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ANALYSIS OF A STAR-FIELD MAPPING TECHNIQUE FOR USE IN DETERMINING THE ATTITUDE OF A SPIN-STABILIZED SPACECRAFT

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ANALYSIS OF A STAR-FIELD MAPPING TECHNIQUE FOR USE IN DETERMINING THE ATTITUDE OF A SPIN-STABILIZED SPACECRAFT*

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SUMMARY

In certain space experiments precise knowledge of the orientation of a spin-stabilized spacecraft is required. This requirement has resulted in the development of a unique technique of star mapping herein discussed.

The star mapping is accomplished by optically scanning a band of the star field about the vehicle's equator. Determination of the orientation of the vehicle's spin axis and roll angle with respect to the celestial sphere is accomplished by cross correlation of the scanned star map with a known reference map of the celestial sphere.

The spinning motion of the vehicle causes star images to pass over a reticle which is configured to generate two groups of coded pulses at the output of the sensor. The amplitudes of the signals out of the optical system will be proportional to the spectral radiance of the scanned stars, and thus classification of stars according to their visual magnitude is possible. The time of the occurrence of the two pulse groups is related to the azimuth angle of the star, and the time separation of the pulse groups is related to the star's elevation angle.

This paper includes a description of the system configuration and system environment. The parameters which limit the performance characteristics of the system are discussed. A typical design is considered and estimated performance characteristics are presented.

INTRODUCTION

Certain space experiments which utilize spin-stabilized spacecraft require precise knowledge of the spacecraft attitude angles. A promising approach to this problem is to

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determine the attitude of the vehicle with respect to the celestial sphere. This paper describes a unique technique for this type of attitude determination in which a strip map of the star field surrounding the vehicle's equator is generated by a passive star telescope that scans the celestial sphere as the vehicle spins. Attitude angles can then be determined by correlating the generated star map with known stellar positions. By using this type of star-field correlation, attitude-angle measurements can be made with as few as two identifiable stars for a body spinning about a principal axis.

SYMBOLS

A effective telescope aperture, centimeters^2

d azimuthal angular width of a group of coded slits

f(m) a stellar constant approximated by 2.5^{-m}

G photomultiplier sensitivity, amperes/lumen

i number of agreements in M noise sample pairs

j number of agreements in M signal sample pairs

K_1 a stellar constant for a star of zero magnitude, 2.1×10^{-10} $\text{lumens/centimeter}^2$

K_2 a constant, $\frac{1}{1.6 \times 10^{-19}}$ electrons/ampere-second

K_3 equivalent number of tenth-magnitude stars per square degree of the effective field of view of the telescope

L code length

M number of 1's or 0's in a code sequence

m stellar visual magnitude

m_1 visual magnitude of the minimum detectable star

N total number of stars in a selected 6° by 360° scan of the celestial sphere

$\overline{N}_B$	average number of electrons produced per unit sample time by background
$\overline{N}_T$	average number of electrons produced per unit sample time by phototube dark current
n	equivalent number of reticle transparent openings of width α
P(m)	probability of occurrence of a star of visual magnitude m
$P(r \geqq T_2)$	probability of detection in the case where a signal is present and probability of false alarm in the case where noise only is present for any star
$P(\overline{S}_T \geqq T_1)$	probability of accepting the signal level of any given sample at the amplitude selector
$P_m(r \geqq T_2)$	probability of detection in the case where a signal is present and probability of false alarm in the case where noise only is present for a star of magnitude m
$P_R(m)$	system probability of response
P_{00}	probability that a noise-only sample will result in a zero
P_{11}	probability that a sample containing signal plus noise will result in a value of 1
$p_{00}(i,M)$	probability of i agreements
$p_{11}(j,M)$	probability of j agreements
R	system figure of merit
r	number of agreements in a correlation of a given code sequence
$\overline{S}$	average number of electrons produced per unit sample time by a source star
$\overline{S}_T$	total average number of electrons produced per unit sample time
T_1	amplitude detection threshold

T_2	correlator detection threshold
t_i	time of occurrence of pulse groups ($i = 1, \ldots, k$)
Δt_i	time lapse between a pair of pulse groups ($i = 1, \ldots, k$)
Z	a random variable with zero mean and standard deviation of 1
Z_1	normalized value of T_1 in presence of noise only
Z_2	normalized value of T_1 in presence of signal and noise
α	width of a reticle transparent slit, degrees
α/ω	unit sample time or observation time, seconds
β	the angle between the two groups of coded slits, degrees
ϵ	vertical field of view, degrees
η	equivalent tube noise, lumens/second$^{1/2}$
ω	vehicle spin rate, degrees/second

PRINCIPLE OF OPERATION

The vehicle under consideration is a spin-stabilized probe. The basic star scanner consists of a telescope, a reticle, and a photomultiplier tube. (See fig. 1.) The telescope is mounted in the probe with the optical axis nominally normal to the spin axis of the vehicle. The reticle is centered in the focal plane of the telescope and the photomultiplier is mounted behind the reticle. The reticle is opaque with two groups of transparent slits. The slits of one group are parallel to the spin axis of the vehicle and the second group is placed at a known angle to this axis. As indicated in figure 1, the spinning motion of the vehicle causes the star image to pass over the reticle, and consequently the input to the photomultiplier will be a series of coded pulses of radiant energy for each group of slits. The number of pulses is dependent upon the number of transparent openings of the reticle. The code possesses special autocorrelation features by which synchronization may be obtained. This pulse coding principle can provide unambiguous indications of a star crossing. The time of occurrence of the two pulse groups provides a

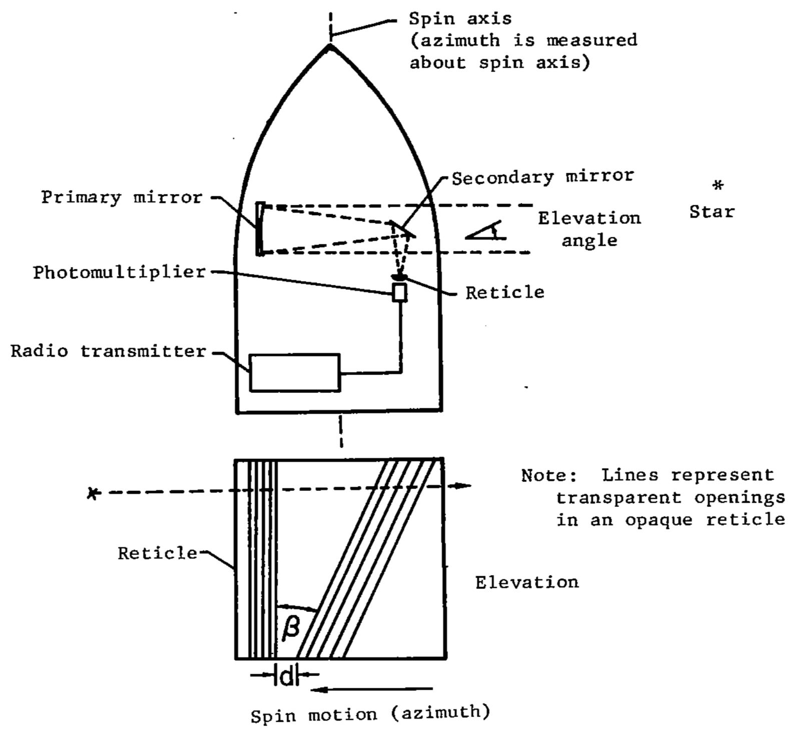

Figure 1.- Simplified drawing of star mapper.

measure of the azimuth angle to the star, and the time separation of a single pair of pulse groups provides a measure of the elevation angle to the star. The amplitude of the pulses of a particular pulse group provides a measure of the intensity of the scanned star. The output of the photomultiplier tube is telemetered to a ground station for data processing, by which the attitude of the vehicle is determined.

System Description

A diagram of the system is shown in figure 2. The star signal which is scanned by the telescope is dependent upon the spectral radiant intensity of the star. Telescope parameters that affect this signal are field of view, which determines whether or not the star is viewed; aperture of the telescope; spectral characteristics, which limit the spectrum of acceptable energy; and efficiency of the optics, which determines the efficiency of transfer of energy to the focal plane of the telescope. The combination of the reticle and vehicle spin causes the signal at the detector to appear as a sampled signal. The signal as sensed by the photomultiplier is also a function of the spectral response characteristics and gain of the photomultiplier tube. The airborne electronics operates on the signal with a gain compatible with transmitter requirements and a band pass compatible with the sampling frequency produced by the combination of reticle and vehicle spin.

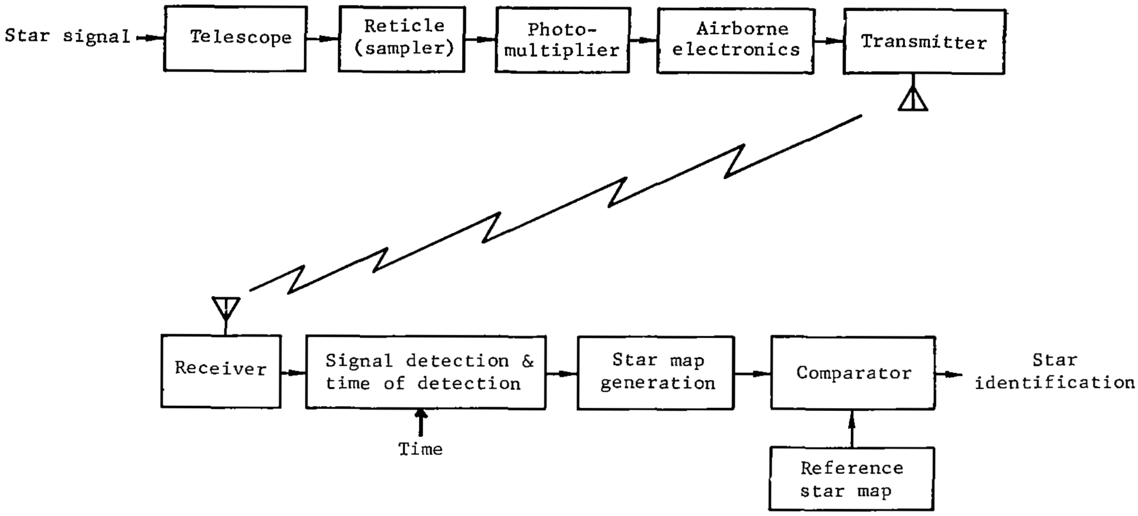

Figure 2.- Simplified block diagram of system.

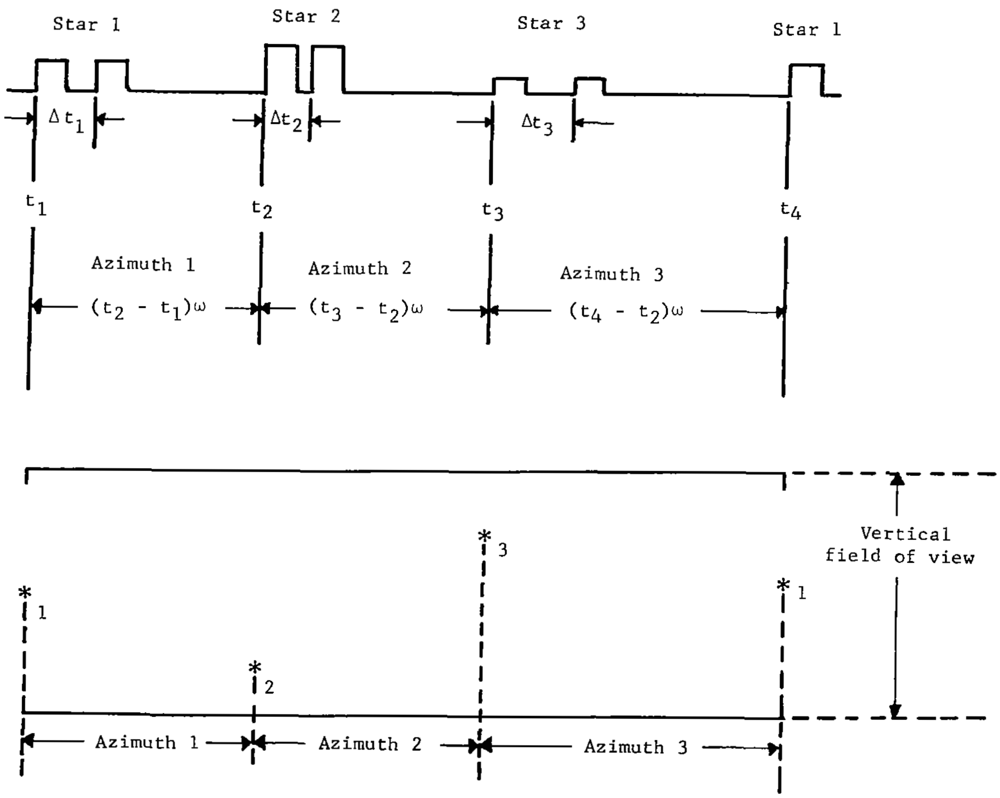

Figure 3.- Incoming signal and computed star map.

In the ground data-handling equipment the signal amplitude is compared with a variable threshold value for signal detection and classification purposes. This classification will also include time of occurrence of the star scan and time elapsed between repeated scans of the same star in a complete revolution of the vehicle. The elevation angle to any scanned star may be determined from the elapsed time between successive pulse groups produced by the star. These times may be converted to angular measurements through a knowledge of the vehicle spin rate and the geometry of the reticle. This time-angle relationship is illustrated in figure 3. The time of occurrence of a pair of pulse groups is indicated by t_1, t_2, . . . t_k, the time lapse between a pair of pulse groups is indicated by Δt_1, Δt_2, . . . Δt_k, and vehicle spin rate is noted as ω. The azimuth angle between two successive stars is defined as $\left(t_{k+1} - t_k\right)\omega$ degrees and the elevation angle of a particular star in the field of view is approximated as $\left(\omega \Delta t_k - 2d\right)\tan(90^{\circ} - \beta)$ degrees.

Data Processing

The time of occurrence of a pulse group generated by a scanned star is determined through the process shown in figure 4. The signals from the receiver pass through the variable-threshold circuit which produces a star presence or star absence signal. This binary signal is then compared with a reference code (a replica of the reticle code) in the decoder. When the proper code sequence is recognized, the time of this recognition is recorded in digital form, and the pulse amplitude is also recorded by activating the analog-to-digital converter at the time of pulse-group recognition. Adjustment of the threshold provides a convenient way of restricting the response of the system to a few of the brighter stars in the scanned field. Figure 3 illustrates the computation necessary to convert the digital record to a star map that has an azimuth dimension of 360° and is

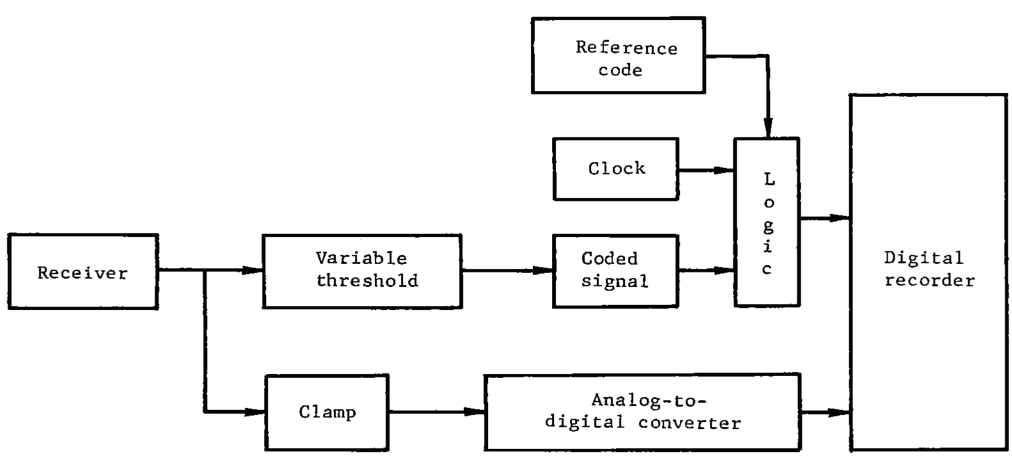

Figure 4.- Data-processing block diagram.

bounded in elevation by the vertical field of view of the telescope. The result of this data processing is a map of the scanned star field with classification of the magnitude of the stars and a measure of their azimuth and elevation angles in the scanned field of view.

Data Interpretation

The generated star map represents a two-dimensional description of the orientation of the field of view of the star mapper in the celestial sphere. The orientation of this field of view and the direction of the vehicle spin axis may be determined by a two-dimensional cross correlation of the generated star map with a reference star map. To minimize processing time, the reference map may be constructed to cover the portion of the celestial sphere that is expected to be scanned during the nominal vehicle trajectory. This reference map must be large enough to accommodate the field of view of the telescope and any coning or nutation about the vehicle spin axis due to unbalance of moments of inertia of the vehicle about the pitch and yaw axes. Nutation can cause considerable difficulty in the interpretation of the data, and a greater number of star sightings may be required for complete attitude determination if nutation amplitudes are appreciable.

SENSITIVITY AND NOISE CONSIDERATION

The performance of the system is best described by its ability to detect and identify an adequate number of stars and by the accuracy with which the orientation of the resulting star field is determined with respect to the field of view of the telescope. The orientation accuracy is a function of angular resolution of an identified star, whereas the identification problem is a function of signal detectability and the number of available stars.

Desired Characteristics

For positive identification of a scanned star field it is necessary to insure that an adequate number of detectable stars will be scanned in a given star field. Figure 5 represents the approximate number of stars brighter than a given magnitude, in a field of view of 1 square degree (ref. 1). Consideration of the requirement of at least two identifiable stars and the knowledge that the star field is 360° in the azimuth direction can lead to a reasonable selection of the optical field of view. For example, if a 6° vertical field of view is selected, the number of stars brighter than a given magnitude in the resultant scanned star field of 2160 square degrees is as shown in figure 6. It can be seen from this figure that in order to encounter at least two stars per vehicle revolution, stars with a visual magnitude of +3 or brighter must be sensed. Ideally the system would be designed to detect, or accept, all scanned stars brighter than the selected magnitude and sharply reject stars which are dimmer than this magnitude. This approach would insure

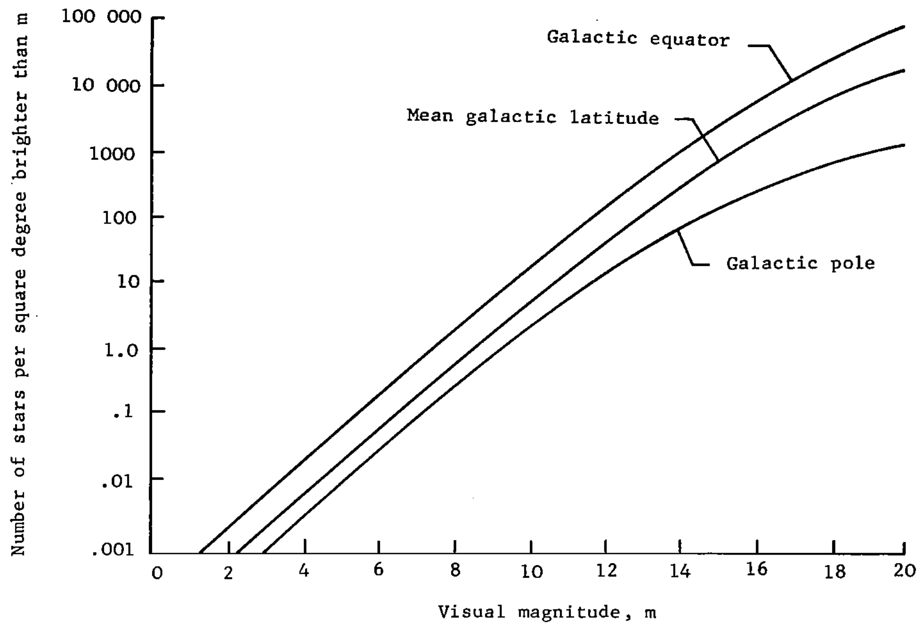

Figure 5.- Spatial density of stars.

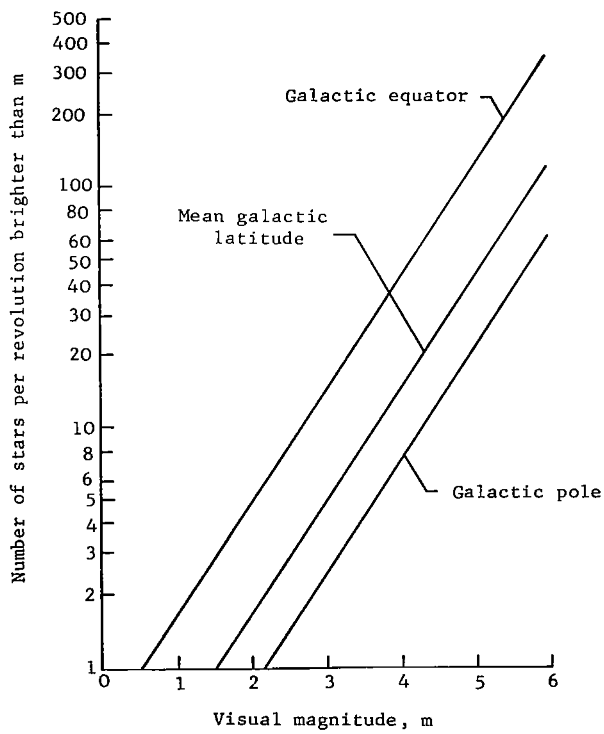

Figure 6.- Spatial density of stars per revolution of a 6^0 vertical field of view.

detection of an adequate number of stars for attitude determination, reduce the number of false alarms due to system noise, and reduce the number of candidate stars considered in the identification of the stars in the scanned star field.

Telescope and Detector Characteristics

A star signal as generated by the photomultiplier is dependent upon the spectral radiant intensity of the star source and the spectral characteristics of the optics and photomultiplier. The spectral radiant intensities of star sources were determined by using source-temperature estimates and assuming black-body spectra for stars with a visual magnitude of +3 or brighter. A study was performed to determine representative source temperatures for the visual magnitudes of interest. The results of this study indicated that the majority of these stars have source temperatures of 6000° K and higher (refs. 1 and 2). In addition the "average" source temperature of the myriad of stars which make up the sky background is 6000° K or lower (refs. 1 and 3). The representative source temperatures clearly indicated that the peak spectral response of the optics and photomultiplier should lie toward the short-wavelength region relative to the visual spectrum. Selection of the telescope aperture and photomultiplier tube gain is not discussed here, but nominal values required for suitable signal amplification are assumed.

Signal and Noise Characteristics

The principal problem involved in a system of this type is signal detection in the presence of system and external noises. Before proceeding to the problem of signal detection it is of interest to examine the characteristics of signal and noise as sensed by the photomultiplier.

Noise sources.- The noise sources considered in this paper are: external noise due mainly to scattered and direct starlight, and galactic light; noise of the photomultiplier tube; and electronic noise of the system. The noise due to starlight is considered to originate from the dimmer stars, that is, the nonprominent stars. Galactic light is emitted starlight or starlight scattered by interstellar dust in the Milky Way. This external light is considered to be of a diffuse nature and hence the noise as sensed by the photomultiplier is dependent upon the effective field of view of the telescope. Effective field of view is defined here as the total clear opening of the reticle in the focal plane. The photomultiplier tube is a quantum detector, sensitive to the rate of interception of light quanta arriving at the photocathode rather than rate of energy arrival or power. For most photomultipliers the quantum efficiency varies continuously over the spectral response region of the tube. Since the probability of emission of an electron, or quantum efficiency, is a function of frequency of the exciting photon, any enhancement of photon noise by the photocathode should be calculated by an integral for other than monochromatic light.

Since the release of electrons from the photocathode is related to the quantum efficiency or to a statistical process, it is important to establish the type of distribution associated with this photocathode current. The Poisson distribution agrees with experimental results and is generally assumed. By definition of the system inputs, the optical parameters, and the photomultiplier tube, signal detectability may be examined in terms of the number of electrons due to signal and noise at the photocathode. The noise considered here will be background noise, photomultiplier dark-current noise, and signal-induced noise. In general, noise sources such as the electronic noise of the airborne equipment and radiofrequency noise are negligible as compared with the other noise sources mentioned here.

Background noise.- Stellar background pertains to the quantity and quality of the stellar irradiation. Quantity refers to the number of stars and their radiance. The quality of irradiation refers to the spectral distribution of the stellar radiance. Since the total number of dim stars is immeasurable, the irradiance must be discussed in terms of the distributed radiance of the celestial sphere. The purpose of this discussion is to describe the background noise that would affect detection of a given star. The background light as viewed by the telescope may be considered as an equivalent signal in terms of total integrated starlight expressed in effective number of tenth-magnitude stars per square degree of effective field of view (refs. 1 and 3). This equivalent signal may be handled in a manner similar to the star signal to determine the effective noise level as sensed by the photomultiplier.

Signal and noise parameters.- In the determination of the number of photocathode electrons (due to signal and noise) which are produced in a given interval of time, certain system parameters must be defined. The following three equations (ref. 4) express the average number of electrons per unit sample time for signal $\overline{S}$, background $\overline{N}_B$, and dark current $\overline{N}_\tau$, respectively:

$$\overline{S} = K_1 K_2 A G \frac{\alpha}{\omega} f(m) \tag{1}$$

$$\overline{N}_B = K_1 K_2 K_3 A G \frac{\alpha^2}{\omega} \epsilon n \tag{2}$$

$$\overline{N}_\tau = \frac{1}{2}\left(K_2 \eta G\right)^2 \frac{\alpha}{\omega} \tag{3}$$

The total expected signal in a unit sample time is defined as

$$\overline{S}_T = \overline{S} + \overline{N}_B + \overline{N}_\tau \tag{4}$$

Coding and Signal Detection

This discussion considers the enhancement of detection through the use of multiple transparent slits in the reticle, where each slit will have a width defined by the angular resolution requirement of the system. By arrangement of the slits in a psuedo-random pattern, this technique may be extended to provide noise suppression. The output of the photomultiplier will now be a pulse-coded modulation of the star signal. The nature of this signal is a series of pulses as generated by the passage of the star image over the pair of coded slit groups in the focal plane of the optical system. The amplitude of these pulses, as discussed earlier, is a function of the brightness of the star and the response of the optical system. The widths of these pulses are determined by the angular width of the transparent slits of the reticle and the spin rate of the vehicle. The principle used here is to correlate a reference code (identical to the code of the reticle) with the output of the photomultiplier. When the correlation process indicates sufficient agreement between the reference code and the coded signal, there will be an indication of signal detection with a confidence level determined by the acceptable level of correlation. This feature can provide unambiguous determination of the times of occurrence of the coded star signals to the resolution afforded by the optical system.

An illustration of the signal-detection technique is shown in figure 7. In this figure the sampler represents the pulse-coded modulation effect of the coded reticle and the spinning vehicle. The function of the amplitude selector and T_1 (first threshold) is to select or gate signals which are greater than a preselected value. The function of the

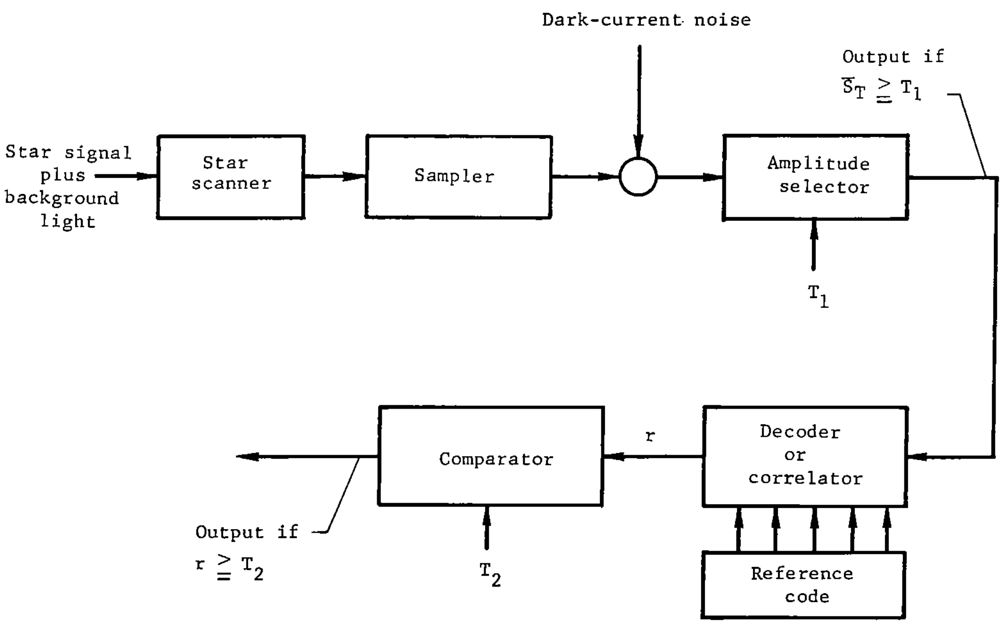

Figure 7.- Block diagram of detection process.

decoder or correlator is to compare a reference code (a binary replica of the reticle code) with the coded signal. The symbol r in this figure represents the number of agreements in a given code sequence. The function of the comparator and T_2 (second threshold) is to act as a gate to permit acceptance of a preselected correlation or agreement count as an indication of a signal representing a star of a given magnitude or brighter.

If the characteristics of the signal and noise are known at the input to the amplitude selector and the first threshold is defined, the probability $P\left(\overline{S}_T \geqq T_1\right)$ of accepting the signal level of any given sample at the amplitude selector may be determined. With this probability determined, the probability of the decoder threshold being exceeded, $P\left(r \geqq T_2\right)$, may be computed.

In the detection technique under consideration a reference code of L elements is assumed and the code is defined as a two-level binary function. The code is further defined to contain an equal number of elements of each state, such as M ones and M zeros. The time duration of a single element is determined by the sampling interval. In the preparation of the coded signal for the decoding process the amplitude selector and the first threshold convert the signal to two levels; that is, each signal level in a sample interval exceeding T_1 will be designated as a 1, and otherwise the signal level of that particular sample interval will be a 0.

<u>Degree of correlation of a single sample pair</u>.- The distribution of the number of electrons in a sample interval has previously been described as a Poisson distribution but for ease of analysis the distribution hereafter will be treated as a normal distribution, with the first- and second-order statistics being those of the Poisson distribution. Inaccuracies introduced by this assumption will be considered negligible so long as the expected number of electrons per unit sample is 10 or greater. The assumed amplitude distributions for the noise-only and signal-plus-noise samples are illustrated in figure 8. With the aid of the normalized standard variate Z, the probability that a noise-only sample will result in a 0 is

$$P_{00} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_1} e^{-Z^2/2}\, dZ \tag{5}$$

where

$$Z_1 = \frac{T_1 - \left(\overline{N}_B + \overline{N}_\tau\right)}{\sqrt{\overline{N}_B + \overline{N}_\tau}} \tag{6}$$

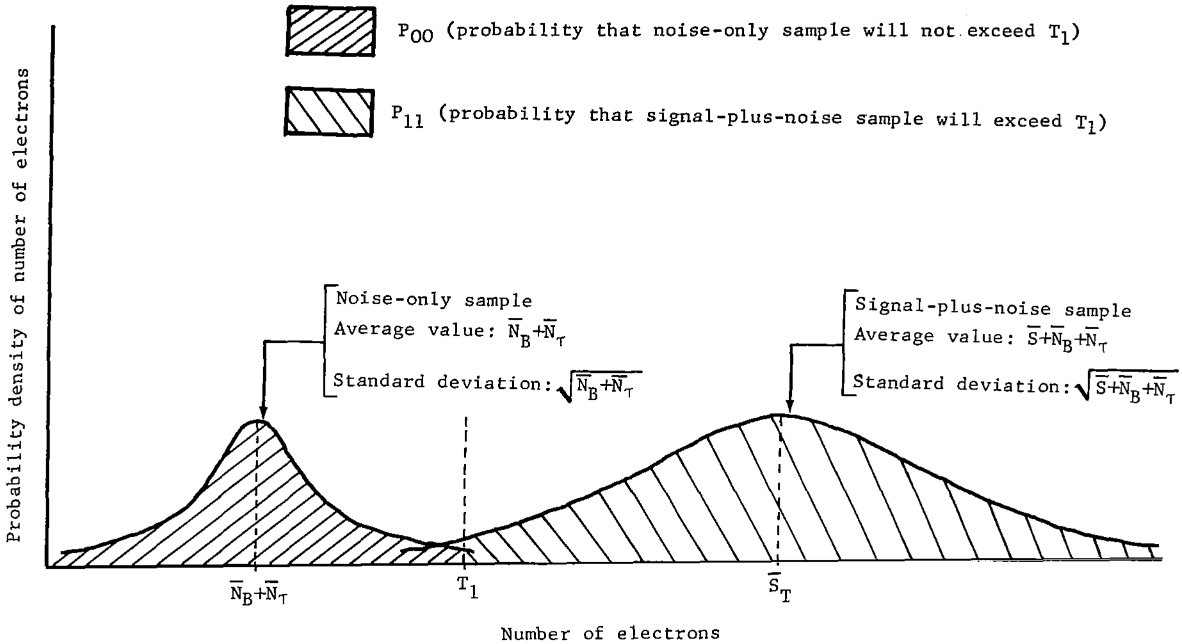

Figure 8.- Assumed electron distribution for noise-only and signal-plus-noise samples. (Each case is assumed to be normally distributed.)

Similarly the probability that a sample containing signal plus noise will result in a value of 1 is

$$P_{11} = \frac{1}{\sqrt{2\pi}} \int_{Z_2}^{\infty} e^{-Z^2/2} \, dZ \tag{7}$$

where

$$Z_2 = \frac{T_1 - \bar{S}_T}{\sqrt{\bar{S}_T}} \tag{8}$$

Now P_{00} and P_{11} are defined as the probability of agreement of a single sample pair of noise and signal samples, respectively.

Degree of correlation of M sample pairs and $P\left(r \geqq T_2\right)$.- It is now assumed that each sample of the photomultiplier signal is independent of any other sample. With this assumption, the distribution of successive correlations of M sample pairs is described by a binomial distribution. The probability of i agreements in M sample pairs representing noise-only samples is

$$p_{00}(i,M) = \frac{M!}{(M - i)!i!} P_{00}^{\ i}\left(1 - P_{00}\right)^{M-i} \tag{9}$$

Similarly, the probability of j agreements out of M sample pairs representing signal plus noise is

$$p_{11}(j,M) = \frac{M!}{(M - j)!j!} P_{11}{}^{j}\left(1 - P_{11}\right)^{M-j} \tag{10}$$

The sum of noise pair agreements and signal pair agreements is defined as

$$r = i + j \tag{11}$$

The probability that r will be greater than or equal to some preselected number of total agreements may be determined from

$$P\left(r \geqq T_2\right) = 1 - P\left(r < T_2\right) = 1 - \sum_{i+j=0}^{T_2-1}\left[\sum_{i=0}^{M} p_{00}(i,M) \sum_{j=0}^{M} p_{11}(j,M)\right] \tag{12}$$

where T_2 is the acceptable level of agreements.

Once $P\left(r \geqq T_2\right)$ has been determined for a range of values of star magnitude, for a range of code lengths, and for assumed noise levels, this detection parameter may be combined with the probability of existence of star magnitudes in a single scan of the vertical field of view. This combined or joint probability function is defined here as the system probability of response $P_R(m)$ to all stars in a single scan. A system figure of merit used in this study is

$$R = \frac{\int_{m_1}^{-\infty} P_R(m)dm}{\int_{\infty}^{-\infty} P_R(m)dm} \tag{13}$$

which is a measure of the system response to stars brighter than m_1 compared with the system response to all star magnitudes. Ideally this parameter would be equal to 1.

This discussion was based upon several assumptions which are listed here in summary:

(a) The azimuth and elevation code groups are independent, but background noise is a function of the open slits in both code groups.

(b) The sample intervals of each code group are independent.

(c) The electrons produced at the cathode of the photomultiplier constitute a Poisson distribution.

(d) The shape of the spectral response of the photomultiplier is that of the standard eye.

(e) Signal and noise from all sources are independent random variables.

(f) Electronic noise such as Johnson noise is negligible compared with background noise, dark-current noise, and signal-dependent noise.

(g) The probability that more than one star of magnitude brighter than m_1 (threshold magnitude) will occur simultaneously in the telescope field of view is negligible.

Advantages and disadvantages of coding.- The advantages of coding lie principally in the enhancement of detection capability through multiple signal pulses for a given star image. The described technique will provide this advantage and yet retain the precision of angular resolution of a single-pulse indication of a star image.

However, as code length is increased the system noise will increase proportionally, since the effective background noise as sensed by the photomultiplier is proportional to the effective field of view, which in turn is proportional to the number of transparent coded openings in the reticle. The optimum number of code elements for a given system is dependent upon the noises of the system and the desired minimum detectable star magnitude.

A second disadvantage in coding results from using very short code lengths. This disadvantage is the inability to design an unambiguous code group, that is, a code group which will result in a cross-correlation function with a uniquely defined peak. The question of desirability of coding in a given system must be answered through an analysis which includes the particular system parameters and environment.

PERFORMANCE ESTIMATES OF A SPECIFIC SYSTEM

Assumed System Parameters and Noise Conditions

A sample design is presented here to demonstrate the effects of increasing code lengths on system performance. Certain assumed noise conditions and fixed system parameters are listed below:

(a) Telescope aperture is 40 centimeters^2

(b) Telescope field of view is 6° by 6°

(c) Field of view of a basic transparent slit of the reticle is 6° by 0.025°

(d) Luminous sensitivity of photomultiplier cathode is 25 microamperes/lumen

(e) Spectral response shape of photomultiplier is that of a standard eye

(f) Photomultiplier dark current (equivalent noise input) is 5×10^{-13} lumens/second$^{1/2}$

(g) Spin rate of vehicle is 90°/second

(h) Star density is assumed to be that which would result from a 6° by 360° scan of the mean galactic latitude

(i) Background light is the equivalent of 160, 500, and 1000 tenth-magnitude stars per square degree

In the construction of the signal and noise models the expected values of signal and noise photoelectrons were computed as a function of the assumed system parameters by use of equations (1) to (4). As previously stated, the distribution of these electrons is considered to be a Poisson distribution. Visual magnitudes ranging from zero to six were assumed for the star signals. Code lengths were varied from two to 24 elements.

In the calculations T_2 was set at 0.75L for all code lengths except $L = 2$. For this value of code length, T_2 was set at 1. For each calculation the value of T_1 used for each noise case was the value that would result in a 90-percent probability that 50 percent or more of all signal-plus-noise samples in each code group would exceed this threshold value with the signal assumed to be a star of +3 visual magnitude. For $L = 2$, the percentage was set at 100 rather than 50, since a single sample must be detected. With this consideration, values of P_{11} were determined from equation (10) for various code lengths by using the three assumed background noise conditions and a signal representing a third-magnitude star. From these values of P_{11} the required values of T_1 were determined by use of equation (7).

By using these first threshold values, P_{11} may be determined for any signal-plus-noise combination and likewise P_{00} may be determined for any noise-only condition. After P_{11} and P_{00} have been determined for a set of signals and an assumed noise case, $P(r \geqq T_2)$ may be determined from equation (12), which is repeated here:

$$P(r \geqq T_2) = 1 - \sum_{i+j=0}^{T_2-1}\left[\sum_{i=0}^{M} p_{00}(i,M) \sum_{j=0}^{M} p_{11}(j,M)\right]$$

A representative plot of $P(r \geqq T_2)$ versus signal level (star visual magnitude) is shown in figure 9 for various code lengths and an assumed background noise of 160 tenth-magnitude stars per square degree.

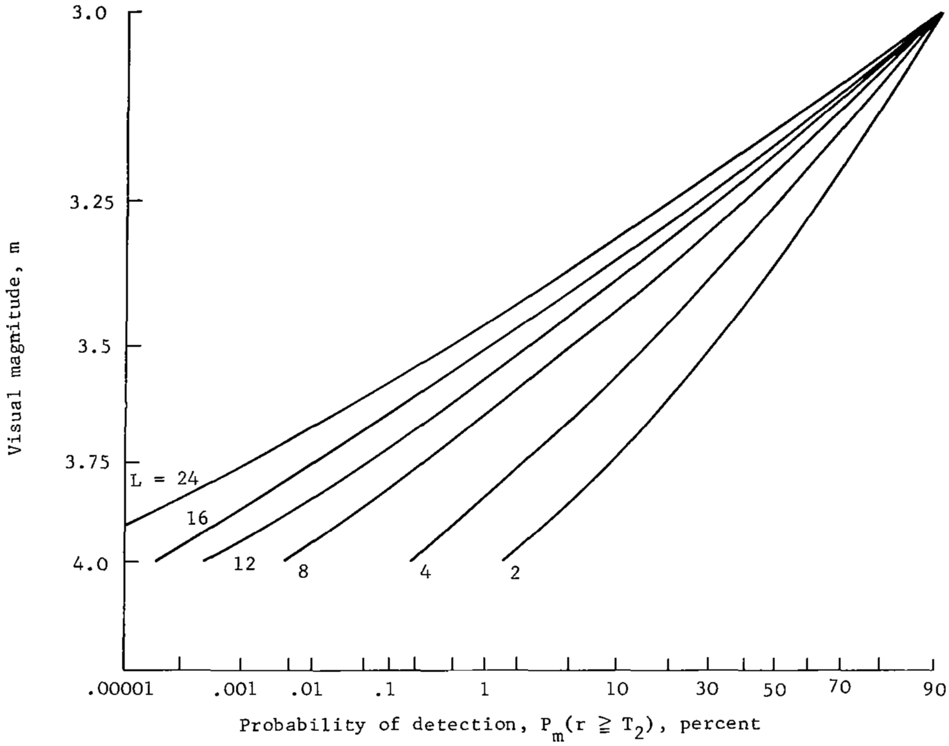

Figure 9.- Probability of detection as a function of visual magnitude. (Background of 160 tenth-magnitude stars per square degree.)

System Probability of Response $P_R(m)$

System probability of response $P_R(m)$ may be determined by a knowledge of $P_m\left(r \geqq T_2\right)$ for various star magnitudes and a knowledge of the probability of occurrence of a star of a given magnitude $P(m)$. For a scan of 6° by 360° the probability of occurrence $P(m)$ of a star of a given magnitude was determined for the mean-galactic-latitude curve of figure 6. This curve was numerically differentiated to obtain the frequency of occurrence of stars of a given visual magnitude in the selected 2160-square-degree sector of the celestial sphere. The probability of occurrence $P(m)$ was defined to be equal to the frequency of occurrence divided by the total number of stars N in the 2160-square-degree sector. The probability of system response to a star of a given magnitude may then be determined from the relationship

$$P_R(m) = P(m)P_m\left(r \geqq T_2\right) \tag{14}$$

Figures 10, 11, and 12 represent $NP_R(m)$ plotted against m at various code lengths for the assumed background noise conditions of 160, 500, and 1000 tenth-magnitude stars per square degree, respectively. A qualitative measure of system performance may be

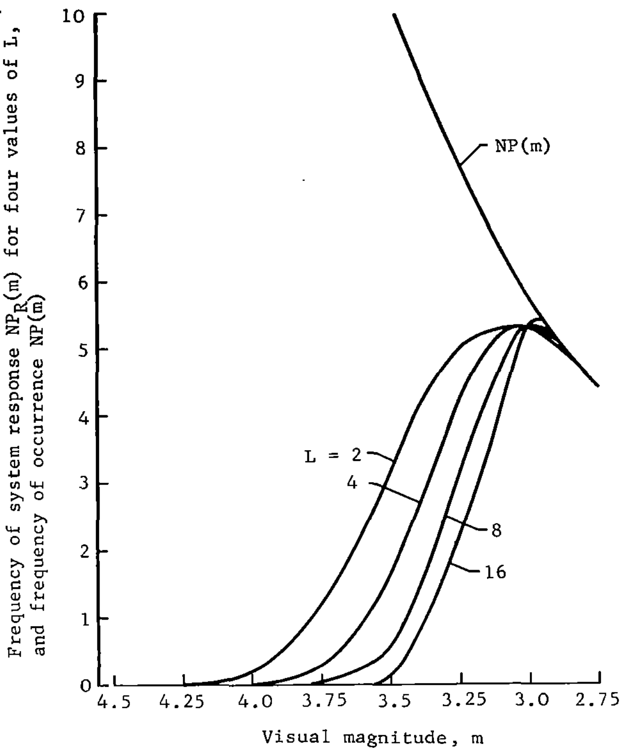

Figure 10.- Frequency of system response as a function of visual magnitude, with a background of 160 tenth-magnitude stars per square degree.

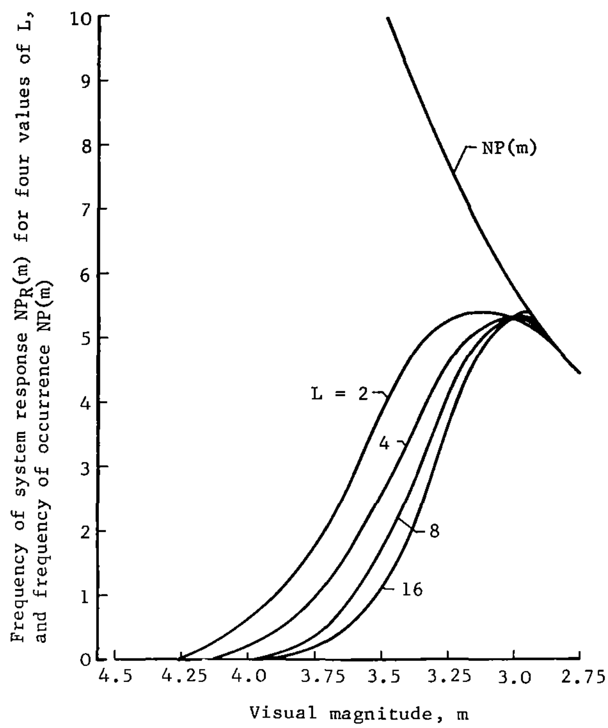

Figure 11.- Frequency of system response as a function of visual magnitude, with a background of 500 tenth-magnitude stars per square degree.

determined by examination of the relationship between system response to stars of magnitude brighter than third magnitude and system response to stars of all magnitudes. This relationship in ratio form is given by equation (13):

$$R = \frac{\int_3^{-\infty} P_R(m)dm}{\int_\infty^{-\infty} P_R(m)dm}$$

This ratio is plotted in figure 13 as a function of code length for the three assumed background-noise conditions. The system shows continuing improvement with increasing code length for the noise conditions and values of L considered, but the rate of improvement decreases for code lengths in excess of four elements. For the assumed problem, a good choice of code length would be about eight elements, as more elements do not greatly increase R but do greatly increase the complexity of design.

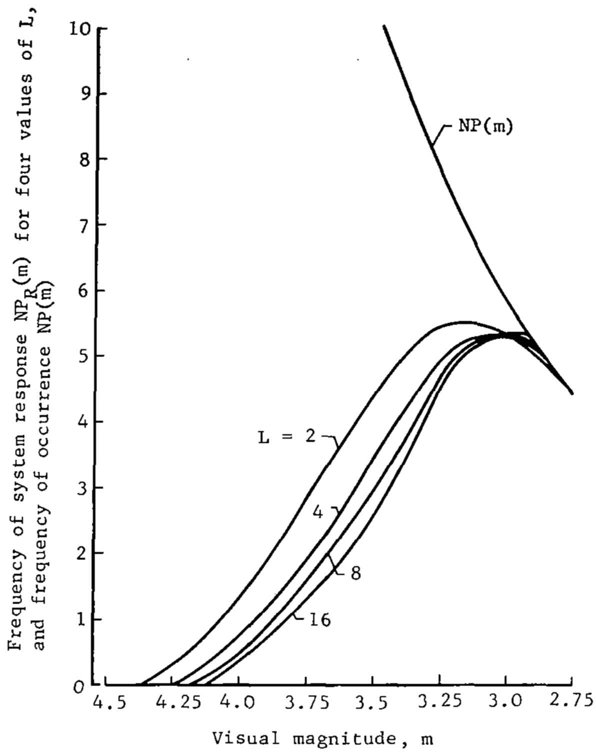

Figure 12.- Frequency of system response as a function of visual magnitude, with a background of 1000 tenth-magnitude stars per square degree.

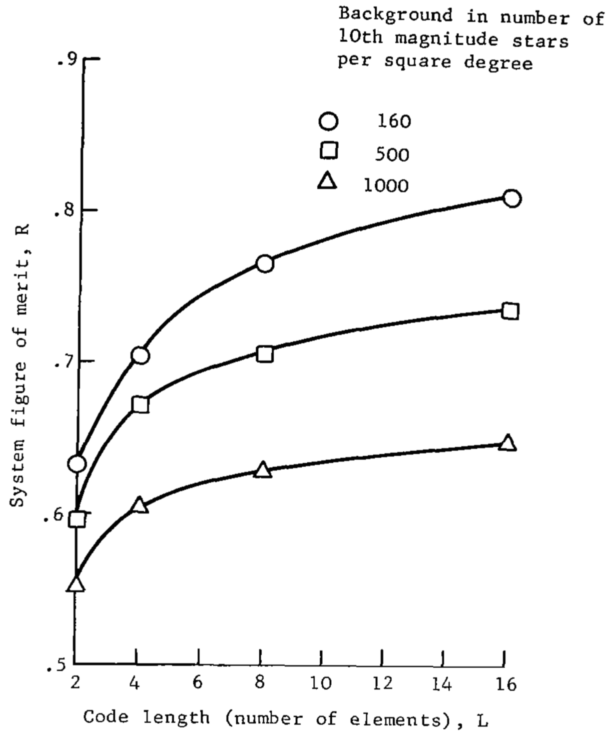

Figure 13.- System figure of merit as a function of code length.

CONCLUDING REMARKS

A method of measuring the attitude angles of a spin-stabilized spacecraft has been presented. This method basically consists of a map-matching procedure in which a strip star map generated about the equator of the vehicle is correlated with a known reference star map of the celestial sphere. This correlation process results in the determination of the orientation of the vehicle spin axis and roll angle about this axis with respect to the celestial sphere.

The star mapper and the necessary data-processing mechanization have been described in block-diagram form. A general discussion of the system sensitivity and factors that limit system performance has been presented. The unique feature of this system is the use of coded optical slits which produce a coded pulse group for each star scanned by the star mapper. This feature reduces the probability of response to noise and to the multitude of stars dimmer than the desired threshold magnitude, and yet retains the degree of resolution obtainable through the use of a single slit.

The performance of a typical design has been estimated for a range of signal and noise conditions. The effects of code length are illustrated, and it is shown that as the number of elements is increased, the improvements in performance become progressively smaller. In this discussion a choice of code length has been made on the basis of diminishing improvement and increasing complexity of mechanization.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., March 18, 1968,
125-17-02-01-23.

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