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To reduce by several orders of magnitude the number of frequency points needed in calculating net fluxes for molecular atmospheres, transmission average opacities are introduced. A power-law representation of these broad-band opacities is suggested for paths at constant pressure and temperature and generalized to nonhomogeneous paths. These broad-band opacities fail to obey the rules of superposition and as a result there is a greater tendency toward convective instability. Approximate formulas are developed for the temperature gradient near the bottom of a cloud layer and at large depths within an atmosphere. Finally, equations are presented to obtain the distribution of solar energy deposition within a planetary atmosphere, which exhibits scattering and absorption by gas molecules, cloud aerosols, and the ground. Such information is needed to find the amount of net thermal flux at any level of the atmosphere.

I. INTRODUCTION

In this paper, nongrey radiative-transfer equations will be developed for determining the time-average structure of planetary atmospheres, particularly in the denser portions of the atmosphere. It will be assumed that the temperature profile can be adequately determined from a local theory, which considers energy exchange within a vertical cross section but neglects the effects of global circulation. Such an approach has been fruitful for the

Earth, where global circulation influences mostly the surface boundary temperature; however, Goody and Robinson (1966) have pointed out that global circulation could play a very significant role when all the solar energy is deposited at the top of an atmosphere. In a companion paper (Pollack, 1968), the application of Goody and Robinson's results to Venus will be commented upon.

Planetary atmospheres differ in several respects from their stellar counterparts. In the case of planets, much of the opacity is supplied by vibration-rotation bands, for which the absorption coefficient varies rapidly with wave number. As a result, a large number of points may be needed to calculate net fluxes. For example, at 1-atm pressure, the half width of rotation lines is typically somewhat less than 10^{-1} cm^{-1}, and so intervals of about 10^{-2} cm^{-1} would be required to ensure that the absorption coefficient is constant over each interval. As the total wave-number domain is between 10^3 cm^{-1} and 10^4 cm^{-1}, 10^5 to 10^6 wave-number points would be required to calculate integrated net fluxes. A feature of this paper is a method of averaging monochromatic opacities in order to greatly reduce the number of wave-number points needed.

Another unusual aspect of planetary atmospheres is the absence of net-flux constancy. Since solar energy is deposited in each atmospheric layer, there will be a variation with altitude in the net flux integrated over those wavelengths in which there is a significant amount of thermal radiation.

A priori, the atmospheric structure of a planet cannot be considered to possess spherical symmetry as a result of the latitudinal and longitudinal variation in the amount of solar energy falling on the planet. Fortunately, the lower portions of atmospheres typically have large heat capacities so that an atmosphere may exhibit little diurnal variation, except for a narrow boundary layer near the surface. For example, only the lower 50 m of the Earth's atmosphere exhibits significant diurnal temperature changes (Goody, 1964). Thus, the time-average temperature solutions presented below may have general applicability to any locale.

One simplifying feature for planets is that once the radiative lapse rate is found to be convectively unstable, the true lapse rate will be adiabatic. This circumstance arises because of the high densities and low net fluxes present, which ensures that the lapse rate of an unstable atmosphere will be exceedingly close to the adiabatic value. For example, Sagan (1960) has shown from the mixing-length theory that under unstable conditions the atmosphere of Venus will be adiabatic to one part in 10^4.

Finally, the influence of localized cloud layers must be considered. Within the small range in altitudes occupied by a condensation cloud, the opacity may be dominated by aerosol absorption and the clouds may radiate approximately as a blackbody in both the downward and the upward directions. The clouds may also act as indicators of the end of a convection zone.

In the first sections of this paper, the temperature structure will be found from a knowledge of the net thermal flux: A radiative equilibrium value of the temperature gradient is obtained and subsequently tested for convective instability. If the solution is unstable, we set the lapse rate equal to the adiabatic value. Various limiting cases will be considered. The following section will be concerned with the calculation of solar-energy deposition in various layers of the atmosphere, as well as at the ground. Scattering and absorption by cloud aerosols, atmospheric molecular constituents, and the ground will be considered.

II. AVERAGE OPACITIES AND NET THERMAL FLUX

An appropriate averaging procedure will be presented for calculating net infrared fluxes from a relatively small number of frequency intervals. A power-law representation will then be suggested for the average opacities for constant pressure paths and subsequently generalized to nonhomogeneous paths. Finally, an important distinction between the average opacities and monochromatic opacities will be discussed.

We wish to find an expression for the net infrared flux integrated over all wavelengths contributing significantly to the thermal radiation. When the source function equals the Planck function B_ν and the atmosphere is plane parallel, the monochromatic specific intensity I_ν is related to B_ν through the formal solution of the equation of transfer:

$$I_\nu(z,\mu) = -\int_{z}^{z_b} B_\nu(z')\, d\left[e^{-\tau_\nu(z',\,z)/\mu}\right] , \qquad (1)$$

where z denotes the vertical coordinate, μ the cosine of the angle between the direction under consideration and the vertical direction, $\tau_\nu(z', z)$ the monochromatic optical depth between z and z′, and z_b the vertical coordinate at the appropriate boundary. We are interested in the specific intensity integrated over some finite frequency interval $\Delta\nu_i$:

$$I_i \equiv \int_{\Delta\nu_i} I_\nu\, d\nu = -\int_{\Delta\nu_i}\int_{z}^{z_b} B_\nu(z')\, d[e^{-\tau_\nu(z',\,z)/\mu}]\, d\nu \quad . \qquad (2)$$

In a manner similar to that employed by Goody (1964), we require that $\Delta\nu_i$ be sufficiently small so that B_ν will not vary appreciably over $\Delta\nu_i$. As a result, the frequency integration can be carried out within the differential:

$$\begin{aligned} I_i &= -\int_{z}^{z_b} B_\nu(z')\, \Delta\nu_i\, d\left[\int_{\Delta\nu_i} e^{-\tau_\nu(z',\,z)/\mu}\, \frac{d\nu}{\Delta\nu_i}\right] \\ &\equiv -\int_{z}^{z_b} B_i(z')\, d\left[e^{-\tau_i(z,\,z',\,\mu)}\right] \\ &= \int_{z}^{z_b} B_i(z')\, e^{-\tau_i(z,\,z',\,\mu)}\, \frac{d\tau_i(z,\,z',\,\mu)}{dz'}\, dz' \quad , \end{aligned} \qquad (3)$$

where τ_i is the average opacity in direction μ over the interval i, whose exponential equals an exponential average of the monochromatic opacities, and B_i equals $B_\nu \Delta\nu_i$. The net thermal flux F is readily found by multiplying through by $2\pi\mu\, d\mu$ and integrating, and then summing over all relevant i regions:

$$F_i = 2\pi \int_{-1}^{1} I_i \mu\, d\mu = 2\pi \int_{-1}^{1} \int_{\tau_i(z)}^{\tau_i(z_b)} B_i(z')\, e^{-\tau_i}\, d\tau_i \mu\, d\mu \tag{4}$$

$$F \equiv \sum_i F_i = \sum_i 2\pi \int_{-1}^{1} \int_{\tau_i(z)}^{\tau_i(z_b)} B_i(z')\, e^{-\tau_i}\, d\tau_i \mu\, d\mu \quad . \tag{5}$$

One obvious advantage of employing the average opacities defined above is that far fewer frequency intervals are required. Since the Planck function varies only slowly with frequency over those frequencies at which it achieves its largest values, only 10 to 100 frequency intervals are required in practice, in contrast to 10^5 to 10^6 such regions for the corresponding monochromatic calculations. Another advantage of the average opacities is that they are simply transmission averages and so can be measured directly in the laboratory for minor constituents of an atmosphere; with present laboratory path lengths of up to 1 km-atm, many wavelength regions for major constituents can also be investigated. On the other hand, one must ultimately use empirical interpolation and perhaps even extrapolation formulas for the average opacities, which will introduce a greater amount of error in the final fluxes. In addition, it is difficult to deal with several sources of opacity unless their monochromatic opacities are uncorrelated. This point will be discussed more fully below. It might be noted that given the necessary information for monochromatic opacities (rotational line strengths, line shapes, etc.) one can always construct theoretical transmissivity averages and so save a large amount of computer time in the next step of obtaining fluxes.

Experience with CO_2 and H_2O has indicated that a power-law representation of τ_i is a good one (Pollack, 1968; Bartko and Hanel, 1968). For paths at constant pressure P and total gas amounts $\underset{\sim}{W}$, τ_i is given approximately by

$$\tau_i = c_i \underset{\sim}{W}^{r_i} P^{s_i} \quad . \tag{6}$$

The total gas amount is proportional to the integral of the density ρ over the path length L

$$\underset{\sim}{W} \propto \int \rho \, dL \quad . \tag{7}$$

We note that unlike the situation for monochromatic opacities, τ_i in general will not vary linearly with $\underset{\sim}{W}$. The physical consequences of this will be explored toward the end of this section. In practice, r_i, s_i, and c_i seem to change very slowly with large changes in $\underset{\sim}{W}$ and P; it would seem that reasonable accuracy could be obtained by the employment of arrays of values for r, s, and c corresponding to various values of $\underset{\sim}{W}$ and P with the selection of appropriate values for a given calculation. Temperature dependence has not been included in Eq. (6). This will be discussed later.

Equation (6) is valid for paths at constant pressure and so we must seek to generalize it to paths over which the pressure varies in order to deal with planetary atmospheres. To accomplish this, we make use of the known equations for the monochromatic opacity τ_ν over variable pressure paths. For permitted transitions and sufficiently high pressures, τ_ν will be given by the sum of the opacity provided by nearby rotation lines, whose profile is assumed to be of the Lorentz form,

$$\tau_\nu = \int \sum_\ell \frac{S_\ell}{\pi} \left[\frac{\delta\nu_\ell}{(\nu - \nu_\ell)^2 - (\delta\nu_\ell)^2} \right] \frac{dW}{\mu} \quad , \tag{8}$$

where S_ℓ is the strength of the ℓth line, $\delta\nu_\ell$ is the line width and proportional to P, ν_ℓ is the frequency of the line center, and W is the gas amount in a vertical column and is equal to $\underset{\sim}{W}\mu$. Inserting Eq. (8) into the definition of τ_i supplied by Eq. (3), we obtain

$$\exp(-\tau_i) = \int_{\Delta\nu_i} \exp\left[-\int K(\nu)\, P^{q_\nu} \frac{dW}{\mu}\right] \frac{d\nu}{\Delta\nu_i} \quad , \tag{9}$$

where $K(\nu)$ includes most of the frequency dependence of Eq. (8) and q_ν is the effective power-law exponent for the pressure dependence of τ_ν . As $\delta\nu_\ell \sim P$, q_ν will vary from almost -1 at line centers to almost +1 far from any line center. For the lower portions of an atmosphere, where the opacity is significantly different from zero at all frequency points of the $\Delta\nu_i$ interval, τ_i will be determined in good part at those frequencies where τ_ν is smallest and hence q_ν is close to +1. We therefore replace q_ν with an average value $\overline{q}$ and so rewrite Eq. (9) as

$$\exp(-\tau_i) \cong \int_{\Delta\nu_i} \exp\left[-\, K(\nu) \int P^{\overline{q}} \frac{dW}{\mu}\right] \frac{d\nu}{\Delta\nu_i} \quad . \tag{10}$$

For the limiting case of a constant pressure path, Eq. (10) becomes

$$\begin{aligned} \exp(-\tau_i) &\cong \int_{\Delta\nu_i} \exp\left[-\, K(\nu)\, P^{\overline{q}} \frac{W}{\mu}\right] \frac{d\nu}{\Delta\nu_i} \\ &= \int_{\Delta\nu_i} \exp\left[-\, K(\nu)\, P^{\overline{q}} \underset{\sim}{W}\right] \frac{d\nu}{\Delta\nu_i} \quad . \end{aligned} \tag{11}$$

According to Eq. (6), τ_i varies as $\underset{\sim}{W}^{r_i}$ under these circumstances; i.e., the effect of varying $\underset{\sim}{W}$, which appears as $\underset{\sim}{W}^1$ in the exponential on the right side of Eq. (11), is to cause a $\underset{\sim}{W}^{r_i}$ dependency in the resultant frequency integration. By symmetry, any other quantity that varies as the first power in the exponential and has no frequency dependence will also vary as the r_i power when the frequency integration is carried out. Thus, $\tau_i \sim (P^{\bar{q}})^{r_i}$ and $\tau_i \sim (W/\mu)^{r_i}$. Comparing this first result with Eq. (6), we see that we can identify $\bar{q}r_i$ as s_i, and this partially justifies our replacement of q_ν with $\bar{q}$. A similar line of argument leads to the following expression for τ_i when the pressure varies over the path:

$$\tau_i = c_i \left(\int P^{s_i/r_i} \frac{dW}{\mu} \right)^{r_i} \quad . \tag{12}$$

We note that just as τ_i varies as W^{r_i} instead of as W, τ_i varies as μ^{-r_i} instead of as μ^{-1}.

Equation (12) is readily generalized to include temperature dependence. If various paths, each at a fixed constant temperature, imply that $\tau_i \propto G_i(T)$, then for a path over which the temperature is varying we can write

$$\tau_i = c_i \left[\int G_i(T)^{1/r_i} \, P^{s_i/r_i} \frac{dW}{\mu} \right]^{r_i} \quad . \tag{13}$$

Significant temperature dependence usually arises when the band supplying the opacity is a hot band, and in that case we can identify $G_i(T)^{1/r_i}$ with the appropriate Boltzmann factor.

Using Eq. (11), we can readily show from a simple example that, in general, r_i is not equal to one. The quantity $K(\nu)$ varies greatly from line centers to regions between line centers. Let us suppose $K(\nu)$ equals ∞ for half the frequency interval and 1 for the other half. If W is varied from 1/2 to 1, τ_i changes only from 1.19 to 1.64 and therefore would not vary so rapidly as W. Estimates of r_i for various CO_2 and H_2O bands always lead to values of r_i less than one, in agreement with this example. We argued above that we generally expected $\bar{q}$ to be close to and somewhat less than 1 for permitted bands, so s_i should be usually somewhat less than r_i. Again, this is in agreement with experience (Pollack, 1968). In the case of pressure-induced bands, $\tau_\nu \propto WP$, q_ν equals 1 at all ν, and $r_i = s_i$.

A very significant difference between the broad-band opacity τ_i and its monochromatic counterpart is that τ_i does not obey the rules of superposition; i.e., the opacity between positions A and B does not equal the difference in the opacity between A and C and B and C. This is readily shown from Eq. (13):

$$
\begin{aligned}
\tau(AB) &= c_i \left[\int_B^A G_i(T)^{1/r_i} \, P^{s_i/r_i} \, \frac{dW}{\mu} \right]^{r_i} \\
&= c_i \left[\int_C^A G_i(T)^{1/r_i} \, P^{s_i/r_i} \, \frac{dW}{\mu} - \int_C^B G_i(T)^{1/r_i} \, P^{s_i/r_i} \, \frac{dW}{\mu} \right]^{r_i} \\
&\neq c_i \left[\int_C^A G_i(T)^{1/r_i} \, P^{s_i/r_i} \, \frac{dW}{\mu} \right]^{r_i} \\
&\quad - c_i \left[\int_C^B G_i(T)^{1/r_i} \, P^{s_i/r_i} \, \frac{dW}{\mu} \right]^{r_i} = \tau(AC) - \tau(BC) \\
&\qquad \text{if } r_i \neq 1 \ . \qquad (14)
\end{aligned}
$$

A consequence of this result is that one cannot derive the usual equation of transfer for I_i in terms of τ_i from Eq. (3). Another implication is the greater tendency of the lower portions of atmospheres toward convective instability or at least a larger temperature gradient. If we let C refer to the top of the atmosphere, then when $\tau(AC) > 1$ and, as is always the case, $r_i < 1$, $\tau(AB)$ will exceed $[\tau(AC) - \tau(BC)]$ by greater amounts at deeper depths of the atmosphere. Consequently, the layers with which a deep layer is exchanging a significant amount of radiation are much closer than would be expected simply on the basis of optical depths with respect to the top of the atmosphere. As a result, the temperature gradient will be larger. This circumstance will be described quantitatively in the next section. Bartko and Hanel (1968) carried through an analysis similar to that in the beginning of this section in using transmissivity averages and a power-law representation of τ_i to treat the temperature structure above the Venus cloud tops. However, they apparently assumed superposition was valid for τ_i, and if so, their numerical results are suspect.

III. TEMPERATURE PROFILE OF THE LOWEST PORTIONS OF THE ATMOSPHERE

Expressions for the temperature gradient will now be found for the lower portions of the atmosphere and compared with corresponding grey and monochromatic expressions. It is supposed that a given layer is sufficiently deep that pressure and temperature vary very little over the first few optical depths away from point z, and so $\tau_i(z, z', \mu)$, given by Eq. (13), can be approximated as:

$$\tau_i \cong c_i\, G_i[T(z)]\, P^{s_i}(z) \frac{[|W(z') - W(z)|]^{r_i}}{\mu^{r_i}} \quad . \qquad (15)$$

Similarly, we set the limits equal to ± ∞ in integrations over optical depth. We restrict the present situation to opacity supplied by only one gas specie at a given wavelength interval so that Eq. (15) is not a sum over various gas species. The form of Eq. (15) suggests that we try to solve Eq. (5) by expanding $B_i(z')$ in a Taylor series for $[W(z') - W(z)]$:

$$B_i(z') \cong B_i(z) + \frac{dB_i}{dW}\Big|_z [W(z') - W(z)] \quad . \tag{16}$$

With these approximations, Eq. (5) is readily evaluated:

$$F(z) = \frac{4\pi}{3} \sum_i \frac{dB_i}{dW} \frac{\Gamma[(1/r_i)+1]}{\left(c_i G_i P^{s_i}\right)^{1/r_i}} \quad , \tag{17}$$

where Γ is the usual gamma function.

We next relate dB_i/dW to the temperature gradient. Locally the temperature T and pressure P can always be represented by a power-law relationship:

$$T \propto P^{\psi} \quad . \tag{18}$$

Differentiating Eq. (18), we find

$$\frac{dT}{dP} = \psi \frac{T}{P} \quad . \tag{19}$$

Thus, the radiative temperature profile as indicated by the parameter ψ can be secured from Eqs. (17) and (19) once we relate dB_i/dW to dT/dP. For the monochromatic Planck function, B_ν again can be locally related by a power-law relation to T:

$$B_\nu = B_{\nu_s} \left(\frac{T}{T_s}\right)^n \quad , \tag{20}$$

where

$$n = \frac{a\,e^a}{e^a - 1} \quad \text{and} \quad a = \frac{(4.965)(2898)}{\lambda_\nu\,T} \tag{21}$$

and T has the units of degrees Kelvin and λ_ν microns. A similar relationship will approximately hold for B_i with λ_ν replaced by $\overline{\lambda}_i$, an average wavelength for band i. Thus,

$$\frac{dB_i}{dW} = \frac{dB_i}{dT}\,\frac{dT}{dP}\,\frac{dP}{dW} = n_i\,\psi\,\frac{B_i}{P\theta} \quad , \tag{22}$$

where $\theta = 1/(dP/dW) = dW/dP$. Inserting Eq. (22) into Eq. (17), we obtain an expression for the temperature gradient parameter ψ:

$$\psi(z) = \frac{3\,F(z)}{4\pi}\,\frac{1}{\sum_i \left[\frac{n_i\,B_i\,\Gamma(1/r_i+1)}{c^{1/r_i}\,P^{(s_i/r_i+1)}\,\theta G^{1/r_i}}\right]} \quad . \tag{23}$$

The analysis so far has operated under the assumption that there is only one source of opacity, e. g., H_2O vapor opacity, or at least that one source is dominant in each band. The latter assumption will usually be true. We now include all sources of opacity for a given band i. The quantity τ_i will be approximately the sum of the individual opacities. For this to be true exactly, there can be no correlation between the positions of the individual rotational lines for the various sources of opacity and, in addition, no large-scale correlations (see, e. g., Goody, 1964):

$$\tau_i = \sum_{j=1}^{J} c_{ij}\, G_{ij}\, P^{s_{ij}} [\,|W_j(z') - W_j(z)|\,]^{r_{ij}} \quad , \tag{24}$$

where the index j refers to the source of opacity. While we cannot exactly derive an analytic equation for F(z) under these circumstances, Eq. (17) suggests that F(z) is given approximately by

$$F(z) = \frac{4\pi}{3} \sum_i 1 \bigg/ \sum_{j=1}^{J} \left\{ \frac{\left(c_{ij}\, G_{ij}\, P^{s_{ij}}\right)^{1/r_{ij}}}{(dB_i/dW_j)\, \Gamma[(1/r_i) + 1]} \right\} \quad . \tag{25}$$

Equation (25) meets certain minimal conditions: The flux in any wavelength interval, F_i, is less than the flux due to any single constituent, and F_i becomes equal to the value implied by Eq. (17) when one source of opacity is dominant.

Proceeding in the same way as between Eqs. (17) and (23), we can readily obtain an equation for ψ from Eq. (25):

$$\psi = \frac{3F}{4\pi} \frac{1}{\sum_i \left(n_i B_i \bigg/ \sum_{j=1}^{J} \left\{ G_{ij}^{1/r_{ij}} P^{[(s_{ij}/r_{ij}) + 1]} \theta_j / \Gamma\left(\frac{1}{r_{ij}} + 1\right) \right\} \right)} \quad , \tag{26}$$

where $\theta_j = dW_j/dP$. According to the equation of hydrostatic equilibrium, θ_j will be constant throughout the atmosphere as long as the mixing ratio of constituent j is constant, and its value will depend on the units chosen for W_j and P, as well as on the mixing ratio of j, α_j. The quantity θ_j is proportional to α_j.

We now compare our results with certain limiting cases. To facilitate the comparison, we set W_j identically equal to the density of the j component integrated over a vertical path length, $\int \rho_j \, dz$. First, let us suppose that the absorption is independent of wavelength, i.e., grey. Then r_i will be 1, as can be readily seen from Eq. (11). Equation (13) gives an expression for τ_i for a single component. For several components we sum over j. With r_i equal to 1, the total optical depth becomes

$$\tau = \sum_{j=1}^{J} \int c_j \, G_j \, P^{s_j} \, a_j \frac{dW_T}{\mu} \equiv \int \kappa \frac{dW_T}{\mu} \quad , \tag{27}$$

where W_T represents the total gas amount, a_j is the mixing ratio of the jth component, and κ is the mass absorption coefficient. In deriving Eq. (27) from Eq. (13), we have set dW_j equal to $a_j \, dW_T$. For the case under consideration, τ has no i dependence. Setting $r_{ij} = 1$ in Eq. (25) and making use of the mass absorption coefficient defined in Eq. (27), we obtain the following flux equation for a grey absorber at large optical depth:

$$F = \frac{4\pi}{3} \frac{(dB/dW)}{\kappa} \quad , \tag{28}$$

where B is the integrated Planck function. Exactly the same result follows from differentiating the expression for the flux derived under the Eddington approximation for the grey case with constant net flux. Similarly, if we let the wavelength intervals i be infinitely small, so that monochromatic absorption coefficients k_{ν_j} are appropriate $a_j c_{ij} G_{ij} P^{s_{ij}} = k_{\nu_j}$, and $r_{ij} = 1$. For this situation Eq.(25) becomes

$$F = \frac{(4\pi/3)(dB/dW)}{\int (dB_\nu/dW) d\nu \Big/ \int [(dB_\nu/dW)(1/\sum_j k_{\nu_j}) \, d\nu]} \quad . \tag{29}$$

The quantity in the denominator of the right-hand side of Eq. (29) is simply the Rosseland mean opacity, which is generally believed to be an appropriate average deep in an atmosphere. Notice that the inferred summation over several sources of opacity in Eq. (25) reduces to the correct summation for the monochromatic case. Thus, the flux equation for the broad-band opacities in the lower portions of an atmosphere reduce to appropriate formulas in certain limiting cases.

IV. TEMPERATURE GRADIENT IN THE VICINITY OF A CLOUD LAYER

In this section, the effect of an opaque condensation cloud on the temperature gradient of the atmosphere below and near to it will be discussed. The bottom of the clouds is assumed to have a sharp boundary, and we wish to estimate the net flux at the atmospheric layer immediately adjacent to the cloud bottoms. The net flux $F(z)$ is formally divided into the flux received from the atmosphere below level z, $F_+(z)$, and the flux radiated downward by the cloud, $F_-(z)$, with $F(z)$ equal to their sum. The quantity $F_-(z)$ is readily evaluated from an equation similar to Eq. (4), but with the limits on the μ integration replaced by -1 to 0. We characterize the cloud with a radiation temperature of T_c, which will be close to but somewhat smaller than its thermometric cloud-bottom temperature, because of the small, nonzero thickness of the cloud, which is contributing significantly to the emergent radiation. The emissivity of the clouds is assumed to be unity. The quantity $F_-(z)$ is then given by

$$F_-(z) = - \pi B(T_c) \quad . \tag{30}$$

The quantity $F_+(z)$ can be evaluated approximately in a manner analogous to that employed in the previous section for the net flux deep in an atmosphere. We expand $B_i(z')$ as

$$B_i(z') \cong B_i(z) + \frac{\overline{dB_i}}{dW} [W(z') - W(z)] \quad , \tag{31}$$

and similarly we write

$$\tau_{ij}(z', z, \mu) \cong \overline{c_{ij} G_{ij} P^{s_{ij}}} \; \frac{[W(z') - W(z)]^{r_{ij}}}{\mu^{r_{ij}}} \quad , \tag{32}$$

where the bar denotes an evaluation of quantities at the field point where $\overset{J}{\underset{j=1}{}}\tau_{ij} = 1$, so as to allow somewhat for variations in the quantities under the bar over the first few optical depths. With these approximations we can obtain $F_+(z)$ in a manner similar to that used for Eqs. (25) and (26):

$$F_+(z) = \pi B[T(z)] + \frac{2\pi}{3}\sum_i \left(\psi_i n_i B_i \Bigg/ \sum_i \left\{ \frac{c_{ij}^{1/r_{ij}} P^{[(s_{ij}/r_{ij})+1]} \theta_j G_{ij}^{1/r_{ij}}}{\Gamma\left(\frac{1}{r_{ij}} + 1\right)} \right\} \right)$$

$$\equiv \pi B[T(z)] + \frac{2\pi}{3}\sum_i \overline{\psi_i Z_i} \quad . \tag{33}$$

The net flux is given by adding together Eqs. (30) and (33):

$$F(z) = \frac{2\pi}{3}\sum_i \overline{\psi_i Z_i} + \pi\left\{B[T(z)] - B(T_c)\right\} \quad . \tag{34}$$

We can relate $B[T(z)]$ to $B(T_c)$ by dividing the radiation field into two streams, I_+ and I_-, at point z; T(z) is the temperature of the air immediately beneath the clouds, and T_c is the effective cloud emission temperature. The quantity I_- will simply be $B(T_c)$, while I_+ can be found from the definition of net flux: $F(z) = \pi\ (I_+ - I_-)$. This enables us to find the average intensity at point z, $(I_+ + I_-)/2$, which in the spirit of the Eddington approximation we set equal to $B[T(z)]$. Thus, we find

$$B[T(z)] = B(T_c) + \frac{F(z)}{2\pi} \quad . \tag{35}$$

Inserting Eq. (35) into Eq. (34), we obtain

$$F(z) = \frac{4\pi}{3}\sum_i \overline{\psi_i Z}_i \quad . \tag{36}$$

If the cloud is very opaque so that T_c equals the cloud-bottom temperature, then, according to Eq. (35), in radiative equilibrium there will be a temperature discontinuity at the cloud bottoms. A similar discontinuity will exist at the ground surface and is familiar from calculations for the Earth (Goody, 1964). Naturally, convective instability will lead to a continuous variation of temperature with distance in the real atmosphere. However, since the mixing length becomes comparable to the distance to the boundary near a surface and possibly a cloud bottom, it becomes quite small close to the boundary. As a result, superadiabatic gradients may exist close to the boundary.

Let us now consider the radiative-equilibrium temperature-gradient parameter ψ within the first few optical depths of the cloud bottoms. For simplicity, we suppose that ψ is independent of i. The quantity Z_i varies inversely as the pressure $\overline{P}$ and the variable θ_j, which is proportional to the mixing ratio of component j, α_j. By increasing $\overline{P}$ and/or α_j, we decrease $\overline{Z}_i$, and for a fixed net flux this leads to an enhanced value of ψ according to Eq. (36). Thus, in principle we can make the radiative equilibrium value of ψ quite large near the bottom of the clouds. The dependence of ψ near the cloud bottom on pressure and mixing ratio allows us to gain valuable information about the sources of opacity from the position of a cloud layer in a given atmosphere. It seems physically reasonable for the cloud bottoms to lie at the end of the convection region of the atmosphere; if the cloud-bottom pressure were too low and so ψ were subadiabatic, it would be difficult to regenerate the cloud through condensation occuring in rising air parcels. On the other hand, if the pressure were too high and ψ substantially superadiabatic, the clouds would tend to be uplifted. Thus, ψ should be close to the adiabatic value near the bottom of the clouds, and we can use Eq. (36) to determine the mixing ratio of one of the absorbing gases, or equivalently to find the cloud pressure.

When an opaque cloud is present at the boundary of the adiabatic portion of the atmosphere, it is far easier to calculate the net flux at the boundary. If the opaque cloud layer is absent, the temperature structure of the radiative equilibrium portion of the atmosphere above the boundary must be known for the calculation to be carried out.

Sagan (1968) showed that the temperature-gradient parameter ψ deep within an atmosphere depended only upon how rapidly the opacity changed with pressure, i.e., upon the exponents s and r, and not upon the absolute value of the opacity. Near the cloud bottoms, the situation is different. Now ψ depends upon such parameters as c_{ij}, G_{ij}, θ_j, and pressure. Two other temperature regions can be distinguished. Radiative exchange within the clouds is controlled chiefly by aerosol opacity; the radiative temperature gradient can generally be expected to be convectively unstable and so a (wet) adiabatic profile will result. Finally, there is the temperature profile above the clouds, which may be considerably different from the profile immediately below the clouds, because of the condensation of one component of the atmosphere to form the clouds. If this component is a good infrared absorber, as H_2O and NH_3 are, the large increase in its mixing ratio between the cloud tops and bottoms will substantially decouple the temperature gradients in the two regions.

V. SOLAR-ENERGY DEPOSITION

We wish to estimate theoretically the fraction of solar energy absorbed by a planet as well as estimate how much energy is deposited in each level of the atmosphere. The former quantity could, in principle, be obtained from reflectivity measurements over a series of phase angles and wavelengths, but it is still useful to calculate this quantity theoretically, both as a check on the model used to calculate the distribution of deposited solar energy and to cover possible gaps in the observational data.

The net infrared flux F(z) at a given level z, which is needed to calculate ψ, can be directly related to the amount of solar energy deposited below this layer through conservation of energy. For the time-independent or average problem, the temperature at any level is constant, and so the net flux integrated over all wavelengths, F_T, must be constant with depth:

$$\frac{dF_T}{dz} = 0 \quad . \tag{37}$$

Since the wavelengths at which there is a substantial amount of solar energy exhibit little overlap with those important for thermal radiation, F_T is simply the sum of the solar energy flux F_s averaged over the day and night sides, and the infrared flux F. Remembering that F_s diminishes with depth and thus is a negative quantity, we can relate the absolute values of F_s and F by means of Eq. (37):

$$\frac{dF}{dz} = \frac{d|F_s|}{dz} \quad . \tag{38}$$

With the boundary condition that the two fluxes have the same absolute value at the top of the atmosphere, $z = \infty$, Eq. (38) can be rewritten as

$$F(z) = |F_s(z)| \quad . \tag{39}$$

The flux $F_s(z)$ can formally be broken into an upward directed flux, which consists of sunlight reflected from levels below level z and reaching level z, $F_+(z)$, and a downward directed flux $F_-(z)$. The flux $F_s(z)$ equals the difference between the absolute values of these two components and so can be equated to the quantity of sunlight absorbed below level z. At the top of the atmosphere, $|F_s(z = \infty)|$ will equal $\int_0^\infty (1 - \mathcal{R}_\lambda) \mathcal{F}_\lambda \, d\lambda/4$, where $\mathcal{R}_\lambda$ is the spherical monochromatic reflectivity, i.e., the fraction of the incident solar energy reflected away to space, and $\mathcal{F}_\lambda$ is the monochromatic solar flux at Venus' orbit. The factor of 4 allows for averaging of the flux over the day and night hemispheres. We can formally express F(z) then as

$$F(z) = f(z) \int_0^\infty (1 - \mathcal{R}_\lambda) \mathcal{F}_\lambda \frac{d\lambda}{4} \quad , \tag{40}$$

where f(z) is the fraction of the deposited solar energy that is absorbed below level z.

Our problem is to find both f(z) and $\mathcal{R}_\lambda$, and in so doing we will consider scattering and absorption by a cloud layer, air molecules, and the ground. We first consider the response of a cloud layer to solar energy falling upon it. As we are interested in quantities that are integrated over both angles of incidence and reflection, it may suffice to use the 2-stream approximation in obtaining the monochromatic transmissivity $\mathcal{T}_\lambda$, reflectivity A_λ, and absorptivity B_λ of the cloud layer. Appropriate equations for these quantities, which allow for anisotropic, nonconservative scattering, are presented by Sagan and Pollack(1967; henceforth called Paper I). The quantities $\mathcal{T}_\lambda$, A_λ, and B_λ are functions of the total optical depth τ_λ, the single scattering albedo $\tilde{\omega}_0(\lambda)$, and the scattering asymmetry parameter 2β. In turn, 2β is a slowly varying function of the average particle size, $\tilde{\omega}_0(\lambda)$ is a function of the indices of refraction, real and imaginary, and the average particle size, while τ_λ can be obtained from the optical depth at some standard wavelength, by scaling as the extinction cross section, which is a function of the average particle size. Thus, if one knows the average particle size, the optical depth at one wavelength, and the chemical composition of the clouds, $\mathcal{T}_\lambda$, A_λ, and B_λ can be computed for all wavelengths of interest.

We now include the effects of Rayleigh scattering by the atmosphere and reflection from the surface in addition to the radiative transfer in the cloud layer. The possibility that Venus' surface pressure is so large that Rayleigh scattering may be important throughout the visible spectrum was first advanced by Sagan (1962). Account of atmospheric absorption will be taken later. Fortunately, for most wavelengths of interest, Rayleigh scattering will occur almost entirely below the cloud layer, and so the cloud layer, Rayleigh layer, and ground are geometrically separated. Since our ultimate goal is to find how much flux is absorbed or reflected, we will approximate the radiative transfer problem by dealing only with fluxes absorbed, reflected, or transmitted by each of the three scattering layers.

A fraction $\mathcal{T}_\lambda$ of the sunlight incident at wavelength λ penetrates the cloud layer. There are three possible pathways or scenarios immediately available to the radiation:

(i) It may be Rayleigh scattered from the atmosphere back through the clouds to be either absorbed there or transmitted to space;

(ii) it may be transmitted by the atmosphere and absorbed by the surface;

(iii) it may be transmitted by the atmosphere and reflected from the surface.

Consider scenario (i). A photon, having penetrated the clouds, has a probability (1 - t) of being Rayleigh scattered back to the clouds, where t is the transmissivity of the Rayleigh layer. The probability of repenetration of the clouds from below is $(1 - A_c)$, and of reflection back into the atmosphere, A_c. This latter fraction may be transmitted or again reflected, and it is clear that we have an infinite but converging series of reflections. Thus, the fraction of the total incident sunlight that initially penetrates the clouds, but that eventually escapes to space, is

$$
\begin{aligned}
(\mathcal{T})f^{(i)} &= \mathcal{T}[(1 - t)(1 - A_c) \\
&\quad + (1 - t)^2 A_c(1 - A_c) + (1 - t)^3 A_c^2(1 - A_c) + \ldots] \\
&= \mathcal{T}\,(1 - A_c)(1 - t)\sum_{j=0}^{\infty} A_c^j(1 - t)^j \\
&= \frac{\mathcal{T}(1 - A_c)(1 - t)}{1 - A_c(1 - t)} \quad . \qquad (41)
\end{aligned}
$$

Similarly, the fractions of the total incident radiation which undergo initial scenarios (ii) and (iii) are

$$\mathcal{T}f^{(ii)} = \frac{t(1 - A_s)\mathcal{T}}{1 - A_c(1 - t)} \qquad (42)$$

and

$$\mathcal{T}f^{(iii)} = \frac{tA_s\mathcal{T}}{1 - A_c(1 - t)} \quad , \qquad (43)$$

where A_s is the surface reflectivity.

It is readily verified that

$$f^{(i)} + f^{(ii)} + f^{(iii)} = 1 \quad , \tag{44}$$

as is demanded by the formulation of the problem.

Consider next the further fate of the radiation of case (iii), once reflected from the surface. The following scenarios are available:

(iv) It may be Rayleigh scattered back to the surface, and there absorbed;

(v) it may be transmitted by the atmosphere, but reflected at the clouds;

(vi) it may be transmitted by the atmosphere and penetrate into the clouds, to either be absorbed or emitted into space.

In a manner exactly analogous to the derivation of Eqs. (41) to (43), one can show that the fractions of the total incident sunlight that option these scenarios are, respectively,

$$\mathcal{T} f^{(iv)} = \frac{f^{(iii)}(1 - t)\mathcal{T}(1 - A_s)}{1 - (1 - t) A_s} \quad , \tag{45}$$

$$\mathcal{T} f^{(v)} = \frac{f^{(iii)}\, tA_c \mathcal{T}}{1 - (1 - t) A_s} \quad , \tag{46}$$

and

$$\mathcal{T} f^{(vi)} = \frac{f^{(iii)}\, t(1 - A_c)\mathcal{T}}{1 - (1 - t) A_s} \quad . \tag{47}$$

We note that

$$f^{(iv)} + f^{(v)} + f^{(vi)} = f^{(iii)} \quad , \tag{48}$$

as it must.

Of the five overall scenarios (i), (ii), (iv), (v), and (vi) considered to this point in the discussion, only scenario (v) leaves the radiation still in the atmosphere. This radiation is indistinguishable from incident radiation that has just penetrated the clouds. Thus, instead of a fraction $\mathcal{T}$ of the total radiation which is incident on the atmosphere below the clouds, we have now a fraction $\mathcal{T}[1 + f^{(v)}]$. Thereby, we have generated an infinite series for the three quantities of primary interest, the fraction $f_a(g)$ of the total incident energy absorbed by the ground, the fraction f_r, which, either directly or indirectly, is reflected back to space, and the fraction $f_a(c)$ of energy absorbed by the clouds. The latter two quantities are derived from $f^{(i)}$ and $f^{(ii)}$ as well as from the radiation initially incident upon the clouds that is not transmitted:

$$f_a(g) = \mathcal{T}\left\{1 + \sum_{j=1}^{\infty}\left[f^{(v)}\right]^j\right\}\left[f^{(ii)} + f^{(iv)}\right]$$

$$= \frac{\mathcal{T}[f^{(ii)} + f^{(iv)}]}{1 - f^{(v)}} \; ; \tag{49}$$

$$f_r = A_c + \frac{\mathcal{T}[f^{(i)} + f^{(vi)}](\mathcal{T})}{1 - f^{(v)} \; (1 - A_c)} \tag{50}$$

$$f_a(c) = B + \frac{\mathcal{T}[f^{(i)} + f^{(vi)}]}{1 - f^{(v)}}\left(\frac{B}{1 - A_c}\right) \; . \tag{51}$$

The term $(B/1 - A_c)$ in Eq. (51) represents the fraction of the nonreflected radiation incident on the bottom of the clouds, which is absorbed. Similarly, $(\mathcal{T}/1 - A_c)$ in Eq. (50) is that fraction that is transmitted to space. If $A_s << 1$, as seems reasonable in many cases, Eqs. (49), (50), and (51) simplify to the following expressions:

$$f_a(g, A_s = 0) = \frac{t\,\mathcal{T}}{1 - (1 - t)\,A_c} \tag{52}$$

$$f_a(c,\ A_s = 0) = B + \frac{B\mathcal{T}(1 - t)}{1 - (1 - t)A_c} \tag{53}$$

$$f_r(A_s = 0) = A_c + \frac{\mathcal{T}^2(1 - t)}{1 - A_c(1 - t)} \quad . \tag{54}$$

The transmissivity of the Rayleigh-scattering layer t can be expressed in terms of the Rayleigh-scattering optical depth τ_R. Setting 2β and $\tilde{\omega}_0$ equal to unity in the general equation for transmissivity of Paper I, we obtain

$$t = \frac{1.16}{1.16 + \tau_R} \quad . \tag{55}$$

Pollack (1967) gives a formula for τ_R in terms of the surface pressure P_s, wavelength λ, and composition of the atmosphere. Note that τ_R is independent of the temperature structure of the atmosphere. For the Venus atmosphere composed principally of N_2, τ_R can be written as

$$\tau_R = \frac{1.1 \times 10^{-2}\ P_s}{\lambda^4} \quad , \tag{56}$$

where P_s has the units of atmospheres and λ of microns. If the atmosphere were principally made up of CO_2, the numerical parameter of Eq. (56) should be multiplied by a factor of 1.55.

We will next present equations for estimating the amount of absorption by atmospheric gases. The amount of energy absorbed above a given layer by a particular absorption band will be given by $\mathcal{F}_{\tilde{\nu}}\ \Delta\tilde{\nu}$, when $\mathcal{F}_{\tilde{\nu}}$ is the solar flux per unit wave number, and $\Delta\tilde{\nu}$ the equivalent width of the band in units of wave number. Gaseous absorption can then be readily estimated once $\Delta\tilde{\nu}$ is known for the various bands. Howard, Burch, and Williams (1956) have obtained empirical expressions for $\Delta\tilde{\nu}$ for the strong-line, weak-band region (i.e., the centers of the individual rotational lines are "black" while the regions between individual lines are comparatively "white") and for the strong-line, strong-band region. The case of the weak-line, weak-band domain will not concern us since little absorption takes place there. For the strong-line, weak-band case, $\Delta\tilde{\nu}$ is represented as

$$(\Delta\tilde{\nu})_{s,w} = c(W\ P^{k/d})^{d} \quad , \tag{57}$$

where W is the amount of gas, P is the pressure, c, d, and k are empirical parameters determined from the laboratory measurements. The corresponding expression for the strong-line, strong-band equivalent width is

$$(\Delta\tilde{\nu})_{s,s} = C + D \log(W\ P^{K/D}) \quad , \tag{58}$$

where again C, D, and K are empirical parameters. It should be noted that a given band investigated by Howard et al., actually consists, in general, of several individual vibrational-rotational bands that are situated close together.

Howard et al. also give values for the equivalent width $\Delta\tilde{\nu}_{\ell}$ that defines the boundary between expressions (57) and (58). If $\Delta\tilde{\nu} > \Delta\tilde{\nu}_{\ell}$ then Eq. (58) is appropriate, while for $\Delta\tilde{\nu} < \Delta\tilde{\nu}_{\ell}$ Eq. (57) is the one to use. In using Eqs. (57) and (58), one must remember that W is the total gas amount seen by photons between the top of the atmosphere and a given point. Because of the nonlinearity of these equations, one cannot substitute the value of W between two given points to find the absorption that takes place between them, but rather must first calculate $\Delta\tilde{\nu}$ for each point with respect to the top of the atmosphere and then subtract the two numbers. To allow for the nonhomogeneous path length, we can employ the Curtis-Godson approximation with the pressure set equal to an average value along the photon path.

Convoluting the gaseous absorption with scattering is difficult, because one is dealing with nonmonochromatic equations, and a useful compromise is to consider scattering as increasing the effective path length. If τ_{iso} is the equivalent isotropic scattering optical depth of the scattering layer, with $\tau_{iso} > 1$, then the equivalent path length will be about τ_{iso} larger than the geometrical path; τ_{iso} will equal the actual optical depth τ times 2β. In making this conversion between τ_{iso} and τ, we allow for the fact that about $(1/2\beta)$ scattering events are needed to make completely random the direction of a photon undergoing scattering strongly biased in the forward direction.

Let us summarize this section by setting out the general strategy for obtaining $\mathcal{T}_\lambda$ and $f(z)$. Equations (52), (53), and (54) are used to calculate the monochromatic solar-energy deposition in the ground $f_a(g)$ and cloud layer $f_a(c)$, as well as the fraction reflected to space f_r and are valid where energy is not removed by gaseous absorption. Equations (57) and (58) are then used to calculate the solar energy deposited in the atmosphere above, below, and within the clouds with allowance made where necessary for an increased path length caused by scattering. The solar beam is multiplied by the cloud transmissivity and reflectivity in estimating gaseous absorption below the clouds and after reflection from the clouds, respectively. Finally, $f_a(g)$, $f_a(c)$, and f_r are weighted by $\mathcal{F}_{\tilde{\nu}}$ and integrated over wavelengths not affected by the relevant gaseous absorption, and similarly the gaseous absorption component is weighted by $\mathcal{F}_{\tilde{\nu}}$ and appropriately integrated. The amount of gaseous absorption above any level is obtained directly from Eqs. (57) and (58), and so $f(z)$ may straightforwardly be found.

VI. SUMMARY

By employing transmission average opacities, which are defined in Eq. (3), we can greatly reduce the number of frequency points needed to calculate net fluxes. Equations (3) and (5) are rigorously correct flux equations that can be used to calculate the temperature structure of a gaseous atmosphere. A power-law dependence on gas amount and pressure is a useful representation for the average opacities, and Eq. (13) presents the generalization to a nonhomogeneous path. Since the average opacity does not vary linearly with gas amount, one cannot write down the usual form of the equation of transfer, and most importantly this implies a greater tendency toward convective instability. Equations (26) and (36) provide approximate formulas for estimating the temperature-gradient parameter ψ deep within an atmosphere and near the bottom of an opaque cloud layer, respectively. The quasi-asymptotic value of ψ deep within an atmosphere is independent of the absolute value of the opacity, but depends only on how rapidly the opacity changes with position (Sagan 1968), while the value of ψ near a cloud bottom depends upon the total pressure and the mixing ratio of the infrared optically active constituents. Condensation cloud layers may serve as useful indications of the end of the convection zone of the atmosphere, and so flux Eq. (36) can be used to gain information about the source of opacity. Finally, the net infrared flux equals the amount of solar energy deposited below the level of interest, and this quantity can be calculated from Eqs. (52), (53), (54), (57), and (58), which allow for scattering and absorption by the atmospheric gases, cloud layer, and ground.

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