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RADIATION AND COUPLING BETWEEN TWO COLLINEAR OPEN ENDED WAVEGUIDES

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SUMMARY

This report describes an experimental and theoretical study of collinear coupled waveguides radiating through a common perfectly conducting ground plane. The problem is formulated as a set of simultaneous integral equations and solved approximately by expanding the aperture field in a Fourier series.

The special case of a single isolated waveguide is treated first, and the results are compared with experiment as well as with the approximate results of Lewin. Attention is given to the edge singularities.

The more general case of two collinear coupled waveguide slots is solved by the same numerical method using the symmetrical properties of the geometry. Phase and amplitude of the mutual coupling are evaluated and compared with experiment. Agreement is excellent.

Considerable attention is given to comparing more simple approximations with this result, and on several occasions it is pointed out how these can be used and what error the additional approximations introduce.

I. INTRODUCTION

The radiating waveguide is a fundamental electromagnetic structure, and one about which a great deal is known. With the realization of large scale microwave arrays, the subject of waveguide radiation and mutual coupling has aroused renewed interest. Since most of the recent theoretical efforts have been concerned with infinite arrays[1,2,3] and since these results are not readily applicable to a consideration of small arrays, or edge effects in large arrays, there is a need to study this problem from the point of view of individually coupled elements. This approach has been found most useful in the case of dipole arrays because the results can be readily compared with experiment and because the analysis may be extended quite naturally to consider the edge behavior of small or large arrays and can be used for infinite arrays as well.

The present work is concerned with the study of two open ended waveguides which radiate through a common perfectly conducting ground plane. The waveguides are arranged collinearly as shown in Figure 1, and the structure has been analyzed by expanding the solution in a truncated set of waveguide modes as an approximation to the integral equation solution. This method was chosen in preference to the computationally simpler variational solutions in order that bounds

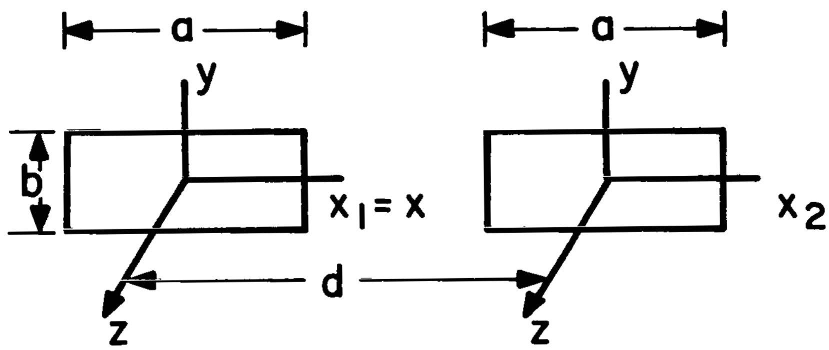


FIGURE I COUPLED WAVEGUIDE GEOMETRY

on the accuracy of the theory could be established by varying the number of terms used in the solution. The theoretical results have been correlated with experimental data in order to justify the basic assumptions of the analysis.

Though many authors have considered the problems of waveguide radiation and mutual coupling, there are still a great many questions which have yet to be answered concerning these subjects. The following discussion is intended to point out some of the gaps in present theoretical knowledge.

Much of the earlier work in this area was concerned with computing the field patterns of single waveguides with assumed aperture distributions. The next subject which attracted general interest, the input admittance of waveguide radiators, has traditionally been studied[4,5] using variational formulas with the lowest order mode as an aperture distribution. These formulations have been quite successful and give reasonably good approximations to both the conductance and susceptance. As mentioned earlier, they cannot easily be made to yield increasingly more accurate solutions. Other stationary formulations[6,7,8,9] have been studied for cases involving waveguides radiating into complex media. These formulations are based upon the use of Fourier integral techniques and use the dominant mode field as the aperture field approximation.

Galejs[10] was apparently the first to mention that the electric field distribution in the waveguide E-plane was not uniform. He correctly observed that the electric field normal to the waveguide edges should have a singularity of the same order as the static field singularity ($r^{-1/3}$) at a 90° edge. His solution, which was formulated by considering the junction between two waveguides as the size of one was made to approach infinity, assumed a trial field which exhibited this singularity. He found differences in admittance of about 5% when comparing this field approximation with the uniform field approximation.

In contrast to the single waveguide radiation problem, relatively little work has been done to describe the coupling of two waveguides. It appears that the first study of this sort was performed by G. W. Wheeler[11] who assumed the coupled radiators were in the far field of one another. By comparing theory and experiment, Wheeler was able to show that a single mode solution was quite adequate for the waveguides which he considered when coupled under this far field condition. In 1956 Levis[12] derived general equations for a variational formulation to obtain the coupling between a number of generally cylindrical waveguides radiating through a common ground plane. He applied the method to a set of coupled annular slots. Galejs[13] applied a stationary formulation due to Richmond[14] to solve the problem of two parallel slots in a ground plane, with both slots backed by waveguides. He also considered the limit in which the slots were the same size as the waveguides. His method yielded usable and convenient formulas; it includes the implicit assumption that the tangential magnetic field at the coupled waveguide aperture is the same as the magnetic field which would be present on the ground plane if the coupled aperture were not present. In this manner Galejs avoided the problem of solving an integral equation.

The most recent work in this area is described in a report by Lyon[15] et.al., and concerns the power coupling between various radiating structures including two open ended waveguides in a common ground plane, with arbitrary orientation and spacing. This experimental and theoretical study resulted in much useful data and some very convenient approximate formulas were based upon a single mode approximation to the coupling and also included the assumption, equivalent to that implicit in the work of Galejs, that the waveguide-backed ground plane slot has a total magnetic field equal to the incident magnetic field. This assumption again eliminated the need to solve an integral equation to determine the coupling.

The recent work on infinite arrays has shown an increased awareness of the importance of higher order modes in scanned array situations. Galindo and Wu[1,2] have studied a limited but important class of infinite arrays for which the boundary value problems could be rigorously solved in terms of scalar functions. Farrell and Kuhn[3] have used two sets of LSE modes to show the existence of a deep null in the array power pattern of a triangular grid array as the array was scanned only a few degress off broadside in the H-plane. This null had been detected experimentally at an earlier date, but was not revealed by theoretical results based upon a single mode approximation. The authors also mention an unpublished study in which the excitation of cross polarized modes is studied as the array is scanned in the E-plane. This appears to be the first mention of this phenomenon.

There are several other mathematical techniques which have been found useful for related problems but do not seem to have been used for rectangular waveguides. The geometrical theory of diffraction has been applied to study the radiation of horns[18,19] and the mutual coupling[18] of parallel plate waveguides. The Weiner-Hopf procedure[19] has been used to study radiation of circular waveguides.

This brief survey emphasizes the fact that there are many areas which should be studied further in order to achieve a better understanding of this basic problem. The present work appears to be the first example of a mutual coupling study (excluding the two infinite array studies) in which an attempt has been made to solve the integral equations which govern the coupling of two waveguides. It also appears to be the first example in which more than a single mode is used to describe the coupling, and in addition it presents the only experimental data currently available showing both the amplitude and phase of the signal coupled between two collinear open ended waveguides. Though this study is intended as an aid in evaluating simpler and more approximate analyses, it cannot be used to estimate the cross polarized field components excited at the array face. These components are not large for the collinear coupled slot case discussed in this report and the analysis presented here has been found to be an excellent approximation for all cases tested experimentally.

II. FORMULATION OF THE BASIC PROBLEM

The basic geometry for this study is shown in Figure 1. Two rectangular open-ended waveguides are mounted flush with an infinite, perfectly-conducting ground plane. The free space field in the half space bounded by the perfectly-conducting plane (with apertures) can be written in the conventional way in terms of the tangential aperture field as:

(for $z \geq 0$)

$$\bar{B}(\bar{r}) = j2\,\omega\mu\varepsilon \sum_{p=1,2} \int_{S_p} \underline{\Gamma}^{o}(\bar{r},\bar{r}') \cdot (\hat{z} \times \bar{E})\,d\,S_p'$$

(1)

$$E(r) = 2 \sum_{p=1,2} \int_{S_p} \bar{\nabla} G(\bar{r},\bar{r}') \times (\hat{z} \times \bar{E})\,d\,S_p'$$

where $\underline{\Gamma}^{o}(\bar{r},\bar{r}') = (\underline{U} + \frac{1}{k_o^2}\underline{\nabla\nabla})G(\bar{r},\bar{r}')$

and $G(\bar{r},\bar{r}') = \dfrac{e^{-jk_o|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|}$

and $|\bar{r}-\bar{r}'| = \sqrt{(x-x)^2 + (y-y')^2 + z^2}$

An $\exp(+j\omega t)$ time dependence is assumed and has been suppressed. Vectors are denoted by a bar above the expression and dyadics by a bar below. $\underline{U}$ is the unity dyadic and $\underline{\Gamma}^{o}$ is the conventional free space dyadic Green's function. An expression which is entirely equivalent to (1), but which helps to explain the major approximation of the analysis to follow, is obtained by defining the magnetic hertzian potential

$$(2) \qquad \bar{\pi}_m(\bar{r}) = \frac{j}{2\pi\omega} \sum_{p=1,2} \int_{S_p} (\hat{z} \times \bar{E}) \frac{e^{-jk_o|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|} \, d\,S_p'$$

The corresponding fields are written

$$(3) \qquad \bar{B}(\bar{r}) = \nabla(\nabla \cdot \bar{\pi}_m) + k_o^2 \, \bar{\pi}_m$$

$$\bar{E}(\bar{r}) = -j\,\omega\,\nabla \times \bar{\pi}_m.$$

The above equations define the fields in free space bounded by the conducting plane with apertures.

The field in each of the rectangular waveguides may be written in a number of ways, but for the purposes of this development it is convenient to expand the field in terms of the magnetic hertzian potential as done for the free space region. This is done by defining two scalar hertzian potentials π_{mx} and π_{my} such that

$$(4) \qquad \bar{\pi}_m = \hat{x}\,\pi_{mx} + \hat{y}\,\pi_{my}.$$

Expressing the fields in this form is convenient because it leads to a very simple and symmetrical form for the integral equations, and also because it emphasizes the formal similarities between this problem and several well-known aperture diffraction problems[20,21,22,23]. This expansion is clearly valid for the exterior half space, and one might expect intuitively that it would also be valid within each of the waveguides, where it is equivalent to two sets of LSE modes. These sets are not sufficiently complete to expand the solution of an obstacle in the waveguide, but it is proven in the appendix that they are adequate to expand the waveguide fields when the matching takes place at a plane perpendicular to the waveguide axis. If the waveguide aperture field is used in equation (2), it is only necessary to equate the tangential magnetic fields on both sides of each aperture in order to obtain integro-differential equations for the coupled fields. In order to distinguish between the free space fields and the waveguide fields the notation π'_{mx}, π'_{my} will be used to designate the free space fields, while the un-primed potentials will be used (later with subscripts) to designate the fields in each waveguide. With these definitions, the equations that express the continuity of the magnetic fields in the apertures can be put into a form similar to that used in the diffraction problems mentioned earlier. At each aperture

$$(4) \qquad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_o^2\right) (\pi'_{mx} - \pi_{mx}) = 0$$

$$(5) \qquad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_o\right) (\pi'_{my} - \pi_{my}) = 0$$

$$(6) \qquad \frac{\partial}{\partial x} (\pi'_{my} - \pi_{my}) = \frac{\partial}{\partial y} (\pi'_{mx} - \pi_{mx}) .$$

These equations have been solved rigorously in only a very small number of cases, most of which are mentioned by Bouwkamp[20]. The radiating waveguide is not among these. All of the published results describe some approximate solution to the above set of integro-differential equations. The analysis which follows is also based upon a simplification of these equations, to the extent that a single hertzian potential function will be used to approximate the input admittance of a single radiating waveguide and the coupling parameters of two collinear radiating waveguides. This assumption is not new; it is implicit in the work of Lewin[4] and Edelberg and Oliner[24] and it is made explicit by Kieburtz and Ishimaru[23]. The study presented here differs from others studies making the same basic assumption in that no other major assumptions are made. Instead, the reduced integro-differential equations are solved by expanding the solution in a truncated infinite series of a restricted set of modes, and then inverting the matrix equation which results. The convergence of this technique has been studied quite carefully and the truncation errors are estimated whenever the data is quoted. By these means, this theoretical study results in a very accurate solution of the approximate model used. Attention is then focused upon the comparison of this model with the experimental results to verify the validity of the model at hand.

The assumption that the single hertzian potential function π_{mx} suffices to expand the field ultimately requires that there be no component of electric field in the $\hat{x}$ direction. Equations (4) and (6) reduce to the following:

$$(7) \qquad \left(\frac{\partial^2}{\partial x^2} + k_o^2\right)(\pi'_{mx} - \pi_{mx}) = 0$$

$$(8) \qquad \frac{\partial}{\partial y}(\pi'_{mx} - \pi_{mx}) = 0$$

At each aperture a solution of these equations is

$$(9) \qquad \pi'_{mx} = \pi_{mx} + A \cos k_o x + B \sin k_o x$$

This integral equation was first presented by Lewin[4] without mention of its approximate nature. The approximation can be understood by noting that the factors A and B are constants and not functions of the spacial parameter y. Copson[21] has pointed out that the constants of integration must be determined so that the fields tangential to an edge vanish at the edge. This is necessary to assure that all fields are square integrable in a three dimensional domain around the edge. The constants of integration for this problem must therefore be determined so that the tangential electric field E_y vanish at the edges $x = \pm{}^a/_2$. This condition is the Babinet equivalent of that ordinarily encountered in dipole theory[25] which states that the current must vanish at both ends of the dipole. However, though this condition is satisfied by a constant for the infinitely thin dipole, the two constants available in this solution are only capable of assuring that this condition is satisfied at one point at each edge. Therefore it is clear that the solution used in this analysis is rigorous for slots whose width "b" equals zero, but that it is only approximate for the present case with finite slot width. The edge condition is not violated in this study however, because the solution is expanded in terms of a finite number of waveguide modes, each of which satisfies the stated condition at $x = \pm {}^{a}/_{2}$.

III. SOLUTION FOR THE SINGLE RADIATING WAVEGUIDE

Subject to this approximation, the electromagnetic field is written below in component form.

(10) $$E_x = 0 \qquad B_x = \frac{\partial^2 \pi_{mx}}{\partial x^2} + k_o^2 \pi_{mx}$$

$$E_y = -j\omega \frac{\partial \pi_{mx}}{\partial z} \qquad B_y = \frac{\partial^2 \pi_{mx}}{\partial x \partial y}$$

$$E_z = +j\omega \frac{\partial \pi_{mx}}{\partial y} \qquad B_z = \frac{\partial^2 \pi_{mx}}{\partial x \partial z}$$

The fields in each waveguide are obtained by writing the total potential function in terms of an incident mode field plus an infinite number of reflected waves. The incident component of the hertzian potential is normalized to unity.

$$\pi_{mx} = \left(e^{\gamma_{10}z} - \Gamma e^{-\gamma_{10}z}\right) \sin \pi\left(\frac{x+a/2}{a}\right) \tag{11}$$

$$+ \sum_{m=1}^{\infty} {\sum_{n=0}^{\infty}}' A_{mn} \sin m\pi\left(\frac{x+a/2}{a}\right) \cos n\pi\left(\frac{y+b/2}{b}\right) e^{\gamma_{mn}z}$$

(12) where $\gamma_{mn} = +\sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - k_o^2}$

$$= \begin{cases} \alpha_{mn} & \text{for } (m,n) \neq (1,0) \\ -j\beta_{10} & \text{for } (m,n) = (1,0) \end{cases}$$

The prime symbol above the summation is used to indicate the omission of the (m,n) = (1,0) term from this double summation. The y component of electric field is given by (10) and (12) and given below at z = 0

$$E_y \Big|_{z=0} = -j\omega \frac{\partial \pi_{mx}}{\partial z}\Big|_{z=0} = -j\omega \left\{ \gamma_{10}(1+\Gamma) \sin \pi\left(\frac{x+a/2}{a}\right)\right.$$

(13)

$$\left. + \sum_{m=1} {\sum_{n=0}}' A_{mn}\, \gamma_{mn} \sin m\pi\left(\frac{x+a/2}{a}\right) \cos n\pi\left(\frac{y+b/2}{b}\right) \right\}$$

This expression is no longer a convergent fourier series because the electric field E_y perpendicular to the top wall of the waveguide exhibits the $\rho^{-1/3}$ singularity typical of a 90^o edge, where ρ is the distance from the edge. The expression (11), however, is convergent everywhere in and outside of the aperture, and this property is required to assure the validity of this solution. Singularities of this same order are exhibited by E_z at $y = \pm b/2$ and by B_z at $x = \pm a/2$.

The basic equation (9) is re-written below using the given hertzian potential form inside of the waveguide.

(14)

$$\frac{-j}{2\pi\omega} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} E_y(x',y') \frac{e^{-jk_o\bar{r}}}{\bar{r}} dx'\, dy' = A \cos k_o x + B \sin k_o x$$

$$+ (1-\Gamma) \sin \pi(\frac{x+a/2}{a}) + \sum_{m=1}^{\infty} {\sum_{n=o}^{\infty}}' A_{mn} \sin m\pi(\frac{x+a/2}{a}) \cos n\pi(\frac{y+b/2}{b})$$

The solution proceeds by using the electric field from (13) in the left side of (14), and setting B = 0 because of the even symmetry. After defining the integrals:

(15)

$$I_{mn}(x,y) = \frac{1}{2\pi} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \frac{e^{-jk_o r_{11}}}{r_{11}} \sin m\pi(\frac{x'+a/2}{a}) \cos n\pi(\frac{y'+b/2}{b}) dx'\, dy'$$

$$\text{where} \quad r_{11} = \sqrt{(x-x')^2 + (y-y')^2}$$

one obtains the reduced form of equation (9) applicable to the single radiating waveguide.

$$-\gamma_{10} (\Gamma+1) I_{10}(x,y) - \sum_{m=1,3}^{\infty} {\sum_{n=0,2}^{\infty}}' A_{mn} \gamma_{mn} I_{mn}(x,y)$$

(16)

$$= A \cos k_o x + (1-\Gamma) \sin \pi(\frac{x+a/2}{a})$$

$$+ \sum_{m=1,3} {\sum_{n=0,2}}' A_{mn} \sin m\pi(\frac{x+a/2}{a}) \cos n\pi(\frac{y+b/2}{b})$$

The set of modes A_{mn} is truncated at finite values of M and N (M_τ, N_τ), and the constant A is treated as an unknown.

This determines a matrix equation when the equation is satisfied at a number of points within the aperture. The number of points chosen is one greater than the number of modes used in the solution. Treating the constant in this manner is equivalent to a procedure which is commonly used in antenna theory[25], namely that of defining a difference potential and assuring that it is zero at some point on the antenna. This method is applicable when the chosen distribution functions of current (or in this case electric field) are such that the fields automatically satisfy the edge condition. One row of the matrix equation is shown below.

$$[-\gamma_{10}\ I_{10}\ (x,y) + \sin \pi(\frac{x+a/2}{a})]\ \Gamma - A \cos k_o x$$

$$(17)\quad - \sum_{m=1,3}^{M_T} \ \sum_{n=0,2}^{'N_T} [\gamma_{mn}\ I_{mn}(x,y) + \sin m\pi(\frac{x+a/2}{a}) \cos n\pi(\frac{y+b/2}{b})]A_{mn}$$

$$= \gamma_{10}\ I_{10}\ (x,y) + \sin \pi(\frac{x+a/2}{a})$$

This equation was repeated at points on a rectangular lattice within the aperture.

IV. SOLUTION FOR TWO COUPLED COLLINEAR ANTENNAS

When two collinear slots couple, the coupled integral equations can be reduced to two uncoupled integral equations by virtue of the geometrical symmetry. The basic integral equations are written below.

At Aperture #1

$$\frac{-j}{2\pi\omega}\int_{-a/2}^{a/2}\int_{-b/2}^{a/2} E_{y_1}\ (x_1',y')\ \frac{e^{-jk_o r_{11}}}{r_{11}}\ dx_1'\ dy'$$

(18)

$$\frac{-j}{2\pi\omega}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} E_{y_2}\ (x_2',y')\ \frac{e^{-jk_o r_{12}}}{r_{12}}\ dx_2'\ dy' = A\cos\ (k_o x_1)$$

$$+ B\sin k_o\ x_1 + S_1(1+\Gamma_1)\ \sin\ \pi(\frac{x_1+{}^a/_2}{a})$$

$$+ \sum_{m=1}^{\infty}\sum_{n=0}^{\infty\ 1} S_1\ A_{mn}\ \sin\ m\pi(\frac{x_1+{}^a/_2}{a})\ \cos\ n\pi(\frac{y+{}^b/_2}{b})$$

At Aperture #2

$$\frac{-j}{2\pi\omega}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} E_{y_1}\ (x_1',y')\ \frac{e^{-jk_o r_{21}}}{r_{21}}\ dx_1'\ dy'$$

$$\frac{-j}{2\pi\omega}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} E_{y_2}\ (x_2',y')\ \frac{e^{-jk_o r_{22}}}{r_{22}}\ dx_2'\ dy' = C\cos(k_o x_2)$$

(19)

$$+D\sin k_0 x_2 + S_2(1+\Gamma_2)\ \sin\ \pi(\frac{x_2+{}^a/_2}{a})$$

$$+ \sum_{m=1}^{\infty}\sum_{n=0}^{\infty\ 1} S_2\ B_{mn}\ \sin\ m\pi(\frac{x_2+{}^a/_2}{a})\ \cos\ n\pi(\frac{y+{}^b/_2}{b})$$

where $r_{11} = \sqrt{(x_1-x_1')^2 + (y-y')^2}$ $\quad r_{12} = \sqrt{(x_2'+D-x_1)^2 + (y-y')^2}$

$r_{22} = \sqrt{(x_2-x_2')^2 + (y-y')^2}$ $\quad r_{21} = \sqrt{(x_2+D-x_1')^2 + (y-y')^2}$

These equations can be uncoupled by considering separate symmetric and anti-symmetric excitations $S_2 = \pm S_1 = \pm 1$. When this is done there is complete symmetry or assymmetry about the line $x_1 = {}^{d}/2$. Setting $-x_2 = +x_1 = x$, $C = \pm A$, $D = \mp B$ and $B_{mn} = \pm A_{mn}$ for m odd and $B_{mn} = \mp A_{mn}$, for m even, one can see that the resulting equations have the solution $E_{y_2}(-x_2,y) = \pm E_{y_1}(x_1,y)$ and that the two equations reduce to a single equation in these two limits. It is then a simple matter to reconstruct the case of arbitrary excitation by taking the appropriate combinations of these two cases. Later the solution to the case of parasitic antenna #2 ($S_2 = 0$) will be considered.

Defining

$$I_{mn}^{\,S}_{\,A}(x,y) = I_{mn}(x,y) \pm I_{mn}(-x + d,y) \tag{20}$$

and using the previous definition (15) of this integral, one obtains the following equations for the symmetric and anti-symmetric problems.

$$\left[-\gamma_{10}\, I_{10}{}^{S}_{A}(x,y) + \sin \pi\left(\frac{x+{}^{a}/2}{a}\right)\right] \Gamma^{S}_{A} - A \cos k_o x - D \sin k_o x$$

(21)

$$- \sum_{m=1,2}^{\infty} \sum_{n=0,2}^{\infty} \left[\gamma_{mn}\, I_{mn}{}^{S}_{A}(x,y) + \sin m\pi\left(\frac{x+{}^{a}/2}{a}\right) \cos n\pi\left(\frac{y+{}^{b}/2}{b}\right)\right] A_{mn}^{S}{}_{A}$$

$$= \gamma_{10}\, I_{10}{}^{S}_{A}(x,y) + \sin \pi\left(\frac{x+{}^{a}/2}{a}\right)$$

As before, this equation was written at a finite number of points within the aperture and a truncated series of modes was used to describe the field. There are now two constants per equation, and these are determined as before by requiring the integral equation to be valid at two extra points in the aperture.

In order to construct the solution for the case when antenna #1 is excited and antenna #2 is kept parasitic and terminated in a matched load, it is simply necessary to superimpose the symmetric and antisymmetric solutions and to divide by two in order to normalize to the source amplitudes. Following the usual procedures for such problems and using the subscript p to denote this case, one obtains the coefficients below:

$$S_{11} = S_{22} = \tfrac{1}{2}(\Gamma^S + \Gamma^A) \tag{22}$$

$$S_{12} = S_{21} = \tfrac{1}{2}(\Gamma^S - \Gamma^A) \tag{23}$$

$$A_{mn_p} = \tfrac{1}{2}(A_{mn}^S + A_{mn}^A) \tag{24}$$

$$B_{mn_p} = \tfrac{1}{2}(B_{mn}^S - B_{mn}^A) \tag{25}$$

V. THEORETICAL AND EXPERIMENTAL RESULTS

The convergence of the matrix solution to equation (21) has been studied in detail in order to determine its accuracy. Furthermore, since this entire mathematical development rests upon the applicability of the scalar approximation (9), an experimental program was undertaken to provide results to compare with the theory. These two subjects are discussed in detail below.

The convergence properties of the solution for single and coupled waveguides are basically different and so they are discussed separately. Table I shows the variation in input reflection coefficient for the single radiating waveguide as various sets of modes are chosen to represent the field distribution. All accuracies are relative to the highest order solution (Case 6). The single mode approximation yields an imaginary part which is in error by about twenty percent and a real part (much smaller than the imaginary part) which has the wrong sign. A solution with four modes (Case 3) but including the 12 and 14 modes to aid in approximating the E-plane distribution is among the more accurate solutions shown. This high accuracy is coincidental and was not generally repeated, but this solution was always within about ten percent of the highest order solution for all cases examined, and so was later used to form the basis of the coupled waveguide study (after the addition of another mode having anti-symmetrical x-dependence). Another four mode solution (Case 2) with one less mode to describe the L-plane distribution was less accurate than the results of Case 3. Nine modes (Case 4) serve to determine the imaginary

TABLE I

A. SINGLE WAVEGUIDE REFLECTION COEFFICIENT

$a/\lambda = 1.0;\ b/\lambda = 0.3$

CASE	MODES	REFLECTION COEFFICIENT
1	10	$\Gamma_{10} = -0.3928 - j0.32680$
2	10, 12, 30, 32	$\Gamma_{10} = 0.10230 - j0.24786$
3	10, 12, 14, 30	$\Gamma_{10} = 0.07742 - j0.26969$
4	10, 30, 50, 12, 32, 52, 14, 34, 54	$\Gamma_{10} = 0.08309 - j0.27193$
5	10, 30, 50, 70, 12, 32, 52, 72, 14, 34, 54, 74	$\Gamma_{10} = 0.08319 - j0.27085$
6	10, 30, 50, 12, 32, 52, 14, 34, 54, 16, 36, 56	$\Gamma_{10} = 0.07729 - j0.27786$

B. MODE AMPLITUDES FOR CASE 6

$\Gamma_{10} = 0.07729 - j0.27786$	$A_{14} = -0.01965 - j0.03589$
$A_{30} = -0.05521 + j0.00195$	$A_{34} = -0.00302 + j0.00051$
$A_{50} = -0.00638 - j0.00151$	$A_{54} = -0.00076 - jo.00034$
$A_{12} = -0.05538 - j0.08904$	$A_{16} = -0.00736 - j0.01418$
$A_{32} = -0.00762 + j0.00171$	$A_{36} = -0.00107 + j0.00030$
$A_{52} = -0.00165 + j0.00022$	$A_{56} = -0.00021 + j0.00021$

part of the reflection coefficient to within about two percent of the highest order solution considered and the real part to about eight percent. A comparison of this solution (Case 5) with Case 6 illustrates that the distribution in the H-plane (x-direction) is fairly well approximated by a few terms because the three extra modes introduced to more precisely approximate the x dependence barely change the value of the reflection coefficient. Table IB lists the mode amplitudes for another twelve mode solution (Case 6) which was chosen to provide the highest order approximation to the E-plane distribution, and hence the most accurate solution for the reflection coefficient. Inspection of these mode amplitudes is a convenient measure of convergence because the integral equation (21) is solved at a number of discrete points within the aperture and therefore one can expect an error in the reflection coefficient which is approximately the size of the modes truncated. Table IB therefore shows why it is especially necessary to include the modes which approximate the E-plane field variation (12,14 etc.).

Figure 2 shows the aperture field variation provided by this twelve mode solution. The approximation of the singularity at $y = +{}^{b}/_{2}$ is evident in this distribution, as is the fact that the H-plane distribution is fairly accurately represented by a single mode.

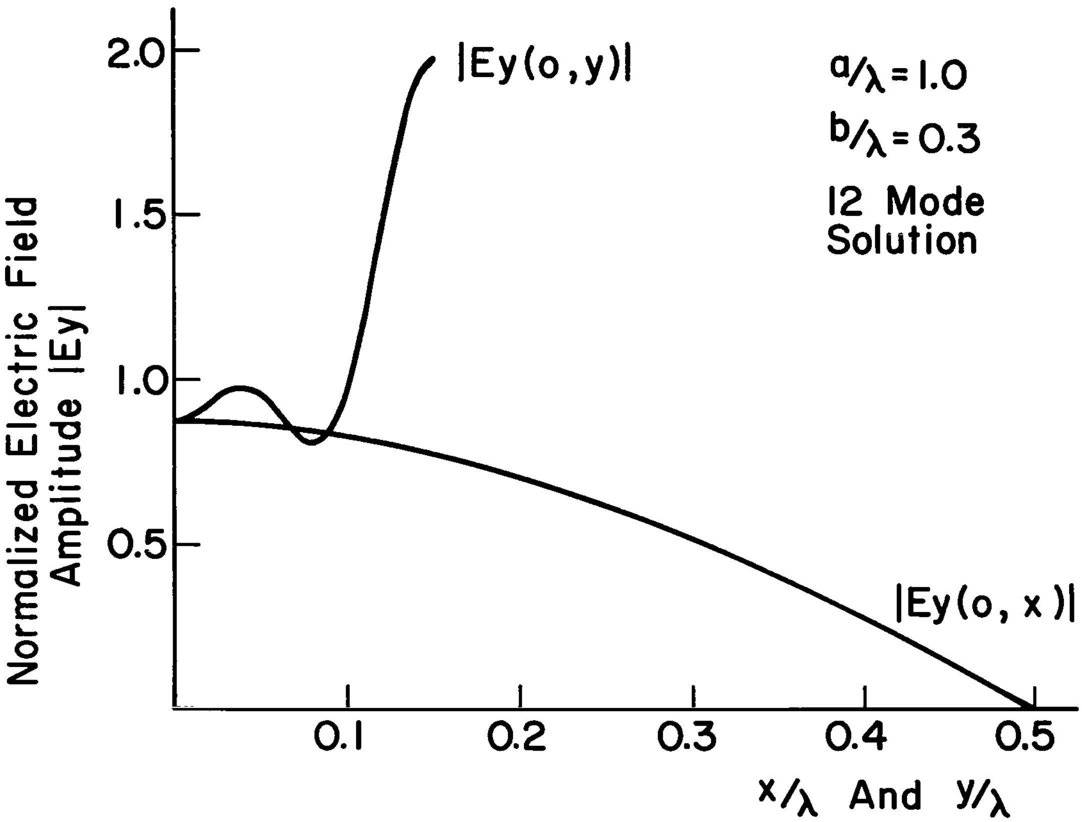
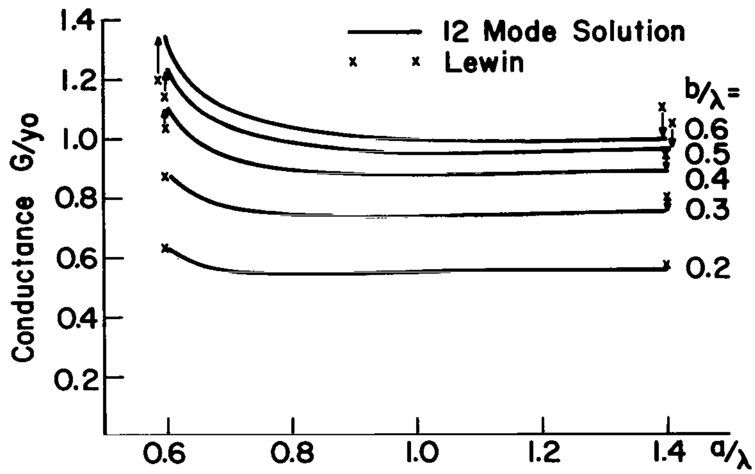


FIGURE 2 APERTURE ELECTRIC FIELD DISTRIBUTION FOR RADIATING WAVEGUIDE

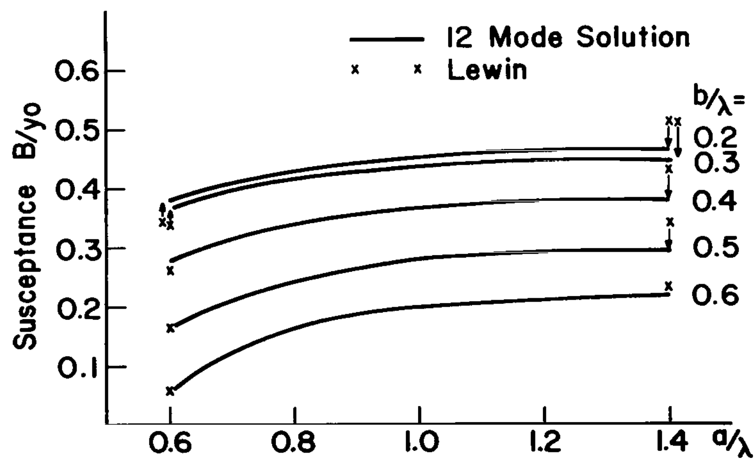
Figure 3 displays a set of curves of input admittance of a radiating rectangular waveguide. These curves are presented in the form of the well known results of Lewin[4] and a few points are shown to illustrate the correlation with Lewin's data. This correlation is very good except for deviations of about ten percent in the conductance for thick slots, and of about thirteen percent in the susceptance for very thin slots. There is no experimental evidence to compare with these two theoretical results, but since they are both derived using the same scalar approximation (9) and since the present analysis is a convergent solution, it is claimed that the data of Figure 3 is a more accurate solution to the integral equation. Whether or not these curves show closer agreement with experiment will determine the usefulness of the scalar approximation itself.

Regarding the usefulness of this scalar approximation for describing waveguide admittance, Figure 4 compares the experimental data of Venema[26,27] with the results of this theory and the work of Lewin. These results indicate that the solution in terms of this single set of hertzian potentials is indeed a valid approximation for describing waveguide admittance. Venema's data does appear to follow the results of this theory more closely than Lewin's theoretical results, but for the size slots chosen there is so little difference between the two that no conclusion can be drawn except that they both agree very closely with experiment.

It is appropriate to point out that Lewin's stationary solution, which uses a single mode, is considerably more accurate than the single mode approximation used with this non-stationary solution. The advantage of the method presented in this note is that enough higher order modes can be conveniently included to bring about a convergent solution.



A. Conductance Vs. Slot Width



B. Susceptance Vs. Slot Width

FIGURE 3 ADMITTANCE OF SINGLE RADIATING WAVEGUIDE

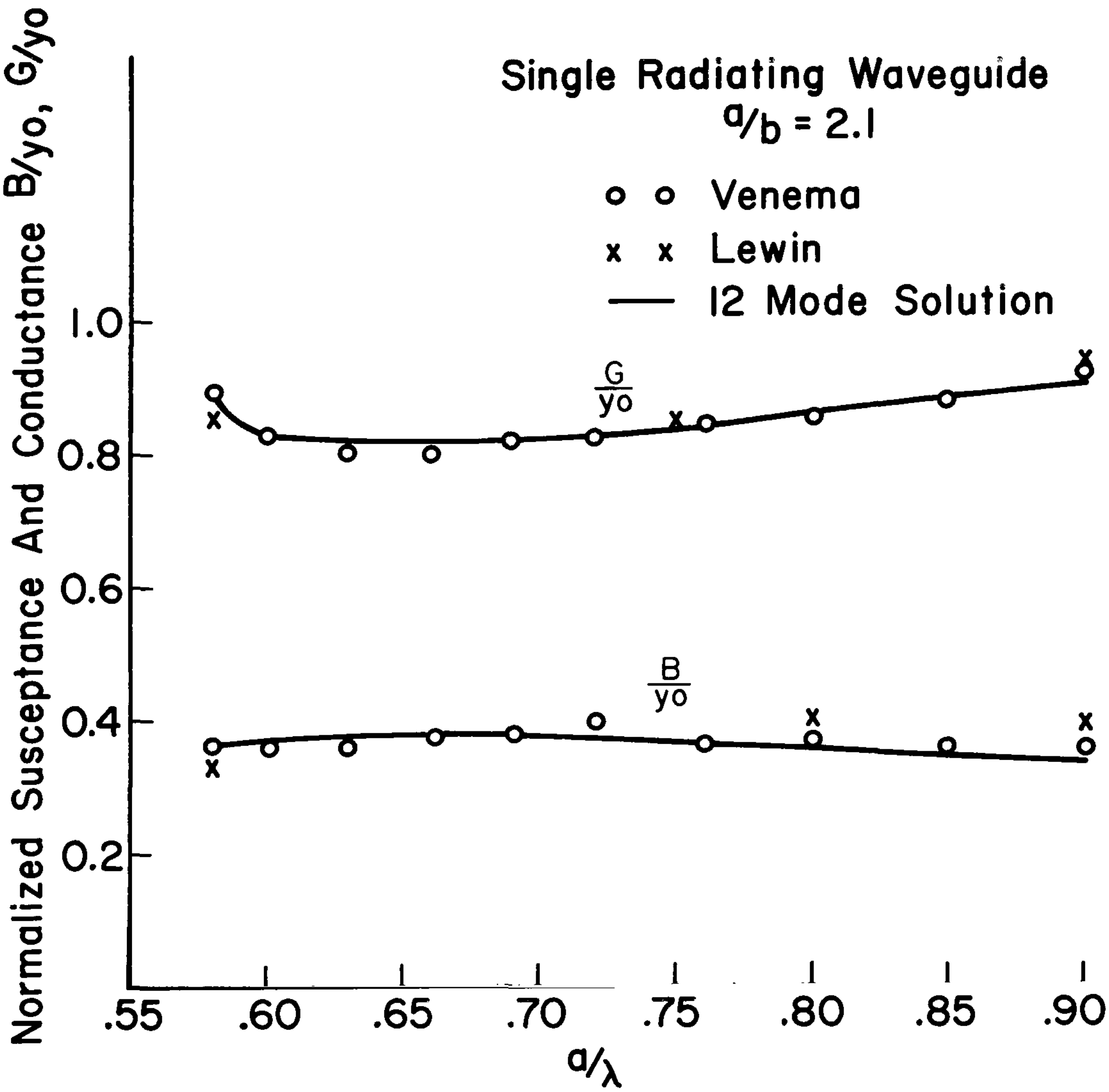


FIGURE 4 COMPARISON BETWEEN THEORY AND EXPERIMENT

The solution of equation 21 for the case of two collinear coupled waveguides involves some of the same convergence properties as the single waveguide, but it also offers the possibility of relaxing the convergence requirements for the coupled parameters. The self admittance terms necessarily behave in a very similar manner to the single waveguide, and so all of the comments about truncation apply here as well. The presence of the second slot in the ground plane (with the second waveguide terminated in a matched load) causes a small change in the input reflection coefficient of the driven antenna. Figure 5 shows that the imaginary part of the reflection coefficient undergoes a change of about eight percent with spacing variations, while the real part being much smaller, changes by about fifty percent. The effect of this coupling is no longer visible when the center to center distance exceeds about 1.2λ. When the spacing $^{d}/_{\lambda}$ exceeds five, the subtraction indicated in equation (23) begins to introduce numerical inaccuracies, but for the range of spacings used throughout this report no additional approximation is introduced by this symmetric-antisymmetric formulation.

A more familiar property of closely spaced radiating slots is the direct coupling between the radiating signal and the parasitic antenna. Table IIA shows that for the case considered this parameter is relatively insensitive to the approximation used in computing the aperture field.

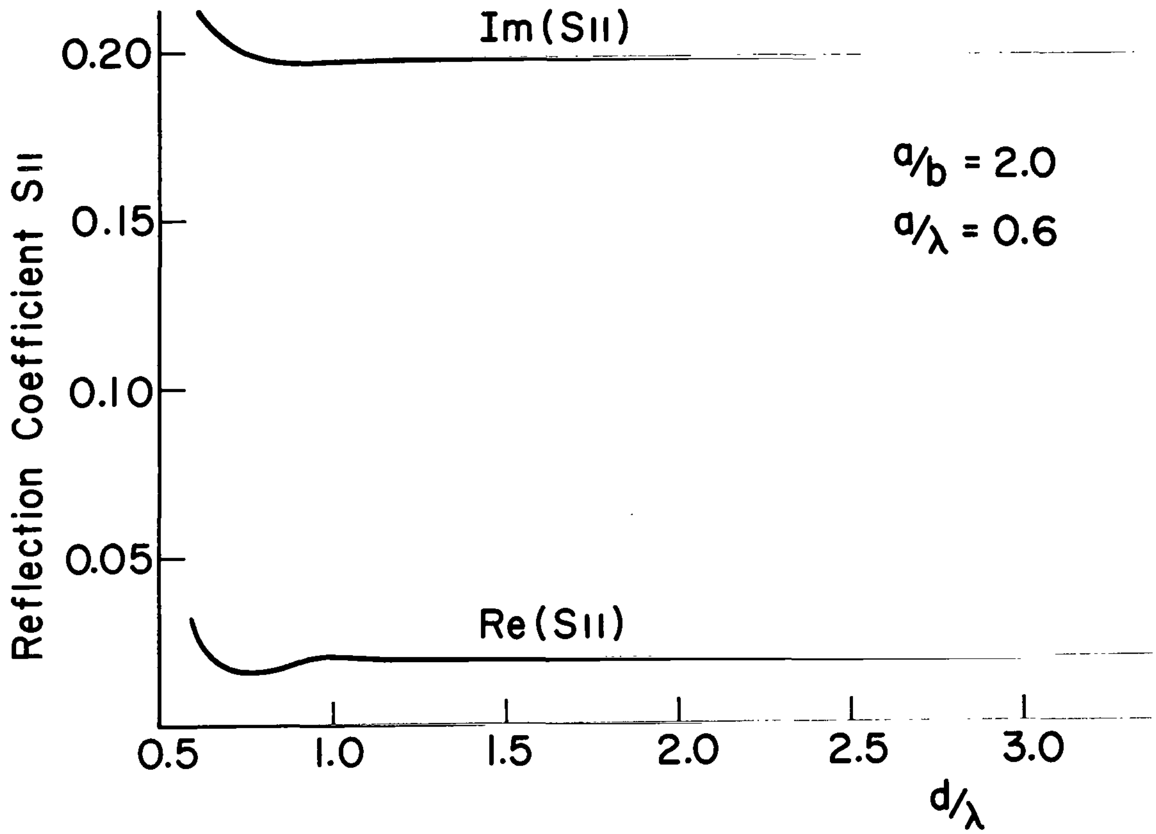


FIGURE 5 REFLECTION COEFFICIENT OF DRIVEN ANTENNA WITH PARASITIC ANTENNA MATCHED

TABLE II

A. COUPLING COEFFICIENT FOR COLLINEAR COUPLED WAVEGUIDES

a = 2.286 cm
d = 2.921

b = 1.016 cm
frequency = 10.0 GHz

CASE	MODES	COUPLING COEFFICIENT S_{12}
1	10	S_{12} = 0.01669 + j0.00754
2	10, 20, 30, 40, 12, 14	S_{12} = 0.01819 + j0.01124
3	10, 20, 30, 12, 14	S_{12} = 0.01786 + j0.01120
4	10, 20, 30, 12, 22, 32, 14, 24, 34	S_{12} = 0.01874 + j0.01047

B. MODE AMPLITUDES FOR CASE 4

S_{11} = 0.05595 - j0.22203	S_{12} = 0.01874 + j0.01047
A_{20_p} = 0.00233 - j0.00210	B_{20_p} = -0.00294 - j0.03949
A_{30_p} = -0.02432 - j0.00741	B_{30_p} = -0.00344 + j0.00266
A_{12_p} = -0.05513 - j0.09512	B_{12_p} = 0.00211 - j0.00130
A_{22_p} = 0.00030 - j0.00033	B_{22_p} = -0.00085 -j0.00569
A_{32_p} = -0.00652 - j0.00053	B_{32_p} = -0.00092 + j0.00095
A_{14_p} = -0.01430 - j0.02800	B_{14_p} = 0.00058 - j0.00041
A_{24_p} = 0.00004 - j0.00010	B_{24_p} = -0.00025 +j0.00161
A_{34_p} = -0.00184 - j0.00009	B_{34_p} = -0.00026 - j0.00027

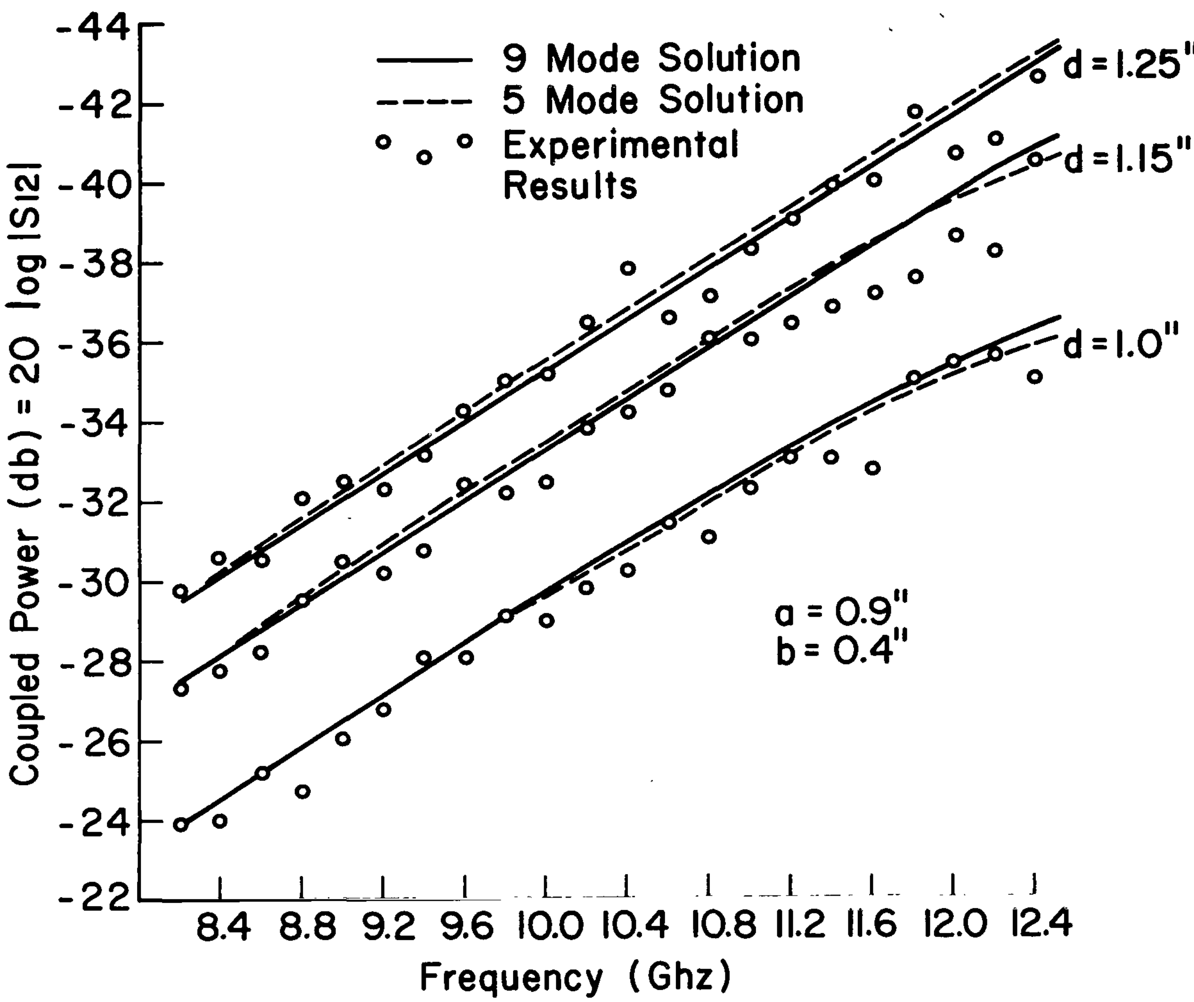
Table IIB shows the coupling coefficients for Case 4 with one antenna driven and the other terminated in a matched load. This data is revealing because it demonstrates that the error involved in estimating S_{11} due to approximating the aperture field singularity is likely to be significantly larger than that involved in evaluating S_{21} for the same field approximation. (Notice that the ratio of $|A_{12}|$ to $|S_{11}|$ is much larger than the ratio of $|B_{12}|$ to $|S_{12}|$). This is because the entire field exhibits this singularity, not merely the reflected wave. Table IIB does show that the ratio $|A_{12}|/|1+S_{11}|$ is indeed about the same (0.1) as the ratio $|B_{12}|/|S_{12}|=(0.115)$.

These comments relate to the convergence of the chosen approximate solution, but reveal nothing about the correspondence of this analysis to the physical problem of collinear slot coupling. It remains to show that the solution based upon this approximate scalar model has relevence to the problem studied.

An experiment has been conducted to test the validity of the analysis for the collinearly coupled waveguide case. Since the coupled signal was so small compared to the transmitted signal, none of the conventional techniques for measuring the phase of the coupled signal sufficed. To solve this problem a new technique[28] was devised which made it possible to measure the phase to within $\pm 4^{\circ}$, even when the coupled signal amplitude was -40 db relative to the transmitted signal. Figure 6 shows that this data compares very closely with two theoretical solutions. The five and nine mode theoretical solutions yield nearly the same coupled power, but the difference in phase between these two approximations can be as much as 15^{o}. This discrepancy is caused by the omission of the higher order modes (22) and (24) in the five mode solution. These modes possess odd symmetry about the center of each waveguide and become very large at the high frequencies where the solutions differ substantially. The amplitude of B_{22_p} is about one third $|S_{12}|$ at 10.0 GHz, but it is about equal to $|S_{12}|$ at 12.5 GHz.

Since the five mode solution does yield reasonably accurate results and requires much less computation time, it has been used in Figures 7a and 7b. These figures show the amplitude and phase of the coupled signal for waveguides with the dimensional ratios $^a/b = 2.0$ and 2.5 and for waveguide widths up to $^a/\lambda = 0.99$. The sets of curves with different $^a/b$ are essentially parallel and sufficiently close together to allow interpolation between the. Therefore, the curves provided cover the range of almost all commercially available wave guides.

Several important points are noted when these results are considered in detail. The first relates to the work of Lyon et.al.[15] in which it is observed that the coupling amplitude $|S_{12}|$ varies very nearly like $^1/d^2$ as the distance d between slots is varied, and that this is approximately



A. Coupled Power - Theory Vs. Experiment

FIGURE 6 COMPARISON BETWEEN THEORY AND EXPERIMENT - COUPLED WAVEGUIDES

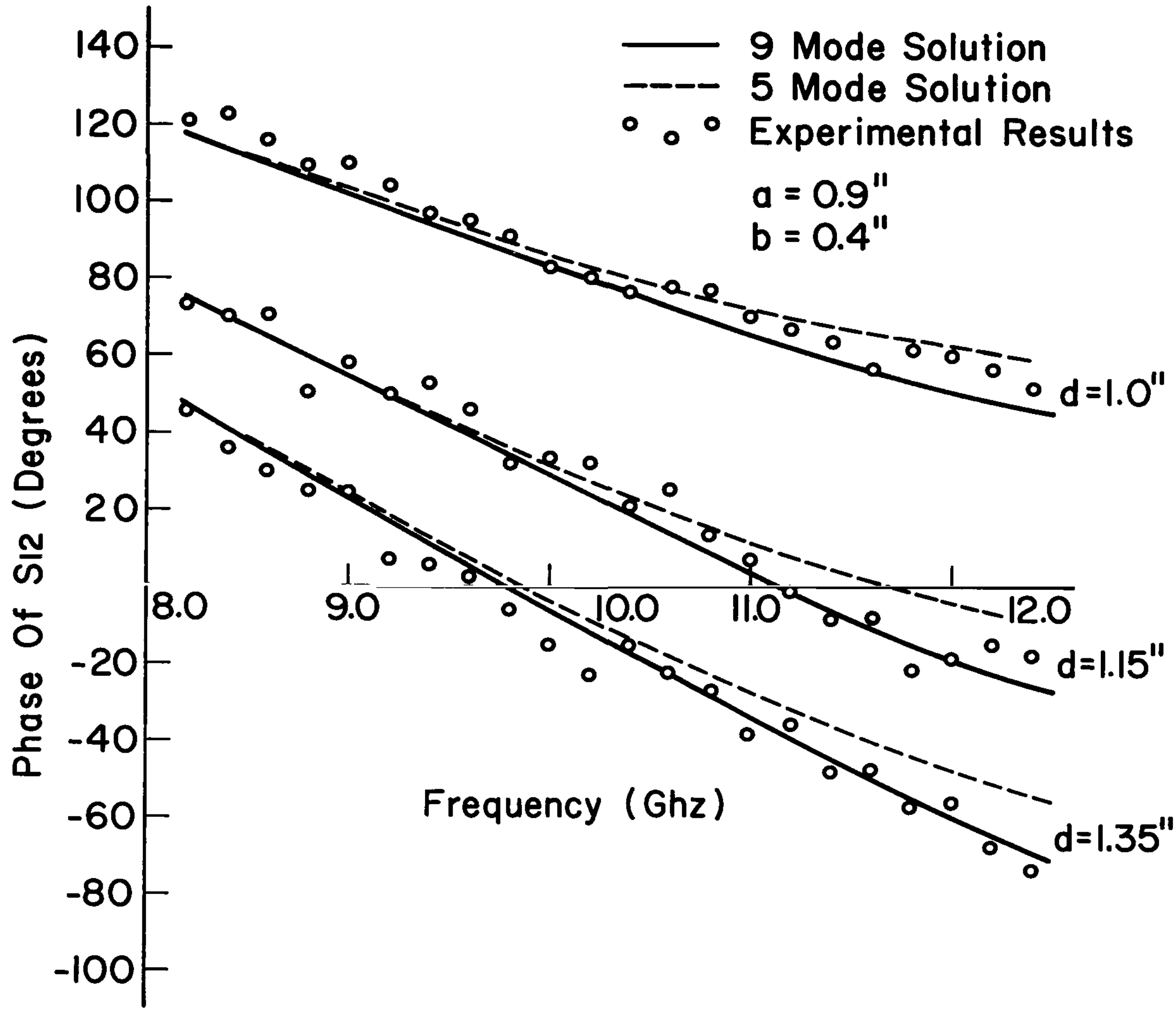
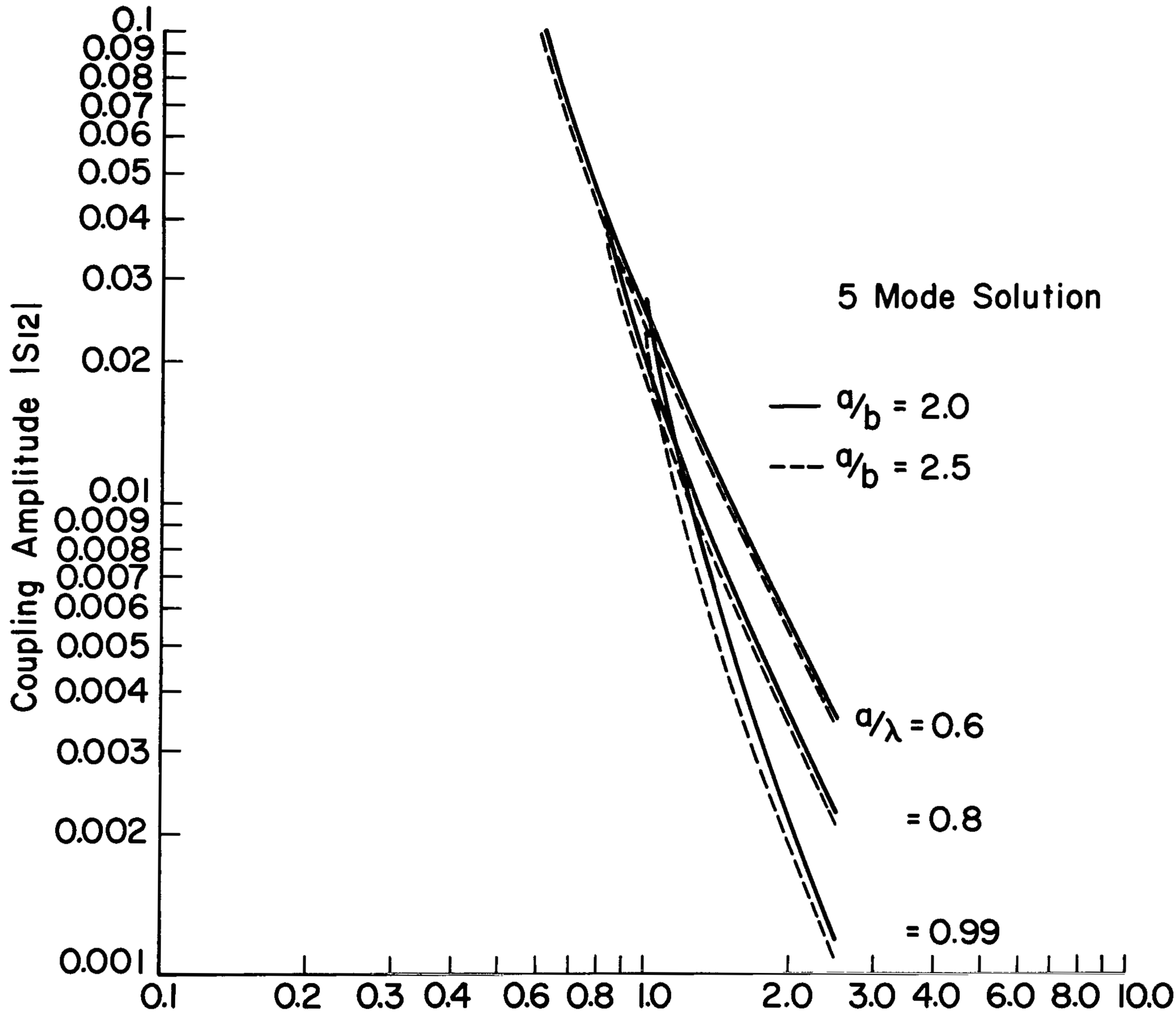


FIGURE 6B PHASE OF S12 - THEORY VS. EXPERIMENT



A. Coupling Amplitude |S12|

FIGURE 7 COUPLING BETWEEN COLLINEAR SLOTS

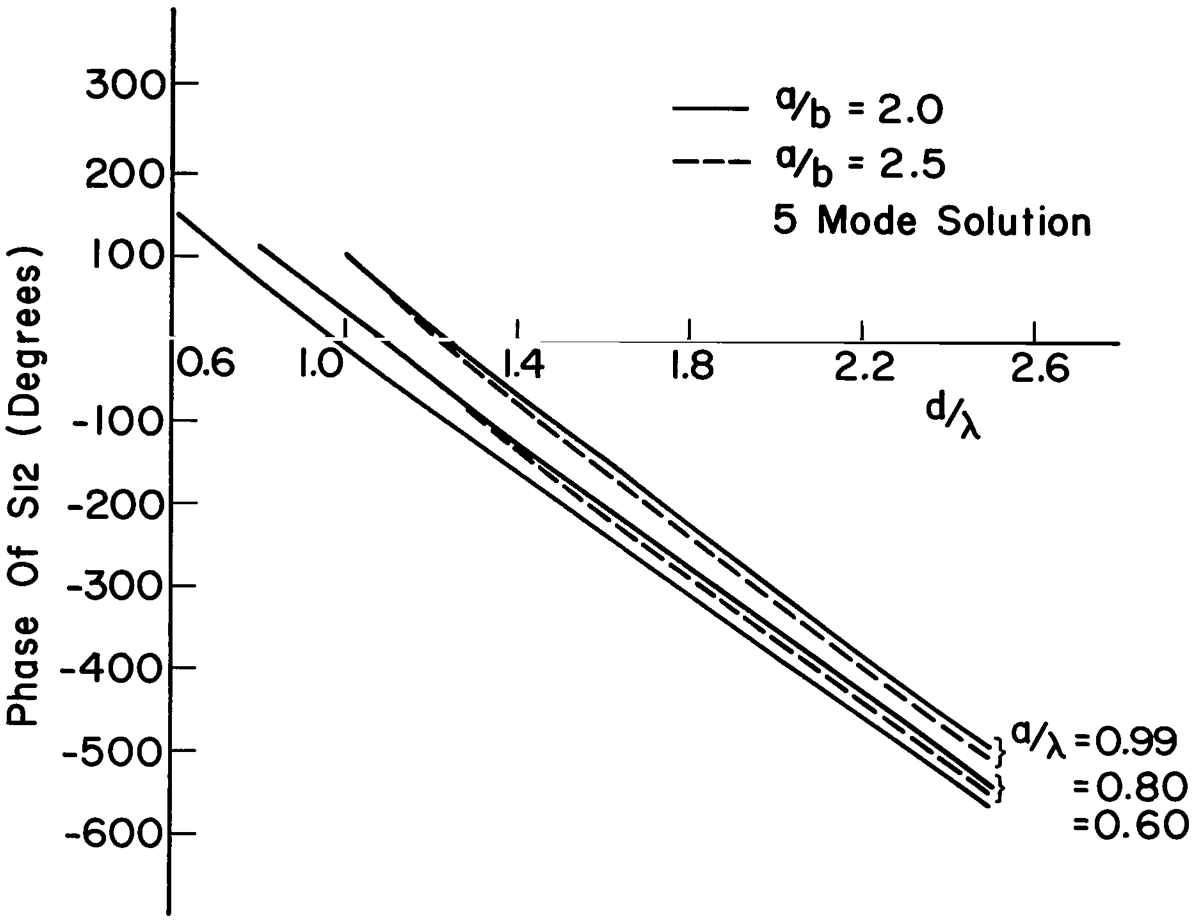


FIGURE 7B PHASE OF S12

true even for very close spacing. Unfortunately, this is only true in a qualitative way and the coupling does not follow this law closely enough to allow its use quantitatively. The complex coupling coefficients S_{12} does vary like $[\exp(-jk_o d)]/d^2$ for the collinear case when the spacing is sufficiently large, but Figure 7a shows a pronounced curvature of the amplitude plot, especially for the larger slots considered, and so it is obvious that the asymptotic behavior of $|S_{12}|$ does not begin to dominate unitl d/λ exceeds 2.0 or 2.5. The convergence of the curves in Figure 7b shows that the phase relationship is even less predictable and, though this is not shown in the Figure, the asymptotic type of variation in phase does not dominate until d/λ exceeds about 5. This same phase nonlinearity is exhibited by the more accurate data of Figure 6b, and so is clearly not caused by the approximations involved in the computations for 7b.

VI. CONCLUSION AND COMMENTS ABOUT ARRAY THEORY

This study has shown that a set of fields derivable from a single component of the hertzian magnetic potential function can provide an extremely accurate solution to the boundary value problem for a single waveguide radiating through a perfectly conducting ground plane or for two collinear radiating slots. The solution is rigorous for idealized slots of zero thickness, and is a good approximation even when the slot is the size of a standard commercial waveguide.

This basic formulation could be used to describe the mutual coupling between elements of an array, especially if the waveguide height is less than one half wavelength, but it can yield no information about the cross-coupled fields that are excited at the array face.

The convergence properties of the solution have been emphasized, and no attempt has been made to simplify the numerical solution. An extensive experimental program was undertaken in an attempt to measure the amplitude and phase of the coupling as accurately as possible so that these could be compared with the theoretical results. The agreement between theory and experiment is excellent.

In addition to the detailed study of the problem at hand, this work has led to certain qualitative conclusions which have application to the development of an approximate array theory for radiating waveguides. These conclusions concern the errors which arise using various sorts of approximations, which errors seem to fall naturally into two categories: those which have to do with specific approximations to the aperture fields or the type of solution used (ie. a variational theory or the truncated series approximation used here), and those fixed by the physical process of coupling independent of the mechanics of solution. Errors of the first type have been discussed in sufficient detail in Section V, and some hints have also been given in that section relative to simplifying the theory. Concerning the physical processes which must be accounted for by any theory, the following conclusions now appear evident.

a) If the waveguide dimensions are such that a cross-polarized TE_{10} mode (ie. one with its electric field in the x-direction) can propagate, then the array theory must allow for coupling between the two senses of polarization.

b) This analysis has shown that coupling between collinear slots is so small that the reflection coefficient at one slot, with the waveguide of the second slot terminated in a matched load, is very nearly the same as for the isolated slot. This is not true for closely spaced parallel slots because they couple much more strongly, and so it will be necessary in general to account for the array geometry in the development of the slot self admittance.

c) The asymptotic approximation $S_{12} = c[\exp(-jk_o d)]/d^2$ provides convenient insight into the mechanism of coupling, but it does not provide accurate amplitude and phase information if the spacing between antennas is less than five wavelengths. An accurate waveguide array theory must therefore be a near field theory.

APPENDIX

Expansion of the Waveguide Fields in terms of two Magnetic Hertzian Potentials.

The complete field in a rectangular waveguide can be expanded rigorously in terms of one component of both the electric and magnetic hertzian potentials when these components are both taken along the same axis. The most common choice of axis is the direction of propagation $(\hat{z})$ but it is sometimes convenient to choose these vectors along either the $\hat{x}$ or the $\hat{y}$ axis. When the fields are to be matched at a plane boundary at the end of a waveguide, the fields can be expanded in terms of two magnetic hertzian potentials. To show this, it is convenient to choose the magnetic and electric hertzian potentials $\hat{x}\pi_{mx}$ and $\hat{x}\pi_{ex}$ as the basis for a rigorous expansion of the waveguide fields for the general waveguide discontinuity problem. The proof will then proceed by showing that a different set of two magnetic hertzian potential functions $\hat{x}\pi_{mx}$ and $\hat{y}\pi_{my}$ can be made to satisfy the same boundary conditions as the original set.

An $\exp[+j\omega t]$ time dependence has been suppressed. The general electromagnetic field is written below as derived from magnetic and electric hertzian potentials.

$$\text{(A1)} \qquad \bar{E}(r) = -j\omega\bar{\nabla} \times \bar{\pi}_m + \nabla(\nabla \cdot \bar{\pi}_e) + k_o^2\,\bar{\pi}_e$$

(A2) $\bar{B}(r) = \nabla(\nabla \cdot \bar{\pi}_m) + k_o^2\pi_m + j\frac{k_o^2}{\omega} \nabla \times \bar{\pi}_e$

As pointed out earlier, the general field inside of a rectangular waveguide can be written in terms of $\hat{x}\pi_{mx}$ and $\hat{x}\pi_{ex}$ as:

(A3) $\bar{E}(r) = -j\omega\nabla \times (\hat{x}\pi_{mx}) + \nabla(\nabla \cdot \hat{x}\pi_{ex}) + \hat{x}\, k_o^2\pi_{ex}$

(A4) $\bar{B}(r) = \nabla(\nabla \cdot \hat{x}\pi_{mx}) + x\, k_o^2\pi_{mx} + j\frac{k_o^2}{\omega} \nabla \times (\hat{x}\pi_{ex})$

The alternate set of magnetic potentials is given below. The prime symbol is used to designate that the magnetic potential π'_{mx} in this expansion need not be the same as π_{mx} in the expansion of equations (A3) and (A4).

(A5) $\bar{E}(r) = -j\omega\nabla \times (\hat{x}\pi'_{mx}) -j\omega\nabla\cdot(\hat{y}\pi_{my})$

(A6) $B(r) = \nabla(\nabla \cdot \hat{x}\pi'_{mx}) + \nabla(\nabla \cdot \hat{y}\pi_{my}) + x\, k_o^2\pi'_{mx} + \hat{y}\, k_o^2\pi_{my}$

The electric fields tangential to the aperture plane are both equated to the tangential aperture field, and therefore are equated to each other. In component form, the two resulting equations are:

(A7) $$j\omega\left(\frac{\partial\pi_{mx}}{\partial z} - \frac{\partial\pi'_{mx}}{\partial z}\right) = \frac{\partial^2\pi_{ex}}{\partial x\partial y}$$

and

(A8) $$\frac{\partial^2\pi_{ex}}{\partial x^2} + k_o^2\pi_{ex} = +j\omega\frac{\partial\pi_{my}}{\partial z}$$

If the field can be expanded in terms of two magnetic potentials, then the above relations which guarantee the equality of the tangential components of $\bar{E}$ in the aperture, should also guarantee that the tangential components of $\bar{B}$ as expanded using either set of functions is also equal at the aperture.

The magnetic fields tangential to the aperture plane are equated to the tangential aperture field and hence to each other. The resulting equations are:

$$(A9) \quad \frac{\partial^2}{\partial x \partial y} \pi_{mx} - \frac{\partial^2}{\partial x \partial y} \pi'_{mx} + j\frac{k_o^2}{\omega} \frac{\partial \pi_{ex}}{\partial z} = \frac{\partial^2 \pi_{my}}{\partial y} + k_o^2 \pi_{my}$$

$$(A10) \quad \left(\frac{\partial^2}{\partial x^2} + k_o^2\right) (\pi_{mx} - \pi'_{mx}) = \frac{\partial^2 \pi_{my}}{\partial x \partial y}$$

If these equalities can be proved to be identically true using (A7) and (A8) then the desired proof will be accomplished. This can be shown to be the case for equation (A10) by taking $\frac{\partial^2}{\partial x \partial y}$ of equation (A8) and by subtracting $j\omega\frac{\partial}{\partial z}$ of equation (A10) from this. The resulting equation can be written in the form below.

$$(A11) \quad \left(\frac{\partial^2}{\partial x^2} + k_o^2\right) \left[j\omega\frac{\partial}{\partial z} (\pi_{mx} - \pi'_{mx})\right] - \frac{\partial^2 \pi_{ex}}{\partial x \partial y} = \theta.$$

This equation is identically satisfied using (A7). Similarly, equation (A9) can be shown to be identically satisfied by starting with equation (A8) and using the fact that π_{ex} satisfies the scalar helmholtz equation to re-write (A8) in the form shown below.

(A12) $$\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2}\right) \pi_{ex} = -j\omega\frac{\partial \pi_{my}}{\partial z}$$

Taking $+\frac{j}{\omega}\left(\frac{\partial^2}{\partial y^2} + k_o^2\right)$ of this equation and subtracting it from $\frac{\partial}{\partial z}$ of equation (A9), simplifying and again using the helmholtz equation one obtains the resulting equation:

(A13) $$\frac{\partial^2}{\partial x \partial y}\left[j\omega\left(\frac{\partial \pi_{mx}}{\partial z} - \frac{\partial \pi'_{mx}}{\partial z}\right)\right] = \frac{\partial^2}{\partial x \partial y}\left(\frac{\partial^2 \pi_{ex}}{\partial x \partial y}\right)$$

This equality is also identically satisfied by equation (A7) and therefore the two expansions are equivalent for the problem considered.

REFERENCES

1. Galindo, V., and Wu, C. P.: Properties of a Phased Array of Rectangular Waveguides with Thick Walls, IEEE Transactions on Antennas and Propagation, Vol. AP-14, pp. 149-158, March 1966.

2. Wu, C. P., and Galindo, V.: Properties of a Phased Array of Rectangular Waveguides with Thin Walls, IEEE Transactions on Antennas and Propagation, Vol. AP-14, pp. 163-173, March 1966.

3. Farrel, G. F. Jr., and Kuhn, D. H.: Mutual Coupling Effects of Triangular-Grid Arrays by Modal Analysis, IEEE Transactions on Antennas and Propagation, Vol. AP-14, pp. 652-654, September 1966.

4. Lewin, L.: Advanced Theory of Waveguides, Iliffe and Sons, Ltd., London, Chapter 6, 1951.

5. Cohen, M. H., Crowley, T. H., Levis, C. A.: The Aperture Admittance of a Rectangular Waveguide Radiating into a Half Space, (ATI-133707) Antenna Laboratory, Ohio State University, Research Foundation, Rept. 339-22, November 14, 1951.

6. Compton, R. T.: The Aperture Admittance of a Rectangular Waveguide Radiating into a Lossy Half Space, Antenna Lab 1691-1, University of Ohio, September 30, 1963.

7. Villeneuve, A. T.: Admittance of a Waveguide Radiating Into a Plasma Environment, IEEE Transactions on Antennas and Propagation, Vol. AP-13, pp. 115-121, January 1965.

8. Rudduck, R. C.: The Admittance of a Rectangular Waveguide Aperture Covered by a Lossless Dielectric Slab, Electro Science Lab, Ohio State University, Rept. 1691-22, May 15, 1967.

9. Croswell, W. F., Rudduck, R. C., and Hatcher, D. M.: The Admittance of a Rectangular Waveguide Radiating into a Dielectric Slab, IEEE Transactions on Antennas and Propagation, Vol. AP-15, No. 5, pp. 627-633, September 1967.

10. Galejs, J.: Admittance of Waveguide Radiating into a Stratefied Plasma, IEEE Transactions on Antennas and Propagation, AP-13, No. 1, pp. 64-70, January 1966.

11. Wheeler, G. W.: Coupled Slot Antennas, Ph.D. Thesis, Harvard University, Cambridge, Mass., May 1950.

12. Levis, C. A.: Variational Calculations of the Impedance Parameters of Coupled Antennas, Ohio State Univ. Research Foundation, Rept. 667-16, Contract AF 33(616)-3353, August 15, 1956.

13. Galejs, J.: Self and Mutual Admittances of Waveguides Radiating into Plasma Layers, Radio Science, J. Res. NBS/USNC-URSI, Vol. 69D, No. 2, pp. 179-189, February 1965.

14. Richmond, J. H.: A Reaction Theorem and Its Application to Antenna Impedance Calculations, IRE Transactions on Antennas and Propagation, Vol. AP-8, pp. 515-520, November 1961.

15. Lyon, J. A. M., Kalafus, R. M., Kwon, Y. K., Diegenis, C. J., Ibrahim, M. A. H., and Chen, C. C.: Derivation of Aerospace Antenna Coupling--Factor Interference Prediction Techniques, Tech. Rept. AFAL-TR-66-57, The University of Michigan, Radiation Laboratory, April 1966.

16. Russo, P. M., Rudduck, R. C., and Peters, L.: A Method for Computing E-Plane Patterns of Horn Antennas, IEEE Transactions on Antennas and Propagation, Vol. AP-13, No. 2, pp. 219-224, March 1965.

17. Yu, J. S., Rudduck, R. C., and Peters, L.: Comprehensive Analysis for E-Plane of Horn Antennas by Edge Diffraction Theory, IEEE Transactions on Antennas and Propagation, Vol. AP-14, No. 2, pp. 138-149, March 1966.

18. Rudduck, R. C.: Application of Wedge Diffraction to Antenna Theory, NASA Contractor Report, CR-372, January 1966.

19. Iijima, Taizo: Theory of Radiation from an Open-Ended Waveguide, Electronics and Communications in Japan, Vol. 48, pp. 21-28, April 1965.

20. Bouwkamp, C. J.: Diffraction Theory, Reports on Progress in Physics, Vol. 17, pp. 35-100, 1954.

21. Copson, E. T.: An Integral Equation Method of Solving Plane Diffraction Problems, Proc. Royal Soc. A., Vol. 186, pp. 100-118.

22. Copson, E. T.: Diffraction by A Plane Screen, Proc. Royal Soc. A., Vol. 202, pp. 277-284.

23. Kieburtz, R. B., and Ishimaru, A.: Aperture Fields of an Array of Rectangular Apertures, IRE Trans. Vol. AP-9, pp. 663-671, 1962.

24. Edelberg, S., and Oliner, A. A.: Mutual Coupling Effects in Large Antenna Arrays: Part I - Slot Arrays, IRE Transactions on Antennas and Propagation, Vol. AP-8, pp. 286-297, May 1960.

25. King, R. W. P., and Sandler, S. S.: The Theory of Broadside Arrays, and the Theory of Endfire Arrays, IEEE Transactions on Antennas and Propagation, Vol. AP-12, pp. 269-275, 276-280, May 1964.

26. Venema, H. J.: Technical Report on the Conductance of a Rectangular Waveguide-fed Slot, W33-038-ac-20778 TR-1, Electrical Engineering Research Laboratory, Engineering Experiment Station, University of Illinois, Urbana, Ill., August 15, 1950.

27. Chase, M. N.: Interim Engineering Report on Antennas for High Speed Aircraft, W33-038-ac-20778 IR-12, Electrical Engineering Research Laboratory, Engineering Experiment Station, University of Illinois, Urbana, Ill., June 1, 1950.

28. Mailloux, R. J. and LaRussa, F. J.: A Microwave Phase Bridge Technique for Measuring the Mutual Coupling of Identical Coupled Antennas, IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-16, February 1968.

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