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FLOW AND WALL-TEMPERATURE SENSITIVITY IN PARALLEL PASSAGES FOR LARGE INLET TO EXIT DENSITY RATIOS IN SUBSONIC FLOW

by Meyer Reshotko

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ABSTRACT

There is a large gas density decrease from reactor inlet to exit in a nuclear rocket. Under certain reactor operating conditions, the momentum pressure drop can be a large portion of the overall pressure drop. As a result, deviations in flow rate or passage diameter can cause the reactor walls to overheat. A simplified analytical investigation has been made to find the sensitivity of the flow and wall temperature in heated passages as a function of Mach number for (1) a change in heat addition, and (2) nonuniform passage diameter. The Mach number range is from 0.2 to 0.9.

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FLOW AND WALL-TEMPERATURE SENSITIVITY IN PARALLEL PASSAGES FOR LARGE INLET TO EXIT DENSITY RATIOS IN SUBSONIC FLOW

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SUMMARY

There is a large gas density decrease from reactor inlet to exit in a nuclear rocket. Under certain reactor operating conditions, the momentum pressure drop can be a large portion of the overall pressure drop. As a result, deviations in flow rate or passage diameter can cause reactor walls to overheat.

A simplified analytical investigation has been made to find the sensitivity of both flow and wall temperature in heated passages as a function of Mach number up to Mach 0.9 for (1) a 6-percent increase in heat addition and (2) a 2-percent decrease in passage diameter.

An analytical investigation has been conducted using reference conditions which are considered typical for nuclear-rocket-reactor applications. The reactor consists of fueled material with many parallel passages through which gas flows. One passage was studied with the assumption of a constant pressure drop across all the passages. The wall temperature is assumed constant.

The results show that for a 6-percent increase in heat addition the sensitivities of flow and wall temperature with respect to Mach number are small. Neither sensitivity parameter increased by more than 10 percent. This same effect takes place for increases in heat addition of up to 12 percent. For a 2-percent decrease in passage diameter the sensitivities of flow and wall temperature with respect to Mach number are also small. Neither sensitivity parameter increased by more than 7 percent. Therefore, for large inlet to exit density ratios, the effect of momentum pressure drop on the flow and wall temperature sensitivities as one goes to increasing Mach numbers is small. However, for a given Mach number of 0.9 with the heat addition kept constant, a 3-percent decrease in passage diameter caused a 10-percent increase in surface temperature.

INTRODUCTION

A pressure drop occurs in any compressible flow system because of the forces necessary to overcome friction and to accelerate the fluid. These two components are generally termed friction pressure drop and momentum pressure drop. Any decrease in density of the compressible fluid, such as heat addition, increases the momentum pressure drop if the flow is to be maintained. Heating the propellant in a nuclear-rocket reactor is one example of a situation where there is a large decrease in density of the fluid as it flows from inlet to outlet. Here, the pressure drop across the reactor core is established by the characteristics of many flow passages acting in parallel. Any decrease in fluid density due to increased heat addition to one passage must be accomplished by a decrease in flow rate through the passage. It has been shown (ref. 1), for example, that, if heat input is increased in one channel, laminar flow through parallel passages may be unstable. This could result in a continuous decrease in flow to the hot channel until failure occurs. Although the turbulent flow case is stable, some maldistribution will occur with nonuniform heating and can be expected to become more severe as the Mach number and momentum pressure drop increase. Maldistribution of flow can also occur with uniform heating but with nonuniform passage diameter.

A simplified analytical investigation has been made to determine both flow and wall-temperature sensitivities in heated passages as a function of Mach number up to Mach 0.9 for a

(1) 6-percent increase in heat addition

(2) 2-percent decrease in passage diameter

This report on flow and wall-temperature sensitivity is an analytical treatment using reference conditions which are considered typical for nuclear-rocket application. The wall temperature is assumed constant with respect to axial position. One flow passage of a multipassage configuration was studied with the assumption of constant pressure drop across all the passages. Simplified equations were developed to calculate the passage operating conditions during steady state before and after a change was made in either heat addition or passage diameter.

METHOD OF CALCULATION

In analyzing the problem of flow and wall-temperature sensitivity the following reactor model was selected. The model consisted of fueled material with many parallel passages through which the gas flows. The passages were all assumed to be uniform with respect to length, diameter, and heat addition except for one passage in which either diameter or heat addition were allowed to vary. This one passage was studied with the assumption that the pressure drop was fixed by the reference operating conditions of the remaining parallel flow passages.

Reference Conditions

Reference conditions chosen to be typical of nuclear-rocket operation using hydrogen as the propellant-coolant were as follows:

Surface temperature, T_s, $^{\circ}R$; K	5000°; 2778
Gas exit total temperature, T_{out}, $^{\circ}R$; K	4500°; 2500
Gas inlet total temperature, T_{in}, $^{\circ}R$; K	320°; 178
Passage length, L, ft; m	3.25; 0.991
Passage equivalent diameter, D, ft; mm	0.0104; 3.17

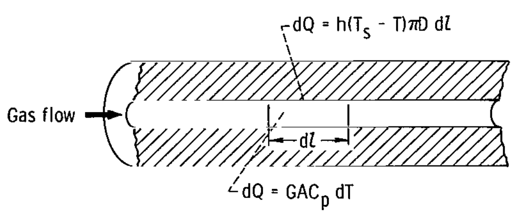
The values given establish mass velocity for all reference conditions. In order to explore the effect of Mach number on flow and wall-temperature sensitivity with a fixed reference mass velocity, the outlet static pressure was allowed to vary to provide a series of reference conditions with outlet Mach numbers varying from 0.2 to 0.9.

Flow and wall-temperature sensitivities were investigated by making incremental changes in the reference values of heat addition and passage diameter. Heat addition was increased by 6 percent and passage diameter decreased by 2 percent. This section develops simplified relations used to calculate the reference conditions and the conditions existing after incremental changes were made to these reference conditions.

From the reference conditions, the mass velocity can be calculated. The total and static temperature, static pressure, heat-transfer coefficient, heat flux, and Mach number profiles as a function of passage length for reference condition operation may then be calculated.

Calculation of mass velocity. - The mass velocity is calculated by taking a heat balance across an increment of passage length (see sketch (a)).

The heat increase of the gas in the control volume is equal to the heat transferred to the gas at the wall.



(a)

$$GAC_p \, dT = h(T_s - T)\pi D \, dl \tag{1}$$

(Symbols are defined in appendix A.) The surface temperature T_s is assumed to be constant along the passage length.

The heat-transfer correlation used to calculate the heat-transfer coefficient was obtained from reference 2. This correlation which offers a good representation of experimental, turbulent heat-transfer data is expressed as follows:

$$h = 0.021 \frac{k}{D} Re^{0.8} Pr^{0.4} \left(\frac{T_s}{T}\right)^{-(0.2900+0.0019l/D)} \tag{2}$$

All gas properties were evaluated at the bulk temperature from the data of reference 3.

Substituting equation (2) into equation (1) yields

$$\frac{\left(\frac{GD}{\mu}\right)^{0.2} Pr^{0.6}}{0.084} \left(\frac{T_s}{T}\right)^{(0.2900+0.0019l/D)} \frac{dT}{T_s - T} = \frac{dl}{D} \tag{3}$$

The reference mass velocity was obtained from equation (3) by trial and error. Equation (3) was numerically integrated from $T = 320^{\circ}$ R (178 K) at $l = 0$ to $l = 3.25$ feet (0.991 m) using an assumed value of mass velocity. The integration was repeated until a mass velocity was found which resulted in the reference outlet temperature of 4500° R (2500 K) at $l = 3.25$ feet (0.991 m). The value of mass velocity found for the reference conditions was 301 pounds mass per square foot per second (1466 kg/(m^2)(sec)). This value is independent of pressure level and therefore applies to all reference conditions regardless of outlet Mach number.

Total temperature was recorded for the final integration and can be plotted as a function of passage length. The local heat-transfer coefficient is calculated by evaluating equation (2), and the local heat flux by the equation

$$q = h(T_s - T)$$

They too can be expressed as a function of passage length.

Static-pressure drop. - The static-pressure drop across the passage is found by integrating a differential static-pressure drop from exit to entrance. The differential-pressure drop is

$$dp = -4f \frac{dl}{D} \frac{\rho V^2}{2g_c} - \frac{G}{g_c} dV \tag{4}$$

The first term on the right represents friction pressure drop and the second term momentum pressure drop. Numerical integration of the differential-pressure drop equation can be accomplished by writing all unknown quantities in terms of p, T, and l. (The relation between T and l is known from the numerical integration of eq. (3).) The resulting equation for static-pressure drop is developed in appendix B.

$$\frac{dp}{dT} = \frac{-\frac{0.092}{0.084} Pr^{0.6}\left(\frac{T_s}{T}\right)^{(0.2900+0.0019l/D)}\left(\frac{\gamma}{\gamma - 1}\right)(-1 + E^{1/2}) \frac{p}{T_s - T} - \frac{G^2R}{g_c} (E^{-1/2}) \frac{1}{p}}{1 - 2 \frac{G^2R}{g_c} (E^{-1/2}) \frac{T}{p^2} + \frac{\gamma}{\gamma - 1} (-1 + E^{1/2})} \tag{5}$$

The term E, is a dimensionless parameter defined as follows:

$$E = 1 + 2\left(\frac{\gamma - 1}{\gamma}\right) \frac{G^2R}{g_c} \frac{T}{p^2}$$

A static-pressure profile is found by numerically integrating equation (5) from the exit to the entrance.

At the exit, the static pressure is determined from the relation

$$p = \frac{G}{M} \sqrt{\frac{RT}{\gamma g_c} \frac{t}{T}} \tag{6}$$

where the values of G and T are the known reference conditions evaluated at the exit, and M is the desired exit Mach number. Corresponding to this Mach number, the isentropic static to total temperature ratio t/T at the exit is found from reference 4. Static pressure can then be plotted as a function of passage length.

With both the static pressure and total temperature as a function of passage length known, a static temperature profile is calculated using the equation which converts total to static temperature.

$$t = \frac{-1 + \left[1 + 2\left(\frac{\gamma - 1}{\gamma}\right)\frac{G^2R}{g_c}\frac{T}{p^2}\right]^{1/2}}{\frac{\gamma - 1}{\gamma}\frac{G^2R}{g_c}\frac{1}{p^2}} \tag{7}$$

The static pressure and static temperature as a function of passage length may be used to calculate a Mach number profile from the following relation:

$$M = \frac{G}{p}\sqrt{\frac{Rt}{\gamma g_c}} \tag{8}$$

The total heat added to the gas is calculated using the equation

$$Q = GAC_p(T_{out} - T_{in}) \tag{9}$$

This completes the reference conditions for the typical nuclear-rocket model.

Flow and Wall-Temperature Sensitivity

Flow sensitivity for a change in heat addition $(\Delta G/G)/(\Delta Q/Q)$ is defined as a fractional change in mass velocity resulting from a given fractional change in heat addition. Flow sensitivity for nonuniform passage diameter $(\Delta G/G)/(\Delta D/D)$ is defined as a fractional change in mass velocity resulting from a given fractional change in passage diameter. Wall-temperature sensitivity for a change in heat addition $(\Delta T_s/T_s)/(\Delta Q/Q)$ is expressed as a fractional change in wall temperature resulting from a given fractional change in heat addition. Wall-temperature sensitivity for nonuniform passage diameter $(\Delta T_s/T_s)/(\Delta D/D)$ is expressed as a fractional change in wall temperature resulting from a given fractional change in passage diameter.

The flow sensitivity and wall-temperature sensitivity was investigated for a

(1) 6-percent increase in heat addition

(2) 2-percent decrease in passage diameter

The sensitivity terms are calculated at exit Mach numbers ranging from 0.2 to 0.9. One flow passage of a multipassage configuration was studied with the assumption of constant pressure drop across all the passages. A change in heat addition or passage diameter to one passage results in a change in flow rate through that passage, while the static-pressure drop and exit static pressure are constant. The static-pressure drop and the exit static pressure are, however, different for each exit Mach number reference condition.

Equation (5) can be used to calculate the pressure drop between two points providing the surface temperature is known and is constant; the gas temperature between the two points can be expressed as a function of passage length. However, when used to determine the flow and wall temperature sensitivity parameters, equation (5) will be one equation of a set of simultaneous equations, the solution of which will necessitate a time-consuming iteration process. Therefore, an approximate formula for pressure drop, which will make future calculations easier, is used instead of equation (5).

Equation (4) can be expressed in incremental form

$$\Delta p = -\left(4f \frac{L}{D} \frac{\rho V^2}{2g_c} + \frac{G}{g_c} \Delta V\right) \tag{10}$$

The perfect gas law $p = \rho Rt$ and the definition of mass velocity $\rho V = G$ may be used to express velocity as $V = GR(t/p)$. By making the substitution

$$V = \frac{V_{out} + V_{in}}{2}$$

which is justified when the variation in velocity is approximately linear with passage length, equation (10) can be rewritten as follows:

$$\Delta p = -\frac{G^2 R}{g_c}\left[f \frac{L}{D}\left(\frac{t_{out}}{p_{out}} + \frac{t_{in}}{p_{in}}\right) + \left(\frac{t_{out}}{p_{out}} - \frac{t_{in}}{p_{in}}\right)\right] \tag{11}$$

For the large density ratios used herein

$$\frac{\rho_{in}}{\rho_{out}} > 35$$

the term t_{in}/p_{in} can be neglected so that equation (10) takes the even simpler form.

$$\Delta p = -\left[\left(f \frac{L}{D} + 1\right) \frac{G^2 R}{g_c} \frac{t_{out}}{p_{out}}\right] \tag{12}$$

The friction factor f is assumed to be constant.

Flow sensitivity. - Equation (12) is rearranged so that all the constant terms are on the left side and the terms affected by the change are on the right side.

$$\frac{\Delta p}{\left(f\frac{L}{D}+1\right)} g_c \frac{p_{out}}{R} = G^2 t_{out} = \text{constant} \tag{13}$$

The product $G^2 t_{out}$ is constant, therefore, t_{out} varies inversely as G^2. A ΔG is obtained by subtracting the reference value from a new value of G. The new value of G was chosen such that the value of ΔG used was that value necessary to make a change in Q of approximately 6 percent. The new exit total temperature is then found using the relation:

$$T_{out} = t_{out} + \frac{\gamma - 1}{2\gamma} \frac{G^2 R}{g_c} \frac{t_{out}^2}{p_{out}^2} \tag{14}$$

The new values of G and T_{out} are inserted in equation (9) to give a new value of Q. When the reference value of Q is subtracted from this new value, a ΔQ is obtained. This is the actual change in heat addition. The flow sensitivity term for a change in heat addition $(\Delta G/G)/(\Delta Q/Q)$ is obtained in this manner.

In a similar manner, the flow sensitivity due to nonuniform passage diameter may be found. This flow sensitivity parameter is calculated for a constant value of heat addition to the gas Q in addition to the condition of constant pressure drop across the flow passage.

Changing the equivalent diameter an amount ΔD allows the following three equations to be solved simultaneously for new values of G, t_{out}, and T_{out}:

$$\frac{\Delta p}{\left(f\frac{L}{D}+1\right)} g_c \frac{p_{out}}{R} = G^2 t_{out} \tag{13}$$

$$T_{out} = t_{out} + \frac{\gamma - 1}{2\gamma} \frac{G^2 R}{g_c} \frac{t_{out}^2}{p_{out}^2} \tag{14}$$

$$Q = G \frac{\pi}{4} D^2 C_p (T_{out} - T_{in}) \tag{9}$$

A ΔG term is obtained by subtracting the reference value from the new value of G. Thus, the flow sensitivity for nonuniform passage diameter $(\Delta G/G)/(\Delta D/D)$ is determined.

Wall-temperature sensitivity. - In determining the wall-temperature sensitivities due to a change in heat addition $(\Delta T_s/T_s)/(\Delta Q/Q)$ and nonuniform passage diameter $(\Delta T_s/T_s)/(\Delta D/D)$, a new surface temperature is found based on the new conditions. The new surface temperature is found by a trial and error technique using equation (3). In order to obtain a starting value for this technique, an approximate value of surface temperature was determined. From the heat balance of equation (1), the following equation is obtained:

$$\frac{dT}{T_s - T} = \frac{h}{G} \frac{4}{C_p} \frac{dl}{D} \tag{15}$$

Integrating this equation for the boundary conditions $T = T_{in}$ at $l = 0$ and $T = T_{out}$ at $l = L$ the approximate surface temperature is

$$T_s = \frac{T_{in} - e^{4(L/D)(h/GC_p)} T_{out}}{1 - e^{4(L/D)(h/GC_p)}} \tag{16}$$

The value of T_{out} has already been calculated in finding the flow sensitivity due to a change in heat addition. The heat-transfer coefficient h at the changed condition is found from the relation

$$h = h_{ref} \left(\frac{G}{G_{ref}} \right)^{0.8}$$

where h_{ref} and G_{ref} are reference values. The term h_{ref} is found by evaluating equation (2) at the inlet and exit and dividing the sum by 2. With this as a starting value, repeated values of T_s are used until equation (3) is balanced. After the new surface temperature is calculated, the difference between the new and reference surface temperature ΔT_s is determined. The surface-temperature sensitivity term $(\Delta T_s/T_s)/(\Delta Q/Q)$ is then calculated where $\Delta Q/Q$ is the same as in the calculation of flow sensitivity.

A similar calculation is made for the wall-temperature sensitivity due to nonuniform passage diameter $(\Delta T_s/T_s)/(\Delta D/D)$. Surface temperature is determined in the same manner as in the previous sensitivity calculations except that the value used for T_{out} is taken from the simultaneous solution of equations (9), (13), and (14) used in the calculation of flow sensitivity due to a decrease in passage diameter. After the surface-temperature difference ΔT_s is determined, the surface-temperature sensitivity term $(\Delta T_s/T_s)/(\Delta D/D)$ is calculated where $\Delta D/D$ is the same as in the calculation of flow sensitivity.

RESULTS AND DISCUSSION

Methods for calculating reference conditions and conditions after changes, have been established in the section METHOD OF CALCULATIONS. Once the reference conditions have been established the sensitivity parameters may be determined for a change in heat addition and nonuniform passage diameter based on the developed simplified equations. Again, it should be emphasized that, during the changes, the static-pressure drop remains constant and the level of the wall temperature increases even though the wall temperature along the passage remains constant.

Reference Conditions

Using the iterative method described in the section METHOD OF CALCULATIONS along with the typical reference conditions for a nuclear rocket, the calculated reference hydrogen mass velocity is 301 pounds mass per square foot per second (1466 kg/ (m^2)(sec)). The total temperature, heat-transfer coefficient, and heat-flux profiles are calculated and shown in figure 1. Exit static pressure and static-pressure drop are shown in table I as a function of exit Mach number.

As an illustrative example for the case of $M = 0.9$, the static pressure, Mach number, and static temperature are plotted as a function of length and are shown in figure 2.

For temperatures up to 2000° R (1111 K), the static temperature is almost the same as the total temperature. The static temperature reaches a peak of 4075° R (2264 K). This corresponds to a point 3.05 feet (0.930 m) from the entrance with a local Mach number of 0.62. The static-temperature peak occurs at this Mach number because of the combination of heat addition and friction in the passage. For an ideal case without friction, this peak occurs at Mach 0.85 (ref. 5).

The heat-transfer correlation used in equation (1) is taken from reference 2. It offers a good fit to experimental, turbulent heat-transfer data. Appendix C shows that different heat-transfer correlations can markedly affect reference design conditions.

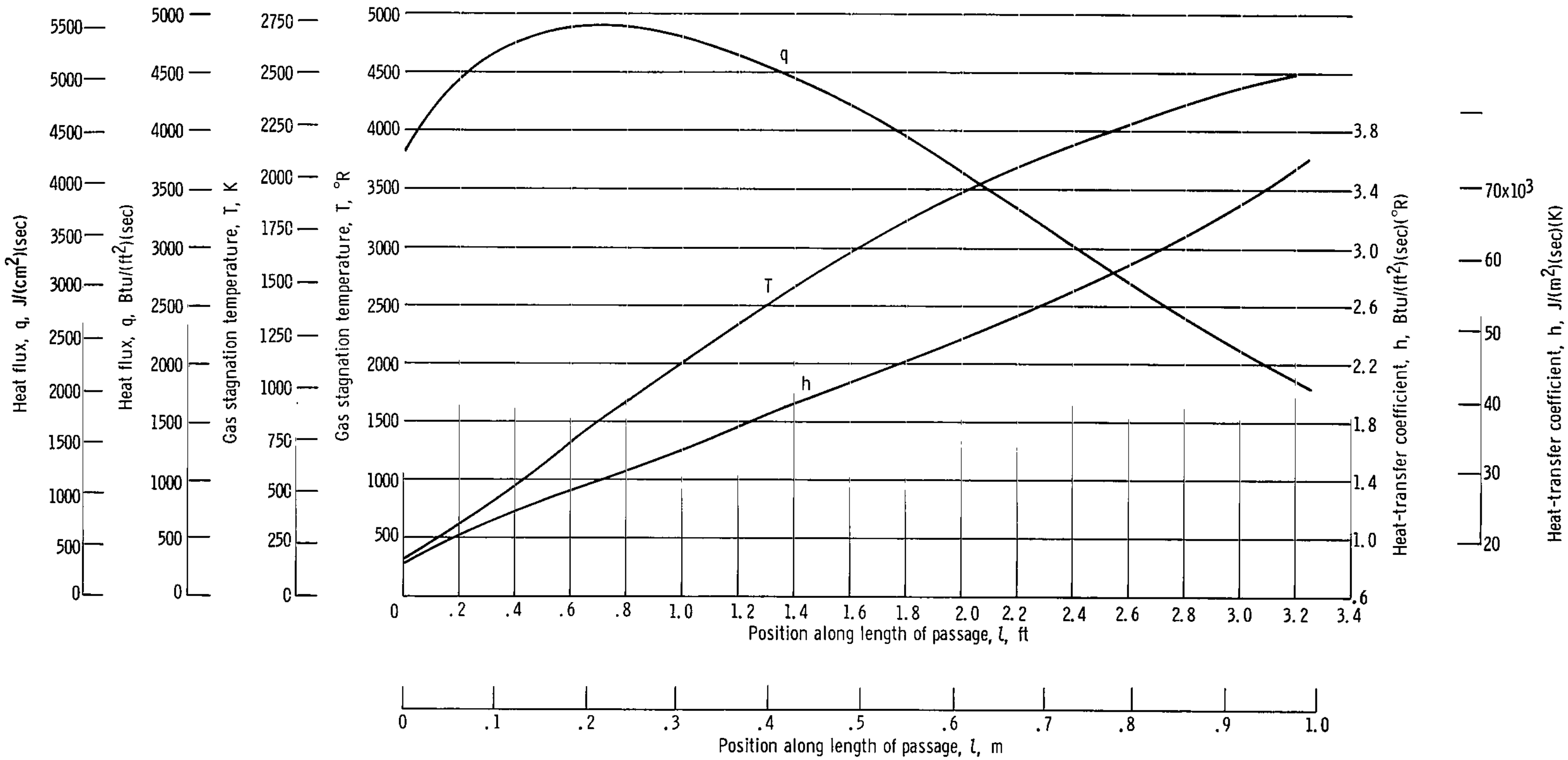


Figure 1. - Total temperature, heat flux, and heat-transfer coefficient as function of passage length. Mass velocity, 301 pounds mass per square foot per second (1466 kg/(m²)(sec)); surface temperature, 5000° R (2778 K); gas, hydrogen.

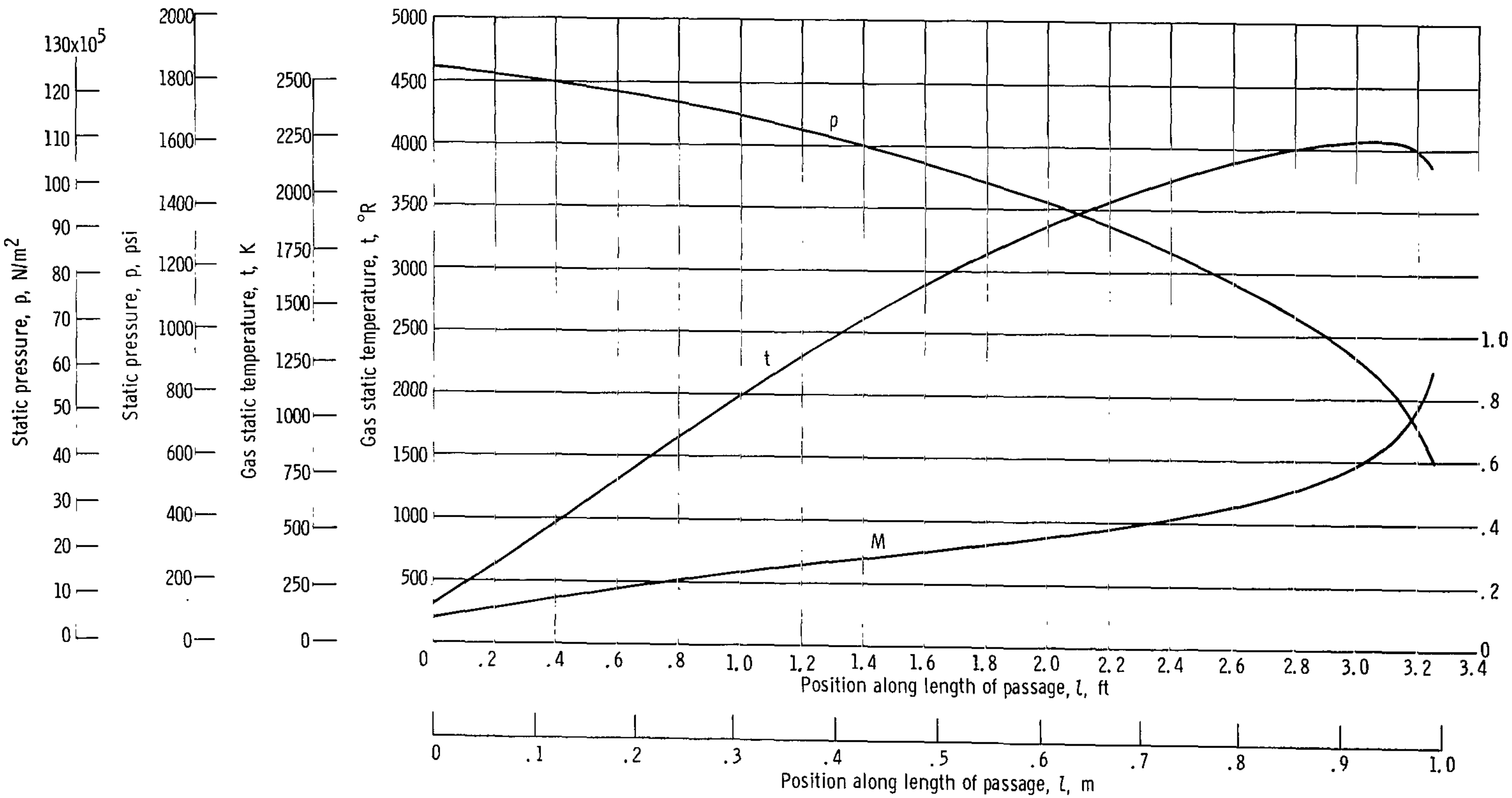


Figure 2. - Static pressure, static temperature, and Mach number as function of passage length for exit Mach number of 0.9. Mass velocity, 301 pounds mass per square foot per second (1466 kg/(m^2)(sec)); surface temperature, 5000° R (2778 K); gas, hydrogen.

TABLE I. - EXIT STATIC PRESSURE AND

STATIC-PRESSURE DROP AS FUNCTION

OF EXIT MACH NUMBER

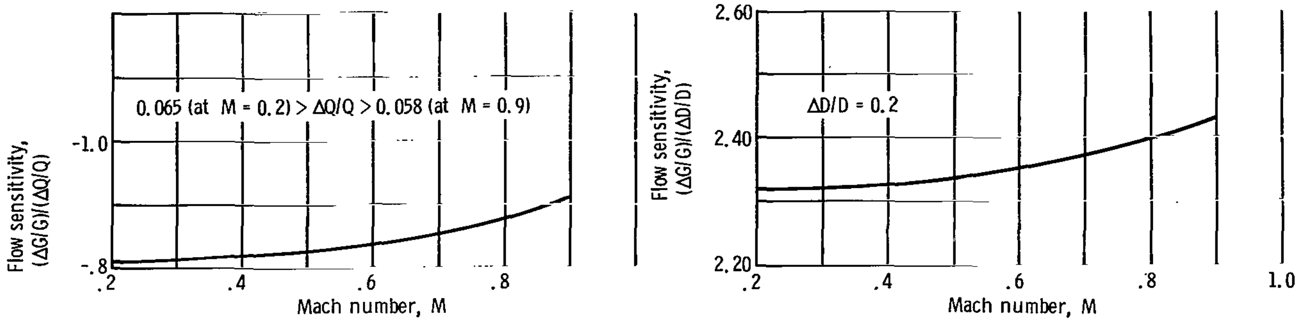
Mach number, M	Exit static pressure, p_{out}		Static-pressure drop, Δp	
	psi	kN/m^2	psi	kN/m^2
0.2	2880	19 857	400	2758
.3	1914	13 200	577	3978
.4	1427	9 839	732	5047
.5	1134	7 819	867	5978
.6	935	6 447	983	6778
.7	792	5 461	1084	7474
.8	681	4 695	1174	8095
.9	600	4 137	1246	8591

Flow Sensitivity

In figure 3 the flow sensitivity parameters $(\Delta G/G)/(\Delta Q/Q)$ and $(\Delta G/G)/(\Delta D/D)$ are plotted against Mach number. The change in flow sensitivity parameter due to an increase in heat addition as a function of Mach number is small, increasing from a value of -0.81 at Mach 0.2 to a value of -0.91 at Mach 0.9. These data are for an increase in heat addition of about 6 percent. The same effect was noted for several points calculated with an increase in heat addition of 12 percent. Thus, it appears that the percent change in mass velocity is nearly equal to the percent change in heat addition and this effect occurs regardless of Mach number. The change in flow-sensitivity parameter due to a decrease in passage diameter as a function of Mach number is also small, increasing from a value of 2.32 at Mach 0.2 to a value of 2.43 at Mach 0.9. These data are for a decrease in passage diameter of 2 percent.

The flow-sensitivity parameter does not vary greatly with Mach number because the expected increase in the ratio of momentum to friction pressure drop with increasing Mach number did not take place. The increase in this ratio did not take place due to the large density decrease of the gas from entrance to exit.

For cases where the inlet density is not much greater than the outlet density, that is, $\rho_{in}/\rho_{out} \approx 2$, any decrease in the outlet density keeping the inlet density fixed, will cause an increase in the ratio of momentum to friction pressure drop. This is noted in the fol-



(a) Sensitivity due to change in heat addition.

(b) Sensitivity due to nonuniform passage diameter.

Figure 3. - Flow sensitivity parameters as function of Mach number.

lowing equation for a given value of $f(L/D)$:

$$\Delta p = -\frac{G^2}{g_c}\left[\overbrace{f\frac{L}{D}\left(\frac{1}{\rho_{out}} + \frac{1}{\rho_{in}}\right)}^{\text{Friction}} + \overbrace{\left(\frac{1}{\rho_{out}} - \frac{1}{\rho_{in}}\right)}^{\text{Momentum}}\right] \tag{17}$$

Equation (17) was obtained by making the substitution

$$\rho = \frac{p}{Rt}$$

in equation (11). As the ratio ρ_{in}/ρ_{out} increases, the ratio of momentum to friction pressure drop increases until it reaches a point where any further increase in the density ratio will cause a very small increase in the pressure-drop ratio. For large changes in density which are typical for nuclear rockets, the term $1/\rho_{out}$ is greater than $1/\rho_{in}$ by at least a factor of 35. Thus, the term $1/\rho_{in}$ can be neglected and equation (17) takes on the following form:

$$\Delta p = -\frac{G^2}{g_c\rho_{out}}\left(f\frac{L}{D} + 1\right) \tag{18}$$

This means that the ratio between momentum and friction pressure drop is a constant equal to the ratio $1/f(L/D)$ irrespective of Mach number. Therefore, for large density changes there is no increase in momentum pressure drop relative to friction pressure drop for an increase in Mach number.

Wall-Temperature Sensitivity

In figure 4, the wall-temperature sensitivity parameters $(\Delta T_s/T_s)/(\Delta Q/Q)$ and $(\Delta T_s/T_s)/(\Delta D/D)$ are plotted against Mach number. The change in wall temperature sensitivity parameter due to an increase in heat addition as a function of Mach number is small, increasing from a value of 1.36 at a Mach number of 0.2 to a value of 1.46 at a Mach number of 0.9. These data are for an increase in heat addition of 6 percent. The change in wall temperature sensitivity due to a decrease in passage diameter as a function of Mach number is also small, increasing from a value of -3.00 at Mach 0.2 to a value of -3.22 at Mach 0.9. These data are for a decrease in passage diameter of 2 percent.

Figure 5 is a plot of surface temperature T_s with respect to passage diameter D for the reference Mach number of 0.9 and with the heat addition Q kept constant. The

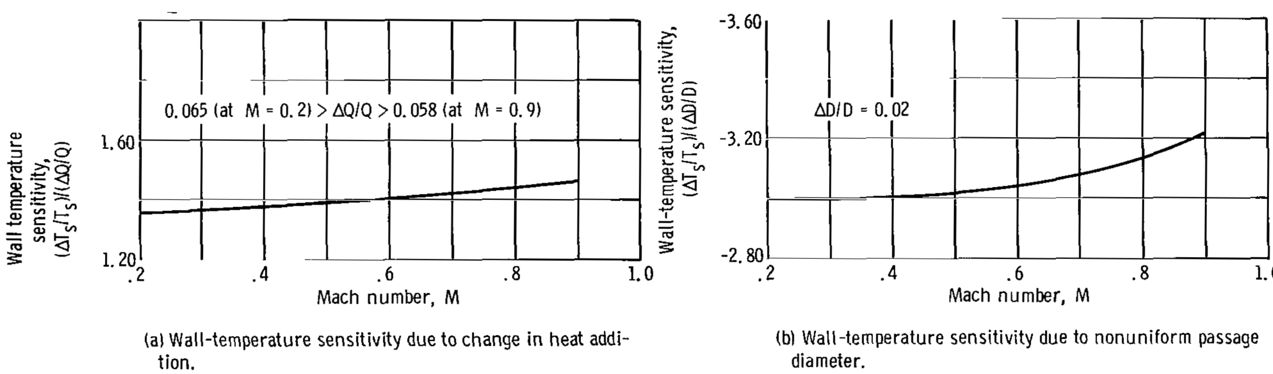


Figure 4. - Wall-temperature sensitivity parameters as function of Mach number.

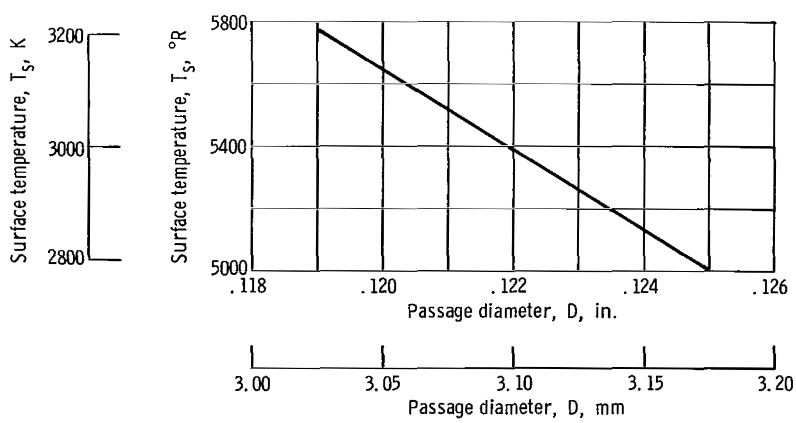


Figure 5. - Surface temperature as function of passage diameter for Mach 0.9. Constant heat addition; constant static pressure drop; gas, hydrogen.

passage surface reaches a temperature of 5500° R (3056 K), an increase of 500° R (278 K), for a decrease in diameter of 0.004 inch (0.102 mm) or a 10-percent increase in temperature for a 3-percent decrease in equivalent diameter.

Pressure Drop

In the section METHOD OF CALCULATION, the pressure drop equation, equation (12), is a good approximation to equation (5).

$$\Delta p = -\left[\left(f \frac{L}{D} + 1\right) \frac{G^2 R}{g_c} \frac{t_{out}}{p_{out}}\right] \tag{12}$$

It uses a simple algebraic relation based on end conditions, to determine changed conditions for a constant pressure drop as a substitute for a complex equation which can only be solved by numerical means. The value of t_{out} as found from equation (13) and based on equation (12), is correct to within a few degrees of the t_{out} found by the exact method using equations (3) and (7). In a check made on the approximate solution, the flow sensitivity was found to be within 1 percent of that calculated using the exact solution.

It should be noted that equation (12) was not intended to be used to evaluate pressure drop. If it were so used it would yield a maximum pressure drop error of 20 percent in the range covered by this report.

In treating the flow sensitivity problem, constant static-pressure drop across the tube passages was used as an operating condition. One may ask why total-pressure drop was not used instead. The answer is that for this particular investigation there are large changes in density which allow the term $1/\rho_{in}$ in equation (17) to be neglected. Thus, whether constant static-pressure or constant total-pressure drop is assumed across the flow passages, the flow change will be influenced in the same manner. This can be shown as follows. Express equation (12) as a pressure ratio which gives

$$\frac{p_{in}}{p_{out}} = \left[\frac{G^2 R}{g_c} \frac{t_{out}}{p_{out}^2}\left(f \frac{L}{D} + 1\right)\right] + 1 \tag{19}$$

Substituting the expression

$$M = \frac{G}{p}\sqrt{\frac{Rt}{\gamma g_c}}$$

into equation (19) yields a static-pressure ratio

$$\frac{p_{in}}{p_{out}} = \gamma M_{out}^2\left(f\frac{L}{D} + 1\right) + 1 \tag{20}$$

For a given change the pressure ratio p_{in}/p_{out} is a constant due to the condition of constant-pressure drop. The term γ is constant, and, for the case of a change in heat addition, the term $f(L/D)$ is constant too. For the case of nonuniform passage diameter, $f(L/D)$ varies slightly due to the variation in D. But because its effect is negligible on equation (20), it is considered constant. Therefore, the term M_{out} must be a constant.

Now, consider the case of a change made at a constant total-pressure drop. Using the isentropic relations for the ratios of total to static pressure and equation (20) enables the ratio of inlet to exit total pressure to be expressed as

$$\frac{\mathscr{P}_{in}}{\mathscr{P}_{out}} = \left[\gamma M_{out}^2\left(f\frac{L}{D} + 1\right) + 1\right]\left(\frac{1 + \frac{\gamma - 1}{2} M_{in}^2}{1 + \frac{\gamma - 1}{2} M_{out}^2}\right)^{\gamma/\gamma-1} \tag{21}$$

In this study the value of $M_{in} \approx 0.06$. Therefore, equation (21) can be approximated by

$$\frac{\mathscr{P}_{in}}{\mathscr{P}_{out}} = \overbrace{\left[\gamma M_{out}^2\left(f\frac{L}{D} + 1\right) + 1\right]}^{p_{in}/p_{out}}\left(\frac{1}{1 + \frac{\gamma - 1}{2} M_{out}^2}\right)^{\gamma/\gamma-1} \tag{22}$$

Now the total-pressure drop is expressed as a ratio similar to the static-pressure ratio. By definition, $\mathscr{P}_{in}/\mathscr{P}_{out}$ is constant in equation (22). The term γ is constant, and, for the case of a change in heat addition, the term $f(L/D)$ is constant too. For the case of nonuniform passage diameter, $f(L/D)$ varies slightly, but, because its effect is negligible on equation (22), it is considered constant. Therefore, the term M_{out} must be a constant. As a result, the term in the first bracket which is the static-pressure ratio p_{in}/p_{out} must also be constant. Furthermore, because $M_{in} \approx 0.06$, it may be assumed that $p_{in} = \mathscr{P}_{in}$, because at that low a Mach number, the isentropic pressure ratio is almost 1. Therefore, p_{in} is a constant and p_{out} must also remain constant. Thus, the assumption of constant total-pressure drop is equivalent to the assumption of constant static-pressure drop for the case investigated in this report.

CONCLUSIONS

A simplified parallel passage flow and wall-temperature sensitivity analysis was made for a typical nuclear-rocket reference condition. The conclusions resulting from changes in heat addition and passage diameter over a Mach number range of 0.2 to 0.9 are as follows:

1. For a change in heat addition with a constant static-pressure drop across the passage, the change in the flow and wall-temperature sensitivity parameters $(\Delta G/G)/(\Delta Q/Q)$ and $(\Delta T_s/T_s)/(\Delta Q/Q)$, respectively, are small with respect to Mach number. The flow sensitivity parameter increases from -0.81 to -0.91 and the wall-temperature sensitivity parameter increases from 1.36 to 1.46 over the Mach number range of 0.2 to 0.9. These data are for an increase in heat addition of 6 percent. This same effect takes place for increases in heat addition of up to 12 percent.

2. For nonuniform passage diameter with a constant static-pressure drop across the passage and constant heat addition, the change in the flow and wall-temperature sensitvity parameters are small with respect to Mach number. The flow sensitivity parameter increases from 2.32 to 2.43, and the wall-temperature sensitivity parameter increases from -3.00 to -3.22 over the Mach number range of 0.2 to 0.9. The equivalent diameter is decreased by 2 percent. However, there is a large increase in surface temperature for a small decrease in passage diameter. At Mach 0.9 a 3-percent decrease in diameter causes a 10-percent increase in surface temperature.

3. For large inlet-outlet density ratios (>35) a formula was developed for approximating pressure drop. This simplified pressure-drop formula is based on end conditions only. Using this simplified formula yielded a flow sensitivity within 1 percent of that calculated using the exact solution.

Lewis Research Center,
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APPENDIX A

SYMBOLS

A flow cross-sectional area, ft^2; m^2

C_p specific heat of gas at constant pressure, Btu/(lb mass)($^\circ$R); J/(kg)(K)

D passage diameter, ft; m (except where otherwise noted)

f average friction factor

G mass velocity, lb mass/(sec)(ft^2); kg/(m^2)(sec)

g_c gravitational constant, 32.2 (lb mass)(ft)/(lb force)(sec^2)

h heat-transfer coefficient, Btu/(ft^2)(sec)($^\circ$R); J/(m^2)(sec)(K)

k thermal conductivity of gas, Btu/(ft)(sec)($^\circ$R); J/(m)(sec)(K)

L length of passage, ft; m

l position along length of passage, ft; m

M Mach number

Nu Nusselt number, hD/k

P diametral perimeter of passage, ft; m

Pr Prandtl number, $C_p\mu/k$

$\mathscr{P}$ total pressure, lb force/ft^2; N/m^2

p static pressure, lb force/ft^2; N/m^2

Q rate of heat transfer to gas, Btu/sec; J/sec

q heat flux, Btu/(sec)(ft^2); J/(sec)(m^2)

R gas constant, (ft)(lb force)/(lb mass)($^\circ$R); (N)(m)/(kg)(K)

Re Reynolds number, DG/μ

T gas stagnation temperature, $^\circ$R; K

T_s surface temperature, $^\circ$R; K

t gas static temperature, $^\circ$R; K

V gas velocity, ft/sec; m/sec

γ ratio of specific heats of gas

μ absolute viscosity of gas, lb mass/(sec)(ft); (N)(sec)/m^2

ρ gas density, lb mass/ft^3; kg/m^3

Subscripts:

b bulk

in inlet

out exit

ref value at reference point

s surface

APPENDIX B

DERIVATION OF PRESSURE DROP DERIVATIVE WITH RESPECT TO TOTAL GAS TEMPERATURES

Pressure drop can be expressed as the sum of the friction and momentum pressure drops

$$dp = -4f \frac{dl}{D} \frac{\rho V^2}{2g_c} - \frac{G}{g_c} dV \tag{B1}$$

The definition of mass velocity $G = \rho V$ is substituted into equation (B1) to form the equation

$$dp = -4f \frac{dl}{D} \frac{GV}{2g_c} - \frac{G}{g_c} dV \tag{B2}$$

Using the definition of mass velocity G, and the perfect gas law

$$\rho = \frac{p}{Rt}$$

results in an expression for velocity

$$V = GR \frac{t}{p} \tag{B3}$$

The equation for total temperature

$$T = t + \frac{1}{2}\left(\frac{\gamma - 1}{\gamma}\right) \frac{G^2 R}{g_c} \frac{t^2}{p^2}$$

may be rearranged to express static temperature as

$$t = \frac{-1 + \left[1 + 2\left(\frac{\gamma - 1}{\gamma}\right) \frac{G^2R}{g_c} \frac{T}{p^2}\right]^{1/2}}{\frac{\gamma - 1}{\gamma} \frac{G^2R}{g_c} \frac{1}{p^2}}$$

Gas velocity can now be expressed in terms of mass velocity, total gas temperature, and static pressure.

$$V = \frac{GR\left\{-1 + \left[1 + 2\left(\frac{\gamma - 1}{\gamma}\right) \frac{G^2R}{g_c} \frac{T}{p^2}\right]^{1/2}\right\}}{\frac{\gamma - 1}{\gamma} \frac{G^2R}{g_c} \frac{1}{p}} \tag{B4}$$

The velocity differential with respect to the variables T and p is

$$dV = GR\left\{\frac{p}{\frac{\gamma - 1}{\gamma} \frac{G^2R}{g_c}} d\left[-1 + \left(1 + 2 \frac{\gamma - 1}{\gamma} \frac{G^2R}{g_c} \frac{T}{p^2}\right)^{1/2}\right]\right.$$

$$\left. + \left[-1 + \left(1 + 2 \frac{\gamma - 1}{\gamma} \frac{G^2R}{g_c} \frac{T}{p^2}\right)^{1/2}\right] \frac{dp}{\frac{\gamma - 1}{\gamma} \frac{G^2R}{g_c}}\right\} \tag{B5}$$

The first term in equation (B5) can be simplified to

$$d\left[-1+\left(1+2\,\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}\,\frac{T}{p^2}\right)^{1/2}\right]=\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}\left(1+2\,\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}\,\frac{T}{p^2}\right)^{-1/2}\frac{dT}{p^2}$$

$$-\,2\,\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}\left(1+2\,\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}\,\frac{T}{p^2}\right)^{-1/2}T\,\frac{dp}{p^3}$$

Substituting it back into equation (B5) gives a more simplified velocity equation.

$$dV=GR\left\{\left(1+2\,\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}\,\frac{T}{p^2}\right)^{-1/2}\frac{dT}{p}-2\left(1+2\,\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}\,\frac{T}{p^2}\right)^{-1/2}T\,\frac{dp}{p^2}\right.$$

$$\left.+\,\frac{\left[-1+\left(1+2\,\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}\,\frac{T}{p^2}\right)^{1/2}\right]dp}{\frac{\gamma-1}{\gamma}\,\frac{G^2R}{g_c}}\right\} \tag{B6}$$

Solve equation (1) for dl

$$dl=\frac{GAC_p}{hP}\,\frac{dT}{T_s-T} \tag{B7}$$

Substituting equations (B4), (B6), and (B7) into equation (B2) yields an equation for pressure drop

$$dp = \frac{-\dfrac{4fGAC_p}{hPD}\,\dfrac{dT}{T_s - T}\,\dfrac{G^2R}{2g_c}\left[-1 + \left(1 + 2\,\dfrac{\gamma - 1}{\gamma}\,\dfrac{G^2R}{g_c}\,\dfrac{T}{p^2}\right)^{1/2}\right]p}{\dfrac{\gamma - 1}{\gamma}\,\dfrac{G^2R}{g_c}}$$

$$-\frac{G^2R}{g_c}\left\{\left(1 + 2\,\frac{\gamma - 1}{\gamma}\,\frac{G^2R}{g_c}\,\frac{T}{p^2}\right)^{-1/2}\frac{dT}{p} - 2\left(1 + 2\,\frac{\gamma - 1}{\gamma}\,\frac{G^2R}{g_c}\,\frac{T}{p^2}\right)^{-1/2} T\,\frac{dp}{p^2}\right.$$

$$\left. + \frac{\left[-1 + \left(1 + 2\,\dfrac{\gamma - 1}{\gamma}\,\dfrac{G^2R}{g_c}\,\dfrac{T}{p^2}\right)^{1/2}\right]}{\dfrac{\gamma - 1}{\gamma}\,\dfrac{G^2R}{g_c}}\,dp\right\} \tag{B8}$$

In equation (B8) the following term can be simplified by making the substitutions $A = (\pi/4)D^2$ and $P = \pi D$

$$\frac{fGAC_p}{hPD} = \frac{f}{4}\,\frac{GC_p}{h} \tag{B9}$$

The Stanton number for this particular correlation (ref. 2) can be expressed as

$$\frac{h}{GC_p} = \frac{Nu}{RePr} = \frac{0.021\ Re^{0.8}Pr^{0.4}\left(\dfrac{T_s}{T}\right)^{-(0.2900+0.0019l/D)}}{RePr}$$

A turbulent friction factor

$$f = \frac{0.046}{Re^{0.2}}$$

taken from reference 6 changes equation (B9) to

$$\frac{fGAC_p}{hPD} = \frac{0.046}{4Re^{0.2}}\ \frac{RePr}{0.021\ Re^{0.8}Pr^{0.4}\left(\dfrac{T_s}{T}\right)^{-(0.2900+0.0019l/D)}}$$

$$= \frac{0.046}{0.084}\ Pr^{0.6}\left(\frac{T_s}{T}\right)^{(0.2900+0.0019l/D)}$$

Inserting this into equation (B8) gives a formula for differential-pressure drop

$$dp = -\frac{\dfrac{0.092}{0.084}\ Pr^{0.6}\left(\dfrac{T_s}{T}\right)^{(0.2900+0.0019l/D)}\ \dfrac{\dfrac{G^2R}{g_c}\left[-1+\left(1+2\dfrac{\gamma-1}{\gamma}\dfrac{G^2R}{g_c}\dfrac{T}{p^2}\right)^{1/2}\right]p}{\dfrac{\gamma-1}{\gamma}\dfrac{G^2R}{g_c}}\ \dfrac{dT}{T_s-T}-\dfrac{G^2R}{g_c}\left(1+2\dfrac{\gamma-1}{\gamma}\dfrac{G^2R}{g_c}\dfrac{T}{p^2}\right)^{-1/2}\dfrac{dT}{p}}{+2\dfrac{G^2R}{g_c}\left(1+2\dfrac{\gamma-1}{\gamma}\dfrac{G^2R}{g_c}\dfrac{T}{p^2}\right)^{-1/2}T\dfrac{dp}{p^2}-\dfrac{\dfrac{G^2R}{g_c}\left[-1+\left(1+2\dfrac{\gamma-1}{\gamma}\dfrac{G^2R}{g_c}\dfrac{T}{p^2}\right)^{1/2}\right]dp}{\dfrac{\gamma-1}{\gamma}\dfrac{G^2R}{g_c}}} \tag{B10}$$

Rearranging terms in equation (B10) yields the derivative of pressure with respect to total gas temperature

$$\frac{dp}{dT} = \frac{-\frac{0.092}{0.084}\,Pr^{0.6}\left(\frac{T_s}{T}\right)^{(0.2900+0.0019l/D)}\left(\frac{\gamma}{\gamma - 1}\right)\left\{-1 + \left[1 + 2\left(\frac{\gamma - 1}{\gamma}\right)\frac{G^2R}{g_c}\frac{T}{p^2}\right]^{1/2}\right\}\frac{p}{T_s - T} - \frac{G^2R}{g_c}\left[1 + 2\left(\frac{\gamma - 1}{\gamma}\right)\frac{G^2R}{g_c}\frac{T}{p^2}\right]^{-1/2}\frac{1}{p}}{1 - 2\frac{G^2R}{g_c}\left[1 + 2\left(\frac{\gamma - 1}{\gamma}\right)\frac{G^2R}{g_c}\frac{T}{p^2}\right]^{-1/2}\frac{T}{p^2} + \frac{\gamma}{\gamma - 1}\left\{-1 + \left[1 + 2\left(\frac{\gamma - 1}{\gamma}\right)\frac{G^2R}{g_c}\frac{T}{p^2}\right]^{1/2}\right\}} \tag{B11}$$

Equation (B11) can be further simplified by the substitution

$$1 + 2\left(\frac{\gamma - 1}{\gamma}\right)\frac{G^2R}{g_c}\frac{T}{p^2} = E$$

The derivative of static pressure with respect to total gas temperature is

$$\frac{dp}{dT} = \frac{-\frac{0.092}{0.084}\,Pr^{0.6}\left(\frac{T_s}{T}\right)^{(0.2900+0.0019l/D)}\left(\frac{\gamma}{\gamma - 1}\right)(-1 + E^{1/2})\frac{p}{T_s - T} - \frac{G^2R}{g_c}(E^{-1/2})\frac{1}{p}}{1 - 2\frac{G^2R}{g_c}(E^{-1/2})\frac{T}{p^2} + \frac{\gamma}{\gamma - 1}(-1 + E^{1/2})}$$

The relation between T and l is known from the numerical integration of equation (3) of the text.

APPENDIX C

DIFFERENCES IN HEAT-TRANSFER CORRELATIONS

Heat-transfer calculations can be no better than the correlation equation which is used. One difficulty is that of choosing a heat-transfer correlation that best represents the conditions under investigation. Different heat-transfer correlations can markedly affect reference design conditions. As an example, a correlation based on properties evaluated at bulk temperature with an l/D effect

$$Nu_b = \frac{h_b D}{k_b} = 0.021\left(\frac{GD}{\mu_b}\right)^{0.8} Pr_b^{0.4}\left(\frac{T_s}{T}\right)^{-(0.2900+0.0019l/D)} \tag{C1}$$

is compared with a correlation based on properties evaluated at surface temperature

$$Nu_s = \frac{h_s D}{k_s} = 0.021\, Re_s^{0.8} Pr_s^{0.4}\left(\frac{T_s}{T}\right)^{-0.8} \tag{C2}$$

For typical reactor operating conditions, keeping tube length, equivalent diameter, temperature and pressure levels the same, a bulk correlation will give a flow rate almost twice as large as that of a surface correlation.

Studying the two correlations as they appear in the following two heat balances explains this difference. The correlation which evaluates properties at the bulk temperature and has a variable exponent with length in the T_s/T term is

$$G^{0.2}\left[\frac{\left(\frac{D}{\mu}\right)_b^{0.2} Pr_b^{0.6}}{0.084}\left(\frac{T_s}{T}\right)^{(0.2900+0.0019l/D)} \frac{dT}{T_s - T}\right] = \frac{dl}{D} \tag{C3}$$

The correlation which evaluates properties at the surface temperature and has a constant exponent in the T_s/T term is

$$G^{0.2}\left[\frac{\left(\frac{D}{\mu}\right)_s^{0.2} Pr_s^{0.6}}{0.084}\left(\frac{T_s}{T}\right)^{0.8}\frac{dT}{T_s - T}\right] = \frac{dl}{D} \tag{C4}$$

The left-hand differential terms in equations (C3) and (C4) are equal to the same increment dl/D. The bracketed term in equation (C4) is larger than that of equation (C3). This occurs because the exponent in equation (C4) is a constant equal to 0.8, while the exponent in equation (C3) is a variable whose lowest value is 0.29 and only exceeds 0.8 near the end of the tube. Because both left-hand terms are equal to the same increment dl/D, the mass flow rate in the bulk correlation (eq. (C3)) is greater than that in the surface correlation (eq. (C4)). It must be remembered that small changes in $(GD/\mu)^{0.2}$ cause large changes in GD/μ.

The following table compares the results of the two:

Correlation	$(GD/\mu)^{0.2}$	GD/μ	Mass velocity, G	
			lb mass/(ft^2)(sec)	kg/(m^2)(sec)
Bulk	10.4	122 000	301	1466
Surface	9.1	62 400	171	833

A bulk correlation will have a 14-percent increase in $(GD/\mu)^{0.2}$ which causes a 76-percent increase in flow rate.

The bulk correlation with the variable exponent, modified Dalle Donne correlation, was used because it offers the best agreement with experimental heat-transfer data (see ref. 2).

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