NASA TECHNICAL NOTE

NASA TN D-4658

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NASA TN D-4658

AXIAL POWER TAILORING TO OBTAIN CONSTANT FUEL-CENTERLINE TEMPERATURE IN A NUCLEAR REACTOR

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Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1968

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ABSTRACT

The total-power output from a nuclear reactor can be increased by tailoring the axial-power shape if reactor power is limited by fuel-centerline temperature. The potential power gains and the axial-power shape required to obtain a constant fuel-centerline temperature are calculated using steady-state heat-transfer calculations on a long fuel rod cooled externally by forced convection. Steady-state two-group neutron diffusion theory is used to predict the axial-fuel loading required to obtain the desired axial-power shape. Fuel loadings and neutron-flux distributions are presented for various axial-power shapes required in a reflected, water moderated, and cooled reactor.

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AXIAL POWER TAILORING TO OBTAIN CONSTANT FUEL-CENTERLINE TEMPERATURE IN A NUCLEAR REACTOR

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Lewis Research Center

SUMMARY

When the power output from a nuclear reactor is limited by the temperature of the fuel, the power can be increased by tailoring the axial-power distribution to produce a constant fuel-centerline temperature. Heat-transfer and neutronic calculations have been made to determine (1) the potential power gain which can be obtained by power tailoring, (2) the axial-power shape required, and (3) the axial-fuel distribution required to obtain the required axial-power shape.

The steady-state heat-transfer calculations were made for a long fuel rod cooled by forced convection to an incompressible coolant. The neutronics calculations are based on the one-dimensional diffusion theory using two energy groups. The results of this study indicate that the power of an unreflected, axially unzoned reactor, limited by fuel-centerline temperature, could be increased by at least 50 percent by tailoring the axial-power shape. When the convective film coefficient is constant, the required axial-power shape is exponential. An exponential-power shape can be obtained by appropriate design of the end reflectors and by adjusting the axial-fuel distributions. When the neutron reflectors at the coolant inlet and outlet ends of the reactor are identical, the maximum-fuel loading is required at the coolant inlet end of the reactor. The maximum and average fuel concentrations required to maintain reactor criticality are larger for small reactors than for large reactors.

Although the required axial-power shape and fuel distribution were calculated for a system having a constant film coefficient, the change in the required axial-power shape caused by small axial variations in the film coefficient was estimated.

INTRODUCTION

In power reactors it is desirable to extract the maximum power per unit volume of the system for most efficient operation. The thermal-power capability of a reactor depends on the magnitude of the operating limits. These limits are imposed on parameters such as fuel or component temperatures, coolant temperature, coolant flow rate, or heat flux to permit safe operation of the reactor. If reactor power is limited by fuel temperature or heat flux, which depend on the power shape in the reactor, reactor power can sometimes be increased by tailoring or shaping the power distribution.

One method of power tailoring relies on distributing or zoning the nuclear fuel within the reactor to obtain the desired power shape. Fuel distributions (fuel zoning) required to produce a flat radial power distribution have been investigated by Goertzel and Loeb (ref. 1) and by Bussard and Delauer (ref. 2). Barth and Haling (ref. 3) investigated axial-fuel distributions required to obtain an optimized-power distribution based on critical heat-flux requirements in a boiling water reactor.

In this report, the focus of attention is on axial-power distributions required when reactor power is limited by fuel-centerline temperature. If reactor power in an unzoned reactor is limited by fuel-centerline temperature, there is usually only one spot in a fuel rod where the centerline temperature reaches the temperature limit. At all other axial positions along the fuel centerline, the temperature is below the temperature limit. The total fuel-rod power and reactor power can be increased by adjusting the axial-power shape so that the fuel-centerline temperature reaches the temperature limit and is constant along the entire length of the fuel rod. Axial-power tailoring to obtain a constant fuel-centerline temperature is desirable to obtain higher reactor powers. Axial-power tailoring would also allow a reduction in reactor size while maintaining the same reactor power, providing the reactor is not criticality limited.

The purpose of this report is

(1) to illustrate the potential gains in reactor power that might be obtained by tailoring the axial-power distribution when reactor power is limited by fuel-centerline temperature

(2) to determine the axial-power distribution (hereinafter called the optimum axial-power shape) required to produce a constant fuel-centerline temperature

(3) to determine the fuel distribution required to produce the optimum axial-power shape.

The potential power gains, as a result of axial-power shaping, and the optimum axial-power shape are determined as a result of an analytical, steady-state, heat-transfer analysis on a fuel rod which is cooled externally by forced convection to an incompressible coolant. The fuel-material distribution required to produce a constant fuel-centerline temperature is calculated using the one-dimensional, two-group, steady-state, neutron diffusion theory.

HEAT-TRANSFER ANALYSIS

The total reactor power that can be obtained with and without a tailored axial-power distribution and the axial-power distribution required to produce a constant fuel-centerline temperature are derived in appendix A. The potential gain in reactor power is based on the following reactor model:

(1) The fuel element is a long cylindrical rod which is cooled externally by incompressible, coaxial, forced-convection flow. The fuel element operates at steady state and is heated internally by fission of the fuel material. The convective film coefficient and thermal conductivity of the fuel are independent of axial position.

(2) The radial power distribution in the reactor is constant and is unaffected by axial-power tailoring.

(3) Reactor power is limited by fuel-centerline temperature, that is, the maximum reactor power is obtained when the temperature of the hottest spot in the hottest fuel element in the reactor reaches a prescribed limit.

The total reactor power that can be obtained with a tailored axial-power shape is given in equation (A31) of appendix A.

$$P_e = FC_p(\theta_L - \theta_2)\left[\exp\left(\frac{1}{\beta}\right) - 1\right] \tag{A31}$$

The dimensionless parameter β is defined in equation (A13). It is a measure of the sum of the temperature rise across the convective coolant film and across the fuel rod relative to the total coolant-temperature rise across the reactor. Beta depends on the coolant flow rate and heat capacity, the convective film coefficient, the radial power distribution in the fuel rod, and the dimensions and thermal conductivity of the fuel. In this study, β will be treated as a thermal-hydraulic-design parameter which influences the axial-power distribution, and, consequently, affects total reactor power. As indicated in equation (A13), β could vary from zero to infinity. For most pressurized water reactors, for example, β is about 10.

The reactor power obtained with a constant fuel distribution (untailored axial-power shape) is given by equation (A39).

$$P_d = \frac{FC_p(\theta_L - \theta_2)\left[\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_2}{x_3}\right)\right]}{\cos\left(\frac{\pi x_2}{x_3}\right) + \left[1 + \left(\frac{\pi\beta}{x_3}\right)^2\right]^{1/2}} \tag{A39}$$

where P_d is a function of the parameters β and the axial-power shape; that is, the values of x_1, x_2, and x_3 depend on the axial-power shape. The axial-power shape in an unreflected reactor having a uniform fuel distribution is described by a cosine function ($x_1 = 0$ and $x_2 = x_3 = 1$). When neutron reflectors are added to both the coolant inlet and outlet ends of the reactor, however, the axial-power shape is flatter and can be approximated by a "chopped cosine" ($x_1 > 0$ and $x_2 < x_3$). The chopped cosine axial-power shape is normally described by the ratio of the maximum to average power generation rates n_p rather than in terms of x_1, x_2, and x_3.

With the same fuel-temperature limit, coolant flow, and coolant outlet temperature, the power that can be obtained by power tailoring relative to that which would be obtained without power tailoring is

$$\frac{P_e}{P_d} = \frac{\left[\exp\left(\frac{1}{\beta}\right) - 1\right]\left\{\cos\left(\frac{\pi x_2}{x_3}\right) + \left[1 + \left(\frac{\pi\beta}{x_3}\right)^2\right]^{1/2}\right\}}{\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_2}{x_3}\right)}$$

Because the coolant flow and outlet temperature are the same, the power gain is achieved by reducing the coolant-inlet temperature.

The ratio P_e/P_d is shown in figure 1 as a function of β and n_p. The potential power gain that can be achieved by power tailoring an unreflected reactor having a constant fuel distribution is described by the curve labeled $n_p = 1.57$. The other curves ($n_p < 1.57$) represent the potential power gain that can be achieved by power tailoring reflected reactors initially having constant fuel distributions and a ratio of maximum-to-average power generation rates of n_p. The potential power gains are less for large values of β than for small values of β. However, even for large values of β ($\beta > 10$) tailoring the power shape in an unreflected reactor would allow about a 50-percent increase in reactor power. When $\beta = 10$ (as it is in many pressurized water reactors) or

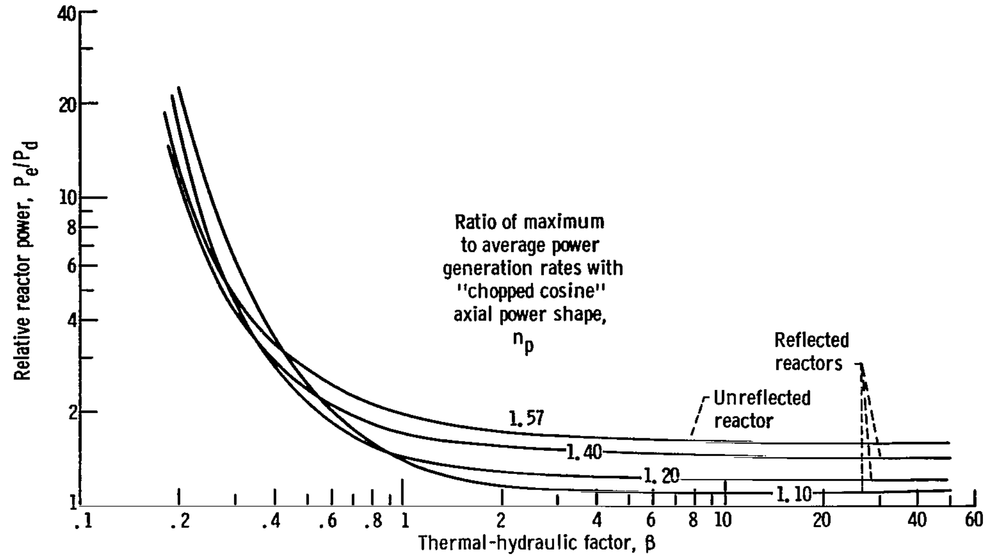

Figure 1. - Potential gain in reactor power which might be achieved by axial power tailoring when reactor power is limited by fuel centerline temperature. Reactor powers are calculated for same coolant flow, coolant effluent temperature, and maximum fuel centerline temperature.

greater, the potential power gain obtained by axial-power tailoring is less for reflected reactors than for unreflected reactors. For example, the power of an unreflected ($n_p = 1.57$), pressurized water-moderated, reactor, which is limited by fuel-centerline temperature, could be increased by at least 50 percent by tailoring the axial-power shape to produce a constant fuel-centerline temperature. If this reactor had neutron reflectors at the coolant inlet and outlet ends such that $n_p = 1.10$, the power could be increased by only about 10 percent by axial-power tailoring. The reactor power gain is smaller in the second example because the initial (untailored) reactor power of a reflected reactor is higher than the initial reactor power of an unreflected reactor. The final (tailored) reactor power would be the same in both cases.

These indicated large power gains may not actually be achievable because reactor power may be limited by some parameter other than fuel temperature before the calculated power gain can be achieved. Although the power gained by axial-power tailoring may not be as large as that indicated in figure 1, a lesser increase in power would still be an advantage.

Having determined the potential reactor-power gains that might be achieved by axial-power tailoring, what is the axial-power distribution required to produce a constant fuel-centerline temperature? The required axial-power shape is exponential, and is defined in equation (A29).

$$\frac{Y(x)}{\bar{Y}} = \left\{\beta\left[\exp\left(\frac{x - x_1}{\beta}\right) - \exp\left(\frac{x - x_1 - 1}{\beta}\right)\right]\right\}^{-1} \tag{A29}$$

This power shape is shown in figure 2 for various values of β. The maximum power is required at the coolant-inlet end of the reactor where the coolant temperature is lowest. The minimum power is required at the outlet end where the coolant temperature is

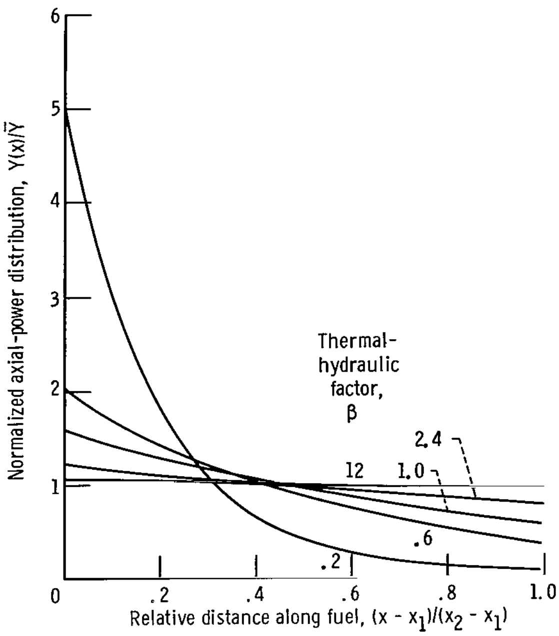

Figure 2. - Axial-power distribution required for constant fuel-temperature distribution.

highest. The power shape required for larger values of β is flatter than that required for smaller β values. This is because larger values of β indicate smaller coolant-temperature rises relative to the temperature rise across the fuel and convective coolant film. Therefore, only a slight reduction in the axial power from inlet to outlet is required to compensate for the small coolant-temperature rise.

The previous discussion is based on the assumption that the parameter β is independent of axial position. Actually, the film coefficient may vary along the length of the fuel element as the coolant temperature increases. The axial-power distribution required for small axial variations in film coefficient is estimated with equation (A24) of appendix A.

$$\frac{Y(x)}{\overline{Y}} = \frac{\exp\left(\frac{\alpha_1}{\beta}\right) - 1}{\alpha_1\left[1 - \exp\left(\frac{\alpha_1}{\beta}\right) + \exp\left(\frac{\alpha_1 + x - x_1}{\beta}\right) - \exp\left(\frac{x - x_1 - 1}{\beta}\right)\right]} \tag{A24}$$

The factor α_1 is a measure of the total change in the film coefficient along the length of the fuel element, and it can be calculated using equation (A22). The change in the required axial-power distribution caused by small variations ($\alpha_1 << \beta$) in the film coefficient is illustrated in figure 3 where the ratio $Y(x, \beta, \alpha_1)/Y(x, \beta, 0)$ is presented as a

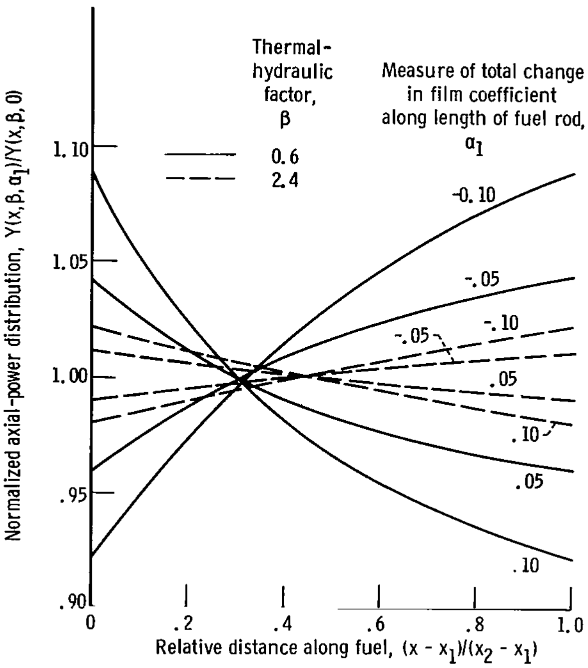

Figure 3. - Effect of total change in film coefficient along length of fuel rod on required axial-power distribution.

function of axial position and α_1. For many reactors α_1 is small ($-0.1 < \alpha_1 < 0.1$). Positive values of α_1 result when the convective film coefficient decreases with temperature. This means that the heat-transfer resistance is lower at the coolant-inlet end than at the outlet. Therefore, the power at the inlet must be higher and that at the outlet lower than would be required with a constant film coefficient ($\alpha_1 = 0$). The result is a steeper power shape when $\alpha_1 > 0$. Conversely, negative values of α_1 are caused when the film coefficient increases with temperature. The required axial-power shape is flatter than that required with a constant film coefficient because a smaller reduction in power is required to compensate for the coolant-temperature rise.

The previous discussion has illustrated the potential reactor power gains that might be achieved by tailoring the axial-power shape and has established the axial-power shape needed to obtain a constant fuel-centerline temperature. The next problem is to determine whether the axial-power shape can be tailored to produce a constant fuel-centerline temperature and how this tailoring can be done. In this analysis it will be assumed that the film coefficient is constant. When the film coefficient is constant, the required axial-power shape is an exponential as indicated in equation (A28).

NEUTRONICS ANALYSIS

There are several methods of power tailoring available to the reactor designer. The most common methods are achieved by design of a neutron reflector or by appropriate distribution of fuel or absorber materials.

In this study, power tailoring will be attempted by adjusting the fuel distribution (fuel zoning) and by reflector design.

Physically, the fuel distribution could be adjusted by varying the isotopic proportion of fissionable material in the fuel rod. For example, if uranium is used as fuel, the proportion of uranium 235 to uranium 238 could be adjusted to obtain the required distribution of fissionable material in a constant-diameter fuel rod. In the following discussion the required fuel distribution is expressed in terms of the macroscopic fission cross section Σ_f. The fuel distribution required to obtain an exponential axial-power shape in a reflected reactor is derived in appendix B.

The neutronics calculations are based on a one-dimensional steady-state diffusion calculation. Two energy groups of neutrons are assumed, such that all fissions result from the absorption of thermal neutrons. The fission neutrons all appear with the same energy, which is higher than the thermal energy. These fast neutrons are lost from a unit volume of reactor by diffusion and by slowing down to thermal energies. The thermal neutrons are formed by the slowing down of the fast neutrons. They are lost from a unit volume of reactor by diffusion and absorption in both fuel and nonfuel materials. The effect of neutron reflectors located at both ends of the reactor is simulated by specifying neutron albedos for both the fast and thermal fluxes. The fast and thermal neutron fluxes are represented by equations (B14) and (B22) in appendix B.

$$\varphi_1(X) = A_1\exp\left(\frac{X}{L_1}\right) + A_2\exp\left(\frac{-X}{L_1}\right) - \frac{L_1^2\gamma_1\exp(-\gamma X)}{\gamma^2 L_1^2 - 1} \tag{B14}$$

$$\varphi_2(X) = A_3\exp\left(\frac{X}{L_2}\right) + A_4\exp\left(\frac{-X}{L_2}\right) - \frac{\gamma_1 L_2^2\left[\dfrac{D_1}{\eta} + \dfrac{\Sigma_R L_1^2}{\left(\gamma^2 L_1^2 - 1\right)}\right]}{D_2\left(\gamma^2 L_2^2 - 1\right)\exp(\gamma X)}$$

$$- \frac{\Sigma_R L_1^2 L_2^2\left[A_1\exp\left(\dfrac{X}{L_1}\right) + A_2\exp\left(\dfrac{-X}{L_1}\right)\right]}{D_2\left(L_2^2 - L_1^2\right)} \tag{B22}$$

The parameters γ_1 and γ can be related to the heat-transfer parameters through equations (B37) and (B35).

$$\gamma_1 = \nu\left\{\beta\epsilon D_1\left[1 - \exp\left(-\frac{1}{\beta}\right)\right]\right\}^{-1}\exp\left[\frac{X_1}{\beta(X_2 - X_1)}\right]\int_0^1 rH(r)dr \tag{B37}$$

$$\gamma = \left[\beta(X_2 - X_1)\right]^{-1} \tag{B35}$$

The coefficients A_1, A_2, A_3, and A_4 are given in equations (B17), (B18), (B27), and (B28). The coefficients A_1 and A_2 depend on the core length, the fast-neutron properties of the core, and the fast-neutron albedo at both ends of the reactor. The coefficients A_3 and A_4 depend on A_1 and A_2, the thermal properties of the core, and the thermal-neutron albedo at both ends of the core. The fuel distribution required to produce an exponential axial-power distribution is

$$\Sigma_f(X) = \frac{D_1\gamma_1\exp(-\gamma X)}{\nu\varphi_2(X)}$$

The required fuel distribution depends on

(1) the fast and thermal-neutron diffusion coefficients D_1 and D_2

(2) the thermal-neutron absorption cross sections of nonfuel materials Σ_a^N

(3) the removal cross section for fast neutrons Σ_R

(4) the number of neutrons produced as a result of thermal-neutron absorptions in the fuel η

(5) the length of the reactor $X_2 - X_1$

(6) the fast and thermal neutron albedo at both ends of the reactor $K_{1,1}$, $K_{2,1}$, $K_{1,2}$, $K_{2,2}$

(7) the exponential coefficient used to describe the axial-power shape γ.

Because of the large number of independent variables involved (eleven) in the determination of fuel tailoring, it is desirable to reducé the number of variables by selecting a specific type of reactor for analysis. A water moderated and cooled reactor was selected for this analysis. The reactor is fueled with uranium 235, and contains aluminum structural components. The following reactor parameters, obtained from references 4 and 5, were used in this study:

Fast neutron diffusion coefficient, D_1, cm	1.42
Thermal neutron diffusion coefficient, D_2, cm	0.262
Fast neutron diffusion length, L_1, cm	5.95
Thermal neutron diffusion length, L_2, cm	4.10
Neutron removal cross section, Σ_R, cm^{-1}	0.0398
Number of neutrons produced per thermal neutron absorbed in fuel, η	2.08
Number of neutrons produced per fission, ν	2.44

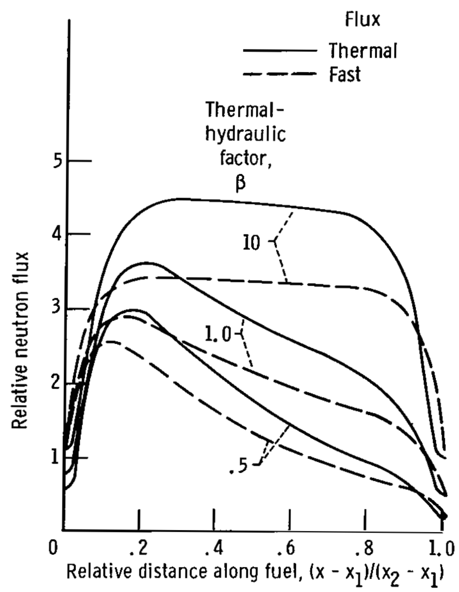

Figure 4. - Neutron flux distribution in reactor with exponential axial power shape $Y(x) = Y_1 exp-(x/\beta)$. Reactor length, 120 centimeters; no thermal leakage from reactor ($K_{1,2} = K_{2,2} = 1$).

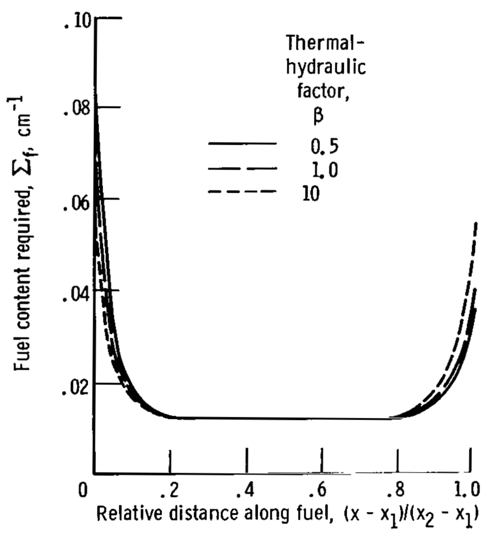

Figure 5. - Fuel distribution required to produce exponential axial-power distribution $Y(x) = Y_1 exp-(x/\beta)$. Reactor length, 120 centimeters; no thermal leakage from reactor ($K_{1,2} = K_{2,2} = 1$).

It will be assumed that the reactor length is 120 centimeters and that it has a moderating reflector at both ends so that the fast-neutron albedos are zero ($K_{1,1} = K_{2,1} = 0$).

The axial neutron-flux distribution and the axial-fuel distribution required to maintain a constant fuel-centerline temperature are illustrated in figures 4 and 5 for three different axial-power shapes (3 values of β). The fast and thermal neutron-flux distributions for three different axial-power distributions are shown in figure 4. The reflector at the coolant-inlet end of the reactor and the reflector at the outlet are identical and are designed such that there is no thermal-neutron leakage from the reactor ($K_{1,2} = K_{2,2} = 1$). Although the two reflectors are identical, the neutron-flux distributions are asymmetric. The asymmetry in the flux distribution is caused by the asymmetry in the fuel distribution (see fig. 5) required to produce an exponential-power shape. More fuel must be added to the inlet end than to the outlet end because a higher power is required at the inlet end. The fuel distribution required to produce a flat power shape is symmetric and is lower in the center than at either end. Although in this example the fuel content remains constant over the central 60 percent of the length of the fuel rod, this region of constant-fuel content depends on the length of the fuel, the neutron removal cross section, the diffusion coefficient, and the diffusion length.

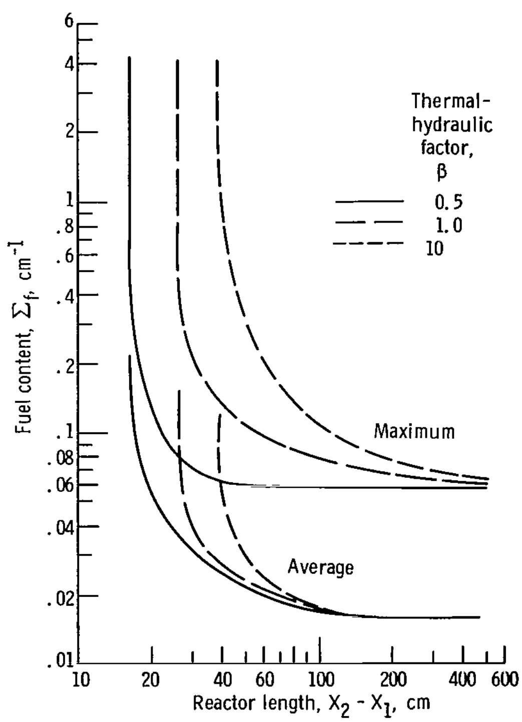

Figure 6. - Fuel content required to produce exponential axial-power distribution $Y(x) = Y_1 \exp-(x/\beta)$. No thermal leakage from reactor ($K_{1,2} = K_{2,2} = 1$).

It is desirable to learn what fuel distributions are required for different reactor lengths. The maximum and axially averaged values of fuel content for various reactor lengths are illustrated in figure 6. Again the thermal albedos at the inlet and outlet ends of the core are one. The smallest reactors that can be made critical while maintaining an exponential axial-power shape ($Y(x) = Y_1 \exp(-x/\beta)$) are represented by the vertical lines. For example, the smallest reactor that could be made critical while maintaining an axial-power shape of the form $e^{-x}(\beta = 1)$ is about 27 centimeters long.

Typically, smaller reactors require larger fuel loadings than larger reactors. For example, the maximum-fuel content and the axially averaged fuel content required to obtain an axial-power shape of the form $e^{-x}(\beta = 1)$ in a 50-centimeter-long reactor are 0.11 and 0.023 reciprocal centimeter, respectively. The maximum and average fuel content required to maintain the same axial-power shape in a 100-centimeter-long reactor are 0.078 and 0.017 reciprocal centimeter, respectively.

If a reactor is to be designed with an exponential-power shape and if the maximum fuel content Σ_{fm} is limited, the reactor length can be increased to obtain the desired power shape. If, however, an exponential-power distribution is desired in an existing (fixed length) reactor, it may be possible to increase the fuel content Σ_{fm} to obtain this power shape.

The previous discussions assume that there is no thermal-neutron leakage from the reactor ($K_{1,2} = K_{2,2} = 1$). The effect of the reflectors on the maximum and average fuel content required are illustrated in figures 7 and 8, respectively. The required fuel content is given as a function of the thermal-neutron albedo at the inlet $K_{1,2}$ and outlet $K_{2,2}$ ends of the reactor. When the reactor is unreflected or when the two reflectors are identical, the maximum-fuel content is required at the inlet ends of the reactor. As more reflector is added to the inlet end of the reactor ($K_{1,2}$ increases), a point is reached where the maximum-fuel content will be required at the outlet end. For example, in figure 7 for no reflectors ($K_{1,2} = K_{2,2} = 0$), the maximum-fuel content (about 0.53 cm^{-1}) is required at the inlet end of the reactor. As more reflector is added to the inlet end, $K_{1,2}$ can be increased to 1.0, and there is no thermal-neutron leakage from the inlet end. The fuel content required at the inlet is about 0.07 reciprocal centimeter. However, the fuel content required at the outlet end is about 0.33 reciprocal centimeter because no reflector has been added there. Further improvements in the inlet-end reflectors have little effect on the fuel content required at the outlet end. The outlet-end reflector must be modified to increase $K_{2,2}$ before the maximum-fuel loading can be reduced further. It should be noted that the thermal-neutron albedo is limited: it depends on the reflector size and material, but can generally have values between zero and slightly greater than one.

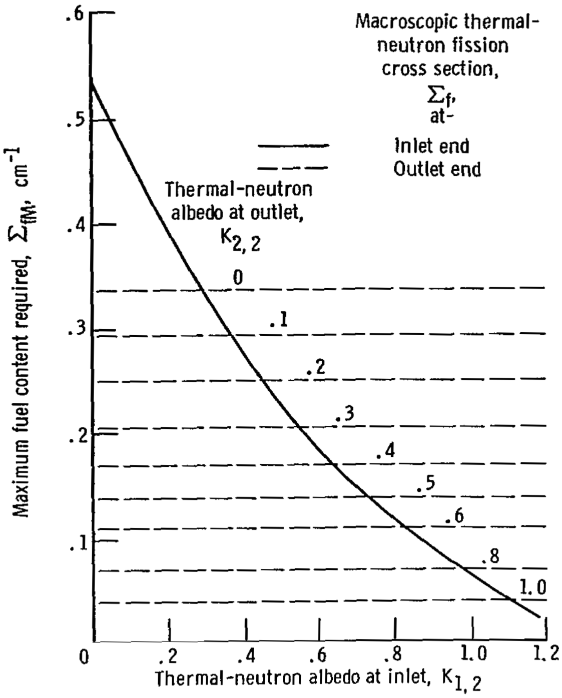

Figure 7. - Effect of reflectors on maximum fuel content required to produce exponential axial power distribution $Y(x) = Y_1 \exp - x$. Reactor length, 120 centimeters; thermal-hydraulic factor, β, is 1.

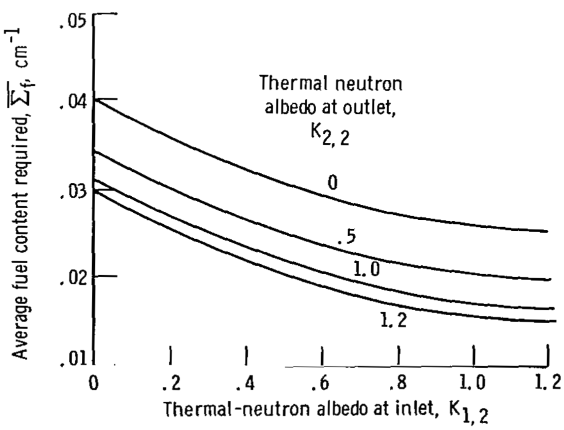

Figure 8. - Effect of reflectors on average fuel content required to produce an exponential axial power distribution $Y(x) = Y_1 \exp - x$. Reactor length, 120 centimeters; thermal-hydraulic factor, β, 1.

CONCLUSIONS

When reactor power is limited by fuel-centerline temperature, the power can be increased by tailoring the power distribution. The power gain depends on the operating conditions. However, the power in an unreflected reactor could be increased by at least 50 percent if the optimum axial-power shape can be obtained. The optimum axial-power shape yields a constant fuel-centerline temperature. When the convective-film coefficient in the reactor is constant, the optimum axial-power shape is of exponential form.

Generally, the axial-power shape required to maintain a constant fuel-centerline temperature is flatter when the coolant-temperature rise is small.

The axial-power distribution can be tailored by proper distribution of the fuel and by proper design of reflectors at the coolant inlet and outlet ends of the reactor. When the two reflectors are identical, the maximum-fuel loading is required at the coolant inlet end of the reactor. As with unzoned reactors, larger fuel concentrations are required for small reactors than for large reactors.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, April 10, 1968,
120-27-06-17-22.

APPENDIX A

AXIAL POWER DISTRIBUTION REQUIRED TO OBTAIN CONSTANT-FUEL-TEMPERATURE DISTRIBUTION

The axial-power distribution required to produce a constant fuel-centerline temperature is determined by calculating the axial-temperature distribution as a function of the unknown axial-power distribution. The functional form of the temperature distribution is differentiated with respect to the axial distance; this derivative is allowed to vanish (constant temperature), and the resulting differential equation is solved to determine the optimum axial-power distribution.

The axial-temperature distribution is calculated for a reactor fuel rod (see fig. 9) operating at steady state and cooled by an incompressible coolant. The fuel assembly is cylindrical, the thermal conductivity is constant, and the length is much greater than the diameter. The temperature distribution in the fuel element is described by the Poisson equation:

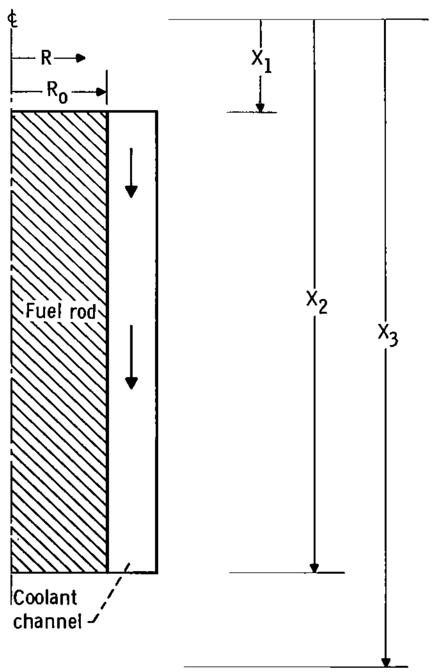

Figure 9. - Reactor fuel rod.

$$\frac{\partial^2\theta}{\partial R^2} + \frac{1}{R}\frac{\partial\theta}{\partial R} + \frac{\partial^2\theta}{\partial X^2} = -\frac{Q(R,X)}{k_M} \tag{A1}$$

This equation can be nondimensionalized to unit order by the following substitutions:

$$T = \frac{\theta}{\theta_{max}}$$

$$r = \frac{R}{R_o}$$

$$x = \frac{X}{\Delta X}$$

The governing equation in terms of the dimensionless parameters is

$$\frac{\theta_{max}}{R_o^2}\frac{\partial^2 T}{\partial r^2} + \frac{\theta_{max}}{R_o^2 r_o}\frac{\partial T}{\partial r} + \frac{\theta_{max}}{(\Delta X)^2}\frac{\partial^2 T}{\partial x^2} = -\frac{Q(R,X)}{k_M}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \left(\frac{R_o}{\Delta X}\right)^2 \frac{\partial^2 T}{\partial x^2} = -\frac{R_o^2 q(r,x)}{\theta_{max} k_M} \tag{A2}$$

Where $q(r,x)$ is $Q(R,X)$ expressed in terms of the nondimensional variables r and x. All variables are of unit order, and $R_o/\Delta X << 1$. When the axial-heat loss is small relative to the radial-heat loss, equation (A2) can be approximated by

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = -\frac{R_o^2 q(r,x)}{\theta_{max} k_M} \tag{A3}$$

For a solid rod, heated internally and cooled externally, the following boundary conditions are used:

$$\frac{dT}{dr} = 0 \qquad \text{at } r = 0$$

$$T = T(1,x) \qquad \text{at } r = 1$$

The temperature anywhere within the rod is

$$T(r,x) = T(1,x) + \frac{R_o^2}{k_M \theta_{max}} \left\{ \int_r^1 \left[\int_0^r rq(r,x)dr \right] \frac{dr}{r} \right\} \tag{A4}$$

The temperature along the axis of the fuel rod is

$$T(0,x) = T(1,x) + \frac{R_o^2}{k_M \theta_{max}} \left\{ \int_0^1 \left[\int_0^r rq(r,x)dr \right] \frac{dr}{r} \right\} \tag{A5}$$

Let the volumetric heating rate be expressed as the product of a function of $r, H(r)$ and a function of $x, Y(x)$. This relation is substituted into equation (A5).

$$T(0,x) = T(1,x) + \frac{R_o^2 Y(x)}{k_M \theta_{max}} \left\{ \int_0^1 \left[\int_0^r rH(r)dr \right] \frac{dr}{r} \right\} \tag{A6}$$

The temperature at the surface of the fuel rod $T(1,x)$ is equal to the local bulk-coolant temperature $T_c(x)$ plus the temperature rise across the convective coolant film. The local bulk-coolant temperature can be expressed in terms of the coolant-temperature rise.

$$T_c(x) = T_1 + m(x)(T_2 - T_1)$$

where $m(x)$ is the fraction of the total fuel-rod power generated up to axial position x.

$$m(x) = \frac{\int_{x_1}^{x} Y(x)dx \int_0^1 rH(r)dr}{\int_{x_1}^{x_2} Y(x)dx \int_0^1 rH(r)dr} = \frac{\int_{x_1}^{x} Y(x)dx}{\int_{x_1}^{x_2} Y(x)dx}$$

The temperature rise across the convective coolant film can be expressed by

$$T(1,x) - T_c(x) = \left(\frac{Q}{A}\right)\frac{n(x)}{h\theta_{max}}$$

where $n(x)$ is the ratio of the local to axially averaged power generation in the fuel rod.

$$n(x) = \frac{H(r)Y(x)}{H(r)\int_{x_1}^{x_2} Y(x)dx} = \frac{Y(x)}{\int_{x_1}^{x_2} Y(x)dx}$$

Therefore, the temperature at the surface of the fuel rod $T(1,x)$ can be expressed by

$$T(1,x) = T_1 + \frac{\int_{x_1}^{x} Y(x)dx}{\int_{x_1}^{x_2} Y(x)dx}(T_2 - T_1) + \frac{FC_p(T_2 - T_1)Y(x)}{2\pi R_o h\Delta X\int_{x_1}^{x_2} Y(x)dx} \tag{A7}$$

When equation (A7) is substituted into equation (A6)

$$\frac{T(0,x) - T_1}{T_2 - T_1} = \frac{\int_{x_1}^{x} Y(x)dx}{\int_{x_1}^{x_2} Y(x)dx} + \frac{FC_pY(x)}{2\pi R_o h\Delta X\int_{x_1}^{x_2} Y(x)dx} + \frac{R_o^2Y(x)\int_0^1\left[\int_0^r rH(r)dr\right]\frac{dr}{r}}{k_M(T_2 - T_1)\theta_{max}} \tag{A8}$$

The total coolant-temperature rise can be expressed by

$$T_2 - T_1 = \frac{2\pi R_o^2\Delta X}{FC_p\theta_{max}}\int_{x_1}^{x_2} Y(x)dx\int_0^1 rH(r)dr \tag{A9}$$

When equation (A9) is substituted into equation (A8), the result is

$$\frac{T(0,x) - T_1}{T_2 - T_1} = \frac{\int_{x_1}^{x} Y(x)dx}{\int_{x_1}^{x_2} Y(x)dx} + \frac{FC_pY(x)}{\Delta X \int_{x_1}^{x_2} Y(x)dx} \left\{ \frac{1}{2\pi R_o h} + \frac{\int_0^1 \left[\int_0^r rH(r)dr\right]\frac{dr}{r}}{2\pi k_M \int_0^1 rH(r)dr} \right\}$$

and

$$\frac{T(0,x) - T_1}{T_2 - T_1} = \frac{\int_{x_1}^{x} Y(x)dx}{\int_{x_1}^{x_2} Y(x)dx} + \frac{FC_pY(x)}{2\pi R_o \Delta X \int_{x_1}^{x_2} Y(x)dx} \left\{ \frac{1}{h} + \frac{R_o \int_0^1 \left[\int_0^r rH(r)dr\right]\frac{dr}{r}}{k_M \int_0^1 rH(r)dr} \right\} \tag{A10}$$

This equation can also be expressed in terms of dimensionless Reynolds, Prandtl and Nusselt numbers.

$$\frac{T(0,x) - T_1}{T_2 - T_1} = \frac{\int_{x_1}^{x} Y(x)dx}{\int_{x_1}^{x_2} Y(x)dx} + \frac{RePrSDeY(x)}{8\pi R_o \Delta X \int_{x_1}^{x_2} Y(x)dx} \left\{ \frac{1}{Nu} + \frac{k_c R_o \int_0^1 \left[\int_0^r rH(r)dr\right]\frac{dr}{r}}{k_M De \int_0^1 rH(r)dr} \right\} \tag{A11}$$

Equation (A11) can be rewritten in the following form:

$$\frac{T(0,x) - T_1}{T_2 - T_1} = \frac{\int_{x_1}^{x} Y(x)dx}{\int_{x_1}^{x_2} Y(x)dx} + \beta \frac{Y(x)}{\int_{x_1}^{x_2} Y(x)dx} \tag{A12}$$

where

$$\beta = \frac{FC_p}{2\pi R_o \Delta X}\left\{\frac{1}{h} + \frac{R_o \int_0^1 \left[\int_0^r rH(r)dr\right]\frac{dr}{r}}{k_M \int_0^1 rH(r)dr}\right\} \tag{A13}$$

$$\beta = \frac{RePrSDe}{8\pi R_o \Delta X}\left\{\frac{1}{Nu} + \frac{k_c R_o \int_0^1 \left[\int_0^r rH(r)dr\right]\frac{dr}{r}}{k_M De \int_0^1 rH(r)dr}\right\} \tag{A14}$$

The axial-power distribution Y(x) required for a constant fuel-centerline temperature is determined by allowing the derivative of the temperature to vanish.

$$\frac{dT(0,x)}{dx} = 0 = Y(x)\left(1 + \frac{d\beta}{dx}\right) + \beta\frac{dY(x)}{dx} \tag{A15}$$

The coolant properties depend on the coolant temperature. Therefore, the film coefficient and β depend on the coolant temperature.

$$\frac{d\beta}{dx} = \left(\frac{d\beta}{d\theta}\right)\left(\frac{d\theta}{dx}\right) \tag{A16}$$

The bulk coolant temperature at any axial position can be expressed by

$$\theta = \theta_1 + \frac{2\pi R_o^2 \Delta X}{FC_p}\int_0^1 rH(r)dr \int_{x_1}^{x} Y(x)dx \tag{A17}$$

$$\frac{d\theta}{dx} = \left[\frac{2\pi R_o^2 \Delta X}{FC_p}\int_0^1 rH(r)dr\right]Y(x) \tag{A18}$$

The functional relation between β and temperature can be approximated by

(1) calculating values of β for various values of coolant temperature
(2) fitting the values of β to a polynominal in coolant temperature.

$$\beta = a_0 + a_1\theta + a_2\theta^2 + \ldots \quad + a_n\theta^n \tag{A19}$$

$$\frac{d\beta}{d\theta} = a_1 + 2a_2\theta + 3a_3\theta^2 + \ldots \quad + na_n\theta^{n-1} \tag{A20}$$

When equations (A18) to (A20) are substituted into equation (A15), the result is an equation describing the axial-power distribution $Y(x)$ as a function of distance along the coolant channel x in integral-differential form. This equation can be solved in closed form when β is constant. For most incompressible coolants such as water and liquid metals, β can be approximated, over the temperature range of interest, by either a constant or a linear function of coolant temperature. When β is a linear function of coolant temperature,

$$\frac{d\beta}{d\theta} = a_1$$

When equation (A18) to (A20) are substituted into equation (A15), the differential equation describing the axial-power shape is

$$\beta \frac{dY}{dx} + \left[1 + \alpha_1 Y\right]Y = 0 \tag{A21}$$

where

$$\alpha_1 = \frac{2\pi a_1 R_0^2 \Delta X}{FC_p} \int_0^1 rH(r)dr \tag{A22}$$

The meaning of α_1 can be more easily recognized by substituting the temperature derivative of equation (A13) into equation (A22). The temperature derivative of equation (A13) is

$$\frac{d\beta}{d\theta} = a_1 = \frac{FC_p}{2\pi R_0 \Delta X} \frac{d}{dT}\left(\frac{1}{h}\right)$$

When this is substituted into equation (A22), the result is

$$\alpha_1 = \left[R_o \int_0^1 rH(r)dr\right] \frac{d}{d\theta}\left(\frac{1}{h}\right)$$

Therefore α_1 is a measure of the change in the film coefficient along the length of the fuel rod. Comparing equations (A15) and (A21) results in

$$\frac{d\beta}{dx} = \alpha_1 Y(x)$$

$$\int_{\beta_1}^{\beta} d\beta = \alpha_1 \int_{x_1}^{x} Y(x)dx$$

The function β can then be expressed as

$$\beta = \beta_1 + \alpha_1 \int_{x_1}^{x} Y(x)dx$$

where β_1 is evaluated from equations (A13) or (A19) using physical property data evaluated at the coolant-inlet temperature. When the axially averaged power distribution is normalized to one

$$\overline{Y} = \frac{\int_{x_1}^{x_2} Y(x)dx}{x_2 - x_1} = 1$$

By definition,

$$x_2 - x_1 = 1$$

Therefore,

$$\int_{x_1}^{x_2} Y(x)dx = 1$$

and

$$0 < \int_{x_1}^{x} Y(x)dx < 1$$

When $\alpha_1 << \beta_1$, the solution of equation (A21) can be approximated by

$$\int_{Y_1}^{Y} \frac{dY}{Y(1 + \alpha_1 Y)} = \frac{1}{\beta} \int_{x_1}^{x} dx$$

the solution is

$$Y(x) = Y_1 \left[(1 + \alpha_1 Y_1) \exp\left(\frac{x - x_1}{\beta}\right) - \alpha_1 Y_1 \right]^{-1}$$

where Y_1 is the relative power at the coolant-inlet end of the reactor. Since

$$\int_{x_1}^{x_2} Y(x)dx = 1$$

Y_1 can be expressed in terms of α_1 and β as follows:

$$Y_1 = \frac{1 - \exp\left(\frac{\alpha_1}{\beta}\right)}{\alpha_1 \left[\exp\left(- \frac{1}{\beta}\right) - 1 \right]} \tag{A23}$$

$$\frac{Y(x)}{\overline{Y}} = \frac{\exp\left(\frac{\alpha_1}{\beta}\right) - 1}{\alpha_1 \left[1 - \exp\left(\frac{\alpha_1}{\beta}\right) + \exp\left(\frac{\alpha_1 + x - x_1}{\beta}\right) - \exp\left(\frac{x - x_1 - 1}{\beta}\right) \right]} \tag{A24}$$

The power distribution in the fuel rod that yields a constant fuel-centerline temperature is

$$q(x, r) = Y_1 H(r) \left[(1 + \alpha_1 Y_1) \exp\left(\frac{x - x_1}{\beta}\right) - \alpha_1 Y_1 \right]^{-1}$$

With this power distribution, the coolant-temperature rise $(\theta_2 - \theta_1)$ and reactor power P_e may be expressed as functions of the fuel temperature limit θ_L using equation (A12).

$$\theta_2 - \theta_1 = (\theta_L - \theta_2)\alpha_1 \left[\exp\left(-\frac{1}{\beta}\right) - 1 \right] \left\{ \beta \left[1 - \exp\left(\frac{\alpha_1}{\beta}\right) \right] + \alpha_1 \left[1 - \exp\left(-\frac{1}{\beta}\right) \right] \right\}^{-1} \tag{A25}$$

$$P_e = (\theta_L - \theta_2) F C_p \alpha_1 \left[\exp\left(-\frac{1}{\beta}\right) - 1 \right] \left\{ \beta \left[1 - \exp\left(\frac{\alpha_1}{\beta}\right) \right] + \alpha_1 \left[1 - \exp\left(-\frac{1}{\beta}\right) \right] \right\}^{-1} \tag{A26}$$

When the film coefficient does not vary along the length of the fuel

$$\frac{d\beta}{dX} = 0$$

and

$$Y_1 = \left\{ \beta \left[1 - \exp\left(-\frac{1}{\beta}\right) \right] \right\}^{-1} \tag{A27}$$

The axial power shape required to produce a constant fuel-centerline temperature is of the following exponential form.

$$Y(x) = Y_1 \exp - \frac{(x - x_1)}{\beta} \tag{A28}$$

$$\frac{Y(x)}{\overline{\overline{Y}}} = \left\{ \beta \left[\exp\left(\frac{x - x_1}{\beta}\right) - \exp\left(\frac{x - x_1 - 1}{\beta}\right) \right] \right\}^{-1} \tag{A29}$$

The coolant temperature rise and reactor power are

$$\theta_2 - \theta_1 = (\theta_L - \theta_2)\left[\exp\left(\frac{1}{\beta}\right) - 1\right] \tag{A30}$$

$$P_e = (\theta_L - \theta_2)\left[\exp\left(\frac{1}{\beta}\right) - 1\right]FC_p \tag{A31}$$

The reactor power that can be achieved by power tailoring can be compared with the power from a reactor having a constant fuel and neutron absorber distribution. A constant fuel and absorber distribution yields an axial-power shape which is symmetric about the reactor midplane and can be described by a cosine function when the origin of the coordinate system is located at the reactor midplane. In this study, however, the origin of the coordinate system is assumed to be above the coolant-inlet end of the reactor as shown in figure 9. Although the axial-power shape is called a chopped cosine to conform with convention, it is described in equation (A32) as a sine function.

$$Y(x) = n_p \sin\left(\frac{\pi x}{x_3}\right) \tag{A32}$$

$$n_p = \pi\left\{x_3\left[\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_2}{x_3}\right)\right]\right\}^{-1}$$

$$\frac{dY(x)}{dx} = \frac{\pi^2 \cos\left(\frac{\pi x}{x_3}\right)}{x_3^2\left[\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_2}{x_3}\right)\right]} \tag{A33}$$

The maximum-fuel temperature and reactor power in a reactor having a chopped cosine power shape is calculated using the same methods used to derive equation (A31). In this case, however, the axial-power shape is known and the position of maximum temperature x_p must be determined in order to evaluate the maximum-fuel temperature from equation (A12). When equation (A32) is substituted into equation (A12) and evaluated at the position x_p of maximum temperature the result is

$$\frac{T(0,x_p) - T_1}{T_2 - T_1} = \frac{\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_p}{x_3}\right) + \frac{\pi\beta}{x_3}\sin\left(\frac{\pi x_p}{x_3}\right)}{\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_2}{x_3}\right)} \tag{A34}$$

The position of maximum temperature is determined by substituting equations (A32) and (A33) into equation (A15). It is assumed that the film coefficient is constant, that is, $d\beta/dx = 0$. The result is

$$\tan\left(\frac{\pi x_p}{x_3}\right) = -\frac{\pi\beta}{x_3}$$

Using this equation and the trignometric identities yield

$$\sin\left(\frac{\pi x_p}{x_3}\right) = \pi\beta\left(\pi^2\beta^2 + x_3^2\right)^{-1/2} \tag{A35}$$

$$\cos\left(\frac{\pi x_p}{x_3}\right) = -x_3\left(\pi^2\beta^2 + x_3^2\right)^{-1/2} \tag{A36}$$

The maximum temperature is determined by substituting the values from equations (A35) and (A36) into equation (A34).

$$\frac{T(0,x_p) - T_1}{T_2 - T_1} = \frac{\cos\left(\frac{\pi x_1}{x_3}\right) + \left[1 + \left(\frac{\pi\beta}{x_3}\right)^2\right]^{1/2}}{\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_2}{x_3}\right)} \tag{A37}$$

With a cosine power distribution, the coolant-temperature rise $(\theta_2 - \theta_1)$ and reactor power P_d can be expressed as a function of the fuel-temperature limit θ_L:

$$\theta_2 - \theta_1 = \frac{(\theta_L - \theta_2)\left[\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_2}{x_3}\right)\right]}{\cos\left(\frac{\pi x_2}{x_3}\right) + \left[1 + \left(\frac{\pi\beta}{x_3}\right)^2\right]^{1/2}} \tag{A38}$$

$$P_d = \frac{(\theta_L - \theta_2)\left[\cos\left(\frac{\pi x_1}{x_3}\right) - \cos\left(\frac{\pi x_2}{x_3}\right)\right]FC_p}{\cos\left(\frac{\pi x_2}{x_3}\right) + \left[1 + \left(\frac{\pi\beta}{x_3}\right)^2\right]^{1/2}} \tag{A39}$$

APPENDIX B

AXIAL-POWER TAILORING BY VARYING AXIAL-FUEL DISTRIBUTION

The method used to determine the fuel distribution required to obtain an exponential power shape is similar to the one discussed by Goertzel and Loeb (ref. 1). The axial neutron-flux distribution is calculated as a function of the known exponential source (power) distribution using a one-dimensional, two-energy group neutron-diffusion model. The required fuel distribution is proportional to the power distribution divided by the flux distribution. The reflector is simulated by specifying the neutron albedo at both ends of the reactor. The neutron population in a unit volume of the reactor can be approximated for each energy group by

$$D_1 \frac{d^2\varphi_1(X)}{dX^2} - \Sigma_R\varphi_1(X) + \nu\Sigma_f(X)\varphi_2(X) = 0 \tag{B1}$$

$$D_2 \frac{d^2\varphi_2(X)}{dX^2} - \Sigma_a^N\varphi_2(X) - \Sigma_a^F(X)\varphi_2(X) + \Sigma_R\varphi_1(X) = 0 \tag{B2}$$

where the subscripts 1 and 2 refer to fast and thermal energy groups, respectively. In the fast group, neutrons are produced in a unit volume by fissions and are depleted from a unit volume by leakage and by slowing down in the moderator. In the thermal group, neutrons are produced in a unit volume by slowing down the fast neutrons. They are depleted by leakage, absorption in the fuel material and absorptions in the nonfuel materials. The neutron diffusion equations can be rewritten as a function of the desired source (power) distribution

$$\frac{d^2\varphi_1(X)}{dX^2} - \frac{\varphi_1(X)}{L_1^2} = -G(X) \tag{B3}$$

$$\frac{d^2\varphi_2(X)}{dX^2} - \frac{\varphi_2(X)}{L_2^2} = \frac{D_1G(X)}{D_2\eta} - \frac{\Sigma_R}{D_2}\varphi_1(X) = G_1(X) \tag{B4}$$

where

$$L_1^2 = \frac{D_1}{\Sigma_R} \tag{B5}$$

$$G(x) = \frac{\nu\Sigma_f(x)\varphi_2(x)}{D_1} \tag{B6}$$

$$L_2^2 = \frac{D_2}{\Sigma_a^N} \tag{B7}$$

$$\eta = \frac{\nu\Sigma_f}{\Sigma_a^F} \tag{B8}$$

The solution to the first equation is

$$\varphi_1(X) = \varphi_1^H(X) + \varphi_1^P(X) \tag{B9}$$

The first term on the right is the solution of the homogeneous equation. The second term is a particular solution. The solution of the homogeneous equation is

$$\varphi_1^H(X) = A_1 \exp\left(\frac{X}{L_1}\right) + A_2' \exp\left(\frac{-X}{L_1}\right) \tag{B10}$$

The particular solution can be found using operator notation.

$$\left(O^2 - \frac{1}{L_1^2}\right)\varphi_1^P(X) = -G(X)$$

where

$$O^2 = \frac{d^2}{dX^2}$$

$$\left(O - \frac{1}{L_1}\right)\left(O + \frac{1}{L_1}\right)\varphi_1^P(X) = -G(X)$$

Let

$$u = \left(O - \frac{1}{L_1}\right)\varphi_1^P(X)$$

$$\left(O + \frac{1}{L_1}\right)u = -G(X)$$

$$u = -\exp\left(\frac{-X}{L_1}\right)\int_{X_0}^{X} G(X)\exp\left(\frac{X}{L_1}\right)dX$$

$$\left(O - \frac{1}{L_1}\right)\varphi_1^P(X) = -\exp\left(\frac{-X}{L_1}\right)\int_{X_0}^{X} G(X)\exp\left(\frac{X}{L_1}\right)dX$$

$$\varphi_1^P(X) = -\exp\left(\frac{X}{L_1}\right)\int_{X_{00}}^{X}\left[\int_{X_0}^{X} G(x)\exp\left(\frac{X}{L_1}\right)dX\right]\exp\left(\frac{-2X}{L_1}\right)dX \qquad \text{(B11)}$$

The lower limits, X_0 and X_{00}, may be arbitrarily selected to yield the most convenient solution. Any particular solution is acceptable provided it satisfies equation (B3). Therefore,

$$\varphi_1(X) = A_1 \exp\left(\frac{X}{L_1}\right) + A_2' \exp\left(\frac{-X}{L_1}\right) - \exp\left(\frac{X}{L_1}\right)\int_{X_{00}}^{X} \exp\left(\frac{-2X}{L_1}\right)\int_{X_0}^{X} G(X)\exp\left(\frac{X}{L_1}\right)dX\, dX \qquad \text{(B12)}$$

When $G(X)$ is an exponential of the form

$$G(X) = \gamma_1 \exp(-\gamma X) \tag{B13}$$

The following limits are selected:

$$X_0 = 0 \quad \text{and} \quad X_{00} = \infty$$

The solution of equation (B12) is

$$\varphi_1(X) = A_1 \exp\left(\frac{X}{L_1}\right) + A_2 \exp\left(-\frac{X}{L_1}\right) - \frac{L_1^2\gamma_1 \exp(-\gamma X)}{L_1^2\gamma^2 - 1} \tag{B14}$$

where A_2 is equal to A_2' plus terms derived from the particular solution $\varphi_1^P(X)$.

The boundary conditions are established by specifying the albedo at the coolant-inlet end of the reactor $K_{1,1}$ and at the coolant-outlet end $K_{2,1}$. The neutron albedo for any energy group is defined as the ratio of the neutron partial current entering the reactor to the neturon partial current leaving the reactor.

$$K_{1,1} = \frac{J_+(X_1)}{J_-(X_1)} \tag{B15}$$

$$K_{2,1} = \frac{J_-(X_2)}{J_+(X_2)} \tag{B16}$$

$$J_+(X_1) = K_{1,1}J_-(X_1)$$

$$J_-(X_2) = K_{2,1}J_+(X_2)$$

The neutron current can be expressed by

$$J_+(X) = \frac{\varphi(X)}{4} - \frac{D_1\, d\varphi(X)}{2\, dX}$$

$$J_-(X) = \frac{\varphi(X)}{4} + \frac{D_1\, d\varphi(X)}{2\, dX}$$

When these equations are substituted into equations (B15) and (B16),

$$(1 - K_{1,1}) \frac{\varphi_1(X_1)}{4} = (1 + K_{1,1}) \frac{D_1}{2} \frac{d\varphi_1(X_1)}{dX}$$

$$(1 - K_{2,1}) \frac{\varphi_1(X_2)}{4} = -(1 + K_{2,1}) \frac{D_1}{2} \frac{d\varphi_1(X_2)}{dX}$$

These equations are solved to determine A_1 and A_2.

$$A_2 = \frac{\gamma_1 L_1^2}{\gamma^2 L_1^2 - 1} \left\{ \frac{\psi_1 \psi_2 \exp(-\gamma X_1) - \psi_3 \psi_4 \exp\left[-\left(\frac{X_2 - X_1}{L_1} + \gamma X_2\right)\right]}{\psi_2 \psi_5 - \psi_4 \psi_6 \exp\left[\frac{-2(X_2 - X_1)}{L_1}\right]} \right\} \exp\left(\frac{X_1}{L_1}\right) \tag{B17}$$

$$A_1 = \frac{\gamma_1 L_1^2}{\gamma^2 L_1^2 - 1} \left\{ \frac{\psi_3 \psi_5 \exp(-\gamma X_2) - \psi_1 \psi_6 \exp\left[-\left(\frac{X_2 - X_1}{L_1} + \gamma X_1\right)\right]}{\psi_2 \psi_5 - \psi_4 \psi_6 \exp\left[\frac{-2(X_2 - X_1)}{L_1}\right]} \right\} \exp\left(\frac{-X_2}{L_1}\right) \tag{B18}$$

$$\left.\begin{aligned}
\psi_1 &= \frac{1 - K_{1,1}}{4} + \frac{(1 + K_{1,1})\gamma D_1}{2} \\
\psi_2 &= \frac{1 - K_{2,1}}{4} + \frac{(1 + K_{2,1})D_1}{2L_1} \\
\psi_3 &= \frac{1 - K_{2,1}}{4} - \frac{(1 + K_{2,1})\gamma D_1}{2} \\
\psi_4 &= \frac{1 - K_{1,1}}{4} - \frac{(1 + K_{1,1})D_1}{2L_1} \\
\psi_5 &= \frac{1 - K_{1,1}}{4} + \frac{(1 + K_{1,1})D_1}{2L_1} \\
\psi_6 &= \frac{1 - K_{2,1}}{4} - \frac{(1 + K_{2,1})D_1}{2L_1}
\end{aligned}\right\} \tag{B19}$$

For the thermal neutron flux

$$\varphi_2(X) = \varphi_2^H(X) + \varphi_2^P(X)$$

$$\varphi_2^H(X) = A_3 \exp\left(\frac{X}{L_2}\right) + A_4' \exp\left(-\frac{X}{L_2}\right) \tag{B20}$$

$$G_1(X) = \frac{D_1\gamma_1}{D_2\eta} \exp(-\gamma X) - \frac{\Sigma_R}{D_2}\left[A_1 \exp\left(\frac{X}{L_1}\right) + A_2 \exp\left(-\frac{X}{L_1}\right) - \frac{L_1^2\gamma_1 \exp(-\gamma X)}{\gamma^2 L_1^2 - 1}\right]$$

As in equation (B11),

$$\varphi_2^P(X) = +\exp\left(\frac{X}{L_2}\right)\int_\infty^X \left[\int_0^X G_1(X)\exp\left(\frac{X}{L_2}\right)dX\right]\exp\left(\frac{-2X}{L_2}\right)dX \tag{B21}$$

$$\varphi_2(X) = A_3 \exp\left(\frac{X}{L_2}\right) + A_4 \exp\left(\frac{-X}{L_2}\right) + \frac{\gamma_1 L_2^2\left[\frac{D_1}{\eta} + \frac{\Sigma_R L_1^2}{\left(\gamma^2 L_1^2 - 1\right)}\right]}{D_2\left(\gamma^2 L_2^2 - 1\right)\exp(\gamma X)}$$

$$- \frac{\Sigma_R L_1^2 L_2^2\left[A_1 \exp\left(\frac{X}{L_1}\right) + A_2 \exp\left(\frac{-X}{L_1}\right)\right]}{D_2\left(L_2^2 - L_1^2\right)} \tag{B22}$$

where A_4 is equal to A_4' plus terms derived from the particular solution $\varphi_2^P(x)$. The constants A_3 and A_4 are determined by specifying the thermal neutron albedo at both ends of the reactor.

$$J_+(X_1) = K_{1,2}J_-(X_1) \tag{B23}$$

$$J_-(X_2) = K_{2,2}J_+(X_2) \tag{B24}$$

$$J_+(X) = \frac{\varphi_2(X)}{4} - \frac{D_2\, d\varphi_2(X)}{2\, dX} \tag{B25}$$

$$J_-(X) = \frac{\varphi_2(X)}{4} + \frac{D_2\, d\varphi_2(X)}{2\, dX} \tag{B26}$$

The constants A_3 and A_4 are determined by simultaneous solution of these equations.

$$A_3 = \left[\frac{\psi_{10}\psi_{12} - \psi_8\psi_{11}\exp\left(\frac{X_2 - X_1}{L_2}\right)}{\psi_9\psi_{11}\exp\left(2\frac{X_2 - X_1}{L_2}\right) - \psi_7\psi_{12}}\right]\exp\left(\frac{-X_1}{L_2}\right) \tag{B27}$$

$$A_4 = \left[\frac{\psi_7\psi_8 - \psi_9\psi_{10} \exp\left(\frac{X_2 - X_1}{L_2}\right)}{\psi_9\psi_{11} \exp\left(2\,\frac{X_2 - X_1}{L_2}\right) - \psi_7\psi_{12}}\right]\exp\left(\frac{X_2}{L_2}\right) \tag{B28}$$

$$\left.\begin{aligned}
\psi_7 &= \frac{1 - K_{1,2}}{4} - \frac{(1 + K_{1,2})D_2}{2\,L_2} \\
\psi_8 &= \frac{(1 - K_{2,2})\varphi_P(X_2)}{4} + \frac{(1 + K_{2,2})D_2}{2}\frac{d\varphi_P(X_2)}{dX} \\
\psi_9 &= \frac{1 - K_{2,2}}{4} + \frac{(1 + K_{2,2})D_2}{2\,L_2} \\
\psi_{10} &= \frac{(1 - K_{1,2})\varphi_P(X_1)}{4} - \frac{(1 + K_{1,2})D_2}{2}\frac{d\varphi_P(X_1)}{dX} \\
\psi_{11} &= \frac{1 - K_{1,2}}{4} + \frac{(1 + K_{1,2})D_2}{2\,L_2} \\
\psi_{12} &= \frac{1 - K_{2,2}}{4} - \frac{(1 + K_{2,2})D_2}{2\,L_2}
\end{aligned}\right\} \tag{B29}$$

where

$$\varphi_P(X) = \frac{\gamma_1 L_2^2\left[\frac{D_1}{\eta} + \frac{\Sigma_R L_1^2}{\gamma^2 L_1^2 - 1}\right]\exp(-\gamma X)}{D_2\left(\gamma^2 L_2^2 - 1\right)} - \frac{\Sigma_R L_1^2 L_2^2\left[A_1 \exp\left(\frac{X}{L_1}\right) + A_2 \exp\left(\frac{-X}{L_1}\right)\right]}{D_2\left(L_2^2 - L_1^2\right)} \tag{B30}$$

$$\frac{d\varphi_p(X)}{dX} = \frac{-\gamma\gamma_1 L_2^2\left[\frac{D_1}{\eta} + \frac{\Sigma_R L_1^2}{\gamma^2 L_1^2 - 1}\right]\exp(-\gamma X)}{D_2\left(\gamma^2 L_2^2 - 1\right)} - \frac{\Sigma_R L_1 L_2^2\left[A_1 \exp\left(\frac{X}{L_1}\right) - A_2 \exp\left(\frac{-X}{L_1}\right)\right]}{D_2\left(L_2^2 - L_1^2\right)} \tag{B31}$$

The fuel distribution required to obtain the desired power distribution $G(X)$ is

$$\Sigma_f(X) = \frac{D_1 G(X)}{\nu\varphi_2(X)} \tag{B32}$$

$G(X)$ can be related to the exponential power distribution $Y(x)$ derived in appendix A (eq. (A29)). From equation (B13)

$$G(X) = \gamma_1 \exp(-\gamma X)$$

and from equation (B6)

$$G(X) = \frac{\nu\Sigma_f(X)\varphi_2(X)}{D_1}$$

Equation (B6) can be rewritten as

$$G(X) = \frac{\nu\overline{\Sigma_f\varphi_2}}{D_1}\left[\frac{\Sigma_f(X)\varphi_2(X)}{\overline{\Sigma_f\varphi_2}}\right] \tag{B33}$$

where $\overline{\Sigma_f\varphi_2}$ is the axially averaged fission rate and

$$\left[\frac{\Sigma_f(X)\varphi_2(X)}{\overline{\Sigma_f\varphi_2}}\right]$$

is the local-to-average fission rate that is also equal to the local-to-average power density $Y(x)/Y$ of equation (A29).

The expression $Y(x)/Y$ must be converted from the nondimensional variable x to the dimensional form

$$x = \frac{X}{X_2 - X_1}$$

Therefore

$$G(X) = \frac{\nu\overline{\Sigma_f\varphi_2}}{D_1}\left\{\beta\left[\exp\left(\frac{1}{\beta}\,\frac{X - X_1}{X_2 - X_1}\right) - \exp\left(\frac{1}{\beta}\,\frac{X - X_1}{X_2 - X_1} - \frac{1}{\beta}\right)\right]\right\}^{-1} \tag{B34}$$

A comparison of equations (B13) and (B34) results in

$$\gamma = \left[\beta(X_2 - X_1)\right]^{-1} \tag{B35}$$

$$\gamma_1 = \nu\overline{\Sigma_f\varphi_2}\,\exp\left[\frac{1}{\beta}\left(\frac{X_1}{X_2 - X_1}\right)\right]\left\{\beta D_1\left[1 - \exp\left(-\frac{1}{\beta}\right)\right]\right\}^{-1} \tag{B36}$$

The average fission rate can be expressed in terms of the radial component of the volumetric heating rate $H(r)$ as follows:

$$\overline{\Sigma_f\varphi_2} = \frac{\int_0^1 rH(r)\,dr}{\epsilon}$$

Equation (B36) becomes

$$\gamma_1 = \nu\int_0^1 H(r)r\,dr\,\exp\left[\frac{1}{\beta}\left(\frac{X_1}{X_2 - X_1}\right)\right]\left\{\beta\epsilon D_1\left[1 - \exp\left(-\frac{1}{\beta}\right)\right]\right\}^{-1} \tag{B37}$$

APPENDIX C

SYMBOLS

A_1 constant used in equations describing fast and thermal neutron flux, neutrons/(m^2)(sec)

a_i coefficients in polynominal describing β as function of temperature, K^{-1}

c_p specific heat of coolant, J/(kg)(K)

D neutron diffusion coefficient, m

De equivalent diameter of coolant channel, m

F mass flow rate of coolant, kg/sec

H(r) radial power distribution in fuel element, W/m^3

h forced convection film coefficient, W/(m^2)(K)

J_+ neutron partial current in positive direction, neutrons/(m^2)(sec)

J_- neutron partial current in negative direction, neutrons/(m^2)(sec)

$K_{i,j}$ neutron albedo at position i for energy group j

k thermal conductivity, W/(m)(K)

L_j neutron diffusion length for energy group j, m

m(x) fraction of total fuel rod power generated up to axial position x

Nu Nusselt number

n(x) ratio of local to axially averaged power generation

n_p ratio of maximum to axially averaged power generation rate for reactor with "chopped cosine" axial-power shape

P_d total reactor power output with cosine axial-power distribution, W

P_e total reactor power output with exponential axial-power distribution, W

Pr Prandtl number

Q volumetric heating rate expressed in dimensional space variables, W/m^3

Q/A average heat flux from surface of fuel rod, W/m^2

q volumetric heating rate expressed in dimensionless space variables

R dimensional radial field point, m

R_o outer radius of fuel element, m

Re Reynolds number

r dimensionless radial field point

S wetted perimeter of coolant channel, m

T dimensionless temperature

X dimensional axial field point

ΔX reactor length, m

x dimensionless axial field point

Y_1 relative local power at inlet end of reactor

$Y(x)$ relative axial power distribution

α_1 factor defined by eq. (A22)

β thermal-hydraulic factor defined by eq. (A13)

γ exponential coefficient used to describe axial-power shape, m^{-1}

ϵ energy released per fission, J

η number of neutrons produced per thermal neutron absorbed in fuel

θ dimensional temperature, K

θ_L prescribed temperature limit for fuel, K

ν number of neutrons produced per thermal fission

Σ_a macroscopic absorption cross section for thermal neutrons, m^{-1}

Σ_f macroscopic thermal-neutron fission cross section, m^{-1}

Σ_R removal cross section for fast neutrons, m^{-1}

φ_j neutron flux for energy group j, neutrons/m^2-sec

ψ arbitrary constant

Subscripts:

c coolant

f fission

M fuel

R removal

1 inlet end of reactor or fast-neutron energy group

2 outlet end of reactor or thermal-neutron energy group

3 extrapolated distance to zero neutron flux

Superscripts:

F fuel material

N nonfuel materials

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