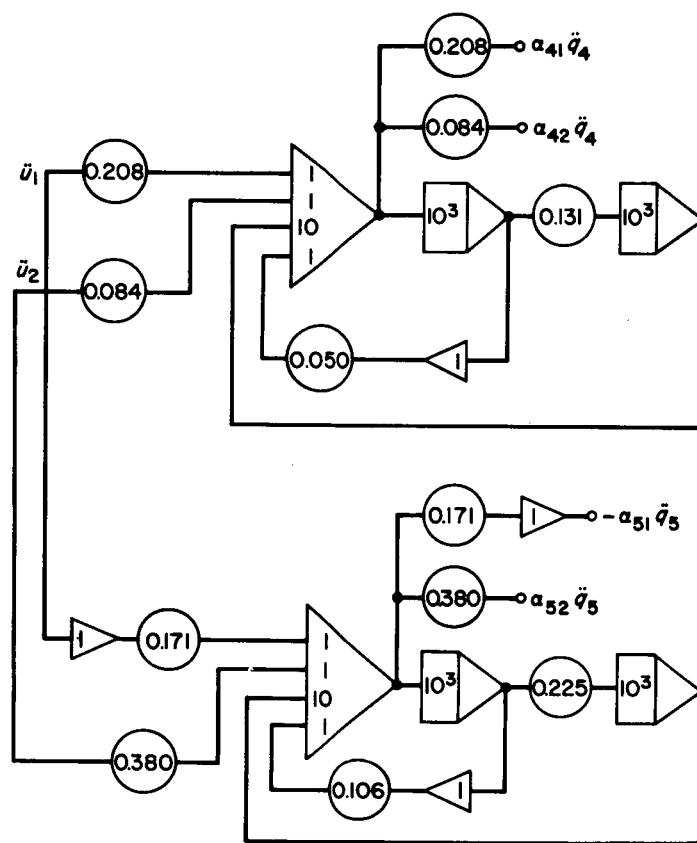Figure A-3. - Analog representation of one-
degree-of-freedom systems, part 1.

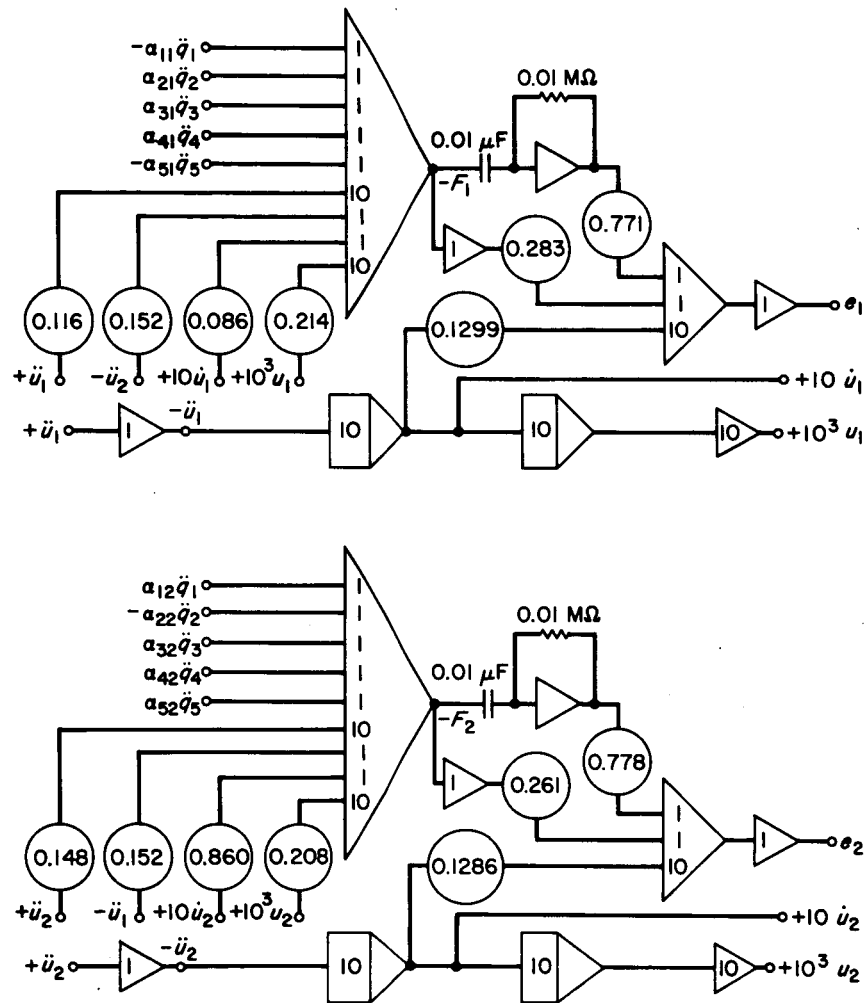Figure A-4.- Analog representation of one-degree-of-freedom systems, part 2.

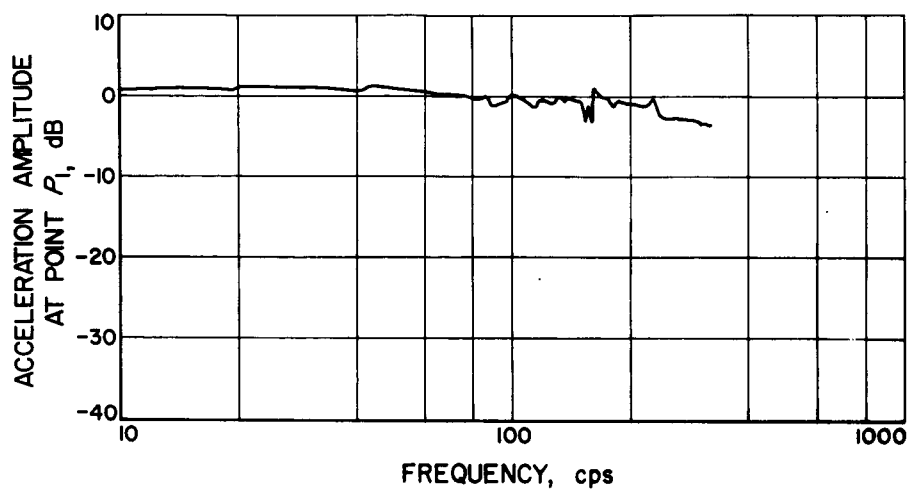Figure A-5. – Combination of modes
and shakers.

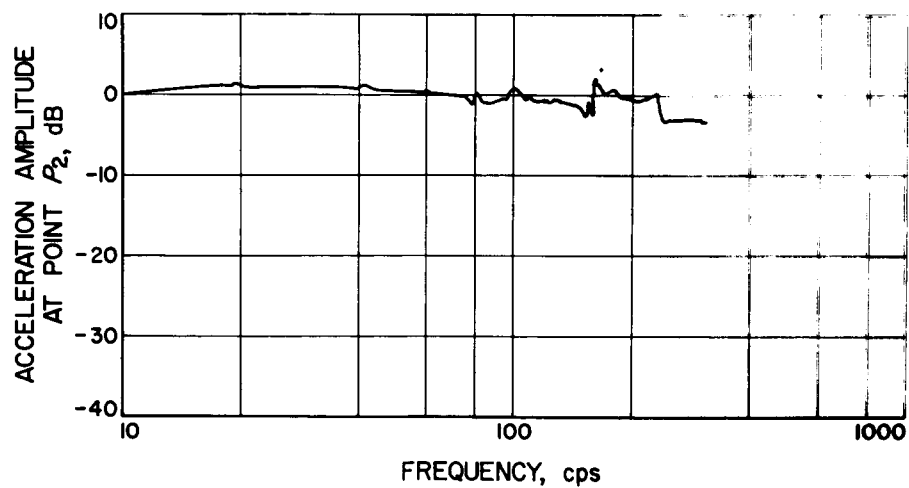Figure A-6. - Equalized acceleration at P_1 for shaker No. 2, locked.

Figure A-7. - Equalized acceleration at P_2 for shaker No. 1, locked.

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