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SOME LABORATORY TECHNIQUES OF WAVE ANALYSIS WITH APPLICATION TO THE APOLLO PROGRAM

by Roy A. Watlington

Manned Spacecraft Center
Houston, Texas

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ABSTRACT

Time and frequency correlation, root-mean-square time histories, and power-spectral density analysis techniques are applied not only in perfecting communication systems but also in gaining vital information on the chemical and physical nature of things. The sophisticated instruments and techniques of the present age have made it possible to record, synthesize, and analyze wave data rapidly and in depth. This paper examines and discusses some of the major techniques of wave analysis being applied to the Apollo Program, describes the instrumentation involved, and provides an illuminating mathematical foundation for the analyses.

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SOME LABORATORY TECHNIQUES OF WAVE ANALYSIS

WITH APPLICATION TO THE APOLLO PROGRAM

By Roy A. Watlington
Manned Spacecraft Center

SUMMARY

The Instrumentation and Electronic Systems Division of the NASA Manned Spacecraft Center operates an analog data analysis facility concerned with the processing and presentation of test data in a form yielding a history of recorded test events. This service is requested by and provided for organizations which originate tests and which design, develop, and mount the necessary instrumentation in the test vehicle to record the desired data. The function of the Wave Analysis Laboratory complements that of other data reduction organizations by handling those data which yield the most useful information when analyzed by analog techniques. An additional service often provided is the analog checking of data which have been reduced primarily by digital techniques or by other analog laboratories.

The laboratory facilities include some of the most advanced equipment in the field of analog data analysis. With this equipment and with a trained staff of engineers and technicians from NASA and a contractor, a number of wave analysis techniques have been refined and applied to the Apollo Program. The operations inherent in these techniques include the following:

1. The provision of graphical time histories of any test data for "quick look" or other purposes

2. The plotting of spectrograms

3. The conducting and recording of 1/3-octave analyses

4. The plotting of autocorrelograms and cross correlograms

5. The plotting of power spectra and cross spectra

Furthermore, the ability to conduct reflectivity studies is under development.

INTRODUCTION

The laboratory techniques employed at any given moment depend wholly upon the mission or missions with which NASA is involved at the time. The procedure in rendering wave analysis services is reviewed in the following sections.

Scale models, mock-ups, and boilerplates representing spacecraft to be used on manned missions such as the Apollo lunar mission are invaluable in testing the general vehicle configuration for specific features. Tests are conducted to establish noise and acceleration levels and to measure stress, strain, temperature, pressure, and many other variables. Models of spacecraft are made full size and full weight or on a reduced scale. They are exposed to the rigors of launching, reentry, and landing simulated by drop tests on land or water and to several other forms of environmental tests. Tests with these models have proven to be very reliable for determining the values of many types of variables and are economically desirable.

Test operators determine the best method of measuring some important variable and mount devices on the test vehicle which measure and record or transmit data on this variable. Transducers are used to sense changes in some physical quantity (such as temperature, pressure, or acceleration) and are capable of transforming the energy received into electronic signals which may be recorded by a tape recorder aboard the test vehicle. For flight tests, the signals may be transmitted by telemetric equipment to listening stations where the signals are recorded. The varying signal voltages from the transducers on board the test vehicle are recorded on magnetic tape with the time code. Most frequently, a 14-channel magnetic tape is used. At least one channel on the tape is reserved for the signal from each transducer. Furthermore, one channel carries the time code, a constant-frequency signal which is amplitude modulated with information that makes it possible to correlate events in the test with changes in the signal voltages. Recordings of the signal from a signal transducer received at several receiving stations are combined to give a complete and continuous record of the signal for the duration of the test. Synchronizing the recordings made by the different receiving stations is most easily accomplished when all stations are using the same time code. Complete tapes of the recorded data are reproduced, and copies are sent to various processing organizations.

In the wave analysis facility, raw test data recorded on magnetic tapes are transformed into comprehensible, graphical records. Data may be displayed in a variety of ways, depending upon the requirements of those organizations interested in the results of the tests. For example, the laboratory can provide either linear-linear, log-log, or semi-log plots as required and can give detail on any segment of the data in which an interesting event or an abrupt, isolated change (called a transient) in the data may occur.

Generally, the organizations conducting the tests provide data processing plans which specify the manner in which the data must be displayed. Specifications for the scale upon which the data must be plotted, the amount of detail desired in particular regions on the tapes, the types of displays required, and the desired time schedule are all provided in the plans. However, a certain amount of flexibility in these instructions and close liaison between the wave data analyst and the organization conducting the test make changes in the schedule possible in order to analyze unexpected results appearing in the processed data. The finished data are returned to the test operators who interpret the output on their transducers to make conclusions about changes in the variables under study.

This report has been prepared to provide a survey of the techniques of wave analysis used in the analog data analysis facility.

SYMBOLS

A	peak amplitude of any constant coefficient
A_n	peak amplitude of the nth component of a set
B	bandwidth
$C_{xy}(f)$	cospectral density function for x and y
d	distance
E_o	fundamental voltage
e	natural logarithmic base
f	frequency, or center frequency (Hz)
f_o	fundamental frequency
G, g	gravitational acceleration, 32 ft/sec^2
$G_x(f)$	power-spectral density function for x
$G_{xy}(f)$	cross-spectral density function for x and y
$G_y(f)$	power-spectral density function for y
$H(f)$	frequency response function of a system
m	infinity is upper limit
n	rational number (as in Fourier series) or 1
P	power, W

$Q_{xy}(f)$	quadrature-spectral density function
R	resistance
R_e	mean-square error of the system
$R_x(\tau)$	autocorrelation function of x
$R_{xy}(\tau)$	cross-correlation function of x and y
$R_y(\tau)$	autocorrelation function of y
T	averaging or observation time, sec
t	time, sec
V	potential difference, V
v	velocity, ft/sec
$x(t), y(t)$	time-dependent variables x and y
$\gamma_{xy}^2(f)$	real frequency-dependent variable or coherence function
Δ	time interval
δ	Dirac delta function
θ	an angle, rad
τ	lag or delay time, sec
ω	angular velocity, rad/sec
ϕ or $\theta_{xy}(f)$	phase angle, deg or rad

Subscripts:

j	discrete or incremental values of y in respect to rms
n	a number times the fundamental frequency
o	fundamental or initial rate (Hz) as in f_o

x independent variable of time

y dependent variable of y(t) or x

TYPES OF DATA SIGNALS

All information-bearing signals may be described in terms of one or more of the basic categories of data. Generally, data may be classified as either deterministic or random (stochastic). Figure 1 is an organizational chart showing the most basic categories for data. The signals which occupy the man- and machine-time in the Wave Analysis Laboratory sometimes may fit precisely into one of these categories. Very often the data being processed exist in a classification comprising two or more of the basic groups.

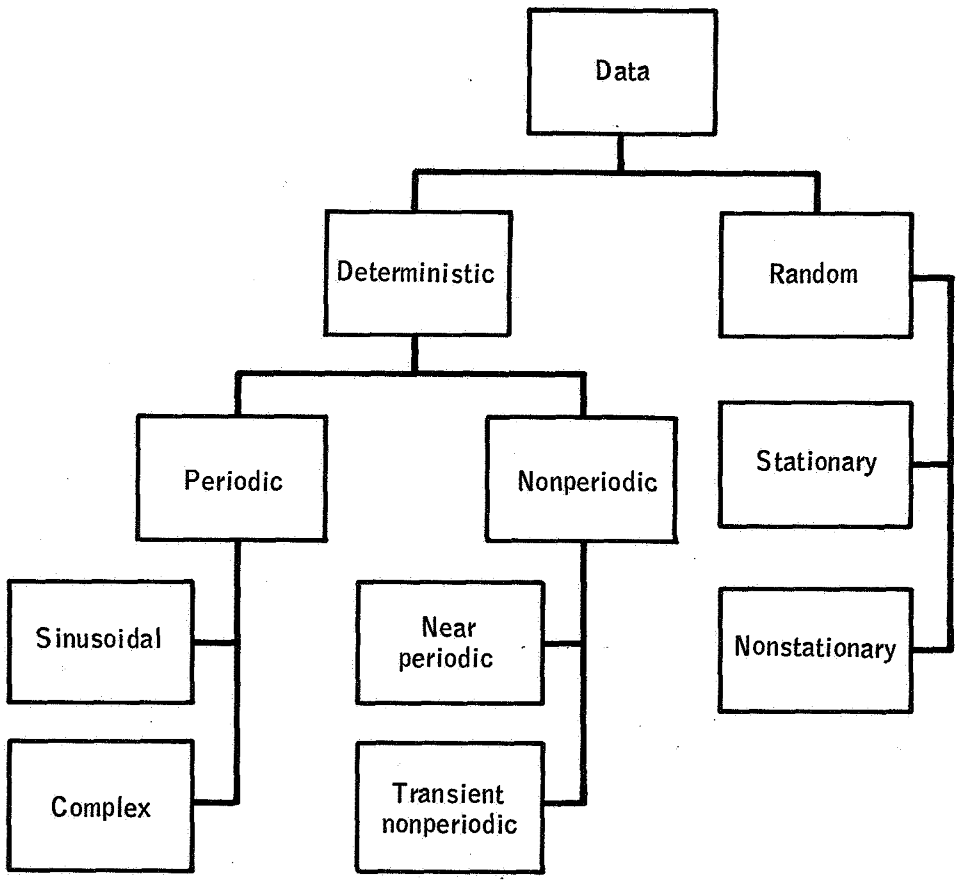


Figure 1.- Organizational chart.

Deterministic Data

Those data from variables which can be described accurately by a mathematical expression and, therefore, are predictable are called deterministic. Natural phenomena such as an apple falling from a tree or the motion of the earth around the sun are deterministic processes. Deterministic data may be more precisely discussed under one of the following categories.

Periodic data. - Periodic data are those data from a dependent process or variable which repeats itself after a certain interval in the independent variable (such as time or displacement). If the variable is being studied with respect to time, the interval after which the dependent process begins to repeat itself is called the period.

Sinusoidal periodic data: Sinusoidal periodic data are data which may be described (for a time-dependent variable) by the mathematical statement

$$y(t) = A \sin\left(2\pi f_o t + \phi\right) \tag{1}$$

where $y(t)$ is the instantaneous value of the variable, A is the peak amplitude, f_o is the frequency of its repetition, and ϕ is the phase angle. The following relationship exists between the frequency and the period T

$$f_o = \frac{1}{T} \tag{2}$$

Complex periodic data: Complex periodic signals are composed of a number of sinusoidal components in addition to a static component. Complex data for a time-dependent variable $y(t)$ may be described by a Fourier series

$$y(t) = A + \sum_{n=1}^{n=\infty} A_n \cos\left(2\pi n f_o t - \phi_n\right) \tag{3}$$

where $n = 1, 2, 3, \ldots$ (an integer); where f_o is the fundamental frequency; and where A_n is the amplitude of the component which has a frequency of n times f_o. The sinusoidal components are known as harmonics; the ratio of the frequencies of any two harmonics equals a rational number. Complex periodic data are more common in natural occurrences than are pure sinusoidal data.

Nonperiodic data. - Nonperiodic data fall into two possible subclassifications.

Near periodic data: Those data consisting of a number of sinusoidal components the frequencies of which are not multiples of a fundamental frequency are called near periodic data. For a time-dependent variable, near periodic data can be described mathematically

$$\left.\begin{aligned} y(t) &= \sum_{n=1}^{n=\infty} A_n \sin\left(2\pi f_n t + \phi_n\right) \\ n &= 1, 2, 3, \ldots \end{aligned}\right\} \tag{4}$$

where the ratios of all possible pairs of component frequencies do not all equal rational numbers.

Transient nonperiodic data: Transient nonperiodic data are produced by a large number of various phenomena. Events isolated or discontinuous in the time domain give rise to transient data and may be described by equations which have limiting or boundary conditions accompanying them. For example, the following expressions define the transient whose time history is shown in figure 2.

$$y = A, \text{ as } 0 \leq t \leq t_1 \tag{5}$$

$$y = 0, \text{ as } t < 0; \quad t > t_1 \tag{6}$$

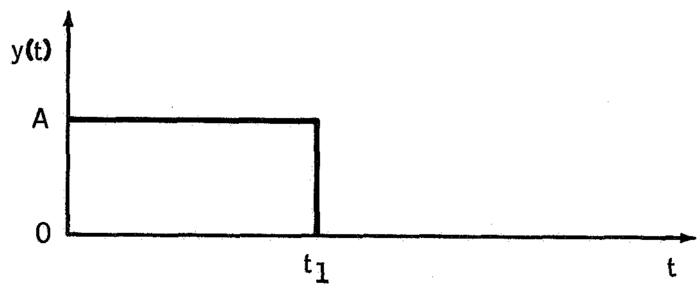


Figure 2. - A transient process y(t).

Transient events may have only continuous spectra, unlike the events described by the other types of data discussed previously, which have discrete spectra.

Random Data

Events which give rise to random data cannot be described by any particular mathematical formula which would be certain to hold true at any two instants of time. Such data cannot be predicted precisely and may be described only by a probability distribution. Random data are said to be stationary or nonstationary as explained in the following paragraphs.

Stationary random data. - A random process is said to be stationary if each of its probability distributions is the same at one time as at another. Alternately, a stationary process may have a mean value, a mean-square value, or an autocorrelation function as a time invariant. When all three of these moments are constants in time, the process is called strongly stationary. If, however, the mean value and the mean-square value are constants but the autocorrelation function is dependent on lag time τ, the process is called weakly stationary.

Nonstationary random data. - The extreme case in which the mean value, the mean-square value, and the autocorrelation function are all time-dependent defines the nonstationary random process. For such a process, a meaningful description of its values can be obtained only by taking averages for every instant of time over the entire set of data concerning the process.

ROOT-MEAN-SQUARE TIME HISTORIES

Among the services provided by the Wave Analysis Laboratory is the development of "root-mean-square (rms) time histories" of test data recorded on magnetic tape (fig. 3). These graphical displays consist of plots of rms values of the signal voltage, from a transducer aboard the test vehicle, as a function of the time elapsed in the test. Time histories of rms values are useful records of net changes in the signal for the duration of the test. They may be compared to and marked with facts known about events in the test. For example, the time when a critical event or activity in the test was initiated and when it was ended is known and may be indicated on the abscissa (x-axis) of the rms time history. In flight tests, the times at which different activities in a sequence are initiated correspond to changes in the rms voltage which are evident in the rms time history. The magnitude and the nature of the changes in signal voltage reveal much about the variance of the parameter being measured.

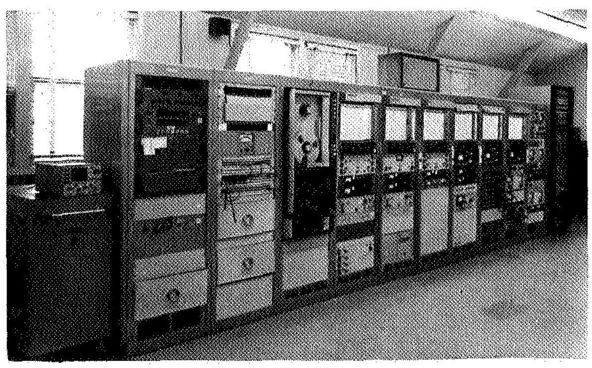


Figure 3. - System for root-mean-square time history.

Figure 4 is a composite plot of several sections of one rms time history. The test was an Apollo flight test. The graph has been marked to show the positions on the

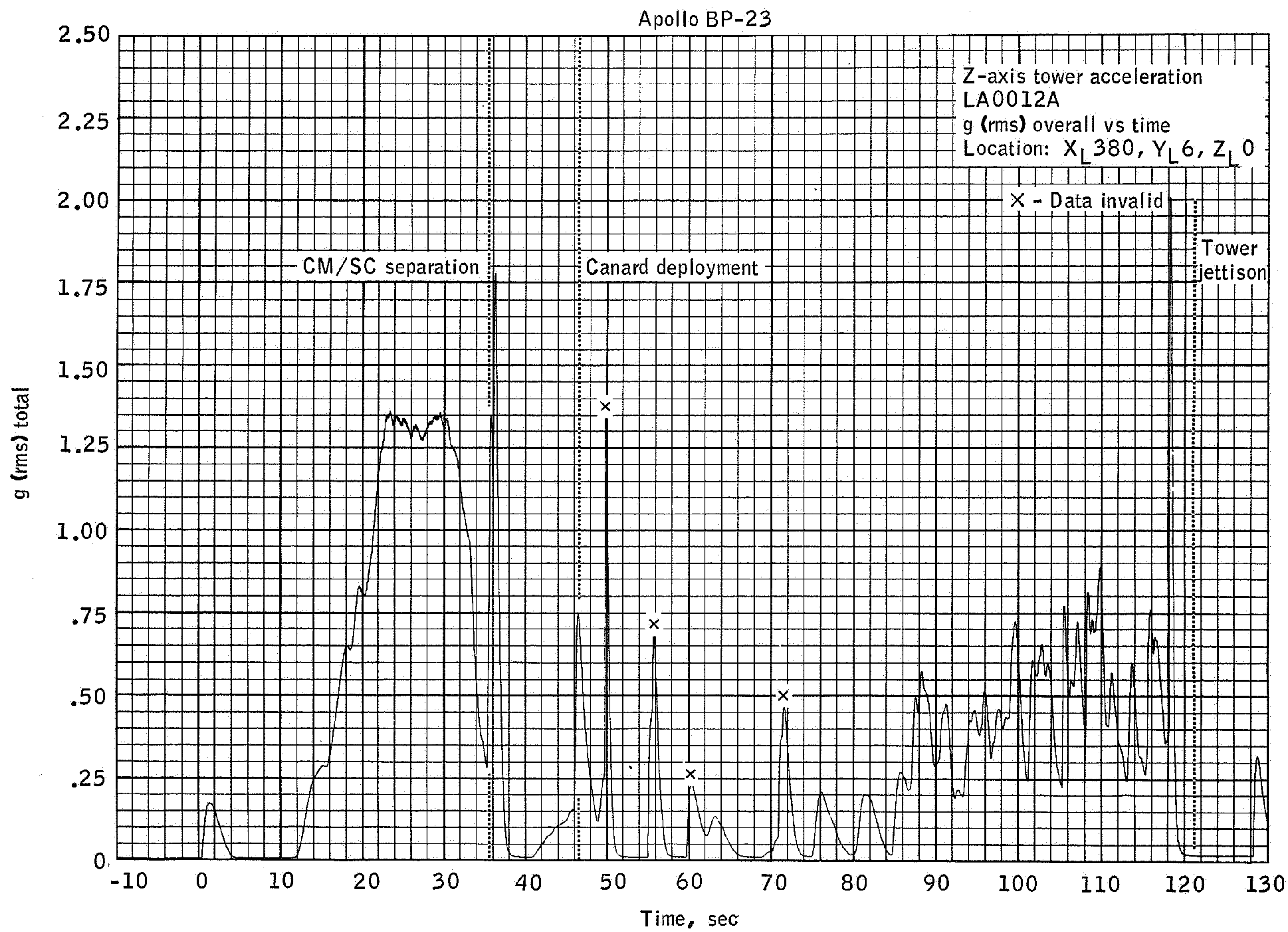


Figure 4. - Time history of root mean square for Apollo Boilerplate 23 test.

time axis at which events such as ignition, separation, and cut-off occurred. Time histories of rms values such as this one are useful in determining which portions of the recorded data are of critical interest and require the most detailed analysis.

The rms value of any quantity y, which is a function of time, is the square root of the time average of the square of the quantity. Expressed symbolically for a continuous function of time, the statements are

$$y_{rms} = \left[\frac{1}{T}\int_0^T y^2(t)dt\right]^{1/2} \tag{7}$$

where $y = y(t)$ (meaning that y is a function of time and T is the time interval over which the rms value of $y(t)$ is desired). If, however, the quantity takes on a number n of discrete values y_j, then the correct mathematical expression for the mean-square value necessitates a summation rather than an integration procedure.

$$y_{rms} = \left(\frac{1}{n}\sum_{j=1}^{j=n} y_j^{\,2}\right)^{1/2} \tag{8}$$

To illustrate the development of the rms value of a quantity which is a continuous function of time, consider the periodic signal described by the equation $y(t) = A \sin t$. If the peak amplitude A of a sinusoidal signal equals 1 volt, the rms value of the signal is 0.707 volt. With random data, however, this generalization is meaningless, and the rms value of a randomly varying voltage is produced only by a full-computation procedure.

The procedure for finding rms values is one of squaring, integrating, multiplying, and taking the square root. In the Wave Analysis Laboratory, this task is performed simply by the use of an analog device as illustrated in figure 5.

The heat generated in the resistive load is proportional to the true-rms input voltage. The low-level dc thermocouple signal is proportional to heat generated in the resistive load because of predetermined conductive and radiative character of the oven. This signal is amplified because of problems in resolving low-level signals, and the output meter is calibrated directly, by empirical means, in terms of rms input.

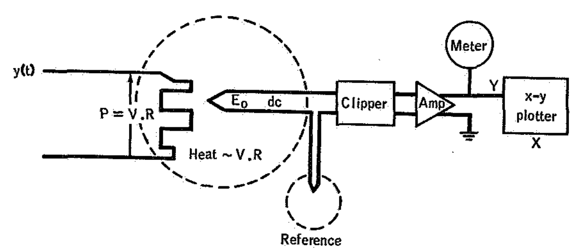
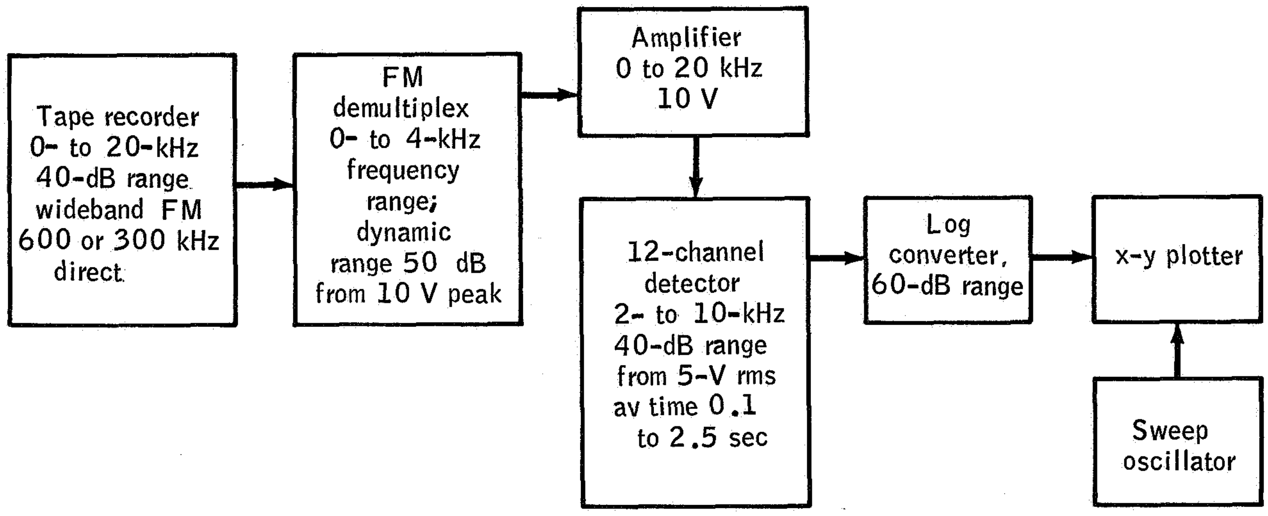


Figure 5.- Analog device for determining root-mean-square values.

The output of the detector is a dc voltage needed to run the x-y plotter. However, the dc voltage is proportional to the rms value of the input signal voltage. When properly calibrated, the x-y plotter describes the correct variance of the rms value of the input signal on the vertical axis of the appropriate graph paper. The horizontal sweep rate of the plotter is matched with the time code recorded on a channel of the same tape as the input signal. Therefore, the graph of the rms value of the signal versus time offers the chronological history of the signal voltage.

The rms over an increment of time also may be obtained from a plot of power-spectral density (PSD) versus frequency by finding the area under the curve and taking the square root.

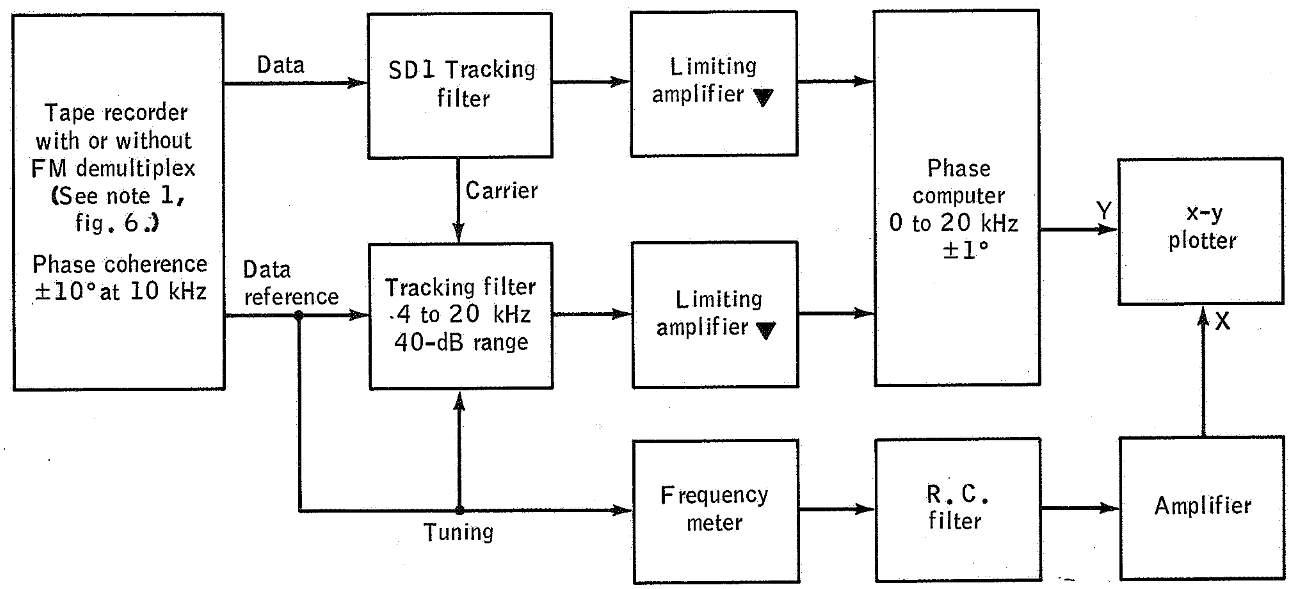
The time-history system is discussed subsequently, with block diagrams in figures 6 to 8 outlining the system.



Notes:

1. The output of the tape recorder in FM reproduce will go directly into the amplifier when the signal is not multiplexed.
2. The x-y plotter has sufficient rise-time compatibility for 0.1-second averaging.
3. The sweep oscillator furnishes a linear ramp.

Figure 6. - Block diagram, time history.



▼ Automatic gain-control units to maintain sufficient signal control

Figure 7. - Block diagram, phase/frequency.

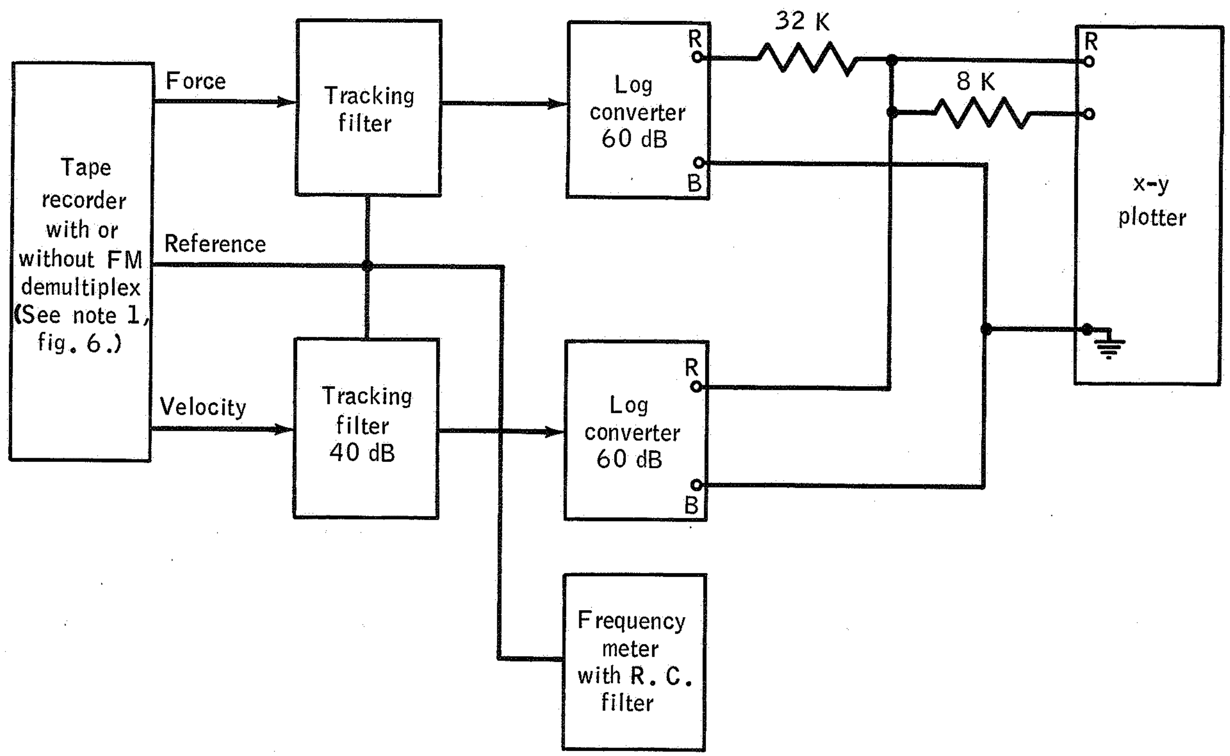


Figure 8. - Block diagram, mechanical impedance versus frequency.

SPECTROGRAMS

In the Wave Analysis Laboratory, a variety of plots is generated by using frequency as the independent variable (plotted on the x-axis). Such plots are called spectrograms and are used to establish the frequency distribution of some property of the dependent variable (in this case, the data signal voltage). (See fig. 9.) A plot of the amplitude of the signal voltage versus frequency is the simplest form of spectrogram. Other spectrograms may have the mean value, the mean-square value, or the rms value of the signal voltage as the dependent variable. In addition to these distinctions, either full-octave or 1/3-octave spectrograms can be plotted in the Wave Analysis Laboratory. The latter capability merits more detail.

Acoustical and vibrational data are often processed by 1/3-octave analysis when the acceleration g levels or the acoustical levels measured by transducers need to be compared to a reference level (often one g rms, in the case of vibrational data). A logarithmic plot is generally used so that the vertical axis reads decibels as units of measure. The following are two of the logarithmic expressions that make this approach convenient:

1. For acoustic level and vibration power level

$$dB = 10 \log_{10}\left(P_1/P_r\right) \tag{9}$$

where P_1 is the power level under consideration and P_r is the reference power level.

2. For acceleration level and acoustic sound pressure level

$$dB = 20 \log_{10}\left(G_1/G_r\right) \tag{10}$$

Here G_1 is the rms acceleration being compared to a known rms acceleration level G_r.

In 1/3-octave spectrograms, a spectrum analyzer divides the frequency spectrum, in which the data signals may be expected to exist, into 1/3-octave bands by using filters with adjustable bandwidths and center frequencies. The signal voltage level is sampled in this bandwidth and, by sweeping center frequencies, the spectrum analyzer develops a continuous plot of signal level versus frequency. In this procedure, as in the processing of rms time histories, the rms of the signal is developed by passing the signal through a detector circuit. If linear scale is to be used, the dc output from the detector will run the x-y plotter. Logarithmic converters are necessary if a log scale (decibels) is desired. Figure 9 is a typical 1/3-octave spectrogram. The system is described in the section of the paper entitled Spectrometer, 1/3- or Full-Octave Acoustic Analysis System.

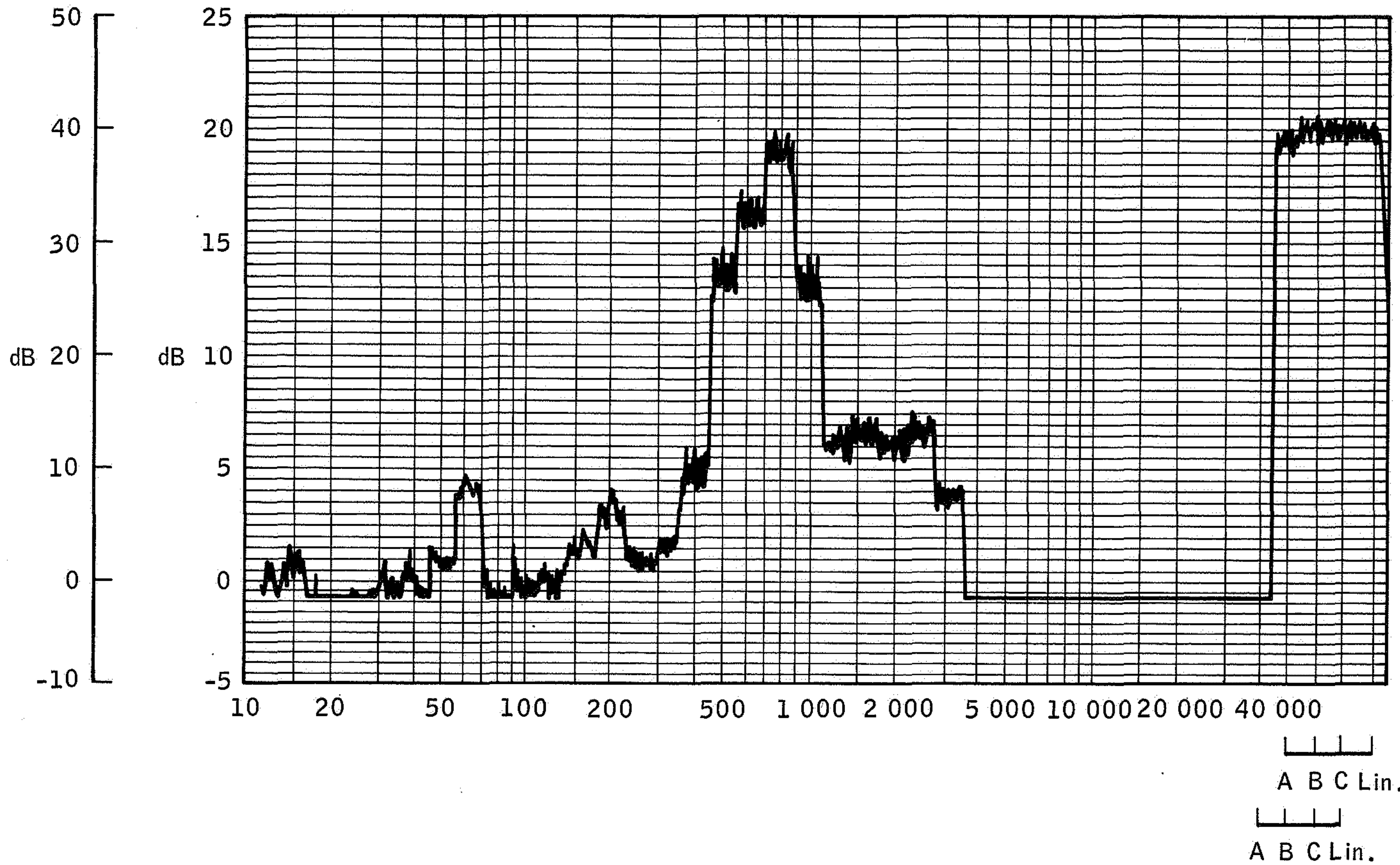


Figure 9.- Apollo spectrogram (1/3 octave).

Another basic capability in the Wave Analysis Laboratory is the development of autocorrelation-function plots (autocorrelograms). (See fig. 10.) The autocorrelation

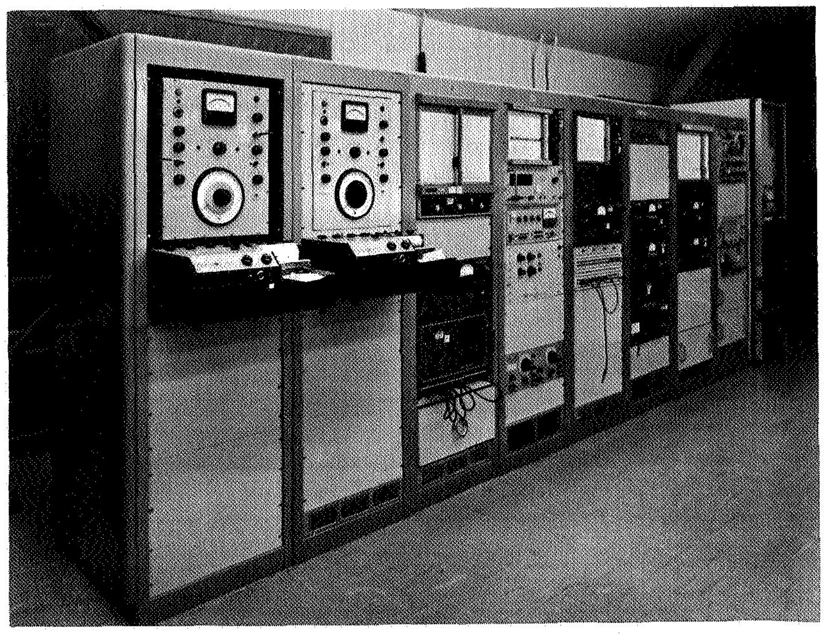


Figure 10. - Time and frequency correlation system.

function of any signal provides a description of the relationship between the values of signal data studied at two different instants. If the value of a variable is known at one time as $y(t)$, and after a time delay τ as $y(t + \tau)$, the autocorrelation between these two values of the same signal may be calculated by using the definition

$$R_y(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T y(t)y(t + \tau)dt \tag{11}$$

where T is the observation or averaging time. Taking T to infinity to get an exact value for $R_y(\tau)$ is not possible in practice; however, a sufficiently accurate estimate is obtained from

$$R_y(\tau) \approx \frac{1}{T} \int_0^T y(t)y(t + \tau)dt \tag{12}$$

The autocorrelation function is a real, even function of the lag time τ. The function has a maximum at $\tau = 0$ and is symmetrical about this point. The maximum value of $R_y(\tau)$ is the mean-square value of the variable $y(t)$ as can be seen when $\tau = 0$ is substituted into equation (11).

$$R_y(0) \approx \frac{1}{T} \int_0^T y^2(t)dt \tag{13}$$

The practical use of the autocorrelation function comes from the autocorrelogram, which is obtained by varying the delay time and plotting the dependence of $R_y(\tau)$ upon τ. Correlograms reveal a variety of useful facts about random as well as deterministic data. They are often used to complement power spectra in areas where the autocorrelation format is more convenient. For example, the autocorrelation function is a very useful device for detecting the presence of periodic signals in a background of random data. Also significant is the fact that the autocorrelation function is the inverse Fourier transform of the PSD function. Consider, for example, the autocorrelation function $R_y(\tau)$ for a signal described by $y(t) = A \sin(\omega t + \phi)$. (See appendix A.)

If $\omega = 2\pi f$, f = frequency, ϕ = phase angle, and A is the peak amplitude of the signal, then

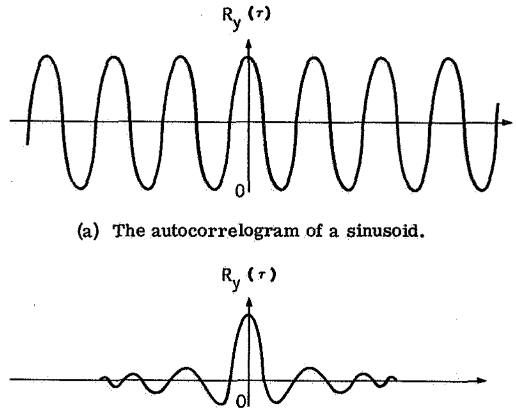
$$R_y(\tau) \approx \frac{1}{T} \int_0^T A \sin(\omega t + \phi) \cdot A \sin[\omega(t + \tau) + \phi]\, dt \tag{14}$$

and

$$R_y(\tau) \approx \frac{A^2}{2} \cos \omega\tau \tag{15}$$

It is clear from this example that the autocorrelation function is an even function, that it has a maximum at $\tau = 0$, that it is a periodic function of τ with the same frequency as the signal $y(t)$, and that it is independent of phase angle ϕ. Figure 11(a) is a plot of this autocorrelation function. An example at the other extreme is the autocorrelation function for a random signal with uniform energy distribution with respect to frequency (hypothetical "white noise"). The autocorrelation function $R_y(\tau)$ in this case is a Dirac delta function which rises to the mean-square value of the signal at $\tau = 0$ and is zero for all other values of τ. However, a signal of true white noise cannot be realized in practice. The autocorrelogram for narrow-band random noise is shown in figure 11(b). In this case, $R_y(\tau)$ goes to zero for large τ. This fact is useful in distinguishing narrow-band random noise from random signals which include errant sinusoids. All of the autocorrelograms shown previously could have been plotted to include negative values of the time lag τ. Since the autocorrelation function is an even function, such plots always would be symmetrical about $\tau = 0$.

The analog instruments used in the Wave Analysis Laboratory perform the mathematical procedures discussed previously in developing autocorrelation functions which are then plotted, as functions of delay time, by an x-y plotter. These procedures are outlined by the block diagram of figure 12.



(a) The autocorrelogram of a sinusoid.

(b) Autocorrelogram of narrow-band random noise.

Figure 11. - Autocorrelation functions.

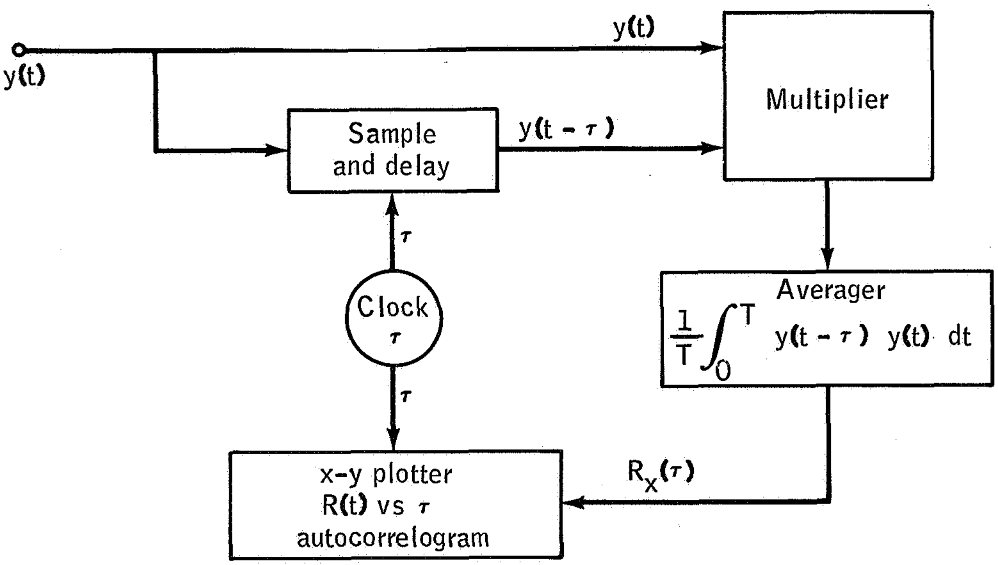


Figure 12. - Block diagram of autocorrelation analysis.

First, the input signal $y(t)$ is delayed by τ seconds. The delayed signal $y(t - \tau)$ is multiplied by the value of the signal $y(t)$ and sampled after τ seconds lag time. Finally, the product $y(t - \tau) \cdot y(t)$ is averaged over the observation time T. The average of this product is identical to the average required in the mathematical expression

$$R_y(\tau) \approx \frac{1}{T}\int_0^T y(t)y(t + \tau)dt \qquad (16)$$

THE CROSS-CORRELATION FUNCTION

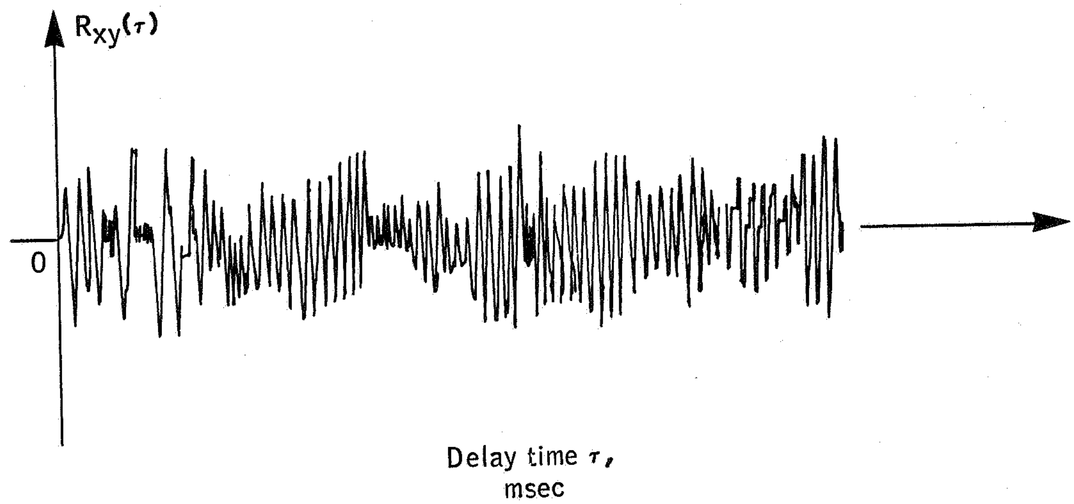
The cross-correlation function for random data from two different processes or variables illustrates the dependence of the values of one variable upon the values of the other variable. For the two time-dependent variables $x(t)$ and $y(t)$, the cross-correlation function is a function of the delay or lag time τ and is defined mathematically by the equation

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)y(t+\tau)dt \tag{17}$$

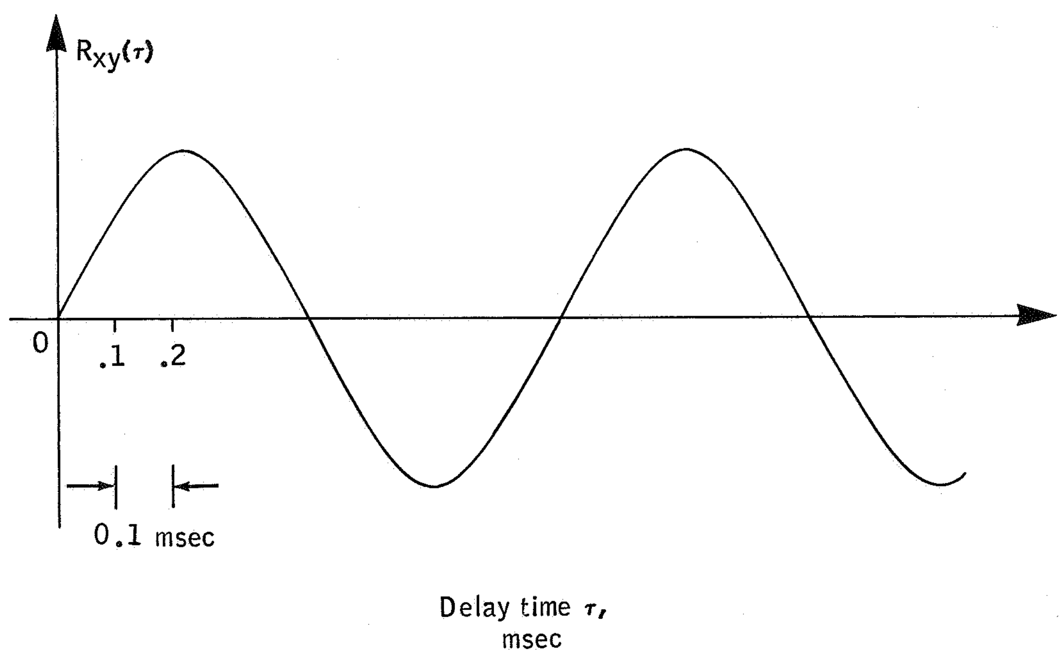
where T is the observation or averaging time, and the variable $y(t+\tau)$ is sampled τ seconds after the variable $x(t)$ is sampled. The value $R_{xy}(\tau)$ is real-valued for real $x(t)$ and $y(t)$ but is not an even function and need not have its maximum at $\tau = 0$ as is true with the autocorrelation function. Because it is not possible to average over infinite time intervals, the cross-correlation function may be estimated as follows

$$R_{xy}(\tau) \approx \frac{1}{T}\int_0^T x(t)y(t+\tau)dt \tag{18}$$

A plot of $R_{xy}(\tau)$ versus τ is called a cross correlogram and is used in measuring time delays, in distinguishing a signal from background noise, and in determining the paths of propagation of noise, vibration, and other disturbances. Cross correlograms display sharp peaks at values of τ at which the two variables correlate. The value of $R_{xy}(\tau)$ is zero when $x(t)$ and $y(t)$ are uncorrelated or if these variables are statistically independent. Figure 13(a) shows the cross correlogram in such a case. Specifically, the variables are a sine wave signal and a wideband random signal. The variation of $R_{xy}(o)$ around zero is attributable to the relatively fast sampling time and the relatively short averaging time. The cross correlogram of two sets of interrelated signals is given in figure 13(b). Figure 13(b) illustrates the cross correlation between two sine waves of the same frequency but of different amplitudes and 90° out of phase. This plot demonstrates the problem worked in appendix B where $x(t) = A_1 \sin\left(\omega_1 t + \phi_1\right)$ and $y(t) = A_2 \sin\left(\omega t + \phi_2\right)$. Here $\phi_2 - \phi_1 = 90°$, $A_1 = 400$ mV, $A_2 = 500$ mV, and $f = 1000$ Hz. As predicted by theory, the cross-correlation function is periodic with the same frequency as the individual signals. The more general problem of appendix B demonstrates that when two signals are completely independent, as is true when $\omega_1 \neq \omega_2$, the cross-correlation function is zero.



(a) Cross correlogram of a 1000-Hz sinusoid with wideband random noise.



(b) Cross correlogram of two 1000-Hz sine waves 90° out of phase.

Figure 13. - Cross-correlation functions.

Measuring time delays and transmission times involves an application of the fact that a cross correlogram shows a peak at that value of τ at which the input signal to a linear system correlates with the output signal from the system. The measurement of the velocity of sound in an isotropic medium appears to be a simple task when using these techniques in a laboratory arrangement as shown in figure 14.

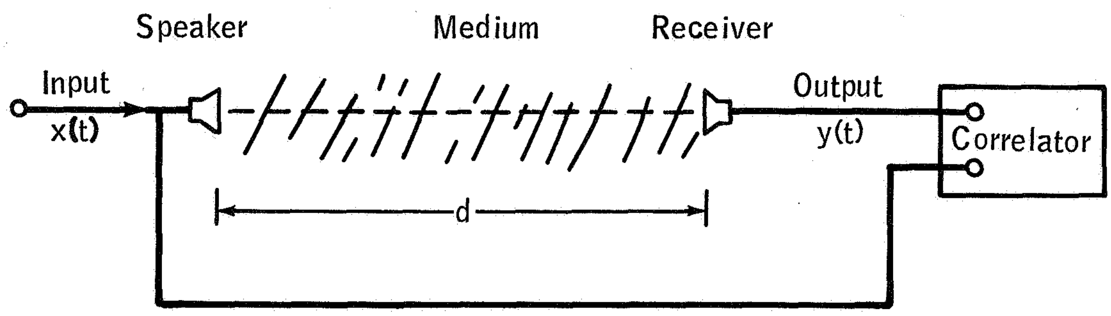


Figure 14. - Measurement of the velocity of sound.

The signal x(t) at the speaker is delayed by τ seconds and is cross correlated with the signal y(t) at the receiver. The cross correlogram is produced by varying τ and shows a peak for $R_{xy}(\tau)$ at that value of τ at which x(t) and y(t) correlate again. This value of τ equals the time of transmission of sound across the distance d between the speaker and the receiver. The familiar equation for velocity $v = d/t$ is used to calculate the average velocity of sound in this medium. This procedure serves as a very straightforward example of the use of cross correlograms but does not always yield reliable results. In the preceding procedure, for example, the frequency or phase characteristics of the sound wave may be altered by the medium so that the output signal does not resemble the input signal well enough for distinct peaks in the correlogram. A precise value of τ is not obtainable in this case.

The cross-correlation function, when used to distinguish signals from background noise, proves itself superior to the autocorrelation function as well as other techniques because the signal does not need to be periodic to be detected. By cross correlating a stored "pure" version of the known signal with the suspected mixture of signal and noise, a more definite verification of the presence of the signal in the mixture is obtained than through autocorrelation. Cross-correlation techniques offer a greater output signal-to-noise ratio than is available by using the autocorrelation function.

Determining the path of propagation of noise or vibration is another useful capability of the cross-correlation function. The path of maximum transmission of noise and/or vibration needs to be identified in a variety of situations including spacecraft systems, factories, and other structures. Once this path is established, steps may be taken to reduce the amount of disturbance transmitted along this path. The cross-correlation function lends itself readily to such a problem.

Consider the situation illustrated in figure 15. A disturbance is generated at point A and traverses a barrier to get to point B. The barrier offers three major paths for the disturbance. To establish which of the available paths accounts for the most transmission, engineers record the input disturbance $x(t)$ at point A and the output disturbance $y(t)$ at point B.

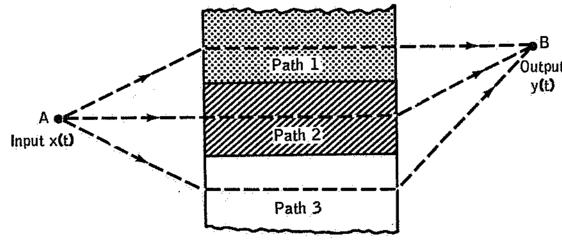


Figure 15. - The propagation of a disturbance through three different paths.

The cross correlogram $x(t)$ and $y(t)$ reveals several peaks of different amplitudes all indicating correlation of the input and output signal (fig. 16). The peak of greatest amplitude is the peak corresponding to maximum transmission. Further correspondence must be established between this peak and a particular path of propagation. This may be done by noting the lag time and making certain safe assumptions about the relative times of transmission for each path. The most limiting feature of this application is the necessary assumption that the important paths of propagation all constitute a linear system. In figure 16, such assumptions lead to the conclusion that path 3 is the path of maximum transmission.

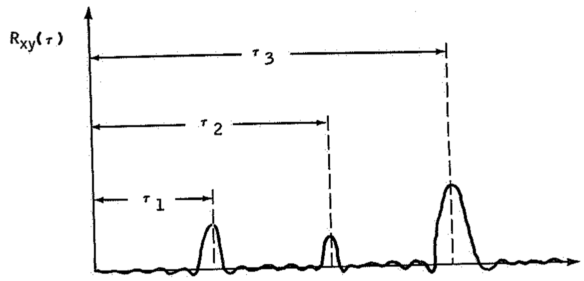


Figure 16. - Cross correlogram of the input $x(t)$ and the output $y(t)$ of the system in figure 15.

The block diagram in figure 17 outlines the steps involved in the development of cross correlograms by analog instruments. Two different signals, $x(t)$ and $y(t)$, are fed into the analyzer. One signal, $x(t)$, is delayed by a time τ to produce $x(t - \tau)$ which is multiplied by the other input signal $y(t)$. This product is averaged over the sampling time T to produce the cross-correlation function $R_{xy}(\tau)$. The same clock that sweeps the value of τ to set the delay on $x(t - \tau)$ runs the horizontal servos on the x-y plotter to produce the graph of $R_{xy}(\tau)$ versus τ. Again, as with the autocorrelation function, the average of the product $x(t - \tau)y(t)$ over T equals the average of $y(t + \tau)x(t)$ as required for $R_{xy}(\tau)$. Figure 18 is a block diagram of the correlation system.

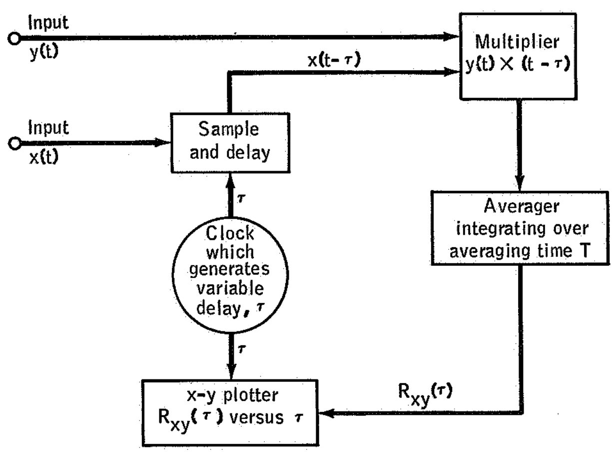


Figure 17. - Block diagram for cross-correlation analysis.

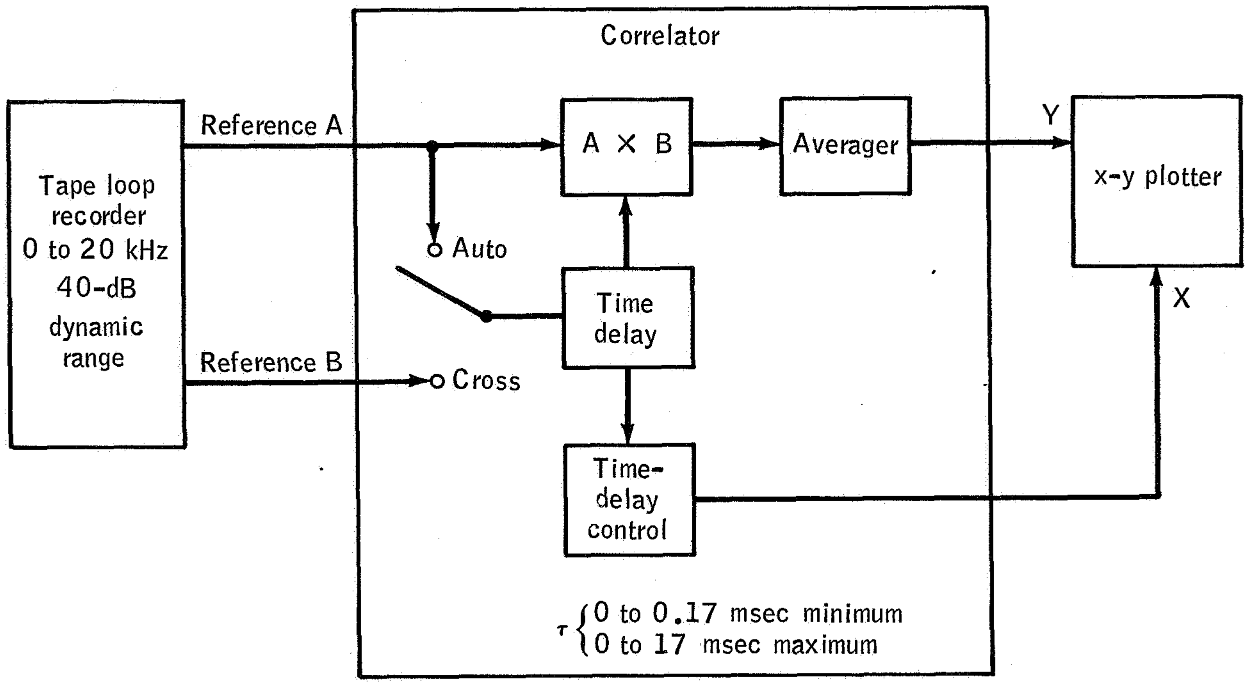


Figure 18. - Block diagram of correlation.

POWER-SPECTRAL DENSITY FUNCTIONS

The output signals from transducers mounted in a test vehicle contain a great quantity of data from the measurement of specific variables during a test. These data yield the maximum of useful information when interpreted in the manner which best suits the variable under consideration. Time histories, for example, are broadly useful as surveys of the behavior of variables for the duration of tests. Frequency spectra are appropriate for determining the frequency distribution of the amplitudes of signals. Further, probability-density function plots are used to verify that random data have normality (that is, a Gaussian-probability-density function) or to determine which theoretical-probability-distribution function best describes the data.

The PSD function, which is a continuous function of frequency for random data, has been found to offer the most meaningful interpretation of the frequency composition of random signals. The PSD function represents the mean-square value of a signal voltage, is measured in V^2/Hz, and is plotted as a function of frequency to yield a power spectrum (figs. 19 and 20). The PSD function describes the manner in which a density function, such as acoustical or vibrational energy-density, is distributed with respect to frequency. Furthermore, PSD analysis may be used to determine the gain factor of the frequency response function from a linear system. Because of the close mathematical relation of the PSD function to other important data-interpreting functions, the PSD function is a convenient tool in many wave analysis situations. Power spectra of signal voltages from transducers measuring such quantities as pressure, velocity, acceleration, current, or voltage reveal the frequency distribution of the energy in the data. Changes in the signal voltages correspond to changes in the

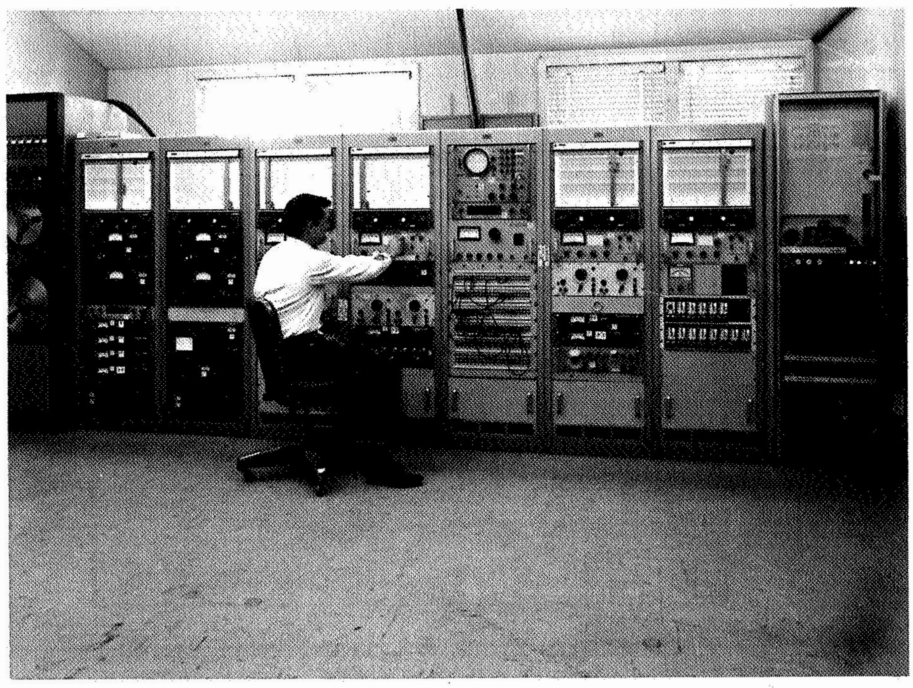


Figure 19. - Power-spectral density analysis system.

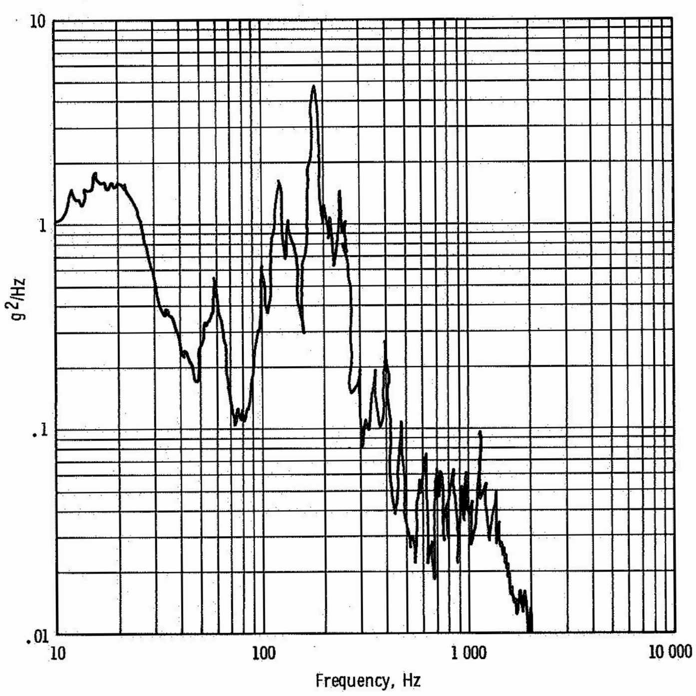


Figure 20. - Typical power spectrum for vibration data from an Apollo mission.

variable being measured by the transducer. The PSD function gives the mean-square value of these signal voltages and is, therefore, directly proportional to the energy (or power per unit of frequency) incident on the transducer.

For acoustical energy-density, for example, transducers are designed to measure acoustical pressure. The PSD function from acoustical tests gives the mean-square value of this pressure per unit of frequency $\left(\text{in psi}^2/\text{Hz}\right)$ and is directly proportional to the acoustical energy-density. A study of power spectra for random acoustical data reveals the frequencies at which the most acoustical energy was generated during the test. Factors known to contribute noise at these frequencies then may be established as the major causes of high noise levels. The vertical scale on a PSD plot may be chosen to read directly in psi^2/Hz once the correspondence between the signal voltage and the acoustical pressure is defined. A more precise treatment of the dimensions of energy-density is usually not necessary for a meaningful analysis of test results but is easily obtainable if certain constants, such as the density of the medium and the velocity of sound in this medium, are known.

For vibrational tests, the PSD function gives the results of the monitored measurements made by accelerometers mounted on the test vehicle. The readout on the vertical axis of a PSD plot for such tests generally has dimensions of g^2/Hz $\left(\text{one } g = 32 \text{ ft/sec}^2\right)$. Figure 20 shows a typical power spectrum for random vibrational test data.

This discussion of the generation of power spectra is brief and leaves many questions concerning the production of PSD functions unanswered. The following outline shows the mathematical calculation of PSD functions and the manner in which analog equipment carries out this calculation.

The PSD function $G_y(f)$ for a signal $y(t)$ is defined mathematically by the equation

$$G_y(f) = \lim_{T \to \infty} \lim_{\Delta f \to 0} \frac{1}{\Delta f} \frac{1}{T} \int_0^T y_{\Delta f}{}^2(t, f)\,dt \tag{19}$$

where $y_{\Delta f}(t,f)$ defines that portion of the signal which is studied in the frequency range between f and $f + \Delta f$ and where Δf is the frequency interval width and T is the averaging time. For real or complex $y(t)$, the PSD function is real and positive.

A convenient feature of the PSD function is its relation to the autocorrelation function given by the expression[1]

$$G_y(f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_y(\tau)e^{-i2\pi f\tau}d\tau \tag{20}$$

This equation shows that $G_y(f)$ is the Fourier transform of $R_y(\tau)$ and may be used to derive the PSD function for the real function $y(t) = 2 \sin 2\pi f_o t$.

In the preceding section, it was shown that the autocorrelation function for the sine wave $y(t)$ is $R_y(\tau) = \frac{A^2}{2} \cos 2\pi f_o \tau$. Using this information with equation (20) yields

$$G_y(f) = \frac{1}{2\pi}\frac{A^2}{2}\int_{-\infty}^{\infty} \cos 2\pi f_o \tau e^{-2\pi i f\tau}d\tau \tag{21}$$

Since $y(t)$ is real, equation (21) may be written

$$G_y(f) = \frac{A^2}{2}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i2\pi f_o \tau} e^{-i2\pi f\tau}d\tau\right) \tag{22}$$

The quantity inside the parentheses defines the Dirac delta function $\delta\left(f - f_o\right)$, which is zero except when $\left(f - f_o\right) = 0$ in which case the function is equal to unity. Therefore, $G_y(f) = \frac{A^2}{2} \delta\left(f - f_o\right)$.

Since taking the limits $T \rightarrow \infty$ and $\Delta f \rightarrow 0$ in equation (19) is not possible in practice, the PSD function for a stationary random signal $y(t)$ has to be approximated as follows

$$G_y(f) \approx \frac{1}{(\Delta f)T}\int_0^T y_{\Delta f}^{\;2}(f,t)dt = \frac{\overline{y_{\Delta f}^{\;2}(f)}}{\Delta f} \tag{23}$$

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[1]Lim, Raymond S.; and Cameron, William D.: Power and Cross-Power Spectrum Analysis by Hybrid Computers. NASA TM-X 1324, 1966.

In this equation, the limit as $\Delta f \to 0$ has been replaced by a finite bandwidth Δf, and a finite averaging time T (in seconds) produces a useful estimate of $G_y(f)$. The center frequency of the bandwidth is f and, like Δf, is measured in hertz. Equation (23) shows that the PSD function is approximately equal to the mean-square value of $y(t)$ in a bandwidth Δf, divided by that bandwidth.

The following is a summary of the most significant mathematical properties of the PSD function $G_y(f)$ for the random variable $y(t)$.

1. The value $G_y(f)$ is proportional to the Fourier transform of the autocorrelation function $R_y(\tau)$.

2. For a stationary random variable $y(t)$, $G_y(f)$ may be considered as the rate of change with respect to frequency of the mean-square value of $y(t)$.

3. The value $G_y(f)$ is related further to the mean-square value $\overline{y^2}$ in that the latter is equal to the area under the power spectrum curve. In the frequency interval $f_2 - f_1 = \Delta f$, the mean-square value is given as follows

$$\overline{y^2(\Delta f)} = \int_{f_1}^{f_2} G_y(f)df \tag{24}$$

4. If a system has a linear-frequency response function $H(f)$ and if a stationary random disturbance $x(t)$, having a PSD function $G_x(f)$, is fed into this system, the output from the system also will be a stationary random disturbance $y(t)$ which has a PSD function $G_y(f)$ given as follows

$$G_y(f) = |H(f)|^2 G_x(f) \tag{25}$$

The function $H(f)$ is complex and is considered in detail in the following sections. Equations (24) and (25) lead to the following relationship

$$\overline{y^2(\Delta f)} = \int_{f_1}^{f_2} |H(f)|^2 G_x(f)df \tag{26}$$

These relationships make possible the determination of the gain factor H(f) of the frequency response function from random data. Establishing the power spectra for the input x(t) and the output y(t) is all that is necessary to solve for |H(f)|.

Equation (23) offers a reliable estimate of the PSD function as follows

$$G_y(f) \approx \frac{1}{(\Delta f)T} \int_0^T y_{\Delta f}^{\,2}(t, f)dt \qquad (27)$$

In developing power spectra, the instruments in the Wave Analysis Laboratory (fig. 19) perform a sequence of operations on the input signal y(t) which is stored on magnetic tape. Figure 21 diagrammatically outlines this sequence. Figure 22 is a block diagram of the PSD system.

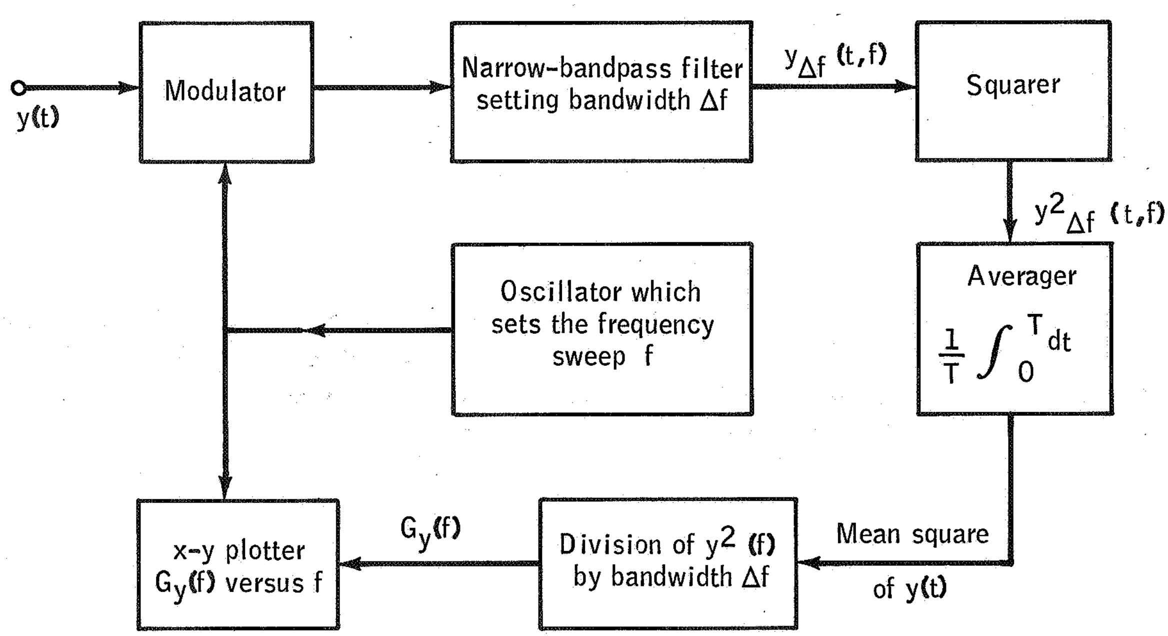


Figure 21. - Block diagram of power-spectral density analysis procedure.

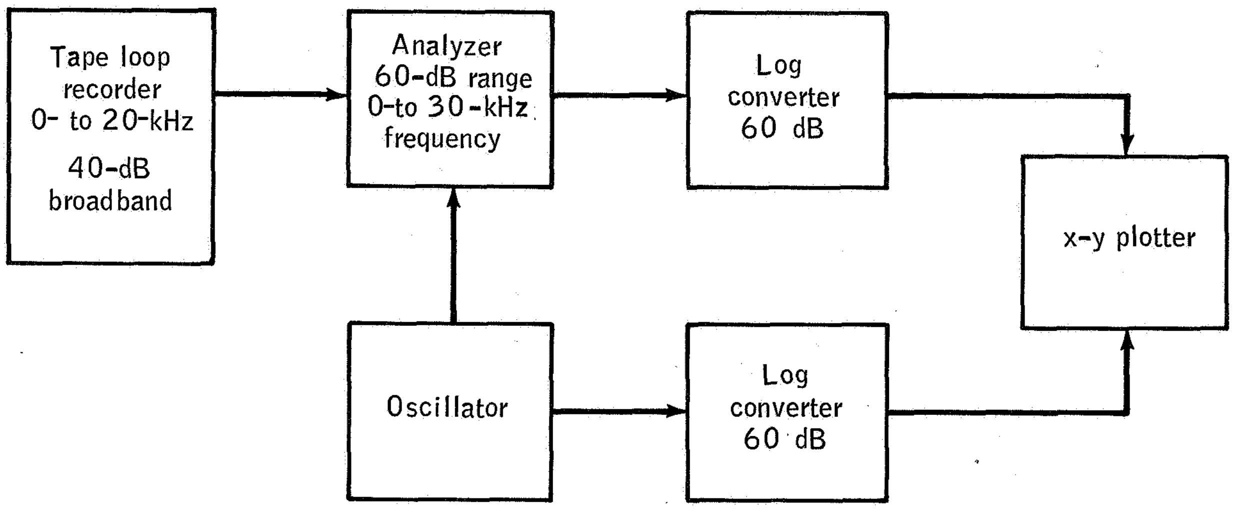


Figure 22. - Block diagram of power spectral density.

CROSS-SPECTRAL DENSITY FUNCTIONS

The cross-spectral density function $G_{xy}(f)$ for random data from two different processes exists in the same relationship with the cross-correlation function $R_{xy}(\tau)$ as does the PSD function $G_y(f)$ with the autocorrelation function $R_y(\tau)$. The value $G_{xy}(f)$ is the Fourier transform of $R_{xy}(\tau)$. The most general representation of the cross-spectral density function is the complex expression[2]

$$G_{xy}(f) = C_{xy}(f) - iQ_{xy}(f) \tag{28}$$

where $C_{xy}(f)$ is called the cospectral density function and $Q_{xy}(f)$ is called the quadrature-spectral density function (fig. 23). These quantities in turn are defined as follows[2]

$$C_{xy}(f) = \lim_{\Delta f \to 0} \lim_{T \to \infty} \frac{1}{(\Delta f)T} \int_0^T x_{\Delta f}(t, f) y_{\Delta f}(t, f) dt \tag{29}$$

[2]Lim and Cameron, Ibid.

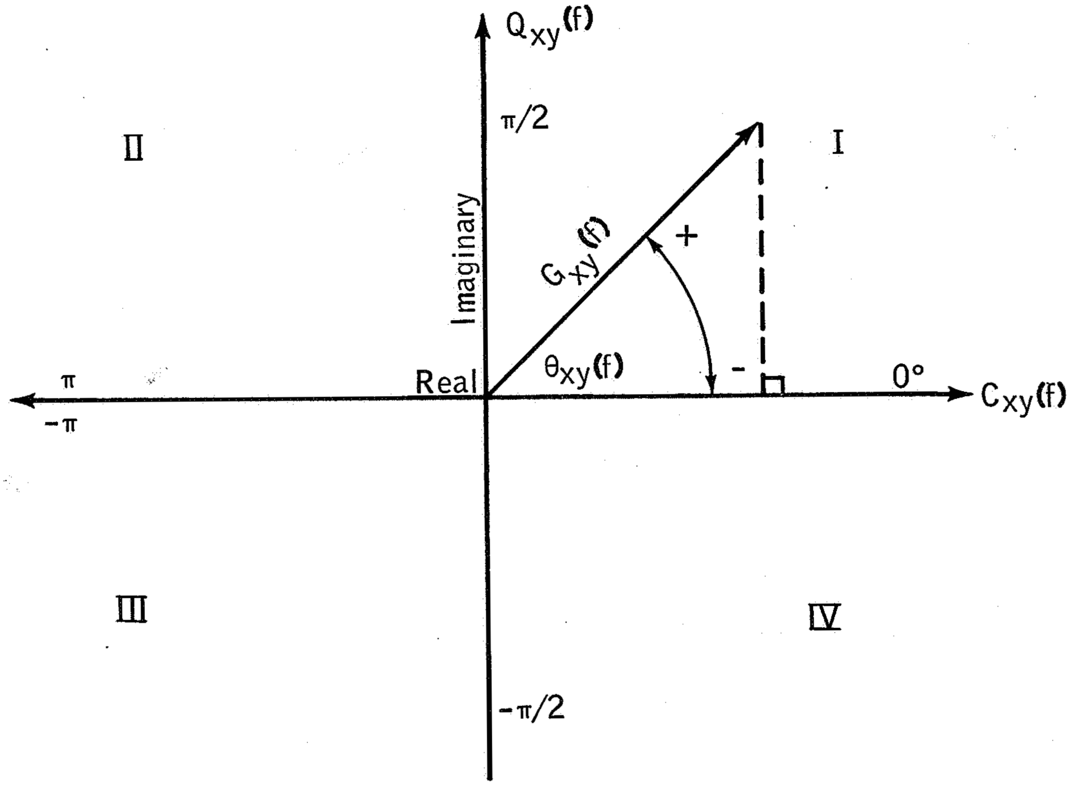


Figure 23. - The significance of the phase angle $\theta_{xy}(f)$.

$$Q_{xy}(f) = \lim_{\Delta f \to 0} \lim_{T \to \infty} \frac{1}{(\Delta f)T} \int_0^T x_{\Delta f}(t, f)\tilde{y}_{\Delta f}(t, f)dt \tag{30}$$

In these expressions, $x_{\Delta f}(t, f)$ and $y_{\Delta f}(t, f)$ represent those portions of the signals $x(t)$ and $y(t)$, respectively, which are found in bandwidth Δf of center frequency f. When shifted in phase by 90°, $\tilde{y}_{\Delta f}(t, f)$ represents $y_{\Delta f}(t, f)$. An alternate expression for the cross-spectral density function is in complex polar notation

$$G_{xy}(f) = \left|G_{xy}(f)\right| e^{i\theta_{xy}(f)} \tag{31}$$

where the magnitude of $G_{xy}(f)$ is

$$G_{xy}(f) = \sqrt{C_{xy}^{2}(f) + Q_{xy}^{2}(f)} \tag{32}$$

and the phase angle (fig. 23) is

$$\theta_{xy}(f) = \arctan\text{-}Q_{xy}(f)/C_{xy}(f) \tag{33}$$

If the roles of $x(t)$ and $y(t)$ are interchanged, the following relationships hold true

$$C_{yx}(f) = C_{xy}(f) \tag{34}$$

$$Q_{yx}(f) = -Q_{xy}(f) \tag{35}$$

$$G_{yx}(f) = G_{xy}(f) = G^*_{xy}(-f) \tag{36}$$

The value $G^*_{xy}(f)$ is the complex conjugate of $G_{xy}(f)$ given as follows

$$\left|G_{xy}(f)\right|^2 \leq G_x(f)G_y(f) \tag{37}$$

Finally, the real frequency-dependent variable $\gamma_{xy}^{2}(f)$, called the coherence function, is defined by the following equation

$$\gamma_{xy}^{2}(f) = \frac{\left|G_{xy}(f)\right|^{2}}{G_x(f)G_y(f)} \leq 1 \tag{38}$$

The coherence function equals unity when the input $x(t)$ to a system and the output $y(t)$ from the system are "fully coherent" and equals zero either when $x(t)$ and $y(t)$ are uncorrelated or when they are statistically independent. This follows from the inequality of equation (37). If the coherence function has a value between zero and unity, this may mean that the system is not truly linear, that $y(t)$ is the response to $x(t)$ plus other inputs, or that instrument noise is appearing in the measurement of $y(t)$. The coherence function then may be considered a measure of the dependence or correlation of $x(t)$ and $y(t)$.

At this point, the reader will anticipate the need for estimates of equations (29) and (30) because of the familiar, unattainable limits required by these equations. Thus

$$C_{xy}(f) \approx \frac{\overline{x(f, \Delta f)y(f, \Delta f)}}{(\Delta f)T} \tag{39}$$

and

$$Q_{xy}(f) \approx \frac{\overline{x(f, \Delta f)y(f, \Delta f)}}{(\Delta f)T} \;\underline{|90^{\circ}} \tag{40}$$

The cross-spectral density function $G_{xy}(f)$ is plotted versus frequency f to produce a cross spectrum. This plot is really two separate displays. One plot is of the real part, the cospectrum $C_{xy}(f)$; the other is of the imaginary part, the quadspectrum $Q_{xy}(f)$, the cross-spectral density function versus frequency. Cross spectra are generated as explained in subsequent paragraphs and illustrated by the block diagram of figure 24.

The practical applications of the cross-spectral density function overlap those of the cross-correlation function. However, the latter are markedly more convenient or fruitful in several cases:

1. In measuring time delays

2. In measuring the frequency response function $H(f)$ for a system

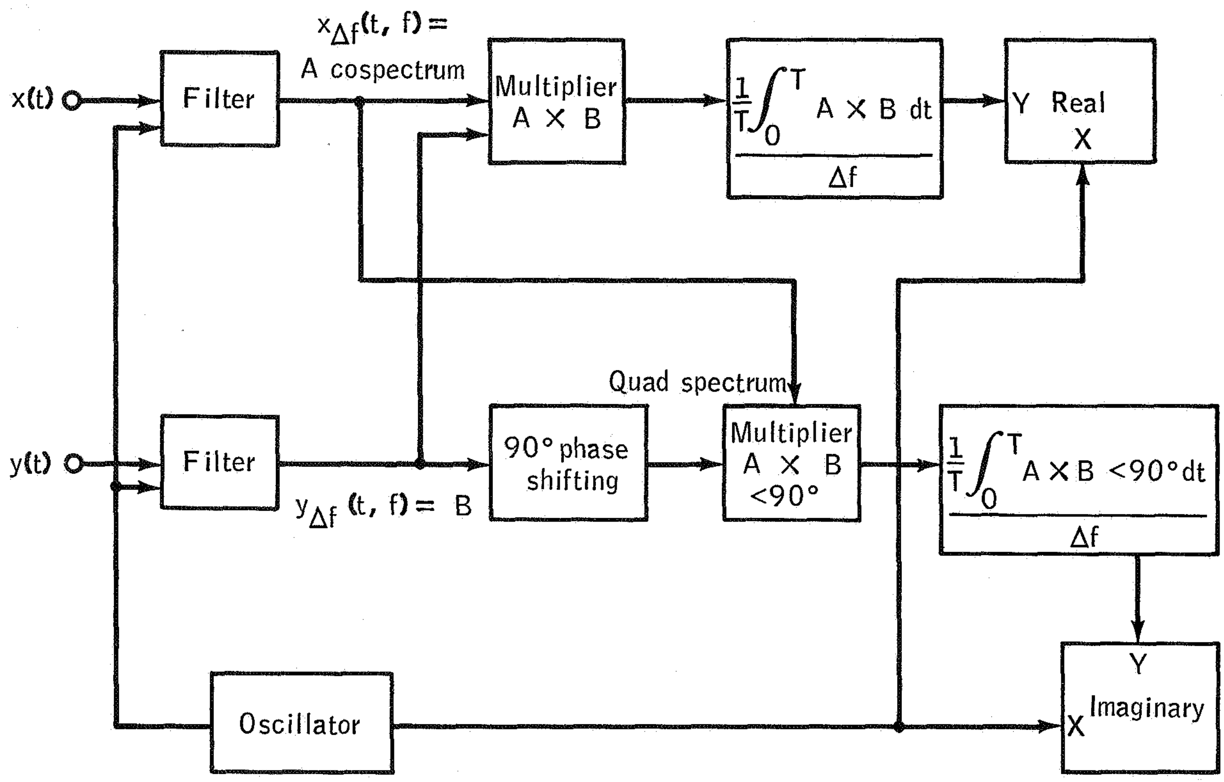


Figure 24. - Cross-spectra block diagram.

3. In determining the best weighting factor to determine the optimum linear filter with reliable characteristics

Measuring time delays by using the cross-spectral density function is preferred over the cross-correlation method because cross spectra provide information on time delays as functions of frequency. If $\theta_{xy}(f)$ gives the phase shift at frequency f produced in the cross-spectral density analysis of the input x(t) and the output y(t) of a system, then the time delay through the system at frequency f is $\tau = \theta_{xy}(f)/2\pi f$.

The frequency response function H(f) of a system into which a stationary random signal x(t) is fed may be obtained from the relationship

$$G_{xy}(f) = H(f)G_x(f) \tag{41}$$

where $G_x(f)$ is the PSD function of x(t) and $G_{xy}(f)$ is the cross-spectral density function of x(t) and output y(t).

In designing the optimum linear filter for transmitting input signal x(t) in a predictable manner minimizing undesirable noise, both cross spectra and power spectra are used along with equation (41). To determine the mean-square error R_e for this system, the coherence function $\gamma_{xy}^2(f)$, defined previously, is used as follows

$$R_e(0) = \int_0^\infty G_y(f)\left[1 - \gamma_{xy}^2(f)\right] df \tag{42}$$

Through a procedure outlined in figure 24, cross-spectral density functions are developed and plotted as cross spectra in the Wave Analysis Laboratory (figs. 25 and 26). The analog instruments perform this function in the following sequence:

1. Filtering $x(t)$ and $y(t)$ separately through identical narrow bandpass filters set at center frequency f

2. Producing an instantaneous product of the two filtered signals $x_{\Delta f}(t, f)$ and $y_{\Delta f}(t, f)$

3. Shifting one filtered signal $y_{\Delta f}(t, f)$ out of phase by 90° for use in producing a second instantaneous product with $x_{\Delta f}(t, f)$

4. Averaging each product over the sampling time T

5. Dividing each resulting mean product by Δf

6. Moving the center frequency f to another value to obtain two plots versus frequency — one of the real mean products divided by Δf, the other of the imaginary mean product (the one shifted by 90°) divided by Δf

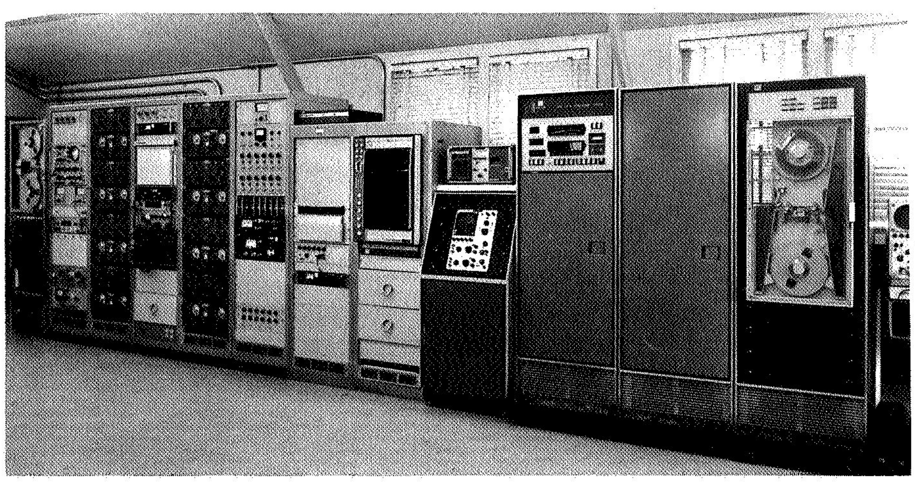
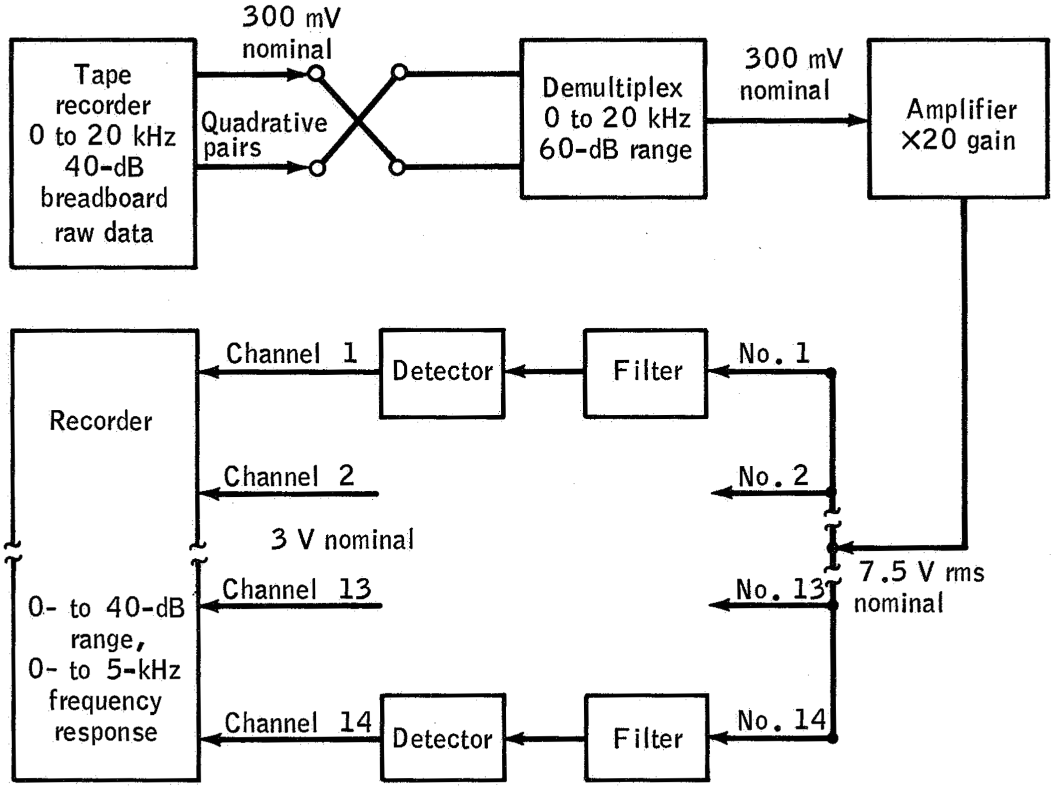


Figure 25. - Variable band filter system with analog-to-digital conversion capability.



Note:

1. Detectors have selectable averaging times from 0.01 to 0.50 seconds. Dynamic range is 50 dB from a 1-volt input.
2. Filter - Q sharpener combination has a bandwidth from a minimum of 1 percent center frequency to 1 octave and can be adjusted to any center frequency from 0 to 20 kHz. The dynamic range is 70 dB from 21 V rms.

Figure 26. - Block diagram of parallel filter system.

PRESENT USE OF THE WAVE ANALYSIS LABORATORY

The capabilities of the Wave Analysis Laboratory include those functions which are currently in the greatest demand. At present, the instrumentation is capable of using such valuable techniques as the following:

1. The rms histories provide further depth studies on behavior of variables.

2. Spectrograms (1/3- or full-octave) are helpful in acoustic analysis.

3. Time and frequency correlations are plotted for comparative analysis.

4. Power-spectral density functions are used for frequency composition of random signals.

5. Parallel time histories are used in the study of Doppler radar return signals.

Through these and other vital tools of wave analysis, NASA programs such as Apollo are receiving useful assistance from the Wave Analysis Laboratory.

Time-History System

The time-history system (fig. 3) has been used in the reduction of Apollo data from both flight and ground tests. The flight data have been received from test flights at Cape Kennedy and White Sands Missile Range. The ground test reduction has consisted of Apollo data that have been generated in tests at White Sands Missile Range, at contractor facilities, and at the Manned Spacecraft Center Vibration and Acoustic Facility under the guidance of the Structures and Mechanics Division. A typical acceleration time history of an Apollo flight mission (BP-23) is shown in figure 4.

The time-history system provides a two- to six-channel simultaneous capability for an assortment of time-varying parameters. The number of simultaneous channels available depends on the function being analyzed. For rms or mean-square time histories, six channels are usable. For phase versus frequency or time, two channels are usable. For mechanical impedance, transmissibility, or other functions where signal division or multiplication is required, the number of system components required limits the simultaneous capability to three. Division is accomplished by subtracting log functions. Block diagrams show the system configuration for rms/rms time history (fig. 6), phase versus frequency (fig. 7), and mechanical impedance (fig. 8). The input-output specifications are included. The diagram in figure 5 is of an analog device that detects the ac voltage at the input and converts it to a dc voltage proportional to the rms of the input ac voltage.

Spectrometer, 1/3- or Full-Octave Acoustic Analysis System

The Wave Analysis Laboratory has supported the Structures and Mechanics Division at MSC with 1/3-octave reduction of acoustic data generated from tests on the Apollo service module, command module, SLA, LM, and other related Apollo tests.

The system also has been used in reduction of data taken from scaled down models of the Apollo configuration. Figure 9 is a typical spectrogram from acoustic data on Apollo ground testing, and figure 10 is a picture of the system along with the correlation capability.

The spectrometer system consists of a tape loop playback recorder, three spectrometers, and calibration and plotting equipment.

The spectrometer consists of a set of 1/3-octave filters covering the 0- to 20-kHz frequency spectrum at standard intervals. By appropriate selection of control settings, the filters may be arranged for full-octave analysis over the same range. A detector scans the output of the filters, and the output of the detector is plotted on an x-y plotter with synchronized time base after conversion to log scale. The system is capable of analyzing three measurements simultaneously. The following block diagram defines pertinent specifications of system components. (The level recorder may be replaced with the x-y plotter 2FRA and the Wave Analysis Laboratory time base generator.)

Tape recorder 0 to 20 kc 40-dB range broadband	→	Band K 2112 spectrometer 0 to 31.5 kc in 1/3 octaves 50-dB range from 3 V rms	→	Band K 2305 level recorder 0 to 20 kc 50-dB range

Correlation System

The correlation system (fig. 18) consists of two correlators and associated equipment. It is capable of two simultaneous autocorrelations or six simultaneous cross-correlations. The cross-correlation is done in each correlator between one reference (input) signal and three delayed or attenuated comparison (output) signals.

Correlation as defined mathematically consists of multiplying signal A by signal B (signal A in autocorrelation) delayed by τ, of integrating this product across τ, and of averaging the summation. The correlator accomplishes this by the use of a variable delay line as shown in the block diagram. The delay line steps in increments between $\tau = 0$ and $\tau = 17$ msec. For each step, the preceding product (signal A $\times$ signal B delayed by τ) is detected, averaged, and plotted versus τ. (Plotter x-axis is synchronized to changes in τ.)

All input signals must be between 300 and 700 mV rms and have appropriate scale factors applied to derive the correct ordinate calibration which is in mean-square units.

Power-Spectral Density System

A two-channel system was procured first and was used in the reduction of Gemini flight data. As the data from Apollo ground tests increased, four more channels were procured. The new system (fig. 19) has been used consistently on data from both ground and flight tests such as Little Joe II, Airframe 002, and many vibration and acoustic tests. Figure 20 shows a typical PSD system for an Apollo mission.

The PSD system has a capability for simultaneous analysis of six PSD measurements. Frequency resolution may be controlled by bandwidth selections of 2, 5, 10, 20, or 50 Hz. One-channel bandwidths of 1 Hz or 100 Hz are also available. Sweep rates and averaging times are selectable for any desirable condition. Loop lengths are limited under normal operating conditions to 90 feet of tape or approximately 18 seconds at 60 inches per second or 36 seconds at 30 inches per second. For special cases, if required, longer loops may be used by using a different tape recorder.

Data analysis is normally accomplished by using 120 degrees of freedom and minimum bandwidth as specified by requestor to determine loop lengths, sweep rates, and averaging times. Unless otherwise specified, where feasible, bandwidth is changed at each decade during the frequency sweep to maintain a constant Q when using the log-log presentation. The block diagram defining system components and pertinent specifications is shown in figure 22. The procedure block diagram in producing a PSD is shown in figure 21.

Parallel Filter System

The parallel filter system (fig. 25) has been used in the reduction of data from studies conducted on the LM radar system at White Sands and from other tests with Apollo applications.

The parallel filter system was designed for the reduction of continuous wave Doppler radar information. The filter center frequencies and bandwidth are adjusted under normal operating conditions to perform time-history studies at selected radar look angles.

The parallel filter system is used for "quick look" real-time spectrum and may be used for any narrow band time-history analysis with simultaneous coverage of 0 to 16 selected bands in the frequency range of 0 to 20 kHz. Figure 26 shows a typical layout of the system for radar data analysis and gives pertinent specifications.

ADDITIONAL CAPABILITIES AND CONCLUDING REMARKS

At this writing, in addition to the functions of the Wave Analysis Laboratory currently in the greatest demand, numerous other capabilities exist, which may be categorized as either latent or potential uses of the laboratory. An example of a latent capability of the Wave Analysis Laboratory is the generating of probability-density function plots. Such plots are valuable in determining the type of probability distribution (in the time domain) that a set of random data may have. This knowledge is necessary so that the validity of the techniques reviewed previously can be established for this particular set of data. It is generally assumed, for example, that most random test data have normal or Gaussian-probability distributions. This is not always true, however, and sometimes this assumption is totally unacceptable.

A potential capability of the facility is the processing of radio-frequency reflectivity data. As a part of the answer to the challenge of manned and unmanned explorations of the Moon, Mars, and the other planets, NASA is developing methods for obtaining highly informative data from reflected radar signals from the surfaces of these planets. In the Wave Analysis Laboratory, systems utilizing prior capabilities and new techniques are being developed for this new and challenging type of data. Many new terms such as "sign sensing" are being included in the vocabulary of wave analysis personnel. New concepts for obtaining the maximum usable information from reflectivity data currently are being developed and tested.

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National Aeronautics and Space Administration
Houston, Texas, March 20, 1968
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APPENDIX A

THE AUTOCORRELATION FUNCTION FOR $y(t) = A \sin(\omega t + \phi)$

In general, the autocorrelation function is defined as

$$R_y(\tau) \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T y(t)y(t+\tau)dt \tag{A1}$$

In this particular case

$$R_y(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T A \sin(\omega t + \phi) A \sin[\omega(t+\tau) + \phi] \tag{A2}$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)] \tag{A3}$$

Therefore

$$\begin{aligned} R_y(\tau) &= \lim_{T\to\infty} \frac{A^2}{2T}\int_0^T \cos \omega\tau \, dt - \lim_{T\to\infty} \frac{A^2}{2T}\int_0^T \cos[2(\omega t + \phi) + \omega\tau]dt \\ &= \lim_{T\to\infty} \frac{A^2}{2T} T \cos \omega\tau - \lim_{T\to\infty} \frac{A^2}{4\omega T}\Big\{\cos \omega\tau[\sin 2(\omega T + \phi) - \sin 2\phi] \\ &\quad + \sin \omega\tau[\cos(\omega T + \phi)2 - \cos 2\phi]\Big\} \end{aligned} \tag{A4}$$

as $T \to \infty$, $\frac{1}{T} \to 0$, which results in

$$R_y(\tau) = \frac{A^2}{2} \cos \omega\tau \tag{A5}$$

APPENDIX B

THE CROSS-CORRELATION FUNCTION

FOR TWO SINUSOIDAL VARIABLES

Case 1

The general case where x(t) and y(t) have different frequencies, amplitudes, and phase angles

$$\left.\begin{aligned} x(t) &= A_1 \sin\left(\omega_1 t + \phi_1\right) \\ y(t) &= A_2 \sin\left(\omega_2 t + \phi_2\right) \end{aligned}\right\} \tag{B1}$$

$$R_{xy}(\tau) \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)y(t + \tau)dt \tag{B2}$$

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T A_1 \sin\left(\omega_1 t + \phi_1\right) A_2 \sin\left[\omega_2(t + \tau) + \phi_2\right] dt \tag{B3}$$

$$\begin{aligned} R_{xy}(\tau) = {} & \lim_{T\to\infty} \frac{A_1 A_2}{2T} \int_0^T \cos\left[t\left(\omega_1 - \omega_2\right) + \phi_1 - \phi_2 - \omega_2\tau\right] dt \\ & - \lim_{T\to\infty} \frac{A_1 A_2}{2T} \int_0^T \cos\left[t\left(\omega_1 + \omega_2\right) + \phi_1 + \phi_2 + \omega_2\tau\right] dt \end{aligned} \tag{B4}$$

$$\begin{aligned} R_{xy}(\tau) = {} & \lim_{T\to\infty} \frac{A_1 A_2}{2\left(\omega_1 - \omega_2\right)T} \left\{\sin\left[T\left(\omega_1 - \omega_2\right) + \phi_1 + \phi_2 - \omega_2\tau\right] - \sin\left(\phi_1 - \phi_2 - \omega_2\tau\right)\right\} \\ & - \lim_{T\to\infty} \frac{A_1 A_2}{2T\left(\omega_1 + \omega_2\right)} \left\{\sin\left[T\left(\omega_1 + \omega_2\right) + \phi_1 + \phi_2 + \omega_2\tau\right] - \sin\left(\phi_1 + \phi_2 + \omega_2\tau\right)\right\} \end{aligned} \tag{B5}$$

From the last equation, it is evident that as $T \to \infty$, $\frac{1}{T} \to 0$. Therefore

$$R_{xy}(\tau) = 0 \tag{B6}$$

when $\omega_1 \neq \omega_2$.

Case 2

In this case, x(t) and y(t) have the same frequency ω.

$$\left.\begin{aligned} x(t) &= A_1 \sin\left(\omega t + \phi_1\right) \\ y(t) &= A_2 \sin\left(\omega t + \phi_2\right) \end{aligned}\right\} \tag{B7}$$

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T A_1 \sin\left(\omega t + \phi_1\right) A_2 \sin\left(\omega t + \phi_2 + \omega\tau\right) dt \tag{B8}$$

$$\begin{aligned} R_{xy}(\tau) &= \lim_{T\to\infty} \frac{A_1A_2}{2T} \int_0^T \left[\cos\left(\phi_1 - \phi_2 - \omega\tau\right) - \cos\left(2\omega t + \phi_1 + \phi_2 + \omega\tau\right)\right] dt \\ &= \lim_{T\to\infty} \left[\frac{A_1A_2}{2T} (T)\cos\left(\phi_1 - \phi_2 - \omega\tau\right)\right] - \lim_{T\to\infty} \left[\frac{A_1A_2}{4\omega T} \sin\left(2\omega t + \phi_1 + \phi_2 + \omega\tau\right)\bigg|_0^T\right] \end{aligned} \tag{B9}$$

as $T \to \infty$, $\frac{1}{T} \to 0$. Therefore

$$R_{xy}(\tau) = \frac{A_1A_2}{2} \cos\left(\omega\tau + \phi_2 - \phi_1\right) \tag{B10}$$

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