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ABSTRACT

Data from the IMP-II magnetic experiment obtained in interplanetary space are analyzed to study the character of the fluctuations in the magnetic field from the magnetohydrodynamic point of view. The fluctuation in interplanetary space is quite anisotropic relative to the steady magnetic field, with the fluctuations transverse to the magnetic line of force much larger than the longitudinal component of fluctuations. This anisotropy gradually decays as the amplitude of the fluctuation increases, and tends to become rather isotropic.
These fluctuations may be related to the thermal anisotropy of solar wind ions, with the conversion of the transverse fluctuations into the longitudinal ones occurring by a non-linear effect when the amplitude of the fluctuation grows above a certain level. Frequency dependence of the mode of fluctuations is also investigated.

INTRODUCTION

Extensive studies of recent satellite measurements in outer spaces have offered a large amount of knowledge about the extraterrestrial environment, and most of fundamental parameters and gross properties of the interplanetary and magnetosphere have been obtained. (Heppner et al., 1963 ; Cahill et al., 1963 ; Ness et al., 1964 ; Holzer et al., 1966 ; Fairfield et al., 1967 ; Ness, 1967). The earth's magnetic field terminates on the sun-lit side of the earth at about 10 ± 2 R_E and is confined to the magnetosphere. Between the magnetosphere and the interplanetary space, there is a transition region called the magnetosheath with large amplitude magnetic fluctuations. In interplanetary space, the magnetic field is rather quiet and steady, but still it fluctuates with various frequencies (Holzer et al., 1966 ; Siscoe et al., 1967). Recent satellite measurements revealed that the ion temperature distribution is anisotropic along the magnetic line of force (Hundhausen et al., 1966), and this anisotropy may contribute to excite magnetic fluctuations in the interplanetary space. In this paper, analysis of the fluctuation of the magnetic field in interplanetary space obtained by the IMP-II satellite is presented, and it will be shown that the fluctuation in the interplanetary magnetic field is quite anisotropic, the transverse fluctuation being much larger than the longitudinal one. This anisotropy is most prominent for low level fluctuation, and the fluctuation becomes rather isotropic as its amplitude increases. The non-linearity effect will be discussed on the basis of magnetohydrodynamics.

ANALYSIS

Details of the IMP-II magnetic experiment have been described in the paper by Fairfield et al. (1967). The satellite was placed in an eccentric orbit with its apogee at 15.9 R_E (R_E: radius of earth). The orbits analyzed in this paper include 1 to 20, and the magnetic field data are obtainable in the magnetosheath and interplanetary space. Apogee of the orbit 1 was near the sun-earth line, and subsequently, it moved toward the dawn meridian, as the earth moved around the sun.

The vector samples of the magnetic data obtained by two monoaxial fluxgate magnetometers are available every 20.5 sec., although four data points are missing after 12 consective data points due to the transmission of the Rubidium magnetometer data from the spacecraft.

Each value of the magnetic field is calculated from 26 digitized values of the spin modulated output taken during a 4.8 second interval. When more than three of 26 digitized values are missing, that sample of data is not used in the analysis. Moreover, in the IMP-II experiment, an additional quantity has been introduced to specify the quality of the data. This quantity, denoted as Δ in the summary paper on IMP-II magnetic field experiment (Fairfield et al., 1967), is the root mean square deviation from the best-fit sinusoidal curve in the raw data reduction process.

In the work presented here, when $\frac{\Delta}{B}$ (B is the magnetic field strength) exceed 0.2, these data are omitted from the analysis.

The running average of the magnetic field is taken over a time interval τ. A typical value of τ is 5.46 minutes. The fluctuating part of the magnetic field is defined as the difference between the original value of the data and this running-averaged value.

The coordinate axis used in the analysis is chosen such that the first axis is along the average magnetic field, and the second axis is in the direction of the vector product of the first axis and the sun-earth line.

Since the sun-earth line is nearly parallel to the motion of ions in the solar wind, the second axis is perpendicular to both the average magnetic field and the motion of ions in the solar wind. The third axis is chosen so that these three axis should form an ordinary right-handed coordinate system. The significance of the choice of the second axis is not so clear in the following analysis, and at present it is for convenience's sake. It is important that the second and third axes are perpendicular to the average magnetic field. The fluctuating magnetic field is projected on these three axis, and analysed in this coordinate system. Three components of the fluctuating magnetic field are denoted as

$$\Delta B_{||}(t_i), \ \Delta B\perp_1(t_i) \ \text{and} \ \Delta B\perp_2(t_i),$$

where $\Delta B_{\parallel}(t_i)$ is the fluctuating magnetic field projected on the average magnetic field, and $\Delta B\perp_1(t_i)$ and $\Delta B\perp_2(t_i)$ are those perpendicular to the average magnetic field.

Most of the following arguments are based on a quantity defined as

$$\eta = \frac{\Delta B_{\parallel}{}^2 - \dfrac{\Delta B\perp_1^2 + \Delta B\perp_2^2}{2}}{\Delta B_{\parallel}{}^2 + \dfrac{\Delta B\perp_1^2 + \Delta B\perp_2^2}{2}}$$

This quantity is positive when the magnetic fluctuations parallel to the average magnetic field is dominant. For an isotropic distribution of fluctuation, η becomes zero, and is negative when the fluctuation is predominantly transverse to the average magnetic field. η is plus or minus unity for either purely longitudinal or purely transverse fluctuation, respectively. Each value of η is evaluated for each data point, and then averaged over a time interval τ, the same as that used in calculating the running average of the magnetic field. When τ = 5.46 minutes is used, we can obtain about 150 averaged values of $\bar{\eta}$ for each orbit, and in Table 1, only the number of $\bar{\eta} > 0$ or $\bar{\eta} < 0$ in each orbit, and an average value of η_{AV} over each orbit are listed. As easily can be seen in this Table, in the interplanetary space the magnetic fluctuation shows much more negative values of η than positive, which means that the magnetic fluctuation is transverse to the average magnetic field. This is quite interesting in relation to the recent discovery of the anisotropy of temperature distribution in the solar wind ions along the magnetic field. (Hundhausen et al., 1967). It is well known that when the ion temperature along the magnetic field is much higher than the transverse temperature, there is a possibility that a firehose instability can be excited in the plasma. (Parker, 1957) The mode of this instability is that the propagation vector is along the magnetic line of force, and the fluctuating magnetic vector is transverse to it. A crude estimate of the criterion for the excitation of the instability

$$(NKT_{\parallel} - NKT_{\perp}) > B^2/2\mu$$

(where B is the magnetic field strength, $T_{\parallel}$ and $T_{\perp}$ the longitudinal and transverse temperature, N the ion density, K Boltzman constant, and μ is the magnetic permeability) is not always satisfied in the solar wind. (Scarf et al., 1967). Scarf and colleagues have also shown the possibility of the excitation of a generalized firehose instability in the solar wind near at the ion cyclotron frequencies. In order to investigate the character of the fluctuation of the magnetic field more in detail, the values of η defined above are calculated in accordance with the amplitude of the fluctuation relative to the average field strength. The relative amplitudes $\frac{\Delta B_{tot}}{B_{AV}}$ are divided into seven groups such as

$$0.15 \cdot (n-1) < \frac{\Delta B_{tot}}{B_{AV}} < 0.15 \cdot m \ (m = 1, 2 \cdots 6)$$

and $$\frac{\Delta B_{tot}}{B_{AV}} > 0.90,$$

where $$\Delta B_{tot} = \sqrt{\Delta B_{\parallel}^2 + \Delta B_{\perp_1}^2 + \Delta B_{\perp_2}^2}$$

They are averaged in each group. In Fig. 1, the value of η is shown for each step of the relative amplitude of the magnetic fluctuation to the average magnetic field. For low level fluctuations, the anisotropy along the average magnetic line of force is very large, and the fluctuations gradually become isotropic as their amplitudes increase. When we look at the cross-correlations between each component, it is clear that when the fluctuation level is very low, the cross correlation between two transverse components is much larger than the other two cross correlations between the longitudinal and transverse fluctuations, as can be seen in Fig. 2, where a new quantity defined as

$$\zeta = \frac{\overline{|\Delta B_{\perp_1} \cdot \Delta B_{\perp_2}|} - \frac{\overline{|\Delta B_{\parallel} \cdot \Delta B_{\perp_1}|} + \overline{|\Delta B_{\parallel} \cdot \Delta B_{\perp_2}|}}{2}}{\overline{|\Delta B_{\perp_1} \cdot \Delta B_{\perp_2}|} + \frac{\overline{|\Delta B_{\parallel} \cdot \Delta B_{\perp_1}|} + \overline{|\Delta B_{\parallel} \cdot \Delta B_{\perp_2}|}}{2}}$$

is introduced. This quantity is positive when the coupling between the two transverse components of fluctuations is much stronger than those between transverse and longitudinal components, and is negative for the opposite case. The anisotropy of this quantity also disappears as the amplitude of the fluctuating magnetic field increases.

This is quite interesting since it is possible that when the amplitude of the instability grows to a certain level, the effect of non-linearity becomes significant, and the transverse mode of oscillation can disintegrate into the longitudinal one.

Magnetohydrodynamic equations which govern a thermally anisotropic plasma are given by

$$\rho \frac{dU_\perp}{dt} = -\nabla_\perp \left(P_\perp + \frac{B^2}{2\mu_0}\right) + (B\cdot\nabla)_\perp B/\mu☆ \quad (1)$$

$$1/\mu☆ = \left(1 + \frac{P_\perp - P_{||}}{B^2/2\mu_0}\right)/\mu_0$$

$$\frac{\partial B}{\partial t} = \nabla \times (U \times B) \quad (2)$$

where

ρ = mass density of plasma,
U = velocity of plasma,
B = magnetic field,
P = pressure,
μ_0 = magnetic permeability in vacuum,

and $||$ and $\perp$ denote parallel or perpendicular to the magnetic field, respectively.

The drift velocity of plasma (solar wind velocity) is neglected in the following argument, since it only affects the Doppler shift in the frequency. When we take $B = B_0 + b$, $B = (B_0, 0, 0)$ $b = (0, 0, b_z)$ $u = (0, 0, U_z)$ and $U_z, b_z \propto exp\ (\omega t - iK_x X - iK_y Y)$, the well-known firehose instability occurs when

$$1 + \frac{P_\perp - P_{||}}{B^2/2\mu_0} < 0$$

is satisfied.

The mode of this instability is purely transverse to the magnetic field lines. When the amplitude of the instability grows, the non-linear terms should be taken into account. Assume that

$$b = (b_x, b_y, b_z) \text{ and } U = (U_x, U_y, U_z),$$

and retain quadratic terms in b_z and U_z and linear terms in other quantities. Then Eqs. (1) and (2) have the form

$$\frac{\partial U_y}{\partial t} = -\frac{\partial}{\partial y}\left(\frac{B_0 b_z}{\mu_0} + \frac{b_z^2}{2\mu_0}\right) + \frac{1}{\mu☆} B_0 \frac{\partial b_y}{\partial x} \tag{3}$$

$$\frac{\partial b_z}{\partial t} = B_0 \frac{\partial U_y}{\partial y} \tag{4}$$

and using div. b = 0, Eqs. (3) and (4) become

$$\frac{\partial^2 b_z}{\partial t^2} - \frac{B_0^2}{\mu_0 \rho} \frac{\partial^2 b_z}{\partial x^2} - \frac{B_0^2}{\mu☆\rho} \frac{\partial^2 b_z}{\partial y^2} = \frac{\partial^2}{\partial y^2} \frac{B_0}{2\mu_0} b_z^2 \tag{5}$$

The left hand side of the Eq. (5) does not necessarily show unstable oscillations even when the condition of the firehose instability

$$1 + \frac{P_\perp - P_{||}}{B^2/2\mu_0} > 0$$

is satisfied. The right hand side, however, is quadratic in the unstable oscillation b_z , and when the instability grows, it helps the growth of the oscillation b_z , the magnetic fluctuation along the magnetic line of force. The criterion for this is

$$\frac{b_z}{B_0} \frac{K_y^2}{K_z^2 + \frac{\mu_0}{\mu☆} K_y^2} > 1 \tag{6}$$

When this condition is fulfilled, the longitudinal fluctuation becomes comparable to the transverse component of fluctuation. This explains the result obtained in Fig. 1, where the fluctuation becomes nearly isotropic, when the amplitude of the fluctuation exceeds the average magnetic field, which is the requirement of the criterion given in Eq. (6).

So far, we have analysed data with the averaging time interval τ of 5.46 minutes. If the mode of fluctuations is dependent on their frequencies, the value η will vary with τ . The result is shown in

Fig. 3. Since the available data are every 20.5 sec, the analysis has been done for $\tau \geq$ 5.46 min. When τ is small, the fluctuations are much more transverse to the steady magnetic field. But as τ is increased, they become rather isotropic. This implies that the mode of fluctuations has a frequency dependence. Since the fluctuations of the magnetic field has a power spectrum of nearly f^{-2}, the fluctuations of lower frequency have much more power. (Holzer et al., 1966) Therefore it can be said from the result shown in Fig. 3 that as the frequencies of fluctuations decrease, they become isotropic.

DISCUSSION

The fluctuation in the magnetic field obtained by the IMP-II satellite was analysed, and it was found that the magnetic fluctuation is anisotropic along the direction of the steady magnetic field, and the magnetic fluctuation transverse to the steady field is much more prominent than the longitudinal one. This anisotropy gradually disappears as the amplitude of the fluctuation increases. This is explained by the conversion of transverse fluctuations into longitudinal one due to non-linear effects, using the magnetohydrodynamic equations. This anisotropy has also a frequency dependence, and becomes isotropic as the frequency decreases.

These phenomena are certainly related to dynamics of waves in plasma and maybe the non-linear coupling between waves should be important. In satellite measurement, we can not obtain knowledge about the direction of wave propagation, which is one of important parameters in the study of waves. Satellite data are limited to measurements at one time and at one position. Accordingly, depending on whether the bulk velocity of solar wind is larger or smaller than the phase velocity of plasma in the direction of solar wind, the obtained spectrum is either wave number spectrum or frequency spectrum. So long as the hydromagnetic waves are concerned, the phase velocity is much smaller than the bulk velocity of solar wind. In order to get both the time and spatial properties of waves, at least two points simultaneous measurements with variable distance is required, and this may be one of the future tasks of space exploration.

One of the successful approaches to non-linear phenomena in physics is the method employed in the analysis of homogeneous isotropic turbulence in ordinary hydrodynamics. The extension of the method to magnetohydrodynamics has been done successfully by Chandrasekhar (Chandrasekhar, 1952). The fundamental equation, similar to the Karman-Howarth equation in ordinary hydrodynamic turbulence, can be expressed by only fluctuating quantities and is irrelevant to the average magnetic field and the convection of plasma, if the fluctuation is homogeneous and isotropic. Usually the fluctuation is not isotropic in the interplanetary field, as described above, but it becomes isotropic for large amplitude fluctuation. The analysis of these large amplitude fluctuations along Chandrasekhar's scheme was tried, but it was not successful, since the data are available only every 20.5 seconds and as can be seen from the correlation functions shown in Fig. 4, they drop to nearly zero before 20.5 seconds time lag.

The fluctuations of extraterestrial magnetic field are interesting not only in geophysics, but also as an example in non-linear plasma dynamics. Our future analysis will be done along this line.

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FIGURE CAPTIONS

Figure 1 : Parameter η which specifies the anisotropy of the magnetic fluctuation as a function of the amplitude of the fluctuation relative to the average magnetic field in the interplanetary space. The magnetic fluctuation is quite anisotropic for low level fluctuations, and it gradually becomes isotropic as the amplitude of fluctuation increases.

Figure 2 : Parameter ζ which specifies the degree of coupling between transverse and longitudinal fluctuations as a function of the amplitude of the magnetic fluctuations relative to the average magnetic field in the interplanetary space. The coupling between transverse and longitudinal fluctuations is very much smaller than the coupling between two components of the transverse fluctuations. This tendency disappears as the amplitude of fluctuation increases.

Figure 3 : Dependence of the anisotropy of fluctuations in the interplanetary magnetic field on the averaging time interval τ

Figure 4 : Autocorrelation functions of the magnetic fluctuations in the interplanetary space averaged over orbits 11-20. $\Delta B_{\parallel}$; fluctuation of the magnetic field along the average magnetic field. $\Delta B_{\perp_1}, \Delta B_{\perp_2}$; fluctuations of the magnetic field transverse to the average magnetic field. The mean square of the fluctuations (i.e. autocorrelation functions at τ = 0) transverse to the average magnetic field is much larger than the longitudinal one.

Orbit No.	Distance from the Earth R_1	R_2	R_3	Number of (−) Values η	Number of (+) Values	Average $\eta_{Av.}$
2	13.7	15.9	13.9	139	23	−.318
3	14.8	15.9	14.4	70	25	−.174
5	15.4	15.5	15.0	30	7	−.238
6	15.6	15.7	13.3	52	6	−.447
7	14.6	15.9	13.3	133	26	−.356
8	15.8	15.9	13.8	87	16	−.252
11	13.5	15.9	12.0	167	25	−.515
12	14.1	15.9	14.8	86	11	−.485
13	15.1	15.9	15.0	78	28	−.193
15	14.2	15.9	14.8	113	17	−.342
20	13.9	15.9	12.4	163	24	−.324

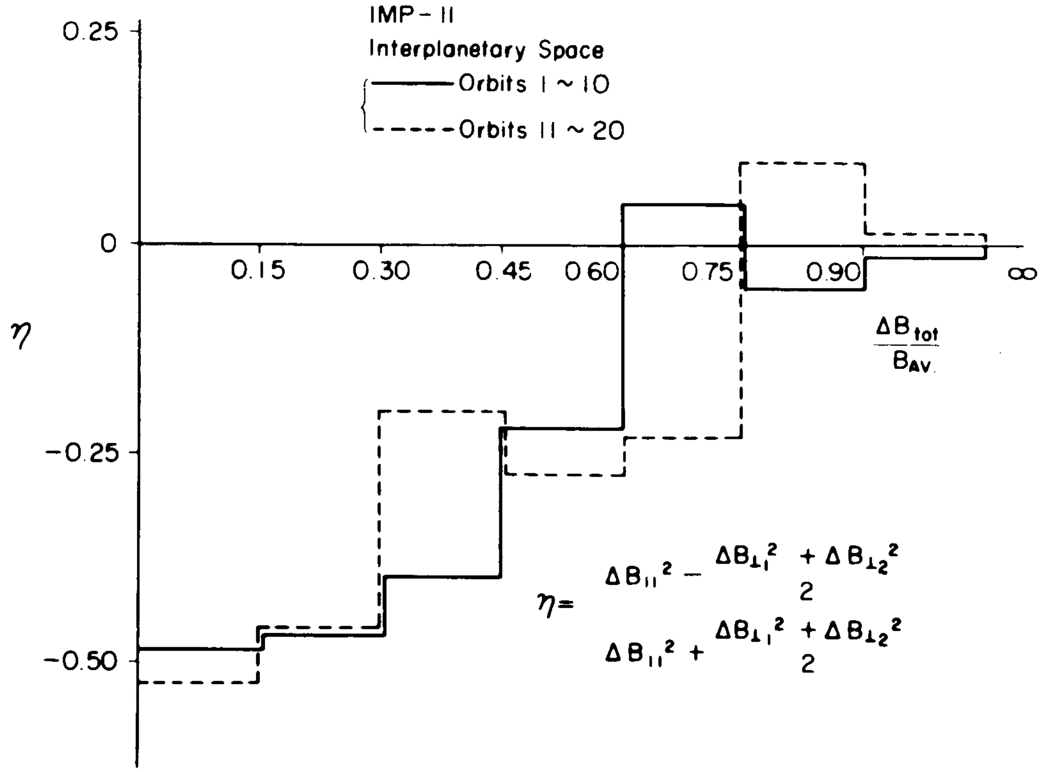
Anisotropy of the Magnetic Field Fluctuation
in the Interplanetary Space

$$\eta = \frac{\overline{\Delta B_{\parallel}^2} - (\overline{\Delta B_{\perp_1}^2} + \overline{\Delta B_{\perp_2}^2})/2}{\overline{\Delta B_{\parallel}^2} + (\overline{\Delta B_{\perp_1}^2} + \overline{\Delta B_{\perp_2}^2})/2}$$

where $\Delta B_{\parallel}$: Fluctuation of the magnetic field parallel to the average magnetic field

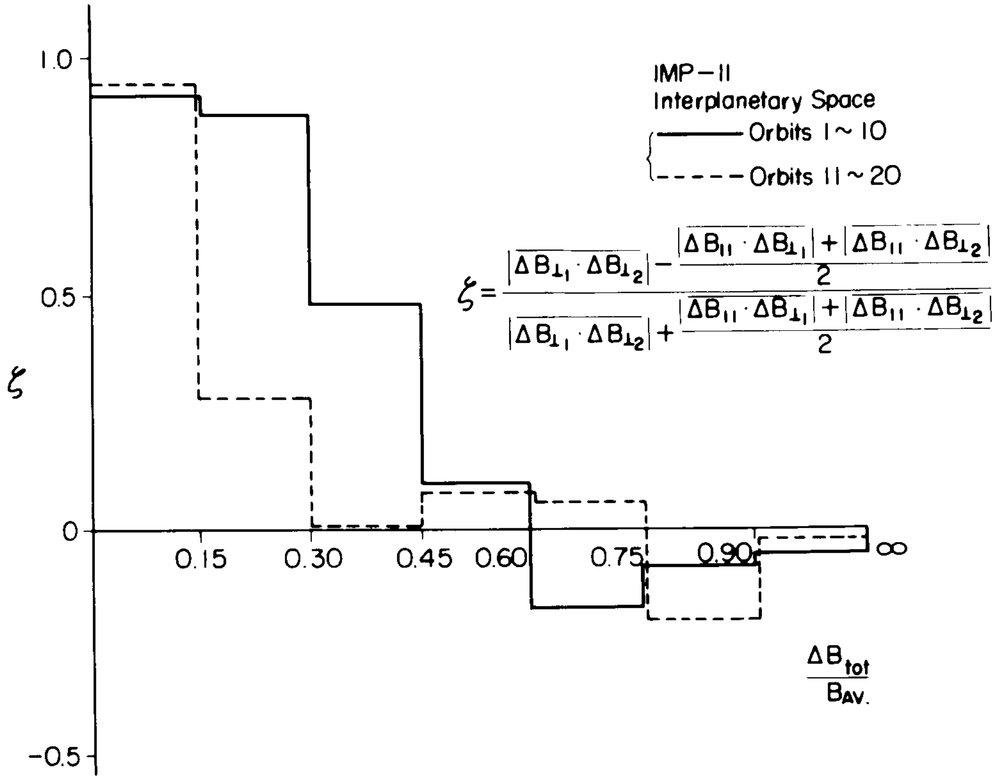
$\Delta B_{\perp_1}$ $\Delta B_{\perp_2}$: Fluctuations of the magnetic field perpendicular to the average magnetic field

Table 1



IMP - II
Interplanetary Space
Orbits I ~ 10
Orbits II ~ 20
0.25
0
−0.25
−0.50
0.15
0.30
0.45
0.60
0.75
0.90
∞
η
$\frac{\Delta B_{tot}}{B_{AV}}$
$\eta = \dfrac{\Delta B_{\parallel}^2 - \dfrac{\Delta B_{\perp 1}^2 + \Delta B_{\perp 2}^2}{2}}{\Delta B_{\parallel}^2 + \dfrac{\Delta B_{\perp 1}^2 + \Delta B_{\perp 2}^2}{2}}$

Fig. 1



IMP—II
Interplanetary Space
Orbits 1 ~ 10
Orbits 11 ~ 20
$\zeta = \dfrac{|\overline{\Delta B_{\perp 1} \cdot \Delta B_{\perp 2}}| - \dfrac{|\overline{\Delta B_{\parallel} \cdot \Delta B_{\perp 1}}| + |\overline{\Delta B_{\parallel} \cdot \Delta B_{\perp 2}}|}{2}}{|\overline{\Delta B_{\perp 1} \cdot \Delta B_{\perp 2}}| + \dfrac{|\overline{\Delta B_{\parallel} \cdot \Delta B_{\perp 1}}| + |\overline{\Delta B_{\parallel} \cdot \Delta B_{\perp 2}}|}{2}}$
1.0
0.5
0
-0.5
ζ
0.15
0.30
0.45
0.60
0.75
0.90
∞
$\frac{\Delta B_{tot}}{B_{AV.}}$

Fig. 2

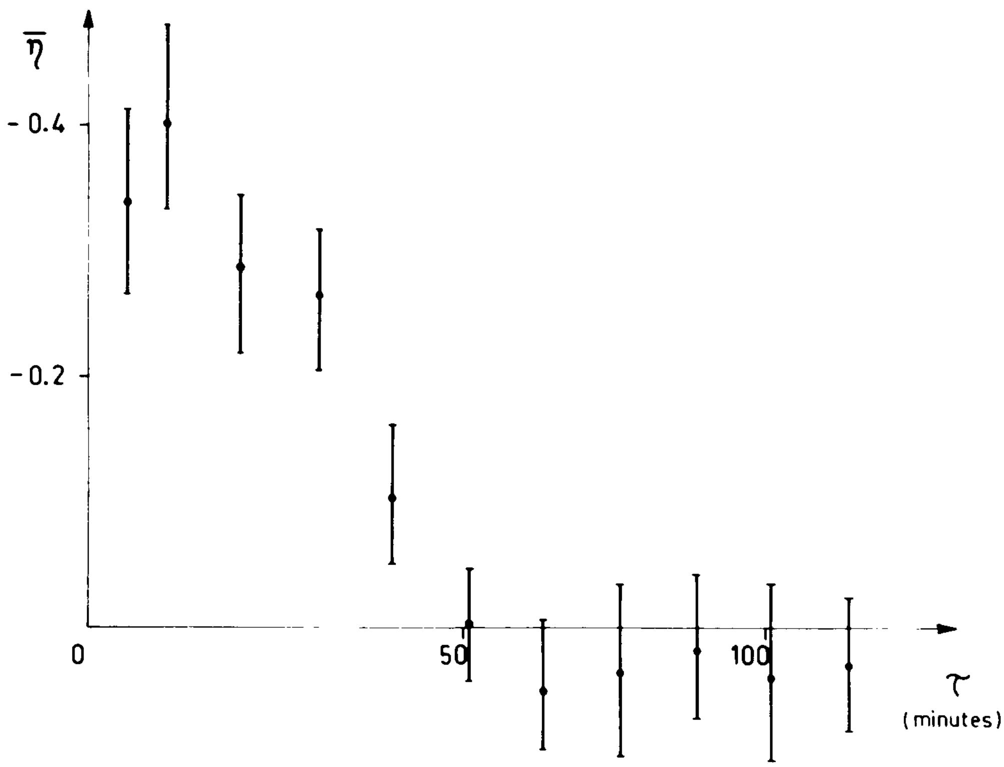


Fig. 3

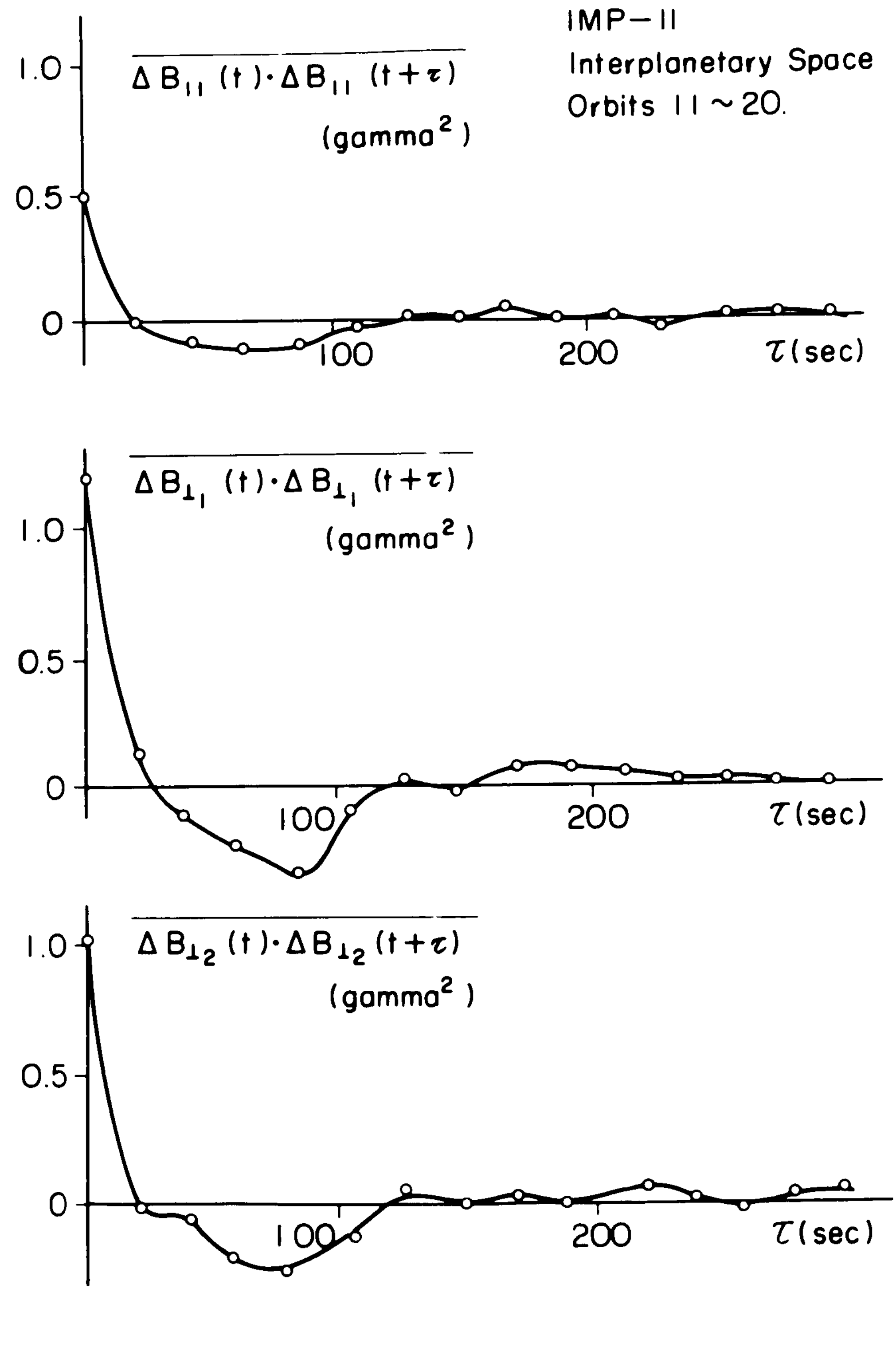

IMP—II
Interplanetary Space
Orbits 11 ~ 20.
$\overline{\Delta B_{\parallel}(t)\cdot\Delta B_{\parallel}(t+\tau)}$
(gamma^2)
1.0
0.5
0
100
200
τ(sec)
$\overline{\Delta B_{\perp_1}(t)\cdot\Delta B_{\perp_1}(t+\tau)}$
(gamma^2)
1.0
0.5
0
100
200
τ(sec)
$\overline{\Delta B_{\perp_2}(t)\cdot\Delta B_{\perp_2}(t+\tau)}$
(gamma^2)
1.0
0.5
0
100
200
τ(sec)

Fig. 4