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ABSTRACT

A survey of element data published in Goddard Space Flight Center's satellite situation reports reveals the existence of a number of interesting objects in strongly resonant orbits. These orbits have not yet been studied for the purposes of satellite geodesy. As of March 1968, new objects in orbits whose resonant beat period (with its dominating gravity harmonic) is greater than 100 days include: 1959 ETA 1 (Vanguard 3), n = 11 revolutions/day; 1959 ALPHA 2, n = 11 revs./day; 1962 A ALPHA 3, n = 14 revs./day; 1963-26A, n = 14 revs./day; 1966-13A, n = 12 revs./day. Newly discovered objects in orbits with a dominant resonance beat period of between 50 and 100 days include: 1961 DELTA 2, n = 12 revs./day; 1962 BETA 2, n = 14 revs./day; 1965-89C, n = 12 revs./day; 1966-13B, n = 12 revs./day; 1966-56A (PAGEOS 1), n = 8 revs./day. Study of the long-term perturbations of these objects should considerably strengthen knowledge of geopotential terms of order (or longitude frequency) 8, 11, 12 and 14.

DISCOVERY OF NEW EARTH SATELLITES IN RESONANT ORBITS

INTRODUCTION

The first earth satellite launched in 1957 has been followed by more than 3000 such objects which have been separately tracked, and whose elements have been catalogued by various institutions and agencies in the United States.[1] About 1000 of these are still in orbit as of March 1968. All such orbiting objects are potentially useful as global earth gravity probes. But because of restrictions on data availability and accuracy imposed by the various tracking networks, and also because of a general need until now for a wide variety of uncontrolled, low drag orbital elements, only a very few of these earth satellites have actually been used for geodetic purposes. It must also be confessed however that the sheer volume of unreduced tracking data on all these objects has also led to a good deal of arbitrary preselection of objects among equally worthy candidates for analysis. Nevertheless, in spite of the rather sparse use, to date, of the earth satellites for geodetic purposes, it is well recognized that the study of these has already defined the large scale features of the geoid to an overall accuracy of less than 30 meters.[2] (The large scale geoid features are those with linear dimensions greater than about 1500 miles).

In order to improve the accuracy of the geopotential, we can look at more of the existing or past decayed satellite orbits, launch new ones with new tracking requirements, or combine these activities judiciously. From a cost effective standpoint every reasonable effort should be expended first to use good new data from existing tracking records to improve the geopotential. Following up this approach, an abbreviated element list of some 1000 presently tracked satellite objects[1] has recently been examined. The purpose was to find new (geodetic) objects in orbits whose periods are close to commensurate with the period of the earth's rotation. These so-called resonant orbits are particularly attractive natural amplifiers of longitude features in the field. Previous use of these special orbits for geodesy has been restricted to the well known controlled communications satellites of high altitude in deep resonance[3] ($n = 1$ and 2 revolutions/day) and a few low altitude objects in very shallow resonance[4,5] ($n = 12$ to 15 revs./day). Three of these low altitude objects are now found to be in strongly resonant orbits. The gravity perturbations of deeply resonant (close to commensurate) orbits are so large that only minimal tracking accuracy and network coverage, together with simplified software, is generally needed to obtain important new geodetic information.

RESONANT ORBIT ELEMENTS

The general specification of a commensurate orbit used implies a repeating ground track for the so-called mean satellite[6,7]:

$$\dot{\lambda} = 0 = \left(\frac{1}{s}\right)(\dot{M} + \dot{\omega}) + \dot{\Omega} - \dot{\theta}_e \ , \tag{1}$$

where $\dot{\lambda}$ is the drift rate of the ascending equator crossing of this "mean satellite," s is the rational number of revolutions per sidereal day the satellite makes in one repetition period, $\dot{M}$ is the mean motion, $\dot{\omega}$ is the rotation rate of the line of apsides, $\dot{\Omega}$ is the progression rate of the line of nodes and $\dot{\theta}_e$ is the rotation rate of the earth.

This specification gives the exact orbit conditions for resonance with the $V_{\ell mp(q=0)}$ terms in Kaula's expansion of the geopotential[8], which satisfy the expression:

$$\frac{m}{\ell - 2p + q} = s \ , \tag{1a}$$

where $\ell \geq m$, and $0 \leq p \leq \ell$. The resonance condition expressed by Equation (1) is not exact but close to exact for those $V_{\ell mpq}$ terms for which[7]:

$$\left| \frac{\dot{\omega} q/m}{(\dot{\theta}_e - \dot{\Omega})} \right| << 1 \ . \tag{2}$$

In the case of critically inclined orbits, $\dot{\omega} = 0$ and Equation (1) is an exact resonance condition for all $V_{\ell mpq}$ terms which satisfy Equation (1a). In the case of circular orbits, $q = 0$ terms are the only non-zero potential terms remaining in Kaula's expansion and once again Equation (1) is an exact resonance condition for all such orbits. Except for orbits of high eccentricity (where the $V_{\ell mpq(large)}$ terms are not negligible) and small semimajor axis (where $\dot{\omega}$ is relatively large), inequality (2) is satisfied. Thus, the commensurability condition Equation (1) is found to be a sufficiently good orbit criteria for almost all resonance effects with longitude terms of the geopotential.

Considering close earth orbits, the well known first order secular effects of the earth's oblateness will be sufficient to determine the mean orbit elements at resonance from Equation (1). The oblateness secular effects are given by[8]:

$$\dot{\Omega} = \frac{3n\, J_{20}\, (a_e/a)^2 \cos i}{2(1 - e^2)^2}$$

$$\dot{\omega} = \frac{3n\, J_{20}\, (a_e/a)^2\, [1 - 5\cos^2 i]}{4(1 - e^2)^2} \tag{4}$$

$$\dot{M} = n - \frac{3n\, J_{20}\, (a_e/a)^2\, [3\cos^2 i - 1]}{4(1 - e^2)^{3/2}} \quad ,$$

where $n^2 a^3 = \mu$.

In Equation (4), $J_{20} \doteq -1.0827 \times 10^{-3}$, $a_e \doteq 6378.165$ km. (The earth's mean equatorial radius), a, e and i are the orbit elements semimajor axis, eccentricity and inclination, n is the unperturbed mean motion of the satellite, and μ is the earth's Gaussian gravity constant (3.98601×10^5 km^3/sec^2).

Let us define the perturbed (or "Mean") mean motion $\bar{n}$ as equal to the secular rate $\dot{M}$ in Equation (4). This appears to conform with the definition of the mean motion given in the 5 line NORAD (North American Air Defense Command, SPADATS or Space Track) mean elements[9] against which the analysis here will be calibrated. This also conforms to the definition of the "Mean" mean motion given by Kozai[10]. Then, also defining (after Kozai[10]) the mean semimajor axis $\bar{a}$ by:

$$\bar{a} = a \left\{ 1 + \frac{3\, J_{20}\, (a_e/a)^2}{4(1 - e^2)^{3/2}} [3\cos^2 i - 1] \right\} \quad , \tag{5}$$

it follows that:

$$\bar{n}^2 \bar{a}^3 = \mu \left\{ 1 + \frac{3\, J_{20}\, (a_e/a)^2}{4(1 - e^2)^{3/2}} [3\cos^2 i - 1] \right\} \tag{6}$$

to a high degree of accuracy. Equation (6) has been calibrated with a number of recent NORAD 5 line elements for widely different satellite orbits, and it has been found that among these sets of elements, the solution for μ in (6) is indeed constant, giving:

$$\mu = 290.483 \text{ earth radii}^3 \times (\text{rev./day})^2.$$

In fact, converting $\mu = 3.98601 \times 10^5$ km^3/sec^2 to these units with $a_e = 6378.165$ km/e.r., 1 day = 86,400 seconds and 2π radians = 1 revolution, gives the same value for μ with a 2 instead of a 3 in the last digit.

With these secular rates and definitions, one can rewrite the commensurability-resonance condition Equation (1) as:

$$\dot{\lambda} = 0 = \bar{n} \left\{ \frac{-1.0827 \times 10^{-3}}{[\bar{a}(1 - e^2)]^2} \frac{[3(1/s)(1 - 5\cos^2 i) + 3\cos i]}{4} \frac{}{2} + \frac{1}{s} \right\}$$

$$(360) - 360.9853 \text{ degrees/day}, \tag{7}$$

where $\bar{a}$ in Equation (7) is in units of earth radii, $\bar{n}$ is in units of revolutions/day (1 rev. = 360°) and $\dot{\theta}_e = 360.9853$ deg./day.

The problem of the specification of resonant orbits then is simply the solution of Equation (7) for zero longitude drift rates. Since the mean motion (or semimajor axis) is the strongest determinant of Equation (7) (except for very high eccentricities) this solution will be for $\bar{n}$ or $\bar{a}$, given s (the commensurate orbit ratio), i and e.

The actual solution for either $\bar{n}$ or $\bar{a}$ from Equation (7) can be accomplished in most cases in one iteration using an unperturbed or Keplerian starting value for $\bar{a}$. In the satellite situation reports[1], the period P is defined from the "mean" mean motion $\bar{n}$ as:

$$P = 1440/\bar{n}, \text{ minutes.} \tag{8}$$

Since there are approximately 1436 minutes in a sidereal day, the Keplerian (two-body) approximation of the resonant orbits mean motion is simply

$$\bar{n}_0 = \left(\frac{1440}{1436}\right) s \quad , \tag{9}$$

since P (res.) $\doteq$ 1436/s minutes.

Then the initial estimate of the semimajor axis ($\bar{a}$ or a) is, from Equation (6):

$$\bar{a}_0 = (290.483)^{1/3} \ (\bar{n}_0)^{-1/2} \quad . \tag{10}$$

With this initial estimate of the mean resonant semimajor axis, one can solve Equation (7) directly for $\bar{n}_1$ and obtain the resonant mean motion which is sufficiently accurate for all but the very closest earth satellites. For the close satellites, another iteration between Equation (6) [to find $\bar{a}_1$ from $\bar{n}_1$, using $\bar{a}_0$ for a in the right side of Equation (6)] and Equation (7) will suffice for an accurate resonant mean motion.

The determination (by this method) of the resonant periods for all commensurabilities from s = 1 (rev./day) to s = 15 (revs./day) is shown in Figure 1. Where two eccentricity curves are shown, the non-zero one is for the drag free orbit of maximum possible eccentricity for that commensurability.

OFF-RESONANT BEAT PERIODS

Also in Figure 1 are shown charts of off resonant (or beat) periods (BP_r) for the dominant perturbation effect as a function of the period distance (ΔP) from exact resonance. The dominant resonant harmonic ($H_{\ell m}$) for a given commensurability s will generally be $H_{\ell s}$ which has a longitude wave length of 360/s degrees.

If $\Delta\dot{\lambda}$ is the longitude drift rate of the mean ground track in degrees/day, then:

$$BP_r = \frac{360/s}{\Delta\dot{\lambda}} \ , \ \text{days} \quad . \tag{11}$$

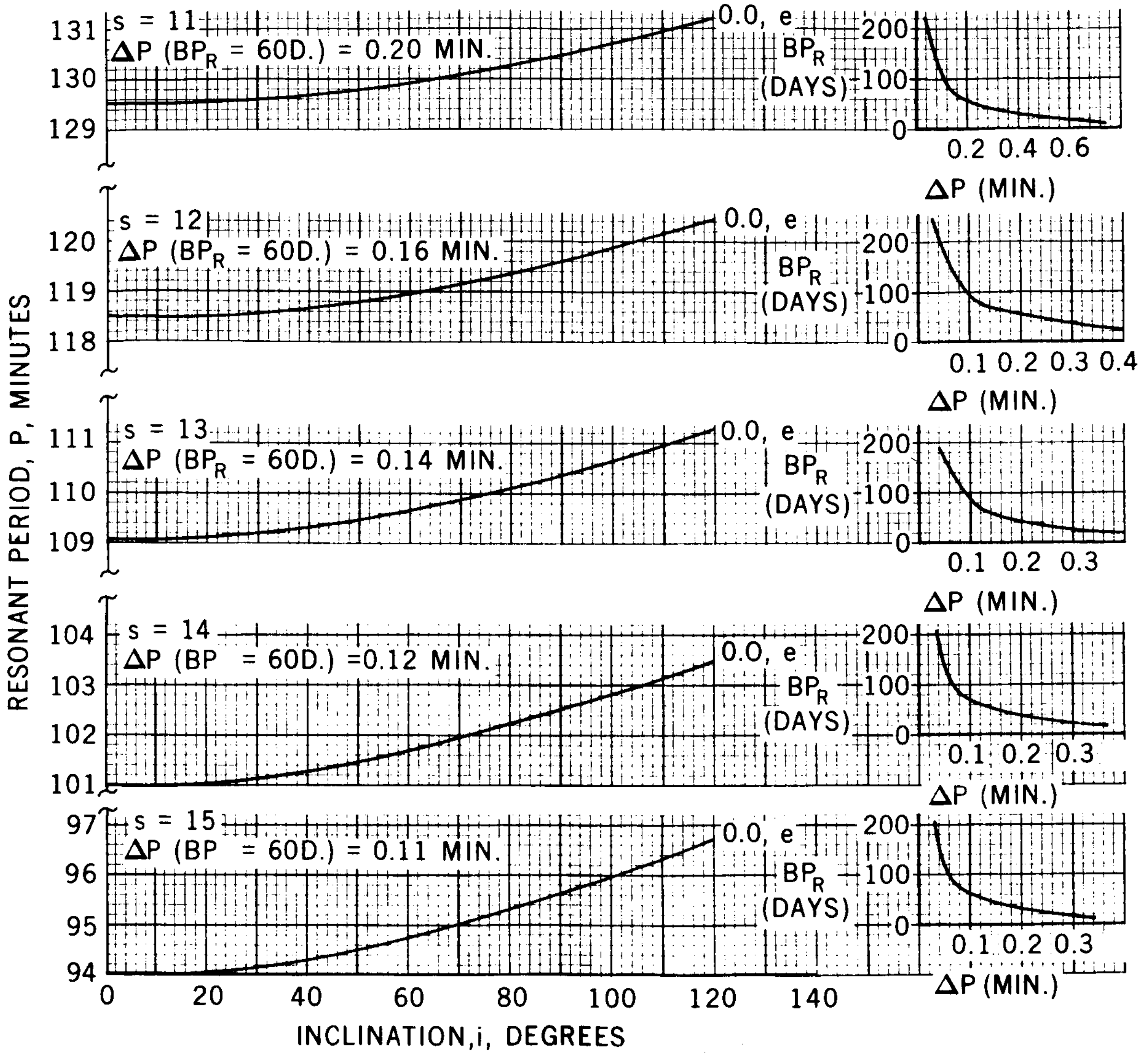

Figure 1. Resonant Periods and Beat Periods for Commensurable and Near Commensurable Earth Satellite Orbits. (Sheet 1 of 3)

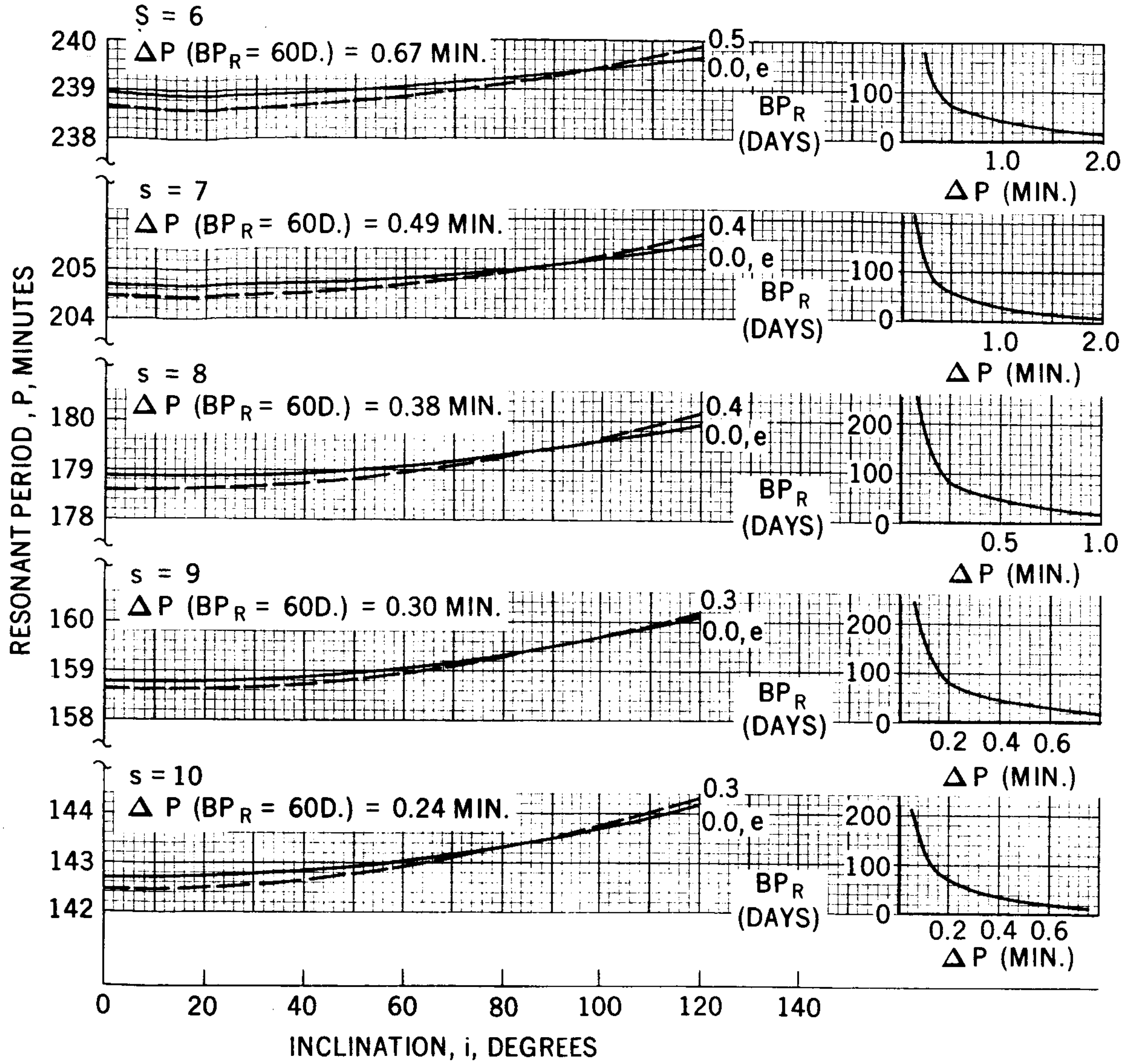

Figure 1. Resonant Periods and Beat Periods for Commensurable and Near Commensurable Earth Satellite Orbits. (Sheet 2 of 3)

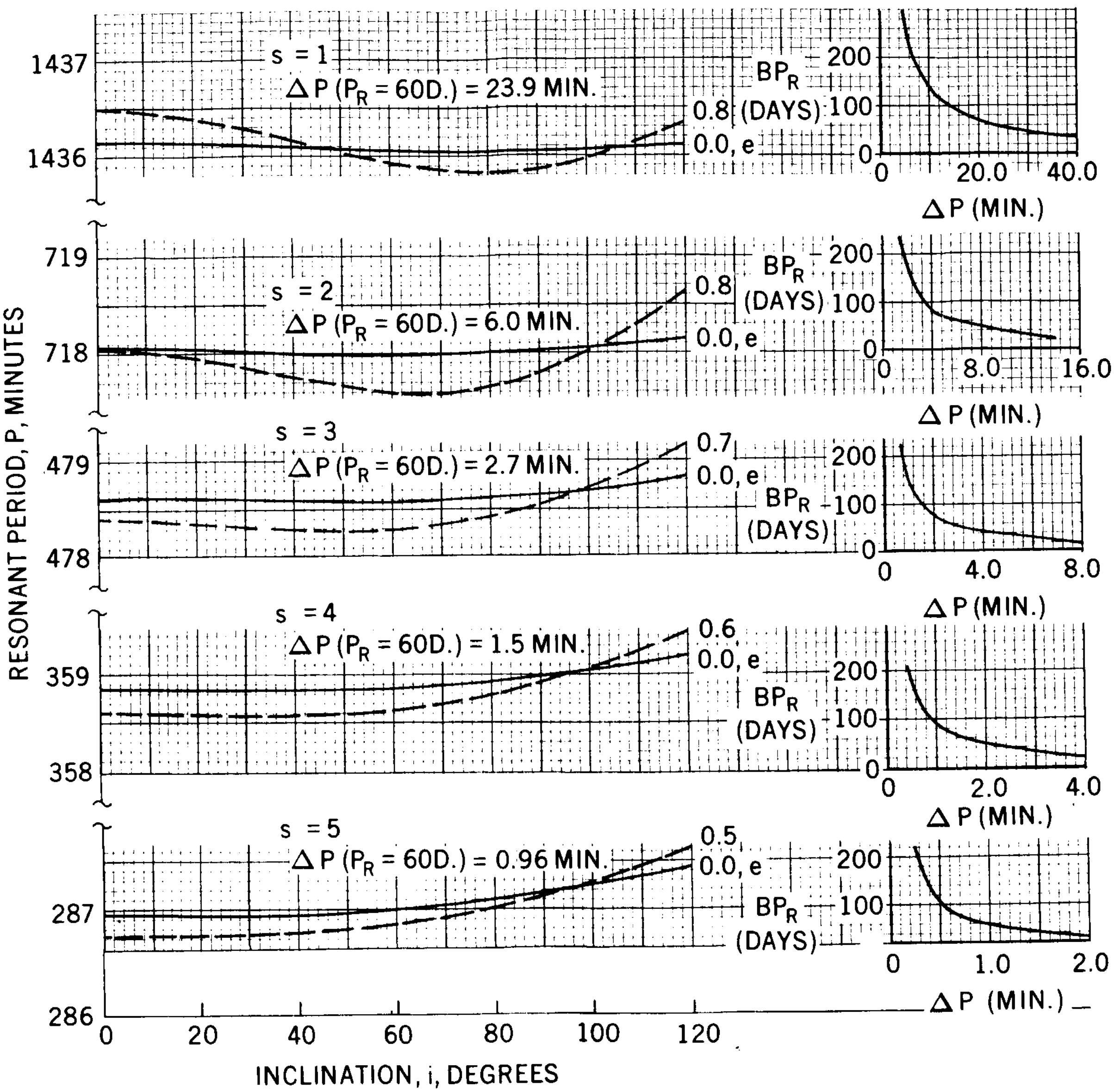

Figure 1. Resonant Periods and Beat Periods for Commensurable and Near Commensurable Earth Satellite Orbits. (Sheet 3 of 3)

(Equation (11) assumes for simplicity that the regime is well away from librational resonance where the period depends strongly on the magnitude of the perturbations). But, from Equation (7), neglecting the oblateness effects:

$$\Delta\dot{\lambda} = \frac{360\ \Delta\bar{n}}{s}, \text{ degrees/day.} \tag{12}$$

Furthermore, from Equation (8):

$$\Delta\bar{n} = -\frac{1440\ \Delta P}{P^2}, \text{ revs./day.} \tag{13}$$

But since, neglecting oblateness again, P (res.) = 1436/s, Equations (13) and (12) in Equation (11) give:

$$BP_r = \frac{1432}{s^2\ \Delta P}, \text{ days,} \tag{14}$$

where ΔP is the resonant period distance in minutes. A single chart of this dominant beat period function is found in Figure 2. Figure 3 can be used to find eccentricity (when this is needed in Figure 1) from the perigee and apogee heights for the objects listed in the satellite situation reports[1].

The importance of this off resonant beat period is as a guide to the relative magnitude of the resonant perturbations which might be expected from these orbits. The close earth resonant satellites (n = 12-15 revs./day) which have already been used for geodesy have been in orbits typically with beat periods less than 10 days. These have shown resonance perturbations of the order of hundreds of meters. On the other hand, the Russian 12-hour satellites with beat periods of over 100 days have shown resonance perturbations of the order of hundreds of kilometers. If fairly good raw tracking data were available on an arbitrary satellite, we might set a lower beat period limit of about 10 days to find interesting new objects on which to do resonant satellite geodesy. However, one can still do valuable satellite geodesy from the NORAD mean elements themselves if these are of sufficiently good quality. These typically come out about once a week. One would probably require at least 8 or these to derive reasonably useful results. Thus, shown in Figure 1 is the period miss distances

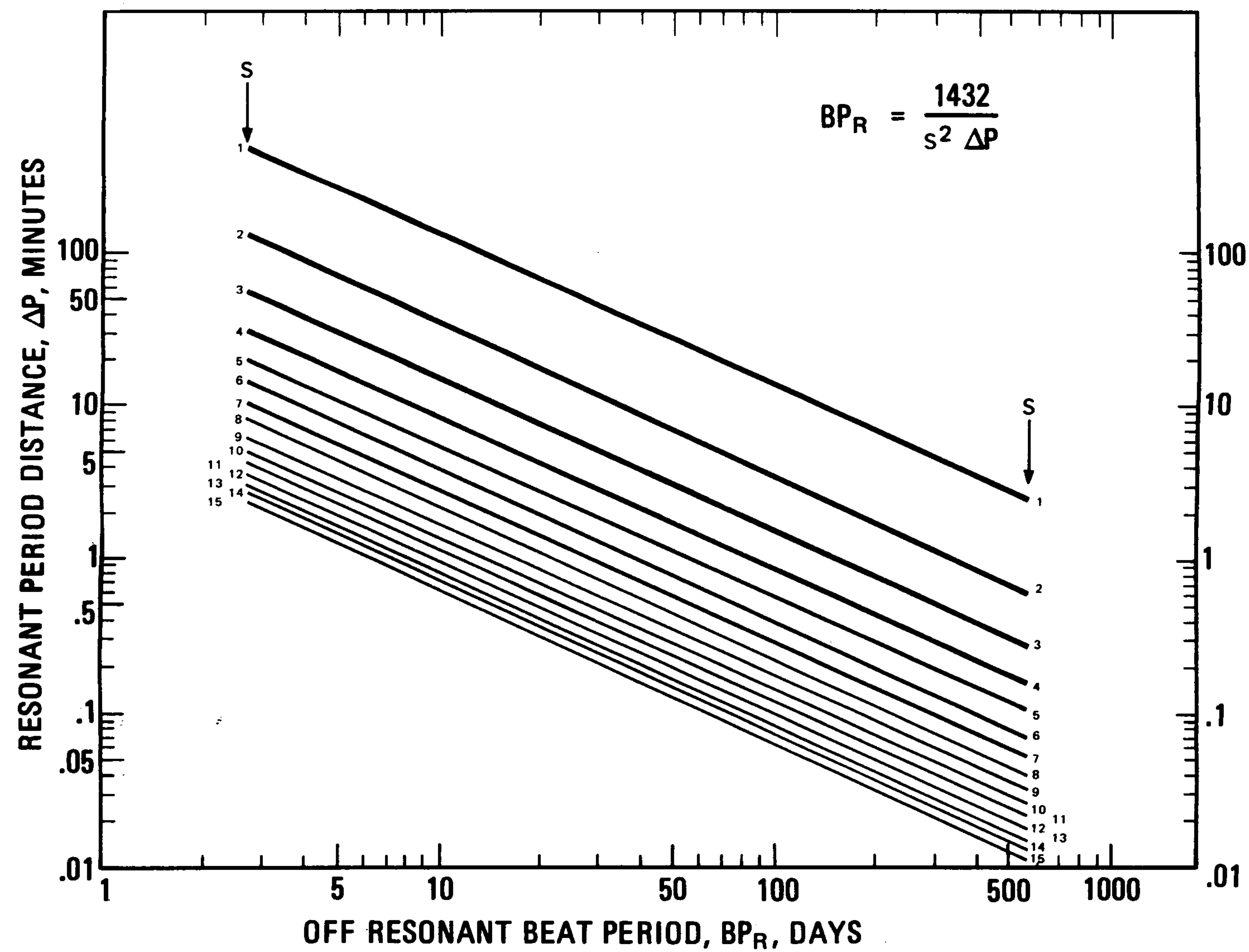

Figure 2. Off Resonant Beat Periods as a Function of Period Distance from Resonance for Near Commensurable Earth Satellite Orbits

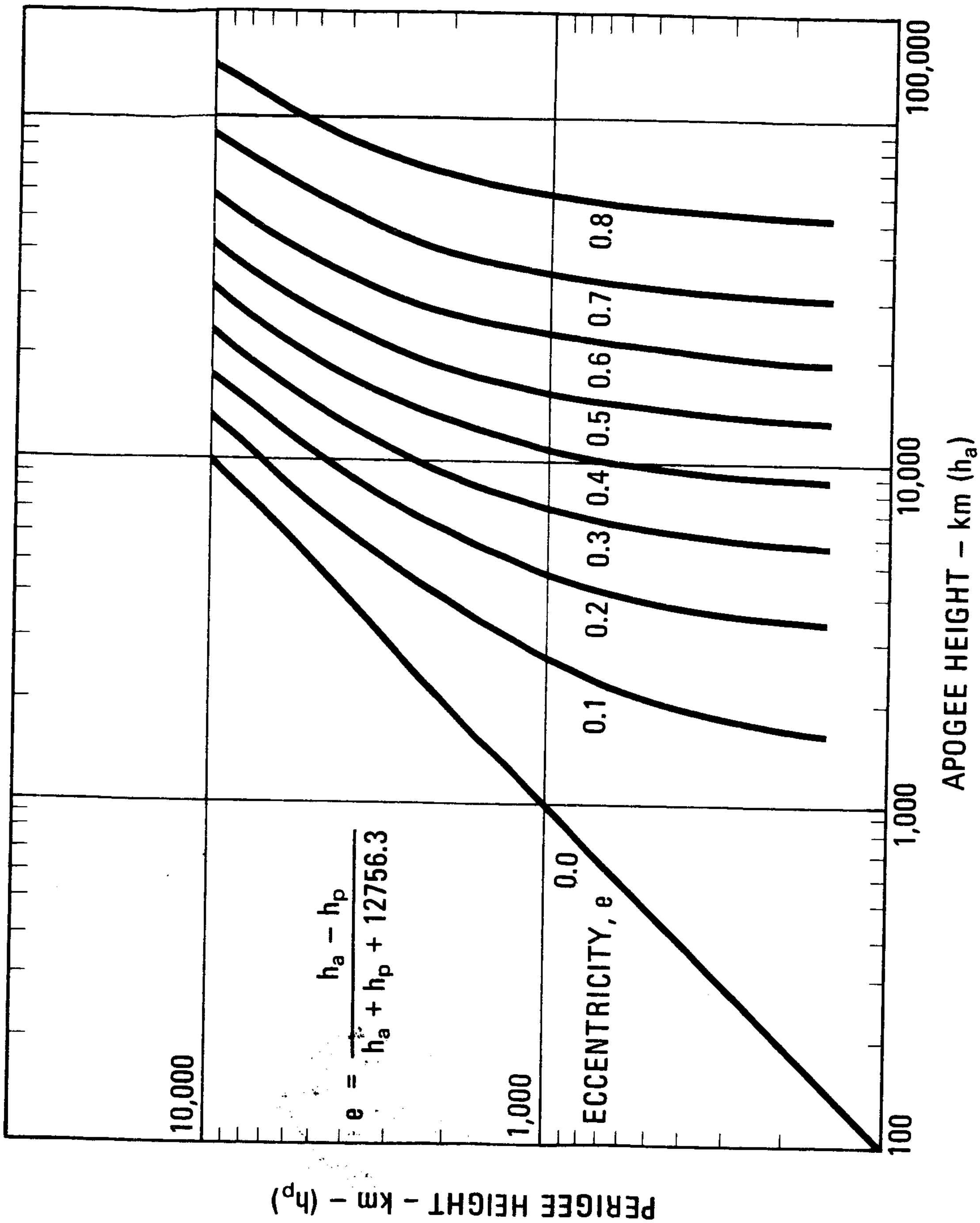

Figure 3. Eccentricity as a Function of Perigee and Apogee Heights

(ΔP) for 60-day beat periods as a lower limit guide for orbits from which interesting geodetic results could be quickly obtained without reanalysis of raw tracking data. (See RESULTS AND CONCLUSIONS).

CALIBRATION OF FIGURES

The individual oblateness perturbations $\dot{M}$, $\dot{\omega}$, and $\dot{\Omega}$ in Equation (4) have been found to agree numerically with the secular rates for these elements as published in a number of recent NORAD 5 line element sets for low and high altitude satellites. Part II of the NORAD special bulletins prints out equator crossing information as well. The daily change of these crossings should be closely related to the drift rate $\dot{\lambda}$ of the mean longitude λ of the satellite, since it can be shown that λ is the geographic longitude of the ascending equator crossing with ω, M and Ω interpreted as osculating elements. For the higher altitude orbits where the actual satellite is closer to the mean satellite, off resonant beat periods derived from satellite situation reports or 5 line elements, through Figures 1 or 2 should be very close to the periods derived directly from this crossing information through Equation (11).

Two high altitude tests were made on objects #2822 ($n = 2$ revs./day) and #2253 ($n = 8$ revs./day). NORAD Bulletin #1 for object #2822 showed $\bar{n}$ = 2.013987 revs./day, $i = 64.7°$ and $e = .74$. The mean period P from this bulletin is then 1440/2.013987 = 715.00 minutes. From Figure 1, the estimated resonant period is 717.6 minutes giving ΔP = 2.6 minutes and BP_r = 125 days. From table data which shows a better discrimination with eccentricity, it is estimated the resonant period is closer to 717.7 minutes which, from Figures 1 or 2 gives BP_r = 130 days. The equator crossing drift rate of this object in Part II of this bulletin was 1.38°/day (absolute value), giving an indicated beat period of 130 days from Equation (11). Thus, the calibration of the scheme with this object is quite good.

NORAD Bulletin #153 for object #2253 showed $\bar{n}$ = 8.01157561 revs./day, $i = 84.95°$ and $e = .195$. The indicated period is 179.74 min, while the resonant period from Figure 1 is about .35 min distant from this. More accurate table data shows ΔP = .36 minutes. From the rough estimation in Figure 1, BP_r = 60 days, while from the more accurate data (and using Figure 2), BP_r = 63 days. From the actual equator crossing drift rate in Part II of this bulletin, BP_r = 64 days. Again, the calibration of the scheme appears to be quite good.

Two lower altitude tests were made on objects #2201 ($n = 10$ revs./day) and #20 ($n = 11$ revs./day). NORAD Bulletin #97 on object #2201 showed $\bar{n}$ = 10.05932911 revs./day, $i = 40.86°$ and $e = .225$. The indicated period is

143.15 min. from which the rough estimates of ΔP and BP_r from Figure 1 are .40 min. and 40 days. The more accurate estimates (using table data and Figure 2) are ΔP = .38 minutes and BP_r = 38 days. But from the actual drift rate in this NORAD Bulletin, BP_r = 33 days. This is not as good a calibration as the previous two.

NORAD Bulletin #258 on object #20 showed $\bar{n}$ = 11.108578 revs./day, i = 33.4° and e = .19. The indicated period is 129.63 min. from which the rough estimates of ΔP and BP_r (Figure 1) are .10 min. and 115 days. The more accurate estimates (using table data and Figure 2) are .09 min. and 120 days. But the equator crossing rate from Part II of this bulletin is .410 deg/day (absolute value) giving an indicated BP_r = 80 days.

Evidently for $\dot{n} > 9$ revs./day the effects of the earth's oblateness are great enough to significantly distort the drift of the mean longitude of a satellite when compared with the drift of its actual longitude. The mean longitude drift itself is still accurately computed by this scheme, as has been checked on this last orbit. The 5 line elements from Bulletin #258, object #20, give $\dot{\Omega}$ = -3.291°/day, $\dot{\omega}$ = 4.901°/day, $\dot{M}$ = (360) (11.108578) = 3999.0881°/day. Thus, from Equation (1), $|\dot{\lambda}|$ = .275°/day, and, from Equation (11), BP_r = 119 days. This compares very favorably with the scheme estimates from Figures 1 and 2. Thus for the low altitude satellites, the beat period estimates from Figures 1 and 2 should still be good while they will only on average agree well with periods calculated from actual equator crossing drift rate data.

RESULTS AND CONCLUSIONS

Using the satellite situation report of March 31, 1968 (listing as satellite orbit elements, the mean period, P, the apogee and perigee heights and the inclination) in conjunction with Figure 1, there have been 5 new objects discovered presently orbiting with off resonant beat periods greater than 100 days. Two have not previously been analyzed for resonant gravity effects. The five objects are:

1959 ETA 1 (Vanguard 3), n = 11 revs./day
1959 ALPHA 2, n = 11 revs./day
1962 A ALPHA 3, n = 14 revs./day
1963-26A, n = 14 revs./day
1966-13A, n = 12 revs./day.

The objects 1959 ETA 1, 1959 ALPHA 2 and 1963-26A were observed (prior to 1966) in shallow resonant orbits, and have already been used (to some extent) for satellite geodesy[4].

All of these objects except Vanguard 3 have orbits with semimajor axes less than that required for exact commensurability. Thus, considering drag, only Vanguard 3 will pass through deeper resonance as time goes on.

Of less immediate interest are the following 5 new objects found whose orbits have resonant beat periods between 50 and 100 days:

1961 DELTA 2, n = 12 revs./day
1962 BETA 2, n = 14 revs./day
1965-89C, n = 12 revs./day
1966-13B, n = 12 revs./day
1966-56A (PAGEOS 1), n = 8 revs./day

Of these objects, only 1965-89C and 1966-56A are above their commensurable mean distances. The semimajor axis of PAGEOS 1 is strongly affected by radiation pressure and at the present time it is being driven down by this pressure so that it's resonant beat period is lengthening.

Orbit analysis of all of the above objects should yield valuable new geodetic information on selected earth harmonics of 8th, 11th, 12th and 14th order (or longitude frequency). A survey still remains to be done on the 2000 or so decayed earth satellites to find those which may have passed through resonances sufficiently slowly (due to drag or radiation pressure) to have suffered geodetically interesting but as yet unrevealed orbit changes.

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