NASA TECHNICAL NOTE

NASA TN D-4531

C.1

NASA TN D-4531

ON THE PERTURBATIONS OF SMALL-ECCENTRICITY SATELLITES

by T. L. Felsentreger

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1968

NASA TN D-4531

ON THE PERTURBATIONS OF SMALL-ECCENTRICITY SATELLITES

By T. L. Felsentreger

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

ABSTRACT

Difficulties have been encountered in the orbit determination of nearly circular earth satellites when "first-order" analytic perturbation theories have been used. "First-order" usually means that periodic terms are developed to order J_2 (the second zonal harmonic coefficient in the earth's gravitational potential), and secular terms to order J_2^2. If these theories are extended to include periodic terms of order J_2^2 and higher, the appearance of eccentricity as a divisor in the perturbations of eccentricity, mean anomaly, and argument of perigee causes many of these "higher-order" terms to be comparable in magnitude with the J_2 terms (for small-eccentricity satellites). Thus, the aforementioned difficulties can be attributed, at least in part, to the omission of these terms from the orbit-determination models.

In this paper, von Zeipel's method is used to derive all such "small divisor" terms inclusive of J_2^3 and $(J_i/J_2)^3$, when J_i is any odd zonal harmonic coefficient. The perturbations in mean anomaly and argument of perigee include terms having J_2/e, $(J_2/e)^2$, $(J_2/e)^3$, $J_i/J_2 e$, $(J_i/J_2 e)^2$, and $(J_i/J_2 e)^3$ as factors; the perturbations in eccentricity include terms having J_2, J_2^2/e, J_i/J_2, $J_i^2/J_2^2 e$, and $J_i^3/J_2^3 e^2$ as factors. In addition, the long-period terms are derived by solving Delaunay's Equations of Type II. The results are applied to the satellites Alouette I, Tiros 8, and Nimbus 2, and are also used to show that the small divisors cause no in-track, cross-track, or along-track errors.

CONTENTS

ON THE PERTURBATIONS OF SMALL-ECCENTRICITY SATELLITES

by

T. L. Felsentreger

Goddard Space Flight Center

INTRODUCTION

To determine the orbits of most artificial earth satellites, "first-order" analytic orbit theories that consider only the non-sphericity of the earth are usually accurate enough ("first-order" means that periodic terms have been developed to order J_2, and secular terms to order J_2^2). However, for a small-eccentricity satellite, the use of such theories for orbit prediction usually results in disagreement with observational data, sometimes intolerable. A principal cause of this is that such theories lack periodic terms of order J_2^2 and higher, in the perturbations of eccentricity, mean anomaly, and argument of perigee; many of these terms have the same magnitude as the J_2 terms because of the presence of eccentricity as a divisor.

Many orbit theories for which these small divisor terms appear have a convergence problem for very small eccentricities—in particular, those based on the von Zeipel method (Brouwer, 1959, and Kozai, 1962). For this reason, various investigators have attacked the problem by using variables for which the singularities do not appear (Kozai, 1961; Lyddane, 1963). However, such approaches are unnecessary when consideration is limited to a satellite whose argument of perigee circulates; for such a satellite, the expansions of Brouwer (1959) and Kozai (1962) are still convergent.

In this study, the von Zeipel method is used to obtain the "small divisor" periodic perturbations in eccentricity, mean anomaly, and argument of perigee through J_2^3 and $(J_i/J_2)^3$ terms. In addition, the long-period terms are shown to arise in a solution to Delaunay's Equations of Type II. The long-period effects on the satellites Alouette I, Tiros 8, and Nimbus 2 are exhibited, and the amplitudes of the short-period terms given. Also, it is shown that the small divisors do not give rise to in-track, cross-track, or along-track errors.

Appendix A lists the symbols employed in the text. Appendix B gives tabular data. Appendix C presents graphs on the eccentricity and argument of perigee for the three satellites.

SHORT-PERIOD TERMS

The essentials of the von Zeipel method as applied to earth satellites will not be discussed here; the reader may refer to Brouwer (1959), Kozai (1962), or Brouwer and Clemence (1961).

It suffices to say that the method involves the derivation of a "determining" function from which the perturbations for a set of canonical variables can be obtained directly. The usual variables are the Delaunay set L, G, H, ℓ, g, and h.

The pertinent parts of the determining functions S_1 and S_2 (see Brouwer, 1959, and Kozai, 1962) are

$$S_1 \simeq \frac{J_2}{16\, a'\sqrt{a'}} \left(2 + 3e'^2\right) \Big\{-2\left(1 - 3\cos^2 i'\right)(f - \ell + e' \sin f) + \sin^2 i' \big[3e' \sin(f + 2g) + 3 \sin 2(f + g) + e' \sin(3f + 2g)\big]\Big\} \cdot \tag{1}$$

$$\begin{aligned} S_2 \simeq \frac{J_2^2}{256\, a'^3 \sqrt{a'}} \Big\{ & -48 \cos^2 i' \left(1 - 5\cos^2 i'\right)(f - \ell) + 6e' \left(29 - 106 \cos^2 i' + 181 \cos^4 i'\right) \sin f \\ & + 6\left(13 - 50 \cos^2 i' + 61 \cos^4 i'\right) \sin 2f + 18e' \left(7 - 22\cos^2 i' + 23 \cos^4 i'\right) \sin 3f \\ & + 12e' \sin^2 i' \left(11 - 69 \cos^2 i'\right) \sin(f + 2g) - 72e' \sin^2 i' \left(1 - 3\cos^2 i'\right) \sin(f - 2g) \\ & - 27e' \sin^4 i' \sin(f + 4g) - 24 \sin^2 i' \left(1 + 3\cos^2 i'\right) \sin 2(f + g) - 9 \sin^4 i' \sin(2f + 4g) \\ & - 4e' \sin^2 i' \left(47 - 121 \cos^2 i'\right) \sin(3f + 2g) - 3e' \sin^2 i' \left(49 - 73 \cos^2 i'\right) \sin(3f + 4g) \\ & - 84 \sin^2 i' \left(1 - 3\cos^2 i'\right) \sin(4f + 2g) - 12 \sin^2 i' \left(5 - 8\cos^2 i'\right) \sin 4(f + g) \\ & - 96e' \sin^2 i' \left(1 - 3\cos^2 i'\right) \sin(5f + 2g) + 3e' \sin^2 i' \left(21 - 13\cos^2 i'\right) \sin(5f + 4g) \\ & + 49 \sin^4 i' \sin(6f + 4g) + 63e' \sin^4 i' \sin(7f + 4g) + 36 \sin^2 i' \left(1 - 3\cos^2 i'\right) \sin 2g \Big\} \quad . \end{aligned} \tag{2}$$

The $\sin 2g$ term has been added by Kozai (1962) to simplify the short period expressions. No terms having eccentricity as a divisor will appear in S_3. Then,

$$\ell = \ell' - \frac{\partial S_1}{\partial L'} - \frac{\partial S_2}{\partial L'} \ , \tag{3}$$

$$g = g' - \frac{\partial S_1}{\partial G'} - \frac{\partial S_2}{\partial G'} \ . \tag{4}$$

Expanding by means of Taylor Series gives (inclusive of J_2^3 terms)

$$
\begin{aligned}
\ell = \ell' &- \frac{\partial S_1}{\partial L'} - \frac{\partial S_2}{\partial L'} - \frac{\partial^2 S_1}{\partial L'\,\partial \ell'}\left[-\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial L'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial L'\,\partial g'}\frac{\partial S_1}{\partial G'} - \frac{\partial S_2}{\partial L'}\right] \\
&- \frac{\partial^2 S_1}{\partial L'\,\partial g'}\left[-\frac{\partial S_1}{\partial G'} + \frac{\partial^2 S_1}{\partial G'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial G'\,\partial g'}\frac{\partial S_1}{\partial G'} - \frac{\partial S_2}{\partial G'}\right] - \frac{1}{2}\frac{\partial^3 S_1}{\partial L'\,\partial \ell'^2}\left(\frac{\partial S_1}{\partial L'}\right)^2 \\
&- \frac{1}{2}\frac{\partial^3 S_1}{\partial L'\,\partial g'^2}\left(\frac{\partial S_1}{\partial G'}\right)^2 - \frac{\partial^3 S_1}{\partial L'\,\partial \ell'\,\partial g'}\frac{\partial S_1}{\partial L'}\frac{\partial S_1}{\partial G'} + \frac{\partial^2 S_2}{\partial L'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_2}{\partial L'\,\partial g'}\frac{\partial S_1}{\partial G'}\,, \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
g = g' &- \frac{\partial S_1}{\partial G'} - \frac{\partial S_2}{\partial G'} - \frac{\partial^2 S_1}{\partial G'\,\partial \ell'}\left[-\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial L'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial L'\,\partial g'}\frac{\partial S_1}{\partial G'} - \frac{\partial S_2}{\partial L'}\right] \\
&- \frac{\partial^2 S_1}{\partial G'\,\partial g'}\left[-\frac{\partial S_1}{\partial G'} + \frac{\partial^2 S_1}{\partial G'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial G'\,\partial g'}\frac{\partial S_1}{\partial G'} - \frac{\partial S_2}{\partial G'}\right] - \frac{1}{2}\frac{\partial^3 S_2}{\partial G'\,\partial \ell'^2}\left(\frac{\partial S_1}{\partial L'}\right)^2 \\
&- \frac{1}{2}\frac{\partial^3 S_1}{\partial G'\,\partial g'^2}\left(\frac{\partial S_1}{\partial G'}\right)^2 - \frac{\partial^3 S_1}{\partial G'\,\partial \ell'\,\partial g'}\frac{\partial S_1}{\partial L'}\frac{\partial S_1}{\partial G'} + \frac{\partial^2 S_2}{\partial G'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_2}{\partial G'\,\partial g'}\frac{\partial S_1}{\partial G'}\,, \qquad (6)
\end{aligned}
$$

in which f and g are to be replaced by f' and g'. Applying the relations

$$\frac{\partial f'}{\partial L'} = \frac{1}{e'}\frac{G'^2}{L'^3}\left(\frac{a'}{r'} + \frac{L'^2}{G'^2}\right)\sin f'\,,$$

$$\frac{\partial f'}{\partial G'} = -\frac{1}{e'}\frac{G'}{L'^2}\left(\frac{a'}{r'} + \frac{L'^2}{G'^2}\right)\sin f'\,,$$

$$\frac{df'}{d\ell'} = \frac{G'}{L'}\frac{a'^2}{r'^2}\,,$$

$$\frac{a'}{r'} = \frac{L'^2}{G'^2}(1 + e'\cos f')\,,$$

$$\frac{\partial e'}{\partial L'} = \frac{1}{e'}\frac{G'^2}{L'^3}\,,$$

$$\frac{\partial e'}{\partial G'} = -\frac{1}{e'}\frac{G'}{L'^2}\,,$$

and retaining only the largest terms gives

$$\ell = \ell' + \Delta\ell , \tag{7}$$

$$g = g' + \Delta g = g' - \Delta\ell , \tag{8}$$

where

$$\begin{aligned}
\Delta\ell = {} & \frac{J_2}{8e' a'^2}\Big[6\left(1 - 3\cos^2 i'\right) \sin f' + 3\sin^2 i' \sin(f' + 2g') - 7\sin^2 i' \sin(3f' + 2g')\Big] \\
& + \frac{J_2^2}{128e'^2 a'^4}\Big[36\sin^2 i' \left(1 - 3\cos^2 i'\right) \sin 2g' + 6\left(13 - 50\cos^2 i' + 61\cos^4 i'\right) \sin 2f' \\
& - 9\sin^4 i' \sin(2f' + 4g') - 84\sin^2 i' \left(1 - 3\cos^2 i'\right) \sin(4f' + 2g') + 49\sin^4 i' \sin(6f' + 4g')\Big] \\
& - \frac{J_2^3}{4096e'^3 a'^6}\Big[12\left(1 - 3\cos^2 i'\right)\left(97 - 266\cos^2 i' + 241\cos^4 i'\right) \sin f' \\
& - 324\left(1 - 3\cos^2 i'\right)\left(9 - 26\cos^2 i' + 25\cos^4 i'\right) \sin 3f' + 3\sin^2 i' \left(263 - 1150\cos^2 i' + 1511\cos^4 i'\right) \sin(f' + 2g') \\
& + 81\sin^2 i' \left(19 - 86\cos^2 i' + 115\cos^4 i'\right) \sin(f' - 2g') + 486\sin^4 i' \left(1 - 3\cos^2 i'\right) \sin(f' + 4g') \\
& - \sin^2 i' \left(1081 - 4610\cos^2 i' + 5977\cos^4 i'\right) \sin(3f' + 2g') - 306\sin^4 i' \left(1 - 3\cos^2 i'\right) \sin(3f' + 4g') \\
& - 81\sin^6 i' \sin(3f' + 6g') + 189\sin^2 i' \left(19 - 86\cos^2 i' + 115\cos^4 i'\right) \sin(5f' + 2g') \\
& + 546\sin^4 i' \left(1 - 3\cos^2 i'\right) \sin(5f' + 4g') + 63\sin^6 i' \sin(5f' + 6g') \\
& - 2646\sin^4 i' \left(1 - 3\cos^2 i'\right) \sin(7f' + 4g') - 147\sin^6 i' \sin(7f' + 6g') + 1029\sin^6 i' \sin(9f' + 6g')\Big] .
\end{aligned} \tag{9}$$

Note that no divisors of e' appear in $\ell + g$. The short-period perturbations in e are obtained from those in L and G. If

$$L = L' + \Delta L , \tag{10}$$

and

$$G = G' + \Delta G , \tag{11}$$

then

$$e = e' + \frac{\partial e'}{\partial L'} \Delta L + \frac{\partial e'}{\partial G'} \Delta G + \frac{1}{2} \frac{\partial^2 e'}{\partial L'^2} (\Delta L)^2 + \frac{1}{2} \frac{\partial^2 e'}{\partial G'^2} (\Delta G)^2 + \frac{\partial^2 e'}{\partial L' \partial G'} \Delta L \Delta G$$

$$+ \frac{1}{6} \frac{\partial^3 e'}{\partial L'^3} (\Delta L)^3 + \frac{1}{6} \frac{\partial^3 e'}{\partial G'^3} (\Delta G)^3 + \frac{1}{2} \frac{\partial^3 e'}{\partial L'^2 \partial G'} (\Delta L)^2 \Delta G + \frac{1}{2} \frac{\partial^3 e'}{\partial L' \partial G'^2} \Delta L (\Delta G)^2, \quad (12)$$

for which

$$\frac{\partial e'}{\partial L'} = \frac{1}{e' \sqrt{a'}},$$

$$\frac{\partial e'}{\partial G'} = - \frac{1}{e' \sqrt{a'}},$$

$$\frac{\partial^2 e'}{\partial L'^2} = - \frac{1 + e'^2}{e'^3 a'},$$

$$\frac{\partial^2 e'}{\partial G'^2} = - \frac{1}{e'^3 a'},$$

$$\frac{\partial^2 e'}{\partial L' \partial G'} = \frac{1 + \frac{1}{2} e'^2}{e'^3 a'},$$

$$\frac{\partial^3 e'}{\partial L'^3} = \frac{3 \left(1 + e'^4\right)}{e'^5 a' \sqrt{a'}},$$

$$\frac{\partial^3 e'}{\partial G'^3} = - \frac{3 \left(1 - \frac{1}{2} e'^2 - \frac{1}{8} e'^4\right)}{e'^5 a' \sqrt{a'}},$$

$$\frac{\partial^3 e'}{\partial L'^2 \partial G'} = - \frac{3 - \frac{1}{2} e'^2 + \frac{9}{8} e'^4}{e'^5 a' \sqrt{a'}},$$

$$\frac{\partial^3 e'}{\partial L' \partial G'^2} = \frac{3 - e'^2}{e'^5 a' \sqrt{a'}}.$$

But

$$L = L' + \frac{\partial S_1}{\partial \ell} + \frac{\partial S_2}{\partial \ell}, \tag{13}$$

and

$$G = G' + \frac{\partial S_1}{\partial g} + \frac{\partial S_2}{\partial g}, \tag{14}$$

which, when expanded, become

$$\begin{aligned} L = L' &+ \frac{\partial S_1}{\partial \ell'} + \frac{\partial S_2}{\partial \ell'} + \frac{\partial^2 S_1}{\partial \ell'^2}\left[-\frac{\partial S_1}{\partial L'} - \frac{\partial S_2}{\partial L'} + \frac{\partial^2 S_1}{\partial L'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial L'\,\partial g'}\frac{\partial S_1}{\partial G'}\right] \\ &+ \frac{\partial^2 S_1}{\partial \ell'\,\partial g'}\left[-\frac{\partial S_1}{\partial G'} - \frac{\partial S_2}{\partial G'} + \frac{\partial^2 S_1}{\partial G'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial G'\,\partial g'}\frac{\partial S_1}{\partial G'}\right] + \frac{1}{2}\frac{\partial^3 S_1}{\partial \ell'^3}\left(\frac{\partial S_1}{\partial L'}\right)^2 \\ &+ \frac{1}{2}\frac{\partial^3 S_1}{\partial \ell'\,\partial g'^2}\left(\frac{\partial S_1}{\partial G'}\right)^2 + \frac{\partial^3 S_1}{\partial \ell'^2\,\partial g'}\frac{\partial S_1}{\partial L'}\frac{\partial S_1}{\partial G'} - \frac{\partial^2 S_2}{\partial \ell'^2}\frac{\partial S_1}{\partial L'} - \frac{\partial^2 S_2}{\partial \ell'\,\partial g'}\frac{\partial S_1}{\partial G'}\,. \end{aligned} \tag{15}$$

$$\begin{aligned} G = G' &+ \frac{\partial S_1}{\partial g'} + \frac{\partial S_2}{\partial g'} + \frac{\partial^2 S_1}{\partial g'^2}\left[-\frac{\partial S_1}{\partial G'} - \frac{\partial S_2}{\partial G'} + \frac{\partial^2 S_1}{\partial G'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial G'\,\partial g'}\frac{\partial S_1}{\partial G'}\right] \\ &+ \frac{\partial^2 S_1}{\partial g'\,\partial \ell'}\left[-\frac{\partial S_1}{\partial L'} - \frac{\partial S_2}{\partial L'} + \frac{\partial^2 S_1}{\partial L'\,\partial \ell'}\frac{\partial S_1}{\partial L'} + \frac{\partial^2 S_1}{\partial L'\,\partial g'}\frac{\partial S_1}{\partial G'}\right] + \frac{1}{2}\frac{\partial^3 S_1}{\partial g'^3}\left(\frac{\partial S_1}{\partial G'}\right)^2 \\ &+ \frac{1}{2}\frac{\partial^3 S_1}{\partial g'\,\partial \ell'^2}\left(\frac{\partial S_1}{\partial L'}\right)^2 + \frac{\partial^3 S_1}{\partial g'^2\,\partial \ell'}\frac{\partial S_1}{\partial L'}\frac{\partial S_1}{\partial G'} - \frac{\partial^2 S_2}{\partial g'^2}\frac{\partial S_1}{\partial G'} - \frac{\partial^2 S_2}{\partial g'\,\partial \ell'}\frac{\partial S_1}{\partial L'}\,. \end{aligned} \tag{16}$$

Again, f and g are to be replaced by f' and g'. Consequently,

$$\begin{aligned} \Delta L = \frac{J_2}{32\,a'\sqrt{a'}} \Big\{ &-2\left(1 - 3\cos^2 i'\right)\left[12e'^2 + 3e'\left(4 + 13e'^2\right)\cos f' + 6e'^2\cos 2f' + e'^3\cos 3f'\right] \\ &+ 3\sin^2 i'\Big[6e'^2\cos 2g' + e'\left(12 + 43e'^2\right)\cos(f' + 2g') + e'^3\cos(f' - 2g') \\ &+ 4\left(2 + 9e'^2\right)\cos 2(f' + g') + e'\left(12 + 43e'^2\right)\cos(3f' + 2g') + 6e'^2\cos(4f' + 2g') + e'^3\cos(5f' + 2g')\Big]\Big\} \end{aligned}$$

$$+ \frac{3J_2^2}{64\, a'^3 \sqrt{a'}} \Big\{ 12 \sin^2 i' \left(1 - 5\cos^2 i'\right) \; (f' - \ell') \left[3e' \sin(f' + 2g') + 2 \sin 2(f' + g') + 3e' \sin(3f' + 2g')\right]$$

$$+ 11 - 34 \cos^2 i' + 47 \cos^4 i' + 2e' \left(29 - 90 \cos^2 i' + 45 \cos^4 i'\right) \cos f'$$

$$- e' \sin^2 i' \left(61 - 99 \cos^2 i'\right) \cos(f' + 2g') - 8 \sin^2 i' \left(4 - 9 \cos^2 i'\right) \cos 2(f' + g')$$

$$- 5e' \sin^2 i' \left(17 - 47 \cos^2 i'\right) \cos(3f' + 2g') + 26e' \sin^4 i' \cos(3f' + 4g')$$

$$+ 5 \sin^4 i' \cos 4(f' + g') + 18e' \sin^4 i' \cos(5f' + 4g') \Big\}$$

$$+ \frac{3J_2^3}{4096\, e' a'^5 \sqrt{a'}} \Big\{ -4 \left(1 - 3\cos^2 i'\right) \left(97 - 266 \cos^2 i' + 241 \cos^4 i'\right) \cos f'$$

$$+ 36 \left(1 - 3\cos^2 i'\right) \left(9 - 26 \cos^2 i' + 25 \cos^4 i'\right) \cos 3f' - \sin^2 i' \left(263 - 1150 \cos^2 i' + 1511 \cos^4 i'\right) \cos(f' + 2g')$$

$$- 3 \sin^2 i' \left(19 - 86 \cos^2 i' + 115 \cos^4 i'\right) \cos(f' - 2g') - 18 \sin^4 i' \left(1 - 3\cos^2 i'\right) \cos(f' + 4g')$$

$$+ \sin^2 i' \left(1081 - 4610 \cos^2 i' + 5977 \cos^4 i'\right) \cos(3f' + 2g') + 306 \sin^4 i' \left(1 - 3\cos^2 i'\right) \cos(3f' + 4g')$$

$$+ 9 \sin^6 i' \cos(3f' + 6g') - 35 \sin^2 i' \left(19 - 86 \cos^2 i' + 115 \cos^4 i'\right) \cos(5f' + 2g')$$

$$- 910 \sin^4 i' \left(1 - 3\cos^2 i'\right) \cos(5f' + 4g') - 105 \sin^6 i' \cos(5f' + 6g')$$

$$+ 686 \sin^4 i' \left(1 - 3\cos^2 i'\right) \cos(7f' + 4g') + 343 \sin^6 i' \cos(7f' + 6g') - 343 \sin^6 i' \cos(9f' + 6g') \Big\} . \tag{17}$$

$$\Delta G = \frac{J_2 \sin^2 i'}{8\, a' \sqrt{a'}} \left(2 + 3e'^2\right) \left[3e' \cos(f' + 2g') + 3 \cos 2(f' + g') + e' \cos(3f' + 2g')\right]$$

$$+ \frac{J_2^2 \sin^2 i'}{32\, a'^3 \sqrt{a'}} \Big\{ 12 \left(1 - 5\cos^2 i'\right) \; (f' - \ell') \left[3e' \sin(f' + 2g') + 3 \sin 2(f' + g') + e' \sin(3f' + 2g')\right]$$

$$- \left(7 - 25 \cos^2 i'\right) - 24e' \left(1 - 3\cos^2 i'\right) \cos f' + 9e' \left(1 - 19 \cos^2 i'\right) \cos(f' + 2g')$$

$$- 18 \sin^2 i' \cos 2(f' + g') - e' \left(29 - 103 \cos^2 i'\right) \cos(3f' + 2g') + 3e' \sin^2 i' \cos(3f' + 4g')$$

$$- 3 \sin^2 i' \cos 4(f' + g') - 3e' \sin^2 i' \cos(5f' + 4g') \Big\}$$

$$+ \frac{J_2^3 \sin^2 i'}{2048\, e' a'^5 \sqrt{a'}} \Big\{ -3 \left(263 - 1150 \cos^2 i' + 1511 \cos^4 i'\right) \cos(f' + 2g')$$

$$+ 9 \left(19 - 86 \cos^2 i' + 115 \cos^4 i'\right) \cos(f' - 2g') - 108 \sin^2 i' \left(1 - 3\cos^2 i'\right) \cos(f' + 4g')$$

$$+ \left(1081 - 4610\cos^2 i' + 5977\cos^4 i'\right)\cos(3f' + 2g') + 612\sin^2 i'\left(1 - 3\cos^2 i'\right)\cos(3f' + 4g')$$

$$+ 27\sin^4 i'\cos(3f' + 6g') - 21\left(19 - 86\cos^2 i' + 115\cos^4 i'\right)\cos(5f' + 2g')$$

$$- 1092\sin^2 i'\left(1 - 3\cos^2 i'\right)\cos(5f' + 4g') - 189\sin^4 i'\cos(5f' + 6g')$$

$$+ 588\sin^2 i'\left(1 - 3\cos^2 i'\right)\cos(7f' + 4g') + 441\sin^4 i'\cos(7f' + 6g') - 343\sin^4 i'\cos(9f' + 6g')\Big\} . \quad (18)$$

Then, from Equation 12,

$$e = e' + \frac{J_2}{8a'^2}\left[-6\left(1 - 3\cos^2 i'\right)\cos f' + 3\sin^2 i'\cos(f' + 2g') + 7\sin^2 i'\cos(3f' + 2g')\right]$$

$$+ \frac{J_2^2}{256e' a'^4}\Big[2\left(47 - 166\cos^2 i' + 191\cos^4 i'\right) + 36\sin^2 i'\left(1 - 3\cos^2 i'\right)\cos 2g'$$

$$- 6\left(13 - 50\cos^2 i' + 61\cos^4 i'\right)\cos 2f' - 120\sin^2 i'\left(1 - 3\cos^2 i'\right)\cos 2(f' + g')$$

$$- 9\sin^4 i'\cos(2f' + 4g') + 84\sin^2 i'\left(1 - 3\cos^2 i'\right)\cos(4f' + 2g')$$

$$+ 42\sin^4 i'\cos 4(f' + g') - 49\sin^4 i'\cos(6f' + 4g')\Big] \; ; \quad (19)$$

all J_2^3/e'^2 terms have dropped out.

LONG-PERIOD TERMS

The most important parts of the long-period determining functions S_1^*, S_2^*, and S_3^* are

$$\Delta S_1^* = e''\sqrt{a''}\, Q\cos g' , \quad (20)$$

$$\Delta S_2^* = \frac{\sqrt{a''}\, Q^2}{4}\sin 2g' , \quad (21)$$

$$\Delta S_3^* = -\frac{\sqrt{a''}\, Q^3}{24e''}\left(3\cos g' + \cos 3g'\right) , \quad (22)$$

where

$$Q = \frac{M}{N} \quad (23)$$

$$N = -\frac{3J_2}{4a''^3\sqrt{a''}}\left(1-5\cos^2 i''\right) + \frac{3J_2^2}{64a''^5\sqrt{a''}}\left(7-114\cos^2 i''+395\cos^4 i''\right)$$

$$-\frac{15J_4}{32a''^5\sqrt{a''}}\left(3-36\cos^2 i''+49\cos^4 i''\right) + \cdots \tag{24}$$

$$M = \frac{3J_3 \sin i''}{8a''^4\sqrt{a''}}\left(1-5\cos^2 i''\right) + \frac{15J_5 \sin i''}{32a''^6\sqrt{a''}}\left(1-14\cos^2 i''+21\cos^4 i''\right)$$

$$+\frac{105J_7 \sin i''}{1024a''^8\sqrt{a''}}\left(5-135\cos^2 i''+495\cos^4 i''-429\cos^6 i''\right)$$

$$+\frac{315J_9 \sin i''}{4096a''^{10}\sqrt{a''}}\left(7-308\cos^2 i''+2002\cos^4 i''-4004\cos^6 i''+2431\cos^8 i''\right)$$

$$+\frac{3465J_{11} \sin i''}{131072a''^{12}\sqrt{a''}}\left(21-1365\cos^2 i''+13650\cos^4 i''-46410\cos^6 i''+62985\cos^8 i''-29393\cos^{10} i''\right) + \cdots \tag{25}$$

As before, the perturbation in g will be the negative of that for ℓ, so only one of them need be derived. Thus,

$$g' = g'' - \frac{\partial(\Delta S_1^*)}{\partial G''} - \frac{\partial(\Delta S_2^*)}{\partial G''} - \frac{\partial(\Delta S_3^*)}{\partial G''}, \tag{26}$$

which, when expanded, is

$$g' = g'' - \frac{\partial(\Delta S_1^*)}{\partial G''} - \frac{\partial(\Delta S_2^*)}{\partial G''} - \frac{\partial(\Delta S_3^*)}{\partial G''} + \frac{\partial^2(\Delta S_1^*)}{\partial G''\,\partial g''}\left[\frac{\partial(\Delta S_1^*)}{\partial G''} + \frac{\partial(\Delta S_2^*)}{\partial G''}\right]$$

$$+\frac{\partial^2(\Delta S_2^*)}{\partial G''\,\partial g''}\,\frac{\partial(\Delta S_1^*)}{\partial G''} - \frac{1}{2}\,\frac{\partial^3(\Delta S_1^*)}{\partial G''\,\partial g''^2}\left[\frac{\partial(\Delta S_1^*)}{\partial G''}\right]^2 . \tag{27}$$

In the right-hand side of Equation 27, the variable g' is to be replaced by g''. Also,

$$\frac{\partial(\Delta S_1^*)}{\partial G''} \simeq -\frac{Q}{e''}\cos g',$$

$$\frac{\partial(\Delta S_2^*)}{\partial G''} \simeq 0,$$

$$\frac{\partial(\Delta S_3^*)}{\partial G''} \simeq -\frac{Q^3}{24e''^3}\left(3\cos g' + \cos 3g'\right).$$

In addition,

$$\cos g' \simeq \cos g'' - \frac{Q}{2e''} \sin 2g'' ,$$

so a complete transformation to double-primed variables yields

$$g' = g'' + \frac{Q}{e''} \cos g'' - \frac{Q^2}{2e''^2} \sin 2g'' - \frac{Q^3}{3e''^3} \cos 3g'' . \tag{28}$$

Therefore,

$$\ell' = \ell'' - \frac{Q}{e''} \cos g'' + \frac{Q^2}{2e''^2} \sin 2g'' + \frac{Q^3}{3e''^3} \cos 3g'' . \tag{29}$$

To obtain the long-period perturbations in e, one need only find those for G, since the long-period terms do not appear in L. Hence,

$$G' = G'' + \frac{\partial (\Delta S_1^*)}{\partial g'} + \frac{\partial (\Delta S_2^*)}{\partial g'} + \frac{\partial (\Delta S_3^*)}{\partial g'} ; \tag{30}$$

which expanded gives

$$G' = G'' + \frac{\partial (\Delta S_1^*)}{\partial g'} + \frac{\partial (\Delta S_2^*)}{\partial g'} + \frac{\partial (\Delta S_3^*)}{\partial g'} - \frac{\partial^2 (\Delta S_1^*)}{\partial g''^2} \left[\frac{\partial (\Delta S_1^*)}{\partial G''} + \frac{\partial (\Delta S_2^*)}{\partial G''} \right]$$

$$- \frac{\partial^2 (\Delta S_2^*)}{\partial g''^2} \frac{\partial (\Delta S_1^*)}{\partial G''} + \frac{1}{2} \frac{\partial^3 (\Delta S_1^*)}{\partial g''^3} \left[\frac{\partial (\Delta S_1^*)}{\partial G''} \right]^2 , \tag{31}$$

in which g'' is substituted for g'. Equation 31 then becomes

$$G' = G'' + \Delta G' , \tag{32}$$

with

$$\Delta G' = - \frac{\sqrt{a''}\, Q^2}{2} - \sqrt{a''}\, e'' Q \sin g'' . \tag{33}$$

Then,

$$e' = e'' + \Delta e' , \tag{34}$$

where

$$\Delta e' \simeq -\frac{\Delta G'}{\sqrt{a''}\; e''} - \frac{(\Delta G')^2}{2a''\, e''^3} - \frac{(\Delta G')^3}{2a''\sqrt{a''}\; e''^5}\ . \tag{35}$$

As a result,

$$e' = e'' + \frac{Q^2}{4e''} + \left(Q - \frac{Q^3}{8e''^2}\right)\sin g'' + \frac{Q^2}{4e''}\cos 2g'' - \frac{Q^3}{8e''^2}\sin 3g''\ . \tag{36}$$

If one sets

$$\overline{\theta} = g'' + \frac{\pi}{2}\ , \tag{37}$$

and

$$e' = e''\left(1 + \frac{Q^2}{4e''^2}\right)\ , \tag{38}$$

then Equations 28 and 36 become

$$g' = g'' + \left(\frac{Q}{e_1} + \frac{Q^3}{4e_1^{\,3}}\right)\sin\overline{\theta} + \frac{Q^2}{2e_1^{\,2}}\sin 2\overline{\theta} + \frac{Q^3}{3e_1^{\,3}}\sin 3\overline{\theta}\ , \tag{39}$$

$$e' = e_1 - \left(Q - \frac{Q^3}{8e_1^{\,2}}\right)\cos\overline{\theta} - \frac{Q^2}{4e_1}\cos 2\overline{\theta} - \frac{Q^3}{8e_1^{\,2}}\cos 3\overline{\theta}\ . \tag{40}$$

These expressions are in a more convenient form to use in the analysis of the satellite data. In addition, Equations 39 and 40 are more readily comparable to those obtained in the solution to Delaunay's Equations of Type II, which are the subject of the next section.

DELAUNAY'S EQUATIONS OF TYPE II

The results of the preceding section can be derived by an alternate method; namely, obtaining a solution to Delaunay's Equations of Type II which appear in his lunar theory. If we neglect all terms having eccentricity as a factor, these equations (see Felsentreger, 1965) are

$$\frac{de'}{dt} = M\sin\theta\ , \tag{41}$$

$$\frac{d\theta}{dt} = N + \frac{M}{e'} \cos\theta , \qquad (42)$$

in which M and N are expressed by Equations 24 and 25, and

$$\theta = g' + \frac{\pi}{2} .$$

The method itself will not be elaborated upon; the reader can refer to Felsentreger (1965). Solutions to Equations 41 and 42 are:

$$e' = e_1 + Q\beta_1 \cos\bar{\theta} + Q^2 \beta_2 \cos 2\bar{\theta} + Q^3 \beta_3 \cos\bar{\theta} , \qquad (43)$$

$$\theta = \bar{\theta} + Q\alpha_1 \sin\bar{\theta} + Q^2 \alpha_2 \sin 2\bar{\theta} + Q^3 \alpha_3 \sin 3\bar{\theta} , \qquad (44)$$

where $Q = M/N$ and $\bar{\theta}$ is a linear function of the time. Modification of the results such that e_1 appears as the eccentricity constant yields

$$\alpha_1 = \frac{1}{e_1} + \frac{Q^2}{4e_1^3} ,$$

$$\beta_1 = -1 + \frac{Q^2}{8e_1^2} ,$$

$$\alpha_2 = \frac{1}{2e_1^2} ,$$

$$\beta_2 = -\frac{1}{4e_1} ,$$

$$\alpha_3 = \frac{1}{3e_1^3} ,$$

$$\beta_3 = -\frac{1}{8e_1^2} ,$$

for which terms with e_1 as a factor have been neglected. Thus, the solutions are the same as Equations 39 and 40.

IN-TRACK, CROSS-TRACK, AND ALONG-TRACK ERRORS

Consider a coordinate system defined in the following manner:

1. The x and y axes are along the directions of the position and velocity vectors, respectively, of the satellite (and therefore are in the orbital plane of the satellite),

2. The z axis is perpendicular to the orbital plane. The in-track, along-track, and cross-track errors in the position of the satellite are simply those errors in the x, y, and z directions, respectively; they are given by the following expressions:

   In-track: Δr ,

   Along-track: $r\left[(\Delta h)\cos i + \Delta f + \Delta g\right]$,

   Cross-track: $r\left[(\Delta i)\sin(f+g) - (\Delta h)\sin i\cos(f+g)\right]$.

Since Δi and Δh contain no terms with e as a divisor and since all such terms cancel out in the sum $\Delta f + \Delta g$, it follows that no along-track or cross-track errors result from small e divisors. The same is true for in-track error also, but some computation is needed to show this. In particular, it will be shown that no divisors of e appear in r through J_2^2 and Q^2. The same procedure can be followed for J_2^3, Q^3 terms, and higher.

The expansion for Δr in terms of primed variables is

$$\Delta r = \frac{\partial r}{\partial \ell'}\Delta\ell + \frac{\partial r}{\partial L'}\Delta L + \frac{\partial r}{\partial G'}\Delta G + \frac{1}{2}\frac{\partial^2 r}{\partial \ell'^2}(\Delta\ell)^2 + \frac{1}{2}\frac{\partial^2 r}{\partial L'^2}(\Delta L)^2 + \frac{1}{2}\frac{\partial^2 r}{\partial G'^2}(\Delta G)^2$$

$$+ \frac{\partial^2 r}{\partial \ell'\,\partial L'}\Delta\ell\,\Delta L + \frac{\partial^2 r}{\partial \ell'\,\partial G'}\Delta\ell\,\Delta G + \frac{\partial^2 r}{\partial L'\,\partial G'}\Delta L\,\Delta G , \tag{45}$$

where $\Delta\ell$, ΔL, and ΔG are given by Equations 9, 15, and 16, respectively. In addition,

$$\frac{\partial r}{\partial \ell'} = a' e' \sin f' ,$$

$$\frac{\partial r}{\partial L'} = -\frac{\sqrt{a'}}{e'}\cos f' + 2\sqrt{a'} ,$$

$$\frac{\partial r}{\partial G'} = \frac{\sqrt{a'}}{e'}\cos f' ,$$

$$\frac{\partial^2 r}{\partial \ell'^2} = a' e' \cos f' ,$$

$$\frac{\partial^2 r}{\partial L'^2} = \frac{1}{e'^2} + \left(\frac{1}{e'^3} - \frac{11}{4e'}\right) \cos f' - \frac{1}{e'^2} \cos 2f' - \frac{1}{4e'} \cos 3f' \;,$$

$$\frac{\partial^2 r}{\partial G'^2} = \frac{1}{e'^2} + \left(\frac{1}{e'^3} + \frac{1}{4e'}\right) \cos f' - \frac{1}{e'^2} \cos 2f' - \frac{1}{4e'} \cos 3f' \;,$$

$$\frac{\partial^2 r}{\partial \ell' \, \partial L'} = \frac{\sqrt{a'}}{e'} \sin f' + \sqrt{a'} \sin 2f' \;,$$

$$\frac{\partial^2 r}{\partial \ell' \, \partial G'} = -\frac{\sqrt{a'}}{e'} \sin f' - \sqrt{a'} \sin 2f' \;,$$

$$\frac{\partial^2 r}{\partial L' \, \partial G'} = -\frac{1}{e'^2} - \left(\frac{1}{e'^3} - \frac{5}{4e'}\right) \cos f' + \frac{1}{e'^2} \cos 2f' + \frac{1}{4e'} \cos 3f' \;.$$

The result is

$$\Delta r = \frac{J_2}{4a'} \left[3\left(1 - 3\cos^2 i'\right) + \sin^2 i' \cos 2(f' + g')\right] \;; \tag{46}$$

all terms with divisor e' have cancelled. In order to use the long-period perturbations in Equations 29 and 33, write

$$\Delta r' = \frac{\partial r'}{\partial \ell''} \Delta\ell' + \frac{\partial r'}{\partial G''} \Delta G' + \frac{1}{2} \frac{\partial^2 r'}{\partial \ell''^2} (\Delta\ell')^2 + \frac{1}{2} \frac{\partial^2 r'}{\partial G''^2} (\Delta G')^2 + \frac{\partial^2 r'}{\partial \ell'' \, \partial G''} \Delta\ell' \, \Delta G' \;, \tag{47}$$

(there are no long-period terms in $\Delta L'$), in which

$$\frac{\partial r'}{\partial \ell''} = a'' e'' \sin f'' \;,$$

$$\frac{\partial r'}{\partial G''} = \frac{\sqrt{a''}}{e''} \cos f'' \;,$$

$$\frac{\partial^2 r'}{\partial \ell''^2} = a'' e'' \cos f'' \;,$$

$$\frac{\partial^2 r'}{\partial G''^2} = \frac{1}{e''^3} \cos f'' \;,$$

$$\frac{\partial^2 r'}{\partial \ell'' \, \partial G''} = -\frac{\sqrt{a''}}{e''} \sin f'' \;.$$

The result is

$$\Delta r' = -a'' Q \sin(f'' + g'') \tag{48}$$

to order J_2. Again, all terms having e'' as a divisor have cancelled.

EXAMPLES

The satellites Alouette I, Tiros 8, and Nimbus 2 were chosen for analysis because their eccentricities are all of order 10^{-3}. In addition, large long-period variations were noticed in the eccentricity and argument of perigee for all three satellites.

Values of e'' and g'', published by Goddard Space Flight Center, first were corrected for lunar and solar effects (Murphy and Felsentreger, 1966) and for the effects caused by lunar and solar tides (Fisher and Felsentreger, 1966). These corrected values, labeled e_c'' and g_c'', appear in Tables 1 to 6, Appendix B. The lunar and solar effects were fairly small except for three near-resonant solar terms influencing the arguments of perigee for Alouette 1 and Nimbus 2. These terms all had periods on the order of 10^4 to 10^5 days; therefore, their effects over the time intervals would appear secular—no attempt was made to include any perturbations that they caused.

The e_c'' and g_c'' values were then fitted, by least squares, to the following models:

$$e_c'' = e_0'' + \sum_{j=1}^{9} A_j \cos j\bar{\theta} \ , \tag{49}$$

$$g_c'' = g_0'' + \dot{g}_c'' \left(t - t_0\right) + \sum_{j=1}^{9} B_j \sin j\bar{\theta} \ . \tag{50}$$

For these calculations, the argument $\bar{\theta}$ was assumed to be a linear function of time with mean motion N (see Equation 42). Values for N were computed from Equation 24 using mean values for a'' and i'', and Kozai's (1964) determination of J_2 and J_4. The mean orbital elements used in this and other calculations are:

	a'' (earth radii)	i''
Alouette 1	1.1589	80°466
Tiros 8	1.1140	58°500
Nimbus 2	1.1782	100°306

The results of the least-squares analyses were

Alouette I

$$\bar{\theta} = 109^\circ.13743 - (2^\circ.5649585/\text{day})(t - t_0) , \tag{51}$$

$$\begin{aligned} e_c'' = {} & 0.0025163652 - 0.0001492876 \cos\bar{\theta} - 0.0001336935 \cos 2\bar{\theta} - 0.0000097969 \cos 3\bar{\theta} \\ & - 0.0000264826 \cos 4\bar{\theta} + 0.0000007387 \cos 5\bar{\theta} - 0.0000042243 \cos 6\bar{\theta} \\ & - 0.0000082012 \cos 7\bar{\theta} - 0.0000070067 \cos 8\bar{\theta} - 0.0000001323 \cos 9\bar{\theta} , \end{aligned} \tag{52}$$

$$\begin{aligned} g_c'' = {} & 17^\circ.74620 - (2^\circ.5618750/\text{day})(t - t_0) + 4^\circ.2076854 \sin\bar{\theta} + 4^\circ.6340045 \sin 2\bar{\theta} \\ & + 0^\circ.1873826 \sin 3\bar{\theta} + 0^\circ.7003276 \sin 4\bar{\theta} - 0^\circ.0066516 \sin 5\bar{\theta} \\ & + 0^\circ.1206393 \sin 6\bar{\theta} + 0^\circ.0102137 \sin 7\bar{\theta} + 0^\circ.0387951 \sin 8\bar{\theta} - 0^\circ.0680971 \sin 9\bar{\theta} . \end{aligned} \tag{53}$$

Tiros 8

$$\bar{\theta} = 213^\circ.61150 + (1^\circ.2452865/\text{day})(t - t_0) , \tag{54}$$

$$\begin{aligned} e_c'' = {} & 0.0034394605 - 0.0004525939 \cos\bar{\theta} - 0.0001389608 \cos 2\bar{\theta} \\ & - 0.0000164065 \cos 3\bar{\theta} - 0.0000155041 \cos 4\bar{\theta} - 0.0000064148 \cos 5\bar{\theta} \\ & - 0.0000043222 \cos 6\bar{\theta} - 0.0000027052 \cos 7\bar{\theta} - 0.0000029382 \cos 8\bar{\theta} \\ & + 0.0000069836 \cos 9\bar{\theta} , \end{aligned} \tag{55}$$

$$\begin{aligned} g_c'' = {} & 125^\circ.65579 + (1^\circ.2412695/\text{day})(t - t_0) + 8^\circ.0967578 \sin\bar{\theta} \\ & + 3^\circ.5255978 \sin 2\bar{\theta} + 0^\circ.5354116 \sin 3\bar{\theta} + 0^\circ.4133096 \sin 4\bar{\theta} \\ & + 0^\circ.1799011 \sin 5\bar{\theta} + 0^\circ.1123452 \sin 6\bar{\theta} + 0^\circ.1127257 \sin 7\bar{\theta} \\ & + 0^\circ.1213947 \sin 8\bar{\theta} + 0^\circ.0427651 \sin 9\bar{\theta} . \end{aligned} \tag{56}$$

Nimbus 2

$$\bar{\theta} = 51^\circ.34033 - (2^\circ.3536297/\text{day})(t - t_0) , \tag{57}$$

$$\begin{aligned} e_c'' = {} & 0.0055936191 - 0.0001200321 \cos\bar{\theta} - 0.0000648964 \cos 2\bar{\theta} + 0.0000044700 \cos 3\bar{\theta} \\ & + 0.0000015399 \cos 4\bar{\theta} + 0.0000017144 \cos 5\bar{\theta} - 0.0000009668 \cos 6\bar{\theta} \\ & + 0.0000034168 \cos 7\bar{\theta} + 0.0000019838 \cos 8\bar{\theta} + 0.0000008763 \cos 9\bar{\theta} , \end{aligned} \tag{58}$$

$$g_c'' = 679\overset{\circ}{.}62285 - (2\overset{\circ}{.}3542110/\text{day})(t - t_0) + 1\overset{\circ}{.}3007803 \sin\bar{\theta}$$
$$+ 0\overset{\circ}{.}8192370 \sin 2\bar{\theta} - 0\overset{\circ}{.}0202026 \sin 3\bar{\theta} + 0\overset{\circ}{.}0196037 \sin 4\bar{\theta}$$
$$+ 0\overset{\circ}{.}0099007 \sin 5\bar{\theta} + 0\overset{\circ}{.}0441686 \sin 6\bar{\theta} + 0\overset{\circ}{.}0034652 \sin 7\bar{\theta}$$
$$+ 0\overset{\circ}{.}0379263 \sin 8\bar{\theta} + 0\overset{\circ}{.}0279919 \sin 9\bar{\theta} \ . \qquad (59)$$

Values for

$$e_0'' + \sum_{j=1}^{9} A_j \cos j\bar{\theta}, \qquad g_c'' - g_0'' - \dot{g}_c''(t - t_0), \qquad \text{and} \qquad \sum_{j=1}^{9} B_j \sin j\bar{\theta}$$

are listed in Tables 1 to 6 (Appendix B). Figures 1 to 6 (Appendix C) reveal the long-period variations in e_c'' and g_c'', and also show the closeness of the least-squares fits.

In order to compare the observed variations with the theory, it is necessary to alter Equations 39 and 40 somewhat. This is because the terms $(Q/e_1)\sin\bar{\theta}$ (Equation 39) and $-Q\cos\bar{\theta}$ (Equation 40) appear in the mathematical model for orbit determination used at Goddard, at least for J_2, J_3, J_4, and J_5 and with e'' in place of e_1. The J_3 and J_5 values used were

$$J_3^* = -2.285 \times 10^{-6}, \qquad J_5^* = -0.232 \times 10^{-6}.$$

It is suggested that Kozai's (1964) determination of the harmonic coefficients form a better set; they are

$$J_2 = 1.082645 \times 10^{-3}, \qquad J_3 = -2.546 \times 10^{-6},$$
$$J_4 = -1.649 \times 10^{-6}, \qquad J_5 = -0.210 \times 10^{-6},$$
$$J_7 = -0.333 \times 10^{-6}, \qquad J_9 = -0.053 \times 10^{-6},$$
$$J_{11} = 0.302 \times 10^{-6}.$$

It is further suggested that the observed variations in e_c'' and g_c'' should reflect the differences between the two sets of values for J_3 and J_5.

As a preliminary, therefore, the following definitions are made:

1. N and M are given by Equations 24 and 25, using Kozai's (1964) determination of the harmonic coefficients;

2. ΔM^* has J_3^* and J_5^* in place of J_3 and J_5 in Equation 25, with $J_7 = J_9 = J_{11} = \cdots = 0$;

3. ΔM has $J_3 - J_3^*$ and $J_5 - J_5^*$ in place of J_3 and J_5 in Equation 25;

4. $\Delta Q^* = \Delta M^*/N$ and $\Delta Q = \Delta M/N$.

Then, from Equations 39 and 40 it is found that the expressions to be compared with the observed variations are

$$\Delta e_c'' = -\left(\Delta Q - \frac{Q^3}{8e_1^2}\right)\cos\bar{\theta} - \frac{Q^2}{4e_1}\cos 2\bar{\theta} - \frac{Q^3}{8e_1^2}\cos 3\bar{\theta} \,, \tag{60}$$

$$\Delta g_c'' = \left(\frac{Q}{e_1} - \frac{\Delta Q^*}{e''} + \frac{Q^3}{4e_1^3}\right)\sin\bar{\theta} + \frac{Q^2}{2e_1^2}\sin 2\bar{\theta} + \frac{Q^3}{3e_1^3}\sin 3\bar{\theta} \,. \tag{61}$$

In the evaluation of $\Delta e_c''$ and $\Delta g_c''$, the eccentricity constants in Equations 52, 55, and 58 were used for e''. Values for e_1 were computed from Equation 38. As a result, the theoretical variations were

Alouette I

$$\Delta e_c'' = -0.0001311878\cos\bar{\theta} - 0.0001183911\cos 2\bar{\theta} - 0.0000250684\cos 3\bar{\theta} \,, \tag{62}$$

$$\Delta g_c'' = 3^\circ\!.4477685\sin\bar{\theta} + 5^\circ\!.1377014\sin 2\bar{\theta} + 1^\circ\!.4504933\sin 3\bar{\theta} \,. \tag{63}$$

Tiros 8

$$\Delta e_c'' = -0.0004504508\cos\bar{\theta} - 0.0001738383\cos 2\bar{\theta} - 0.0000380812\cos 3\bar{\theta} \,, \tag{64}$$

$$\Delta g_c'' = 8^\circ\!.0064801\sin\bar{\theta} + 5^\circ\!.4989977\sin 2\bar{\theta} + 1^\circ\!.6061560\sin 3\bar{\theta} \,. \tag{65}$$

Nimbus 2

$$\Delta e_c'' = -0.0001173404\cos\bar{\theta} - 0.0000497609\cos 2\bar{\theta} - 0.0000046725\cos 3\bar{\theta} \,, \tag{66}$$

$$\Delta g_c'' = 1^\circ\!.2480738\sin\bar{\theta} + 1^\circ\!.0103390\sin 2\bar{\theta} + 0^\circ\!.1264920\sin 3\bar{\theta} \,. \tag{67}$$

The theoretical and observed amplitudes are compared in Table 7 (Appendix B). The overall agreement is fairly good, if we bear in mind that the formulas are sensitive to e_1 and the differences $J_3 - J_3^*$ and $J_5 - J_5^*$. In addition, it was found that even the J_{11} harmonic was important in the computation, so possibly the inclusion of higher harmonics would improve the agreement.

AMPLITUDES OF SHORT-PERIOD TERMS

From the mean orbital elements given earlier, the derived amplitudes of the short-period terms in Equations 9 and 19 are as follows:

Alouette I

$$\Delta\ell(J_2\,\mathrm{part}) = 12\overset{\circ}{.}633\sin f' + 6\overset{\circ}{.}6939\sin(f'+2g') - 15\overset{\circ}{.}619\sin(3f'+2g') ,$$

$$\Delta\ell(J_2^2\,\mathrm{part}) = 3\overset{\circ}{.}2172\sin 2f' - 0\overset{\circ}{.}39101\sin(2f'+4g') - 3\overset{\circ}{.}4435\sin(4f'+2g') + 2\overset{\circ}{.}1288\sin(6f'+4g') ,$$

$$\begin{aligned}\Delta\ell(J_2^3\,\mathrm{part}) = {} & -0\overset{\circ}{.}45512\sin f' + 1\overset{\circ}{.}1355\sin 3f' - 0\overset{\circ}{.}31203\sin(f'+2g') - 0\overset{\circ}{.}60585\sin(f'-2g') \\ & - 0\overset{\circ}{.}19397\sin(f'+4g') + 0\overset{\circ}{.}42885\sin(3f'+2g') + 0\overset{\circ}{.}12213\sin(3f'+4g') \\ & + 0\overset{\circ}{.}034261\sin(3f'+6g') - 1\overset{\circ}{.}4137\sin(5f'+2g') - 0\overset{\circ}{.}21792\sin(5f'+4g') \\ & - 0\overset{\circ}{.}026648\sin(5f'+6g') + 1\overset{\circ}{.}0561\sin(7f'+4g') + 0\overset{\circ}{.}062177\sin(7f'+6g') \\ & - 0\overset{\circ}{.}43524 + \sin(9f'+6g') ,\end{aligned}$$

$$10^4 \times \Delta e(J_2\,\mathrm{part}) = -5.5481\cos f' + 2.9399\cos(f'+2g') + 6.8597\cos(3f'+2g') ,$$

$$\begin{aligned}10^4 \times \Delta e(J_2^2\,\mathrm{part}) = {} & -0.70653\cos 2f' - 1.0803\cos 2(f'+g') - 0.085871\cos(2f'+4g') \\ & + 0.75625\cos(4f'+2g') + 0.40073\cos 4(f'+g') - 0.46752\cos(6f'+4g') .\end{aligned}$$

Tiros 8

$$\Delta\ell(J_2\,\mathrm{part}) = 1\overset{\circ}{.}9723\sin f' + 3\overset{\circ}{.}9618\sin(f'+2g') - 9\overset{\circ}{.}2441\sin(3f'+2g') ,$$

$$\Delta\ell(J_2^2\,\mathrm{part}) = 0\overset{\circ}{.}67311\sin 2f' - 0\overset{\circ}{.}13697\sin(2f'+4g') - 0\overset{\circ}{.}31822\sin(4f'+2g') + 0\overset{\circ}{.}74570\sin(6f'+4g') ,$$

$$\begin{aligned}\Delta\ell(J_2^3\,\mathrm{part}) = {} & -0\overset{\circ}{.}020985\sin f' + 0\overset{\circ}{.}050382\sin 3f' - 0\overset{\circ}{.}030686\sin(f'+2g') \\ & - 0\overset{\circ}{.}054994\sin(f'-2g') - 0\overset{\circ}{.}010608\sin(f'+4g') + 0\overset{\circ}{.}044446\sin(3f'+2g') \\ & + 0\overset{\circ}{.}0066795\sin(3f'+4g') + 0\overset{\circ}{.}0071030\sin(3f'+6g') - 0\overset{\circ}{.}12832\sin(5f'+2g') \\ & - 0\overset{\circ}{.}011918\sin(5f'+4g') - 0\overset{\circ}{.}0055243\sin(5f'+6g') + 0\overset{\circ}{.}057754\sin(7f'+4g') \\ & + 0\overset{\circ}{.}012890\sin(7f'+6g') - 0\overset{\circ}{.}090229\sin(9f'+6g') ,\end{aligned}$$

$$10^4 \times \Delta e\left(J_2 \text{ part}\right) = -1.1841 \cos f' + 2.3784 \cos(f' + 2g') + 5.5494 \cos(3f' + 2g') ,$$

$$10^4 \times \Delta e\left(J_2^2 \text{ part}\right) = -0.20202 \cos 2f' - 0.13644 \cos 2(f' + g') - 0.041108 \cos(2f' + 4g')$$
$$+ 0.095507 \cos(4f' + 2g') + 0.19184 \cos 4(f' + g') - 0.22381 \cos(6f' + 4g') .$$

Nimbus 2

$$\Delta \ell\left(J_2 \text{ part}\right) = 5^\circ\!.4162 \sin f' + 2^\circ\!.8999 \sin(f' + 2g') - 6^\circ\!.7661 \sin(3f' + 2g')$$

$$\Delta \ell\left(J_2^2 \text{ part}\right) = 0^\circ\!.59845 \sin 2f' - 0^\circ\!.073384 \sin(2f' + 4g') - 0^\circ\!.63959 \sin(4f' + 2g')$$
$$+ 0^\circ\!.39953 \sin(6f' + 4g') ,$$

$$\Delta \ell\left(J_2^3 \text{ part}\right) = -0^\circ\!.036495 \sin f' + 0^\circ\!.090986 \sin 3f' - 0^\circ\!.025075 \sin(f' + 2g')$$
$$- 0^\circ\!.048651 \sin(f' - 2g') - 0^\circ\!.015608 \sin(f' + 4g') + 0^\circ\!.034483 \sin(3f' + 2g')$$
$$+ 0^\circ\!.0098274 \sin(3f' + 4g') + 0^\circ\!.0027855 \sin(3f' + 6g') - 0^\circ\!.11352 \sin(5f' + 2g')$$
$$- 0^\circ\!.017535 \sin(5f' + 4g') - 0^\circ\!.0021665 \sin(5f' + 6g') + 0^\circ\!.084975 \sin(7f' + 4g')$$
$$+ 0^\circ\!.0050553 \sin(7f' + 6g') - 0^\circ\!.035387 \sin(9f' + 6g') ,$$

$$10^4 \times \Delta e\left(J_2 \text{ part}\right) = -5.2876 \cos f' + 2.8310 \cos(f' + 2g') + 6.6058 \cos(3f' + 2g') ,$$

$$10^4 \times \Delta e\left(J_2^2 \text{ part}\right) = -0.29213 \cos 2f' - 0.44603 \cos 2(f' + g') - 0.035821 \cos(2f' + 4g')$$
$$+ 0.31222 \cos(4f' + 2g') + 0.16717 \cos 4(f' + g') - 0.19503 \cos(6f' + 4g') .$$

Clearly, the J_2^2 and J_2^3 terms are comparable to the J_2 terms. Certainly, there is not a 10^{-3} difference between terms of succeeding "orders" of J_2.

CONCLUSIONS

The preceding results clearly show that, for small eccentricity satellites, "first-order" analytic orbit theories give rise to orbit-determination models that are lacking in substantial periodic perturbations comparable in size to the J_2 terms. These additional perturbations come from the "higher-order" terms that have eccentricity as a divisor; moreover, the effects of the long-period terms are easily discernible in the motions of existing satellites. It might provide better agreement between the observed and theoretical long-period amplitudes if we were to include the J_2^2 and J_2^3 short-period terms in the orbit determination model used to derive the mean elements. These results are expected to be compared with numerical integration; it is anticipated that this will further substantiate the findings.

ACKNOWLEDGMENTS

The author wishes to thank Mr. Eric L. Victor and Miss Ellen L. Steinberg for their help in checking many of the computations, preparing tables and graphs, and making computer runs.

Goddard Space Flight Center
National Aeronautics and Space Administration
Greenbelt, Maryland, March 11, 1968
311-07-21-01-51

REFERENCES

Brouwer, D., "Solution of the Problem of Artificial Satellite Theory Without Air Drag," *Astron. J.* 64(9):378-397, November 1959.

Brouwer, D., and Clemence, G. M., "Methods of Celestial Mechanics," New York: The Academic Press, 1961.

Felsentreger, T. L., "Solution of Delaunay's Equations of Type II," Goddard Space Flight Center Document X-547-65-160, April 1965 (Revised May 1965).

Fisher, D., and Felsentreger, T. L., "Effects of the Solar and Lunar Tides on the Motion of an Artificial Satellite," Goddard Space Flight Center Document X-547-66-560, November 1966.

Kozai, Y., "Note on the Motion of a Close Earth Satellite with a Small Eccentricity," *Astron. J.* 66(3):132-134, April 1961.

Kozai, Y., "Second-Order Solution of Artificial Satellite Theory Without Air Drag," *Astron. J.* 67(7):446-461, September 1962.

Kozai, Y., "New Determination of Zonal Harmonics Coefficients of the Earth's Gravitational Potential," Special Report 165, Smithsonian Institution Astrophysical Observatory, November 2, 1964.

Lyddane, R. H., "Small Eccentricities or Inclinations in the Brouwer Theory of the Artificial Satellite," *Astron. J.* 68(8):555-558, October 1963.

Murphy, J. P., and Felsentreger, T. L., "Analysis of Lunar and Solar Effects on the Motion of Close Earth Satellites," NASA Technical Note D-3559, August 1966.

Appendix A

Symbols

a	Semimajor axis of satellite's orbit
A_j $(j = 1, 2, \cdots, 9)$	Constants, defined in Equation 49
B_j $(j = 1, 2, \cdots, 9)$	Constants, defined in Equation 50
e	Eccentricity of satellite's orbit
e_1	Eccentricity constant defined by Equation 38
e_0''	Eccentricity constant in Equation 49
e_c''	Eccentricity corrected for lunar and solar effects
f	True anomaly of satellite
g	Argument of perigee of satellite's orbit
g_0''	Argument of perigee constant in Equation 50
g_c''	Argument of perigee corrected for lunar and solar effects
$\dot{g}_c''$	Secular motion of g_c'' (Equation 50)
G	$\left[\mu a\left(1 - e^2\right)\right]^{1/2}$
h	Longitude of ascending node
H	$G \cos i$
i	Inclination of satellite's orbital plane to earth's equatorial plane
$J_2, J_3, \cdots$	Zonal harmonic coefficients in earth's gravitational potential
J_3^*, J_5^*	J_3, J_5 values used at Goddard Space Flight Center in orbit-determination program (i.e., $J_3^* = -2.285 \times 10^{-6}$, $J_5^* = -0.232 \times 10^{-6}$)
ℓ	Mean anomaly of satellite
L	$(\mu a)^{1/2}$
M	Defined by Equation 25
ΔM	Defined by Equation 25, with $J_3 - J_3^*$ and $J_5 - J_5^*$ in place of J_3 and J_5
ΔM^*	Defined by Equation 25, with J_3^* and J_5^* in place of J_3 and J_5, and with $J_7 = J_9 = J_{11} = \cdots = 0$
N	Defined by Equation 24

Q	M/N
ΔQ	$\Delta M/N$
ΔQ^*	$\Delta M^*/N$
r	Geocentric distance of satellite
S_1, S_2	Short-period determining functions (See Equations 1 and 2)
S_1^*, S_2^*, S_3^*	Long-period determining functions
$\Delta S_1^*, \Delta S_2^*, \Delta S_3^*$	Defined by Equations 20, 21, and 22
$t - t_0$	Elapsed time since initial epoch
$\alpha_1, \alpha_2, \alpha_3$	Coefficients defined in Equation 44
$\beta_1, \beta_2, \beta_3$	Coefficients defined in Equation 43
θ	$g' + \pi/2$
$\overline{\theta}$	$g'' + \pi/2$

Notes on Symbols:

Primed elements (e', ℓ', etc.) are osculating elements *less* first-order short-period terms.

Double-primed elements (e'', ℓ'', etc.) are primed elements *less* J_2, J_3, J_4, J_5 long-period terms.

The symbol "Δ" prefixing an element designates a perturbation or variation in the element.

Appendix B

Tables

Table 1

Eccentricity of Alouette I.

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0'' + \sum_{j=1}^{9} A_j \cos j\bar{\theta}\right) \times 10^2$
0	0.26144376	0.26537030
7	0.25871777	0.26220757
14	0.25607286	0.25939988
21	0.24449395	0.24958975
28	0.23735904	0.23887395
35	0.22244876	0.22744022
49	0.22755925	0.22533918
56	0.24002016	0.23735173
70	0.26071055	0.25837643
77	0.26551042	0.26191449
84	0.26744275	0.26482671
98	0.25913103	0.26195223
119	0.25383359	0.25431977
126	0.25626008	0.26072945
133	0.26025462	0.26565796
140	0.26297148	0.26553041
147	0.25871509	0.26231250
154	0.26134549	0.25967895
161	0.24748761	0.25019792
168	0.24020682	0.23935735
182	0.21334841	0.21800599
189	0.22432010	0.22467505
196	0.23247997	0.23685073
201	0.24577516	0.24391138
208	0.26312523	0.25540848
215	0.26812333	0.26129415
222	0.26386234	0.26360335
236	0.26792922	0.26376418
243	0.25908024	0.25783860
250	0.24963096	0.25201272
257	0.25192732	0.25250961
264	0.25645127	0.25862181
271	0.26108098	0.26434214
278	0.26628269	0.26629414
285	0.25685775	0.26320387
292	0.25121652	0.26099828
299	0.25179609	0.25412753
306	0.23714491	0.24265445
313	0.22717008	0.23241403
320	0.22112633	0.22003809
327	0.22437962	0.22070388
334	0.23871339	0.23323816
341	0.25044991	0.24337748
348	0.25785615	0.25488266
362	0.27062429	0.26343142
369	0.27130551	0.26636918
376	0.26925108	0.26401216

Table 1 (Continued)

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0'' + \sum_{j=1}^{9} A_j \cos j\bar{\theta}\right) \times 10^2$
383	0.26114939	0.25816855
390	0.25307810	0.25220931
397	0.25245587	0.25228939
404	0.25637716	0.25829464
411	0.26291397	0.26410523
418	0.26170294	0.26635163
425	0.26389721	0.26336706
432	0.25789832	0.26113067
439	0.25016203	0.25467635
446	0.24019515	0.24317523
455	0.22589934	0.22945224
462	0.21591834	0.21838892
469	0.22068942	0.22338346
476	0.23533547	0.23582200
483	0.24822932	0.24595138
489.70486	0.25030181	0.25678984
496.70486	0.26066031	0.26157692
503.70486	0.26765384	0.26411447
510.70486	0.27155964	0.26640223
517.70486	0.26437484	0.26302577
524.70486	0.25939428	0.25689890
531.70486	0.25239343	0.25155922
538.70486	0.25124586	0.25321344
545.70486	0.25564045	0.25953301
552.70486	0.26061170	0.26496173
557.70486	0.26295045	0.26641240
564.70486	0.26095131	0.26368288
571.70486	0.25934972	0.26134336
578.70486	0.25650770	0.25564022
584.70486	0.24500476	0.24573405
591.70486	0.23457316	0.23562205
598.70486	0.22485878	0.22314570
605.70486	0.22265337	0.21848485
612.70486	0.23192097	0.22970239
619.70486	0.24308295	0.24051097
626.70486	0.25162352	0.25163704
633.70486	0.26184213	0.26025111
640.70486	0.26530320	0.26259063
647.70486	0.27353441	0.26587053
654.70486	0.27184750	0.26524760
661.70486	0.26447327	0.25999302
667.70486	0.25222628	0.25445884
674.70486	0.25103885	0.25119352
681.70486	0.25335351	0.25548038
688.70486	0.25820461	0.26182112
695.70486	0.26029715	0.26613031
702.70486	0.26490152	0.26490888
709.70486	0.26249207	0.26195477
716.70486	0.25713385	0.25853996

Table 1 (Continued)

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0'' + \sum_{j=1}^{9} A_j \cos j\bar{\theta}\right) \times 10^2$
723.70486	0.24794743	0.24798143
730.70486	0.23423279	0.23757589
737.70486	0.22400092	0.22564218
744.70486	0.21910726	0.21786121
751.70486	0.23313638	0.22713393
758.70486	0.24311689	0.23865496
765.70486	0.24894646	0.24931496
772.70486	0.26140113	0.25926586
779.70486	0.25985893	0.26216213
786.70486	0.26040271	0.26529515
794.70486	0.27028544	0.26542908
804.70486	0.26504396	0.25756433
808.70486	0.25435655	0.25390550
822.70486	0.25505307	0.25608246
829.70486	0.25681805	0.26234663
836.70486	0.26084080	0.26628561
843.70486	0.25794681	0.26457214
850.70486	0.25903076	0.26179389
857.70486	0.26195972	0.25784642
884.70486	0.21750902	0.21782818
891.70486	0.22434360	0.22645646
898.70486	0.24179434	0.23816861
905.70486	0.24720073	0.24870863
911.70486	0.26352571	0.25792895
918.70486	0.26893127	0.26181188
925.70486	0.27281346	0.26461062
932.70486	0.27063842	0.26627094
940.70486	0.26949408	0.26146684
954.70486	0.25533530	0.25116558
961.70486	0.25193124	0.25483685
975.70486	0.26561079	0.26589761
982.70486	0.26585615	0.26526020
996.70486	0.25891672	0.25920282

$e_0'' = 0.0025163652$ $\quad A_1 = -0.0001492876$ $\quad A_2 = -0.0001336935$

$A_3 = -0.0000097969$ $\quad A_4 = -0.0000264826$ $\quad A_5 = 0.0000007387$

$A_6 = -0.0000042243$ $\quad A_7 = -0.0000082012$ $\quad A_8 = -0.0000070067$

$A_9 = -0.0000001323$ $\quad \bar{\theta} = 109^\circ\!.13743 - (2^\circ\!.5649585/\text{day})(t - t_0)$

$t - t_0$ = days since 3 Nov. 1963, 7 hrs 05 min U.T.

Table 2

Argument of Perigee of Alouette I.

$t - t_0$ (days)	g_c'' (degrees)	$g_c'' - [g_0'' + \dot{g}_c''(t - t_0)]$ (degrees)	$\sum_{j=1}^{9} B_j \sin j\bar{\theta}$ (degrees)
0	1099.31099	1.56480	1.66824
7	1084.38172	4.56862	3.79416
14	1067.61000	5.73005	5.97897
21	1050.38279	6.43597	7.32940
28	1032.26639	6.25270	7.45176
35	1013.54196	5.46142	5.25543
49	967.47511	-4.73922	-4.62688
56	947.84411	-6.43709	-7.30885
70	911.67829	-6.73666	-6.27662
77	896.39081	-4.09102	-4.13321
84	880.80939	-1.73931	-2.02219
98	848.42732	1.74487	1.94181
119	791.28301	-1.60007	-1.98001
126	773.83660	-1.11335	-2.11188
133	757.18526	0.16843	-0.77541
140	740.11901	1.03531	1.55155
147	724.76300	3.61242	3.68696
154	707.99420	4.77675	5.87760
161	692.34621	7.06188	7.29129
168	674.36622	7.01502	7.48468
182	627.80422	-3.68073	0.69798
189	609.52750	-4.02433	-4.41088
196	588.52382	-7.09488	-7.24896
201	575.71039	-7.09894	-7.55947
208	558.96521	-5.91099	-6.80886
215	542.91323	-4.02985	-4.88339
222	526.50266	-2.50729	-2.73991
236	496.45747	3.31427	1.55125
243	479.05674	3.84616	2.28482
250	459.97451	2.69706	1.15849
257	439.03768	-0.30665	-1.42359
264	420.29035	-1.12085	-2.26965
271	403.05640	-0.42168	-1.37480
278	386.63545	1.09050	0.74730
285	370.72010	3.10827	2.98848
292	354.69434	5.01564	5.15365
299	338.16865	6.42307	6.96186
306	320.44346	6.63101	7.57280
313	300.43327	4.55394	6.48325
320	278.97242	1.02622	2.47439
327	254.45608	-5.55699	-2.82822
334	233.25165	-8.82830	-6.65280
341	215.49481	-8.65202	-7.56748
348	198.67583	-7.53787	-6.87546
362	167.44797	-2.89948	-2.84445
369	152.10532	-0.30901	-0.57744

Table 2 (Continued)

$t - t_0$ (days)	g_c'' (degrees)	$g_c'' - \left[g_0'' + \dot{g}_c''(t - t_0)\right]$ (degrees)	$\sum_{j=1}^{9} B_j \sin j\bar{\theta}$ (degrees)
376	136.67553	2.19433	1.47965
383	119.89750	3.34942	2.28144
390	100.44572	1.83077	1.27344
397	80.31454	-0.36729	-1.31618
404	61.59278	-1.15592	-2.27901
411	44.11512	-0.70046	-1.45130
418	27.74386	0.86141	0.62450
425	12.30852	3.35919	2.88440
432	- 3.65805	5.32575	5.04053
439	- 19.80376	7.11316	6.90004
446	- 38.01440	6.83565	7.56960
455	- 62.10665	5.80027	5.79301
462	- 85.22013	0.61992	1.24025
469	-109.03395	-5.26078	-3.96046
476	-129.72493	-8.01863	-7.10704
483	-147.85210	-8.21268	-7.50396
489.70486	-163.46894	-6.65250	-6.60175
496.70486	-179.87633	-5.12677	-4.56244
503.70486	-195.74486	-3.06199	-2.44097
510.70486	-210.88356	-0.26775	-0.11219
517.70486	-225.88114	2.66780	1.73491
524.70486	-243.75366	2.72840	2.26852
531.70486	-262.58594	1.82925	0.80781
538.70486	-283.03946	-0.69115	-1.69490
545.70486	-301.55471	-1.27327	-2.22192
552.70486	-318.52399	-0.30943	-1.13936
557.70486	-330.59113	0.43281	0.39964
564.70486	-346.42503	2.53203	2.69249
571.70486	-362.28373	4.60646	4.83200
578.70486	-377.44955	7.37376	6.77763
584.70486	-392.75477	7.43979	7.51143
591.70486	-410.97344	7.15425	7.07658
598.70486	-434.21879	1.84202	3.87229
605.70486	-455.35533	-1.36139	-1.34258
612.70486	-478.35659	-6.42953	-5.85601
619.70486	-498.33821	-8.47802	-7.54058
626.70486	-515.50363	-7.71032	-7.18929
633.70486	-531.34169	-5.61525	-5.62738
640.70486	-546.94966	-3.29010	-3.43513
647.70486	-562.43236	-0.83967	-1.26893
654.70486	-576.75933	2.76648	1.00699
661.70486	-594.06069	3.39825	2.18588
667.70486	-609.07338	3.75681	1.99732
674.70486	-629.97554	0.78777	-0.22544
681.70486	-649.07326	-0.37682	-2.16137
688.70486	-667.32226	-0.69270	-1.96313
695.70486	-683.65836	0.90433	-0.38545
702.70486	-699.55967	2.93614	1.97156

Table 2 (Continued)

$t - t_0$ (days)	g_c'' (degrees)	$g_c'' - [g_0'' + \dot{g}_c'' (t - t_0)]$ (degrees)	$\sum_{j=1}^{9} B_j \sin j\bar{\theta}$ (degrees)
709.70486	- 715.70662	4.72232	4.08336
716.70486	- 732.30086	6.06120	6.23510
723.70486	- 750.52649	5.76870	7.41737
730.70486	- 768.25612	5.97219	7.33361
737.70486	- 788.66003	3.50141	4.72236
744.70486	- 811.32247	-1.22791	-0.30179
751.70486	- 834.31196	-6.28427	-5.16844
758.70486	- 853.71462	-7.75381	-7.43487
765.70486	- 872.67092	-8.77692	-7.34571
772.70486	- 888.45325	-6.62619	-6.02391
779.70486	- 903.40289	-3.64270	-3.84290
786.70486	- 920.41593	-2.72262	-1.72051
794.70486	- 936.98729	1.20102	0.91193
804.70486	- 959.72034	4.08672	2.28405
808.70486	- 971.13202	2.92254	1.88870
822.70486	-1010.85152	-0.93069	-2.22030
829.70486	-1029.19917	-1.34526	-1.87294
836.70486	-1046.28979	-0.50274	-0.17727
843.70486	-1062.14499	1.57519	2.17480
850.70486	-1078.27671	3.37661	4.28622
857.70486	-1094.47075	5.11564	6.39915
884.70486	-1168.06836	0.68866	-0.02841
891.70486	-1192.05474	-5.36451	-4.97071
898.70486	-1213.49333	-8.86999	-7.39306
905.70486	-1231.15939	-8.60296	-7.37985
911.70486	-1245.52583	-7.59811	-6.38134
918.70486	-1260.73638	-4.87559	-4.26335
925.70486	-1275.36111	-1.56716	-2.15224
932.70486	-1291.62282	0.10426	0.20143
940.70486	-1309.48614	2.73596	2.01669
954.70486	-1345.35347	2.73481	0.05570
961.70486	-1367.39337	-1.37196	-2.07316
975.70486	-1402.75694	-0.86921	-0.60340
982.70486	-1419.34065	0.48011	1.74421

$g_0'' = 1097\overset{\circ}{.}74620$ $\quad \dot{g}_c'' = -2\overset{\circ}{.}5618750/\text{day}$ $\quad B_1 = 4\overset{\circ}{.}2076854$ $\quad B_2 = 4\overset{\circ}{.}6340045$

$B_3 = 0\overset{\circ}{.}1873826$ $\quad B_4 = 0\overset{\circ}{.}7003276$ $\quad B_5 = 0\overset{\circ}{.}0066516$ $\quad B_6 = 0\overset{\circ}{.}1206393$

$B_7 = 0\overset{\circ}{.}0102137$ $\quad B_8 = 0\overset{\circ}{.}0387951$ $\quad B_9 = -0\overset{\circ}{.}0680971$

$\bar{\theta} = 109\overset{\circ}{.}13743 - (2\overset{\circ}{.}5649585/\text{day})(t - t_0)$

Table 3

Eccentricity of Tiros 8.

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0'' + \sum_{j=1}^{9} A_j \cos j\theta\right) \times 10^2$
0	0.37506961	0.37628641
4	0.37619047	0.37570213
10.99306	0.37460858	0.37520643
17.99306	0.37238024	0.37446838
24.99306	0.37085765	0.37171128
31.99306	0.36733611	0.36676298
38.99306	0.36039354	0.36119998
45.99306	0.35743823	0.35587962
52.99306	0.35615808	0.34972838
59.99306	0.33757149	0.34161870
66.99306	0.33314402	0.33233538
73.99306	0.31740621	0.32357566
80.99306	0.31274407	0.31560048
87.99306	0.30286917	0.30718518
94.99306	0.29095610	0.29789457
101.99306	0.28879741	0.28924569
108.99306	0.28080994	0.28324962
115.99306	0.27858500	0.28074478
122.99306	0.27612262	0.28169755
129.99306	0.28875311	0.28616007
136.99306	0.29812405	0.29382323
143.99306	0.30691754	0.30309784
150.99306	0.30825442	0.31197276
157.99306	0.31510139	0.31998188
164.99306	0.33099291	0.32829756
171.99306	0.33561471	0.33751327
177.99306	0.33987896	0.34518655
183.99306	0.35051776	0.35155119
190.99306	0.35399237	0.35731930
196.99306	0.36010977	0.36188160
203.99306	0.36691971	0.36745437
210.99306	0.36883817	0.37219755
217.99306	0.37460394	0.37464193
224.99306	0.37351657	0.37524503
231.99306	0.37470128	0.37581423
238.99306	0.37580827	0.37690387
245.99306	0.37346974	0.37725685
252.99306	0.37382165	0.37616075
259.99306	0.37218264	0.37497208
266.99306	0.37528776	0.37532137
273.99306	0.37495581	0.37672327
280.99306	0.37767549	0.37729502
287.99306	0.37789616	0.37646770
294.99306	0.37447423	0.37550382
301.99306	0.37530401	0.37510934
308.99306	0.37283971	0.37396844
315.99306	0.37287115	0.37052801

Table 3 (Continued)

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0'' + \sum_{j=1}^{9} A_j \cos j\bar{\theta}\right) \times 10^2$
322.99306	0.36830645	0.36523473
329.99306	0.36135782	0.35974035
336.99306	0.35292034	0.35435431
343.99306	0.34782399	0.34771606
350.99306	0.34033867	0.33912574
357.99306	0.33043465	0.32984273
364.99306	0.32312787	0.32136150
378.99306	0.30475120	0.30470470
385.99306	0.29380197	0.29537618
392.99306	0.28598036	0.28728450
396.99306	0.28557000	0.28395876
414.99306	0.28209103	0.28312168
421.99306	0.28255986	0.28901708
428.99306	0.29979616	0.29761177
435.58889	0.30805258	0.30639272
442.58889	0.31621209	0.31489632
449.58889	0.31967711	0.32285454
456.58889	0.33049635	0.33152425
463.58889	0.34108957	0.34082264
470.58889	0.34535766	0.34909678
477.58889	0.35227630	0.35539594
484.58889	0.35844634	0.36072574
491.58889	0.36343811	0.36627238
498.58889	0.37040260	0.37134748
505.58889	0.37353149	0.37432617
512.58889	0.37415101	0.37517826
519.58889	0.37646365	0.37563213
526.58889	0.37427000	0.37667850
533.58889	0.37487808	0.37731041
540.58889	0.37637825	0.37647366
547.58889	0.37514961	0.37512945
554.58889	0.37589851	0.37510403
561.58889	0.38017558	0.37643276
568.58889	0.37948983	0.37730770
575.58889	0.37910182	0.37671023
582.58889	0.37713322	0.37565452
589.58889	0.37896258	0.37518764
595.58889	0.37454717	0.37458753
603.58889	0.37299622	0.37146791
610.58889	0.37132891	0.36643251
617.58889	0.36500245	0.36087979
624.58889	0.35629898	0.35555421
631.58889	0.35118739	0.34930429
638.58889	0.34392614	0.34108286
645.58889	0.33500286	0.33178773
652.58889	0.32298911	0.32308859
659.58889	0.31421141	0.31512612
666.58889	0.30859248	0.30665164
673.58889	0.29948474	0.29734106

Table 3 (Continued)

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0'' + \sum_{j=1}^{9} A_j \cos j\overline{\theta}\right) \times 10^2$
680.58889	0.28822450	0.28880022
687.58889	0.28074034	0.28300203
694.58889	0.28409691	0.28070556
701.58889	0.28624389	0.28186334
708.58889	0.28773906	0.28653461
715.58889	0.29714635	0.29435071
722.58889	0.30602984	0.30365109
729.58889	0.31626900	0.31246574
736.58889	0.31880058	0.32045329
743.58889	0.32718453	0.32882380
750.58889	0.33949012	0.33806790
757.58889	0.34328959	0.34682916
764.58889	0.35768636	0.35369119
771.58889	0.35838231	0.35913877
778.58889	0.36159859	0.36459420
792.58889	0.37257410	0.37370595
799.58889	0.37840063	0.37505557
806.58889	0.37739372	0.37543823
813.58889	0.37577636	0.37633588
820.58889	0.37751534	0.37725826
827.58889	0.37799798	0.37686126
834.58889	0.37371363	0.37546057
841.58889	0.37525164	0.37492026
848.58889	0.37966114	0.37599023
855.58889	0.38152254	0.37719711
862.58889	0.37475111	0.37701033
869.58889	0.37627181	0.37592491
876.58889	0.37252805	0.37528131
883.58889	0.37785191	0.37477231
890.58889	0.37263262	0.37260474
897.58889	0.36647209	0.36807100
904.58889	0.36525288	0.36250619
911.58889	0.35752890	0.35716509
918.58889	0.35083585	0.35136084
925.58889	0.34323831	0.34372833
932.58889	0.34242113	0.33456261
939.58889	0.32710771	0.32557333
946.58889	0.32017274	0.31749620
952.58889	0.31498104	0.31052414
959.58889	0.30196193	0.30148605
966.58889	0.28533687	0.29233065
973.58889	0.28808723	0.28514244
980.58889	0.28253207	0.28128755
987.58889	0.28178518	0.28092661
994.58889	0.28628806	0.28403749
1001.58889	0.29114240	0.29058966

$e_0'' = 0.0034394605$ $A_1 = -0.0004525939$ $A_2 = -0.0001389608$

$A_3 = -0.0000164065$ $A_4 = -0.0000155041$ $A_5 = -0.0000064148$

$A_6 = -0.0000043222$ $A_7 = -0.0000027052$ $A_8 = -0.0000029382$

$A_9 = 0.0000069836$ $\overline{\theta} = 213°61150 + (1°2452865/\text{day})(t - t_0)$

$t - t_0$ = days since 21 Dec. 1963, 9 hr 52 min U.T.

Table 4

Argument of Perigee of Tiros 8.

$t - t_0$ (days)	g_c'' (degrees)	$g_c'' - [g_0'' + \dot{g}_c'' (t - t_0)]$ (degrees)	$\sum_{j=1}^{9} B_j \sin j\bar{\theta}$ (degrees)
0	-236.38850	-2.04429	-1.53353
4	-231.73110	-2.35197	-1.93639
10.99306	-223.82704	-3.12818	-2.73801
17.99306	-216.04005	-4.03008	-3.67624
24.99306	-208.48143	-5.16034	-4.75277
31.99306	-200.83191	-6.19971	-5.88175
38.99306	-193.33678	-7.39347	-6.90784
45.99306	-185.33747	-8.08305	-7.74952
52.99306	-177.78234	-9.21680	-8.47321
59.99306	-169.81386	-9.93721	-9.14937
66.99306	-161.14324	-9.95548	-9.66197
73.99306	-152.21856	-9.71968	-9.76938
80.99306	-143.75699	-9.94700	-9.37769
87.99306	-133.50278	-8.38168	-8.63201
94.99306	-124.28197	-7.84975	-7.63543
101.99306	-114.91644	-7.17311	-6.16535
108.99306	-105.35505	-6.30061	-3.86079
115.99306	- 89.66564	0.69991	-0.74903
122.99306	- 75.61633	6.06034	2.54710
129.99306	- 67.09591	5.89187	5.25347
136.99306	- 57.01023	7.28867	7.06225
143.99306	- 47.03913	8.57088	8.22353
150.99306	- 36.88613	10.03499	9.07706
164.99306	- 20.06909	9.47426	9.77187
171.99306	- 11.35478	9.49968	9.41251
177.99306	- 4.33190	9.07494	8.87943
183.99306	2.62983	8.58906	8.28431
190.99306	10.66657	7.93690	7.54004
196.99306	16.95969	6.78141	6.78911
203.99306	24.78787	5.92170	5.74262
210.99306	32.64922	5.09417	4.61202
217.99306	39.80285	3.55891	3.55044
224.99306	48.05398	3.12115	2.63039
231.99306	56.30848	2.68677	1.84372
238.99306	63.83417	1.52357	1.18418
245.99306	72.45159	1.45210	0.67331
252.99306	80.74838	1.06000	0.31552
259.99306	88.97464	0.59739	0.06574
266.99306	97.86244	0.79630	-0.15698
273.99306	105.75508	0.00005	-0.44144
280.99306	114.11662	-0.32730	-0.85767
287.99306	122.33638	-0.79643	-1.43047
294.99306	130.00547	-1.81622	-2.14224
301.99306	137.77430	-2.73628	-2.97953
308.99306	145.60725	-3.59222	-3.95778
315.99306	153.02459	-4.86376	-5.06178

Table 4 (Continued)

$t - t_0$ (days)	g_c'' (degrees)	$g_c'' - \left[g_0'' + \dot{g}_c''(t - t_0)\right]$ (degrees)	$\sum_{j=1}^{9} B_j \sin j\bar{\theta}$ (degrees)
322.99306	161.08507	-5.49217	-6.17801
329.99306	168.64011	-6.62602	-7.15528
336.99306	176.39026	-7.56475	-7.95375
343.99306	184.02055	-8.62335	-8.66309
350.99306	192.56864	-8.76415	-9.31534
357.99306	200.29325	-9.72843	-9.74030
364.99306	209.28824	-9.42232	-9.70909
378.99306	227.47850	-8.60984	-8.38776
385.99306	237.73345	-7.04377	-7.29803
392.99306	248.87318	-4.59293	-5.62938
396.99306	254.09261	-4.33858	-4.28094
414.99306	281.67401	0.89997	3.77656
421.99306	293.92654	4.46362	6.10914
428.99306	304.88601	6.73420	7.59980
435.58889	314.60638	8.26736	8.55505
442.58889	323.79059	8.76269	9.32326
449.58889	332.82721	9.11042	9.75414
456.58889	341.20804	8.80237	9.69090
463.58889	349.83079	8.73623	9.20441
470.58889	357.87348	8.09003	8.53474
477.58889	365.59878	7.12644	7.81628
484.58889	373.55167	6.39044	6.98932
491.58889	381.87638	6.02627	5.97841
498.58889	389.04468	4.50568	4.85216
505.58889	397.09801	3.87012	3.76595
512.58889	403.69348	1.77671	2.81483
519.58889	412.20186	1.59621	2.00134
526.58889	420.73651	1.44197	1.31337
533.58889	428.68714	0.70372	0.76898
540.58889	437.28478	0.61247	0.38057
547.58889	445.77413	0.41293	0.11376
554.58889	453.39280	-0.65729	-0.10716
561.58889	462.25789	-0.48109	-0.37148
568.58889	470.68661	-0.74125	-0.75567
575.58889	478.61534	-1.50141	-1.29559
582.58889	486.65216	-2.15348	-1.97982
589.58889	494.53630	-2.95823	-2.78966
595.58889	500.92154	-4.02060	-3.59222
603.58889	510.16508	-4.70721	-4.81970
610.58889	517.66721	-5.89397	-5.94698
617.58889	525.25424	-6.99583	-6.96291
624.58889	533.15538	-7.78358	-7.79459
631.58889	540.96597	-7.66187	-8.51467
638.58889	548.93491	-9.38182	-9.18656
645.58889	557.13811	-9.86751	-9.68179

Table 4 (Continued)

$t - t_0$ (days)	g_c'' (degrees)	$g_c'' - [g_0'' + \dot{g}_c'' (t - t_0)]$ (degrees)	$\sum_{j=1}^{9} B_j \sin j\bar{\theta}$ (degrees)
652.58889	565.22543	-10.46908	-9.75950
659.58889	575.08680	- 9.29659	-9.34122
666.58889	584.65118	- 8.42110	-8.58024
673.58889	594.21800	- 7.43117	-7.56515
680.58889	604.07334	- 6.37672	-6.05440
687.58889	616.27122	- 2.86772	-3.69494
694.58889	626.58839	- 1.23944	-0.55013
701.58889	638.92826	2.41154	2.73047
708.58889	650.86627	5.66067	5.38560
715.58889	660.95223	7.05774	7.14523
722.58889	671.07217	8.48879	8.28075
729.58889	680.65811	9.38585	9.12046
736.58889	689.57999	9.61884	9.67293
743.58889	698.54485	9.89481	9.76275
750.58889	706.80434	9.46542	9.38007
757.58889	714.97396	8.94614	8.74158
764.58889	722.38052	7.66383	8.03699
771.58889	730.82908	7.42350	7.25423
778.58889	738.96423	6.86976	6.29846
792.58889	753.42075	3.94850	4.07828
799.58889	760.48241	2.32128	3.08342
806.58889	769.22128	2.37126	2.23113
813.58889	777.17153	1.63262	1.50499
820.58889	785.09547	0.86768	0.91505
827.58889	793.83962	0.92294	0.48126
834.58889	802.00001	0.39445	0.18468
841.58889	809.85732	- 0.43712	-0.04066
848.58889	819.50980	0.52646	-0.28291
855.58889	828.02533	0.35310	-0.62523
862.58889	835.39453	- 0.96658	-1.11808
869.58889	843.93954	- 1.11045	-1.76196

$g_0'' = -234°34421$ $\quad \dot{g}_c'' = 1°2412695/\text{day}$ $\quad B_1 = 8°0967578$ $\quad B_2 = 3°5255978$

$B_3 = 0°5354116$ $\quad B_4 = 0°4133096$ $\quad B_5 = 0°1799011$ $\quad B_6 = 0°1123452$

$B_7 = 0°1127257$ $\quad B_8 = 0°1213947$ $\quad B_9 = 0°0427651$

$\bar{\theta} = 213°61150 + (1°2452865/\text{day})(t - t_0)$

Table 5

Eccentricity of Nimbus 2.

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0'' + \sum_{j=1}^{9} A_j \cos j\bar{\theta}\right) \times 10^2$
0	0.55332889	0.55309715
7	0.54728658	0.54689101
14	0.54168522	0.54263530
21	0.54233060	0.54216936
28	0.54606277	0.54230895
35	0.54525536	0.54553649
42	0.54996346	0.55172237
49	0.55606657	0.55742556
55	0.55957010	0.56257593
62	0.56235129	0.56728237
69	0.56480941	0.56868472
76	0.56463475	0.56837489
83	0.56371704	0.56742564
90	0.56290166	0.56565369
97	0.56267565	0.56413156
104	0.56470174	0.56492539
111	0.56764970	0.56686812
118	0.57106100	0.56810100
125	0.57244757	0.56866868
132	0.57036198	0.56814157
139	0.56764492	0.56465560
146	0.56238443	0.55874767
153	0.55574878	0.55306014
160	0.54816344	0.54685197
167	0.54255936	0.54262321
174	0.54007409	0.54216969
181	0.54265645	0.54231507
188	0.54266910	0.54557165
195	0.55178840	0.55176106
202	0.55480257	0.55746235
209	0.56466682	0.56344980
216	0.56788385	0.56768853
223	0.56750003	0.56869821
230	0.56620046	0.56827280
237	0.56768515	0.56721607
244	0.56621362	0.56534977
251	0.56548537	0.56408524
258	0.56634020	0.56520973
264	0.56908258	0.56687891
271	0.57008731	0.56810638
278	0.57087450	0.56867023
285	0.56981932	0.56813067
292	0.56534614	0.56462217
299	0.55772303	0.55870967
306	0.55368415	0.55302309

$e_0'' = 0.0055936191$ $A_1 = -0.0001200321$ $A_2 = -0.0000648964$ $A_3 = 0.0000044700$

$A_4 = 0.0000015399$ $A_5 = 0.0000017144$ $A_6 = -0.0000009668$ $A_7 = 0.0000034168$

$A_8 = 0.0000019838$ $A_9 = 0.0000008763$ $\bar{\theta} = 51^\circ.34033 - (2^\circ.3536297/\text{day})(t - t_0)$

$t - t_0$ = days since 22 May 1966, 0 hr U.T.

Table 6

Argument of Perigee of Nimbus 2.

$t - t_0$ (days)	g_c'' (degrees)	$g_c'' - [g_0'' + \dot{g}_c''(t - t_0)]$ (degrees)	$\sum_{j=1}^{9} B_j \sin j\overline{\theta}$ (degrees)
0	681.34033	1.71748	1.81008
7	664.56280	1.41943	1.42397
14	647.43303	0.76913	0.98474
21	630.18123	-0.00319	0.12833
28	612.91141	-0.79353	-0.84045
35	596.17881	-1.04665	-1.32491
42	579.25093	-1.49506	-1.74587
49	562.60975	-1.65676	-1.81638
55	548.70462	-1.43662	-1.60563
62	532.58137	-1.08040	-1.22130
69	516.59107	-0.59122	-0.63771
76	500.47859	-0.22422	-0.25701
83	484.09840	-0.12494	0.00321
90	467.60543	-0.13843	0.15816
97	450.82827	-0.43611	0.03784
104	434.13663	-0.64828	-0.14006
111	417.82345	-0.48198	-0.08924
118	401.75990	-0.06605	0.16244
125	385.67054	0.32406	0.46239
132	369.87252	1.00552	1.01109
139	354.07590	1.68838	1.49166
146	337.83500	1.92696	1.77110
153	321.44596	2.01739	1.80866
160	305.04411	2.09502	1.42115
167	287.97163	1.50202	0.98117
174	270.32661	0.33647	0.12129
181	252.80550	-0.70516	-0.84488
188	236.12142	-0.90976	-1.32760
195	218.76501	-1.78669	-1.74801
202	202.22859	-1.84364	-1.81532
209	186.04263	-1.55012	-1.56182
216	169.99133	-1.12194	-1.13988
223	153.95519	-0.67861	-0.56128
230	137.79547	-0.35885	-0.21829
237	121.45899	-0.21585	0.04051
244	105.04672	-0.14865	0.15577
251	88.51341	-0.20248	0.00728
258	71.94073	-0.29568	-0.15223
264	57.97534	-0.13581	-0.08798
271	41.83007	0.19842	0.16406
278	25.69696	0.54477	0.46502
285	9.76031	1.08759	1.01492
292	- 6.29829	1.50843	1.49380
299	-22.50038	1.78586	1.77253
306	-38.95179	1.81392	1.80722

$g_0'' = 679°62285$ $\quad \dot{g}_c'' = -2°3542110/\text{day}$ $\quad B_1 = 1°3007803$ $\quad B_2 = 0°8192370$ $\quad B_3 = -0°0202026$

$B_4 = 0°0196037$ $\quad B_5 = 0°0099007$ $\quad B_6 = 0°0441686$ $\quad B_7 = 0°0034652$ $\quad B_8 = 0°0379263$

$B_9 = 0°0279919$ $\quad \overline{\theta} = 51°34033 - (2°3536297/\text{day})(t - t_0)$

Table 7

Theoretical and Observed Amplitudes in the Perturbations of Eccentricity and Argument of Perigee.

Trigonometric Term	Amplitudes for Alouette I		Amplitudes for Tiros 8		Amplitudes for Nimbus 2	
	Observed	Theoretical	Observed	Theoretical	Observed	Theoretical
$\sin \overline{\theta}$	4°.2076854	3°.4477685	8°.0967578	8°.0064801	1°.3007803	1°.2480738
$\sin 2\overline{\theta}$	4°.6340045	5°.1377014	3°.5255978	5°.4989977	0°.8192370	1°.0103390
$\sin 3\overline{\theta}$	0°.1873826	1°.4504933	0°.5354116	1°.6061560	-0°.0202026	0°.1264920
$\cos \overline{\theta}$	-0.0001492876	-0.0001311878	-0.0004525939	-0.0004504508	-0.0001200321	-0.0001173404
$\cos 2\overline{\theta}$	-0.0001336935	-0.0001183911	-0.0001389608	-0.0001738383	-0.0000648964	-0.0000497609
$\cos 3\overline{\theta}$	-0.0000097969	-0.0000250684	-0.0000164065	-0.0000380812	0.0000044700	-0.0000046725

NOTE: The sine and cosine terms appear in the perturbations of argument of perigee and eccentricity, respectively.

Appendix C

Figures

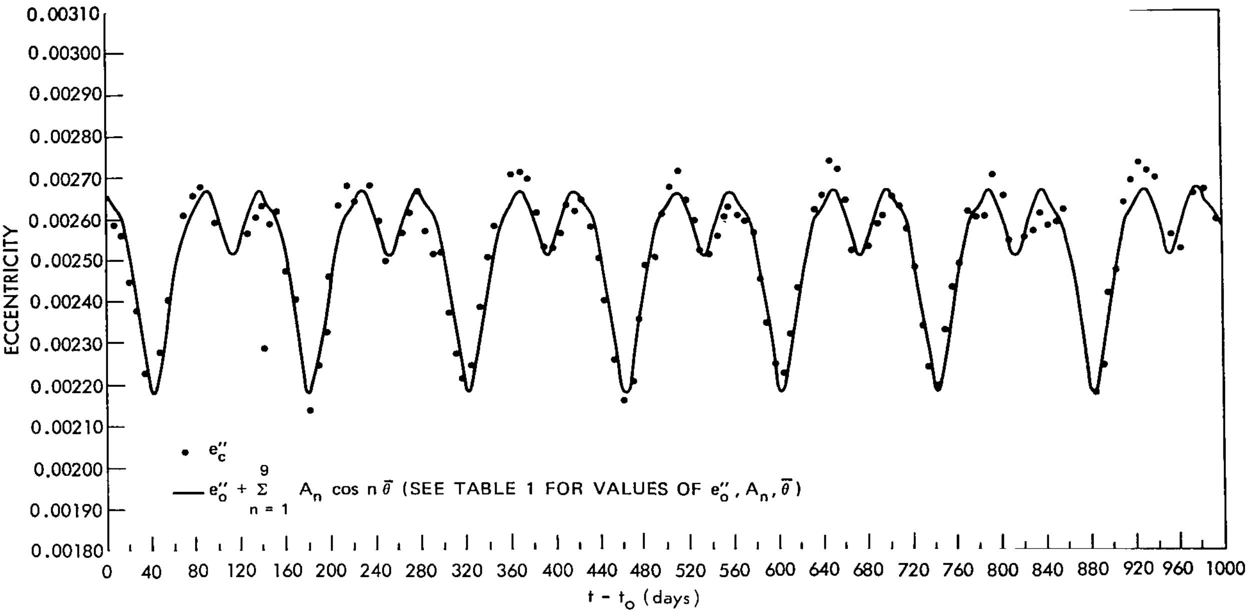

Figure 1—Eccentricity of Alouette 1.

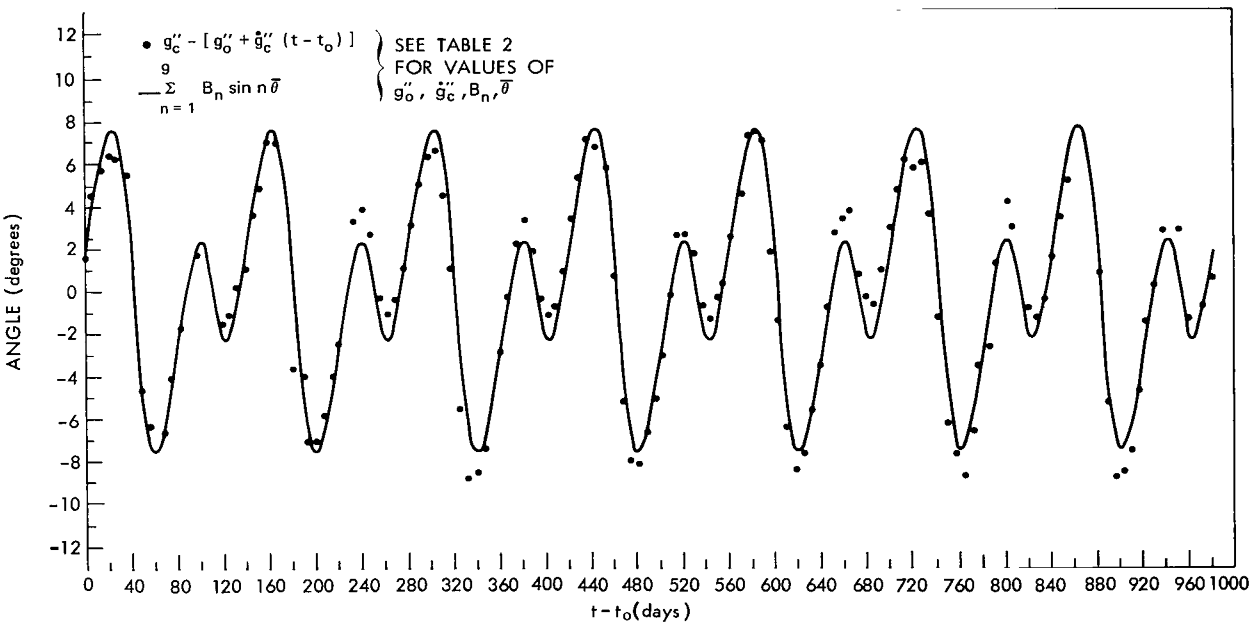

Figure 2—Argument of perigee of Alouette 1.

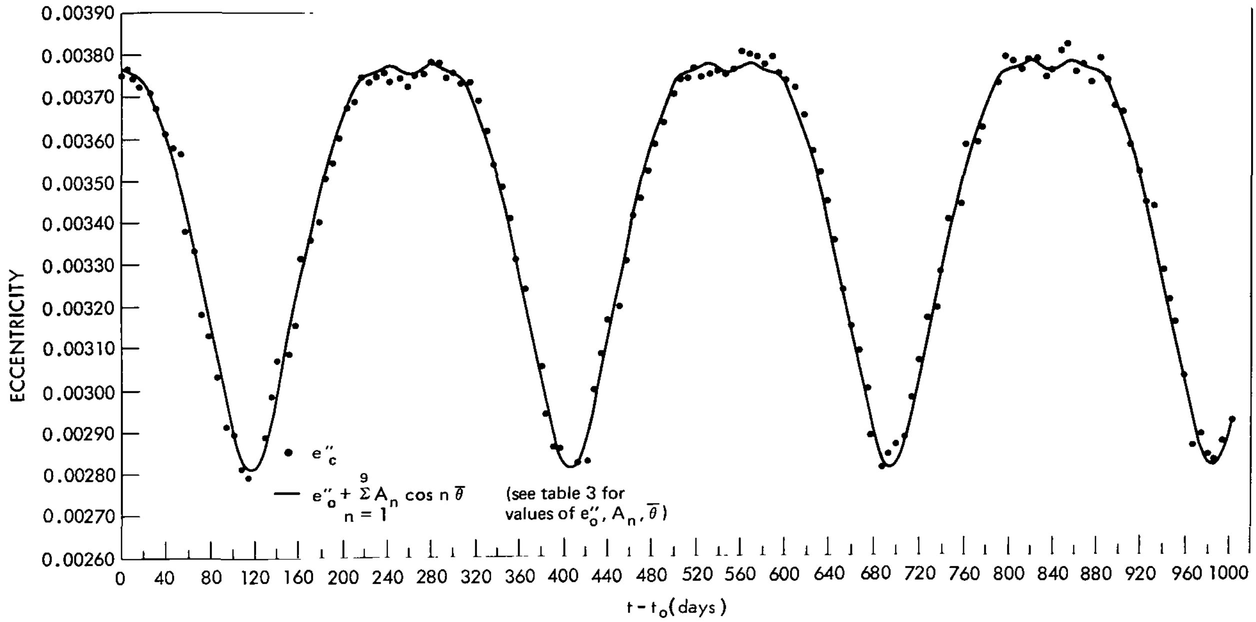

Figure 3—Eccentricity of Tiros 8.

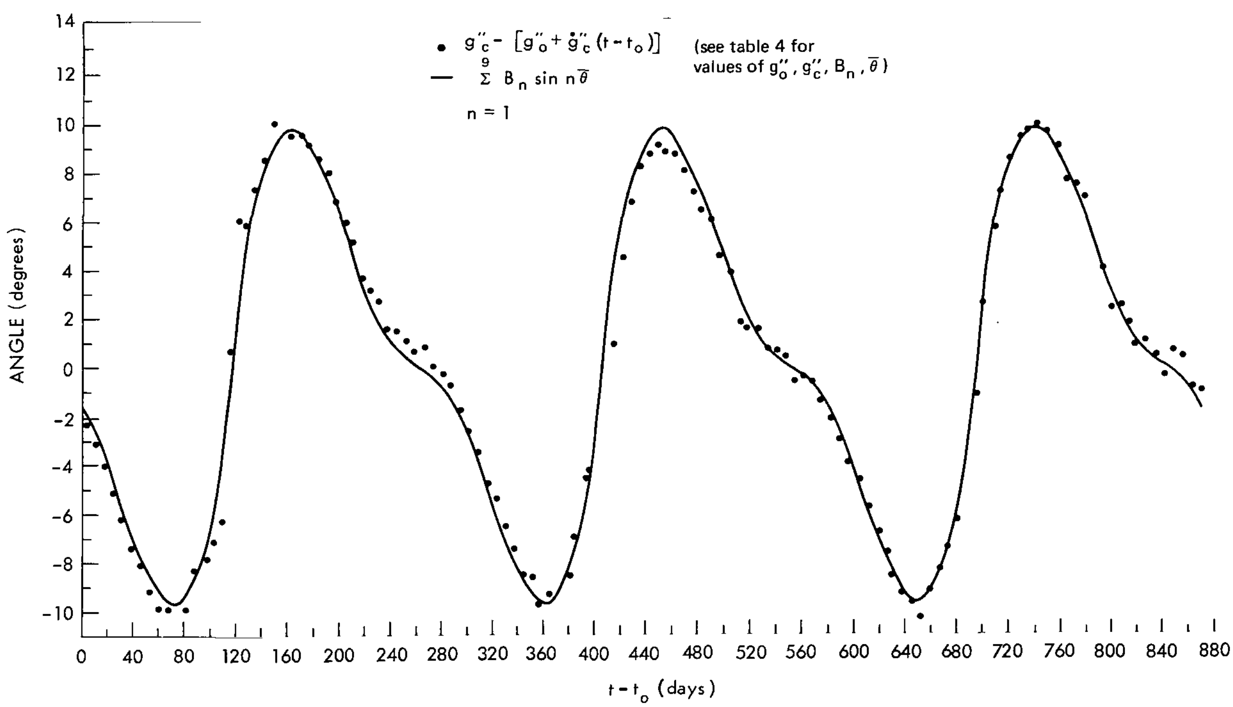

Figure 4—Argument of perigee of Tiros 8.

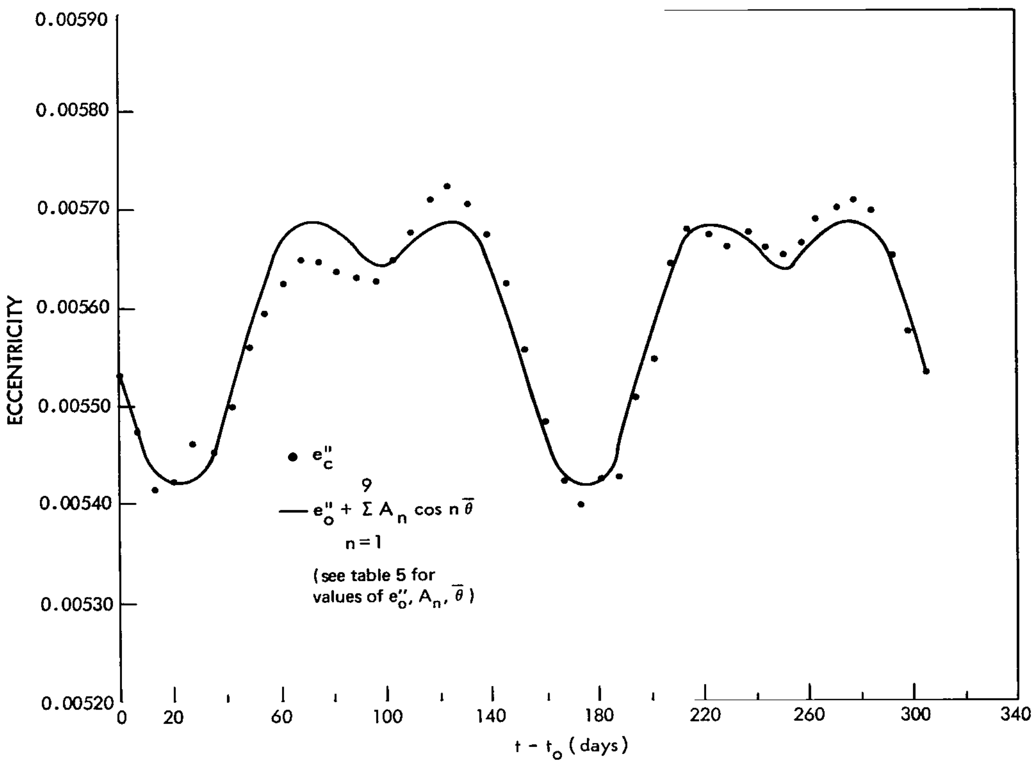

Figure 5—Eccentricity of Nimbus 2.

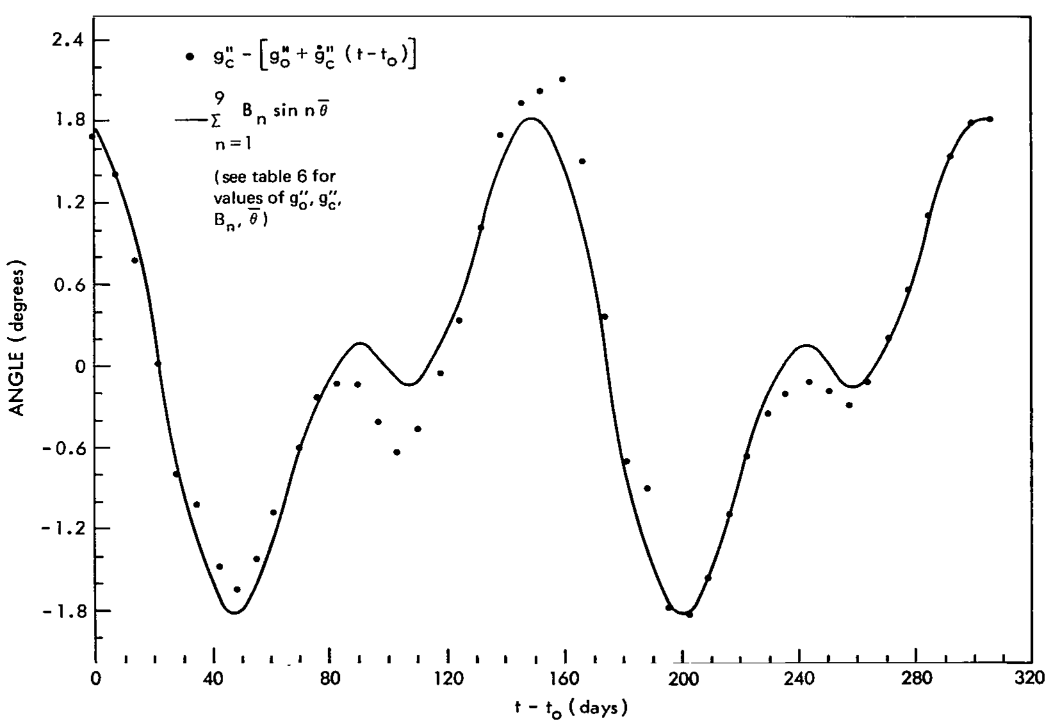

Figure 6—Argument of perigee of Nimbus 2.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546

OFFICIAL BUSINESS

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

CAL OCI 55 51 3DS
AIR FORCE WEAPONS LABORATORY
KIRTLAND AIR FORCE BASE

ATT MISS MADELINE F. CANOVA
LIBRARY /WLIL/

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546