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VELOCITY AXIS RESPONSE OF AN ION MASS SPECTROMETER ON A SPHERICAL SATELLITE

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ABSTRACT

Title of Thesis: Velocity Axis Response of an Ion Mass Spectrometer on a Spherical Satellite

Michael G. Toner, Master of Science, 1968.

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The ratio of the ion current collected by an ion mass spectrometer when the normal to the spectrometer's orifice is aligned parallel to the vehicle's velocity vector, to that current collected when they are aligned anti-parallel, is known as the ram to wake ratio. This ratio is calculated as a function of ion temperature, first under the assumption that there is no sheath around the vehicle, and secondly in the presence of a model sheath, whose structure is not a function of ambient conditions. In both cases, the results show that the ratio is a rapidly varying function of ion temperature, and that for H^+, which has the smallest and most easily measured ratios, this approach could well provide a convenient means of measuring such temperatures.

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CHAPTER I

INTRODUCTION

The Explorer 32 satellite is a spherical body of radius 45.7 cm which was launched on May 25, 1966 into an orbit of apogee 2800 km and perigee 270 km. The spin rate of the satellite is 0.5 revolutions per second and the spin axis is aligned perpendicular to the velocity vector, to within about five degrees. One of the experiments being performed on board is the measurement of positive ion concentrations in the upper atmosphere using a Bennett radio frequency mass spectrometer (Taylor and Brinton, private communication). Ions enter the spectrometer by passing through a circular orifice of radius 1.65 cm in the satellite's surface. We let Θ be the angle between the velocity vector and the vector normal to the circular orifice, and since the orifice is located near the spin equator, Θ takes on nearly all values from 0^{o} to 180^{o}. Due to the fact that the satellite is moving through the ionosphere at speeds comparable to the ion mean thermal velocities, the ion current collected depends on the instantaneous value of Θ. This spin modulation of the ion current is superimposed upon the tuning curve of the spectrometer as shown in Fig. 1. With certain exceptions, to be discussed later, the largest currents occur at $\Theta = 0^{o}$, and the smallest at $\Theta = 180^{o}$. Although the spectrometer takes measurements for the four different ions O^{+}, N^{+} He^{+} and H^{+}, only the H^{+}, because of its greater mean thermal speed, ever shows a measureable current at 180^{o}. The ratio of the current collected at 0^{o} to the current collected at 180^{o} will be called the ram to wake ratio. It is to be expected that this ratio will be a strong function of temperature for a given ion. The purpose of this paper is to calculate this relationship, using a simple model of the sheath structure. Once this relationship is known, it can then serve as a means of measuring ion temperatures. In order to obtain a complete solution to the problem, it would be necessary to know the sheath structure as a function of ion and electron temperatures and densities. Such an approach is beyond the scope of this paper. It is very probable, however, that the main features of the ram to wake ratio versus temperature relationship can be found using the assumption of a non-varying sheath; and this is the assumption that will be used here.

CHAPTER II

NO SHEATH, ONE DIMENSIONAL CALCULATION

The first case to be considered will be the simplest possible one, namely that there is no sheath at all. The assumption of no sheath is equivalent to saying, among other things, that the electric potential at the satellite's surface and in the space surrounding the satellite is zero. If in addition to this we consider only that component of ion velocity lying along the vehicle's velocity vector, a very simple calculation of the ram to wake ratio, which will be designated R/W, is possible.

Let A be the area of the circular orifice through which ions must pass to reach the spectrometer. Also let the satellite's velocity vector be the direction of the positive z axis of a coordinate frame 0, moving with the satellite. A similarly oriented frame at rest with respect to the plasma will be termed 0'. Then in this one dimensional model, the only ions which can reach A when it is centered at $\Theta = 0^{\circ}$ or $\Theta = 180^{\circ}$, are those in an infinite cylinder with cross section equal to A, lying along the z axis. If the velocity distribution of the ions in 0' is taken to be the Maxwellian distribution,

$$F'(v_z')dv_z' = \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-mv_z'^2/2kT} \, dv_z'$$

then the distribution in 0 is given by

$$F(v_z)dv_z = \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-m(v_z + v_s)^2/2kT} \, dv_z$$

where v_s is a positive number equal to the satellite's speed.

Now if the ambient density of ions is n, the number of ions striking A per unit time with velocities in a small range dv_z about v_z is $nA|v_z|F(v_z)dv_z$. For $\Theta = 0^o$, only particles with $v_z < 0$ can strike A, and for $\Theta = 180^o$, it is those with $v_z > 0$ which can do so. Therefore

$$R/W = \frac{\int_{-\infty}^{0} (-v_z) \quad F(v_z)dv_z}{\int_{0}^{\infty} v_z \; F(v_z)dv_z}$$

This expression reduces to

$$R/W = \frac{s \circ \operatorname{erf}(s) + \frac{1}{\sqrt{\pi}} e^{-s^2} + s}{s \cdot \operatorname{erf}(s) + \frac{1}{\sqrt{\pi}} e^{-s^2} - s}$$

where $s = \sqrt{\frac{m}{2kT}} \; v_s$

and $\operatorname{erf}(s) = \frac{2}{\sqrt{\pi}} \int_0^s e^{-t^2} dt$

This result is plotted in Fig. 2 as a function of temperature for m = 1, 4, 14 and 16 A. M. U., and for v_s = 7.8 km/sec.

CHAPTER III

THE SHEATH STRUCTURE

It is known in general from Langmuir probe data and from other sources that conducting bodies in the ionosphere develop surface potentials anywhere from -15 to +1 v, while more typical values are from -3 to -1.5 v. At an ambient ion temperature of 2000°, -1.5 v represents over five times the mean ion energy. It is clear, then, that the sheath will have a major effect on the trajectories of ions approaching the satellite, and that its thickness and structure need to be taken into account in calculating the ram to wake ratio. The attempt here will be to find a mathematical representation only of the general, overall features of the potential, and not of its detailed structure. Three functions will be sought, the potential, Φ, as a function of r at $\Theta = 0^{\circ}$, $\Phi(r)$ at 180°, and $\frac{1}{r}\left(\frac{\partial r}{\partial \Theta}\right)_{\Phi}$ as a function of both r and Θ. The radial electric fields, E_r, can be calculated at 0° and 180° from the $\Phi(r)$'s. The radial field at points between will then be found by taking a linear average. That is,

$$(E_r)_{\Theta} = \frac{1}{\pi}\left[(E_r)_o\,[\pi - \Theta] + (E_r)_{\pi} \circ \Theta\right]$$

Walker (1965) has treated the general problem of the potential distribution around an axially symmetric body moving rapidly through a neutral plasma. His approach depends upon the assumptions that the mean thermal velocity of the ions, $\bar{v}_i$, is much smaller than the vehicle velocity, v_s. In the case of Explorer 32, the vehicle velocity ranges from 8.34 km/sec at perigee to 6.05 km/sec at apogee. In order to see how these compare to the thermal velocities, a set of "average" conditions, which will be used to determine the sheath structure, will now be stated. These conditions are

Satellite velocity, v_s	-	7.8 km/sec
Surface potential	-	-1.5 volts
Altitude	-	800 km
Density of H^+, $(N_{H+})_o$	-	$2.5 \cdot 10^4$ ions/cm^3
Density of O^+, $(N_{O+})_o$	-	$2.5 \cdot 10^4$ ions/cm^3
Electron denisty, $(N_e)_o$	-	$5 \cdot 10^4$ electrons/cm^3
Ion temperature	-	2000° K
Electron temperature	-	3500° K

At 2000°, the mean thermal velocities of hydrogen and oxygen are respectively 6.35 km/sec and 1.58 km/sec. The assumption that $v_s >> \bar{v}_i$ is therefore, at best, only a fair one. Nevertheless, the use of Walker's approach in the present problem is a valid approximation. That this is true can be seen from his graphs of electrical potential energy (in terms of the mean ion energy) vs. radius (in terms of the Debye length). In these units, the effect of the temperature on the sheath structure is largely removed, as evidenced by the great similarity between his plots for ion temperatures of 0° K and 1500° K.

In order to utilize Walker's approach, I attempted to run the same program that he had used, under the ambient conditions listed above. The use of the program is essentially a trial and error procedure A surface on which the potential is equal to 10^{-2} to 10^{-3} times the surface potential is chosen as the outer sheath boundary. This surface is only defined from $\Theta = 0^\circ$ to $\Theta = 90^\circ$. (It is assumed that no particles with positive v_z can reach the satellite.) The shape of this equipotential, the value of Φ on it, and the ambient conditions are the input required by the program to calculate as many inner equipotentials as desired. The problem, which must be solved by a trial and error technique, is to match the inner equipotential, on which Φ equals the surface potential, to a sphere with a radius equal to that of the satellite. Due to the difficulty of this technique, and to time limitations, the use of this program was limited to a determination of $\Phi(r)$ at $\Theta = 0^o$.

From both the graphs presented by Walker for ion temperatures of 0^oK and 1500^oK, I estimated that the ratio of the surface electric field at $\Theta = 0^o$ to the field at 180^o was 3. This ratio can be used here if one again assumes that it will not change significantly as one goes to higher temperatures. Then using this ratio, the surface electric field in the wake is specified from that already found for 0^o. If the ion and electron densities along the negative z axis were also known, then $\Phi(r)$ could also be calculated in the wake. Due to the great speed, the electrons may be assumed to be distributed according to the Boltzman equations, $N_e = (N_e)_o\, e^{-e\Phi/kT}$. On the other hand, the O^+, because of its low speed, will be taken to be totally absent from the space immediately behind the vehicle. To obtain an expression for the density of H^+, assume as in the previous chapter that there is no sheath. Then N_{H^+} at a radius of r is given by

$$N_{H^+} = (N_{H^+})_o \left[\int_0^\infty v^2 dv \int_0^{2\pi} d\phi \int_\alpha^\pi \sin\Theta d\Theta \; v \; F(v,v_s,T) \right] / A$$

$$\alpha = \sin^{-1} \frac{r_o}{r}$$

where F(v) is the velocity distribution in the satellite's frame as in Chapter 2, and r_o is the satellite radius. A is the value of the integral when $\alpha = 0$.

α is the half angle of the cone subtended by the satellite at r, as shown in Fig.3.At $\alpha=90^{\circ}(r=r_o)$, $N_{H^+} = 0.13(N_{H^+})_o$ and at $\alpha = 45^{\circ}$, $N_{H^+} = 0.29(N_{H^+})_o$. A straight line was drawn though these two points, and using this linear approximation for $N_{H^+}(r)$, a simple program was written to iterate the potential outward along the negative z axis. Appendix 1 gives this program, as well as details on the evaluation of the above integral. The resulting $\Phi(r)$ from this program, and the result of Walker's program for 0°, are graphed in Fig. 4.

To find a form for the function $D = \frac{1}{r}\left(\frac{\partial r}{\partial \Theta}\right)_{\Phi}$, it is first necessary to note the following two limiting conditions.

1) D must go to 0 as r approaches r_o.
2) D must go to 0 as Θ approaches 0° or 180°. This is due to the fact that the potential must be axially symmetric and continuous.

It was therefore decided to let D be a function of r alone between $\Theta = 15^{\circ}$ and $\Theta = 165^{\circ}$, while from 15° to 0°, and from 165° to 180°, at constant r, D would go linearly to 0. The final forms chosen for D are shown in Figs. 5 and 6.

CHAPTER IV

TRAJECTORY CALCULATION

The knowledge of $\Phi(r)$ and $\frac{1}{r}\left(\frac{\partial r}{\partial \Theta}\right)_\Phi$ in the space surrounding the satellite permit one to calculate the forces on an ion as it approaches the vehicle surface. If we let

$$e = \text{charge on a proton}$$

$$C = -e \left({}^{\partial\Phi}/_{\partial r}\right)_\Theta$$

$$D = \frac{1}{r} \left({}^{\partial r}/_{\partial \Theta}\right)_\Phi$$

$$SDZ = \sqrt{x^2 + y^2}\ /z$$

then using the relation

$$\frac{1}{r}\left({}^{\partial r}/_{\partial\Theta}\right)_\Phi = -\frac{1}{r}\left({}^{\partial\Phi}/_{\partial\Theta}\right)_r \Big/ \left({}^{\partial\Phi}/_{\partial r}\right)_\Theta$$

as well as

$$F_x = F_r \sin\Theta \cos\phi + F_\Theta \cos\Theta \cos\phi$$

$$F_y = F_r \sin\Theta \sin\phi + F_\Theta \cos\Theta \sin\phi$$

$$F_z = F_r \cos\Theta + F_\Theta \sin\Theta$$

the cartesian force components acting on a simply charged positive ion are

$$F_x = \frac{xC}{r}\left[1 - D/ SDZ\right]$$

$$F_y = \frac{yC}{r}\left[1 - D/ SDZ\right]$$

$$F_z = \frac{zC}{r}\left[1 + D \circ SDZ\right]$$

The z axis is aligned as before, along the satellite's velocity vector. The x and y axes are perpendicular to it and to each other. Their exact position is immaterial due to the cylindrical symmetry.

In order to obtain simple mathematical forms for quantities like C and D, the graphs in Figs. 4 and 5, or graphs derived from them, were subjected to least square curve fit. Thus a graph of $-\left(\frac{\partial\Phi}{\partial r}\right)_{\Theta=0^{\circ}}$ versus r, with both Φ and r expressed in Walker's nondimensional units, was fitted with a cubic polynomial. At 180°, $-e\,\Phi$ (in electron volts) versus r(cm) was fitted using a quartic. And finally, $\left(\frac{\partial r}{\partial\Theta}\right)_{\Phi}$ versus r was fitted with a cubic, for values of Θ between 15° and 165°.

Using these polynomials, the above force equations can be used as the basis of a computer subroutine to calculate the trajectories of ions in the sheath. The program which was written for this purpose uses an iteration procedure. From a knowledge of the position of the particle, the forces on it may be calculated, as above. Then using the second integration of Newton's law,

$$x = x_o + v_{xo}\,t + \tfrac{1}{2}\,\frac{Fx}{m}\,t^2$$

plus the particles velocity, a new position may be calculated. t may be thought of as time or simply as a movement parameter. The new velocity may also be found $v_x = v_{xo} + \frac{Fx}{m}\,t$. The same equations, of course, hold for the y and z components. The process continues until either the particle strikes the satellite surface, or until it leaves the sheath.

Appendix 2 contains the program as well as comments on its operation. The required inputs are an initial position and velocity, x, y, z, v_x, v_y, v_z, the curve fitting coefficients A(i,j), and t, the time increment. The outputs are either the Θ at which the particle strikes the vehicle surface, or else Θ = 22 for a particle which leaves the sheath.

CHAPTER V

CALCULATION OF RAM TO WAKE RATIOS

General Approach

When the circular orifice through which ions enter the spectrometer is centered at 0°, it encompasses all values of Θ from 0 to .0360, while the corresponding limits at 180° are 3.1056 to 3.1416. In order to calculate the ram to wake ratio, it is necessary to find the normal flux of ions to each of these areas on the vehicle's surface. The ions making up this flux will all be taken to originate in a thin spherical shell, concentric with the satellite, and of radius 64 cm, which is large enough to put it everywhere outside the sheath. This shell is then broken up into a series of rings, with each ring consisting of all the area between two values of Θ, Θ_1 and Θ_2. A single point within each ring is then chosen to be representative of its entire ring. Because the problem has axial symmetry, all values of the azimuth angle, ϕ, are equivalent. For simplicity, ϕ will be set equal to 0 for all these points. Θ for the representative points, $\overline{\Theta}(\Theta_1, \Theta_2)$, will be taken to be the average value of Θ for particles within the ring,

$$\overline{\Theta}(\Theta_1, \Theta_2) \quad \frac{\int_{\Theta_1}^{\Theta_2} \Theta \circ \sin\Theta \, d\Theta}{\int_{\Theta_1}^{\Theta_2} \sin\Theta \, d\Theta}$$

This equation assumes that the particles are distributed on the spherical shell with constant density. The values of Θ_1, Θ_2, $\overline{\Theta}$, and A_n, where A_n is the area of the n^{th} ring, are given below for some rings.

Ring #	Θ_1	Θ_2	$\bar{\Theta}$	A (arbitrary units)
R_1	0°	3.6°	2.38°	1.97
R_2	3.6°	7.2°	5.57°	5.92
R_3	7.2°	10.8°	9.13°	9.82
⋮	⋮	⋮	⋮	⋮
R_{98}	169.2°	172.8°	170.87°	.82
R_{99}	172.8°	176.4°	174.4°	5.92
R_{100}	176.4°	180.0°	177.62°	1.97

The problem, then, is to compute, for a series of different ion temperatures, the normal flux to the orifice at 0° and/or 180° from each of the points $P_n(r = 64\text{ cm}, \phi = 0^\circ, \Theta = \bar{\Theta}_n)$, to weight these fluxes according to the areas A_n, and to add them together for a final answer.

Use of the Trajectory Program

If we let the area on the satellite's surface from $\Theta=0$ to $\Theta= .036$ be termed B, and that from 3.1056 to 3.1416 B^1, then in order to find the flux from a point, P_n, to $B(B^1)$, one must first find and define, at P_n, the region of velocity space containing those velocities which will take a particle from P_n to $B(B^1)$. If we call these velocity space regions Δv_B and Δv_{B^1}, then the normal fluxes to B and B^1 from P_n are respectively

$$\int_{\Delta v_B} v_r \, F(v_x, v_y, v_z, v_s) \, d^3v$$

and

$$\int_{\Delta v_{B^1}} v_r \, F(v_x, v_y, v_z, v_s) \, d^3v$$

where

$$F(v_x, v_y, v_z, v_s) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}(v_x^2 + v_y^2 + [v_z + v_s]^2)}$$

$$v_r = \vec{v} \circ \hat{r}$$

$$v_s = \text{satellite speed (7.8 km/sec)}$$

The problem of finding the regions Δv_B and/or Δv_{B^1} for a given P_n can be solved using the trajectory routine, subroutine TRAJ. It has been found that a point P_n which contributes a significant amount of flux to B contributes a negligible amount to B^1, and vice-versa. Therefore, at each P_n, only one Δv need be considered, Δv_B for $\Theta < 90^\circ$ and Δv_{B^1} for $\Theta > 90^\circ$. Considering a P_n with $\Theta < 90^\circ$, v_y is first set equal to 0 in the main routine used in conjunction with TRAJ. Then v_z is allowed to take on a series of values such as -1, -3, -5, etc. (all km/sec) and for each v_z, v_x is also run over a similar but more closely spaced series. The purpose is to discover, at a given v_z and with $v_y = 0$, the upper ($\Theta = .036$, $x > 0$), lower ($\Theta = .036$, $x < 0$), and central ($\Theta = x = 0$) values of v_x for reaching the indicated parts of B. These will be called v_{x1}, v_{x2}, and v_{xo} respectively. Then for the same v_z and with $v_x = v_{xo}$, a series of v_y's is run, until the largest one that can reach B is found. Let D_y be this largest v_y value, and let D_x equal $\frac{1}{2}(v_{x1} - v_{x2})$. Because each P_n has $\phi = 0^\circ$, the Δv's will be symmetric with respect to the $v_x - v_z$ plane. At a given v_z, the section of Δv_B or Δv_{B1} through the $v_x - v_y$ plane will be assumed to be an ellipse with center at $v_x = v_{xo}$ amd $v_y = 0$, and with D_x and D_y as the same major and minor axes. This assumption was checked at several P's and v_x's and was found to be quite accurate.

When, at a given P_n, v_{xo}, D_x and D_y have been found for several v_z, mathematical formulas for them as functions of v_z are obtained using least square curve fits. In most cases, in order to provide good fits at lower orders, the curves are broken up into two regions and each is fitted separately. Appendix 3 lists two main routines used in conjunction with subroutine TRAJ to carry out the above procedure.

Integration in Velocity Space

With the boundariss of the regions Δv_B and Δv_{B1} thus suitably defined, it is straight forward to perform the required numerical integrations in velocity space. At each P_n, the space is broken up into cubical volumes, 0.08 km/sec on a side, and the sum of $v_r\ F(v_x,\ v_y,\ v_z,\ v_s)$ over all the cubes is found. The program for finding this sum is given in Appendix 4. This program also multiplies the sum at each P_n by its proper weight, and computes the final ram to wake ratios for eight temperatures between 800° and 4300°.

CHAPTER VI

RESULTS AND CONCLUSIONS

In order to obtain complete results for all temperatures considered, it was necessary to calculate 9 positions in the ram (0° to 32.4°) and 11 in the wake (140.4° to 180°). The results of this calculation are presented in Fig. 7, which shows a ram to wake ratio decreasing monotonicaly with increasing temperature, from 2600 at 800° to 14 at 4300°. At a given temperature, the ratio is always lower than it is in the no sheath model (Fig. ` In general this lowering can be ascribed to the gr .er focusing power of the sheath in the wake.

With regard to the ram to wake ratio, we may divide the data sample supplied by H. Brinton from Explorer 32 into three groups:

1) Those showing no measurable flux in the wake, corresponding roughly to ratios greater than 50 to 1.

2) Those having ratios between 50 to 1 and 15 to 1.

3) Those having ratios less than 15 to 1.

The first group represents, according to Fig. 7, H^{+} temperature less than 2200°. Nearly all of the nighttime and a few of the daytime samples fall into this category. The second group comprises many (probably a majority) of the daytime data units (turn-ons with each assigned a number). The table below presents ten group two turn-ons in the altitude range 671-1000 km.

TURN ON #	ALTITUDE (km)	LOCAL TIME	GM LATITUDE	RATIO	H^+ TEMPERATURE
2163	671	15:51	-19.7°	34	2600°
3115	677	9:56	-33.4°	35	2572°
2787	682	11:59	-21.3°	51	2200°
2792	726	11:50	-13.3°	37	2500°
2148	731	15:47	-30.5°	16.2	3925°
2490	737	13:51	-33.3°	18.3	3600°
2514	771	13:39	-14.2°	46	2275°
3127	832	9:30	-31.1°	44	2325°
3473	833	7:21	-33.8°	33	2625°
3477	1000	6:51	-30.8°	18.3	3600°

Although no real conclusions can be drawn from the above short listing, the temperatures do seem to fall roughly within the ranges of expected daytime ion temperatures around 800 km.

Group three turn-ons, which make up a significant number of daytime samples, seem to represent ion temperatures in excess of 4000°. A few of these turn-ons show ratios as low as 4 to 1, while 6 and 7 to 1 are fairly common. These ratios seem to occur most often at high altitudes and latitudes. Rather than accept the existence of the implied temperatures of 8000° or greater, it seems more reasonable that the model breaks down at higher temperatures. In addition to low ratios, some group three turn-ons exhibit the additional feature of small sub peaks in the flux near 180°. While there is no way that the present type of calcualtion could reveal such a phenomenon, the connection of these sub peaks with high ion temperatures does seem likely.

In summary, then, the analysis of ram to wake ratios, as presented in this paper, seems to provide a means of determining hydrogen ion temperatures in the range of approximately 2000° to 4000°. The accuracy of the present model has yet to be determined. To include lower temperatures, a more sensitive detector would be necessary; while to extend the range above 4000°, the calculation would have to be redone using a sheath model more appropriate to these higher temperatures.

APPENDIX 1

ESTIMATION OF WAKE POTENTIAL

To evaluate

$$f = \int_0^\infty v^2 dv \int_0^{2\pi} d\phi \int_\alpha^\pi \sin\Theta d\Theta \quad v\left(\frac{m}{2\pi kT}\right)^{3/2} \quad e^{-\frac{m}{2kT}\left(v_x^2 + v_y^2 + (v_z + v_s)^2\right)}$$

Letting $x = \frac{mv_s^2}{2kT}$

$$f = e^{-x} \frac{m}{kT} \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} \int_0^\infty dv \int_\alpha^\pi d\Theta \; v^3 e^{-\frac{mv^2}{2kT}} \sin\Theta e^{-\frac{mv_s v}{kT}\cos\Theta}$$

$$f = \frac{e^{-x}}{v_s} \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} \int_0^\infty v^2 e^{-\frac{mv^2}{2kT}} \left[e^{\frac{mv_s v}{kT}} - e^{-\frac{mv_s v}{kT}\cos\alpha}\right]$$

We have two integrals of the form

$$I(a,b) = \int_0^\infty v^2 e^{-av^2 - bv} dv$$

with

$$f = I\left(\frac{m}{2kT}, -\frac{mv_s}{kT}\right) - I\left(\frac{m}{2kT}, \frac{mv_s\cos\alpha}{kT}\right)$$

I reduces to

$$I = \frac{e^{s^2}}{a^{3/2}} \left[\frac{1}{2}\left(\frac{\sqrt{\pi}}{2} - \int_0^{s^2} t^{\frac{1}{2}} e^{-t} dt\right) - se^{-s^2} - \sqrt{\pi}\, s^2 \left(\operatorname{erf}(\sqrt{2}\, s) - \frac{1}{2}\right)\right]$$

where $s^2 = \frac{b^2}{4a}$

The integral is an incomplete gamma function. Letting

$$P(s^2) = \frac{1}{2} \int_0^{s^2} t^{\frac{1}{2}} e^{-t} dt$$

$$f = \frac{vs}{\sqrt{\pi}\ x} \left[P(\sqrt{x}) + \sqrt{x}\, e^{-x} + \sqrt{\pi}\ \ x\left(\tfrac{1}{2} + \mathrm{erf}(\sqrt{2x}\ ) \right) + e^{-x} \sin^2\alpha \right.$$

$$P\left(\sqrt{x}\ \cos\ \alpha\right) + \sqrt{x}\ \cos\ \alpha\ e^{-x} - \sqrt{\pi}\ x\ \cos^2\alpha e^{-x}\ \sin^2\alpha$$

$$\left. \left(\tfrac{1}{2} - \mathrm{erf}(\sqrt{2x}\ \ \cos\ \alpha)\right) \right]$$

Normalizing the integral by dividing by $f(\alpha = o)$, we find that

$f(90^o) = 0.13$

$f(45^o) = 0.29$

for $x = .704$

A knowledge of the ion and electron densities and the surface electric field in the wake permit one to calculate the potential along the negative z axis. A program which uses an iteration procedure was written for this purpose. From the charge densities and electric field at R, the field at R + DR can be estimated, as well as the change in the potential between R and R + DR. New charge densities at R + DR are then calculated, and the process continues. Each iteration is carried to second order in DR, which is set equal to 0.05 cm. The surface potential (at R = 45.72cm) is -1.5ev. The program is listed on the following page.

This and other programs in this paper are written in Fortran IV for the IBM 7094.

```
C             WAKE POTENTIAL
    8 FORMAT(6E16.5)
    9 FORMAT(65HO              PHI                       R                  N(E)
      WRITE(6,9)
     1  N(I)        )
      AZ=4.0*3.14159*4.802*10.0**(-10)
      E=-6.0*10.0**(-4)
      TK=4.03*10.0**(-13)
      B=0.12-1.8*0.4572
      D=2.0*3.14159*23.04*10.0**(-20)
      R=45.72
      PHQ=-2.403*10.0**(-12)
      DR=0.05
      DO 28 J=1,300
      XNE=5.0*10.0**4*EXP(PHQ/TK)
      XNI=2.5*10.0**4*(0.018000*R+B)
      PKT=PHQ/(1.6019*10.0**(-12))
      WRITE(6,8) PKT,R,XNE,XNI,PHQ,E
      PHQ=PHQ-4.802*10.0**(-10)*E*DR*(1.-DR/R)-D*DR**2*(XNI-XNE)
      E=(E+AZ*DR*(XNI-XNE))/(1.0+2.0*DR/R)
      R=R+DR
   28 CONTINUE
      STOP
      END
```

APPENDIX 2

TRAJECTORY SUBROUTINE

Subroutine TRAJ is listed on the next two pages. This routine calcualtes the trajectory of a H^+ ion from the edge of the sheath to the satellite surface, as explained in Chapter 4. The value of Θ at which the ion hits the surface is returned. If the H^+ enters and then leaves the sheath without striking the satellite, Θ is set equal to 22.

```
      SUBROUTINE TRAJ(X,Y,Z,VX,VY,VZ,TIME,TM,A,THT)
      DIMENSION A(4,3)
      KK=1
      S=X**2+Y**2
      RSQ=S+Z**2
C
C     CALCULATION OF THT(THETA)
   98 SDZ=SQRT(S)/Z
   97 THT=ATAN(SDZ)
      IF(Z.GE.0.0) GO TO 99
   92 THT=THT+3.1415927
C
C     CALCULATION OF C(RADIAL ELECTRIC FIELD) FROM IT'S VALUE IN THE RAM(C1) AND
C     IN THE WAKE(C2)
   99 R=SQRT(RSQ)
   94 C1=A(1,1)+A(2,1)*R+A(3,1)*R**2+A(4,1)*R**3
      C2=A(1,2)
      DO 93 K=1,3
   93 C2=C2+A(K+1,2)*R**K
      C=(5.099*C2*THT+2.5197*C1*(3.14159-THT))*10.**(-13)
C
C     CALCULATION OF D (SEE TEXT)
   91 D=A(1,3)/R+A(2,3)+A(3,3)*R+A(4,3)*R**2
      IF(THT.LT.0.2618) GO TO 90
   89 IF(2.8798.GE.THT) GO TO 87
   88 D=D*(3.14159-THT)/.2618
      GO TO 87
   90 D=D*THT/.2618
C
C     CALCULATION OF ACCELERATIONS
   87 EM=5.9791*10.**23*C/R
      IF(X.NE.0.0) GO TO 86
   85 ACX=0.
      ACY= EM*Y*(1.-D/SDZ)
      GO TO 34
   86 ACX= EM*X*(1.-D/SDZ)
      ACY=Y*ACX/X
   84 ACZ= EM*Z*(1.+SDZ*D)
C     TM=0.5*TIME**2
      X=X+VX*TIME+ACX*TM
      Y=Y+VY*TIME+ACY*TM
      Z=Z+VZ*TIME+ACZ*TM
      VX=VX+ACX*TIME
      VY=VY+ACY*TIME
      VZ=VZ+ACZ*TIME
      S=X**2+Y**2
      RSQ=S+Z**2
C
C     TESTING OF RSQ(R**2). IF TOO LARGE, TRAJ RETURNS THT=22 INDICATING A MISS.
C     IF PARTICLE HAS REACHED SURFACE OF VEHICLE, MOST RECENT THT VALUE IS
C     RETURNED. IF NEITHER HAS HAPPENED, LOOP STARTING WITH STATEMENT 98 IS
C     REPEATED. TIME INCREMENT IS MADE SMALLER NEAR SURFACE.
      IF(RSQ.GE.3900.0) GO TO 82
      IF(RSQ.GT.2120.0) GO TO 98
      IF(KK.GT.1) GO TO 83
      TIME=TIME/3.0
      TM=TM/9.0
      KK=2
   83 IF(RSQ.GT.2090.3184) GO TO 98
```

```
C
C    RESTORATION OF TIME VALUES
      TIME=TIME*3.0
      TM=TM*9.0
   80 RETURN
   82 THT=22.0
      RETURN
      END
```

APPENDIX 3

TRAJECTORY MAIN ROUTINE

This appendix lists two main routines used in conjunction with subroutine TRAJ. Deck AAA 1 first, at a given spatial position, sets $v_y = o$. Then a series of uniformly spaced v_z values are run, and for each v_z, a similar and usually more closely spaced series is run over v_x. The object of this procedure is explained in Chapter 5. Briefly, it is to find, for $v_y = o$ and for a given v_z, the largest and smallest v_x's which will cause a particle to strike the satellite with $\Theta \leq .036$ or $\Theta \geq 3.10559$. Also sought is the v which will result in $\Theta = o$ or $\Theta = \pi$. This value of v_x is later called x_o.

The purpose of Deck AAA 2 is to find, with $v_x = x_o$, and for a given v_z, the value of v_y which results in $\Theta = .036$ or $\Theta =$ .10559. This v_y is known as dy, and its value is found by linear interpolation.

```
C     DECK AAA1
        DIMENSION A(4,3)
      1 FORMAT(4E20.8/4E20.8/4E20.8)
      2 FORMAT(I5,2F10.4,3E12.4)
      3 FORMAT(I5,E12.4)
     14 FORMAT(18H1 POSITION NUMBER   ,I5// 4H   X=,F10.4/ 4H   Z=,F10.4/ 11H
       1  THETA(1)=,F10.4/ 11H  THETA(2)=,F10.4/ 12H  THETA BAR=,F10.4/ 7H
       2  AREA=,F10.4)
    101 FORMAT(48H0      V(Z)         V(Y)         V(X)       THETA      )
    102 FORMAT(/3F12.3,F10.5)
C
C     IREP IS THE NUMBER OF SPATIAL POSITIONS TO BE DONE. LETTER P AFTER A
C     VARIABLE (VYP) MEANS A QUANTITY EQUAL TO THE VARIABLE (VY) BUT NOT ALTERED
C     BY SUBROUTINE TRAJ. PF INDICATES SAME THING IN KM/SEC.
        READ(5,3) IREP,TIME
        VYP=0.0
        VYPF=VYP*10.0**(-5)
        TM=0.5*TIME**2
        READ(5,1) (A(J,1),J=1,4),(A(K,2),K=1,4),(A(L,3),L=1,4)
C
C     ADJUSTING A'S TO DEFINITION IN TRAJ
        DO 11 ML=2,4
     11 A(ML,1)=A(ML,1)*0.183046**(ML-1)
        DO 6 I=1,IREP
C
C     NUM IS POSITION NUMBER. TH1P IS THETA(1) IN DEGREES. RSH IS SHEATH RADIUS
C     SQUARED.
        READ(5,2) NUM,TH1P,RSH
        TH1=TH1P/57.29578
        TH2=TH1+0.0628319
        B=COS(TH1)-COS(TH2)
        THBAR=(SIN(TH2)-SIN(TH1)+TH1*COS(TH1)-TH2*COS(TH2))/B
        TH2P=TH2*57.29578
        THBP=THBAR*57.29578
        AREA=B*10.0**3
        XP=64.0*SIN(THBAR)
        ZP=64.0*COS(THBAR)
        WRITE(6,14) NUM,XP,ZP,TH1P,TH2P,THBP,AREA
        WRITE(6,101)
C
C     K1 IS TWICE NUMBER OF VZ'S TO BE DONE. BK2 IS SPACING, BK1 IS STARTING VALUE
C     MINUS BK2, ALL IN KM/SEC. SIMILARLY FOR VX BELOW.
        READ(5,2) K1,BK1,BK2
        VZP=BK1*10.0**5
        DO 6 K=1,K1,2
        VZP=VZP+BK2*10.0**5
        VZPF=VZP*10.0**(-5)
        READ(5,2) L1,BL1,BL2
        WRITE(6,102) BL2
        VXP=BL1*10.0**5
        DO 6 L=1,L1,2
        VXP=VXP+BL2*10.0**5
        VXPF=VXP*10.0**(-5)
        X=XP
        Z=ZP
        VX=VXP
        VY=VYP
        VZ=VZP
C
```

```
C     NEXT 10 STATEMENTS BRING PARTICLE FROM ORIGINAL POSITION TO EDGE OF SHEATH.
C     THETA=21 IS RETURNED IF PARTICLE MISSES SHEATH.
      VSQ=VX**2+VY**2+VZ**2
      RV=X*VX+Z*VZ
      ARG=RV**2-VSQ*(4096.0-RSH)
      IF(ARG.GE.0.0) GO TO 36
      THT=21.0
      GO TO 38
   36 T=-(RV+SQRT(ARG))/VSQ
      X=X+VX*T
      Y=VY*T
      Z=Z+VZ*T
      CALL TRAJ(X,Y,Z,VX,VY,VZ,TIME,TM,A,THT)
   38 WRITE(6,102) VZPF,VYPF,VXPF,THT
    6 CONTINUE
      STOP
      END
```

```
C    DECK AAA2
        DIMENSION A(4,3),DD(100),X0(100)
      1 FORMAT(4E20.8/4E20.8/4E20.8)
      2 FORMAT(I5,2F10.4,3E12.4)
      3 FORMAT(I5,E12.4,I5)
     14 FORMAT(18H1 POSITION NUMBER    ,I5// 4H   X=,F10.4/ 4H   Z=,F10.4/ 12H
       1   THETA BAR=,F10.4)
     17 FORMAT(8F10.4)
    101 FORMAT(48H0      V(Z)          V(Y)          V(X)        THETA      )
    102 FORMAT(/3F12.3,F10.5)
        READ(5,3) IREP,TIME,JREP
C
C    DD'S ARE STARTING VALUES OF VY IN KM/SEC. ALSO READ IN ARE CORRESPONDING
C    X0'S. JREP IS TOTAL NUMBER OF THESE (DD,X0).
        KKK=0
        TM=0.5*TIME**2
        READ(5,17)  (DD(J),J=1,JREP)
        READ(5,17) (X0(K),K=1,JREP)
        READ(5,1) (A(J,1),J=1,4),(A(K,2),K=1,4),(A(L,3),L=1,4)
        DO 11 ML=2,4
     11 A(ML,1)=A(ML,1)*0.183046**(ML-1)
        DO 6 I=1,IREP
        READ(5,2) NUM,THBAR,RSH
        THBAR=THBAR/57.29578
        XP=64.0*SIN(THBAR)
        ZP=64.0*COS(THBAR)
        THBAR=THBAR*57.29578
        WRITE(6,14) NUM,XP,ZP,THBAR
        WRITE(6,101)
        READ(5,2) K1,BK1,BK2
        VZP=BK1*10.0**5
        DO 6 K=1,K1,2
        VZP=VZP+BK2*10.0**5
        VZPF=VZP*10.0**(-5)
        KKK=KKK+1
        VXPF=X0(KKK)
        VXP=VXPF*10.0**5
        VYPF=DD(KKK)
        DO 7 L=1,15
        VYPF=VYPF+0.05
        VYP=VYPF*10.0**5
        X=XP
        Z=ZP
        VX=VXP
        VY=VYP
        VZ=VZP
        VSQ=VX**2+VY**2+VZ**2
        RV=X*VX+Z*VZ
        T=-(RV+SQRT(RV**2-VSQ*(4096.0-RSH)))/VSQ
        X=X+VX*T
        Y=VY*T
        Z=Z+VZ*T
        CALL TRAJ(X,Y,Z,VX,VY,VZ,TIME,TM,A,THT)
        WRITE(6,102) VZPF,VYPF,VXPF,THT
C
C    SECTION BELOW INTERRUPTS THE LOOP 'DO 7' WHEN THETA BECOMES GREATER THAN
C    0.036 OR LESS THAN 3.10559. USING PREVIOUS THETA AND VY VALUES, A LINEAR
C    INTERPOLATION IS MADE TO FIND VY (=DY) WHEN THETA EQUAL   . 36 OR  .
        IF(NUM.LT.50) GO TO 20
```

```
      IF(THT.LT.3.10559) GO TO 19
      GO TO 21
   20 IF(THT.GE.0.036) GO TO 23
   21 GTHT=THT
      GVY=VYPF
    7 CONTINUE
   19 DY=GVY+0.05*(GTHT-3.10559)/(GTHT-THT)
      GO TO 66
   23 DY=GVY+0.05*(0.036-GTHT)/(THT-GTHT)
   66 WRITE(6,17) DY
    6 CONTINUE
      STOP
      END
```

APPENDIX 4

INTEGRATION PROGRAM

The appendix contains the program (a main routine and a short subroutine) to carry out the integration of $v_r\ F(v_x, v_y, v_z, v_s)$ over a volume of velocity space defined by the two dimensional array B. The unit volume used is a cube 0.08 km/sec on a side. In the summation of the contributions from the various cubes, all multiplicative factors which would eventually cancel out in the calculation of a ram to wake ratio are ignored. If the integrations at all relevant spatial positions are done simultaneously, a short sequence could be added to the main routine to directly produce ram to wake ratios.

```
C   PROGRAM FOR DOING INTEGRATION IN VELOCITY SPACE. KA (DO 69 KA=) SPECIFIES A
C   SPACIAL POSITION. J (DO 70 J=) SPECIFIES A TEMPERATURE. AT EACH POSITION,
C   DX, XO, AND DY ARE DEFINED AS FUNCTIONS OF VZ THRU COEFFICIENTS B. EACH
C   FUNCTION IS BROKEN UP INTO TWO RANGES, VZ LESS THAN CH (READ(5,8) CH...),
C   AND VZ GREATER THAN CH.  THE FIRST SUBSCRIPT OF B IS 1,2,3 FOR DX,XO,DY
C   WITH VZ LESS THAN CH / 4,5,6 FOR SAME WITH VZ GREATER THAN CH. NIT'S ARE
C   CONSECUTIVE SERIES OF INTEGERS SPECIFYING NUMBER OF CURVE FITTING
C   COEFFICIENTS USED IN EACH CASE (SECOND SUBSCRIPT OF B). WT'S ARE THE AREAS
C   EACH POSITION REPRESENTS.
      DIMENSION B(6,6),NIT(96),WT(16),BB(16,8)
    4 FORMAT(4E20.8)
    7 FORMAT(40I2)
    8 FORMAT(8F10.5)
   18 FORMAT(16H1 FOR A TEMP OF ,F8.1,23H  DEGREES RATIO EQUALS ,E16.6)
   22 FORMAT(I5,F10.5)
  101 FORMAT(20H1  THESE ARE BB'S       )
  999 FORMAT(2E16.5)
      READ(5,7) (NIT(J),J=1,42)
      READ(5,8) (WT(J),J=1,7)
      JJJ=0
      DO 69 KA=1,7
      DO 27 NL=1,6
      JJJ=JJJ+1
      LQ=NIT(JJJ)
   27 READ(5,4) (B(L,NL),L=1,LQ)
      READ(5,8) CH,X,Z
      DO 70 J=1,8
      BJ=J
C
C   T IS TEMPERATURE, VT IS MAXIMUM V**2 THAT WILL BE CONSIDERED AT EACH
C   TEMPERATURE. SUM IS VALUE OF INTEGRAL FOR A GIVEN T AND POSITION
      T=500.0*BJ+300.0
      VT=0.148*T
      TEX=-60.554/T
      SUM=0.0
      READ(5,22) K1,BK1
      VZ=BK1
      WRITE(6,4) T,VT,CH,X,Z
      DO 68 K=1,K1
      VZ=VZ+0.08
      VZL=(VZ+7.8)**2
C
C   POLY CALCULATES DX, XO, DY
      CALL POLY(VZ,B,JJJ,NIT,CH,DX,XO,DY)
      IF(((XO+DX)**2+VZL).GT.VT) GO TO 68
      RPV=Z*VZ
      DXSQ=DX**2
      DYSQ=DY**2
      N=DX/0.08
      BN=N
      A=BN/12.5+0.08
      NN=DY/0.08
      BNN=NN
      AN=BNN/12.5+0.08
      L1=2*N+1
      VX=-A+XO
      IF(NN.NE.0) GO TO 10
      DO 62 LP=1,L1
      VX=VX+0.08
```

```
      VASQ=VX**2+VZL
      RV=RPV+X*VX
   62 SUM=SUM+RV*EXP(TEX*VASQ)
      GO TO 68
   10 DO 64 L=1,L1
      VX=VX+0.08
      VASQ=VX**2+VZL
      IF(VASQ.GT.VT) GO TO 64
      RV=RPV+X*VX
C
C   BECAUSE OF SYMMETRY ABOUT VX-VZ PLANE, ONLY POINTS ON THIS PLANE (NEXT
C   STATEMENT), PLUS 2* POINTS ON ONE SIDE OF IT (STATEMENT BEFORE 60) NEED TO
C   BE ADDED.
      SUM=SUM+RV*EXP(TEX*VASQ)
      VXT=(VX-X0)**2
      VY=-AN
      RV2=2.0*RV
      DO 60 M=1,NN
      VY=VY+0.08
      VYSQ=VY**2
      IF((VXT/DXSQ+VYSQ/DYSQ).GT.1.0) GO TO 60
      VLSQ=VASQ+VYSQ
      IF(VLSQ.GT.VT) GO TO 60
      SUM=SUM+RV2*EXP(TEX*VLSQ)
   60 CONTINUE
   64 CONTINUE
   68 CONTINUE
      WRITE(6,4) SUM
   70 BB(KA,J)=SUM*WT(KA)
   69 CONTINUE
      WRITE(5,101)
      WRITE(6,4) ((BB(I,J),I=1, 7),J=1,8)
C
C   AT THIS POINT A SHORT SEQUENCE COULD BE ADDED TO COMBINE THE BB'S SO AS TO
C   PRODUCE RAM TO WAKE RATIOS, ASSUMING ALL POSITIONS ARE RUN AT ONCE.
      STOP
      END
$IBFTC DECKB
      SUBROUTINE POLY(VZ,B,JJJ,NIT,CH,DX,X0,DY)
      DIMENSION B(6,6),NIT(96)
      M1=NIT(JJJ-5)
      M2=NIT(JJJ-4)
      M3=NIT(JJJ-3)
      M4=NIT(JJJ-2)
      M5=NIT(JJJ-1)
      M6=NIT(JJJ)
      IF(VZ.GT.CH) GO TO 11
      DX=B(1,1)
      DO 12 K=2,M1
   12 DX=DX+B(K,1)*VZ**(K-1)
      X0=B(1,2)
      DO 13 K=2,M2
   13 X0=X0+B(K,2)*VZ**(K-1)
      DY=B(1,3)
      DO 14 K=2,M3
   14 DY=DY+B(K,3)*VZ**(K-1)
      RETURN
   11 DX=B(1,4)
      DO 15 K=2,M4
```

```
15 DX=DX+B(K,4)*VZ**(K-1)
   X0=B(1,5)
   DO 16 K=2,M5
16 X0=X0+B(K,5)*VZ**(K-1)
   DY=B(1,6)
   DO 17 K=2,M6
17 DY=DY+B(K,6)*VZ**(K-1)
   RETURN
   END
```

REFERENCE CITED

Walker, E. H., "Plasma Sheath and Screening Around A Stationary Charged Sphere and a Rapidly Moving Charged Body," in Interactions of Space Vehicles with an Ionized Atmosphere, edited by S. F. Singer, pp. 134-140, Pergamon Press, Oxford, (1965).

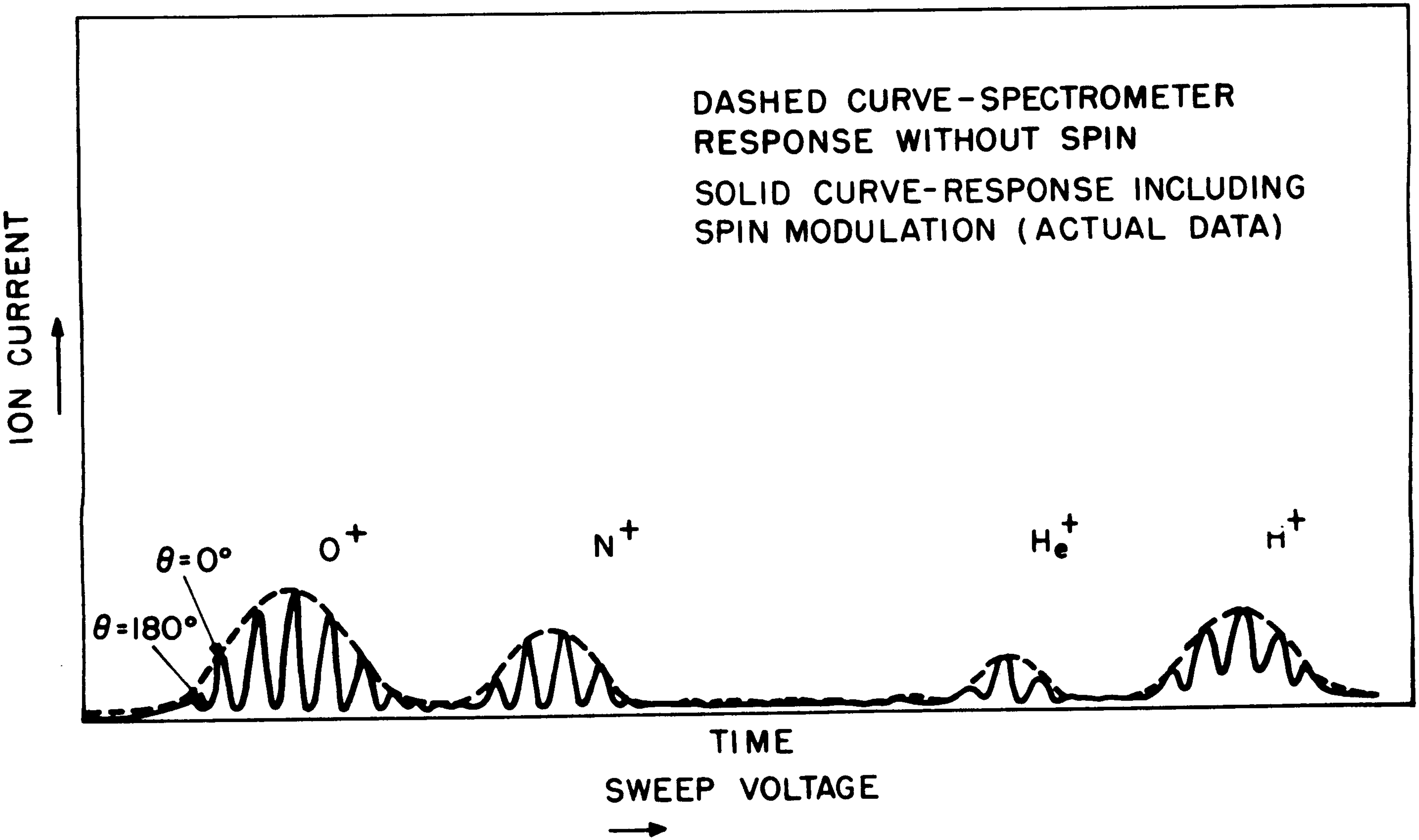

DASHED CURVE-SPECTROMETER RESPONSE WITHOUT SPIN
SOLID CURVE-RESPONSE INCLUDING SPIN MODULATION (ACTUAL DATA)
ION CURRENT
θ=0°
θ=180°
O+
N+
He+
H+
TIME
SWEEP VOLTAGE

FIG. 1

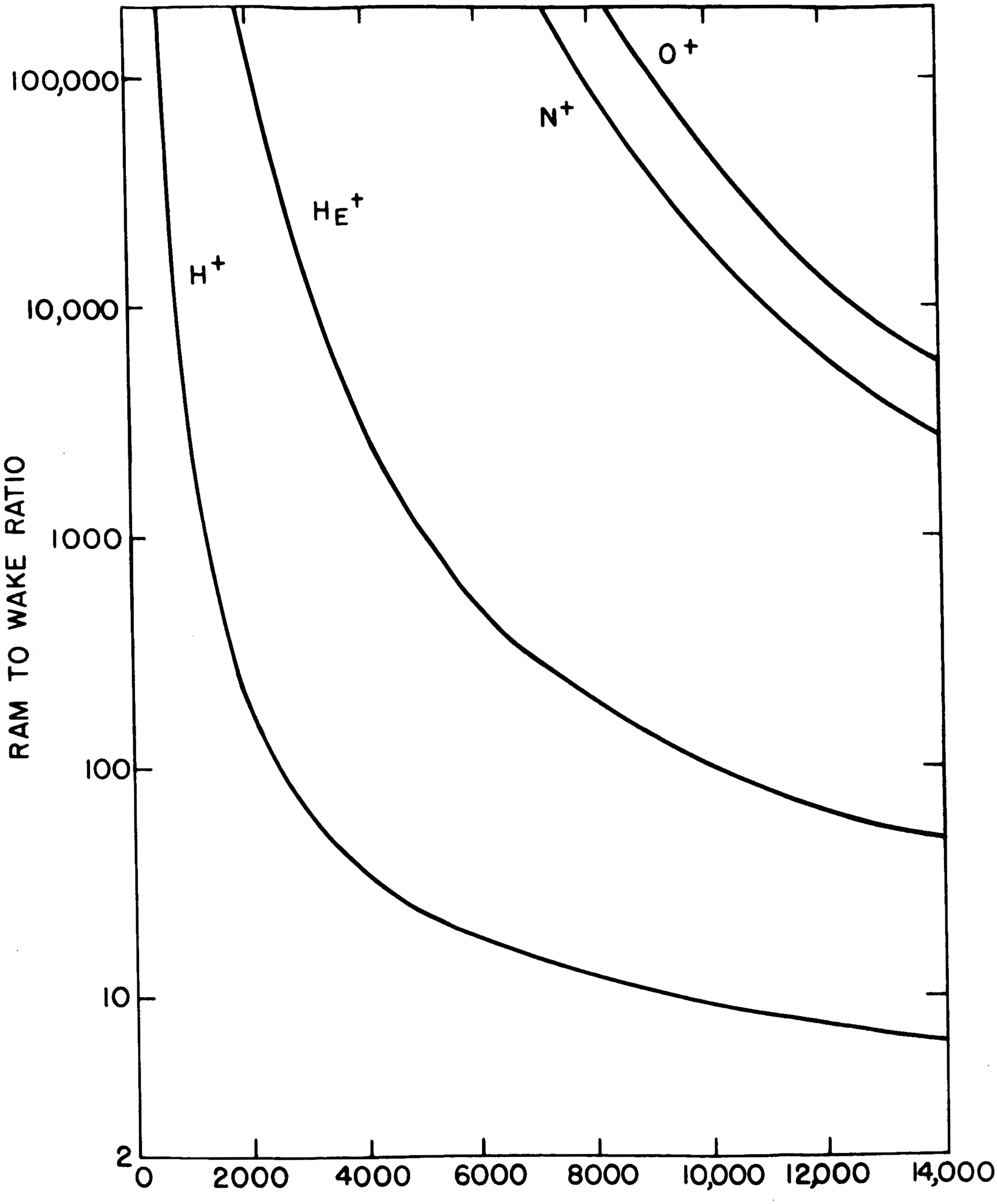

O+
N+
HE+
H+
100,000
10,000
1000
100
10
2
RAM TO WAKE RATIO
0
2000
4000
6000
8000
10,000
12,000
14,000
ION TEMPERATURE

FIG. 2

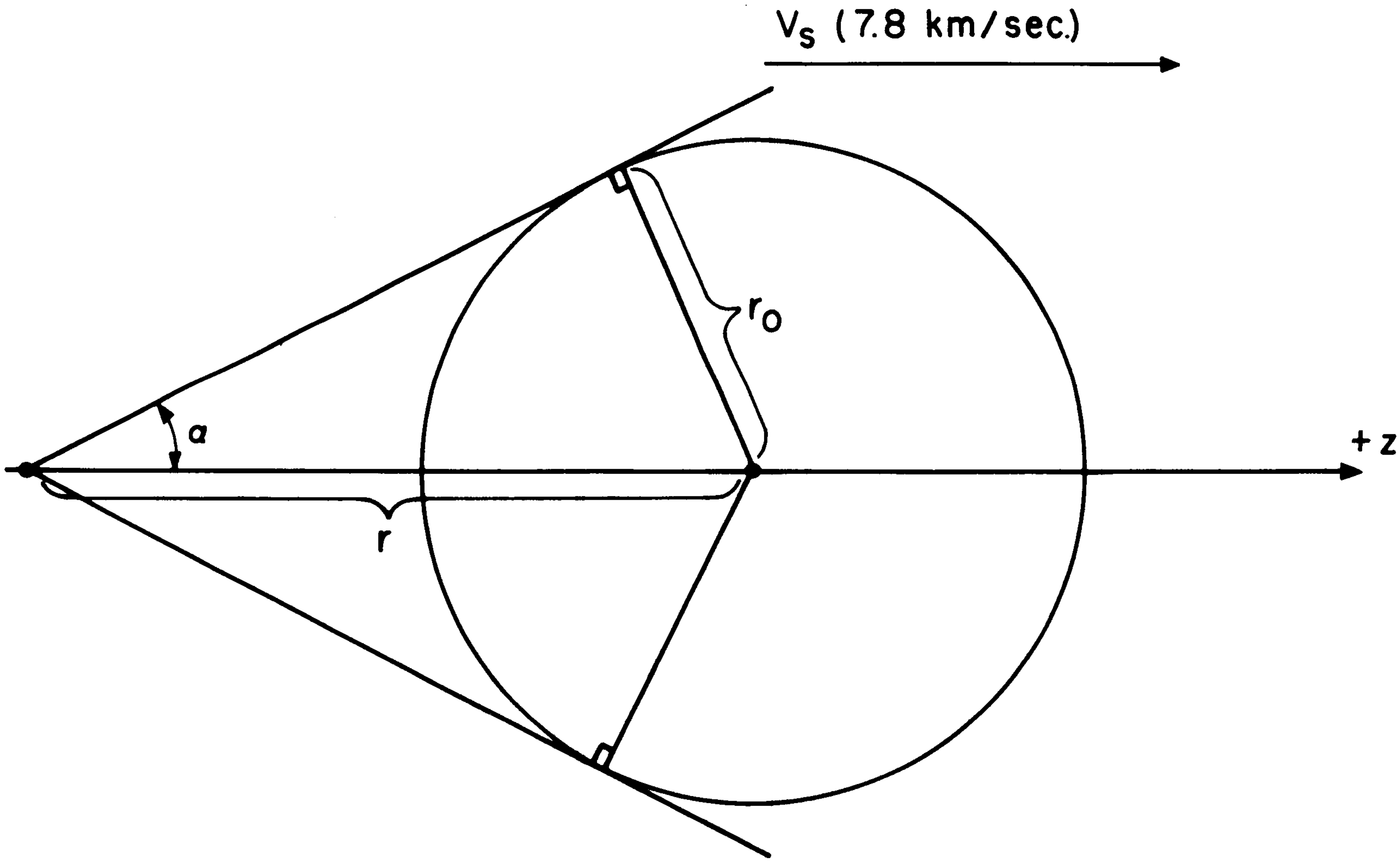


FIG. 3

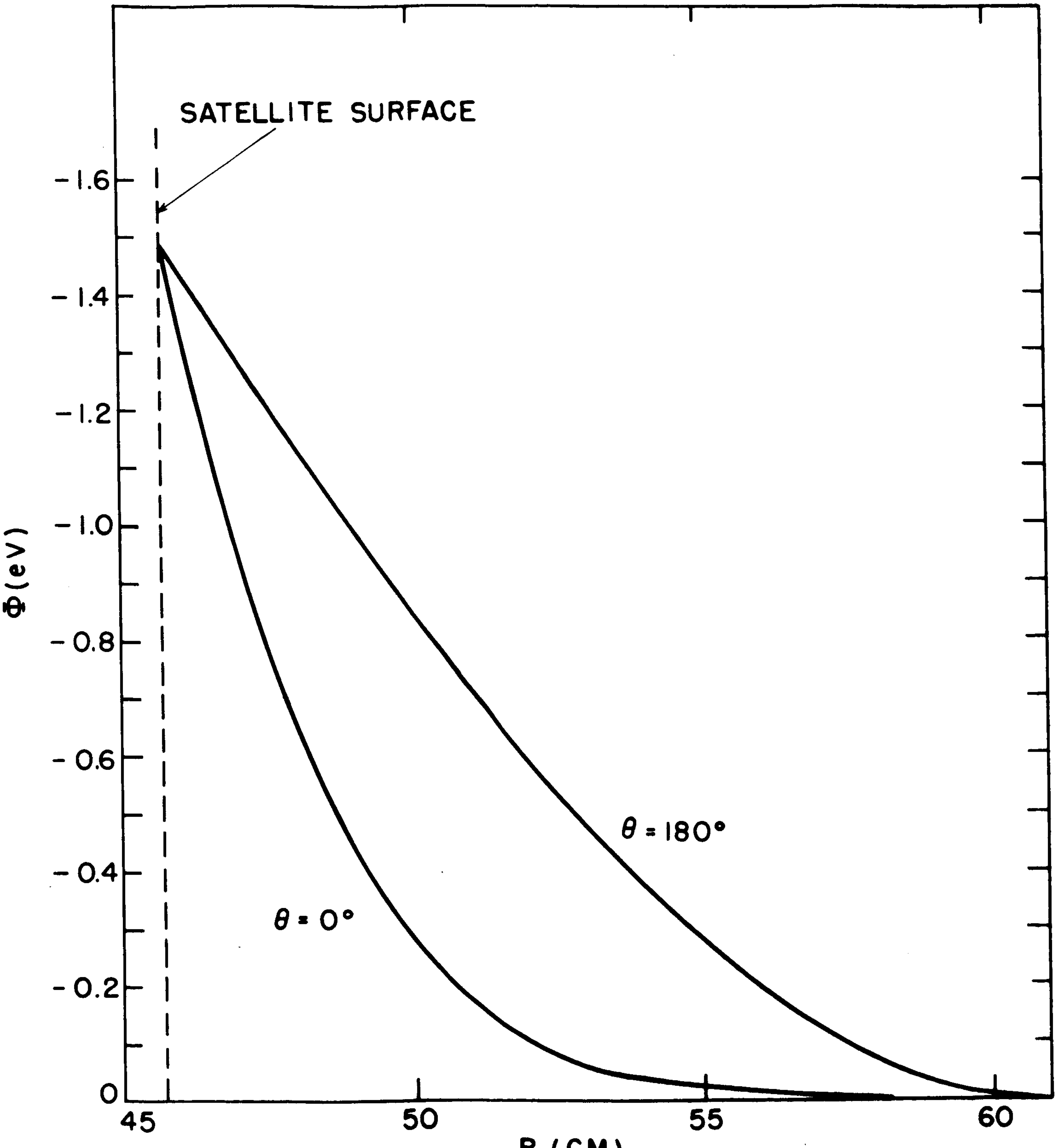

SATELLITE SURFACE
-1.6
-1.4
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0
Φ(eV)
θ = 180°
θ = 0°
45
50
55
60
R (CM)

FIG. 4

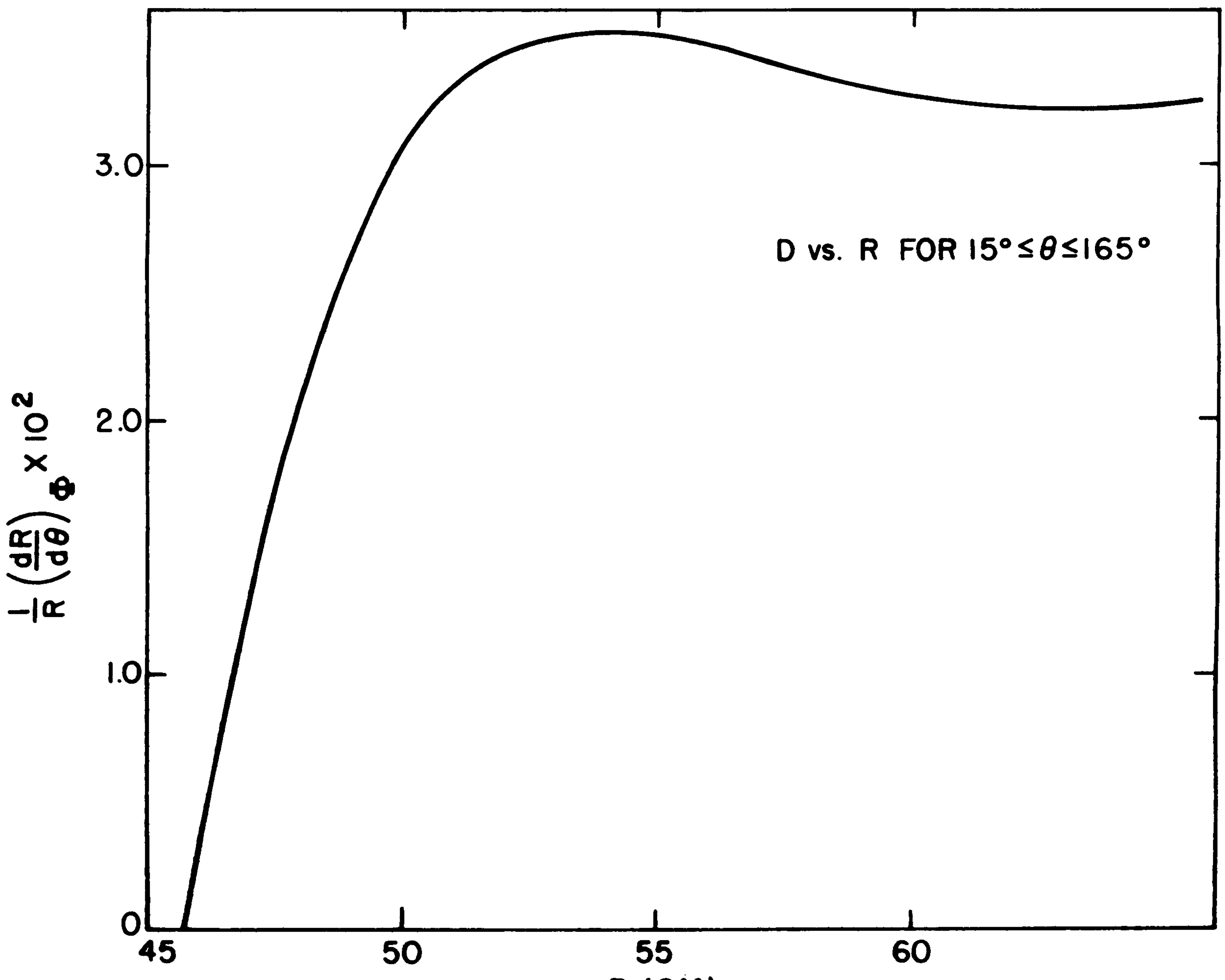
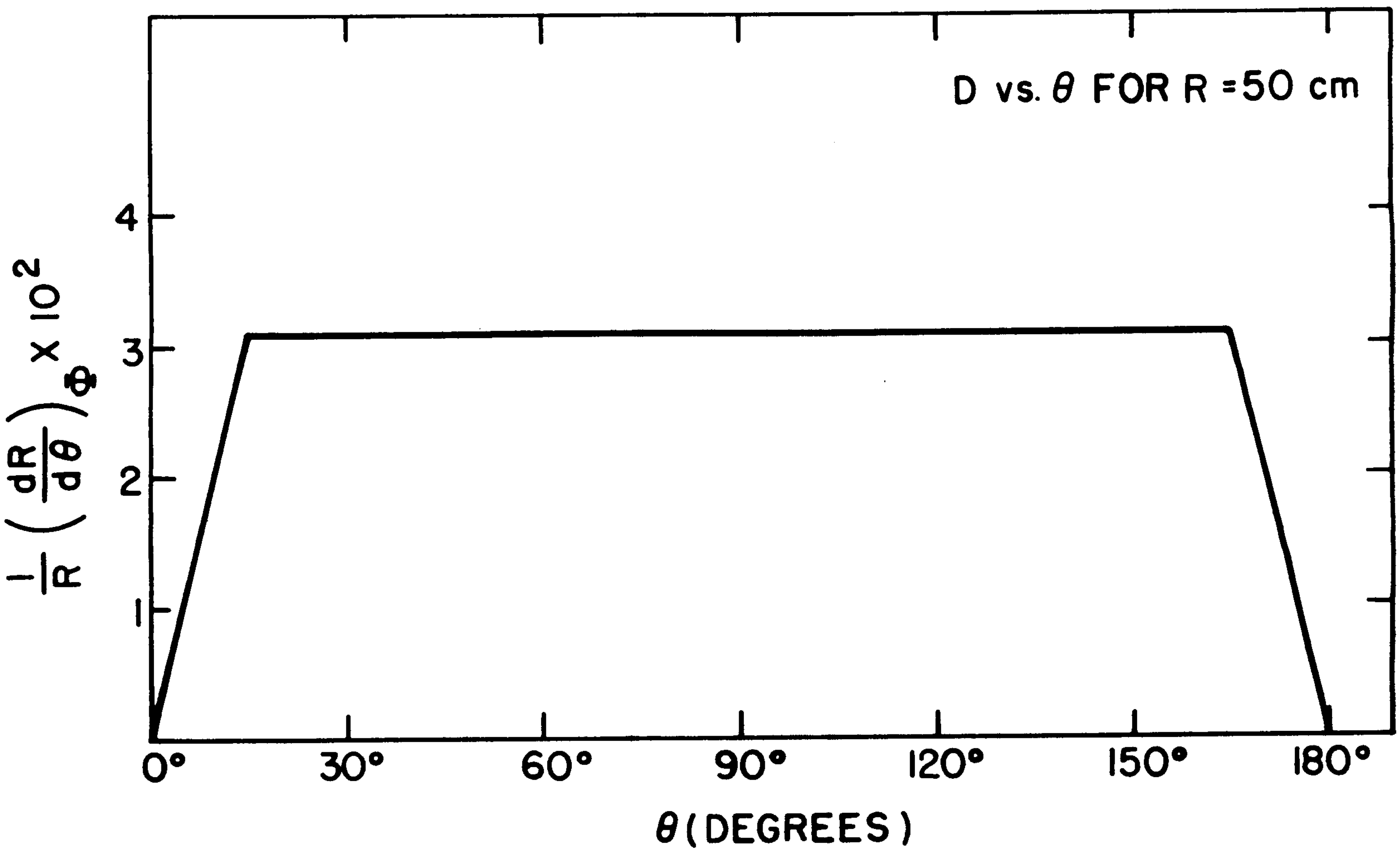
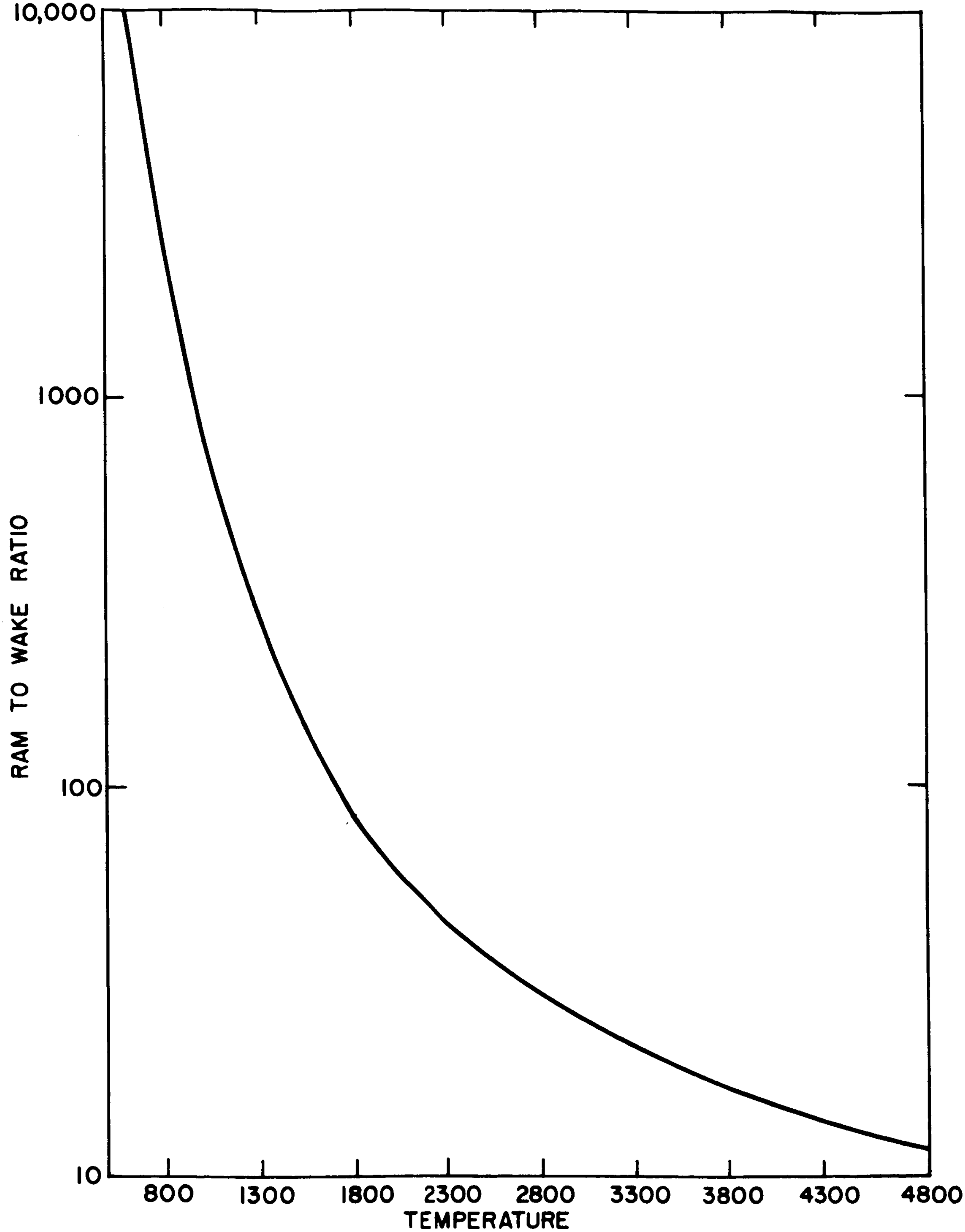


FIG. 5



D vs. θ FOR R =50 cm
4
3
2
1
$\frac{1}{R}\left(\frac{dR}{d\theta}\right)_{\Phi} \times 10^2$
0°
30°
60°
90°
120°
150°
180°
θ (DEGREES)

FIG. 6


10,000
1000
100
10
RAM TO WAKE RATIO
800
1300
1800
2300
2800
3300
3800
4300
4800
TEMPERATURE
FIG. 7