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THEORETICAL STUDY OF THE BALLISTICS AND HEAT TRANSFER IN
SPINNING SOLID PROPELLANT ROCKET MOTORS

By R. Harold Whitesides, Jr. and B. Keith Hodge

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PREFACE

This program was conducted for the Langley Research Center of the National Aeronautics and Space Administration by the Huntsville Division of Thiokol Chemical Corporation under Contract NAS 1-7034. Mr. G. Burton Northam was Technical Monitor for the contract. This work was performed during the period April 3, 1967 through July 3, 1968.

R. Harold Whitesides, Jr. was the Principal Investigator for this program. B. Keith Hodge performed the gas dynamics and burning rate model studies. W. I. Dale, Jr. was Program Manager for this contract.

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ABSTRACT

This study consisted of an analytical investigation of the effects of spin-induced radial acceleration on (1) the ballistics of a motor with a circumferential slot, (2) the evolution of the burning surface, (3) the convective heat transfer to the nozzle and head-end dome, and (4) the burning rate of metallized and non-metallized propellants.

Techniques and computer programs that were developed to predict the amount of metal/metal oxide retained within a circumferential slot of a spinning rocket motor and to predict the effects of the slot-port flow interaction, implied that the spin effects on circumferential slot ballistics are secondary.

A computerized technique for predicting the regression of an internal burning surface with radial acceleration effects showed that the specific effects of spin rate on surface regression are (1) the non-uniform surface evolution of star grains, (2) increasing motor pressures with spin rate, and (3) increasing progressiveness with spin rate of pressure-time histories for star grains.

Two semi-empirical analyses and computer programs were developed that predict the convective heat transfer to the nozzle wall and to the head-end dome of a spinning rocket motor. The trends predicted in parametric studies were in agreement with known experimental results.

The existing analytical burning rate models for metallized and non-metallized propellants were modified to obtain better correlation with experimental data.

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NOMENCLATURE

a	radial acceleration, $\Omega^2 R/g_o$
A	nozzle flow area
b	slot width
C_o	sonic velocity, total conditions
c_p	specific heat, constant pressure
ΔC_v	concentration difference
C_D	drag coefficient
C^*	characteristic velocity
d	port diameter, particle diameter
D	nozzle diameter, local
F	force on particle
g_o	gravitational constant
G_{ij}	influence coefficient
h	heat transfer coefficient
h_D	diffusion coefficient
H	enthalpy
k	gas thermal conductivity
K	vortex strength constant
L	characteristic length of fuel vapor pocket
$\dot{m}$	mass flow rate
M	Mach number
MM	moment of momentum

M_{wt}	molecular weight of gases
n	burning rate exponent, $r_b = \alpha P^n$
N	number of walls
N_s	number of pits per unit area
P	static pressure
r	radial coordinate
r_b	burning rate
r/r_o	r_b/r_{b0} -- burning rate ratio
R	nozzle radius, port radius
R_g	gas constant, $\bar{R}/M_{wt}$
$\bar{R}$	universal gas constant
s	distance along grain
S	particle surface area
Δs	distance traveled in time, Δt
t	time
T	temperature
U	axial velocity of gas
V	total relative velocity
w	tangential velocity of gases
w_m	percent aluminum content of propellant
W	relative tangential velocity component
z	axial coordinate
α	burning rate coefficient, $r_b = \alpha P^n$

α^*	swirl strength
β	pitting parameter = $(N_s \pi d^2)^{-1} \cos \Phi$
γ	ratio of specific heats
δ_r	thickness of gas phase reaction zone
η	dimensionless radial coordinate, r/r_c
θ	tangential coordinate
λ_g	thermal conductivity
Λ	volume of fuel vapor pocket
μ	dynamic viscosity
ξ_o	radius ratio
ρ	density
σ	variance in a log normal distribution
Φ	angle between surface normal and acceleration vector
Ψ	swirl function defined by equation (A10a)
Ω	motor spin rate
G_r	Grashof number, $a \Delta \rho d_{fv}^3 / \mu^2 g$
N_u	Nusselt number, hD/k
P_r	Prandtl number, $c_p \mu/k$
R_e	Reynolds number, $VD\rho/\mu$
S_c	Schmidt number, $\mu/\rho D$
S_h	Sherwood number, $h_D d_{fv}/D_g$

Subscripts:

adw	adiabatic wall
B	buoyant
c	core
D	drag
f	film
fv	fuel vapor
g	gas
inc	incompressible
M	metal particle
o	port, zero-spin conditions
ox	oxidizer
p	propellant
r	radial
s	surface, spin
t	total
ts	total condition with spin
w	wall
z	axial
θ	tangential
1	one-dimensional
2	two-dimensional
2s	slot flow at station 2
2p	axial flow at station 2
1s	slot flow at station 1

Subscripts: (Cont'd)

1p axial flow at station 1

3 station 3

Superscripts:

* throat

' dimensionless variable

— weight mean

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SUMMARY

This study consisted of an analytical investigation of the effects of spin-induced radial acceleration on (1) the ballistics of a motor with a circumferential slot, (2) the evolution of the burning surface, (3) the convective heat transfer to the nozzle and head end dome, and (4) the burning rate of metallized and non-metallized propellants.

Techniques were developed to predict the amount of metal/metal oxide retained within a circumferential slot of a spinning rocket motor and to predict the ballistic effects of a circumferential slot in a spinning rocket motor. Two computer programs were written using the techniques developed. The study of metal/metal oxide retention showed that particles could be retained within the slot and that the amount of metal/metal oxide retained is strongly dependent upon the assumed particle size and distribution. The metal retention analysis is qualitative in nature. The ballistic effects of a circumferential slot were predicted by using a swirl ballistics program in conjunction with a slot-port flow interaction analysis. A parametric study, which was made using the computer program with the slot-port flow analysis, implied that the spin effects of circumferential slots are small, if not negligible.

A computerized technique for predicting the regression of an internal burning surface with radial acceleration effects was developed. The computer program may be used to predict the pressure-time history of a motor with a star grain at any given spin rate. This analysis predicts that the specific effects of spin rate on surface regression are (1) the non-uniform surface evolution of star grains, (2) increasing motor pressures with spin rate, and (3) increasing progressiveness with spin rate of pressure-time histories for star grains.

Two semi-empirical analyses and computer programs were developed which predict the convective heat transfer to the nozzle wall and to the head end dome of a spinning rocket motor. The nozzle heat transfer analy-

sis was developed by applying the Bartz analogy to swirling flow in a spinning nozzle and employing Mager's solution for isentropic swirling flow through a nozzle to obtain local density, axial velocity, and tangential velocity of the gases along the nozzle wall. Nozzle wall heat transfer coefficients are predicted to increase significantly with spin rate, as expected. The increase in coefficients is primarily due to the increase in motor pressure with spin rate; however, a significant portion of the increase is due to the increase in the total relative velocity between the swirling gases and the wall. The computerized head end dome heat transfer analysis was developed with a semi-empirical approach similar to that used for the nozzle analysis. This analysis predicts sharp increases in heat transfer coefficients in a region near the center of the head end dome. This trend is in agreement with known experimental results and thus supports qualitatively the model upon which the head end analysis was based.

An extensive examination of both the Thiokol non-metallized and metallized burning rate models was made. A modification concerning the particle retention criteria was made to the metallized model. This modification, which enabled the model to better correlate experimental data, did not change the dependence of the burning rate upon the orientation of the acceleration vector. Several anomalies between the non-metallized burning rate model and experimental data were discussed. Evidence was presented to substantiate the assumptions of the model and to restrict the validity of the original model to a low acceleration regime. The concept of a stratified combustion model for high acceleration was introduced and formulated. Because of the lack of mass diffusivity data, no comparison of experimental data and the stratified combustion model was possible.

INTRODUCTION

Captive spin tests and flight tests of solid propellant rocket motors have indicated that the performance characteristics of spinning motors can deviate substantially from the static performance characteristics and that the magnitude of the deviations increases with spin rate. These spin-induced deviations have in many cases caused motor failure where the motor design had been previously proven in static tests. The major causes of the observed spin-induced performance deviations can be associated with one or more of the following: (1) increases in the propellant burning rate, (2) changes in the motor and nozzle gas dynamics, (3) retention of metal and metal oxide within the motor, and (4) increases in heat transfer rate in local areas. Consequently, an understanding of the results of spin tests of actual motors and the development of analytical motor prediction techniques must be founded upon knowledge of the individual causes and the interactions between them.

During the past two years, the Huntsville Division of Thiokol Chemical Corporation has conducted analytical studies directed at understanding and interrelating the effects of acceleration on (1) solid propellant burning rate, (2) regression of the burning surface, (3) internal gas dynamics, and (4) convective heat transfer. The results of these studies will serve as a basis for developing comprehensive techniques and guidelines for designing spin-stabilized solid propellant rocket motors.

Under Contract NAS-7-406 for the Langley Research Center, the Huntsville Division developed burning rate models that described the combustion of both metallized and non-metallized composite propellants subjected to acceleration fields and developed a procedure to predict the internal ballistics-at-fixed-time of spinning rocket motors with axisymmetric grains. The results of that work were reported in NASA Report 66218 (Thiokol Report 42-66, U-66-42A).

A follow-on program, which is the subject of this report, has been conducted under Contract NAS-1-7034, also for the Langley Research Center. This program was divided into four phases numbered III through VI. (The first two phases were performed under Contract NAS-7-406.) The goals of the program phases are described below:

Phase III - Flow in Circumferential Slots with Spin Effects -
Develop a computer program to describe the flow field in a circumferential slot and determine the effects of the slot on the internal ballistics and retention of metal/metal oxide for a spinning motor.

Phase IV - Surface Regression Analysis - Develop a computerized technique for predicting the evolution of the burning surface of a star grain under the influence of an acceleration-induced non-uniform burning rate and use the technique to study the effects of spin rate and acceleration level on mass generation rate and chamber pressure.

Phase V - Theoretical Heat Transfer Study - Develop computer programs for predicting the convective heat transfer to the nozzle wall and head-end dome of a spinning rocket motor and conduct a study of the effect of spin rate on the predicted heat transfer rates for a model motor.

Phase VI - Effect of Acceleration on Burning Rate - Extend and/or revise analytical burning rate models developed under Contract NAS-7-406 as required to account for observed anomalies between the theory and current experimental data and correlate new experimental data with parameters obtained from the analytical models.

These phases are discussed separately in the following sections of this report.

Five computer programs were developed during the course of this contract. General aspects of these programs are discussed in this report; details of the programs that are not in appropriate form for inclusion in a bound volume were transmitted to NASA Langley under separate cover. That information included the following: (1) card decks, (2) source listings, (3) sample cases, and (4) user's instructions regarding input format.

ANALYSIS OF FLOW IN CIRCUMFERENTIAL SLOTS WITH SPIN EFFECTS

Introduction

The objective of this phase was to develop analytical techniques to predict the effects of circumferential slots in spinning rocket motors. In spinning rocket motors with circumferential slots, two areas are of current interest: (1) the ballistic effects resulting from the interaction of the main axial flow and the slot flow and (2) the amount of metal/metal oxide retained within the slot. An exact solution to either of the problems is extremely difficult, if not impossible, since the former requires treatment of the turbulent, compressible mixing of two streams and the latter requires consideration of a compressible vortex with mass addition and two-phase flow. Thus, approximations were made in order to facilitate solutions.

A technique to estimate the amount of metal/metal oxide retained within the slot was examined first. An order-of-magnitude analysis revealed that to a good approximation the flow within the slot is one-dimensional. Using this as a basis for computing the flow field, the metal/metal oxide retained can be estimated by considering the trajectories of evolved metallic particles.

The ballistic effects of a circumferential slot were treated by developing a subroutine for an available equilibrium-at-fixed-time computer program. By assuming conservation of both linear and angular momentum, the axial stream and slot flow were interacted and the pressure drop across the slot computed. The determination of the pressure drop across the slot permits the flow properties downstream of the slot to be estimated. Thus, the new subroutine when used in conjunction with the available internal ballistics program permits the performance effects of slots to be studied.

Metal/Metal Oxide Retention

Flow Field. - One of the problems of interest (and concern) with spinning motors containing a circumferential slot is the amount of metal/metal oxide retained within the slot. Since the combustion products evolved within the slot are normally removed from the slot, any retention of metal/metal oxide particles must result from the inability of the particles to follow the slot flow field within a spinning motor. Thus metal/metal oxide retention must be treated as a two-phase flow effect.

Numerous books, reports, technical articles, etc. treating two-phase flow systems are available. Available two-phase flow analyses tend to consider systems in which the condensed phase "lags" the gaseous phase (ref. 1, 2). The lag, which is particle size dependent, refers to differences in velocity, temperature, etc. between the gaseous and condensed phases of the flow system. For some flow situations encountered in solid rocket motors the lag hypothesis is valid and yields good results. However, the two-phase flow system encountered in the circumferential slot does not fall within this category. For the "lag" solutions the particle follows the flow, but for the two-phase flow within the slot the particle may or may not follow the flow. A particle deposited on the slot bottom, for instance, does not follow the flow. Thus the conventional lag solution is invalid within the slot.

Consider the probable physical happenings within the slot. As the burning surface of a metallized propellant regresses, the evolved combustion products constitute a two-phase flow system. Figure 1 schematically represents the path that a metal/metal oxide particle might describe as the burning surface regresses and the particle is released into the slot.

The solid line represents the hypothetical path a particle would have relative to a fixed reference if no forces (aerodynamic, inertial, etc.) were acting on it after it is expelled from the surface. The particle velocity for this case is $r\Omega$ and since no forces are acting on the particle the velocity will remain constant until impact with the wall. The broken line represents a typical trajectory the metal/metal oxide particle might have when drag and buoyancy forces are considered.

Since the magnitude and direction of the drag force are explicit functions of the relative velocity between the particle and the gas, it is apparent that the flow field within the slot must be known before the trajectory of a particle can be computed.

Then the problem of the two-phase flow within the slot can be treated by considering the flow field of the gaseous phase and the trajectories of the condensed phase. The major purpose of this analysis is to determine the

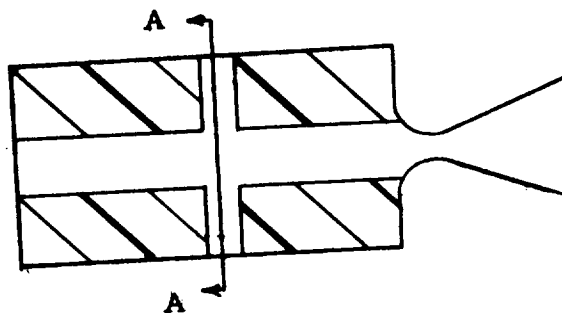
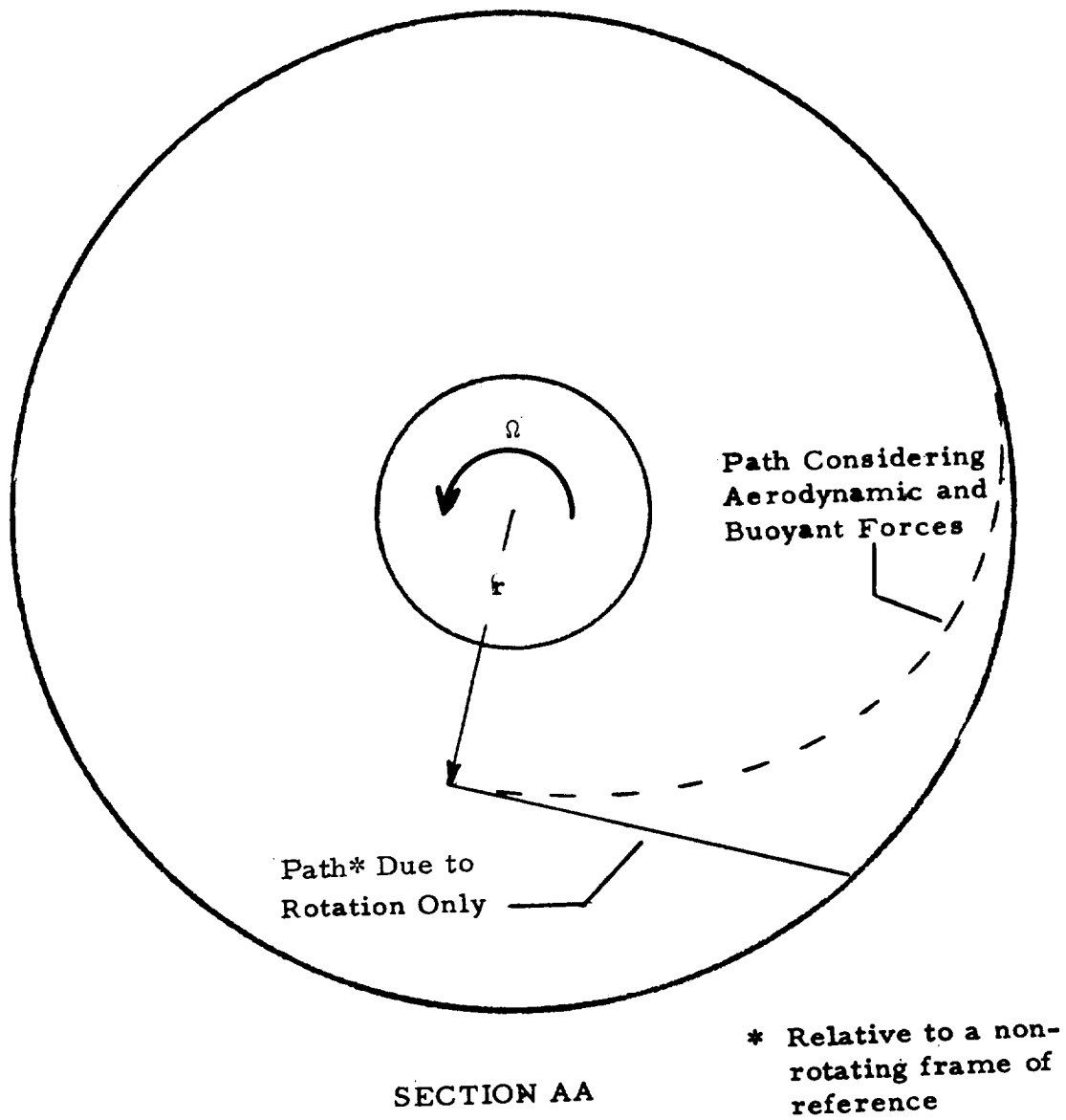


Figure 1 . Schematic of Particle Trajectories

retention of metal/metal oxide particles in the circumferential slots. The error induced by separate consideration of the two phases comes primarily through the neglect of heat transfer between the phases. For conventional motors, losses of from 0.3% to 4.5% in C^* are caused by thermal and velocity lags between the phases (ref. 1). Thus it is concluded that the errors induced by the separate consideration of the phases will be small, if not negligible.

The problem of the flow field within the slot will now be examined. Figure 2 shows the system under consideration and illustrates the coordinate system, the control surface, and the nomenclature employed. Conventional cylindrical coordinates are employed with r denoting the radius and z denoting the axial distance from a wall of the slot. The components of velocity in the r , θ , and z directions are V_r , V_θ , and V_z , respectively. The flow is assumed to be inviscid, steady, adiabatic, and axisymmetric, and the fluid a perfect gas with constant specific heat and molecular weight.

Then the equations of motion in dimensionless form (non-dimensionalized on r , V_r) for the slot reduce to (ref. 3)

$$V_r' \frac{\partial V_r'}{\partial r'} - \frac{V_\theta'^2}{r'} + V_r' \frac{\partial V_r'}{\partial z'} = -\frac{1}{\rho'} \frac{\partial P'}{\partial r'} \quad (1)$$

$$V_r' \frac{\partial V_\theta'}{\partial r'} + \frac{V_r' V_\theta'}{r'} + V_z' \frac{\partial V_\theta'}{\partial z'} = 0 \quad (2)$$

$$V_r' \frac{\partial V_z'}{\partial r'} + V_z' \frac{\partial V_z'}{\partial z'} = -\frac{1}{\rho'} \frac{\partial P'}{\partial z'} \quad (3)$$

Whence by definition: $O(V_r') = 1$ and $O(r') = 1$. The concern is with spin effects, hence $O(V_\theta') = 1$. For realistic slots, $r > z$, thus $O(z') = \delta$ (where $\delta < 1$). The area available for discharge of mass into the axial stream for thin, deep slots ($r > z_0$) is significantly less than the area available for mass generation. Therefore, the axial velocity of the gases in the slot (V_z) is of a lower order than the radial discharge velocity, V_r ; i.e., $O(V_z') = \delta$. An order of magnitude analysis of equations (1), (2), and (3), using the above as indicative of the magnitude of the terms, results in no simplification to equations (1), (2), or (3). Equation (3), however, is of a lower order of magnitude than equations (1) and (2). Thus,

* $O(\)$ denotes the order of magnitude of the dimensionless argument.

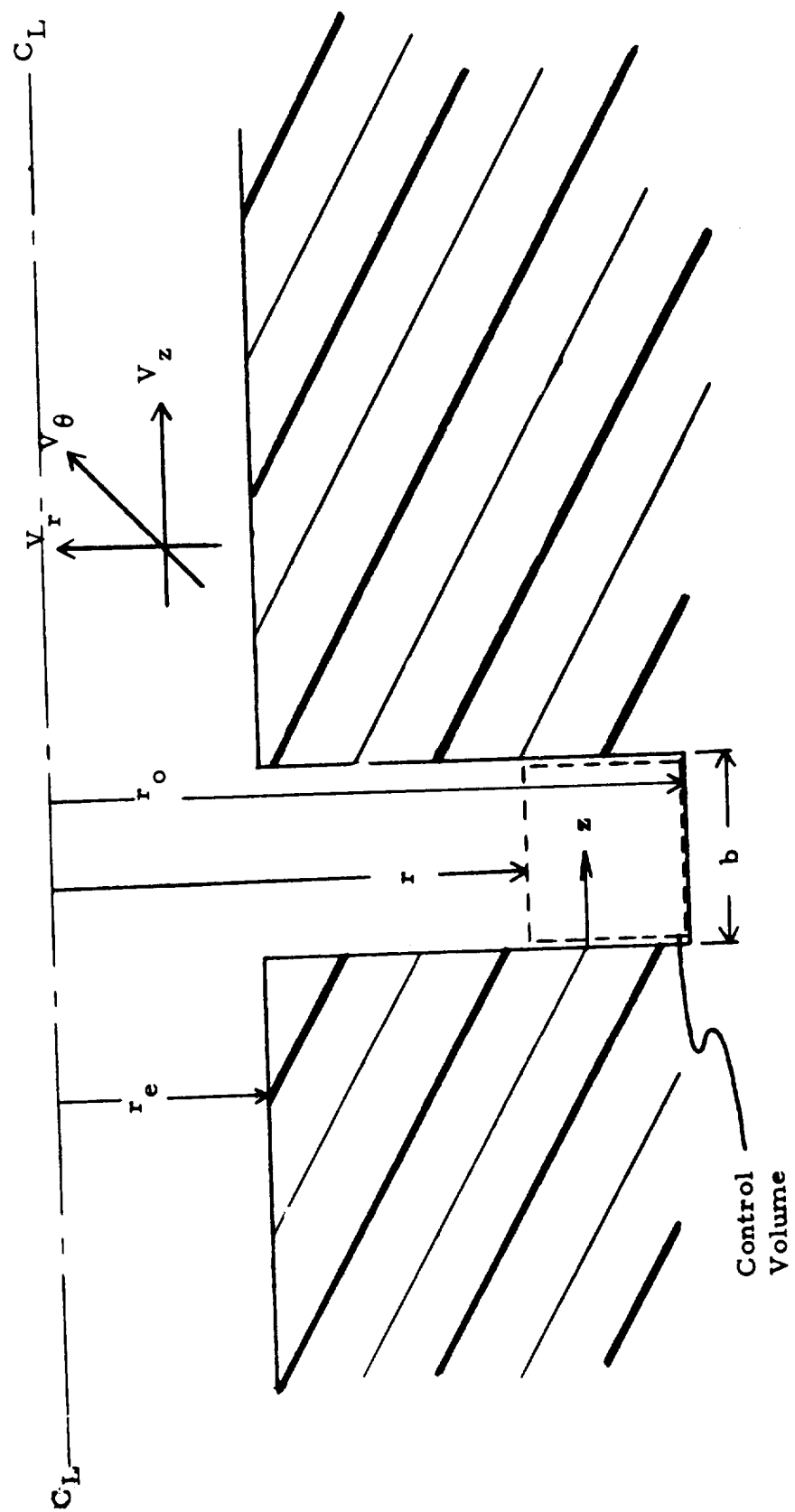


Figure 2 . Schematic of Variables for Flow Field

$$O\left(\frac{\partial P'}{\partial r'}\right) > O\left(\frac{\partial P'}{\partial z'}\right) \quad (4)$$

i. e., the pressure gradient across the slot is unimportant when compared to the radial pressure gradient within the slot.

Since both the axial pressure gradient ($\partial P/\partial z$) and the axial component of velocity (V_z) are of a lower order of magnitude than the radial pressure gradient ($\partial P/\partial r$) and the remaining velocity components (V_θ and V_r), the variation in pressure and velocity across the slot can be neglected for a first order analysis. Hence, the slot can be treated as a one-dimensional flow problem: $V_R = V_R(r)$, $P = P(r)$, and $T = T(r)$.

A system of seven equations and seven unknown describing the flow can then be examined (ref. 4):

$$\text{STATE:} \quad \frac{P}{\rho} = RT \quad (5)$$

$$\text{CONTINUITY:} \quad \oint \rho \bar{V} \cdot d\bar{A} = 0 \quad (6)$$

$$\text{ANGULAR MOMENTUM:} \quad \oint \rho r V_\theta \bar{V} \cdot d\bar{A} = 0 \quad (7)$$

$$\text{ENERGY:} \quad T_t = T \cdot \left(1 + \frac{\gamma-1}{2} M^2\right) \quad (8)$$

$$\text{ISENTROPIC RELATION:} \quad \frac{P}{P_o} = \left(\frac{T}{T_o}\right)^{\frac{\gamma}{\gamma-1}} \quad (9)$$

$$\text{MACH NUMBER:} \quad M^2 = \frac{V_\theta^2 + V_R^2}{C^2} \quad (10)$$

$$\text{SPEED OF SOUND:} \quad C = \sqrt{\gamma R T} \quad (11)$$

The continuity equation, (equation (6)) for the slot control volume (see figure 2) reduces to

$$\int_{\text{sides}} \rho \bar{V} \cdot d\bar{A} + \int_{\text{bottom}} \rho \bar{V} \cdot d\bar{A} - \int_{\text{exit}} \rho \bar{V} \cdot d\bar{A} = 0 \quad (12)$$

But

$$\rho \bar{V} \cdot d\bar{A} = \rho_p \alpha_t P^n dA \quad (13)$$

Then

$$2\pi W \int_r^{r_o} \rho_p \alpha_t P^n r dr + 2\pi r_o b \rho_p \alpha_t P_o^n - 2\pi r b \frac{P}{RT} V_R = 0 \quad (14)$$

The radial velocity can be expressed as

$$V_R = \frac{RT_o \rho_p \alpha_t P_o^{n-1}}{r b} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}} \left\{ N \int_r^{r_o} \frac{r dr}{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{n\gamma}{\gamma-1}}} + b r_o \right\} \quad (15)$$

where the energy equation (8) and an isentropic relation, equation (9) are utilized. The integral expression in equation (15),

$$\int_r^{r_o} \frac{r dr}{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{n\gamma}{\gamma-1}}} \quad (16)$$

takes into account the change in static pressure resulting from the fluid

velocity. For low Mach numbers $\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{n\gamma}{\gamma-1}}$ is approximately

one (1) and can be treated as a constant. Thus equation (15) (for the restriction of low Mach numbers) becomes

$$V_R = \frac{R T_o \rho_p \alpha_t P_o^{n-1}}{r b} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1}{\gamma-1}} \left\{ \frac{N}{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{n\gamma}{\gamma-1}}} \left(\frac{r_o^2}{2} - \frac{r^2}{2} \right) + b r_o \right\} \quad (17)$$

If no mass evolution (generation) takes place in the slot and if low Mach numbers are encountered then the radial velocity becomes

$$V_R = \frac{R T_o \rho_p \alpha_t P_o^{n-1}}{b} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1-n\gamma}{\gamma-1}} \frac{N}{2} r \left[\left(\frac{r_o}{r} \right)^2 - 1 \right] \quad (18)$$

And in a similar manner conservation of angular momentum for the slot control volume can be written:

$$\int_{\text{sides}} \rho r V_\theta \bar{V} \cdot d\bar{A} + \int_{\text{bottom}} \rho r V_\theta \bar{V} \cdot d\bar{A} - \int_{\text{exit}} \rho r V_\theta \bar{V} \cdot d\bar{A} = 0 \quad (19)$$

which becomes

$$P_o^n \omega \rho_p \alpha_t \left\{ N \int_r^{r_o} \frac{1}{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{n\gamma}{\gamma-1}}} r^3 dr + b r_o^3 \right\} - \frac{r^2 b V_r V_\theta}{R T_o} P \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1}{\gamma-1}} = 0 \quad (20)$$

The tangential velocity, V_θ , can be solved explicitly from equation (17):

$$V_\theta = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}} \frac{\omega \rho_p \alpha_t P_o^{n-1} R T_o}{r^2 b V_r} \left\{ N \int_r^{r_o} \frac{r^3}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{n\gamma}{\gamma-1}}} dr + b r_o^3 \right\} \quad (21)$$

With the assumption of low Mach numbers [i. e. $\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{n\gamma}{\gamma-1}} \approx 1$] equation (21) reduces to

$$V_\theta = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}} \frac{\omega \rho_p \alpha_t P_o^{n-1} R T_o}{r^2 b V_R} \left\{ \frac{N}{4} \frac{|r_o^4 - r^4|}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{n\gamma}{\gamma-1}}} + b r_o^3 \right\} \quad (22)$$

and for the case of no mass evolution in the slot bottom (using equation (18) for V_R)

$$V_\theta = \frac{r \omega}{2} \left[\left(\frac{r_o}{r}\right)^2 + 1 \right] \quad (23)$$

With a negligible axial velocity (V_z) in the slot, the Mach number is

$$M^2 = \frac{V_\theta^2 + V_R^2}{C^2} \quad (24)$$

where

$$C = \sqrt{\gamma R T} \quad (25)$$

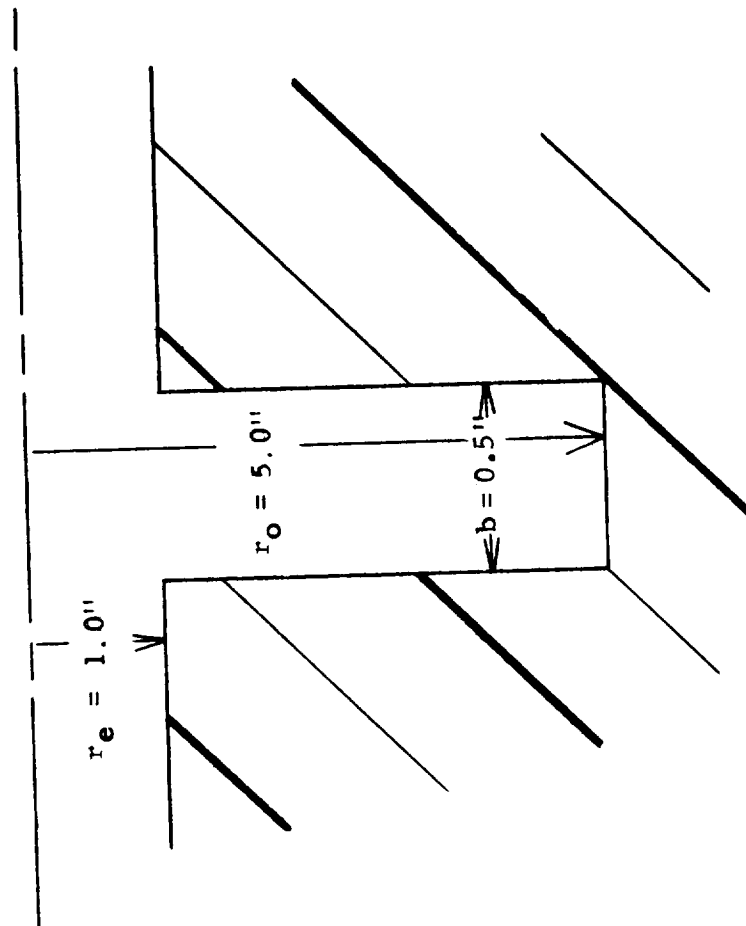
An expression for Mach number can be obtained by using equations (15) and (21) as V_R and V_θ respectively in equation (24) and simplifying the results:

$$M^2 = \left\{ \left(1 + \frac{\gamma-1}{2} M^2 \right) \frac{1}{\gamma R T_o} \right\} \frac{\omega^2}{r^2} \left[\frac{\int_r^{r_o} \frac{r^3}{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{n\gamma}{\gamma-1}}} dr + b r_e^3}{\int_r^{r_o} \frac{r}{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{n\gamma}{\gamma-1}}} dr + b r_e} \right]^2$$

$$+ \frac{P_o^{2n-2} (R T_o \rho_p \alpha_t)^2}{r^2 b^2} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{2}{\gamma-1}} \left[N \int_r^{r_o} \frac{r}{\left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{n\gamma}{\gamma-1}}} dr + b r_e \right]^2 \quad (26)$$

Equation (26) cannot be solved to yield Mach Number explicitly, hence an iterative procedure must be used. A computer program has been written which will solve equation (26) for Mach number as a function of radius. The program as currently written requires the slot geometry (width, inside radius, and outside radius), propellant properties (ρ_t , n , flame temperature, and gas constant), and total pressure in the slot as input. Once the Mach number at a radial station is evaluated, the tangential velocity and the relative velocity as well as the density, static pressure, and static temperature can be calculated using equations (15), (21), (5), (9), and (8), respectively. Thus the flow field is completely specified.

In order to obtain an indication of the effects of spin on the internal flow field of a slot, a limited parametric study was conducted. Figure 3 is a schematic of the circumferential slot that was used. The effect of spin on the velocity profile in the slot is illustrated in figure 4. Thus spin increases the total velocity at every point within the slot. Tangential velocity profiles for various spin rates are presented in figure 5. The strong dependence of tangential velocity upon spin rate is evident from this figure. The increase in tangential velocity as the flow nears the



$$\begin{aligned}
 T_f &= 5000 \text{ } ^\circ\text{F} \\
 \rho_p &= 0.0623 \text{ lbm/in}^3 \\
 P_o &= 1500 \text{ psi} \\
 \gamma &= 1.16 \\
 \alpha_t &= 0.05 \\
 n &= 0.25 \\
 Rg &= 53.3 \frac{\text{lb}_b - \text{ft}}{\text{lb}_m \cdot \text{R}}
 \end{aligned}$$

Figure 3. Input for Flow Field Study

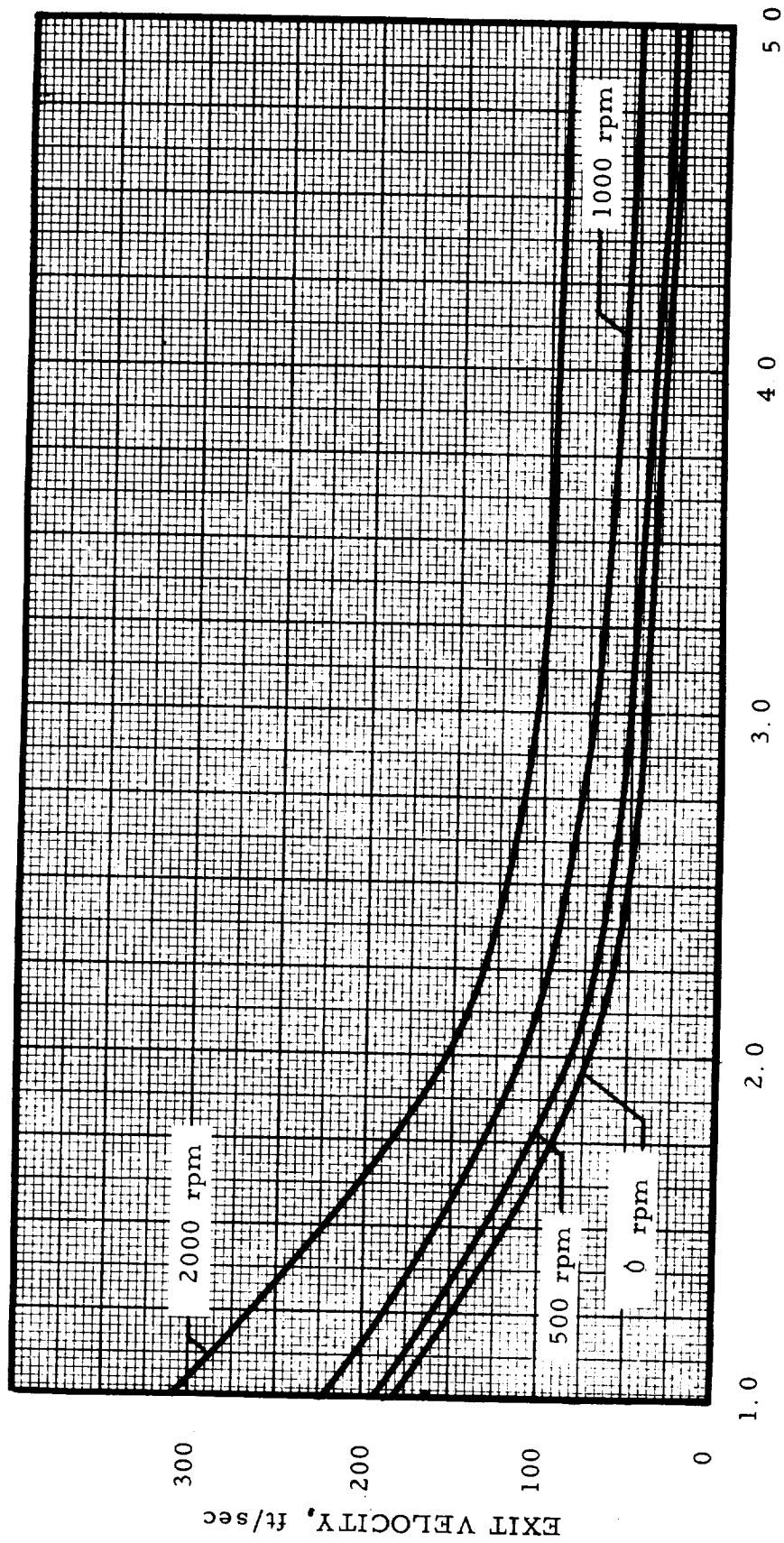


Figure 4. Velocity Profile for Slot, $r/b = 5/0.5$

slot exit is necessary in order to conserve angular momentum. Figure 6 contains a table of the radial velocities at the slot exit and bottom for varying spin rates. The lack of significant variation in the radial velocities indicates that spin has essentially no effect on the radial velocity component and that, therefore, any change in total velocity (figure 4) of the slot flow is due to change in the tangential velocity. The effect of slot width on the exit velocity of the flow field is examined in figure 7. The increase in velocity at zero spin rate is due to increasing depth/width ratios while the increase in velocity at constant depth/width rates is due to increased tangential velocity caused by an increased spin rate.

Particle Dynamics. - This portion of the analysis is concerned with trajectories of the metal/metal oxide particles. The forces acting on a condensed particle in the slot result from (1) drag force due to relative velocity between the gas and particle flow and (2) buoyancy forces due to density difference between the gas and particle phases of the flow.

The drag force acting on a particle can be represented as

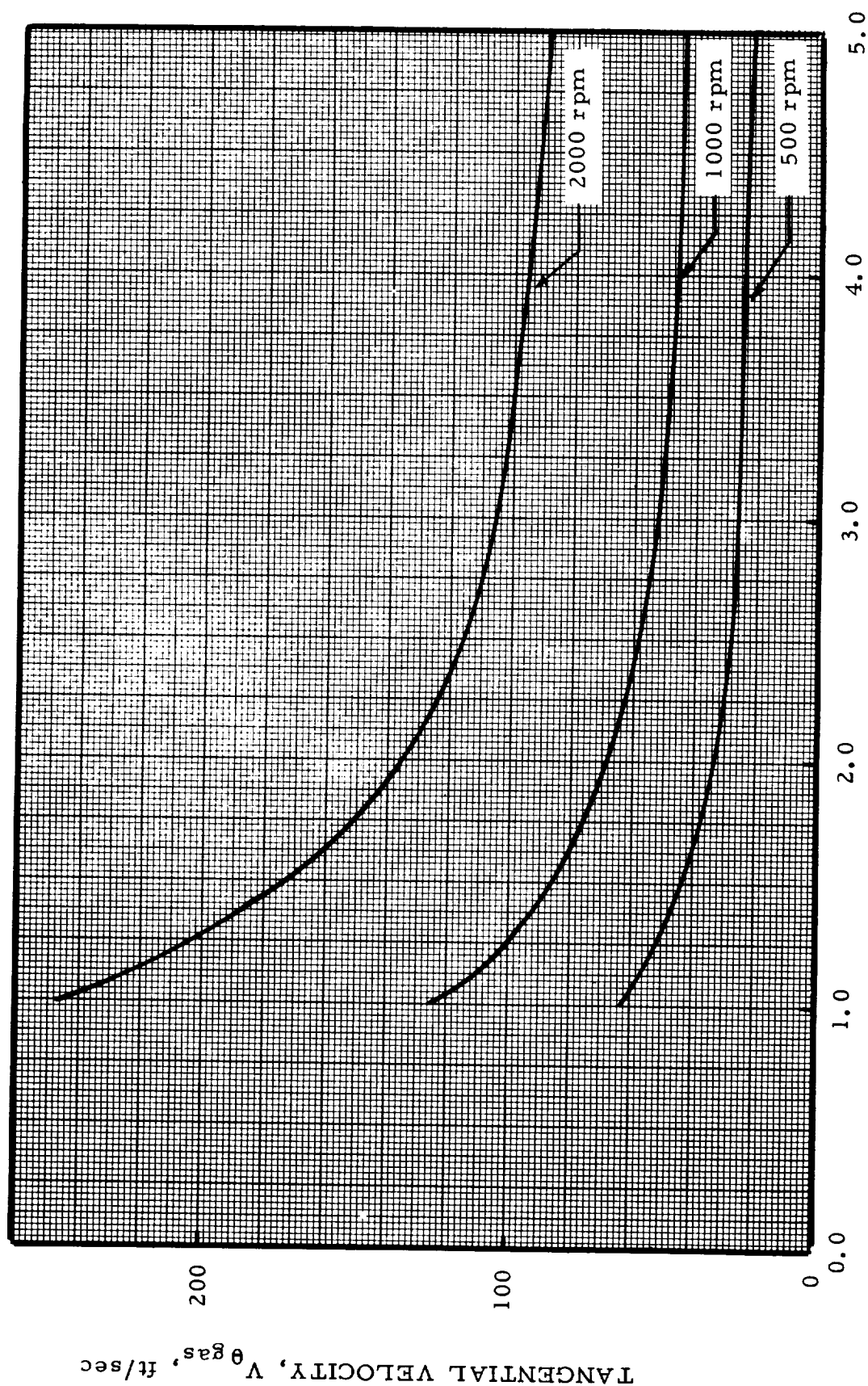
$$F_D = \frac{1}{2} \rho V_{rel}^2 C_D S \quad (27)$$

where V_{rel} is the relative velocity between the particle and the fluid. Thus the accurate determination of the drag coefficient, C_D , is paramount to the accurate prediction of the particle drag. In general, the value of the drag coefficient is dependent upon the Reynold's number (based upon the relative velocity)

$$Re_{rel} = \rho \frac{V_{rel} d}{\mu} \quad (28)$$

and the Mach number (based on the relative velocity)

$$M_{rel} = \frac{V_{rel}}{\sqrt{\gamma R T}} \quad (29)$$



DISTANCE FROM MOTOR CENTERLINE, inches

Figure 5 . Tangential Velocity Profiles for Slot, $r_o/b = 5/0.5$

ω	$V_{R_{exit}}$ ft/sec	$V_{R_{bottom}}$ ft/sec
0	184.56	5.4613
500	184.59	5.4613
1000	184.58	5.4613
1500	184.57	5.4612
2000	184.56	5.4611
10000	184.53	5.4586

$$\frac{r_o}{b} = \frac{5}{0.5}$$

$$T_f = 5000 \text{ }^\circ\text{F}$$

$$\gamma = 1.16$$

$$\rho_p = 0.0623 \text{ lb}_m/\text{in}^3$$

$$P = 1500 \text{ psi}$$

$$Rg = 53.3$$

$$\alpha_t = 0.05$$

$$M = .25$$

Figure 6 . Table Showing Variations in Radial Velocity Due to Swirl

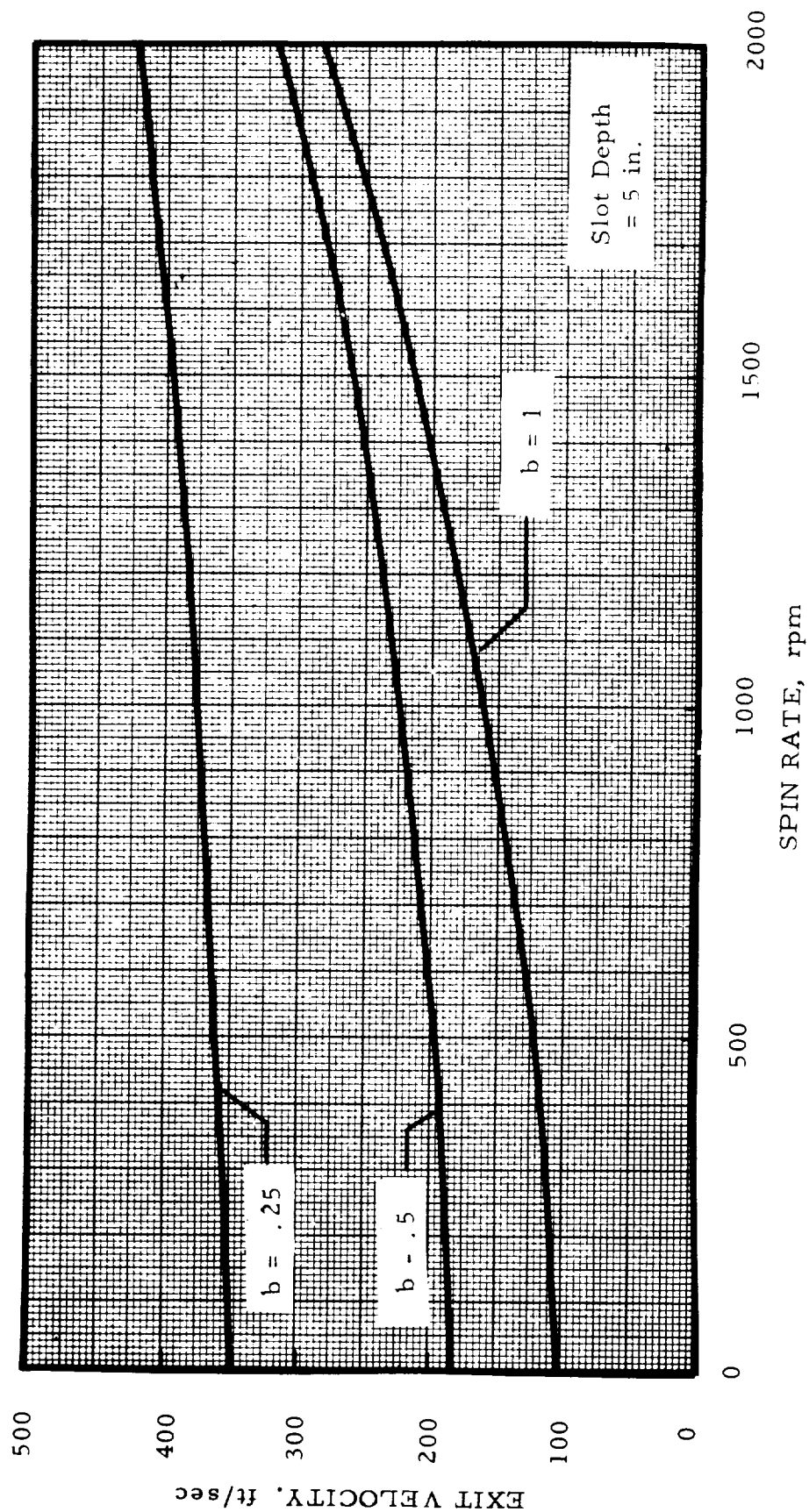


Figure 7. Exit Velocity for Various Slot Widths

Crowe and Willoughby of UTC have obtained an expression for drag coefficients valid for the range of flow regimes from free molecule flow to continuum flow. The relationships are presented here for completeness (ref. 1):

$$C_D = (C_{D_{inc}})^{-2} e^{[-3.07\sqrt{\gamma} \frac{M}{R_e} g(R_e)] + \frac{2.4}{M} e^{-\frac{R_e}{2M}} + 2} \quad (30)$$

where

$$\log_{10} g(R_e) = 1.255 [1 + \tanh(0.767 \log_{10} R_e - 1.917)] \quad (31a)$$

$$\log_{10} C_{D_{inc}} = 1.422 - 9.924 \log_{10} R_e + 0.117 (\log_{10} R_e)^2 - 0.00145 (\log_{10} R_e)^3 \quad \text{and} \quad (31b)$$

$$\log_{10} C_{D_{inc}} = 1.38 (1 + 0.0338 R_e) - \log_{10} R_e \quad (31c)$$

Equation (31b) is valid for $0.7 \leq R_e \leq 10^3$ and equation (31c) is valid for $R_e < .7$). Reference 3 states that the discrepancy between these empirical formulae and the experimentally established curve is less than 2%.

The direction of the drag force vector can be evaluated by consideration of the difference between the velocity components of the condensed phase and the gas phase:

$$\Delta V_\theta = V_{\theta_{gas}} - V_{\theta_{part}} \quad (32a)$$

$$\Delta V_r = V_{r_{gas}} - V_{r_{part}} \quad (32b)$$

(see figure 8 for a schematic of the gas/particle system.)

Then

$$V_{rel} = \Delta V = (\Delta V_\theta^2 + \Delta V_r^2)^{1/2} \quad (33)$$

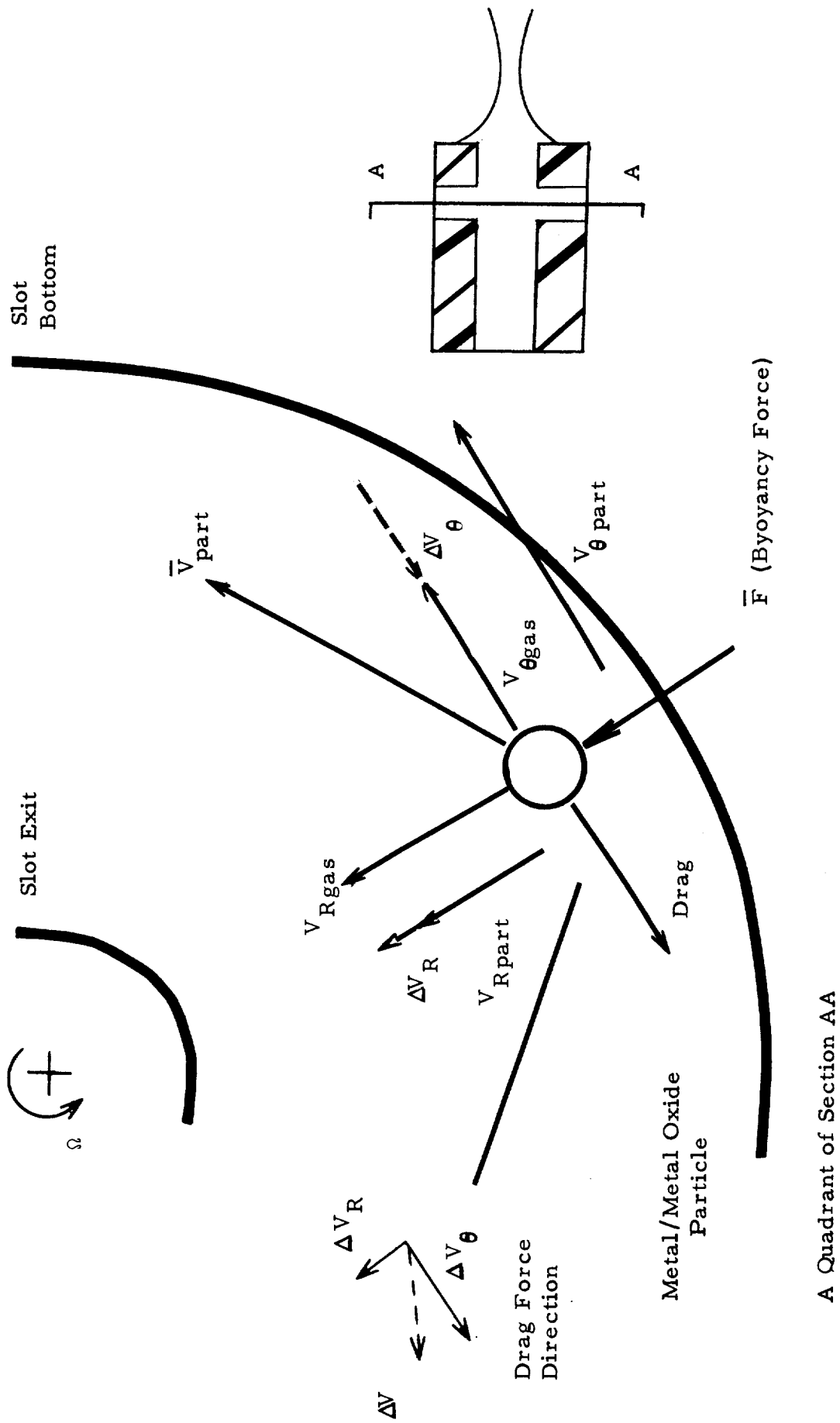


Figure 8. Nature of Forces Acting on Particle

This acceleration is directed radially inward. The acceleration due to buoyancy then becomes

$$a_F = \frac{\rho}{\rho_a} \cdot \frac{v_{\theta \text{ gas}}^2}{r} \quad (39)$$

The acceleration in component form is

$$a_\theta = a_D \cos \psi \quad (40a)$$

$$a_R = a_D \sin \psi + \frac{\rho}{\rho_a} \frac{v_{\theta \text{ gas}}^2}{r} \quad (40b)$$

At some time Δt later the particle velocity is (ref 5)

$$v_{R \text{ part}} \Big|_{t + \Delta t} = v_{R \text{ part}} \Big|_t + a_R \Delta t \quad (41a)$$

$$v_{\theta \text{ part}} \Big|_{t + \Delta t} = v_{\theta \text{ part}} \Big|_t + a_\theta \Delta t \quad (41b)$$

And the distance traveled during this time increment Δt is

$$\Delta S_\theta = \left[\frac{v_{\theta \text{ part}} \Big|_t + v_{\theta \text{ part}} \Big|_{t + \Delta t}}{2} \right] \Delta t \quad (42a)$$

$$\Delta S_R = \left[\frac{v_{R \text{ part}} \Big|_t + v_{R \text{ part}} \Big|_{t + \Delta t}}{2} \right] \Delta t \quad (42b)$$

Figure 9 is a schematic of the happenings during the time interval Δt . The radius vector at $t + \Delta t$ is

$$r_{t + \Delta t} = [r_t^2 + \Delta S^2 - 2 r_t \Delta S \cos \xi]^{1/2} \quad (43)$$

Thus the Reynolds number and the Mach number (both based on the relative velocity) can be calculated from equations (28) and (29). Once the Mach number and Reynolds number are known, equations (30) - (31) can be utilized to give C_D for the particle. Then the drag force is

$$F_D = \frac{\pi}{8} \rho_g C_D d^2 \Delta V^2 \quad (34)$$

The particle, assumed to be spherical, has a mass of $\frac{1}{6} \pi d^3 \rho_a$ where ρ_a is the effective density of the metal/metal oxide particle. Then the acceleration due to the drag force ($\bar{D} = m a_D$) is

$$a_D = \frac{3}{4} \frac{\rho_g}{\rho_a} \frac{C_D}{d} \Delta V^2 \quad (35)$$

The direction of the acceleration is specified by the velocity components.

$$\psi = \arctan \frac{\Delta V_R}{\Delta V_\theta} \quad (36a)$$

$$a_{D\theta} = a_D \cos \psi \quad (36b)$$

$$a_{DR} = a_D \sin \psi \quad (36c)$$

The buoyancy force felt by the particle is:

$$F_B = \frac{1}{6} \pi d^3 \rho_g a_g \quad (37)$$

where a_g is the acceleration to which the fluid is subjected and is given by

$$a_g = \frac{v_{\theta \text{ gas}}^2}{r} \quad (38)$$

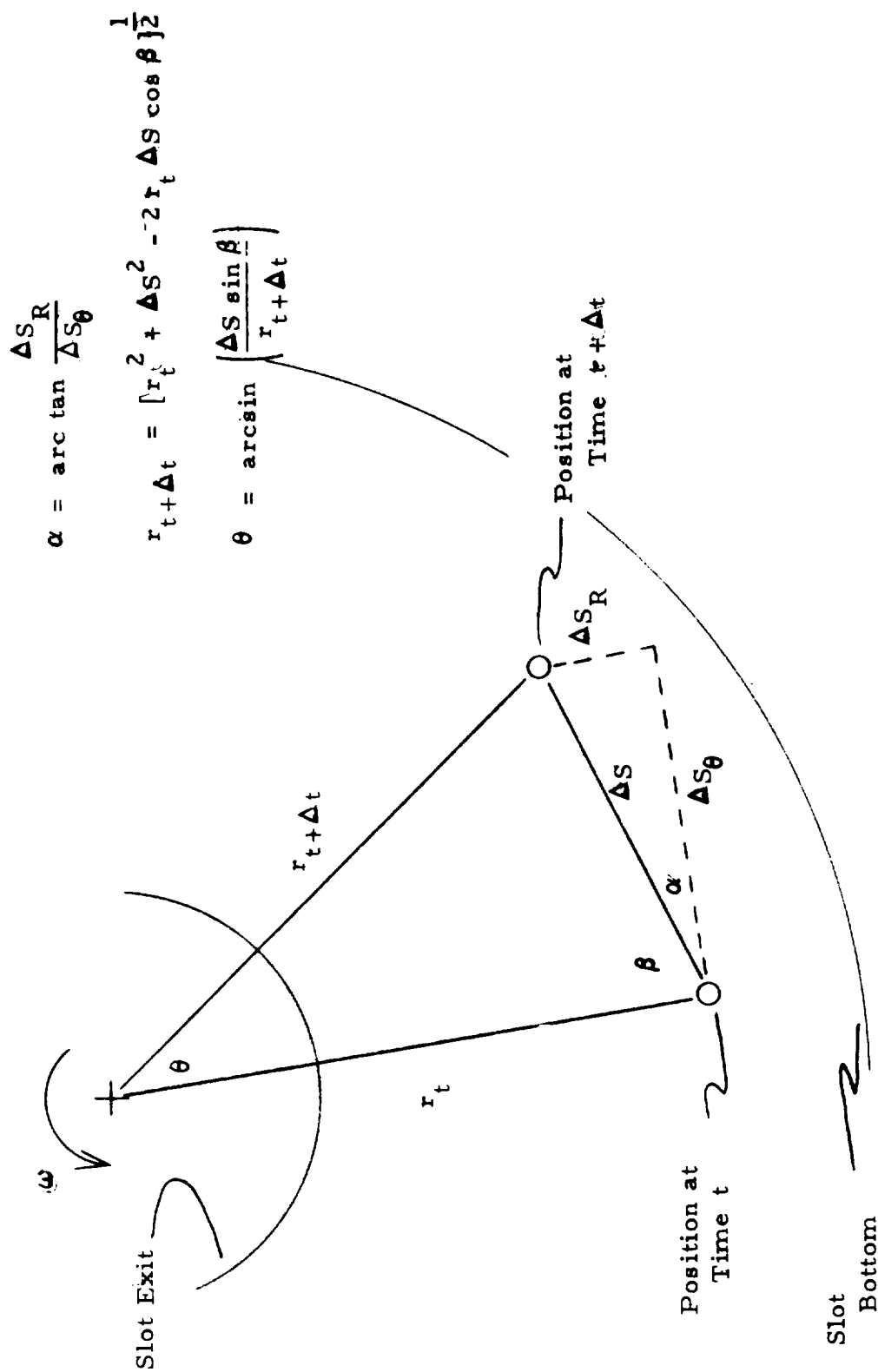


Figure 9. Schematic of Particle Displacement

where

$$\Delta S = (\Delta S_{\theta}^2 + \Delta S_R^2)^{1/2} \quad (44a)$$

$$\xi = 90 - \alpha \quad (44b)$$

$$\alpha = \arctan \frac{\Delta S_R}{\Delta S_{\theta}} \quad (44c)$$

and the angle rotated through

$$\theta_{t + \Delta t} = \arcsin \left(\frac{\Delta S \sin \xi}{r_{t + \Delta t}} \right) \quad (45)$$

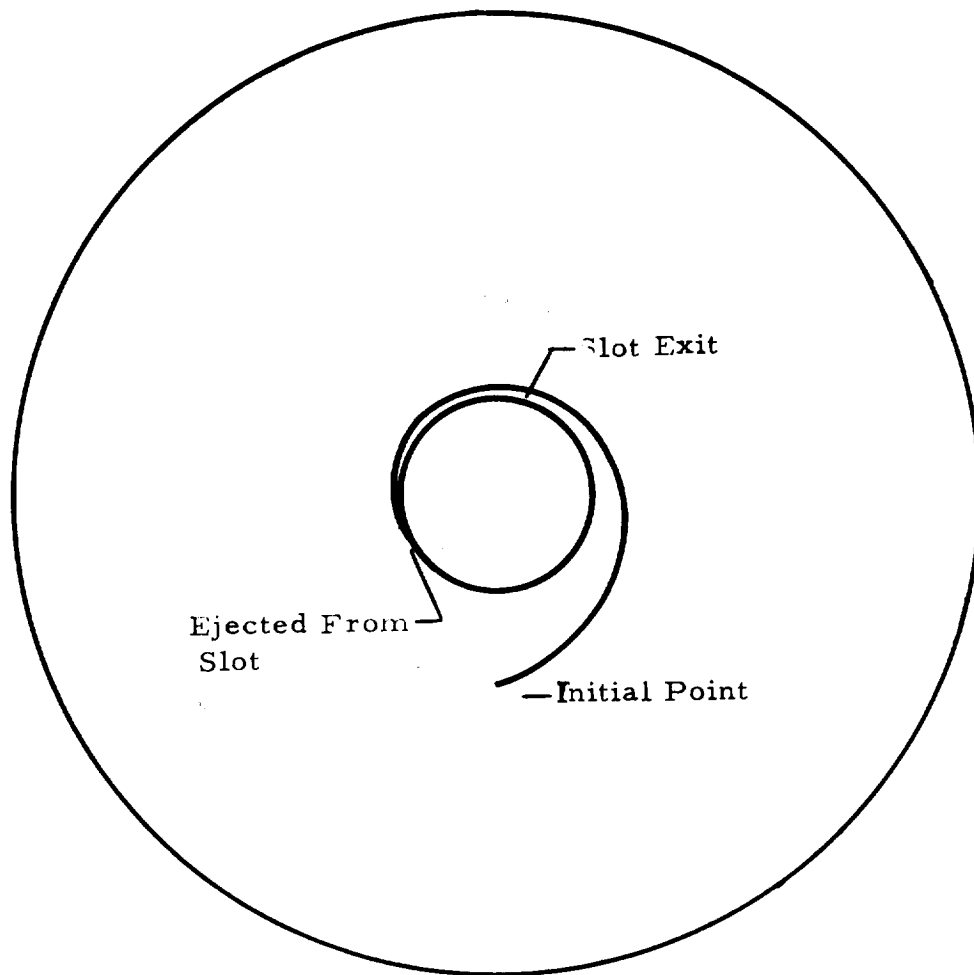
Equations (28) - (45) will "fly" the particle through the slot. At time $t = 0$, i.e., the time of particle evolution, the particle velocity is

$$v_{\theta \text{ part}} \Big|_{t=0} = r \omega \quad (46)$$

$$v_{R \text{ part}} \Big|_{t=0} = 0 \quad (47)$$

A computer program has been written which utilizes equations (28)-(45) with equations (46) and (47) as initial conditions. Figures 10 and 11 are typical trajectories of particles in a circumferential slot. The densities and diameters of the particles were chosen such that both trajectories are near the critical case.

Whether or not a particle is retained or ejected depends upon a number of variables: (1) spin rate, (2) particle density, (3) particle diameter, (4) slot geometry, and (5) propellant properties. Thus for a given motor configuration, propellant, and spin rate, critical particle diameter as a function of radial distance within the slot can be determined by using the flow field and particle trajectory analyses. The critical particle diameter is defined as the diameter of a particle that will barely impact the bottom of the slot. All particles smaller than the critical particle size will be ejected from the slot and all particles larger than the critical particle size will be retained within the slot.



Particle Properties

$$\rho_a = .1073 \text{ 0 lbm/in}^3$$

$$r_p = 75 \mu$$

Propellant Properties

$$T_t = 5000^\circ\text{R}$$

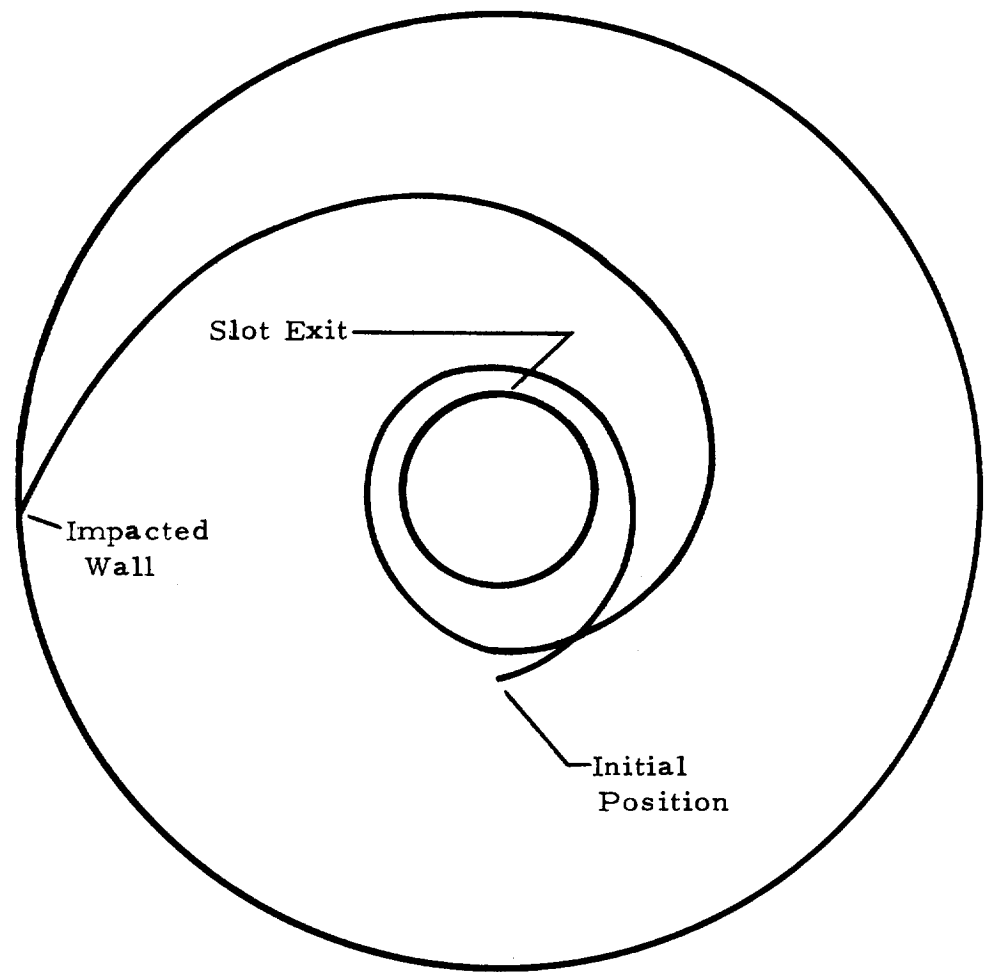
$$P_o = 1500 \text{ psi}$$

$$\rho_p = 0.063 \text{ lbm/in}^3$$

$$\Omega = 2000 \text{ RPM}$$

Initial Radius - 2.0 inches

Figure 11. Trajectory of a Particle Ejected From the Slot



Particle Properties

$$\rho_a = .0873 \text{ lbm/in}^3$$

$$r_p = 120 \mu$$

Propellant Properties

$$T_t = 5000^\circ\text{R}$$

$$P_o = 1500 \text{ psi}$$

$$\rho_p = 0.063 \text{ lbs/in}^3$$

$$\Omega = 2000 \text{ RPM}$$

$$\text{Initial Radius} = 2.0 \text{ in.}$$

Figure 10. Trajectory of a Particle Retained Within the Slot

Figure 12 illustrates the variation in critical particle size with spin rate and radial location at ejection. Before a quantitative estimate of the retained metal/metal oxide can be made, a particle distribution must be specified.

Crump (ref. 6) used high speed photography in an effort to ascertain the particle size and distribution of aluminum and aluminum oxide particles in the flame zone of composite propellants. He postulated that aluminum particles accumulated and agglomerated on the surface and were then burned in the flame zone. However, while this data is indicative of the mechanism involved, no inference about the size and distribution of metal/metal oxide particles outside the flame zone is possible. Although Crump's data is qualitatively useful, it provides no quantitative information about particle size and distribution except in the flame zone.

Once a distribution of metal/metal oxide within the slot is known, the fraction of mass (M/M_T) contained in particles less than the critical particle radius can be computed. Then:

$$\left(\frac{M}{M_T} \right) = f(\text{size, distribution}) \quad (48)$$

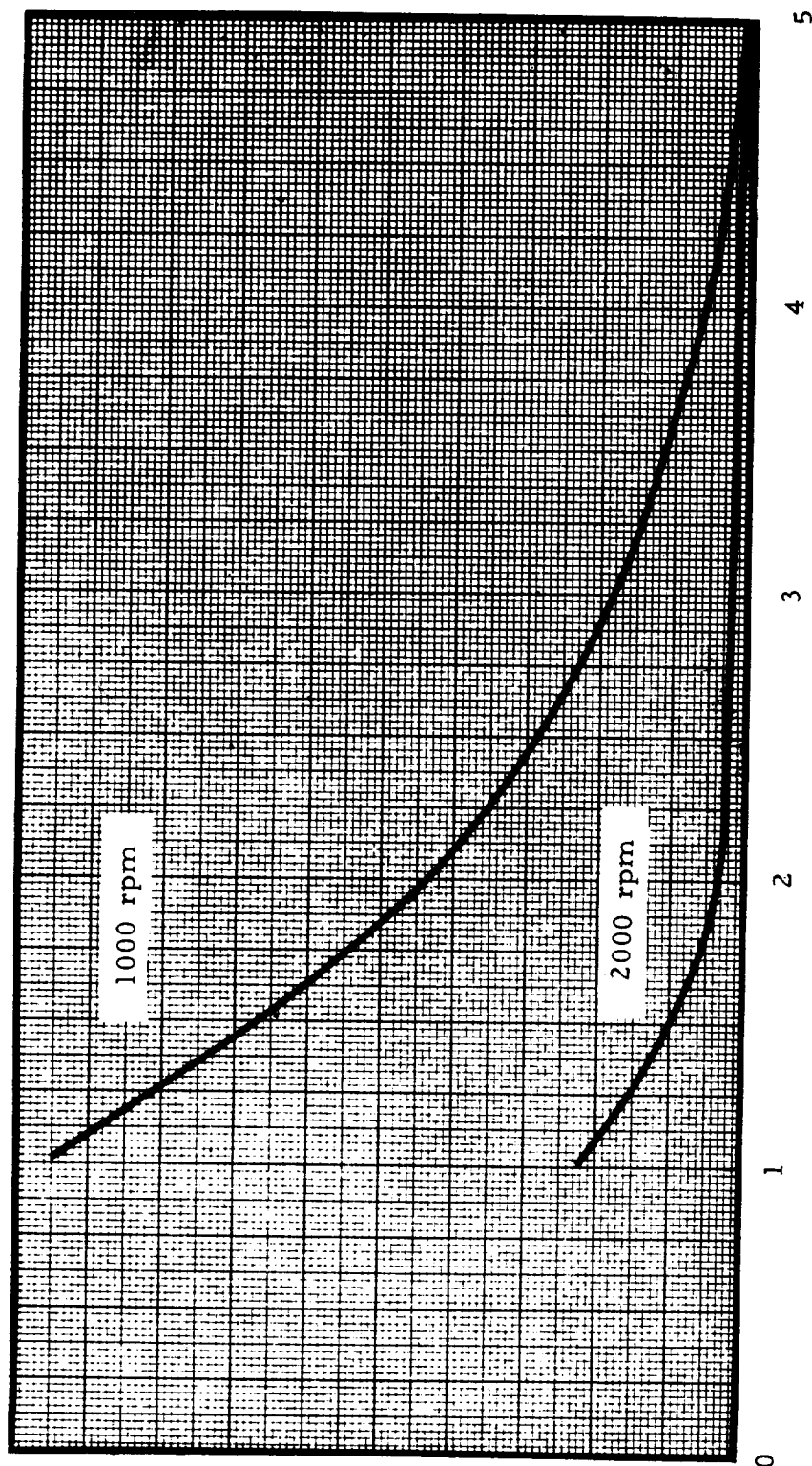
The amount of metal evolved is

$$d \dot{m}_{\text{metal}} = w_n \rho_p \alpha_t P^n d A_b \quad (49)$$

and the amount of metal retained within the slot becomes

$$d \dot{m}_r = \left(\frac{M}{M_T} \right) w_n \rho_p \alpha_t P^n d A_b \quad (50)$$

METAL/METAL OXIDE PARTICLE DIAMETER, microns



DISTANCE FROM MOTOR CENTERLINE, inches

Figure 12. Critical Particle Diameter for Two Spin Rates

The amount of metal/metal oxide that is retained from the sides of the slot is

$$\dot{m}_{r_s} = N \int_{r_e} \left[1 - \left(\frac{M}{M_T} \right) \right] w_n \rho_p \alpha_t P^n 2\pi r dr \quad (51)$$

The metal/metal oxide from the bottom of the slot that is retained is

$$\dot{m}_{r_b} = \left[1 - \left(\frac{M}{M_T} \right) \right] w_n \rho_p \alpha_t^2 P^n 2\pi r_b \quad (52)$$

The critical particle radius, r_{pc} , for the slot bottom is the same as the critical particle radius at $r = r_o$ since the flow field is one-dimensional in nature. That is

$$r_p) \text{ bottom} = r_p) \text{ side} \quad (53)$$

And the total metal/metal oxide retained within the slot is

$$\dot{m}_t = \dot{m}_{r_s} + \dot{m}_{r_b} \quad (54)$$

Slot-Port Flow Interaction

The preceding analysis developed a method of estimating the amount of metal/metal oxide that would be retained within the circumferential slot of a spinning rocket motor. In addition to the problem of metal retention, the ballistic effects of a circumferential slot in a spinning rocket motor are largely unknown. This portion of the analysis is directed toward developing a theoretical technique for predicting the ballistic effects of such a slot.

Reference 8 develops a technique for analyzing the ballistic effects of a circumferential slot in a non-spinning rocket motor. Figure 13 is a schematic of the analytical model considered. The slot flow, entering at right angles to the main axial flow, causes a separation region downstream of the slot. The two components can be considered as separate streams at the point of separation since there is very little mixing taking place within several port diameters of the slot. References 9 and 10 show that some discrete distance is needed before an appreciable amount of mixing takes place.

The theoretical analysis was directed at the region bounded by the control surface; namely, the port upstream of the slot, the exit of the slot, and the surface where the primary flow area is a minimum. For the region inside the control surface, the following was assumed:

1. The flow is steady, adiabatic, and inviscid.
2. The primary and secondary fluids are perfect gases with the same composition and total temperature.
3. There is no mass or heat exchange between the primary and secondary flows.
4. The flows at the inlet and exit of the control volume are quasi-one-dimensional with a specified radial density, pressure, and tangential velocity distribution.
5. The static pressure is continuous in the radial direction.
6. The effects of mass addition in the separation region are negligible.
7. Angular momentum is conserved.

Within the framework of the above assumptions the governing equations become

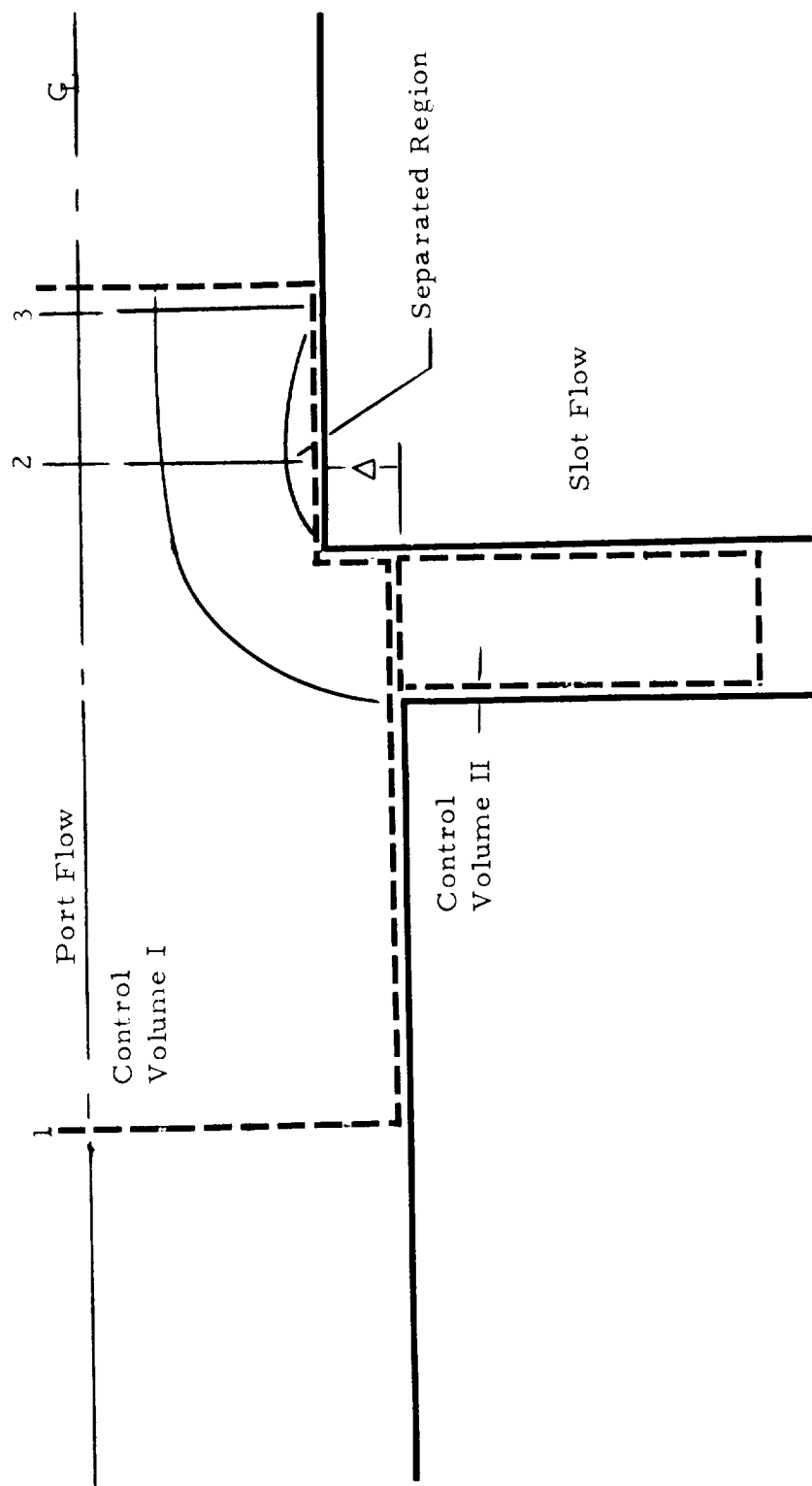


Figure 13. Schematic Diagram of the Slot

CONSERVATION OF MASS:

$$\dot{m}_p = \int_{A_{ip}} v_{1p} \rho_{1p} dA_{1p} = \int_{A_{2p}} v_{2p} \rho_{2p} dA_{2p} \quad (53)$$

$$\dot{m}_s = \int_{A_{1s}} v_{1s} \rho_{1s} dA_{1s} = \int_{A_{2s}} v_{2s} \rho_{2s} dA_{2s} \quad (54)$$

CONSERVATION OF LINEAR MOMENTUM:

$$\int_{A_1} p_{1p} dA_1 - \int_{A_2} p_2 dA_2 = \int_{A_{2p}} \rho_{2p} v_{2p}^2 dA_{2p} - \int_{A_1} \rho_{1p} v_{1p}^2 dA_{1p} + \int_{A_{2s}} \rho_{2s} v_{2s}^2 dA_{2s} \quad (55)$$

ENERGY:

$$T_t = \frac{v^2}{2C_p} + T \quad (56)$$

STATE:

$$p = \rho RT \quad (57)$$

CONSERVATION OF ANGULAR MOMENTUM

$$MM_s = \int_{A_s} \rho_{1s} v_{\theta 1s} v_{R 1s} r dA_{1s} = \int_{A_{2s}} \rho_{2s} v_{\theta 2s} v_{2s} r dA_{2s} \quad (58)$$

$$MM_p = \int_{A_{1p}} \rho_{1p} v_{\theta 1p} v_{1p} r dA_{1p} = \int_{A_{2p}} \rho_{2p} v_{\theta 2p} v_{2p} r dA_{2p} \quad (59)$$

and considering the flow processes from station 1 to station 2 to be isentropic for each stream

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2} \right)^{\frac{\gamma}{\gamma-1}} \quad (60)$$

The energy equation written in terms of the local Mach number becomes

$$T_f = T \left(1 + \frac{\gamma-1}{2} M^2 \right) \quad (61)$$

$$\text{where} \quad M^2 = \frac{V_\theta^2 + V_z^2}{C^2} \quad (62)$$

and for the two streams

$$\frac{T_s}{T_p} = \left[\frac{1 + \left(\frac{\gamma-1}{2} \right) M_p^2}{1 + \left(\frac{\gamma-1}{2} \right) M_s^2} \right] \quad (63)$$

The radial pressure and density gradient resulting from the tangential velocity distribution is (ref. 11)

$$P = P_o \left\{ 1 - \frac{\gamma-1}{2} M_\theta^2 \left[\left(\frac{r_o}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma}{\gamma-1}} \quad (64)$$

$$\rho = \rho_o \left\{ 1 - \frac{\gamma-1}{2} M_\theta^2 \left[\left(\frac{r_o}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma-1}{\gamma}} \quad (65)$$

A potential vortex also gives rise to a "core" region because of the pressure drop as the tangential velocity is increased radially. Eventually with a compressible fluid a zero pressure is reached and the resulting core diameter is

$$r_c = r_o \left(\frac{\frac{\gamma-1}{2} M_\theta^2}{1 + \frac{\gamma-1}{2} M_\theta^2} \right)^{\frac{1}{2}} \quad (66)$$

Since the pressure and density are zero within the core, no flow either axial or tangential is possible within the core region.

As the schematic of the mathematical model indicates (figure 13) the two streams are considered to remain intact - at least to the point of the maximum contraction. Thus, it is necessary to resort to subscripts to denote the different streams at the various stations. The main axial stream is subscripted "p", the slot flow is subscripted "s". The station is identified as "1", the flow before the interaction; "2", the flow at the maximum separation point; or "3" the flow at the re-attachment point. For example, the tangential velocity of the slot gases at station 2 is denoted by $V_{\theta 2s}$ and the axial velocity by V_{Z2s} . Conditions at the interface between the two streams are subscripted with an "i"; for example, the radius of the interface is denoted by r_{i2} .

The conditions at station 1 are assumed to be known. Therefore,

$\dot{m}_s, \dot{m}_p, \rho_p, \rho_{1s}, P_1, V_{\theta 1p}, V_{r1s}, V_{\theta 1s}, V_{Z1p}, T_{1p}, T_{1s}, MM_s$ and MM_p are known.

Mass conservation at station 2 becomes

$$\dot{m}_p = \int_{r_{i2}}^{r_{c2}} \rho_{o21} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta 2p}^2 \left[\left(\frac{r_{c2}}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma-1}{\gamma}} V_{2p} 2\pi r dr \quad (67)$$

$$\dot{m}_s = \int_{r_{o2}}^{r_{i2}} \rho_{2s} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta 2s}^2 \left[\left(\frac{r_{o2}}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma-1}{\gamma}} V_{2s} 2\pi r dr \quad (68)$$

The components of the angular momentum are

$$MM_s = \int_{r_{o2}}^{r_{i2}} \rho_{2s} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta 2s}^2 \left[\left(\frac{r_{o2}}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma-1}{\gamma}} K_2 V_{2s} 2\pi dr \quad (69)$$

$$MM_p = \int_{r_{i2}}^{r_{c2}} \rho_{12p} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta 2p}^2 \left[\left(\frac{r_{i2}}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma-1}{\gamma}} K_1 V_{2p} 2\pi dr \quad (70)$$

The pressure at the interface is

$$P_{i2p} = P_{2s} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta 2s}^2 \left[\left(\frac{r_{o2}}{r_{i2}} \right)^2 - 1 \right] \right\}^{\frac{\gamma}{\gamma-1}} \quad (71)$$

Then the radial pressure variation for the axial stream becomes (since pressure continuity is assumed)

$$P = P_{i2p} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta 2p}^2 \left[\left(\frac{r_{i2}}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma}{\gamma-1}} \quad (72)$$

and the corresponding "core" radius

$$r_{c2} = r_{i2} \left[\frac{\frac{\gamma-1}{2} M_{\theta 2p}^2}{1 + \frac{\gamma-1}{2} M_{\theta 2p}^2} \right]^{\frac{1}{2}} \quad (73)$$

Equations 60 through 73 constitute the governing equations for the slot-port interaction. The integration of the pressure terms in the momentum equation must be done using the radial pressure distributions. For example:

$$\int_{A_{lp}}^{P_{ip}} dA_{ip} = \int_{r_{ol}}^{r_{cl}} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta_{lp}}^2 \left[\left(\frac{r_{ol}}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma}{\gamma-1}} 2\pi r dr \quad (74)$$

The remaining pressure integrations are carried out in an analogous manner.

A closed form solution to the interaction problem was not possible, so an iteration procedure was developed, and a computer program that will solve the system of equations has been written. A block diagram of the iteration technique is presented in figure 14.

Utilization of this technique permits the flow conditions and properties at station 2 to be computed from the flow at station 1.

As the schematic diagram (figure 13) shows, the flow will expand to fill the port area at some point downstream of the interaction point. The specification of the flow field at this point will complete the interaction. Water table studies have shown that for circumferential slots in conventional (non-spinning) rocket motors the pressure recovery across the interaction zone is small. Thus to a good approximation $P_2 = P_3$. Conservation of mass and angular momentum become

$$\begin{aligned} \dot{m}_3 &= \dot{m}_s + \dot{m}_p \\ &= \int_{r_{o3}}^{r_{c3}} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta_3}^2 \left[\left(\frac{r_{o3}}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma-1}{\gamma}} V_3 2\pi r dr \quad \text{and} \quad (75) \end{aligned}$$

$$\begin{aligned} MM_3 &= MM_s + MM_p \\ &= \int_{r_{o3}}^{r_{c3}} \left\{ 1 - \frac{\gamma-1}{2} M_{\theta_3}^2 \left[\left(\frac{r_{o3}}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma-1}{\gamma}} K_3 V_3 2\pi r dr \quad (76) \end{aligned}$$

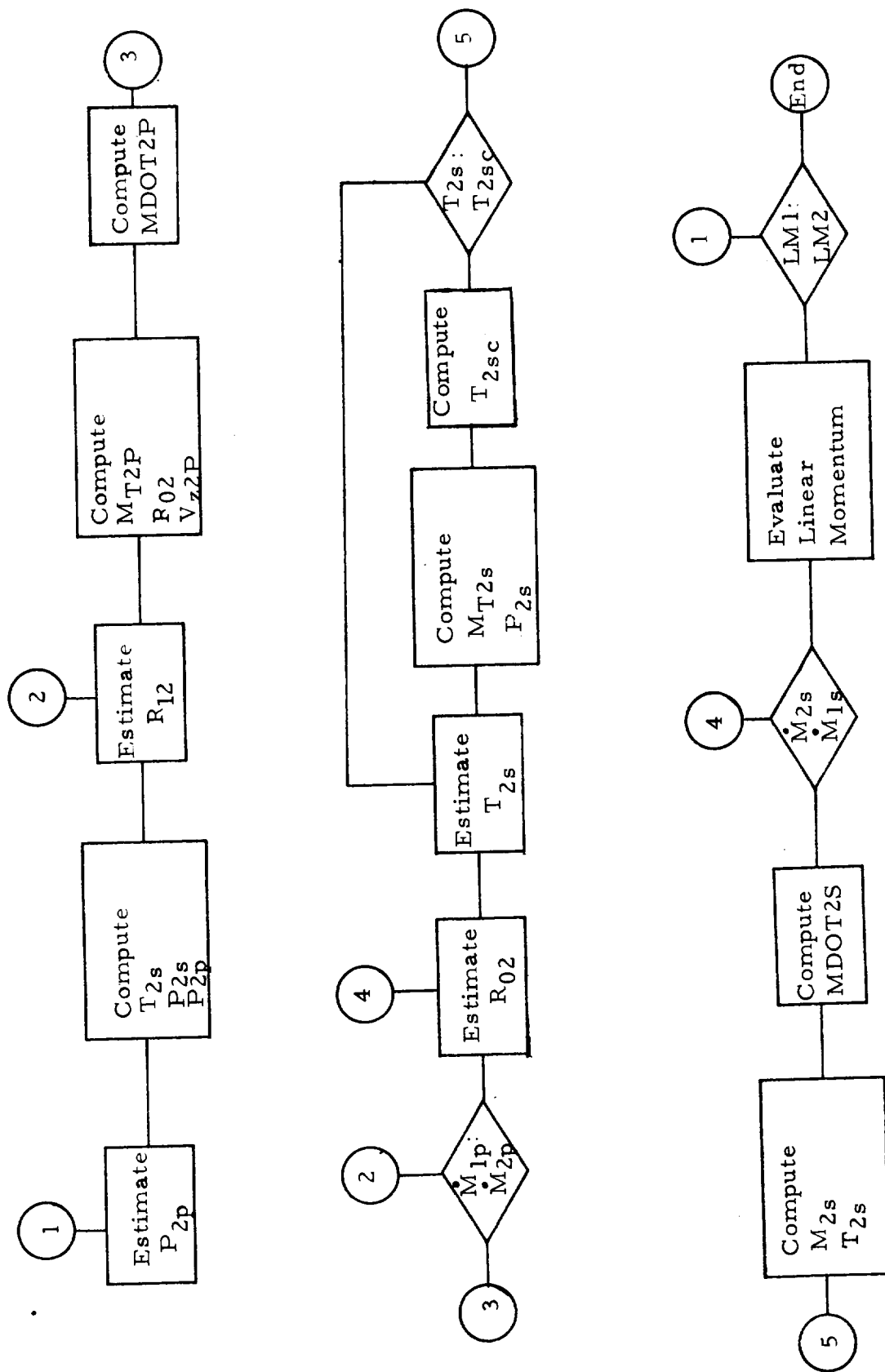


Figure 14. . Block Diagram for Slot-Port Flow Interaction Technique

The total temperature will remain constant

$$T_3 = \frac{T_t}{1 + \frac{\gamma-1}{2} M_3^2} \quad (77)$$

and the core radius at station 3 is

$$r_{c3} = r_{o3} \left[\frac{\frac{\gamma-1}{2} M_{\theta 3}^2}{1 + \frac{\gamma-1}{2} M_{\theta 3}^2} \right]^{\frac{1}{2}} \quad (78)$$

where:

$$M_{\theta 3} = \frac{K_3}{r_{o3} \sqrt{\gamma R T_3}} \quad (79)$$

Equations 75 through 79 will yield the flow conditions at station 3. The ballistics effect of a circumferential slot can be calculated by using the preceding analysis.

By using the analysis developed for the slot-port flow interaction in conjunction with the equilibrium-at-fixed-time computer program (see Appendix A) a computer program capable of predicting the ballistic effects of circumferential slots was generated. The governing differential equations for a swirling flow are integrated down the propellant grain using a fourth-order Runge-Kutta numerical procedure. At the last integration increment before the slot, the mass discharge and moment of momentum, as well as the properties of the flow, are stored and the slot routine called. The geometry of the slot and the propellant properties are input, and the slot flow field is computed using equations 5 through 26. At the slot exit, mass discharge and moment of momentum are computed and stored with the state variables of the flow. The mass discharge, moment of momentum, and flow variables of the primary (axial) and slot flows constitute the requisite information needed to compute the slot-port flow interaction. The development of the slot-port flow interaction technique is presented using equations 53 through 79.

The slot-port interaction yields the pressure aft of the slot as well as the value of the vortex strength (K) necessary to conserve moment of momentum. This information is returned to the main program and the integration of the swirl ballistic equations continued to the end of the grain. A mass discharge/mass generated criterion is used to establish the equilibrium operating pressure of the motor.

A parametric study was made using the generated computer program to ascertain the effect of spin on motor performance. The study was made with a CP (cylindrical port) grain 29 inches long and 4 inches in port diameter. Slot depth was varied from 3 to 4 inches and slot location of 25 inches to 14 inches from the head-end of the motor. A constant slot width of one inch was considered. Table I is a summary of the results of the parametric study. The results of this study imply that spin-induced effects of circumferential slots are negligible. Regardless of the spin rate, the pressure drop across the slot is approximately the same. For the slot located in the aft-most position (25 inches) the effect of spin rate is to decrease slightly the pressure drop for the 3-inch-deep slot and increase slightly the pressure drop for the 4-inch-deep slot. These increases and decreases are so small that they can be neglected. The slot, when located near the grain midpoint (14 inches), again shows no appreciable effect with spin rate. Variations in slot depth exhibit the same trends.

Conclusions

The study of the effects of spin on the performance of motors with circumferential slots indicates that:

1. Metal/metal oxide particles can be retained within the slot.
2. The amount of metal/metal oxide retained depends upon the mean particle size and the particle distribution.
3. Because of limited information on the size and distribution of particles any attempt to predict the metal/metal oxide retained within a slot is qualitative in nature.
4. The pressure drop across circumferential slots is nearly independent of spin rate.
5. Regardless of the slot depth and location, spin effects on ballistic performance can be neglected for a first order analysis.

Also, two computer programs were generated. One is capable of computing the flow field of a circumferential slot in a spinning rocket motor and then, using the flow field results, predicting particle trajectories. The other is an internal ballistics program for predicting the effects of circumferential slots on gas flow within spinning rocket motors.

TABLE I
PARAMETRIC STUDY OF CIRCUMFERENTIAL SLOTS
IN SPINNING ROCKET MOTORS

Grain Length (in)	Throat Radius (in)	Slot Location* (in)	Head-End Pressure (psi)	Slot Depth (in)	Pressure Drop (psi)	Spin Rate (rpm)
29	1.05	25	734	3	16.66	1
29	1.05	25	735	3	16.65	500
29	1.05	25	735	3	16.62	1000
29	1.05	25	737	3	16.53	2000
29	1.05	25	1084	4	34.02	1
29	1.05	25	1085	4	33.97	500
29	1.05	25	1088	4	33.79	1000
29	.70	25	2835	4	20.43	1
29	.70	25	2842	4	20.51	500
29	1.05	14	1054	4	21.47	1
29	1.05	14	1055	4	21.49	500
29	1.05	14	1058	4	21.51	1000
29	1.05	14	1069	4	21.94	2000
29	1.05	7	1039	4	14.03	1
29	1.05	7	1041	4	14.57	500
29	1.05	7	1045	4	15.79	1000

$$\rho_p = 0.064 \text{ lbm/in}^3$$

$$\alpha_t = 0.12$$

$$n = 0.23$$

$$T_t = 5899 \text{ }^\circ\text{R}$$

$$C^* = 5169 \text{ ft/sec}$$

$$R_g = 57.554 \text{ ft-lbf/lbm }^\circ\text{R}$$

$$b = 1 \text{ in.}$$

$$\gamma = 1.147$$

* From the head-end of the motor.

SURFACE REGRESSION ANALYSIS

Introduction

In the absence of acceleration effects, the evolution of a burning surface may be determined by conventional techniques derived from Piobert's Law for solid propellant burning. The burning rate is assumed to be constant over the entire surface regardless of the complexity of the grain design and the surface is assumed to regress in a direction normal to the surface at every point at any instant of time. However, when an acceleration field is induced by motor spin, the burning rate becomes variant over the surface and regression occurs in a nonuniform manner. When acceleration effects are present, the burning rate is a function of both the magnitude and the direction of the acceleration vector with respect to the burning surface. Thus, for an internal burning surface, the burning rate would differ for every point on the surface.

The only exception is a surface defined by a circular arc with its center coincident with the axis of rotation. The burning rate would be constant over such a surface because every point would be subjected to the same magnitude of acceleration, since every point is the same distance from the axis of rotation, and the angle between the acceleration vector and the surface normal is identical for each point on the surface. Thus, a cylindrical port grain design would regress in a uniform manner with acceleration effects present although the rate of regression would be greater due to the enhanced burning rate. However, all other grain designs will regress nonuniformly due to the varying distance between points on the surface and the axis of rotation and varying angles between the acceleration vector and the surface normal for points on the surface.

A computerized technique for predicting the regression of an internal burning surface with radial acceleration effects has been developed. At any instant of time, the burning surface is described by a set of coordinates for points on the surface. A local burning rate for each point is calculated from an empirical burning rate function which was deduced from experimental data. The constants in this function may be adjusted to yield agreement between the calculated burning rate and the experimental data for a given propellant. All points are regressed normal to the surface over a small increment of time at burning rates which vary for each point to form a new set of coordinates which describe the surface at a later time. This process is repeated until the web burn time is achieved. The propellant grain volume at any time is determined from a

numerical integration of the set of coordinates describing the burning surface at that time. The mass generation rate is determined from a numerical differentiation of a table of values describing the grain volume as a function of time.

Burning Rate Function

The formulation of a burning rate function is first necessary in order to apply the method outlined above to predict the regression of a burning surface with acceleration effects. The burning rate must be determined as a function of the acceleration magnitude, a , and the angle between the acceleration vector and the surface normal, Φ .

The data of Anderson (ref. 12) indicate that the relationship between the burning rate ratio, the ratio of the burning rate with acceleration to the burning rate without acceleration, and the magnitude of acceleration normal to the surface is strongly influenced by the propellant composition. Aluminum content, aluminum particle size distribution, ammonium perchlorate particle size distribution, and binder type are some of the most important propellant characteristics which affect the amount of burning rate augmentation. However, Glick's theory (ref. 13), for acceleration effects on the burning rate of composite propellants and Anderson's data both reveal that the shape of curves describing burning rate ratio as a function of acceleration are similar for various propellants and that the burning rate ratio tends to approach an asymptotic value with increasing acceleration. The asymptotic value for the burning rate was found to be dependent on propellant formulation. Accordingly, a function was sought which could be used to determine the burning rate ratio at a given acceleration magnitude for a number of propellants by evaluating constants in the function from data for a particular propellant. A function of the following form satisfied the above criteria and closely approximated Anderson's data and Glick's theory.

$$r / r_o = 1 + \left[\frac{(r / r_o)_o a_o - 1}{\log_e (a_o + 1)} \right] \log_e (a + 1) \quad (1)$$

where

$$r / r_o = \frac{r_b a}{r_b a_o} = \text{burning rate ratio}$$

$$a_o = \text{a value for acceleration from data}$$

$$\text{and } r / r_o) a_o = \text{burning rate ratio for } a_o \text{ from data}$$

A comparison of this function with Anderson's data for a nonmetallized propellant is shown in figure 15.

$$\text{where } a_o = 800 \text{ g's}$$

$$\text{and } r / r_o) a_o = 1.314$$

This function can be easily altered to approximate burning rate data for another propellant by simply inserting a value for a_o along with the corresponding value of r / r_o from data for the propellant.

The effect of the angular orientation of the burning surface with respect to the acceleration vector was investigated experimentally by Northam (ref. 14). His results indicated that the acceleration effect on burning rate is a maximum for normal acceleration into the surface ($\Phi = 0^\circ$) and that the effect of acceleration was not discernable for angles of 30° , 60° , or 90° . This implies that the burning rate ratio is a maximum for $\Phi = 0^\circ$ and decreases to a value of 1.0 as Φ approaches 30° . Glick's theory predicts a critical angle above which there is no acceleration effect on burning rate and shows that the burning rate continuously decreases from a maximum value at $\Phi = 0$ to a value of 1.0 at the critical angle. He showed that the critical angle is a function of the asymptotic value of burning rate ratio, $r / r_o)_{\max}$. The approximate predicted range of the critical angle is 30° to 50° . The exact value would be dependent on the propellant in question.

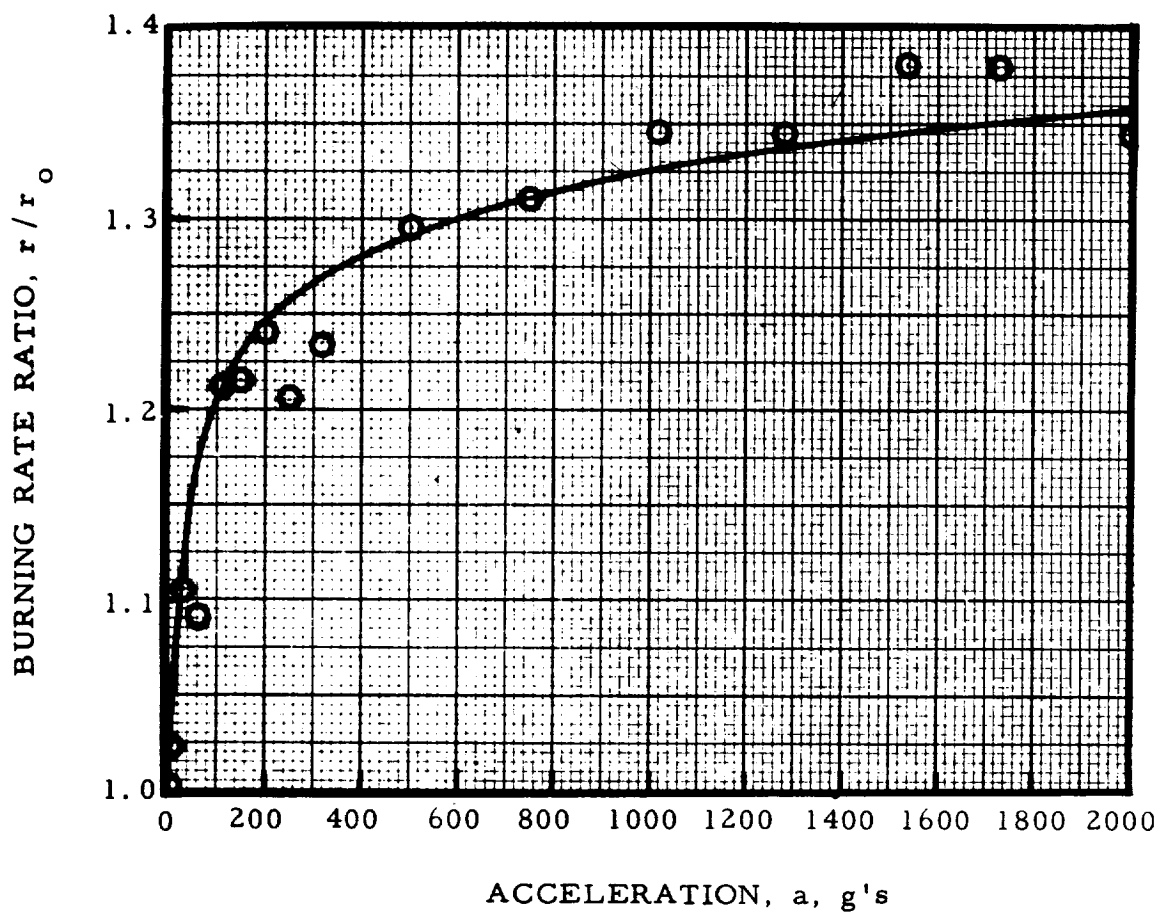


Figure 15. Correlation of Burning Rate Data from reference 12

Due to the uncertainty of the exact dependence of the burning rate ratio on the angle, Φ , equation (1) was modified to include angle effects as follows:

$$r / r_o = 1 + F(a) F(\Phi) \quad (2)$$

where

$$F(a) = \left[\frac{(r/r_o) a_o - 1}{\log_e (a_o + 1)} \right] \log_e (a + 1)$$

and

$$F_1(\Phi) = \cos(2\Phi)$$

for

$$0^\circ \leq \Phi \leq 45^\circ$$

The following exponential angle function was also postulated.

$$F_2(\Phi) = e^{-c\Phi^2}$$

where

$$c = 7.47 \text{ and } \Phi \text{ is expressed in radius}$$

for

$$0^\circ \leq \Phi \leq 45^\circ$$

Burning rate functions (equation (2)) containing $F_1(\Phi)$ and $F_2(\Phi)$ were compared with Glick's theory and with each other in figure 16. Acceleration is constant for this figure where

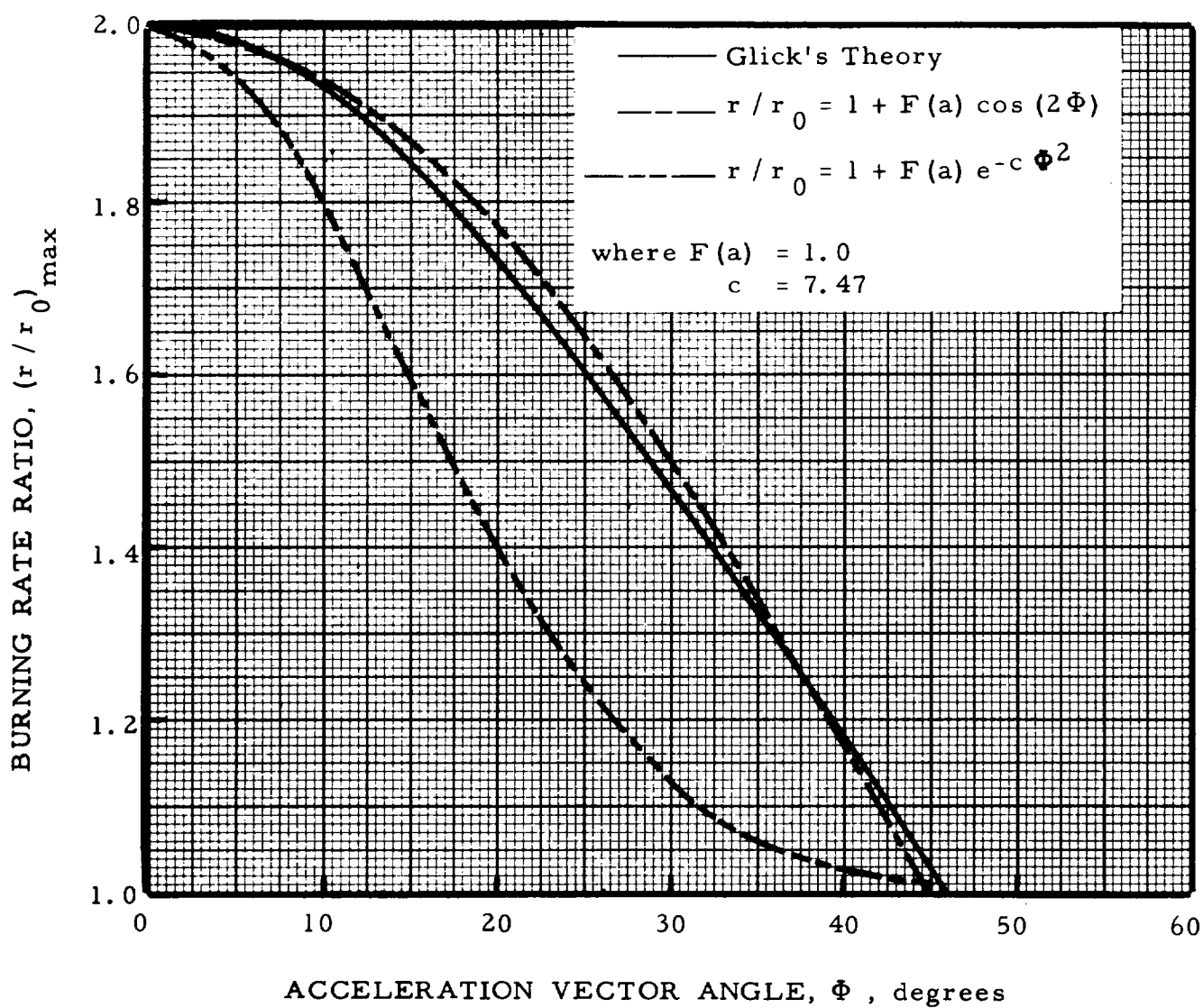


Figure 16. Comparison of Empirical Burning Rate Function with Theory

$$a = a_o$$

$$r / r_o) a_o = 2.0$$

$$\therefore F(a) = 1.0$$

The burning rate function containing $F_1(\Phi)$ agrees remarkable well with Glick's theory and was chosen in preference to the function containing $F_2(\Phi)$. The resulting equation for burning rate ratio which was used in the analysis is

$$r / r_o = 1 + \left[\frac{r / r_o) a_o - 1}{\log_e (a_o + 1)} \right] \log_e (a + 1) \cos (2 \Phi) \quad (3)$$

$$\text{for } 0 \leq \Phi \leq 45$$

Thus, an empirical burning rate function that includes both the effects of acceleration and orientation has been determined. While it may be an oversimplification of a very complex phenomenon, it has served as a model for the development of the grain regression technique and may be easily modified to include other variables or replaced with another function depending on the results of future experimental data.

Surface Regression Technique

An illustration of the nomenclature employed in the regression technique is shown in figure 17. The prediction of the location of the burning surface as a function of time is accomplished by performing calculations for each point in a set of coordinates which describe the surface at some time. The procedure is described in the following steps.

1. An input set of coordinates describing the grain surface at the initial time over an angle of π / N_s must be individually selected from a layout of the propellant grain design. The number of star points is N_s .
2. A second set of coordinates is calculated by interpolation from the input set such that the straight line distance between consecutive points is equal to λ , an input constant. This set of coordinates also describes the surface at the initial time.

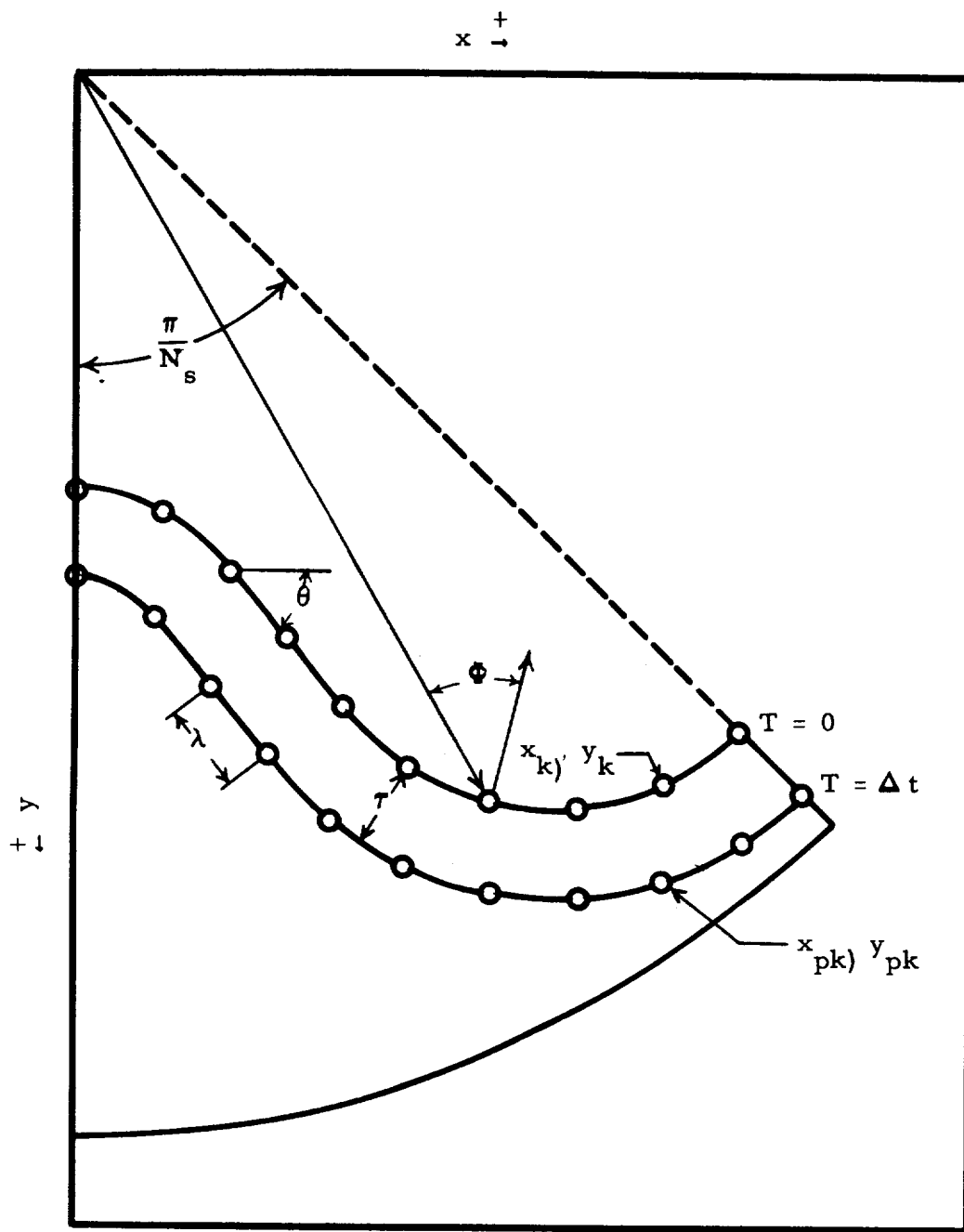


Figure 17. Nomenclature Schematic for Regression Analysis

3. The slope of the surface at each point is next determined by a numerical differentiation.
4. The angles θ and Φ are then calculated for each point from the following equations written for the kth point.

$$\theta_k = \arctan \left(\frac{dy}{dx} \right)_k \quad (4)$$

$$\Phi_k = \arctan (x_k / y_k) + \theta_k \quad (5)$$

5. The radial distance from the axis of rotation for the kth point is given by

$$R_k = (x_k^2 + y_k^2)^{1/2} \quad (6)$$

6. The local burning rate ratio is determined by the condition

$$r / r_{o)k} = 1.0$$

if Φ_k is greater than $\pi / 4$, or from equation (3) if Φ_k is less than $\pi / 4$.

7. The cross sectional area, A_g , of the grain at time, t , is then determined by a numerical integration over an area bounded by the grain boundary adjacent to the motor case and the x, y set of coordinates which describe the burning surface at time, t .

8. The motor port area is

$$A_o = \pi R_g^2 - A_g \quad (7)$$

9. The grain perimeter at time, t , is calculated by numerical techniques from the equation

$$S_g = \int_{x_1}^{x_k} \left[1 + (dy/dx)^2 \right]^{1/2} dx \quad (8)$$

10. The chamber pressure is then estimated from the relation

$$P = \left[\rho_p \alpha (r/r_o)_{av} A_s C^* / 32.174 A^* \right]^{1/1-n} \quad (9)$$

where

$(r/r_o)_{av}$ = average of values for r/r_o from step 6 for all points at time, t

and A_s = surface area ($S_g \times L_g + N_e A_g$), where N_e is the number of uninhibited grain ends

11. The local burning rate is calculated from the equation

$$r_k = (r/r_o)_k \alpha P^n \quad (10)$$

12. The mass of the grain at time, t , is

$$m_g = \rho_p L_g A_g \quad (11)$$

13. The mass generation rate is obtained from a numerical differentiation of a set of values for grain mass, m_g , as a function of time.

$$\dot{m}_g = (dm_g / dt)_t \quad (12)$$

14. The nozzle mass discharge rate is

$$\dot{m}_n = (32.174 A^* P) / C^* \quad (13)$$

15. The estimation of chamber pressure in step 10 is evaluated by checking for agreement of mass flow rates calculated in steps 13 and 14.

16. The distance burned normal to the surface in the time, Δt , is

$$\tau_k = r_k \Delta t. \quad (14)$$

17. The coordinates of the points which define the surface at the time $t + \Delta t$ can be calculated from the equations

$$x_{pk} = x_k - \tau_k \sin(\theta_k) \quad (15)$$

and

$$y_{pk} = y_k + \tau_k \cos(\theta_k). \quad (16)$$

18. The new grain length at time $t + \Delta t$ is

$$L_{g(t+\Delta t)} = L_{g_t} - \alpha P^n \Delta t N_e \quad (17)$$

19. The coordinates of the points at time $t + \Delta t$, x_p , y_p , which were generated in step 17 are not spaced equidistance and so are used to calculate another set of coordinates such that λ is the equidistance spacing as in step 2.

20. The remaining steps 3 through 19 are then repeated for this new set of coordinates at time $t + \Delta t$.

21. All above steps repeated for each succeeding time increment until the burning surface intersects the grain boundary adjacent to the motor case wall, i. e., until some R_k becomes greater than R_g .

Results of Regression Analysis

In order to check the validity of solutions obtained by the computerized regression technique, a cylindrical core grain geometry with a zero spin rate was chosen as an initial test case. For this case exact solutions for grain cross sectional area, grain perimeter, and mass generation rate are easily obtained for comparison with the computer program results.

The values for these three parameters calculated by the computer program were all within 0.27 percent of the exact values after a burn time of 8 seconds through a web thickness of 4 inches. This agreement verified the calculation techniques of the computer program which are the same regardless of the motor spin rate except for the actual calculation of the local burning rate.

Values for the local burning rate were checked independently at several spin rates and angular orientations with satisfactory results.

In order to determine the effect of radial acceleration on the surface regression of a typical star configuration, a star grain with neutral characteristics at zero spin was designed with the following parameters.

Grain diameter = 12.0 in.

Grain length = 50.0 in.

Web thickness = 1.5 in.

No. of star points = 6

Total burn time = 3.0 sec.

Burning rate ($\Omega = 0$) = 0.5 in/sec @ 2,000 psia

A solution for the surface regression of this star grain at a spin rate of zero is shown in figure 18. The accuracy of the solution was checked by a comparison of the results with values obtained from an existing ballistics computer program. This ballistics program contains closed form mathematical expressions for grain perimeter and motor port area and thus these values are exact. Table II presents the results of this comparison.

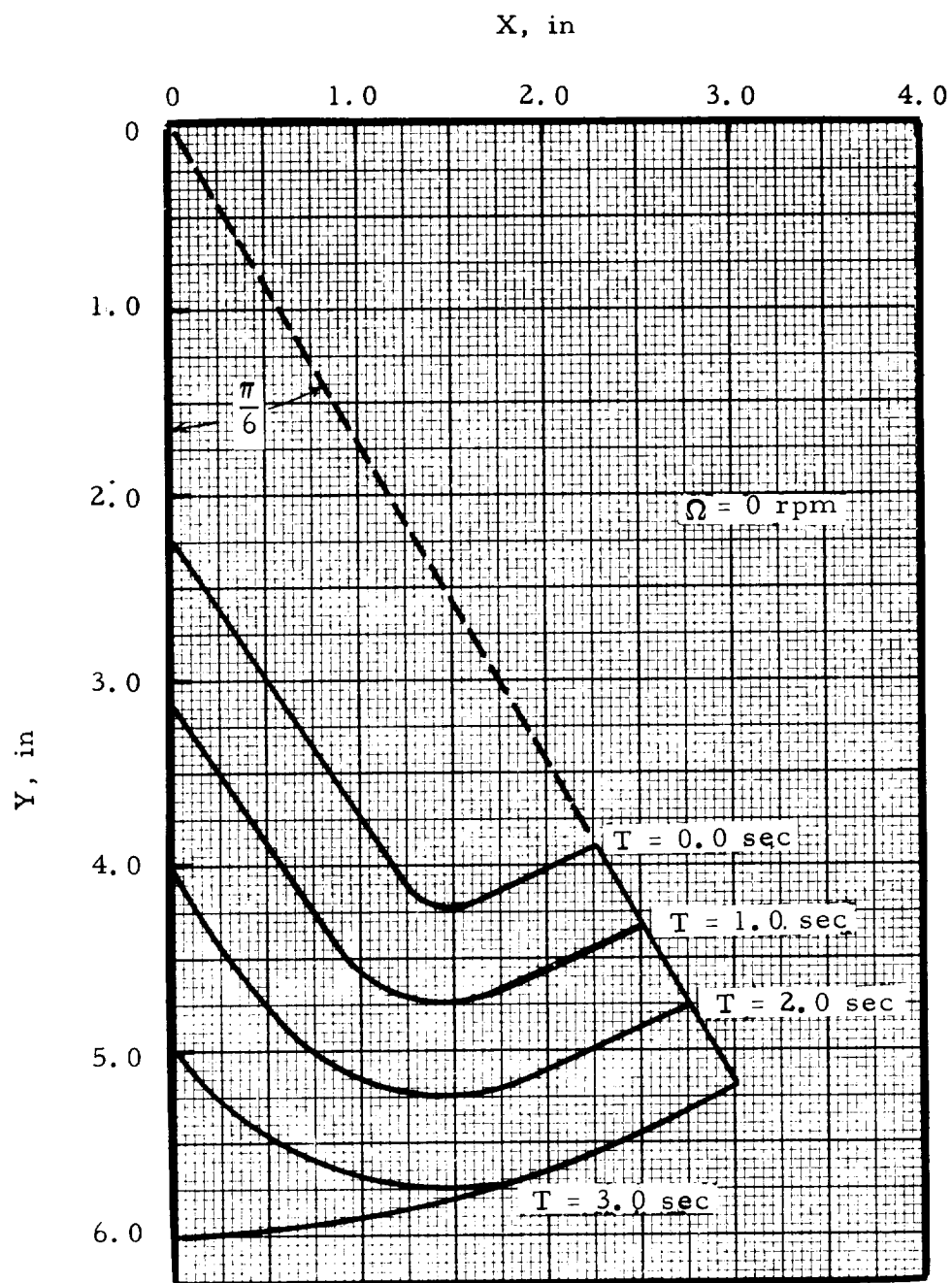


Figure 18. Effect of Radial Acceleration on Surface Regression of a Star Grain

TABLE II

COMPARISON OF ZERO SPIN SOLUTION WITH EXACT SOLUTION

<u>Time = 0 seconds</u>	<u>Perimeter (in.)</u>	<u>Port Area (in.²)</u>	<u>Mass Flow (lbm/sec)</u>	<u>Pressure (psia)</u>
Surface regression analysis	40.540	44.592	65.878	3979
Internal ballistics program	40.526	44.580	65.855	3981
<u>Time = 3.0 seconds</u>				
Surface regression analysis	40.469	105.37	65.748	3972
Internal ballistics program	40.526	105.37	65.854	3981

Solutions for the surface regression of this star grain at spin rates of 400 rpm and 1,000 rpm are shown in figures 19 and 20. The values for the constants $r / r_o a_o$ and a_o that were used in the burning rate function were 2.0 and 149.12, respectively. A spin rate of 1,000 rpm produces an average acceleration of approximately 149 g's on the web surface of this particular grain design which results in a burning rate ratio of approximately 2.0 on the web surface. The burning rate on the web surface at a spin rate of 400 rpm is approximately 1.64.

It may be noted from figures 19 and 20 that the burning rate on the sides of the star point is not affected by the radial acceleration due to the greater than 45° angle between the acceleration vector and the surface normal. However, the burning rate on the web surface is 1.64 and 2.0 times greater than the burning rate on the star point sides for the two figures, respectively. The progressively higher burning rates for the web surface in figures 18, 19, and 20 is also reflected by the web burn time which is decreased from 3.0 seconds for zero spin to values of 1.65 and 1.31 seconds for spin rates of 400 and 1,000 rpm, respectively.

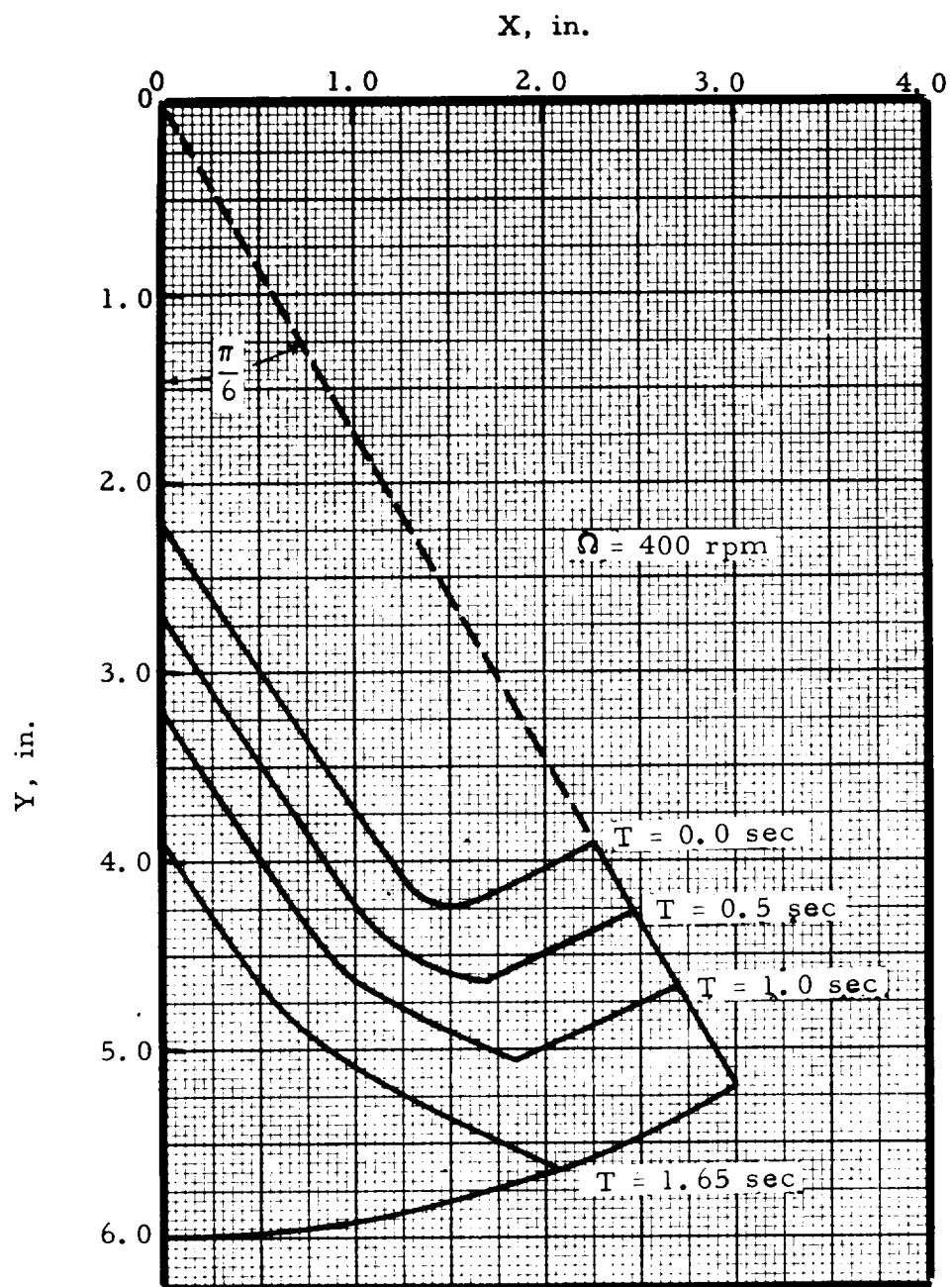


Figure 19. Effect of Radial Acceleration on Surface Regression of a Star Grain

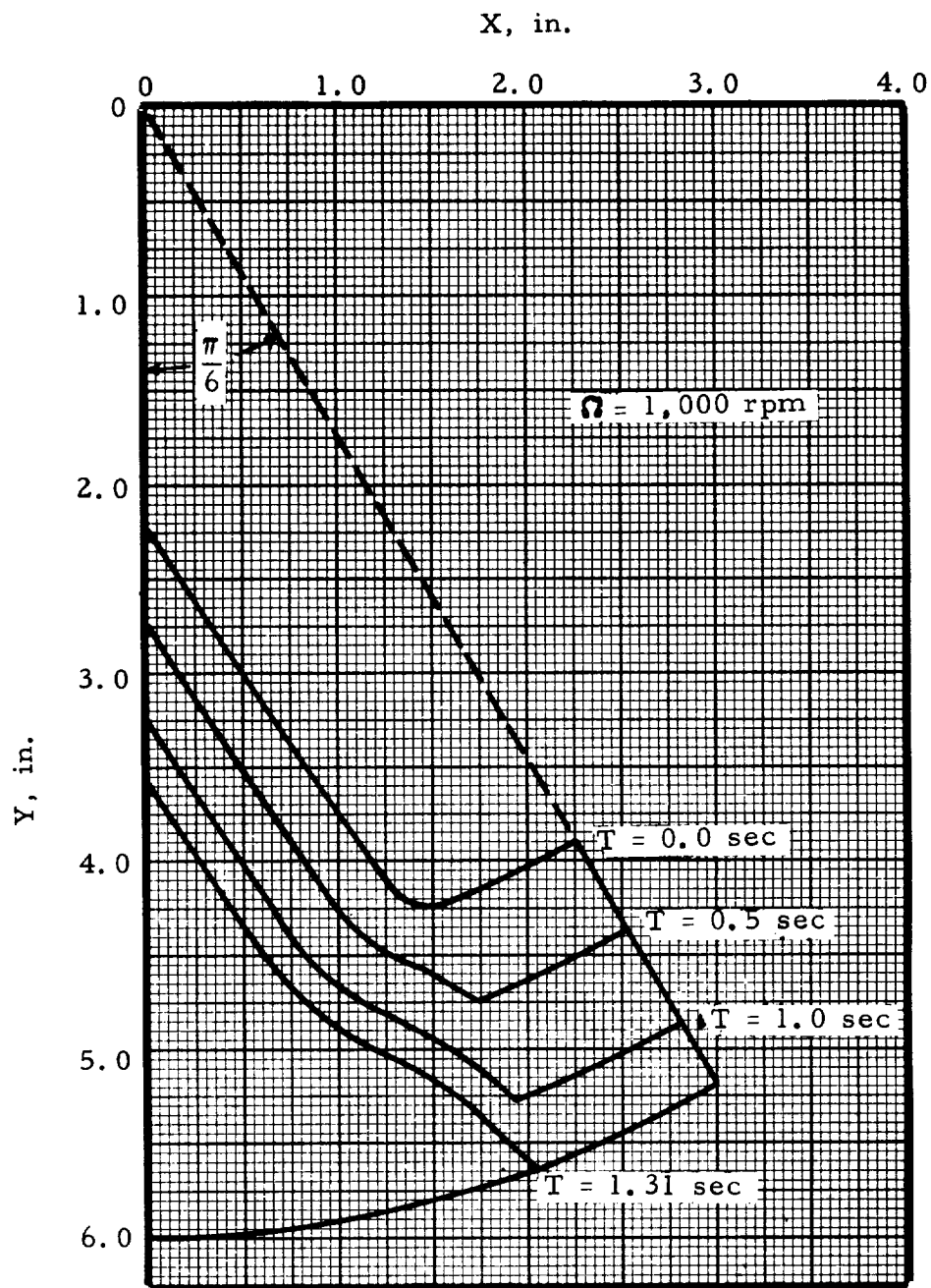


Figure 20. Effect of Radial Acceleration on Surface Regression of a Star Grain

The effect of spin rate on the pressure-time history of a motor with the star grain design described previously is shown in figure 21 for seven spin rates from zero to 1,000 rpm. The neutrality of the star grain at zero spin rate is evidenced by the pressure remaining constant at the design value of 2000 psia for the entire web burn time of 3.0 seconds. As the spin rate was increased, the average pressure level, as well as the rate of pressure rise also increased. However, web burn time decreased with increasing spin rate since the average burning rates increased with spin rate.

The pressure-time predictions are for web burn times only and do not consider ignition or tail-off transients. Also, the effects of nozzle plugging are not included since a one-dimensional flow equation is used to calculate the mass discharge rate through the nozzle. Thus, the increases observed in pressure during web burn time are due solely to acceleration induced effects on burning rate and grain regression.

Conclusions

The following conclusions may be drawn from the preceeding surface regression analysis.

1. A computerized surface regression technique for predicting the evolution of a burning surface under the influence of a radial acceleration field has been successfully developed.
2. This analysis predicts that the specific effects of spin rate on surface regression are (1) the non-uniform surface evolution of star grains, (2) increasing motor pressures with spin rate, and (3) increasing progressiveness with spin rate of pressure traces for star grains. These effects, of course, were expected and agree with experimentally observed trends.
3. A comparison of the surface evolution history for an actual motor with a prediction of the surface evolution history for the same motor should be made to verify the analysis.

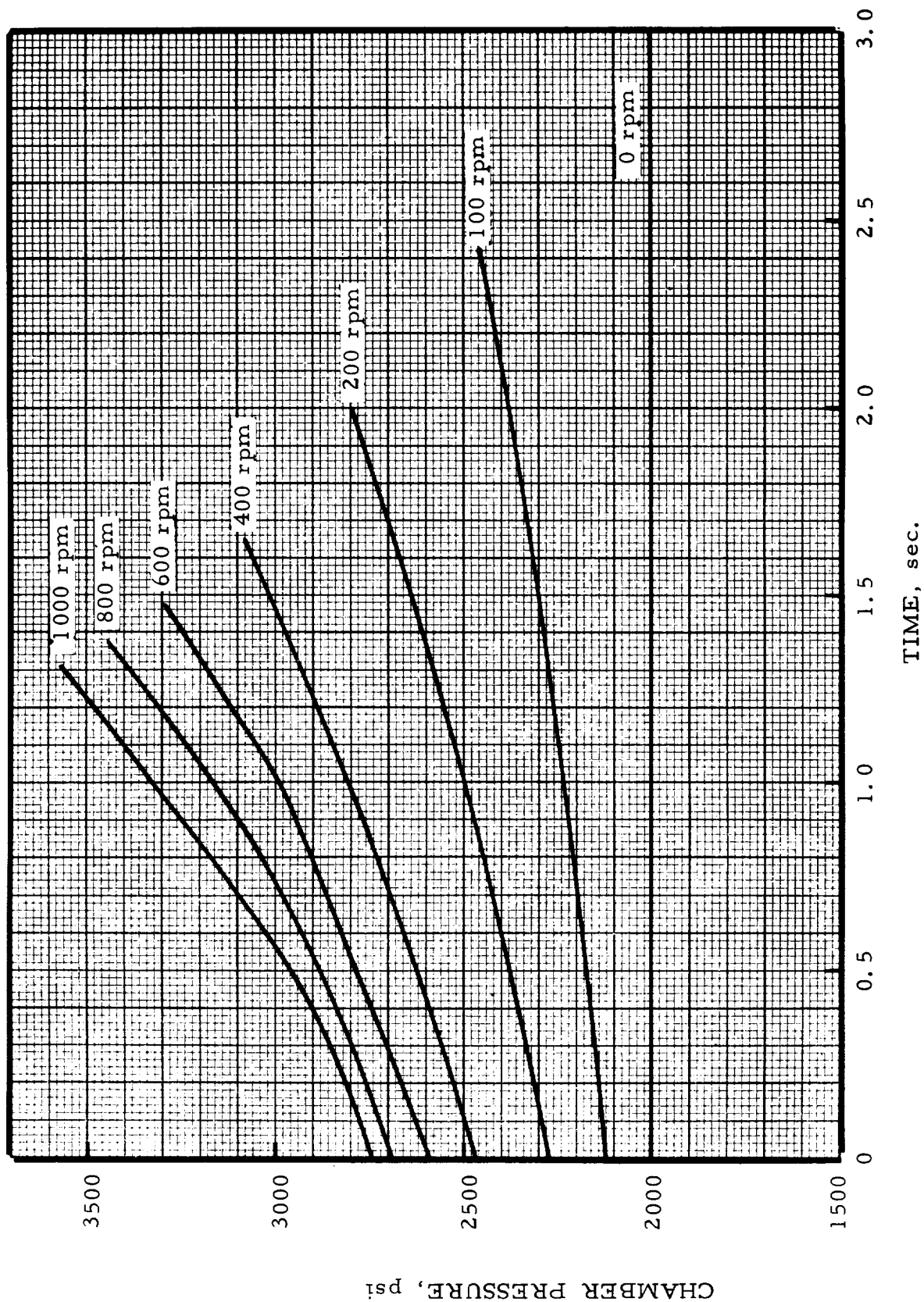


Figure 21. The Effect of Spin Rate on Pressure-Time History

THEORETICAL HEAT TRANSFER STUDY

Nozzle Heat Transfer

Introduction. - The purpose of this study was to develop a computerized analysis for predicting convective heat transfer in the nozzle of a spinning rocket motor and to determine the effects of spin rate on nozzle heat transfer.

Two technical approaches were considered: (1) a boundary layer approach, and (2) a semi-empirical approach. The current boundary layer theories for swirling flow in a conical nozzle are discussed and the procedure for developing a boundary layer analysis for swirling flow in a spinning rocket motor nozzle are presented in Appendix B. It was anticipated that the extension of an existing boundary layer analysis by employing a form of Reynold's analogy would yield a method whereby the heat transfer coefficients could be calculated along the nozzle wall. However, the procedure outlined for developing an analysis for a turbulent, compressible boundary layer on a spinning nozzle wall with swirling freestream flow was judged to yield, at best, only a rough approximation to the solution due to the assumptions necessary for a solution as discussed in Appendix B. Consequently, an alternate approach to an analysis of nozzle heat transfer based on a semi-empirical method was developed. The advantages of this approach are shown in the following section by comparing the current integral boundary layer methods with the semi-empirical methods for one-dimensional flow in a nozzle.

Approach justification. - Several analyses of the convective heat transfer in convergent-divergent nozzles with one-dimensional flow have been published. These analyses can be categorized as either (a) integral analyses, or (b) semi-empirical correlations.

The integral analyses are approximate in nature since the integral equations themselves are approximate. This follows from the fact that in the derivation of these equations, the usual simplifying boundary layer assumptions are made. Furthermore, a relationship between the momentum and displacement thicknesses, a relationship between the flow characteristics and the local wall shearing stress, and the velocity and temperature profiles all must be assumed. The main value of the integral analyses is that they can be applied to a wide range of geometries. Also, the boundary layer parameters are by-products of the solution. Usually, however, either heat

transfer or drag calculations are the desired end result and the boundary layer parameters are only the result of intermediate steps in the solution.

The semi-empirical correlations are obtained by first deducing an appropriate form for a correlation equation from dimensional arguments, theory, and/or experience and then adjusting values of experimental constants so that the results predicted by the correlation equation agree with experimental results. The main advantages of the semi-empirical methods is their closed form and adaptability to rapid calculations without a sacrifice in accuracy for specific applications.

A comparison of the results of the integral methods of Bartz (ref. 15) and Sibulkin (ref. 16) with results from semi-empirical methods and with experimental data has been made by Rose (ref. 17) for a conical converging-diverging nozzle. The results indicate no clear-cut advantage to either method. Moreover, both techniques gave good agreement with the experimental results. In reference 18 a comparison is made between nozzle heat transfer coefficients calculated by employing Bartz's turbulent boundary layer method (ref. 15) and coefficients calculated from Bartz's semi-empirical method (ref. 18). The agreement is remarkable considering the difference in complexity of the calculation techniques of the two methods. Also, both methods showed reasonable agreement with the available experimental data.

The foregoing discussion has served to compare the integral boundary layer methods with the semi-empirical methods for calculating heat transfer coefficients in nozzles with one-dimensional flow and to point out the advantages of the former approach. A semi-empirical approach to a heat transfer analysis for swirling flow in a spinning nozzle would have the same advantages especially since there is more uncertainty regarding the validity of the assumptions necessary to obtain a boundary layer solution for the swirling three-dimensional boundary layer as discussed in Appendix B. Also there is an apparent lack of friction factor data and correlations for flow conditions similar to those in a spinning nozzle with internal swirling flow. Hence, it is expected that the accuracy of results from the semi-empirical approach described below will be comparable to or exceed the accuracy of a boundary layer technique. Of course, the boundary layer thicknesses will not be determined with the semi-empirical approach, but these parameters are at most of secondary interest to the current problem of estimating heat transfer rates in spinning motor nozzles.

Approach. - The approach to a method to determine heat transfer coefficients along the nozzle wall of a spinning motor consists of applying the Bartz analogy (ref. 18) to swirling flow in a spinning nozzle and employing Mager's analysis (ref. 11) to obtain local density, axial velocity, and

tangential velocity of the gases along the nozzle wall. In reference 18 Bartz made the observation, based on results from his turbulent boundary layer calculation methods and experimental data, that the mass flow rate per unit area or the mass velocity, ρU , is the dominant factor governing heat transfer coefficients and that boundary layer parameters exert only a secondary effect. Thus he suggested an equation of the form

$$h \sim (\rho U)^m \quad (1)$$

This relationship was then put into the nondimensional form

$$Nu = C R_e^m Pr^n \quad (2)$$

although the assumption of fully developed pipe flow had not been made. The correct value for "m" can be shown to be 0.8 and a value for "n" of 0.4 was arbitrarily selected. Equation (2) was solved for "h" to give

$$h = \frac{C}{D^{.2}} \left(\frac{\mu^{.2} C_p}{P_r^{.6}} \right) (\rho U)^{.8} \sigma \quad (3)$$

where σ = factor containing corrections for property variations across the boundary layer.

Equation (3) was further developed in reference 18 by employing the one-dimensional isentropic flow equations to evaluate the mass velocity, ρU , in terms of C^* and A/A^* .

A recent revision of the turbulent boundary layer analysis of Bartz is described in reference 19. The method developed by this analysis may be applied to unusual nozzles with two-dimensional flow fields by using Mach numbers near the wall as input for the freestream conditions instead of Mach numbers based on one-dimensional flow. Therefore, it is postulated that equation (3) may be used to predict heat transfer coefficients for swirling flow in a conventional converging-diverging nozzle if local values of mass velocity at the wall are used in the equation. The density, which has a strong radial gradient for swirling flow, must be evaluated at the wall and the velocity must be the total velocity relative to the wall, V . The total relative velocity at the wall for swirling flow is given by

$$V^2 = W^2 + U^2 \quad (4)$$

where W = tangential gas velocity at the wall relative to the wall
and U = axial gas velocity at the wall relative to the wall

The tangential gas velocity for a potential vortex flow field is simply

$$w_g = \frac{K}{R_w} \quad (5)$$

The constant, K , is evaluated at the nozzle inlet. For a cylindrical port grain design

$$K = w_g R_o = (R_o \Omega) R_o$$

$$K = R_o^2 \Omega \quad (6)$$

The tangential velocity of the wall is simply

$$w_w = R_w \Omega \quad (7)$$

The relative tangential velocity between the gases and the wall is

$$W = w_g - w_w \quad (8)$$

Equations (5), (6), and (7) substituted into equation (8) yield the equation for the relative tangential velocity component.

$$W = \Omega / R_w (R_o^2 - R_w^2) \quad (9)$$

The form for the equation to predict heat transfer coefficients in a spinning nozzle with swirling flow is obtained by substituting equation (9) into equation (4) and using the total relative velocity, V , for the one-dimensional axial velocity in equation (3).

$$h = \frac{C}{D^{.2}} \left(\frac{\mu^{.2} c_p}{P_r^{.6}} \right) \rho^{.8} \left[\frac{\Omega^2}{R_w^2} (R_o^2 - R_w^2)^2 + U^2 \right]^{.4} \quad (10)$$

The gas density, ρ , and the axial component of velocity, U , are functions of local nozzle radius for a given spin rate and are determined by applying Mager's approximate solution of two-dimensional swirling potential flow through a nozzle (ref. 11). Although his analysis is primarily used to determine the effect of swirl on the mass flow rate and specific impulse, it may also be used to calculate the axial component of velocity along the nozzle as well as the local density at the wall. The calculation procedure for the axial velocity is not in closed form; and, due to the length of the procedure, it will not be described here. However, the procedure has been outlined in the following section.

Formulation. - The equations used in the computer program flow diagram for the nozzle heat transfer analysis are listed below. The computer program input information includes the propellant ballistic properties, motor operating conditions, and a table of axial and radial coordinates which describes the nozzle contour.

1. The vortex strength constant is derived by writing the equation for the potential vortex tangential velocity at the motor port radius

$$W = K/R_o = R_o \Omega$$

$$\therefore K = R_o^2 \Omega \quad (11)$$

2. The swirl strength is computed from equation (11) of reference 11.

$$\alpha^* = \frac{K}{R_o^* C_o} \sqrt{(\gamma - 1)/2} \quad (12)$$

3. The non-dimensional radius ratio, ξ_o^* , at the throat is determined from equation (20) of reference 11 by employing an iterative technique.

$$\alpha^* = \xi_o^* \left\{ 1 - \frac{1}{\frac{3-\gamma}{\gamma-1} + 2 \frac{[1 - (\xi_o^*)^2]^{1/(\gamma-1)}}{I^*}} \right\}^{1/2} \quad (13)$$

$$\text{where } I^* = \frac{2}{R^{*2}} \int_{r_o}^R \left[1 - \left(\frac{r_o}{r} \right)^2 \right]^{1/(\gamma-1)} r dr$$

4. The Mach number of the related flow is from equation (19) of reference 11.

$$M^* = \left\{ \frac{2}{\gamma-1} \left[\left(\frac{\xi_o^*}{\alpha^*} \right)^2 - 1 \right] \right\}^{1/2} \quad (14)$$

5. The non-dimensional flow is determined from equation (5c) of reference 11.

$$m = \frac{M^* I^*}{\left[1 + \frac{\gamma-1}{2} (M^*)^2 \right]^{(\gamma+1)/2 (\gamma-1)}} \quad (15)$$

6. The ratio of the actual nozzle mass flow with swirl to the one-dimensional mass flow may be developed from equation (26) of reference 11.

$$m_r = \frac{\dot{m}_2}{\dot{m}_1} = m \left(\frac{\gamma+1}{2} \right)^{\gamma+1/2 (r-1)} \quad (16)$$

7. The mass generation rate of the motor under spin conditions in terms of the motor equilibrium pressure at zero spin is given by

$$\dot{m}_{gen} = \frac{g_o A^*}{C^*} P_t \left(\frac{P_{ts}}{P_t} \right)^n \quad (17)$$

8. The nozzle mass discharge rate under spin conditions is from equation (16)

$$\dot{m}_2 = m_r \dot{m}_1 = m_r \frac{g_o P_{ts} A^*}{C^*} \quad (18)$$

9. The motor equilibrium pressure with spin is determined by equating equations (17) and (18).

$$P_{ts} = \frac{P_{ts}}{m_r^{1/1-n}} \quad (19)$$

10. The nozzle mass discharge rate is then

$$\dot{m}_2 = m_r \dot{m}_1 = m_r \frac{g_o P_{ts} A^*}{C^*} \quad (20)$$

11. The calculation of the local values of the non-dimensional radius ratio, ξ_o , and the Mach number of the related flow, M , is accomplished by a simultaneous numerical solution of equation (24)

$$\frac{(\alpha^*)^2}{m} \frac{M}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{(3-\gamma)/2 (\gamma-1)}} = \frac{\xi_o^2}{I} \quad (21)$$

and equation (25) of reference 11.

$$\frac{A^*}{A} = \frac{(\xi_o/\alpha^*)^2}{1 + \frac{\gamma-1}{2} M^2} \quad (22)$$

12. Once the local values of Mager's parameters have been determined, the local axial velocity component may be calculated using equation (9) of reference 11 which defines the related flow Mach number.

$$U = \frac{M C_o}{\left[M^2 \left(\frac{\gamma-1}{2}\right) + 1\right]^{1/2}} \quad (23)$$

13. The relative tangential velocity component between the swirling gases and the spinning nozzle is given by

$$W = \frac{K}{R_w} - R_w \Omega$$

or by substituting for the vortex strength constant, equation (11)

$$W = \frac{\Omega}{R_w} (R_o^2 - R_w^2) \quad (24)$$

14. The total relative velocity is then the vector sum of the angular and axial components.

$$V = [U^2 + W^2]^{1/2} \quad (25)$$

15. The local value of the radius of the void core in the vortex is given by equation (7) of reference 11.

$$r_c = \left[\frac{K^2 (\gamma - 1) / 2}{C_o^2 - U^2 (\gamma - 1) / 2} \right]^{1/2} \quad (26)$$

16. The density variation is expressed by equation (6a) of reference 11.

$$\rho = \frac{P_{ts} M_{wt}}{\bar{R} T_t} \left\{ \left[1 - \left(\frac{U}{C_o} \right)^2 (\gamma - 1) / 2 \right] \left[1 - (r_c / R)^2 \right] \right\}^{1/(\gamma-1)} \quad (27)$$

17. The following equation for the temperature variation was derived in a manner similar to that Mager employed to derive the density variation for irrotational flow.

$$T = T_t \left[1 - \left(\frac{U}{C_o} \right)^2 (\gamma - 1) / 2 \right] \left[1 - (r_c / R)^2 \right] \quad (28)$$

18. Prandtl number, thermal conductivity, and dynamic viscosity of the gases are evaluated using equations (8) and (9) of reference 18.

$$\mu = (46.6 \times 10^{-10}) (M_{wt})^{1/2} (T_f)^{0.6} \quad (29)$$

$$P_r = \frac{4\gamma}{9\gamma - 5} \quad (30)$$

$$k = \frac{\mu C_p}{P_r} \quad (31)$$

19. The heat transfer coefficient is calculated by

$$h = \frac{.026}{D^{0.6}} (k)^{0.6} \left(\frac{C_p}{\mu}\right)^{0.4} (\rho V)^{0.8} \left(\frac{D^*}{R_c}\right)^{0.1} \quad (32)$$

where all properties including density are evaluated at the film temperature given by

$$T_f = (T_t + T_w) / 2 \quad (33)$$

20. The adiabatic wall temperature is given by

$$T_{adw} = (T_t - T) P_r^{1/3} + T \quad (34)$$

21. The nozzle wall heat flux may be determined as

$$q = h (T_{adw} - T_w) \quad (35)$$

Results of nozzle heat transfer analysis. - In order to check the validity of the computerized nozzle heat transfer analysis, values obtained from a computer program test case for the various parameters used in Mager's solution, M^* , ξ_o , α^* , and I , were compared with the cross-plots of those parameters in reference 11. The heat transfer parameters were checked by comparing the results from a zero spin test case with results obtained using the Bartz analysis of reference 18. Those test case checks verified the analysis and the computer program.

The effects of spin rate on nozzle heat transfer were investigated by considering at various spin rates a particular nozzle geometry and set of propellant ballistic properties referred to as the datum case or conditions. The design parameters for the conical nozzle considered are:

$$\begin{aligned} \text{Throat radius, } R^* &= 2.0 \text{ in,} \\ \text{Throat radius of curvature, } R_c &= 2.0 \text{ in,} \end{aligned}$$

Nozzle contraction ratio = 4.0,
 Nozzle contraction half-angle = 45°,
 Nozzle expansion ratio = 10,
 Nozzle divergence half-angle = 15°,
 and Nozzle wall temperature, T_w = 3500°R.

The propellant ballistic properties for the datum case are:

Characteristic velocity, C^* = 5000 ft/sec,
 Molecular weight, M_{wt} = 27.11,
 Ratio of specific heats, γ = 1.14,
 Specific heat, C_p = 0.5 Btu/lbm°R,
 Burning rate exponent, n = .25,
 and Total temperature, T_t = 5500°R.

Datum case parameters relating to the motor design are:

Motor port radius, R_o = 8.0 in
 and Total equilibrium pressure,
 zero spin, P_t = 500 psi

Profiles of nozzle wall heat transfer coefficient and heat flux, with spin rate as a parameter, are presented in figures 22 and 23. Both heat transfer coefficient and heat flux are seen to increase with spin rate. This effect is due to the increase in local wall values of mass velocity, ρV , with spin rate. Due to the nozzle vortex plugging effect, the nozzle inlet total pressure and, consequently, the nozzle mass discharge rate increase with spin rate. The rate of increase in pressure and flow rate is dependent on motor and nozzle geometry and the propellant burning rate exponent. As spin rate varies from 0 to 10,000 rpm, the nozzle inlet total pressure varies from 500 to 1700 psi and nozzle flow rate varies from 40.4 to 54.9 lbm/sec for the datum case.

It might appear that the increase in heat transfer coefficient is due solely to the increase in pressure and mass flow rate. Accordingly, an attempt was made to isolate the effects of pressure and nozzle mass flow. The results are presented in figure 24 which shows nozzle throat heat transfer coefficient as a function of spin rate. The upper curve is for the datum case previously discussed where the nozzle throat static pressure increases from 300 to 711 psi as spin rate increases from 0 to 10,000 rpm. The lower curve was generated for the datum conditions by setting the propellant burning rate

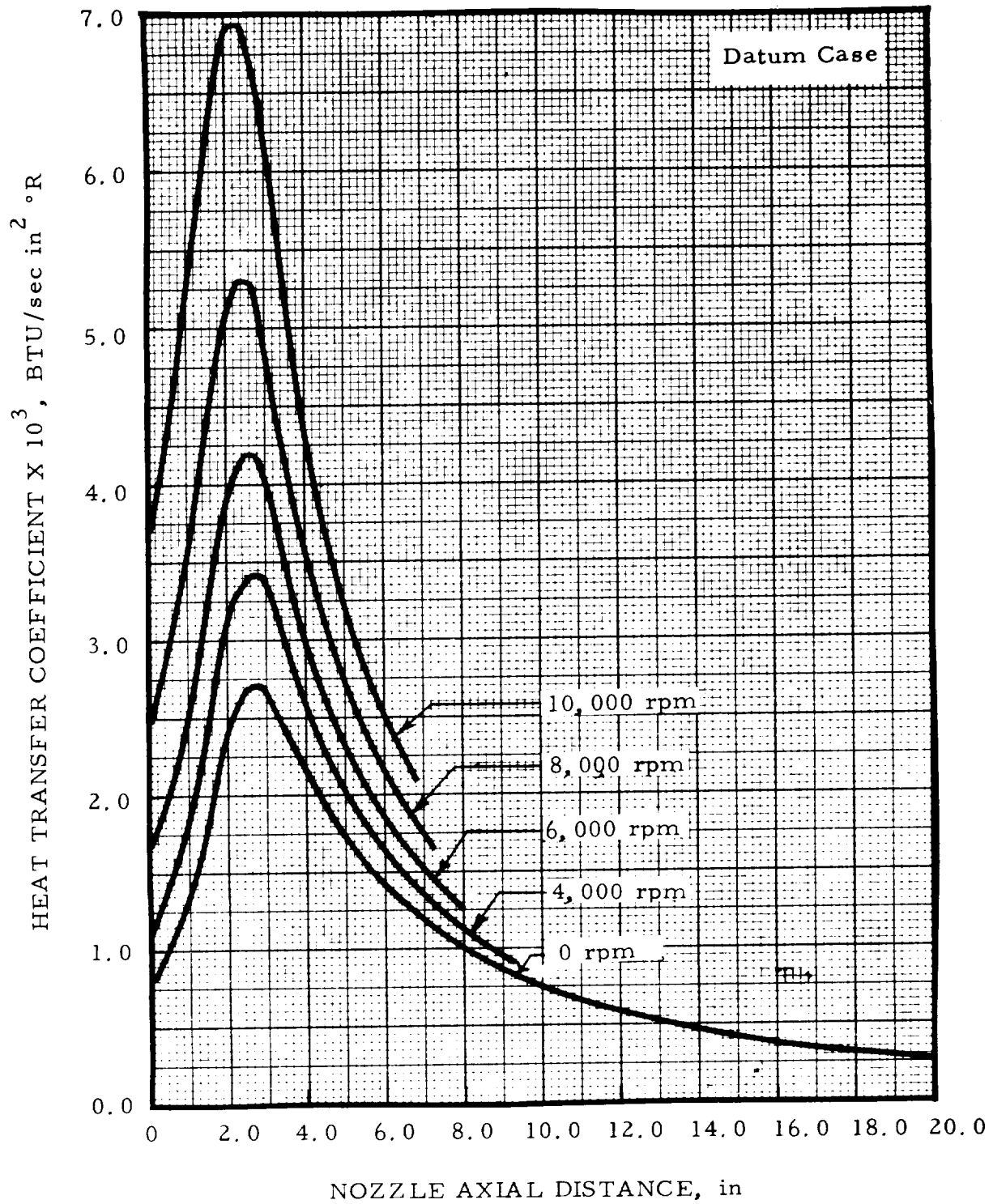


Figure 22. Effect of Spin Rate on Nozzle Heat Transfer Coefficient Profiles

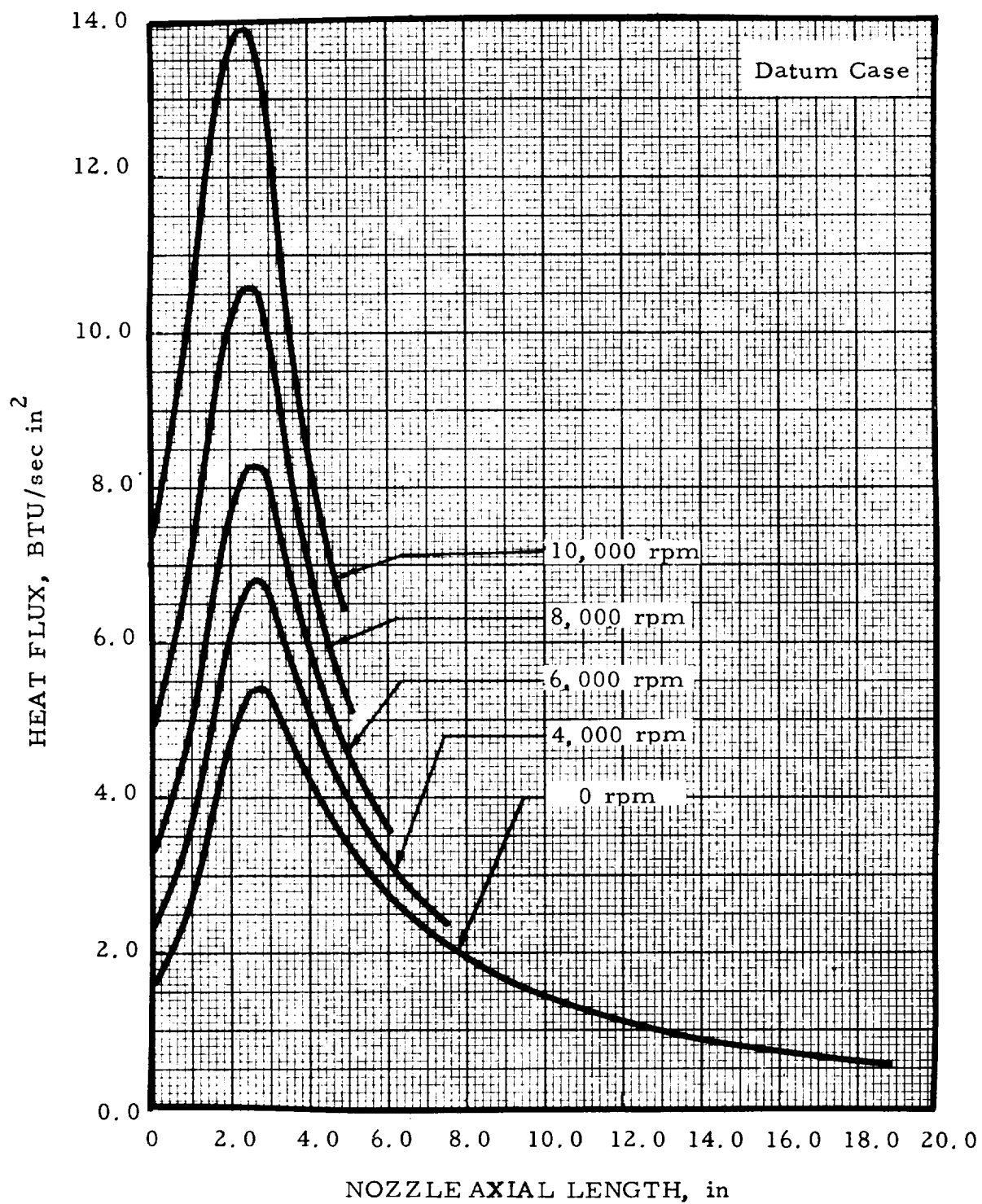


Figure 23. Effect of Spin Rate on Nozzle Heat Flux

exponent equal to zero and multiplying the resulting heat transfer coefficients by the ratio of the throat pressure at zero spin, 300 psi, to the actual throat pressure raised to the 0.8 power. Using a propellant exponent of "0" insured that the mass generation rate and thus the actual nozzle mass discharge rate would not change with spin rate. Performing the pressure correction eliminated the effect on the heat transfer coefficient of density increases due to pressure increases. Thus the nozzle mass flow rate and nozzle throat static pressure are essentially constant for the lower curve in figure 24.

However, it can be seen that the heat transfer coefficient still increases by approximately 31 percent as spin rate varies from 0 to 12,000 rpm. This can be explained by the variation of the velocity components with spin rate shown in figure 25. The significant increase in the tangential velocity component with spin rate results in an increase in the total velocity with spin rate. Thus the mass velocity, ρV , continues to increase with spin rate even though density is held essentially constant. This, of course, results in an increasing heat transfer coefficient with spin rate at constant pressure and nozzle mass flow rate.

The effect of spin rate on the ratio of the actual Nusselt number at the throat to the throat Nusselt number at zero spin is shown in figure 26. The throat Nusselt number ratio has been corrected in the same manner as described above for figure 24 such that the nozzle mass flow rate and throat static pressure are essentially constant. It may be observed that spin effects are quite progressive with spin rate even at constant pressure and flow rate. The predicted increase in throat Nusselt number is 13 percent at 5,000 rpm and 59% at 10,000 rpm.

Figure 27 shows the effect of the ratio of the motor port radius to the nozzle throat radius on the throat Nusselt number ratio. Spin rate is a constant 10,000 rpm for this curve and it is also corrected such that mass flow rate and throat static pressure are constant. Port-to-throat area ratios greater than 10:1 are necessary to get an increase in the throat Nusselt number of 10%. Larger port-to-throat area ratios have a significant effect on the percentage increase in the throat Nusselt number over the zero spin value.

Nusselt number ratio versus axial distance from the nozzle inlet is presented in figure 28 with spin rate as a parameter. The curves in this figure are all for constant nozzle mass flow rate; however, pressure is varying with nozzle axial distance. This figure shows that the Nusselt number increases are much higher at the nozzle entrance than at the nozzle throat ($Z = 2.8$ in.). At 6,000 rpm the Nusselt number is predicted to be 80 percent higher than for zero spin at the nozzle entrance ($R = 4.0$ in.), 36 percent higher at the throat, and only 5 percent higher at an area ratio

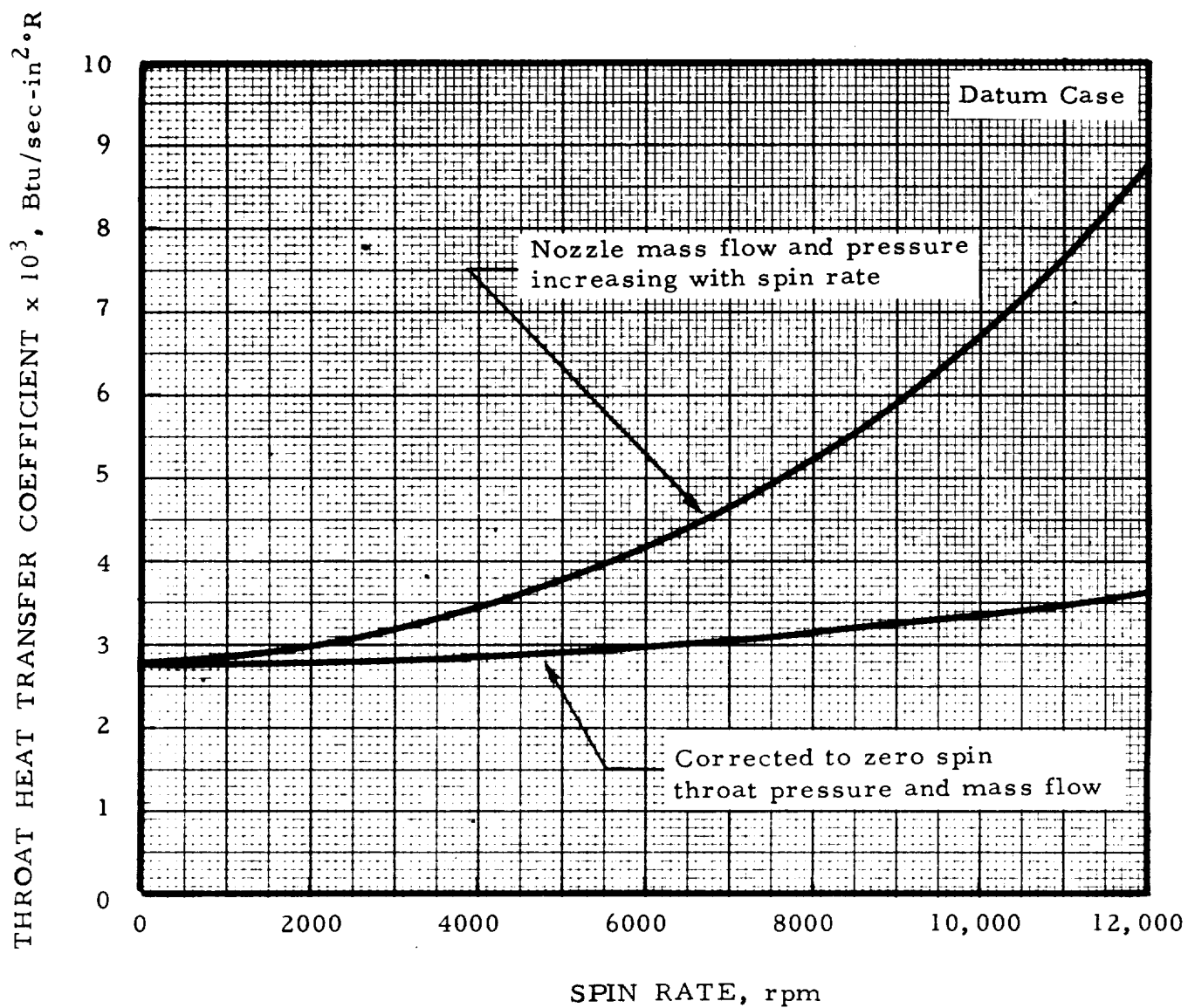


Figure 24. Effect of Spin Rate on Nozzle Throat Heat Transfer Coefficient

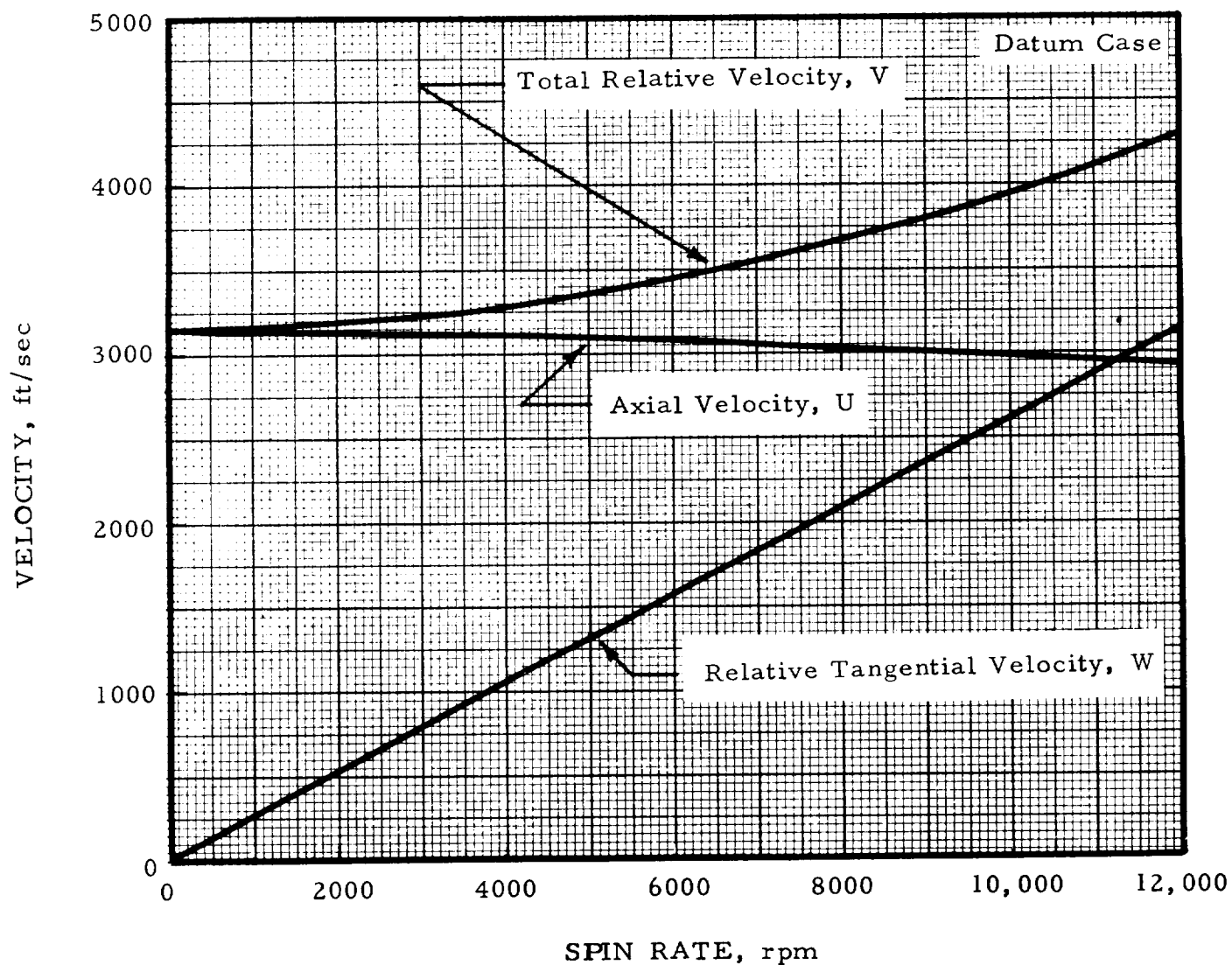


Figure 25. Effect of Spin Rate on Velocity Components

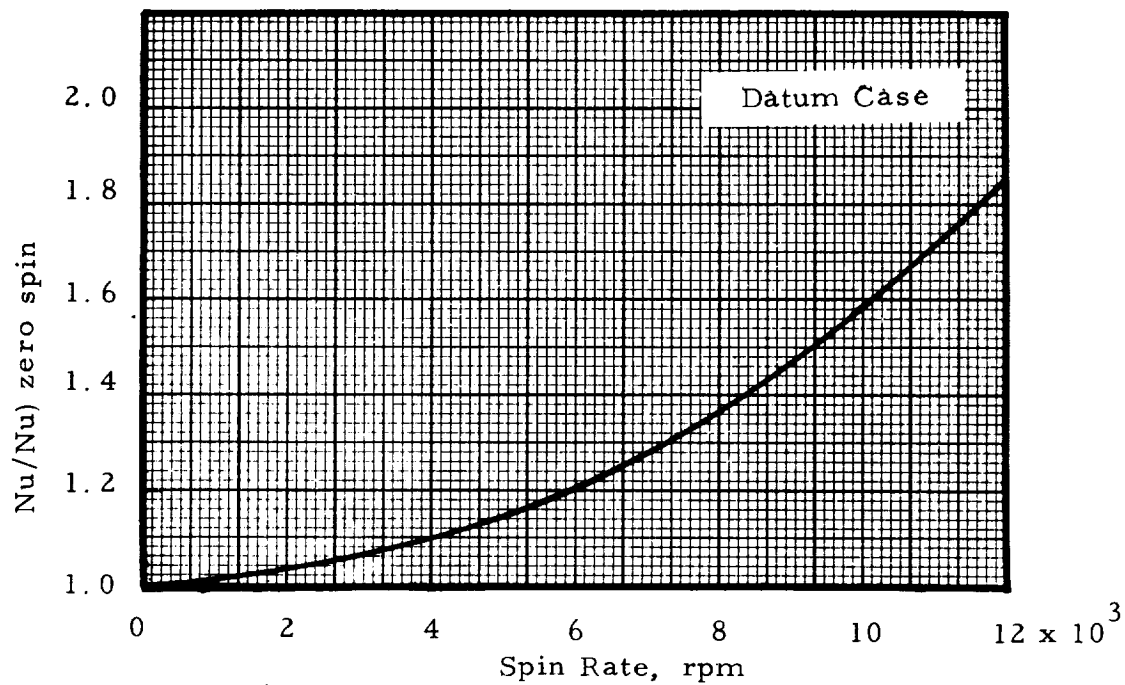


Figure 26 . Effect of Spin Rate on Throat Nusselt Number Ratio

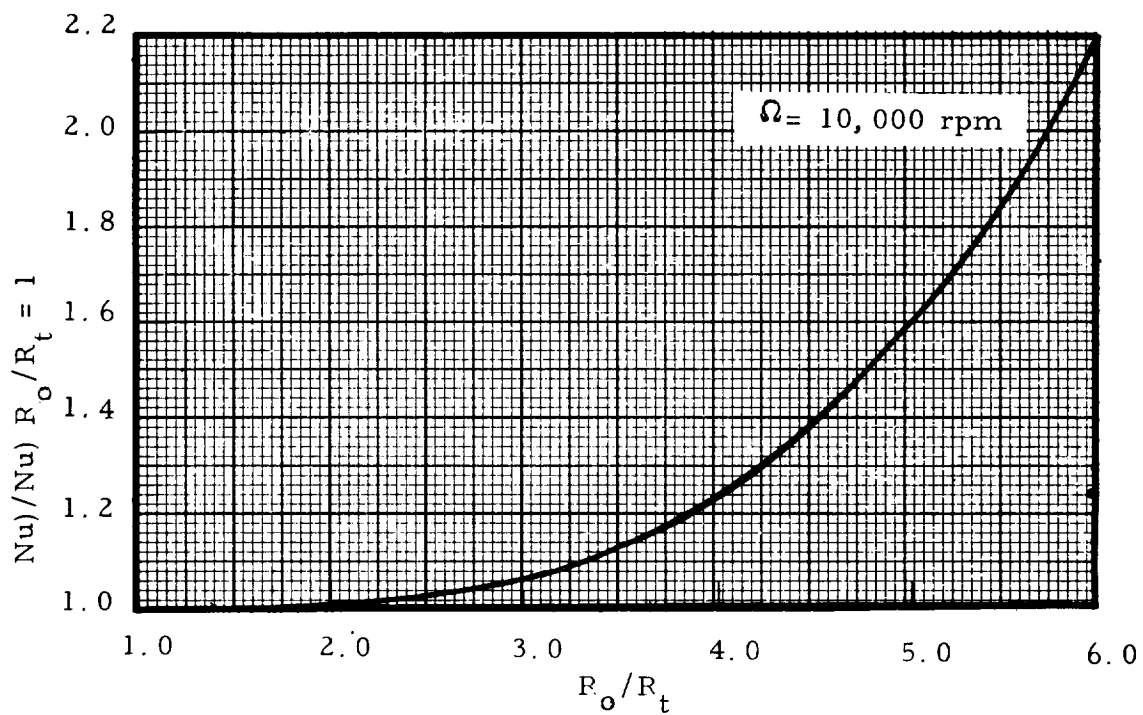


Figure 27 . Effect of Port to Throat Radius Ratio on Throat Nusselt Number Ratio

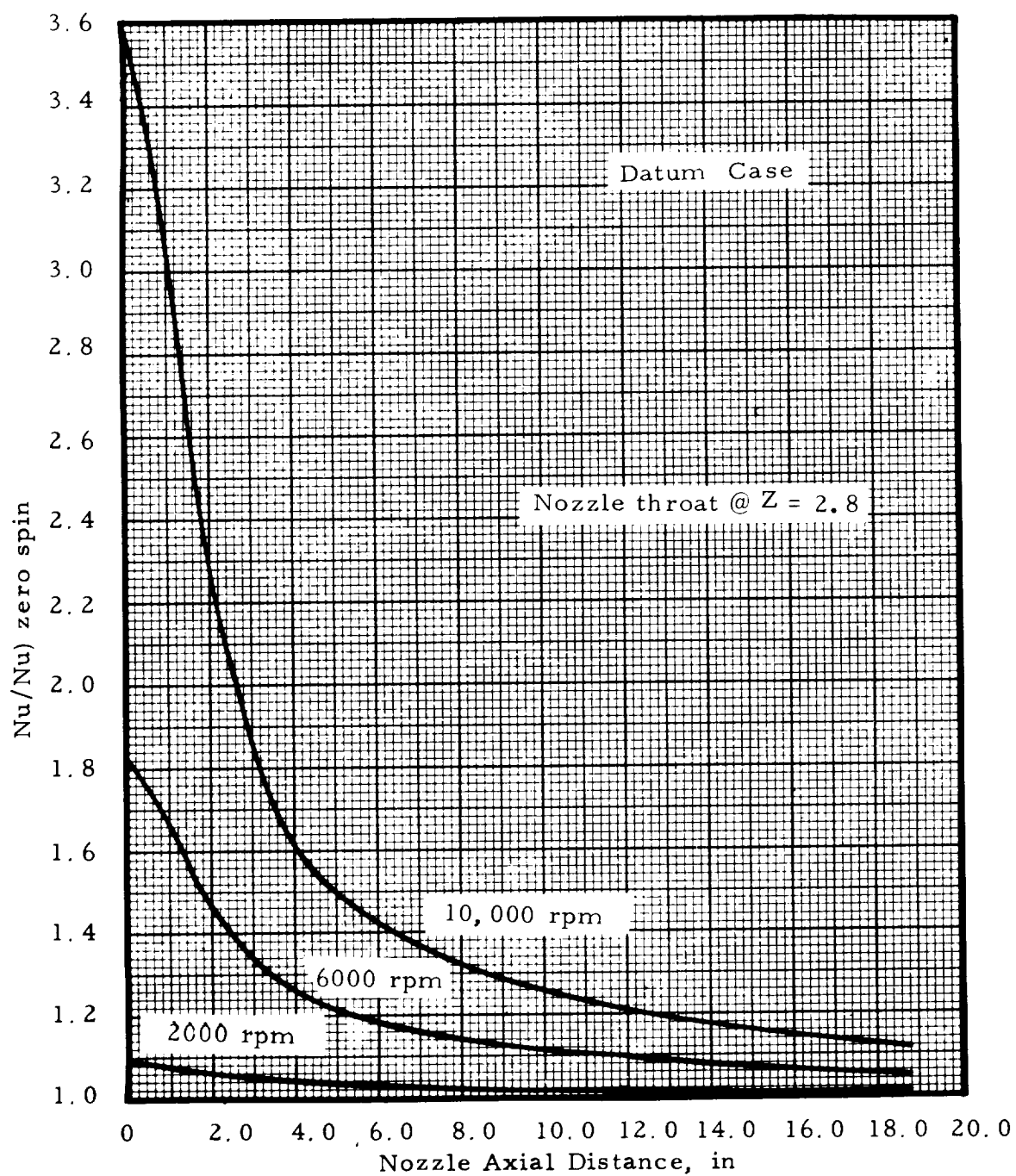


Figure 28 Effect of Spin Rate on Nusselt Number Ratio Profile

of 10. This effect is partially due to the fact that the total pressure increases at a faster rate than the throat pressure due to the vortex nozzle plugging effect. At 6,000 rpm the total pressure increased from 500 to 755 psia, an increase of 51 percent while the throat static pressure increased from 300 to 401 psia, an increase of only 34 percent. Other factors which vary with spin rate such as static temperature at the throat and total relative velocity at the throat are also responsible for causing the greatest percentage increase in Nusselt number to occur at the nozzle entrance.

Head End Dome Heat Transfer

Introduction. - A review of current analyses of problems involving rotating discs in the presence of a rotating flow field has revealed that the existing solutions for rotating flow fields have limited usefulness for application to describing the flow field near the head end of a spinning rocket motor. Analysis of the flow field in a rotating motor chamber is complicated by secondary flows caused by the interaction of the head end wall boundary layer with the primary vortex flow field. Usually only a weak interaction exists between the main flow field and the wall boundary layer; however, it appears that the flow field in a spinning motor chamber may be completely controlled by such interactions. The phenomenon of vortex breakdown or reversed axial flow, which is thought to result from either these viscous effects or dynamic instability effects, is probably the most important distortion of the free vortex flow field as far as effects on head end heat transfer are concerned. The existence of regions of locally reversed flow near the axis for vortex flows has been verified experimentally and analytically. Harvey (ref. 20) reported the existence of reversed axial flows in experimental work with vortex tubes. Burgers (ref. 21) obtained solutions for various cases of vortex flows for converging and diverging flows. His results are quite interesting in that regions of negative or reversed axial flows are predicted under certain conditions for both converging and diverging flows. Although there has been considerable work done on vortex flows, stability criteria and viscous interaction effects have not been defined to a state which would enable a prediction to be made for the flow field inside a given spinning motor chamber and nozzle geometry.

The anticipated boundary layer heat transfer solution would have been based on the consideration of a spinning disc in a normal free vortex flow field. It is now apparent that such a solution might not be useful for predicting head-end dome heat transfer due to the axial flow reversal. It is obvious that a reversed axial flow along the motor centerline could result in the impingement of gases at stagnation conditions on the head end dome. Of course, this would result in considerably higher heat transfer rates in the area of impingement. The development of a meaningful, valid boundary

layer heat transfer solution is precluded by a lack of knowledge concerning these flows which may occur near the head end dome of a motor. Thus it seems that before a sophisticated heat transfer solution is attempted, analyses which more clearly define the flow field in the motor chamber must be developed.

However, an approximate semi-empirical heat transfer solution to the originally posed model of a free vortex over a spinning disk with no secondary flows will be obtained with an approach similar to that used in the nozzle heat transfer analysis. The main difference between the head end analysis and the nozzle analysis is the elimination, in the head end analysis, of the axial velocity component from Mager's solution and the assumption of the existence of a solid body vortex inside the void core of Mager's potential vortex. Of course, the semi-empirical solution will not account for the flow reversal and consequent impingement on the head end dome; but by comparing the predictions of this analysis with qualitative experimental results obtained from eroded head end motor parts, the significance or contribution of any possible flow impingement at the center of the head end dome to the heat transfer process can be evaluated.

Analysis. - The semi-empirical heat transfer solution for the head end dome consists of modifying an existing heat transfer analysis for an isothermal rotating disk with a turbulent boundary layer. The modification will account for relative effects between the spinning head end dome and the swirling gases.

The solution of the integral momentum equations for a turbulent boundary layer on a rotating disk was first obtained by von Karman (ref. 22). The resulting equation for the turning moment coefficient, C_m , is

$$C_m = 0.146 R_e^{-1/5} \quad (36)$$

Cobb and Saunders (ref. 23) applied the Reynold's analogy to obtain a solution for the turbulent heat transfer from an isothermal disk by using the above equation for the turning moment coefficient. They derived the following equation for the local Nusselt number.

$$N_u = .0268 R_e^{0.8} P_r \quad (37)$$

Dorfman (ref. 24) showed that the analogy between the turning moment coefficient and the heat transfer coefficient actually applied to a nonisothermal

disk of a particular wall temperature distribution and not to an isothermal disk. However, Dorfman extended his analysis to other wall temperatures and the following equation for an isothermal disk may be obtained from his analysis by assuming that the Nusselt number varies directly as the Prandtl number.

$$N_u = .0257 R_e^{0.8} P_r \quad (38)$$

Dorfman's results were judged to be the most accurate and are used in the following analysis.

Since it has been established that the local mass velocity at the wall is the governing factor for heat transfer, equation (38) was modified to predict heat transfer to the spinning head end dome from the hot swirling gases by redefining the mass velocity in the Reynold's number to be the relative mass velocity between the gases and the wall. In equation (38) the Nusselt number is

$$N_u = h R / k \quad (39)$$

and the Reynold's number is

$$R_e = \frac{\Omega R^2 \rho}{\mu} = \frac{(\Omega R) R \rho}{\mu} \quad (40)$$

The tangential velocity of the disk, (ΩR) , is the relative velocity between the gases and the disk for the case of a rotating disk in a stationary fluid. For the case of a rotating disk in a swirling fluid the relative velocity is, of course, the difference between the tangential disk and gas velocities. The gas velocity is obtained from the free vortex velocity distribution.

$$W = K / R \quad (41)$$

The relative velocity is then

$$V = K / R - R \Omega = (K - R^2 \Omega) / R \quad (41)$$

and the Reynold's number becomes

$$R_e = (K - R^2 \Omega) \rho / \mu \quad (42)$$

The density, ρ , is taken from the theoretical density distribution for an isentropic free vortex given by equation (43).

$$\rho = \rho_t \left\{ 1 - \frac{\gamma - 1}{2} \frac{K^2}{c_o^2 R^2} \right\}^{1/(\gamma - 1)} \quad (43)$$

The equation for the local heat transfer coefficient on the head end dome is obtained from equations (38), (39), and (42).

$$h = \frac{.0257 k \rho}{\mu} (K - R^2 \Omega) P_r \quad (44)$$

The vortex strength constant, K , is given by equation (11) and the gas properties, μ , P_r , and k are given by equations (29), (30), and (31). These properties are evaluated at the film temperature.

The assumptions made in deriving equation (43) impose two major conditions on its use. The equations for a free vortex must approximate the freestream conditions and the boundary layer on the head end dome must be turbulent. These conditions imply that equation (43) is only valid in a region bounded by a maximum and a minimum radius for any given head end dome radius.

The maximum radius is defined by the minimum Reynold's number for turbulent flow on a rotating disk which has been established as approximately 2.8×10^5 . The Reynold's number decreases as the radius increases since the relative velocity decreases to zero as the radius approaches the port radius, R_o . This fact is evident from the following equation for the relative velocity which was obtained by substituting equation (11) into equation (41).

$$V = \Omega / R (R_o^2 - R^2) \quad (45)$$

The minimum radius is defined by the radius of the void core in the center of the potential vortex. Mager (ref. 11) locates the core radius where the density vanishes. However, in this analysis the core radius is located by a specified static pressure in the core. The core radius then becomes from equation (43)

$$R_e = [(2 - 1) K^2 / 2 c_o^2 (1 - X)]^{1/2} \quad (46)$$

where $X = (\rho_c / \rho_t)^{\gamma-1}$

The core radius in the analysis can thus be controlled by the specified core static pressure. The radius of the core wall determines the boundary between the outer potential vortex flow field and the inner solid body vortex. The potential vortex is assumed to transform to a solid body vortex at the core radius due to the viscous effects.

Although it is not essential to the foregoing analysis, calculation of the heat transfer coefficients in the core region may be performed by making the following assumptions which may be supported to some extent by experimental data.

Inside the solid body vortex the tangential velocity of the gases was assumed to be linear with the radius and decreased from a maximum value at the core radius to zero at the centerline. Thus the equation for the relative velocity in the core is given by

$$V = V_c \frac{R}{R_c} - R \Omega \quad (47)$$

The static pressure is assumed to be specified and constant in the core. The static temperature is assumed to be linear with the radius and increase from the minimum value at the core radius to the total temperature at the centerline. The static temperature distribution is given by

$$T_t = T_c - \frac{R}{R_c} (T_t - T_c) \quad (48)$$

The static density may be determined by

$$\rho = \frac{P}{R T} \quad (49)$$

Values from equations (47), (48), and (49) in conjunction with equation (44) may then be used to determine heat transfer coefficients in the core.

Results of analysis. - The effects of spin rate on head end dome heat transfer were investigated by employing the foregoing analysis and

considering a particular motor port radius and set of propellant ballistic properties referred to as the datum case or conditions. The datum case parameters are:

Motor port radius, R_o	= 8.0 in.
Wall temperature, T_w	= 3500°R
Total temperature, T_t	= 5500°R
Total pressure, P_t	= 500 psia
Ambient (core) pressure, P_{am}	= 14.7 psia
Molecular weight, products, M_{wt}	= 27.11
Specific heat ratio, γ	= 1.14
Specific heat, c_p	= 0.5

Radial profiles of heat flux with spin rate as a parameter are presented in figure 29. Nusselt number profiles at the same spin rates are shown in figure 30. It is immediately apparent that, although the analysis predicts steep gradients with heat flux increasing as local head end radius decreases from the motor port radius, it also predicts a sharp decrease in heat transfer rates close to the center of the head end dome. However, experimental results reveal high heating rates at the center of the head end which decrease radially outward. Of course, the results of the analysis are questionable in areas near or inside the core radius. However, the possibility exists that two-dimensional heating could account for the effects of high heating rates extending to the center of the head end dome, especially for cases where the core radius was small.

In figure 30, the location of the point of maximum Nusselt number is shown to shift radially outward as spin rate increases, and in figure 29 the point of maximum heat flux is shown to shift likewise. This is due to the increase in core radius with spin rate as shown in figure 31. It may be observed that the points of maximum heat flux and Nusselt number always occur at radii greater than the core radius for a particular spin rate. The points of maximum Nusselt number do not correspond to points of maximum heat flux due to the fact that the points of maximum Nusselt number do not occur at the same radii as points of maximum heat transfer coefficients. The heat transfer coefficient continues to increase as the radius is decreased from its value at the maximum Nusselt number since it is inversely proportional to head end radius according to equation (39). However, since the Nusselt number approaches zero at the centerline, the heat transfer coefficient and heat flux accordingly reach a maximum and decrease to zero at the centerline.

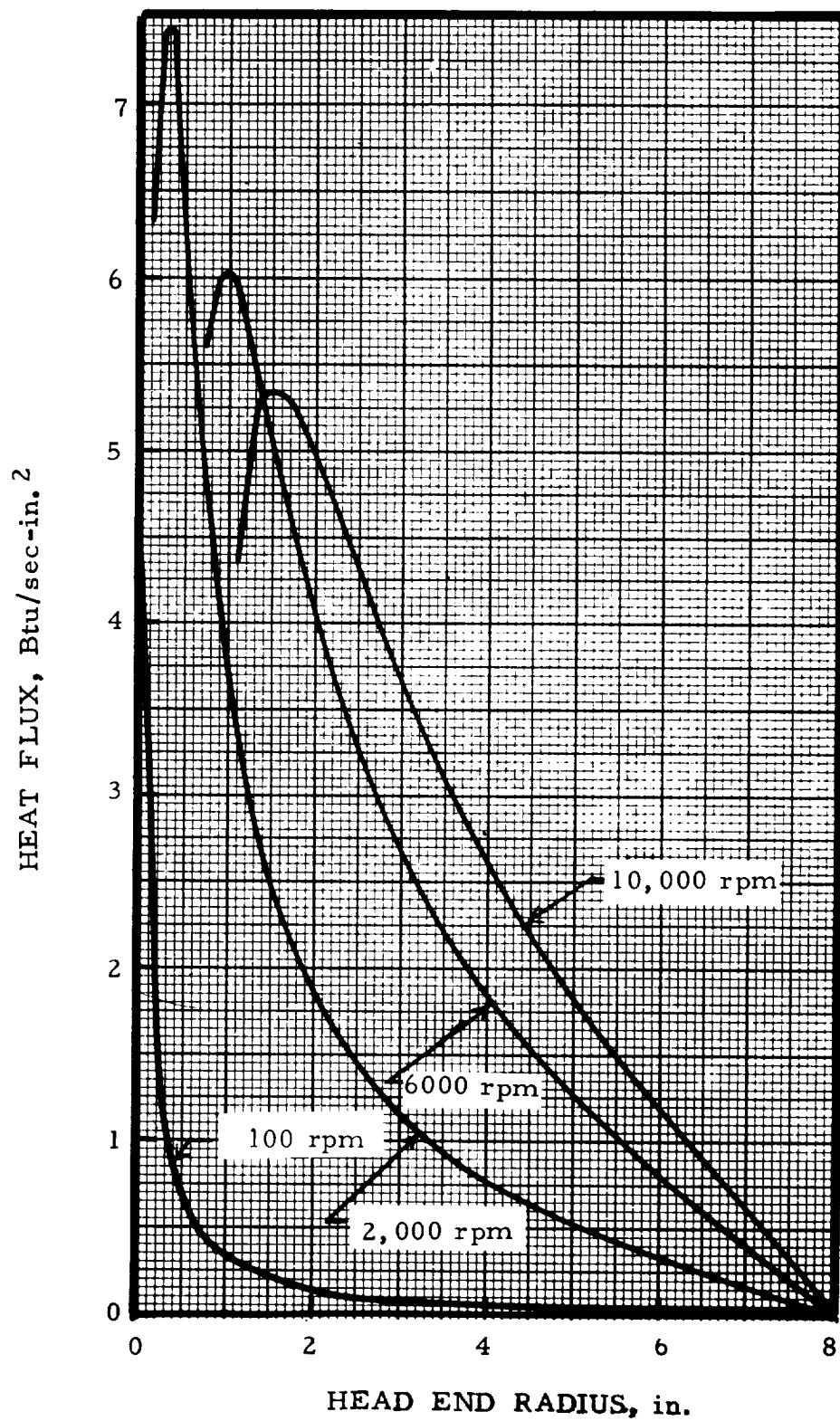


Figure 29. Effect of Spin Rate on Heat Flux

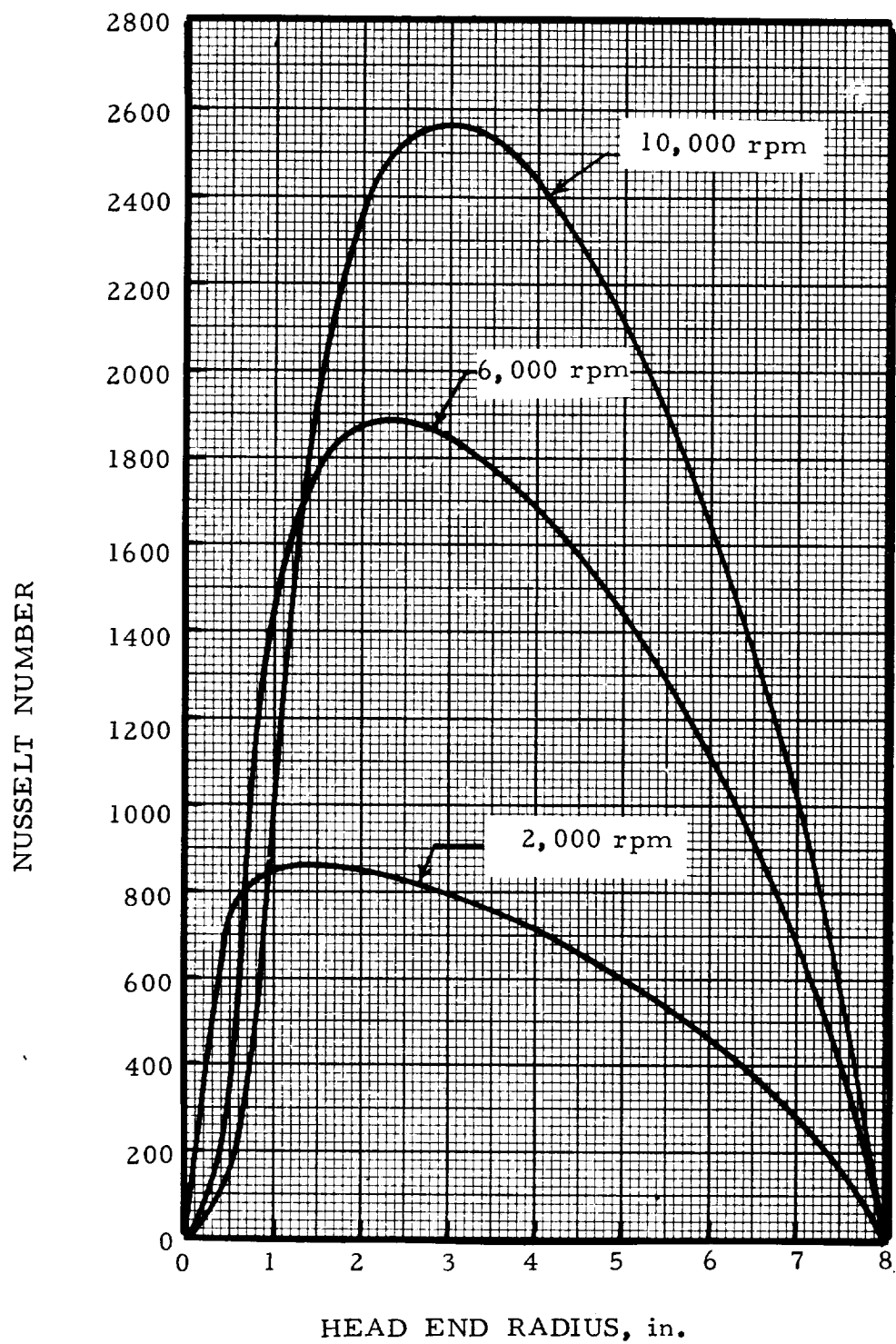


Figure 30 . Effect of Spin Rate on Nusselt Number Profile

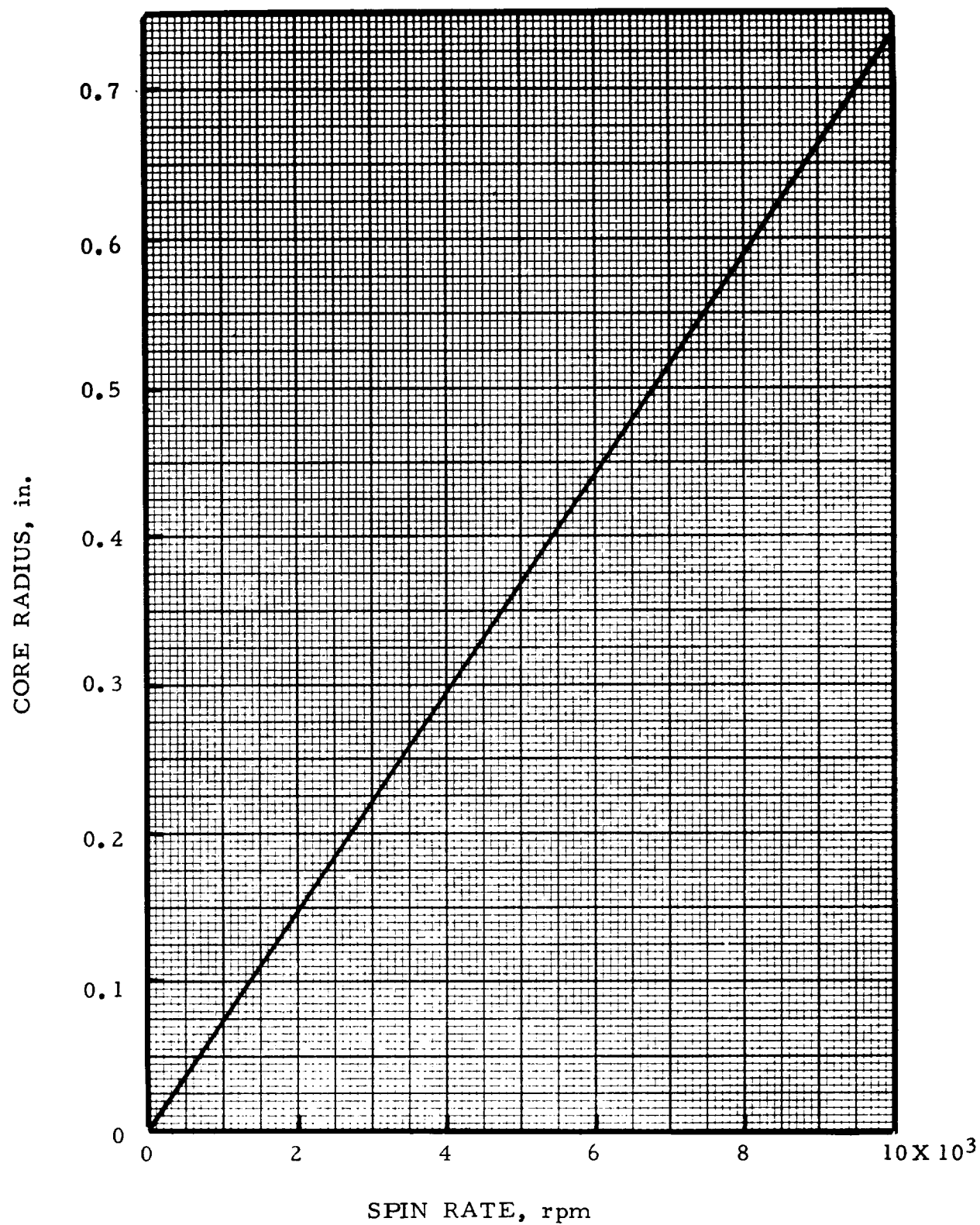


Figure 31. Effect of Spin Rate on Core Radius

It is interesting to note from figures 30 and 32 that values for the peak or maximum Nusselt number always increase with spin rate whereas in computer data generated for figure 29, the values for the maximum heat increased with spin rate up to approximately 200 rpm and then decreased with spin rate. This behavior is due to a complex interaction of variables caused by a shifting of the core radius with spin rate and the fact that the velocity and gas temperature profiles reverse slope across the core radius. Of course, at a spin rate of zero there is no relative velocity between the gases and the wall and the Nusselt number and, consequently, the heat flux are zero for all radii.

The effect of motor port radius or head end dome radius on the maximum Nusselt number is shown in figure 33. This curve was generated at a constant spin rate of 10,000 rpm.

An attempt was made to acquire from the available literature experimental data on head end erosion which could be used to estimate heat flux to the head end dome of the spinning motor. The validity of the preceding analysis could then be evaluated especially on the question of the importance of impinging axial backflow on head end heat transfer. However, this attempt was unsuccessful due to the fact that the information found in the literature on head end erosion was insufficient for making a prediction with the present analysis. All of the required motor operating conditions, head end material properties, and propellant properties were not reported in the literature checked containing reports of head end erosion. Thus an analytical prediction of the head end heat flux using the developed analysis was precluded.

Conclusions

The following conclusions may be drawn from the preceding analyses of heat transfer to the nozzle wall and head-end dome of a spinning rocket motor.

1. A computer program based on a semi-empirical analysis of nozzle wall heat transfer in a spinning rocket motor has been successfully developed.
2. Nozzle wall heat transfer coefficients are predicted to increase significantly with spin rate, as expected. The increase in coefficients is primarily due to the increase in motor pressure with spin rate; however, a significant portion of the increase is due to the increase in the total relative velocity between the swirling gases and the wall.

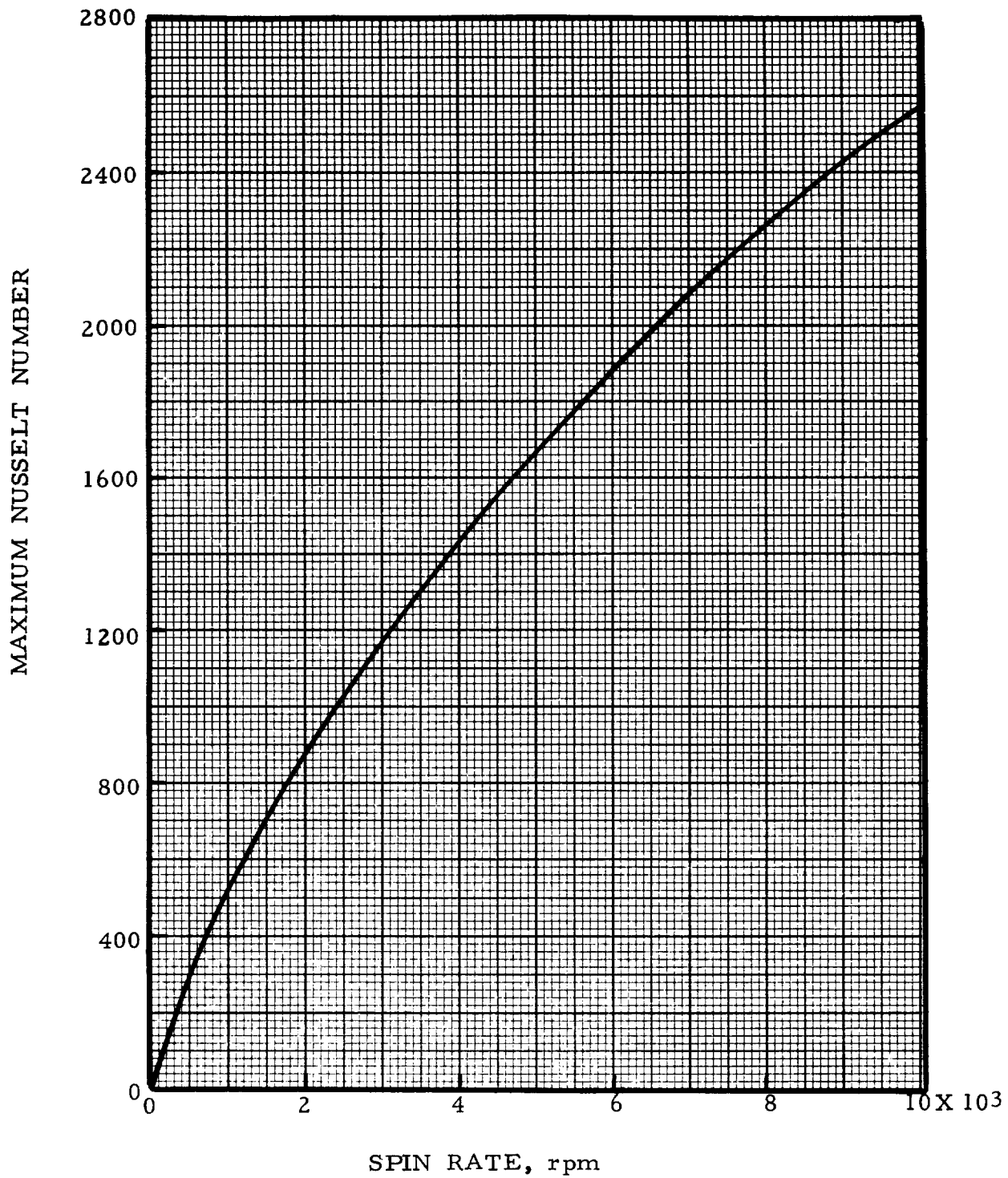


Figure 32 . Effect of Spin Rate on Maximum Nusselt Number

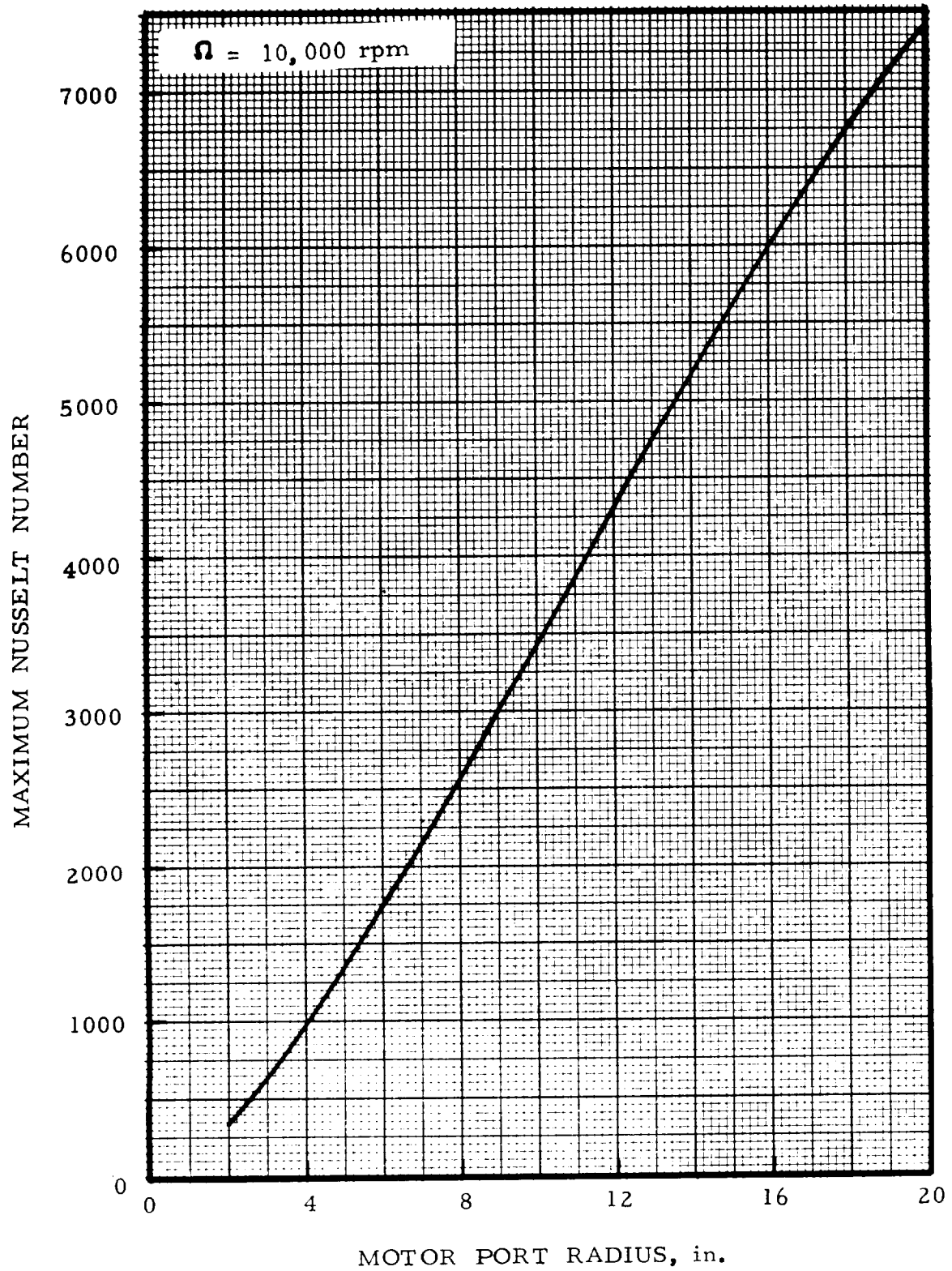


Figure 33 . Effect of Port Radius on Maximum Nusselt Number

3. The maximum percentage increase in the Nusselt number due to spin effects occurs at the nozzle entrance or inlet and the percentage increase declines from that point through the remainder of the nozzle.
4. A semi-empirical analysis of head-end dome heat transfer which does not account for secondary flows in the motor chamber has been developed and computerized.
5. The head-end heat transfer analysis predicts sharp increases in heat transfer coefficients in a region near the center of the head-end dome. This trend is in agreement with known experimental results and thus supports qualitatively the model upon which the head-end analysis was based.
6. Comparison of experimental heat transfer data with predictions from the nozzle wall and head-end dome computer programs should be made to further evaluate these analyses.

EFFECT OF ACCELERATION ON BURNING RATE

General

The objective of this phase was to modify the burning rate models developed under Contract NAS-7-406. Additionally, any new data was to be examined and correlated with parameters obtained from the burning rate models. Both the metallized and the non-metallized burning rate models that had been developed showed discrepancies when compared with available experimental data. A brief description of each burning rate model is presented before the discussion of its modification.

Metallized Propellant

The metallized burning rate model is based on the hypothesis that a fraction of the metal evolved at the burning surface during the combustion process is retained and burned there (ref. 25). Some of the energy released by this metal combustion is transferred to the burning surface thereby increasing the burning rate. Extinguished strands of metallized propellant have shown a pitted surface suggesting that increases in propellant burning rate are local phenomena. Additionally, particulate residue of a size that is large compared with the particle size of the metal additive is retained in the motor after firing. This suggests that agglomeration occurs on the burning surface. The following assumptions were made in the development of the metallized burning rate model.

1. Condensed phase particles that are initially retained on the burning surface agglomerate and remain on the burning surface.
2. The interaction between the retained condensed phase material and the burning surface occurs only at a finite number of points.
3. The line of descent of the agglomerated metal through the propellant is colinear with the acceleration vector.
4. The process is steady in the mean. (That is, the agglomeration rate is assumed equal to the metal combustion rate.)
5. The particle retention criteria is based upon the relative magnitudes of the buoyant and drag forces acting on a particle.

Figure 34 is a schematic of the model.

The rate of descent of a single agglomerated particle through the propellant was computed from a balance between the amount of energy required to increase the burning rate above the base rate and the energy supplied through metal combustion. The energy required to increase the base rate is

$$E_r = (r_a - r_o) h_v \rho_p A_i \quad (1)$$

where A_i is assumed to be the projected area where burning rate is increased. Therefore,

$$A_i = \pi (\xi_a \cos \theta_c)^2 \quad (2)$$

However, $\cos^2 \theta_c = 1 - \sin^2 \theta_c$ and $\sin \theta_c = r_o / r_a$

Therefore,

$$A_i = \pi \xi_a^2 [1 - (r_o / r_a)^2] \quad (3)$$

Since the burning surface is pitted, any metal added to the agglomerated particle must originate within its own pit. If there are N_s pits per unit area of the mean burning surface, the effective area swept by each pit is

$$A_s = N_s^{-1} \cos \Phi \quad (4)$$

Assuming that all metal in the interaction area, A_i , is agglomerated, the amount of metal added to the agglomerated particle in unit time is

$$\dot{m} = r_a \rho_p w_m [A_i + (A_s - A_i) G] \quad (5)$$

where G is the retained fraction of the metal evolved from the wall of the pit. The process is steady-state; therefore, this metal must be consumed. The energy released over the whole particle is $\dot{m} \Delta H_c$. However, only the energy released in the clearance above A_i is effective in increasing burning rate. Assuming that the energy release is uniform over the particle, the

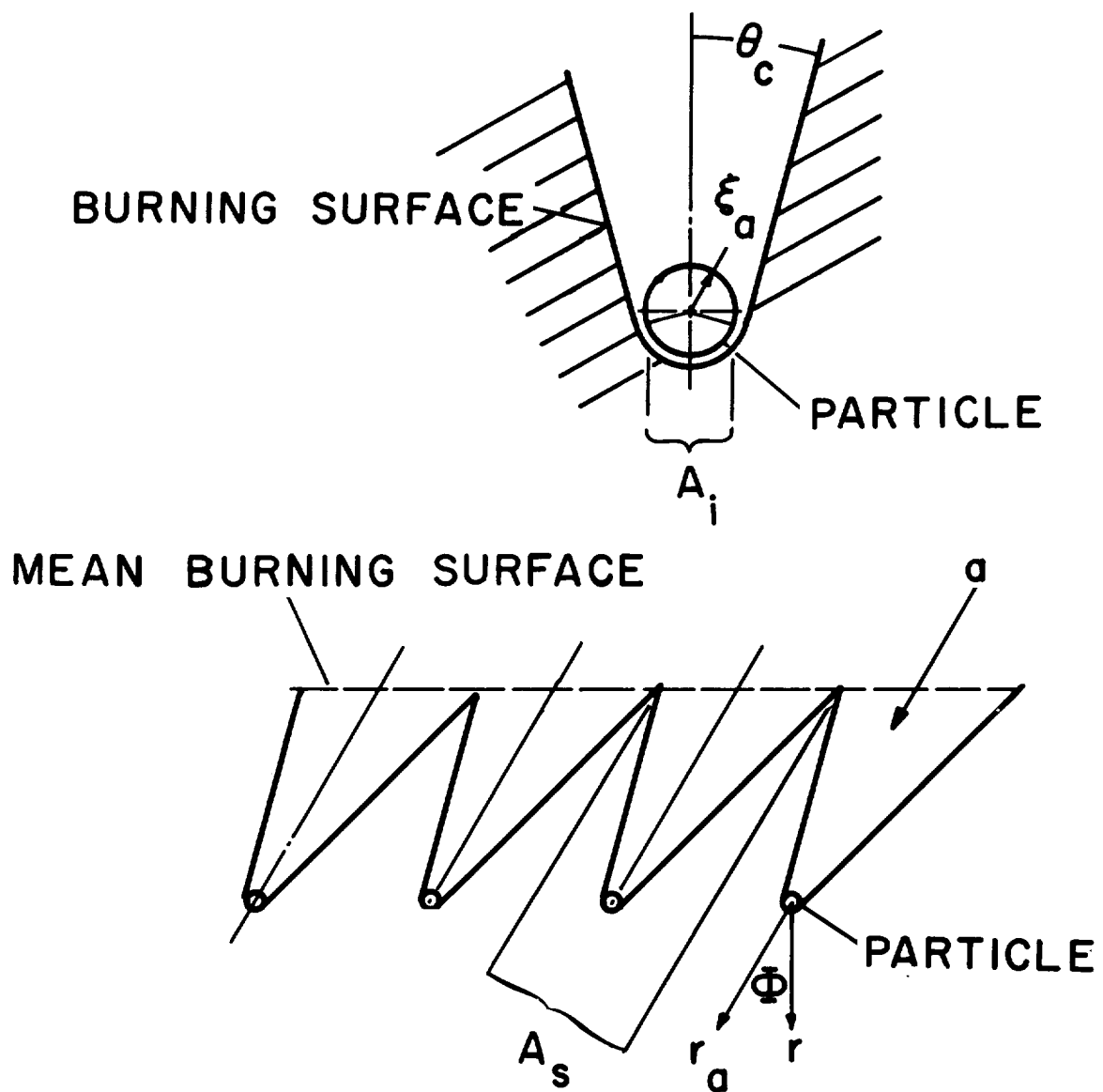


Figure 34. Analytical Model for Particle Burning.

increase in energy feedback to the burning surface is

$$\dot{E}_s = m \Delta H_c (1 - r_o/r_a) / 2 \quad (6)$$

The rate of descent of the particle may be obtained by equating equations 1 and 6 and employing equations (3)(5). This yields

$$r_a/r_o = \left[\frac{1 - \Pi(1-G)}{1 - \Pi[1 + (\beta-1)G]} \right]^{1/2} \quad (7)$$

where the dimensionless parameters Π and β are $\Pi = w_M \Delta H_c / (2h_v)$ and $\beta = (N_s \pi \xi_a^2)^{-1} \cos \Phi$.

The burning rate desired, however, is the regression rate of the mean burning surface. Examination of figure 34 shows that $r = r_a \cos \Phi$. Therefore, the burning rate is

$$r/r_o = \left[\frac{1 - \Pi(1-G)}{1 - \Pi[1 + (\beta-1)G]} \right]^{1/2} \cos \Phi \quad (8)$$

To obtain the function G , it was assumed that the evolving particles were solid spheres and that their diameters followed a log normal distribution. Therefore, if all particles with $\xi_M > \xi'_M$ are retained on the burning surface, it can be shown (ref. 1) that

$$G = \text{erfc} [\ln (\xi'_M / \bar{\xi}_M) \sigma] / 2 \quad (9)$$

For particles with $\xi_M = \xi'_M$ inertial and viscous forces are equal and a force balance shows that (the gas velocity at the conical surface of the pit is $r_o \rho_p / \rho_{gs}$)

$$\xi'_M = [9 \mu_{gs} r_o \rho_p / (2 \rho_a \rho_{gs})]^{1/2} \quad (10)$$

Assuming that the Stokes drag coefficient applies and denoting conditions where $\xi'_M = \bar{\xi}_M$ with a subscript c, the radius ratio $\xi'_M / \bar{\xi}_M$ for a particular propellant (μ_{gs} , ρ_p , T_s are essentially constant) becomes

$$\xi'_M / \bar{\xi}_M = [r/(a\rho)] / [r/(a\rho)]_c^{1/2} \quad (11)$$

For a particular motor this reduces to

$$\xi'_M / \bar{\xi}_M = (a_c/a)^{1/2} \quad (12)$$

A complete discussion of the properties of this model can be found in reference 25. The parameters (Π , β and G) involved in this model are each concerned with a different phenomena. The parameter Π , or $W_m \Delta H_e / 2h\nu$, can be reviewed as an energy parameter; β , or $(N_s \pi \xi^2)^{-1} \cos \Phi$, is descriptive of the pitting; and G is indicative of the agglomeration. The energy parameter, Π , was evaluated using data from Reference 26 and the heat of formation of Al_2O_3 (ref. 26). Equations(9) and (10) together with the propellant properties were used to compute the value of G . The remaining parameter, β , was not amenable to analysis. No method presently exists by which the value of β as a function of spin rate, propellant, etc., can be calculated. In order to obtain some idea of the dependence of β upon propellant composition and spin rate the data of Anderson (ref. 12) was used in conjunction with equations(8) (9) and (10) to determine the values of β necessary to fit Anderson's data. The results of such a computation are shown in figure 35. Over the range of accelerations of interest, the functional dependence of β for various propellants bears little resemblance to each other; i.e., very little correlation between the various propellants was achieved. This lack of correlation suggests a closer examination of the model.

Available information suggests that the primary features of the Thiokol model are correct. In particular, the assumption that heat transfer to the propellant occurs at a discrete number of points seems to be the dominant phenomenon. Presently the Thiokol model postulates burning of agglomerated metal additive as the source of heat. Thus the accurate computation of the metal agglomerated is one of the more important aspects of the current theory. Metal is agglomerated into the particle from two sources: (1) any metal evolved below the particle and (2) a portion of the metal evolved from the side of the pit.

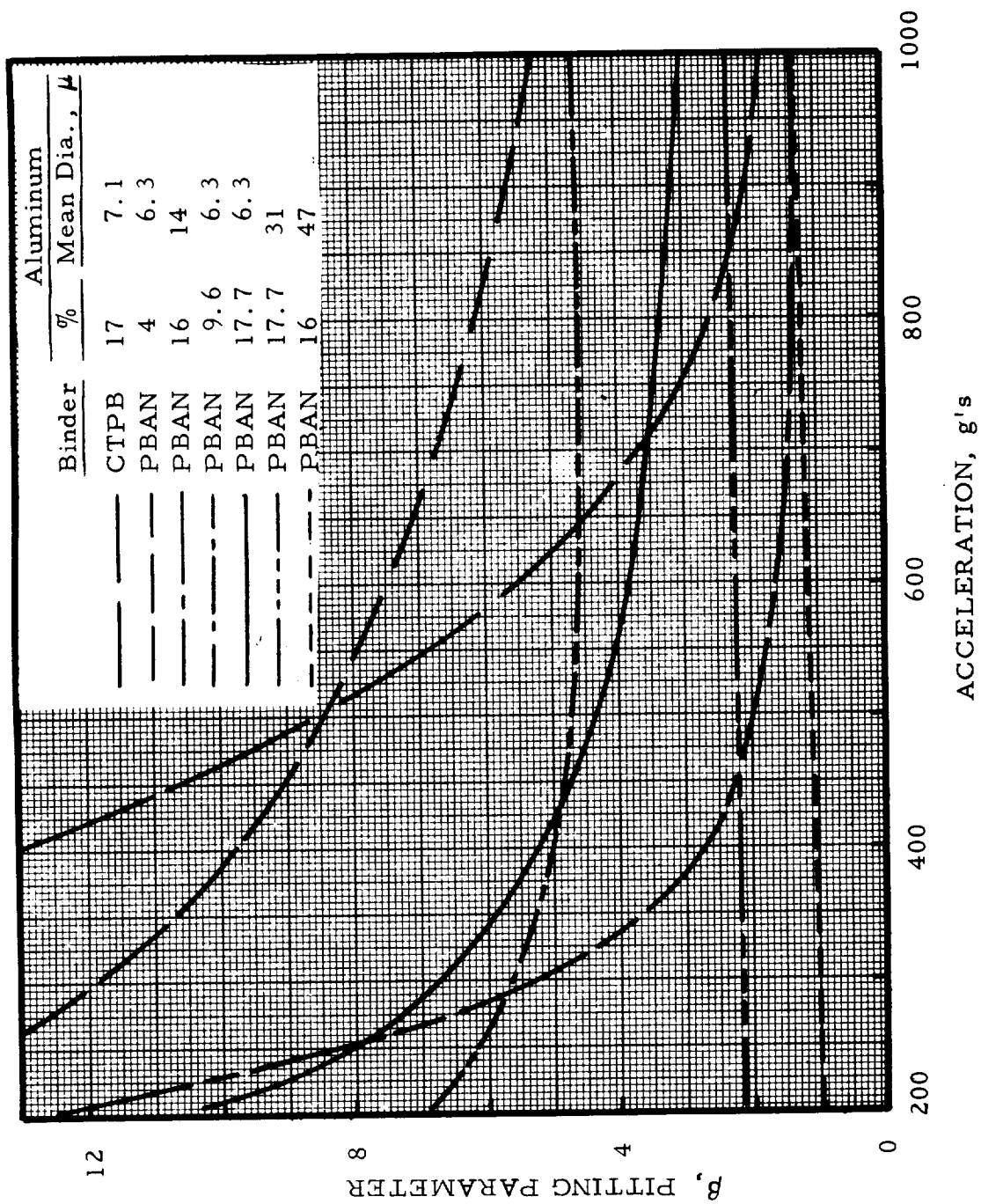


Figure 35. Functional Dependence of β on Acceleration

Therefore, some criteria for agglomeration into the particle or escape from the pit must be postulated for the evolving metal additive particles. The model originally formulated uses the concept of a critical particle radius; i. e. all particles larger than the particle of critical radius are agglomerated and all particles smaller are removed from the pit. The determination of the critical particle size is dependent upon the relative magnitudes of the viscous and inertia forces. The projected modification to the metallized burning rate model involved a change in the calculation procedure for the critical particle diameter.

A particle on the surface is subjected to a viscous force (drag), which tends to remove the evolved particle, and to an inertia force (due to the acceleration felt by the particle), which tends to retain the particle on the surface. The inertia force in the present theory is given by

$$F_I = \frac{1}{6} \pi d^3 \rho_a \quad (13)$$

where a is the acceleration felt by the particle:

$$a = r\omega^2 \quad (14)$$

And the viscous force is represented by

$$F_D = \frac{\pi}{8} \rho_g V^2 C_D d^2 \quad (15)$$

If the drag force (F_D) is greater than the inertia force (F_I) then the particle is blown off the surface and is assumed to be unavailable for agglomeration. Thus the critical particle diameter occurs when the drag (viscous) and inertia forces are equal:

$$\frac{\pi}{8} \rho_g V^2 C_D d^2 = \frac{1}{6} \pi \rho_a d^3 a \quad (16)$$

$$\text{or } d = \frac{3}{4} \frac{\rho_g V^2 C_D}{\rho_a a} \quad (17)$$

With the further assumption of Stokes Flow (ref. 3)

$$C_D = \frac{24}{Re} \quad (18)$$

Therefore,

$$r_c = \left[\frac{9}{2} \frac{\mu_g r \rho_p}{\rho_a \rho_g a} \right]^{\frac{1}{2}} \quad (19)$$

A particle with a radius less than r_c is blown off the surface while a particle with a radius greater than r_c is retained on the surface and agglomerated.

However, it has been pointed out that a particle evolved from the cone side has only a portion of the drag force with which to overcome the inertia force. Since the cone side is not perpendicular to the radius vector (radial acceleration vector) only the radial component of the drag force is available to overcome the inertia force. Figure 36 illustrates by means of free body diagrams the difference between the two concepts. Thus the component of the inertia force in the direction of the drag force is

$$F_I = \frac{1}{6} \pi d^3 a \rho \sin \theta_c \quad (20)$$

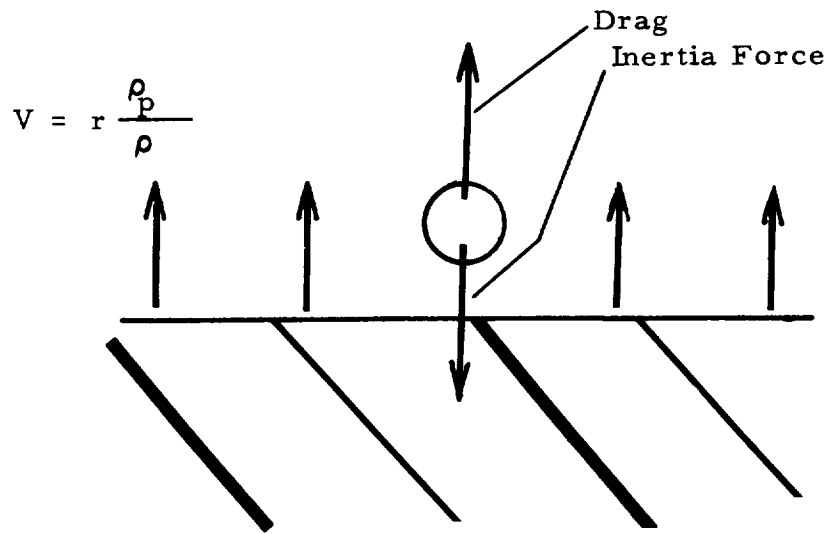
$$\text{But } \sin \theta_c = r_o / r$$

$$\text{and } F_I = \frac{1}{6} \pi d^3 \rho a \frac{r_o}{r} \quad (21)$$

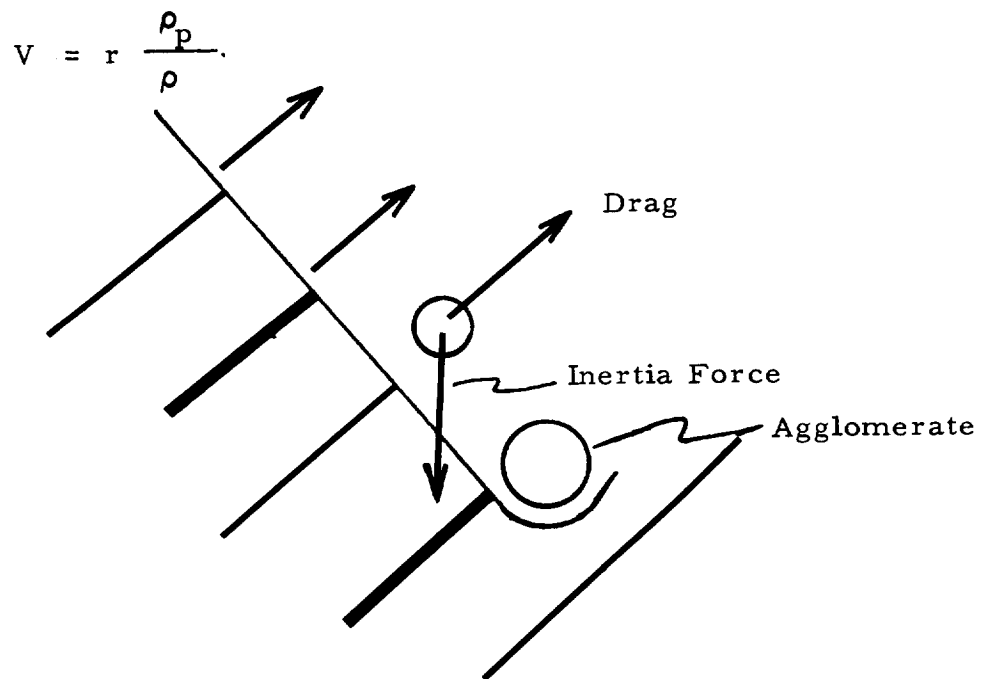
The critical radius, when only the radial component of drag is utilized, becomes

$$r_c = \left[\frac{9}{2} \frac{\mu_g \rho_p}{\rho_a \rho_g a} r \right]^{\frac{1}{2}} \quad (22)$$

Equation (22) is the criterion for movement of a particle off the wall of the pit, but this indicates nothing about movement of the particle out of the pit. Merely blowing the particle off the surface is no longer a criteria for agglomeration. If the particle is blown off the surface, it can still move to the top or to the bottom of the pit depending upon the direction of the resultant force. A movement to the bottom will result in agglomeration while a movement out of the pit would remove the particle from consideration of agglomeration. Thus the critical particle radius is determined by the inertia force and the radial component of the drag force.



(a) Particle Retention Criteria for Present Model



(b) Particle Retention Criteria for Modification to Model

Figure 36. Particle Retention Criteria

If the particle is carried out of the pit

$$F_D \sin \theta \geq F_I \quad (23)$$

So that the critical particle radius becomes

$$r_c = \left[\frac{9}{2} \mu_g \frac{\rho_p}{\rho_g} \frac{1}{\rho_a} r_o \right]^{\frac{1}{2}} \quad (24)$$

Thus equation (8), which gives the relationship between the parameters, is the same for both the old and the new Thiokol burning rate models. But a new criterion for particle retention has to be developed and this development will result in a different behavior of the model.

Glick (ref. 25) parametrically studied the metallized burning rate model using a/a_c and $(ap/r_b) / (ap/r_b)_c$ (where the subscript c refers to conditions necessary to hold the mean particle on the surface) as the independent variables. The modification to the models yields $ap/(ap)_c$ and $[a/a_c]$ as variables for the general case and for the same motor at constant pressure. Thus the case studies for $[a/a_c]$ are the same for the model regardless of the version (for Π, β constant). Figure 37 presents a comparison of the modified model with the previous model using $[ap/(ap)_c]$ as the parameter. The parametric study was made using constant β and Π parameters, so that the full effects of particle retention criteria could be evaluated. The effect of acceleration upon the burning rate of a propellant is dependent upon the orientation of the acceleration vector with respect to the mean propellant burning surface (ref. 14).

The particle retention criterion of the original model was independent of the cone sides. Therefore, regardless of the orientation of the acceleration vector the retention criterion for the pit walls was the same. If the modification made to the model does not result in a change in the dependence of the criterion, then the effect of acceleration orientation for the original model and the modified model will be the same. Figure 38 is an expanded view of a typical cone in a burning metallized propellant with a non-normal acceleration vector. A free body diagram of particles from opposed sides is shown in this figure. From the figure it can be seen that regardless of which side the particle is evolved from the retention criteria is

$$F_D \sin \theta \geq F_I \quad (25)$$

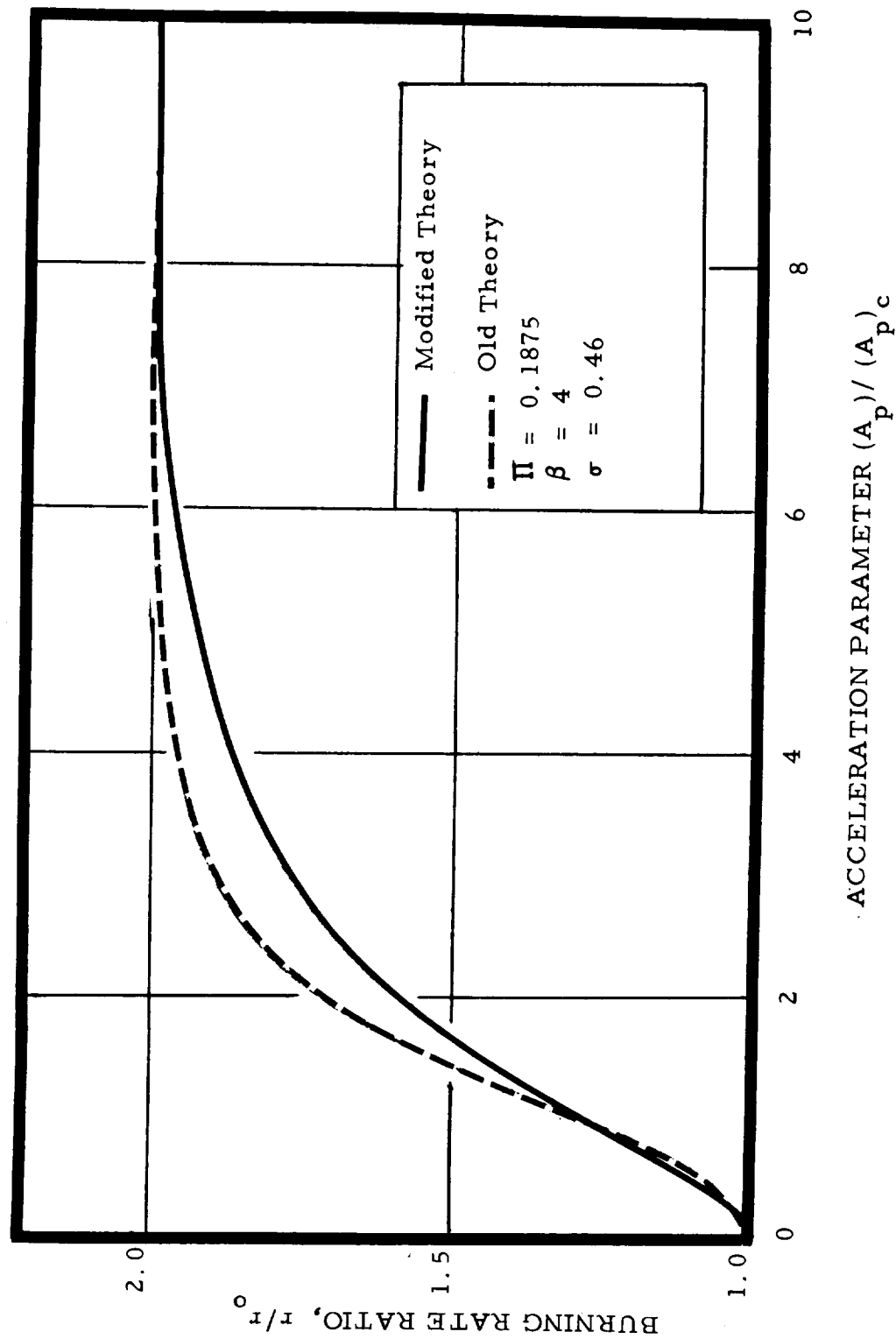
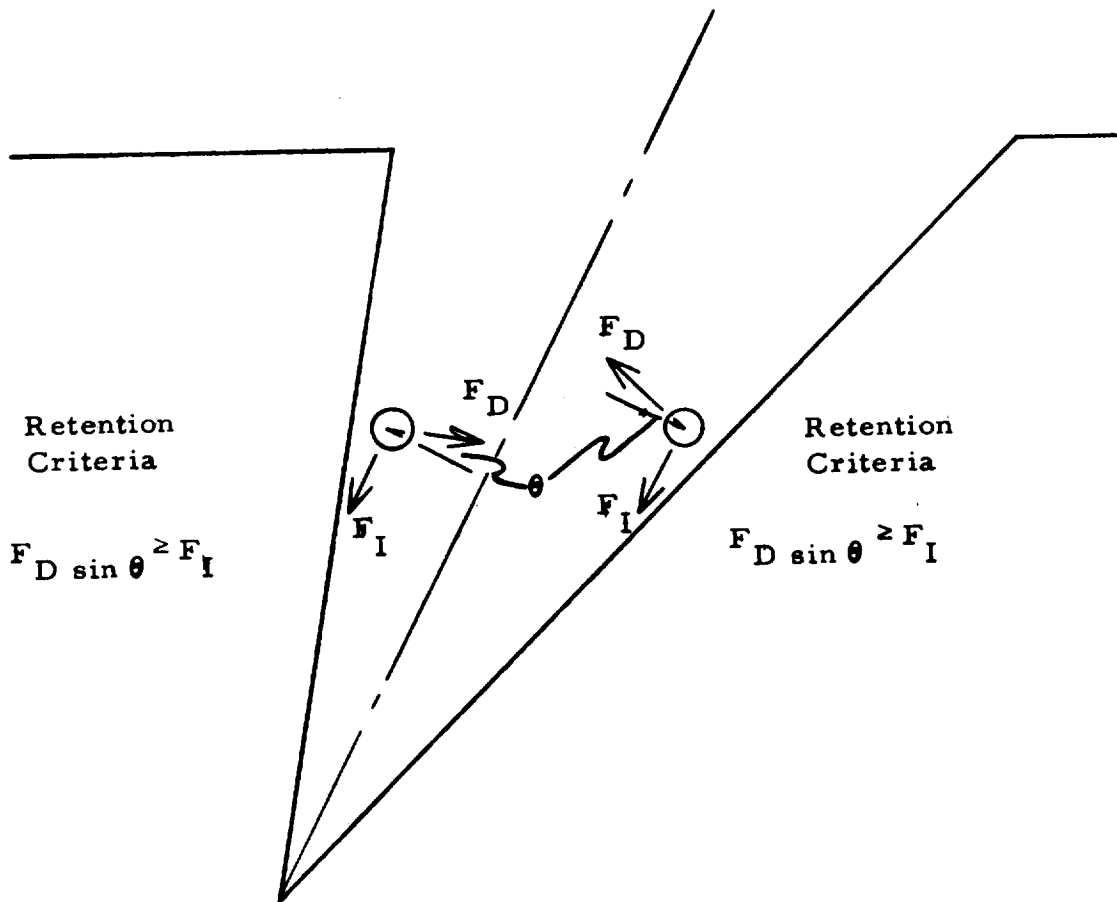


Figure 37. Comparison of Burning Rate Models



Typical Pit in Burning Metallized Propellant

Figure 38. Free-Body Diagram of Particles Evolved from Different Sides of the Pit

Thus, the modification to the metallized burning rate model does not change the angular dependence of the model.

The effect the orientation of the acceleration vector has on the burning rate can be examined by assuming that all the metal is retained and burned, i. e., $G = 1$.

In that case:

$$(r/r_0)_{\max} = [1/(1 - \Pi\beta_0 \cos \Phi)]^{1/2} \cos \Phi \quad (26)$$

Equation 26 gives the theoretical angular dependence of the model. This equation is valid only for angles less than the critical angle. The critical angle is defined as the angle for which r/r_0 is equal to unity and can be considered as the angle beyond which acceleration effects are not felt. Figure 39 illustrates this dependence for two values of burning rate ratio at $\Phi = 0$. The angular dependence is less than a normal cosine curve (shown for comparison in figure 39).

As with the previous metallized burning rate model, no a priori method is available by which the value of β can be computed. As with the previous burning rate model, the data of Anderson (ref. 12) and equation (8) were used to calculate the values of β required to fit the data of Anderson. The results of the computation are presented in figure 40. The close grouping of the β 's for several propellant compositions suggest that the modified model more nearly fits the physical situation for PBAN binder. A comparison of figure 40 with figure 35 shows that the β 's for various compositions when computed using the modified theory exhibit trends similar to each other, while the β 's for various compositions when computed using the old theory do not exhibit the same trends.

Non-Metallized Propellant

The non-metallized burning rate model was developed by extending the granular diffusion flame model (GDFM) of Summerfield (ref. 28) to include acceleration effects. With the GDFM, acceleration-induced burning rate changes must originate from accelerated-induced effects in the gas phase reaction zone. For the pressure range of interest ($p > 600$ psia), gaseous diffusion is the burning rate controlling mechanism. Two effects appear possible: an acceleration-induced pressure difference and acceleration-induced relative motion caused by density inhomogeneities.

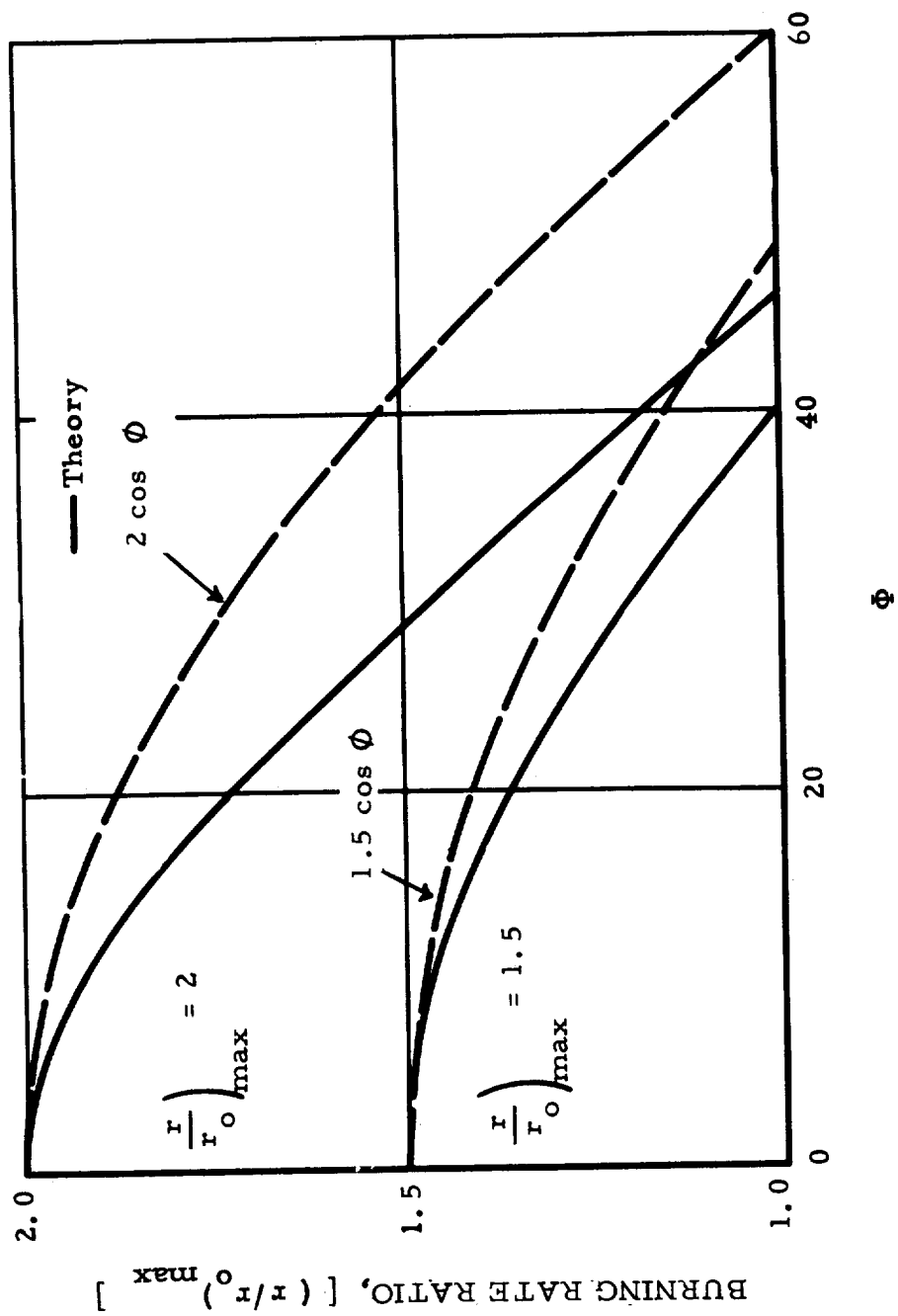


Figure 39. Effect of the Direction of Acceleration Force on the Maximum Burning Rate Ratio.

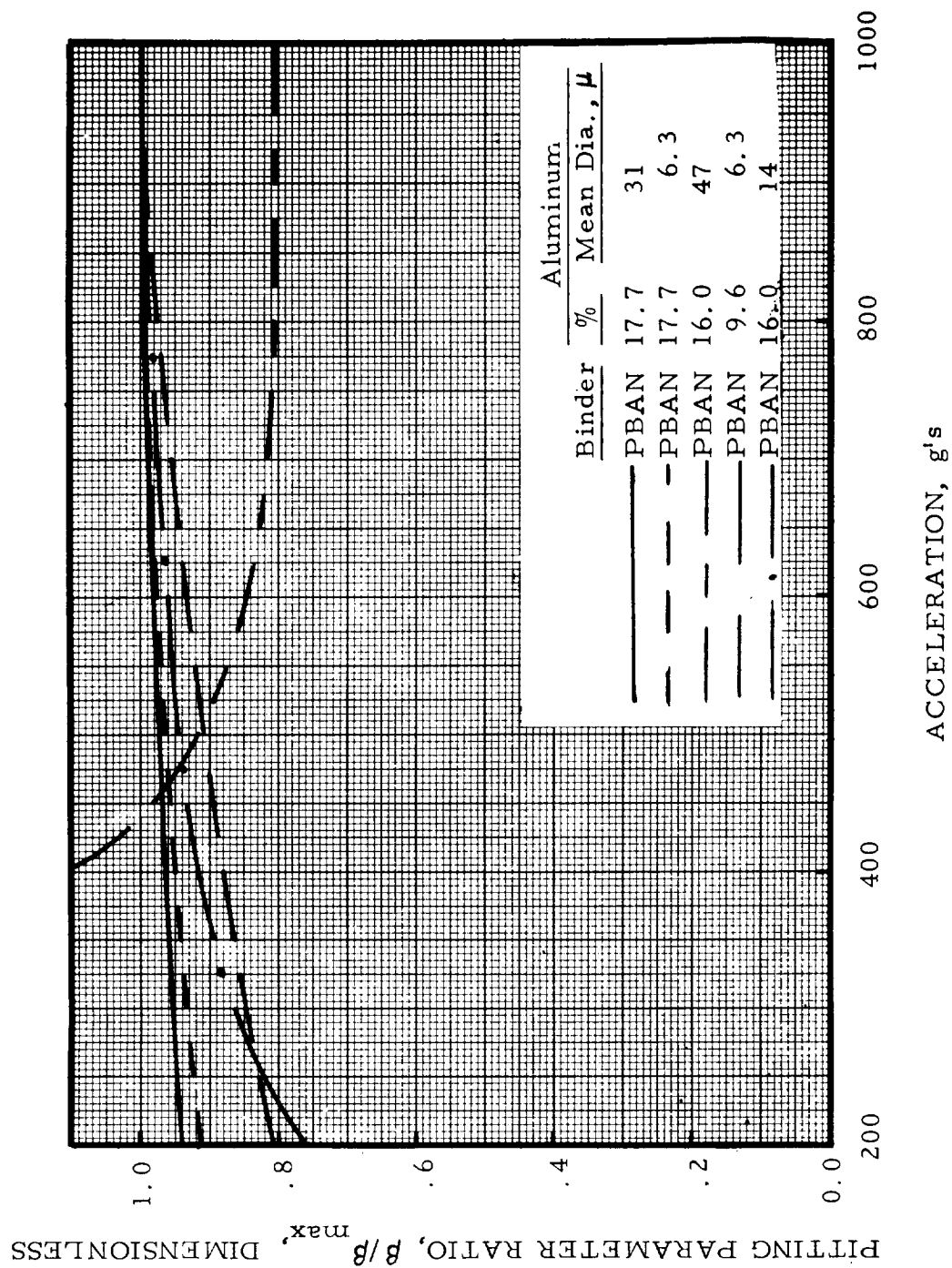


Figure 10. $\beta/\beta_{\max}$ Versus Acceleration for Several Compositions

The acceleration-induced pressure difference is determined by applying the momentum theorem. This yields for $\Phi = 0$

$$p_{gs} - p_f = r_b \rho_p (v_f - v_{gs}) + \delta_r \rho_{gs} a \quad (27)$$

Application of the typical condition data shows that, even for extreme accelerations

$$(a = 50,000 \text{ g}), O(p_{gs} - p_f) = 0.1 \text{ psi}$$

Since this case produces the largest pressure differences, acceleration-induced burning rate changes must arise from density inhomogeneities.

Density inhomogeneities arise from two sources: the heterogeneous structure of the zone and the temperature gradient through it. Thus, the zone may be pictured as having a mean structure dependent only upon distance from the burning surface and a heterogeneous structure dependent upon spatial location and time (i.e., fuel vapor pockets embedded in oxidizer vapor). The acceleration field acting on the mean structure will produce free convection effects. However, it was shown (ref. 33) that, for $a < 1000 \text{ g}$, these effects are negligible. Note that, when $\Phi = 0$ and acceleration effects are a maximum, no free convection is possible. Thus, acceleration-induced burning rate changes must result from the action of the acceleration field on the heterogeneous structure.

The effect of an acceleration field on the heterogeneous structure of the reaction zone will be typified by the action of an acceleration field on a pocket of fuel vapor embedded in a steady flow of oxidizer vapor (figure 41). In the GDFM, burning rate is evaluated from a balance between the energy required to heat up and gasify the propellant and the heat conducted to the burning surface through the reaction zone. This balance yields

$$r_b = \lambda_g (T_f - T_s) / \rho_p \delta_r [c_p (T_s - T_i) - Q_s] \quad (28)$$

In the high pressure limit

$$\delta_r = v_{fv} t_{fv} \quad (29)$$

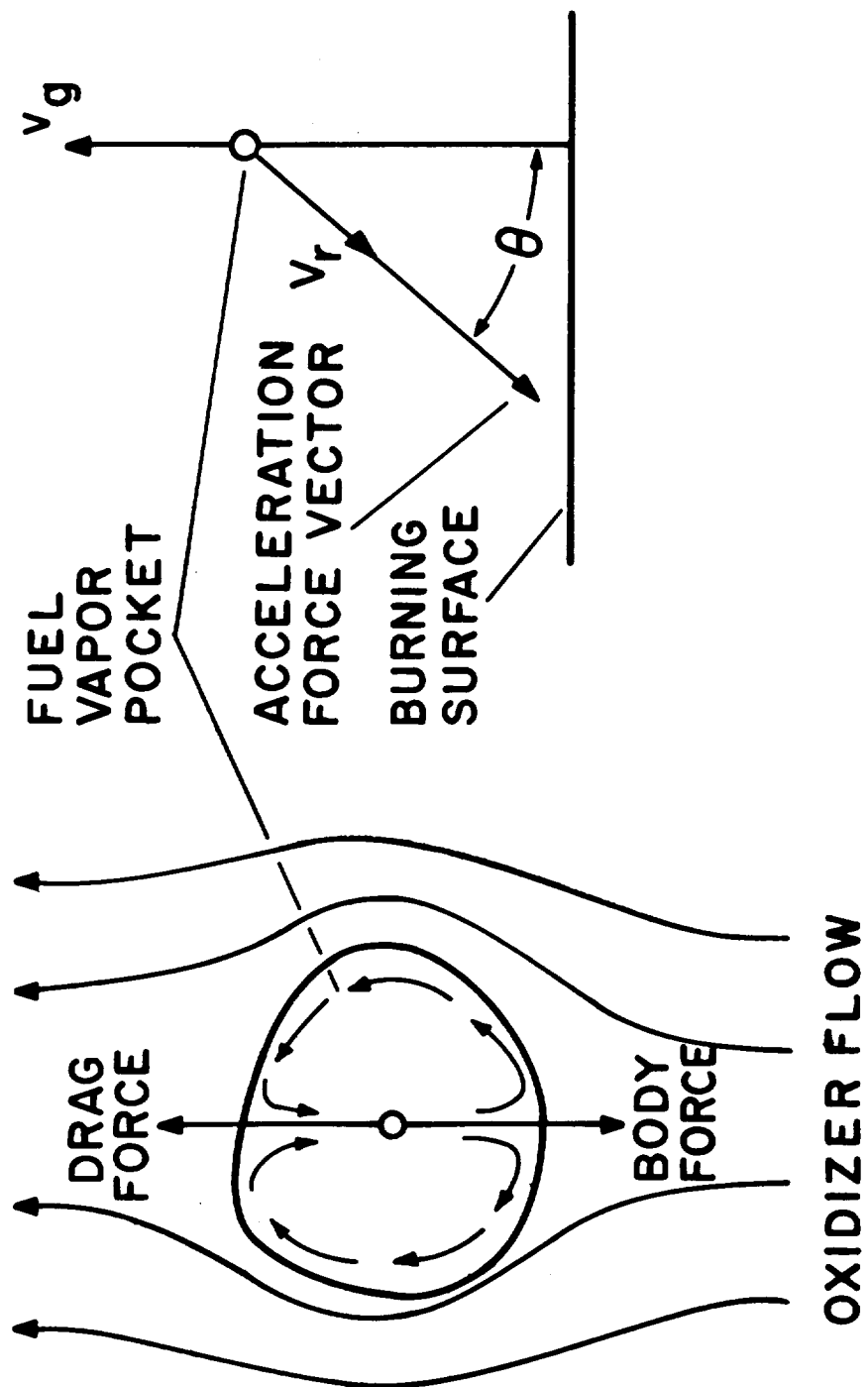


Figure 41. Schematic of Mathematical Model for Non-Metallized Propellants

Therefore, for a particular propellant, $r_b \propto (v_{fv} t_{fv})^{-1}$. Since the densities of the oxidizer and fuel vapors are different, in an acceleration field the fuel vapor pocket will move relative to the oxidizer vapor and the direction of the relative movement will depend upon the acceleration force vector. Therefore, v_{fv} depends upon both the magnitude and direction of the acceleration force vector. In addition, relative movement will increase the rate of inter-diffusion thereby decreasing t_{fv} . Assume now that $\rho_{fv} > \rho_{ox}$; therefore, when $\Phi = 0$, $v_{fv} < v_g$, $t_{fv} < t_{fv,0}$, and $r > r_0$. On the other hand, when $\Phi = 180^\circ$, $v_{fv} > v_g$, $t_{fv} < t_{fv,0}$, and r can be greater than, equal to, or less than r_0 depending on the magnitude of the two preceding effects. Finally, when $\Phi = 90^\circ$, $v_{fv} = v_g$, $t_{fv} < t_{fv,0}$, and $r > r_0$. However, the increase is not so great as when $\Phi = 0$, because $v_{fv} = v_g$. Note that the available data support this trend.

The motion of the pocket is governed by the reversed effective force, $\Lambda \Delta \rho a$ and the drag force, $g C_d A V_r [V_r]$; it is assumed that these forces are equal and opposite. The volume and frontal area of the pocket are $\Lambda \propto d_{fv}^3$ and $\Lambda \propto d_{fv}^2$.

Therefore, the relative velocity is

$$V_r \propto [a d_{fv} \Delta \rho / (\rho_g C_d)]^{1/2} \quad (30)$$

and the velocity of the fuel vapor pocket is

$$v_{fv} = v_g - V_r \cos \Phi \quad (31)$$

The mean rate of mass transfer is

$$\dot{m} = m/t_{fv} = h_D A \Delta C_v \quad (32)$$

Thus, the lifetime of the fuel vapor pocket is

$$t_{fv} = m / (h_D A \Delta C_v). \quad (33)$$

The surface coefficient of mass transfer is usually expressed in terms of the Sherwood number and the concentration difference is ρ_{fv} . Therefore, equation (33) can be rewritten as

$$t_{fv} = d_{fv}^2 / (D_g Sh) \quad (34)$$

The burning rate can be determined by employing equations (28), (31) and (34). This yields

$$r = \left\{ \frac{\lambda_g (T_f - T_s)}{\rho_p [c (T_s - T_i) - Q_s]} \right\} \frac{D_g}{d_{fv}^2} \times \left\{ \frac{C_1 Sh}{r \rho_p / \rho_g - C_2 [a d_{fv} \Delta \rho / (\rho_g C_d)]^{1/2} \cos \Phi} \right\} \quad (35)$$

When $a \rightarrow 0$, $r \rightarrow r_o$. Therefore, the braced term in equation (35) must be $r_o^2 \rho_p / (\rho_g C_1 Sh_o)$. With this result and some rearrangement, equation (35) becomes

$$r/r_o = C_3 \frac{Gr_d^{1/2} \cos \Phi}{C_d^{1/2} Re_o} + \left[C_3^2 \frac{Gr_d \cos^2 \Phi}{C_d Re_o} + \frac{Sh}{Sh_o} \right]^{1/2} \quad (36)$$

Dimensional analysis suggests that $C_d = f(Re_r)$ and $Sh = g(Re_r, Sc)$. The functional form will largely depend on $O(Re_r)$. Since $O(v_{fv}) = O(v_g)$, $O(d_{fv}) = O(d_{ox})$, and the mass of the fuel vapor pocket is constant,

$$O(Re_r) < r_b \rho_p d_{ox} \mu_g^{-1} (\rho_p / \rho_g)^{1/3} \quad (37)$$

Substitution of typical values into equation 37 shows that $O(Re_r) < 20$. Therefore, the relative flow is dominated by viscous forces and should be in the Stokes regime. For the Stokes regime, Redfield and Houghton (ref. 29) found that the drag coefficient for single bubbles in a liquid obey the relationship

$$C_d = Re_r^{-1} \quad (38)$$

They also report that this relationship is obeyed for liquid-liquid drop systems. The mass transfer results of Redfield and Houghton are scattered for low Reynolds numbers. Therefore, a relation for solid spheres, which has the correct asymptotic limit as $Re_r \rightarrow 0$, will be employed,

$$Sh = Sh_o + 0.57 Re_r^{0.5} Sc_g^{0.35} \quad (39)$$

where $Sh_o = 2$. The relative velocity is obtained by substituting equation (39) into equation (36). This yields

$$V_r = C_4 ad_{fv}^2 \Delta \rho / \mu_g \quad (40)$$

A more explicit form of equation (37) can now be obtained by employing the definitions of Re_r and Gr_d and equations (36), (38)-(40). This yields

$$\begin{aligned} r/r_o = C_5 Gr_d \cos \Phi / Re_o + \\ [(C_5 Gr_d \cos \Phi / Re_o)^2 + 0.28 Gr_d^{1/2} Sc_g^{0.35} + 1]^{1/2} \end{aligned} \quad (41)$$

Equation (41) should be valid for the regime where $Gr_d < 10$. For conditions exceeding this criteria equation 41 can be expected to fail because the drag coefficient becomes a function of the size of the bubble as well as the properties of both fluids. Unfortunately, neither data nor analyses are available for C_d when $Gr_d > 10$.

Anderson and Reichenback (ref. 30) investigated the use of the non-metallized burning rate model in fitting their data. Although good agreement was obtained, several anomalies between the extended GDFM and experi-

mental data were observed. These anomalies are concerned with: (1) the relative densities of ρ_{ox} and ρ_{fv} , (2) the pressure dependence predicted by the model, and (3) the behavior of the propellant at low acceleration levels.

The fitting of equation(41) to Anderson's data required the oxidizer density to be greater than the density of the fuel vapor. This is counter to the assumptions made in the derivation because the implications of assuming $\rho_{ox} > \rho_{fv}$ are rather far reaching an examination of the derivation of equation (13) was made using $\rho_{ox} > \rho_{fv}$. This resulted in a negative term, $-C_1 Gr_d \cos \Phi / Re_o$, which was positive for the original assumption of $\rho_{fv} > \rho_{ox}$. Acceleration directed inward would then tend to force the fuel vapor pocket away from the surface while acceleration directed outward would tend to retard the motion of the pocket. Phenomenologically, the flame zone thickness for acceleration out of the surface would be less than the thickness for acceleration directed into the surface. The smaller flame zone thickness implies a higher burning rate. Hence, for $\rho_{ox} > \rho_{fv}$ acceleration effects would be greater for acceleration out of the surface. This is contrary to the trends of available data.

The gaseous products of the ammonium perchlorate and binder decomposition in the granular diffusion flame zone are not known. However, the molecular weight of the gaseous fuel component is approximately twice the molecular weight of the decomposed gaseous oxidizer component. Also, for the assumption that both fuel and oxidant products are at the same temperature and pressure, the perfect gas law indicates that $\rho_{fv} \approx 2\rho_{ox}$. Hence, the available information supports the original assumption that $\rho_{fv} > \rho_{ox}$. The above discussion indicates the correct assumption regarding the relative density of the evolved gases and suggests a more subtle reason for the discrepancy.

The pressure dependency predicted by the model has also been questioned. This was recognized and mentioned in the final report (NASA CR 66218) of Contract NAS-7-406 (ref. 25). The Schmidt number is not strongly pressure dependent, therefore, any pressure dependency must arise from the Grashof and Reynolds numbers. The mass of a fuel vapor pocket is, $m \propto \rho_{g_{fv}} d_{fv}^3$. Hence, the Grashof and Reynolds numbers can be written as

$$Gr_d \propto a \Delta \rho_m / \mu_g^2 \quad (42)$$

and

$$Re_o \propto r_b \rho_p m^{1/3} / (\mu_g \rho_g^{1/3}) \quad (43)$$

Since $\Delta\rho \propto p$, $\rho_g \propto p$, $r_b \propto p^{1/3}$, and viscosity are not strongly pressure dependent, $Re_o \propto f(p)$ and $Gr_d \propto p$. Thus

$$\frac{r}{r_o} = C_6 a p \cos \Phi + (C_6^2 a^2 p^2 \cos^2 \Phi + C_2 a^{1/2} p^{1/2} + 1)^{1/2} \quad (44)$$

Equation(44) shows that, in general, burning rate enhancement is strongly pressure dependent. This is in marked contrast to the weak pressure dependency exhibited by the data. In equation(44) pressure dependency is controlled by the ratio of Gr_d/C_d , since Reynolds number is not a function of pressure. Thus, the pressure dilemma is directly traceable to the assumed drag relationships.

Considerable effort has been expended in an attempt to obtain a more realistic expression for drag. Factors which could affect the drag coefficient are (1) deviation of fuel vapor from a spherical geometry and (2) interaction between combustion fuel vapor and oxidizer. Typical values of the thermo-physical properties suggest that $O(Re_r) < 20$. Thus a Stokes flow regime can be safely assumed. Any deformation of the pocket due to acceleration would tend to "flatten" the sphere into an oblate spheroid with the limiting case a disc. Hence any change in drag coefficient due to the shape must lie between the drag coefficient of a sphere and the drag coefficient of a disc. A comparison of the two extremes is presented in figure 42 for Reynolds numbers of interest (ref. 3). This indicates that no significant variation in drag coefficient will result from changes in the shape of the fuel vapor pocket.

Thus, it is concluded that any changes in drag coefficient must be the result of interaction between the combusting fuel vapor and oxidizer vapor. These changes are the result of the (1) effect of gas/gas two phase flow on the drag and/or effect of a sheath of gases surrounding the fuel vapor pocket. An extensive literature survey failed to contribute any additional information on drag relations between gas and gas bubbles for the regime of interest.

Glick's extended GDFM predicts an increase in burning rate for any value of acceleration. However, experimental data (ref. 12) show no perceptible increase until about 100 g's are felt on the propellant surface. Anderson suggested that a critical acceleration level (below which the model is invalid) exists. The concept and use of a critical acceleration level is

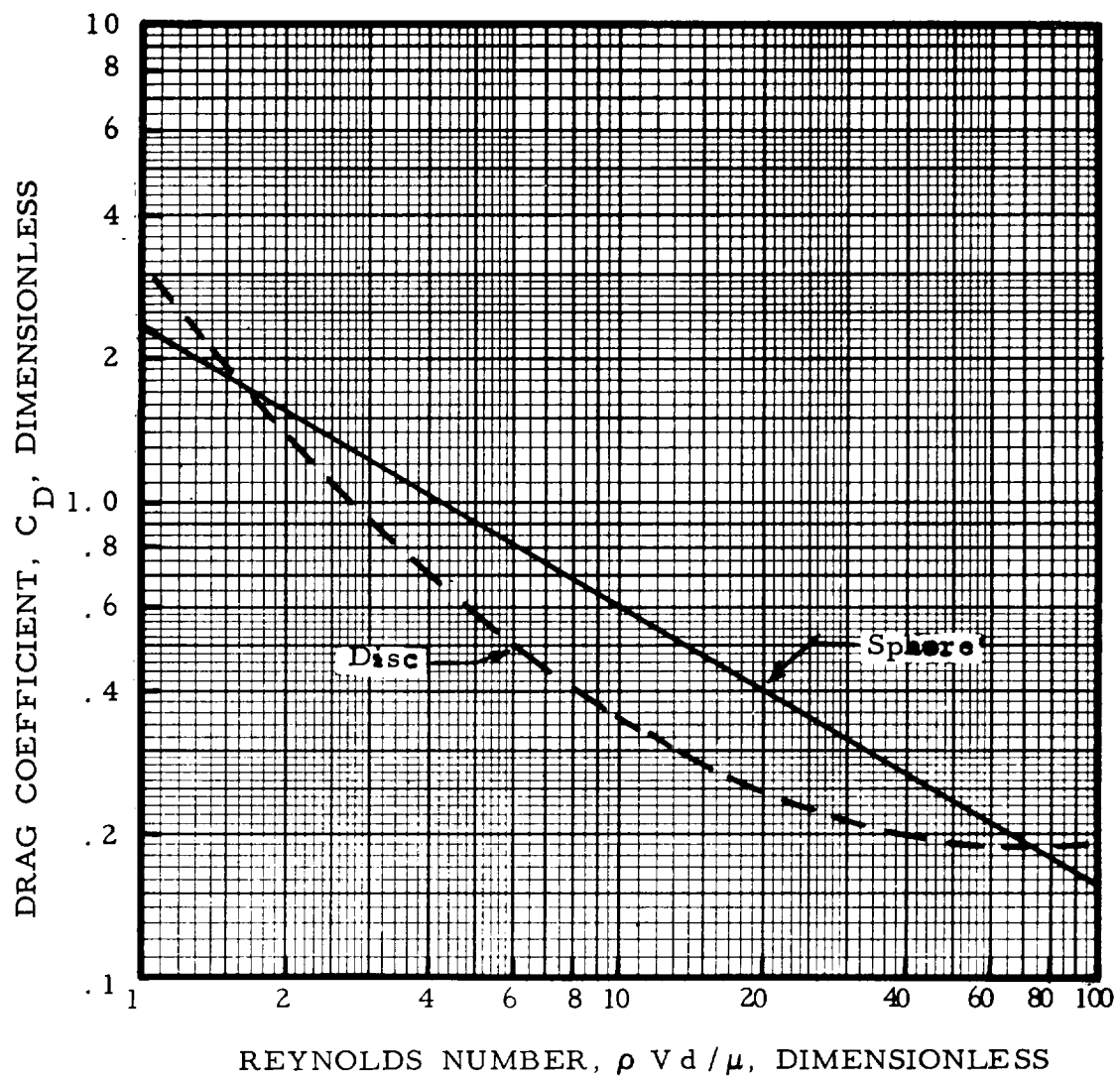


Figure 42. A Comparison of the Drag Coefficients of a Sphere and a Disc

investigated in references 12 and 30. The model discussed in those references postulates that acceleration effects arise from the "granular" structure of the gas phase reaction zone as a result of "slippage" between fuel vapor pockets and the lighter oxidizer vapors (see figure 41). Physically the slippage causes two effects: an increased interdiffusion rate (decreased flame zone thickness) and an altered pocket velocity normal to the burning surface.

Consider further the case for which $\rho_{fv} > \rho_{ox}$. As acceleration increases the velocity of the fuel vapor pocket decreases. A limiting (but finite) acceleration exists for which the pocket velocity is zero and for which the pocket of fuel vapor cannot escape from the burning surface unreacted. Hence, at high accelerations the gas phase reaction zone exchanges its "granular" character for a more stratified one. Moreover, once this point is reached, further increases in acceleration should induce only small changes in burning rate. Therefore, physical reasoning suggests that an upper bound to burning rate exists for which Glick's extended granular diffusion model is valid. Thus, the theory as developed under Contract NAS-7-406 appears to be limited to low accelerations. Existing data suggest that burning rate increases are bounded and that the bound is approached asymptotically.

Assuming Glick's extended GDFM to be valid only in the lower acceleration regime, fitting equation(41) to Anderson's data required $\rho_{fv} > \rho_{ox}$.

Figure 43 illustrates the curve fit for low values of acceleration. By assuming the extended GDFM to be invalid at high acceleration levels a good fit for $0 < a < 300$ g's was obtained. This assumption requires $\rho_{fv} > \rho_{ox}$ and removes one of the anomalies confronting the model.

The behavior of the model for values of acceleration near zero also shows considerable improvement. The dashed line in figure 43 indicates the behavior of the model when curve fitted to Anderson's data at high g levels. Although there is some data scatter the assumption of validity for low acceleration results in more realistic behavior.

The expectation that the present model is invalid for large accelerations raises the question of how an analytical model valid for $0 \leq a \leq \infty$ could be obtained. The asymptotic behavior of the present model differs from the asymptotic behavior of experimental data. Namely the data suggest

$$\lim_{a \rightarrow \infty} \frac{r}{r_0} < \left(\frac{r}{r_0} \right)_{\max} \quad (45)$$

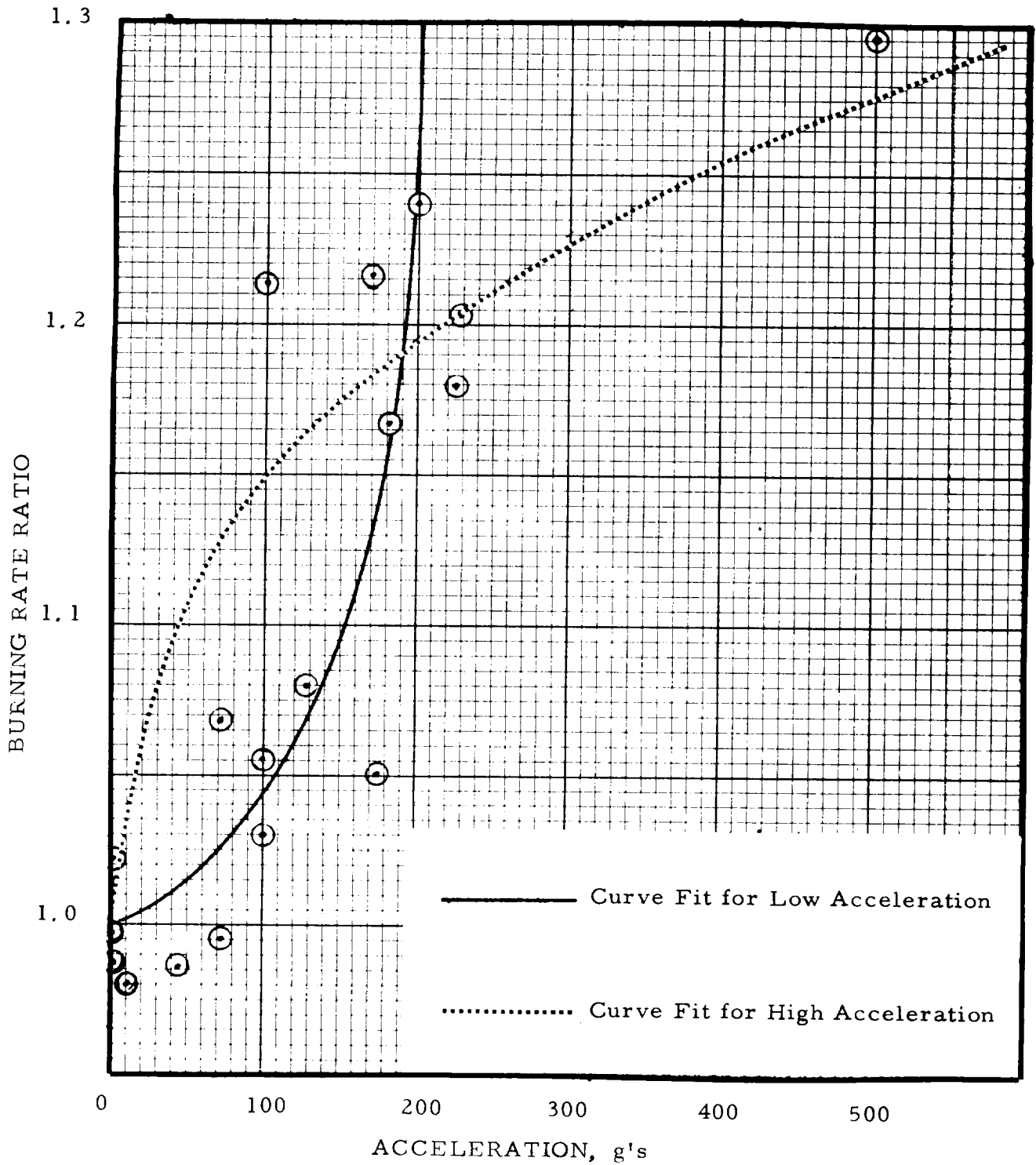


Figure 43 .GDFM for Low Acceleration Level

i. e., the burning rate enhancement has an upper bound in the limit; while in the present extended GDFM

$$\lim_{a \rightarrow \infty} \frac{r}{r_0} = \infty \quad (46)$$

i. e., the burning rate enhancement increases without bound.

Physically, the gas phase reaction zone must become stratified as $a \rightarrow \infty$. The present model, however, retains the granular diffusion zone as $a \rightarrow \infty$. Figure 44 schematically depicts the stratified nature of the GDFM at high accelerations. At higher accelerations the drag force on the fuel vapor pocket is unable to overcome the buoyant force resulting from the density differences between the oxidizer and fuel vapor. Thus, the heavier fuel vapor pocket is held on the burning surface and reacted on the burning surface. The oxidizer vapors are streaming around the sides and top of the fuel vapor pocket. Because there is a concentration difference at these interfaces, mass diffusion is also taking place. The region immediately above the fuel vapor pocket is assumed to be composed of reacting gases and is further assumed to be very nearly the flame temperature of the propellant.

As with the previous burning rate models, a thermal model is proposed for the stratified combustion model. In the GDFM, burning rate is evaluated from a balance between the energy required to heat up and gasify the propellant and the energy conducted to the burning surface through the reaction zone. The same phenomenon is postulated for the stratified combustion model. Then

$$r_b = \frac{k_g (T_f - T_s)}{\rho_p \delta_{fv} [C_p (T_s - T_i) - Q_s]} \quad (48)$$

where

δ_{fv} = fuel vapor pocket thickness

and k_g = Effective thermal conductivity of the fuel vapor

Because the oxidizer vapor is streaming away from the burning surface, no appreciable heat can be conducted through the oxidizer flow back to the burning surface.

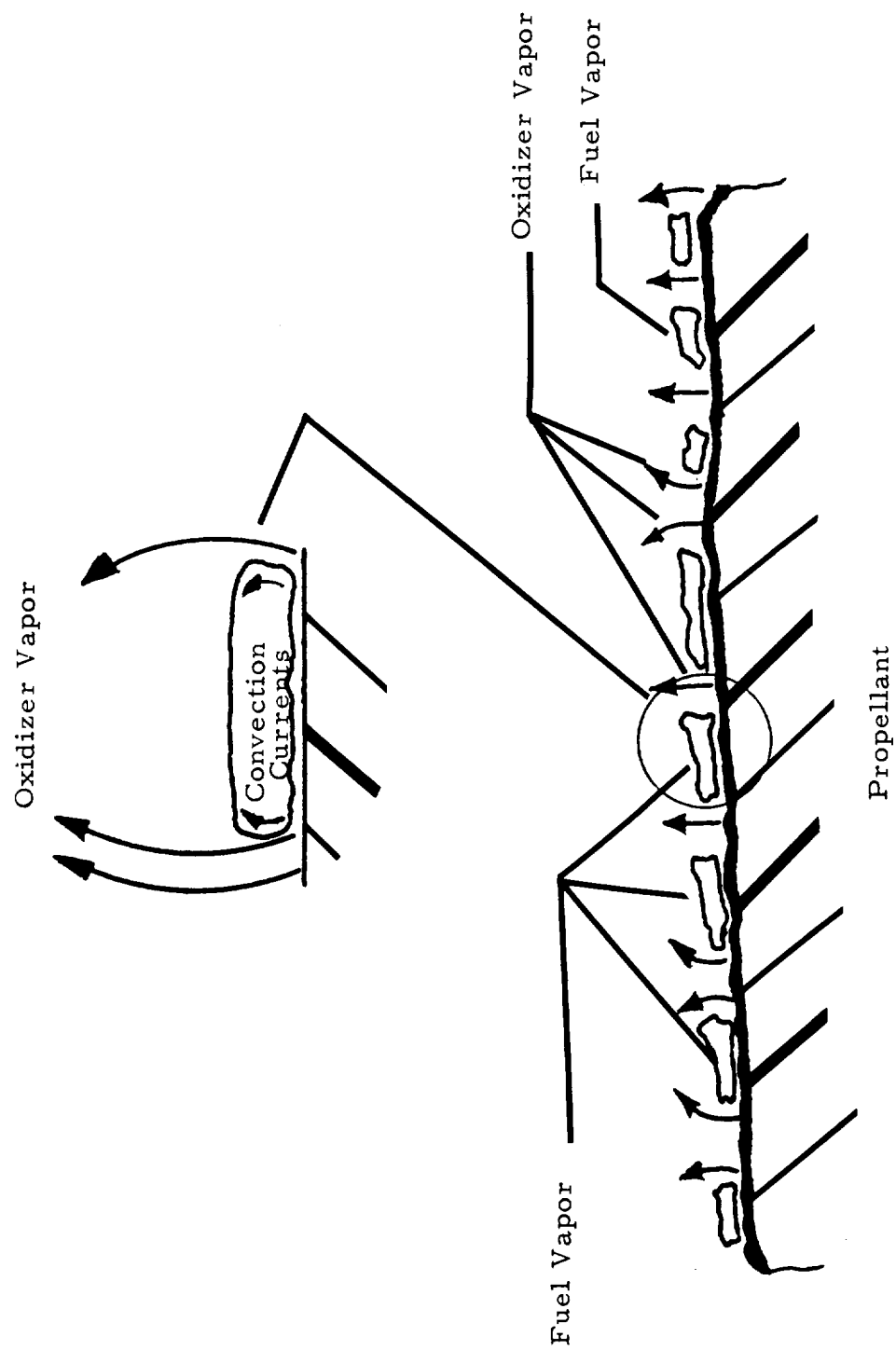


Figure 44. Schematic of Proposed Stratified Combustion Model

Thermal conductivity, k_g , and the thickness of the fuel vapor pocket, δ_{fv} are then the controlling parameters for the stratified combustion model. Summerfield (ref. 28) has previously shown radiation heat transfer to be of secondary importance in the formulation of his model. Thus, conduction and convection are the important modes by which heat is transferred through the fuel vapor pocket. Any convection within the fuel vapor pocket must be of the free convective type. But an unstable density gradient must exist before a free convective flow becomes possible.

In a compressible fluid with a temperature gradient, a density gradient will exist. The heavier fluid will originate at the cold side; the lighter fluid at the hot side. Acceleration will put the heavier fluid to the low potential. If this datum is lower than the cold side, convection currents will exist.

The stratified combustion model will thus have no convection currents since the cold side is actually the propellant and the acceleration vector will keep the colder, more dense fluid "pinned" to the wall. Therefore, a secondary flow system within the fuel vapor pocket can be ruled out. Further, the temperature and density profiles will be stratified.

The thickness of the fuel vapor pocket is governed by the diffusion rate between the fuel vapor and the oxidizer vapor. Fuel vapor is added to the pocket at the rate of

$$\dot{m}_{fv} = V_{fv} \rho_{fv} L^2 \quad (49)$$

where L = characteristic length of fuel vapor pocket, and the rate at which the fuel vapor is diffused into the oxidizer vapor is

$$\dot{m}_{fv} = C_1 h_D \delta_{fv} L \Delta C_v + C_2 h_D' L^2 \Delta C_v \quad (50)$$

where: h_D, h_D' = surface coefficient of mass transfer
 ΔC_v = Concentration difference
 and C_1, C_2 = Constants of proportionality

An expression for δ_{fv} obtained by using equations (49) and (50) and realizing that $\Delta C_v = \rho_{fv}$ is

$$\delta_{fv} = \frac{r_b \rho_p L - C_3 \rho_{fv} h_D'}{C_4 \rho_{fv} h_D} \quad (51)$$

The expression for burning rate in the stratified combustion model then becomes

$$r_b = \frac{k_g (T_f - T_s) (C_3 \rho_{fv} h_D)}{\rho_p [C_p (T_s - T_i) - Q_s] [r_b \rho_p L - C_4 \rho_{fv} h'_D]} \quad (52)$$

Equation(52) which is quadratic in r_b , can be solved to yield the burning rate:

$$r_b = \frac{C_4 \rho_{fv} h'_D}{2 \rho_p^2 [C_p (T_s - T_i) - Q_s]} + \left\{ \left(\frac{C_4 \rho_{fv} h'_D}{2 \rho_p^2 [C_p (T_s - T_i) - Q_s]} \right)^2 + \frac{C_s k_g (T_f - T_s) h_D \rho_{fv}}{\rho_p^2 [C_p (T_s - T_i) - Q_s] L} \right\}^{\frac{1}{2}} \quad (53)$$

But ρ_p , C_p , T_s , T_i and Q_s are all concerned with the propellant in solid form. Incorporating these quantities into the constants C_5 and C_6 the expression for burning rate becomes

$$r_b = C_5 \rho_{fv} h'_D + \left\{ (C_5 \rho_{fv} h'_D)^2 + \frac{C_6 k_g \rho_{fv} h_D}{L} \right\}^{\frac{1}{2}} \quad (54)$$

The Schmidt and Sherwood numbers are defined respectively as

$$S_c = \frac{\mu}{\rho D} \quad (55)$$

and

$$S_h = \frac{h_D L}{D_g} \quad (56)$$

Then:

$$\rho h_D = \frac{\mu}{L} \frac{S_h}{S_c} \quad (57)$$

and the burning rate equation reduces to

$$r_b = (L)^{-1} \left\{ C_5 \frac{S'_h}{S'_c} + \left[(C_5 \frac{S'_h}{S'_c})^2 + C_b k_g \frac{S_h}{S_c} \right] \right\}^{1/2} \quad (58)$$

The pressure dependence predicted by equation (58) is of interest. The Schmidt number is largely independent of pressure. Kinetic theory predicts the thermal conductivity of a gas to be

$$k_g = \frac{9\gamma - 5}{4(\gamma - 1)} \mu R \quad (59)$$

Therefore, thermal conductivity of a gas is independent of pressure.

Reference 38 presents a relationship for the mass transfer to or from a sphere under forced convection conditions:

$$Sh = 2.0 (1 + 0.276 Re^{1/2} Sc^{1/3}) \quad (60)$$

For the stratified combustion model $Re \propto \rho V$ since L and μ are independent of pressure. The Reynolds number dependence upon pressure is then:

$$\begin{aligned} Re &\propto (p) \frac{\rho_p}{\rho_g} r_b \\ &\propto p^{1/3} \end{aligned} \quad (61)$$

Then the dependence of the Sherwood number is:

$$Sh \propto p^{1/6} \quad (62)$$

since the Schmidt number is largely independent of pressure.

Thus

$$r_b \propto p^{1/6} \quad (63)$$

This indicates that the pressure dependency predicted by the stratified combustion model is in a feasible range.

With all other parameters constant, equation 58 reduces to

$$r_b \propto (L)^{-1} \quad (64)$$

The parameter, l , is dependent upon the distance between the oxidizer particles. The smaller the particle size the smaller the distance between the particle and hence the smaller l . Burning rate as predicted by equation (58) increases as l decreases. This is in accord with experimental data.

Equation 58 predicts a reasonable pressure dependence and a reasonable dependence on the oxidizer particle size. The stratified combustion model formulation involves two diffusion coefficients; one is evaluated for the oxidizer/fuel vapor interface on the pocket side and the other is evaluated in the turbulent region on the oxidizer/fuel vapor interface on the top of the fuel vapor pocket. The latter is subjected to a high acceleration while the direction of mass diffusion of the former is perpendicular to the acceleration vector. The difference between the two mass diffusion coefficients is not known. Consequently, equation (58) cannot be further reduced.

Conclusions

The study of the burning rate models indicates:

Metallized Propellant:

1. Values of the pitting parameter, β , when calculated for the original model and Anderson's data showed several deviations.
2. The modification to the model retained the pertinent features of the original model but resulted in a better correlation of Anderson's data.
3. The modification to the model did not change the dependence of the burning rate ratio on angular orientation of the acceleration vector.

Non-Metallized Propellant:

1. An extensive literature survey did not reveal any new information on mass diffusivities and drag coefficients for gas/gas systems.

2. Glick's extended GDFM is valid only for low levels of acceleration. The assumed density ratio ($\rho_{fv} / \rho_{ox} > 1$) will fit Anderson's data if an upper limit of acceleration is assumed.
3. Physical reasoning suggests that at higher acceleration's the flame zone takes on a stratified appearance.
4. A stratified combustion model was proposed and formulated. However, the lack of diffusivity information made comparison with burning rate data impossible.

Burning Rate Data

The second objective of this Phase was the correlation of new experimental data with parameters obtained from the analytical models. No new data on either metallized or non-metallized propellant has become available. However, in an effort to glean as much information as possible from the data of Anderson, additional analysis has been performed on the data.

One of the most interesting and least understood phenomena encountered in the study of acceleration effects on burning rate is the significant change in burning rate augmentation due to binder composition and metal additive content. Anderson presents strand data for metallized PBAN and CTPB propellant as well as for non-metallized PBAN propellant. The effect of acceleration on the burning rate coefficient, α_t , and the pressure exponent, n , has been calculated from the data of Anderson. The results for these propellants are presented in figure 45. The burning rate coefficient was calculated by using

$$r_b = \alpha_t \left(\frac{P}{500} \right)^n$$

With α_t known for a given acceleration it is possible to compute an average value for n over the three pressures, 500, 1000, and 1500 psi. The technique was applied to Anderson's data. The non-metallized PBAN propellant data was also analyzed using the above procedure. The effects of acceleration on α_t and n for the non-metallized propellant are presented in Figure 46. A very interesting comparison can be made between the metallized and non-metallized PBAN propellants. The ratio of burning rate coefficients, α_t/α_{t_0} , for both PBAN propellants follows the same general trends -- although the magnitudes vary. However, the trends for $(n-n_0)$ for the non-metallized and metallized PBAN propellants are vastly different. While no attempt is made to explain the difference, they are indicative of different mechanisms acting on the propellants.

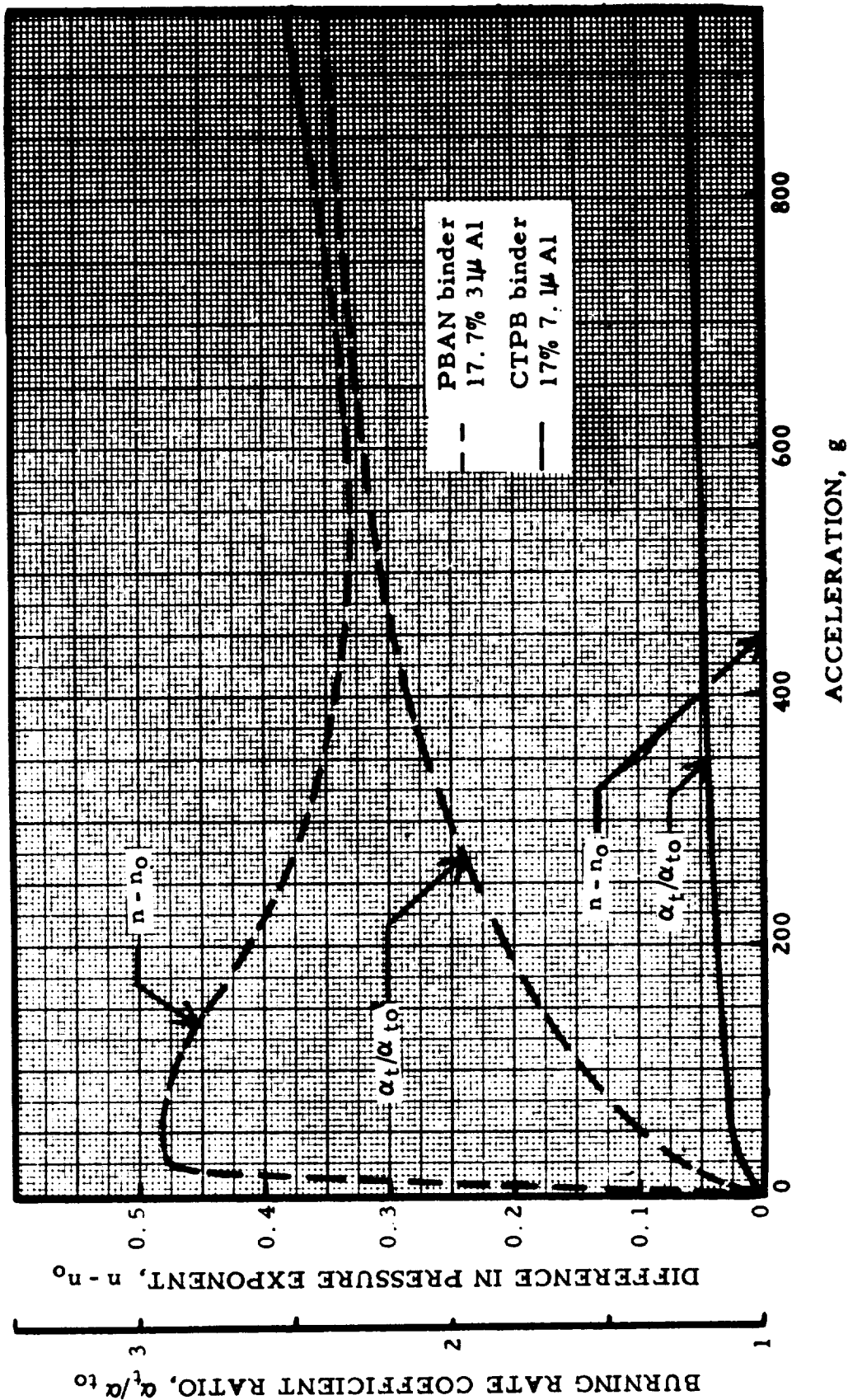


Figure 45. α_t and n for Metallized PBAN and CTPB Propellants

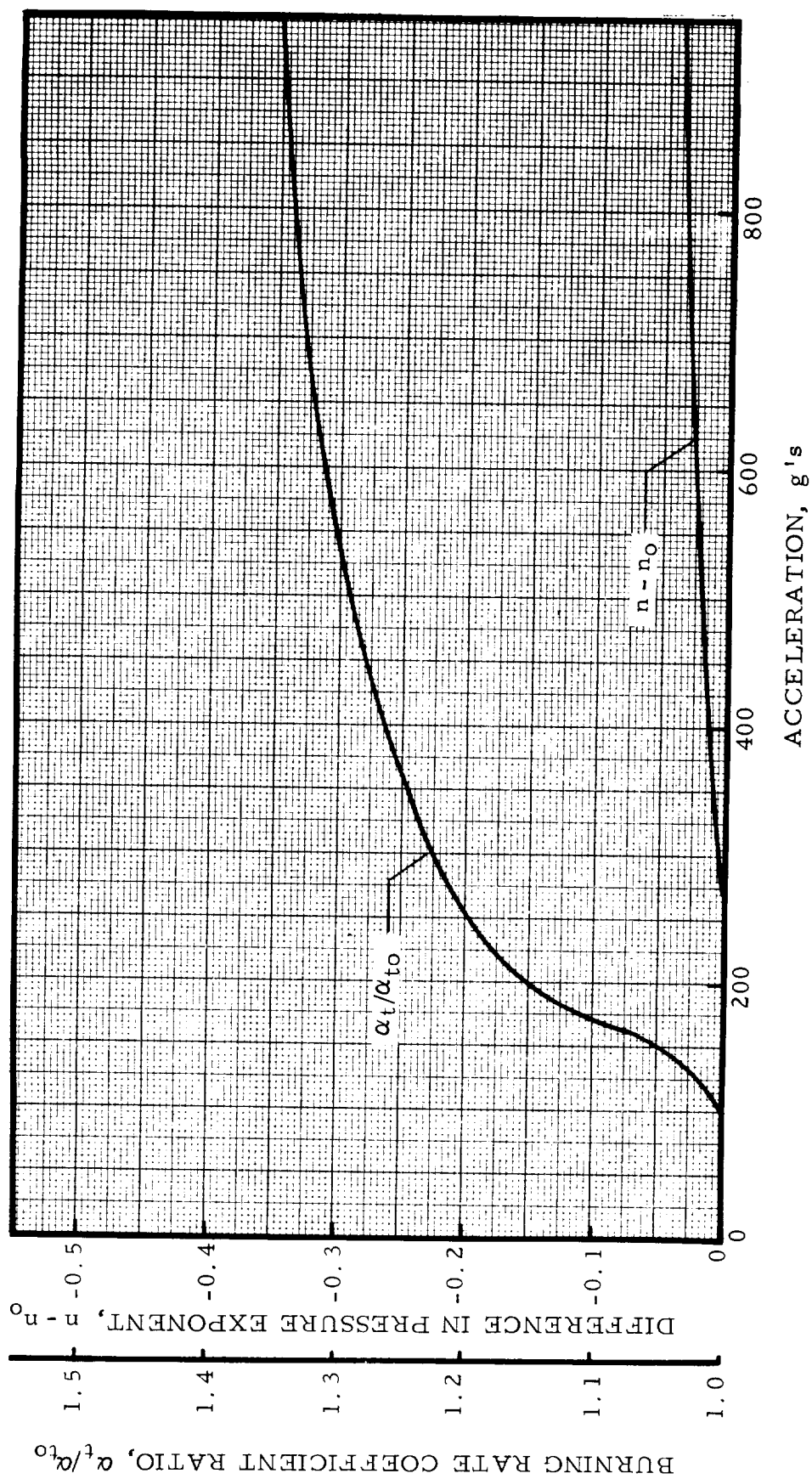


Figure 46. α_t and n for Non-Metallized PBAN Propellants

CONCLUSIONS

Analysis of Flow in Circumferential Slots with Spin Effects

The study of the effects of spin on the performance of motors with circumferential slots indicates that:

1. Metal/metal oxide particles can be retained within the slot.
2. The amount of metal/metal oxide retained depends upon the mean particle size and the particle distribution.
3. Because of limited information on the size and distribution of particles, any attempt to predict the metal/metal oxide retained within a slot is qualitative in nature.
4. The pressure drop across circumferential slots is nearly independent of spin rate.
5. Regardless of the slot depth and location, spin effects on ballistic performance can be neglected for a first order analysis.

Also, two computer programs were generated. One is capable of computing the flow field of a circumferential slot in a spinning rocket motor and then, using the flow field results, predicting particle trajectories. The other is an internal ballistics program for predicting the effects of circumferential slots in spinning rocket motors.

Surface Regression Analysis

The following conclusions may be drawn from the surface regression analysis.

1. A computerized surface regression technique for predicting the evolution of a burning surface under the influence of a radial acceleration field has been successfully developed.
2. This analysis predicts that the specific effects of spin rate on surface regression are (1) the non-uniform surface evolution of star grains, (2) increasing motor pressures with spin rate, and (3) increasing progressiveness with spin rate of pressure traces for star grains. These effects, of course, were expected and agree with experimentally observed trends.

3. A comparison of the surface evolution history for an actual motor with a prediction of the surface evolution history for the same motor should be made to verify the analysis.

Theoretical Heat Transfer

The following conclusions may be drawn from the analyses of heat transfer to the nozzle wall and head-end dome of a spinning rocket motor.

1. A computer program based on a semi-empirical analysis of nozzle wall heat transfer in a spinning rocket motor has been successfully developed.
2. Nozzle wall heat transfer coefficients are predicted to increase significantly with spin rate, as expected. The increase in coefficients is primarily due to the increase in motor pressure with spin rate; however, a significant portion of the increase is due to the increase in the total relative velocity between the swirling gases and the wall.
3. The maximum percentage increase in the Nusselt number due to spin effects occurs at the nozzle entrance or inlet and the percentage increase declines from that point through the remainder of the nozzle.
4. A semi-empirical analysis of head-end dome heat transfer which does not account for secondary flows in the motor chamber has been developed and computerized.
5. The head-end heat transfer analysis predicts sharp increases in heat transfer coefficients in a region near the center of the head-end dome. This trend is in agreement with known experimental results and thus supports qualitatively the model upon which the head-end analysis was based.
6. Comparison of experimental heat transfer data with predictions from the nozzle wall and head-end dome computer programs should be made to further evaluate these analyses.

Effect of Acceleration on Burning Rate

The study of the burning rate models indicates:

Metallized Propellant:

1. Values of the pitting parameter, β , when calculated for the original model and Anderson's data showed considerable deviations.
2. The modification to the model retained the pertinent features of the original model but resulted in a better correlation of Anderson's data.
3. The modification to the model did not change the dependence of the burning rate ratio on angular orientation of the acceleration vector.

Non-Metallized Propellant:

1. An extensive literature survey did not reveal any new information on mass diffusivities and drag coefficients for gas/gas systems.
2. Glick's extended GDFM is valid only for low levels of acceleration. The assumed density ratio ($\rho_{fv}/\rho_{ox} > 1$) will fit Anderson's data if an upper limit of acceleration is assumed.
3. Physical reasoning suggests that at higher acceleration's the flame zone takes on a stratified appearance.
4. A stratified combustion model was proposed and formulated. However, the lack of diffusivity information made comparison with burning rate data impossible.

APPENDIX A

INTERNAL BALLISTICS
SWIRL BALLISTICS PROGRAM

Computer Program Development

An equilibrium-at-fixed-time ballistics program including swirl effects was developed under Contract NAS-7-406 for the Langley Research Center. For completeness, a description of the development of the swirl ballistics program is presented here. The final report for Contract NAS-7-406 (NASA Report 66218 or Thiokol Report 42-66) presents a complete discussion of the gas dynamic aspects of swirling flow in the port and nozzle of a rocket motor, as well as a description of the equilibrium-at-fixed-time ballistics program.

Figure A1 shows the system that is under consideration and illustrates the coordinate system, the control surface, and the nomenclature that are employed. Conventional cylindrical coordinates are employed with r denoting the burning surface and z denoting the axial distance from the head-end of the motor. The radius of the burning surface is r and the radius of the throat is r^* . The components of velocity in the r , θ , and z directions are V_r , V_θ , and V_z , respectively, and s denotes the distance along the surface of the grain.

A typical rocket motor has a radial Reynolds Number ($Re_r = \rho V_r r / \nu$) of 500. Experiments with vortex chambers (ref. 32) have shown that, for μ radial Reynolds numbers greater than 5, viscous effects may be neglected. Thus, the flow is assumed to be inviscid. Additionally, it was assumed that the flow was steady, axisymmetric, and adiabatic, and that the fluid was calorically perfect. With these assumptions the conservation equations for the control volume illustrated in figure A1 become

$$\text{Continuity:} \quad \oint \rho \bar{V} \cdot d\bar{A} = 0 \quad (\text{A1a})$$

$$\text{Energy:} \quad \oint \rho H \bar{V} \cdot d\bar{A} = 0 \quad (\text{A1b})$$

$$\text{Axial Momentum:} \quad \oint \rho V_z \bar{V} \cdot d\bar{A} + \oint P dA_x = 0 \quad (\text{A1c})$$

$$\text{Moment of Momentum:} \quad \oint \rho r V_\theta \bar{V} \cdot d\bar{A} = 0 \quad (\text{A1c})$$

$$\text{State:} \quad P_0 = \rho_0 RT \quad (\text{A1c})$$

A radial velocity, pressure, and density profile must be established before equations (A1) can be integrated. An order of magnitude analysis of the inviscid equations of motion suggest that to a first approximation both the total pressure and the total enthalpy are the same for each stream line in the flow field; i. e., the flow field is nearly isentropic. Thus, to a good approximation the required velocity profile may be obtained from potential flow.

In an axisymmetric flow field the curl of the velocity vector is zero. That is

$$\frac{\partial}{\partial r} (r V_{\theta}) = 0 \quad (a) \quad \frac{\partial}{\partial z} (r V_{\theta}) = 0 \quad (c) \quad (A2)$$

$$\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} = 0 \quad (b)$$

Equations (A2a) and (A2c) yield the radial variation of the tangential flow field:

$$V_{\theta} = \frac{K}{r} \quad (A3)$$

Equation (A2b) suggests that the axial velocity is essentially constant across the port:

$$V_z = U(z) \quad (A4)$$

and the corresponding radial profiles for pressure and density are

$$P = P_o \left\{ 1 - \frac{\gamma-1}{2} M_{\theta}^2 \left[\left(\frac{r_o}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma}{\gamma-1}} \quad (A5)$$

$$\rho = \rho_o \left\{ 1 - \frac{\gamma-1}{2} M_{\theta}^2 \left[\left(\frac{r_o}{r} \right)^2 - 1 \right] \right\}^{\frac{\gamma-1}{\gamma}} \quad (A6)$$

where $M_{\theta} = K/(r_o C_o)$

Equations (A5) and (A6) show that at some radius, r_c , the density and pressure become negative. Therefore, only flow in the region $r_c \leq r \leq r_o$ can have physical significance. The lower limit of integration then becomes

$$r_c = r_o \left[\frac{\frac{\gamma-1}{2} M_\theta^2}{1 + \frac{\gamma-1}{2} M_\theta^2} \right]^{\frac{1}{2}} \quad (A7)$$

A set of useful swirl parameters can be generated by defining

$$\eta = \left(\frac{r}{r_o} \right)^2 \quad (A8)$$

Then

$$\left\{ 1 - \frac{\gamma-1}{2} M_\theta^2 \left[\left(\frac{r_o}{r} \right)^2 - 1 \right] \right\} = 1 - \frac{\eta_c}{1-\eta_c} \frac{1-\eta}{\eta} + f(\eta_c, \eta) \quad (A9)$$

and

$$\psi = \int_c^1 [f(\eta, \eta_c)]^{\frac{1}{\gamma-1}} d\eta \quad (A10a)$$

$$\Phi = \int_{\eta_c}^1 [f(\eta, \eta_c)]^{\frac{\gamma}{\gamma-1}} d\eta \quad (A10b)$$

Then using equations (A1) through (A10) and according them a treatment similar to the treatment Shapiro used in the derivation of the one-dimensional coefficients a set of influence coefficients for "swirl" flow was developed. The influence coefficients for swirling flow are presented in Table A1 (see reference 33 for detailed development).

The differential equations describing the flow in the port of a spinning rocket motor written in terms of the influence coefficients are

$$\frac{dM_z^2}{ds} = M_z^2 \left[\left(G_{11} + G_{13} \frac{H_s - H}{C_p T_o} + G_{14} \frac{V_{\theta s} - V_{\theta o}}{V_{\theta o}} \right) \frac{2\pi r_o r_b}{\dot{m}} + \frac{G_{15}}{A_o} \frac{dA_o}{ds} \right] \quad (A11a)$$

$$\frac{dM_\theta^2}{ds} = M_\theta^2 \left[\left(G_{21} + G_{23} \frac{H_s - H}{C_p T_o} + G_{24} \frac{V_{\theta s} - V_{\theta o}}{V_{\theta o}} \right) \frac{2\pi r_o r_b}{\dot{m}} + \frac{G_{25}}{A_o} \frac{dA_o}{ds} \right] \quad (A11b)$$

$$\frac{dP_o}{ds} = P_o \left[\left(G_{31} + G_{33} \frac{H_s - H}{C_p T_o} + G_{34} \frac{V_{\theta s} - V_{\theta o}}{V_{\theta o}} \right) \frac{2\pi r_o r_b}{\dot{m}} + \frac{G_{45}}{A_o} \frac{dA_o}{ds} \right] \quad (A11c)$$

$$\frac{dT_o}{ds} = T_o \left[\left(G_{41} + G_{43} \frac{H_s - H}{C_p T_o} + G_{44} \frac{V_{\theta s} - V_{\theta o}}{V_{\theta o}} \right) \frac{2\pi r_o r_b}{\dot{m}} + \frac{G_{35}}{A_o} \frac{dA_o}{ds} \right] \quad (A11d)$$

$$\text{and} \quad \frac{dm}{ds} = 2\pi r_o r_b \rho_b \quad (A11e)$$

The integration of equations 11 is performed numerically using a fourth-order Runge-Kutta procedure.

Experimental evidence indicates that nozzle plugging due to vortex action may occur when a swirling flow is passed through a nozzle. The plugging phenomena must be treated mathematically if the effect of nozzle plugging is included in the ballistic program. Various theories are available depending upon the assumed form of the vortex flow. An end-burning grain would initially possess a forced vortex, $V_\theta = r\Omega$, while a cylinder propellant configuration would give rise to a potential vortex. $V_\theta = \frac{K}{r}$.

The theory of Mager (ref. 11) assumes a potential vortex flow and the influence coefficients were developed using a potential vortex. Hence, the Mager analysis will be used to account for nozzle plugging effects.

In particular Mager's analysis shows that the ratio of mass discharged with swirl to mass discharged without swirl is

$$\frac{\dot{m}}{\dot{m}_{1-D}} = f(\alpha^*, \gamma) \quad (A12)$$

$$\text{where} \quad \alpha^* = \frac{K}{r^* A_t} \sqrt{\frac{\gamma - 1}{2}} \quad (A13)$$

Whence Mager's analysis provides an estimate of the nozzle plugging due to swirl.

The preceding contains the theoretical background for determining the flow fields in the port and nozzle of a spinning rocket motor. By employing a mass balance between mass generated and mass discharged the equilibrium operating point can be determined.

A block diagram of the resulting computer program is illustrated in figure A2. The basic computational procedure is:

1. Estimate the head-end pressure BLOCK 1
2. Integrate equation (A11) along the port to determine mass flow, K, and total conditions BLOCK 2
3. Compute the mass flow through the nozzle using Mager's theory BLOCK 3
4. Iterate on head-end pressure until the mass flow balances BLOCK 4

Parametric Study

A parametric study of spin effects in rocket motors was made using the equilibrium-at-fixed-time ballistics computer program developed under Program 7-406. Figure A3 illustrates the geometry of the "motor" simulated in this study. The pertinent propellant properties are shown in figure A3.

Chamber conditions can be affected in three different ways: (1) swirl effects in the port due to tangential velocity, (2) choking of the throat due to swirl, and (3) effects of burning rate enhancement caused by acceleration. For this particular configuration, swirl effects in the port were small for spin rates up to 4000 RPM. At 4000 RPM, 908 g's of acceleration are present on the grain surface. It was understood that 1000 g's was the upper bound on acceleration of interest to NASA-Langley. In all, some 45 cases were run (and the results examined) to generate data for this study.

Since port effects due to swirl were small for the grain geometry used, the port effects and nozzle choking effects will be presented together. Both effects arise from the gas dynamics of the system and, are, thus, related. The effects of swirl will be presented on a head-end pressure versus spin rate basis.

Figure A4, which considers gas dynamic effects and nozzle plugging, illustrates the increase in head-end pressure required to maintain equilibrium as the spin rate increases. Both port effects and nozzle plugging effects are small to about 1500 rpm; then a gradual, but noticeable, increase in head-end pressure begins. As the spin rate increases, the head-end pressure required for equilibrium increases; and, as the spin rate increases, the increase in pressure per rpm also increases.

The next effect examined is the burning rate augmentation due to radial acceleration. The qualitative effects of burning rate augmentation due to acceleration are well documented and won't be examined here. Since the metal content of the propellant causes a wide variation in the magnitude of the enhancement, two types of propellant (with the same base rate) were considered. A highly metallized propellant ($r/r_0 = 3.72$ at 908 g's) and a non-metallized propellant ($r/r_0 = 1.33$ at 908 g's) were used to examine the effects of burning rate enhancements due to spin. A burning rate dependence on acceleration similar to that predicted by equation (1), page 44, was used in this study.

The results obtained for the high metal content propellant are presented in figure A5. The consideration of burning rate effects causes a drastic increase in head-end pressure, especially at high spin rates and clearly illustrates the catastrophic results that the negligence of spin effects could cause. Another interesting parameter is the pressure drop down the grain. This pressure drop results from acceleration of the evolved gases and increases as the mass generated increases. Figure A6 is a plot of the pressure drop versus the spin rate for a highly metallized propellant. It follows very closely the head-end pressure versus the spin rate. The inflection point on the pressure-drop curve is caused by the asymptotic behavior of the burning rate enhancement at high levels of acceleration.

The next propellant examined is a non-metallized propellant with relatively small enhancement rates. Figure A7 presents the results of a study using the non-metallized propellant. The available data for non-metallized propellants suggest a critical acceleration below which spin induced burning rate changes are small. The data of Anderson exhibited this characteristic. The relatively flat trace for acceleration less than 1000 g's resulted from the low magnitudes of the burning rate enhancements. The non-metallized propellants also exhibit an asymptotic behavior with acceleration thus resulting in an inflection point for the non-metallized propellant. Both the highly metallized and the non-metallized propellants exhibit the same trends with increasing acceleration, but the effects are much more severe for the metallized propellants.

The next phase of the study involved varying α_t , the burning rate coefficient and n , the pressure exponent, in the empirical equation $r = \alpha_t P_n$. Figure A8 was generated by doubling the pressure exponent (from 0.23 to 0.46). In addition to the extremely high head-end pressures encountered, a noticeable increase in head-end pressure with spin rate was experienced. The behavior of this curve can be contrasted with the behavior of the curve in Figure A4.

The burning rate coefficient was increased from 0.06 to 0.09. The resulting curve is presented in figure A9. The behavior of figure A9 can be contrasted with the behavior of figure A4. The increased sensitivity of the head-end pressure to the pressure exponent can be seen. Spin effects in figure A6 starts at 500 rpm, but in figures A4 and A9 no appreciable effect is noted until about 1500 rpm.

The results of figures A4 through A7 are indicative of the trends to be expected when parameters are varied in motors subjected to moderate spin rates and accelerations. Although no new or unexpected trends are in evidence, it is felt that this study qualitatively indicates the areas where spin effects must be considered.

TABLE A1.
INFLUENCE COEFFICIENTS FOR SWIRLING FLOW

j →	Independent Variable →	1 $\frac{d\dot{m}}{\dot{m}}$	2 $-2\gamma \frac{d\dot{m}}{\dot{m}}$	3 $\frac{d\bar{H}}{C_{p_0}}$	4 $\frac{dA_0}{A_0}$	5 $\frac{d\bar{V}_\theta}{V_{\theta_0}}$
	Dependent Variable ↓					
1	$\frac{dM_z^2}{M_z^2}$	$\frac{\gamma-1}{2} \frac{M_z^2}{1-M_z^2} \zeta$	$\frac{\gamma-1}{2} \frac{M_z^2}{1-M_z^2} \zeta$	$1 + \frac{\bar{\psi}}{\phi} \gamma M_z^2 + \frac{1-M_z^2 \zeta}{1-M_z^2}$	$\frac{\gamma-1}{2} M_\theta^2 (1 + \frac{\bar{\psi}}{\phi} \gamma M_z^2) - \frac{2}{\phi} (1 + \frac{\gamma-1}{2} M_z^2) + \frac{1-M_z^2 \zeta}{1-M_z^2}$	$\frac{(\gamma-1) M_\theta^2 (1 + \frac{\bar{\psi}}{\phi} \gamma M_z^2) + 4 \left[1 + \frac{\gamma-1}{2} (M_z^2 + M_\theta^2) \right] (g_\psi - g_\phi)}{1-M_z^2 \zeta}$
2	$\frac{dM_\theta^2}{M_\theta^2}$	$\frac{(\gamma-1) M_z^2 (1 + \frac{\bar{\psi}}{\phi} \gamma M_z^2)}{1-M_z^2 \zeta}$	$\gamma \frac{\gamma-1}{2} \frac{\bar{\psi}}{\phi} \frac{M_z^4}{1-M_z^2 \zeta}$	$\frac{\bar{\psi}}{\phi} \gamma M_z^2 - 1 + \frac{\gamma-1}{2} M_\theta^2 (1 + \frac{\bar{\psi}}{\phi} \gamma M_z^2) - (\gamma-1) M_z^2 (1 + \frac{1}{\phi})$	$\frac{1-M_z^2 \zeta}{1-M_z^2}$	$\frac{2 \left[(\gamma-1) M_z^2 + (1 + \frac{\gamma-1}{2} M_\theta^2) (1 - \frac{\bar{\psi}}{\phi} \gamma M_z^2) \right]}{1-M_z^2 \zeta}$
3	$\frac{dp_0}{p_0}$	$-\frac{2 \frac{\bar{\psi}}{\phi} \gamma M_z^2 (1 + \frac{\gamma-1}{2} M_z^2)}{1-M_z^2 \zeta} + \frac{(\gamma-1) M_z^2 (g_\phi + g_\psi \frac{\bar{\psi}}{\phi} \gamma M_z^2)}{1-M_z^2 \zeta} \left[\frac{1 + (\gamma-1) M_z^2 (1 + g_\psi)}{1-M_z^2 \zeta} \right]^{1/2}$	$-\frac{\frac{\bar{\psi}}{\phi} \gamma M_z^2}{1-M_z^2 \zeta}$	$\frac{g_\phi - \frac{\bar{\psi}}{\phi} \gamma M_z^2 (1 + g_\psi)}{1-M_z^2 \zeta}$	$\frac{1-\phi}{\phi} (1-M_z^2 \zeta) + (1 + \frac{\gamma-1}{2} M_\theta^2) (g_\phi - g_\psi \frac{\bar{\psi}}{\phi} \gamma M_z^2) + \frac{1-M_z^2 \zeta}{1-M_z^2}$	$\frac{2(1 + \frac{\gamma-1}{2} M_\theta^2) (\frac{\bar{\psi}}{\phi} \gamma M_z^2 g_\psi - g_\phi) + (\gamma-1) M_z^2 (\frac{\bar{\psi}}{\phi} \gamma M_\theta^2 - 2 g_\phi)}{1-M_z^2 \zeta}$
4	$\frac{dt_0}{t_0}$	$-\frac{(\gamma-1) M_z^2 (1 + \frac{\bar{\psi}}{\phi} \gamma M_z^2)}{1-M_z^2 \zeta}$	$-\gamma \frac{\gamma-1}{2} \frac{\bar{\psi}}{\phi} \frac{M_z^4}{1-M_z^2 \zeta}$	$\frac{1-\frac{\bar{\psi}}{\phi} \gamma M_z^2}{1-M_z^2 \zeta}$	$\frac{(\gamma-1) \left[M_\theta^2 (1 - \frac{\bar{\psi}}{\phi} \gamma M_z^2 / 2 + M_z^2 (1/\phi - g_\psi + g_\phi)) \right]}{1-M_z^2 \zeta}$	$\frac{(\gamma-1) [2 M_z^2 (g_\psi - g_\phi) - M_\theta^2 (1 - \frac{\bar{\psi}}{\phi} \gamma M_z^2)]}{1-M_z^2 \zeta}$

$$\zeta = \frac{\bar{\psi}}{\phi} \gamma - (\gamma-1) (1 + g_\psi - g_\phi)$$

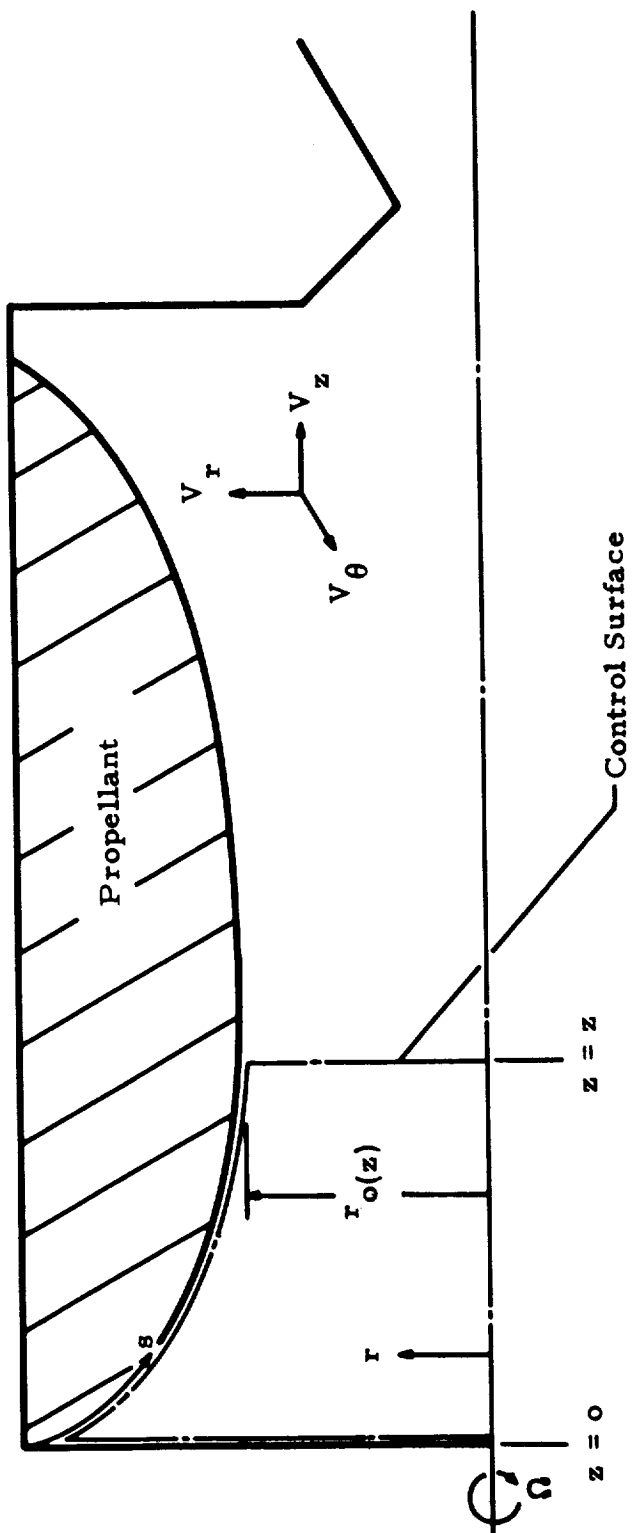


Figure A1. Schematic Diagram Illustrating System.

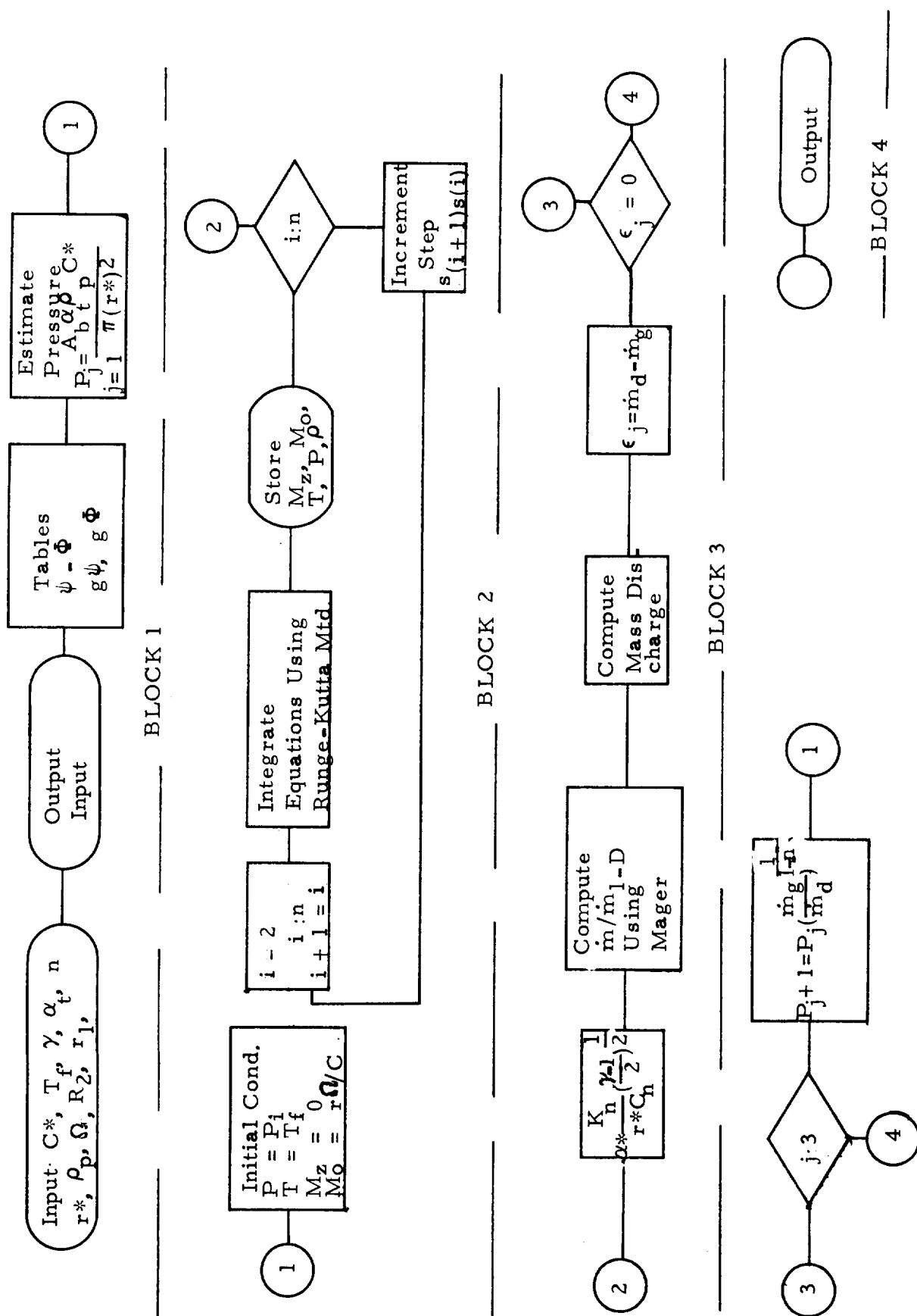


Figure A2. Flow Diagram for Swirl Ballistic Computer Program

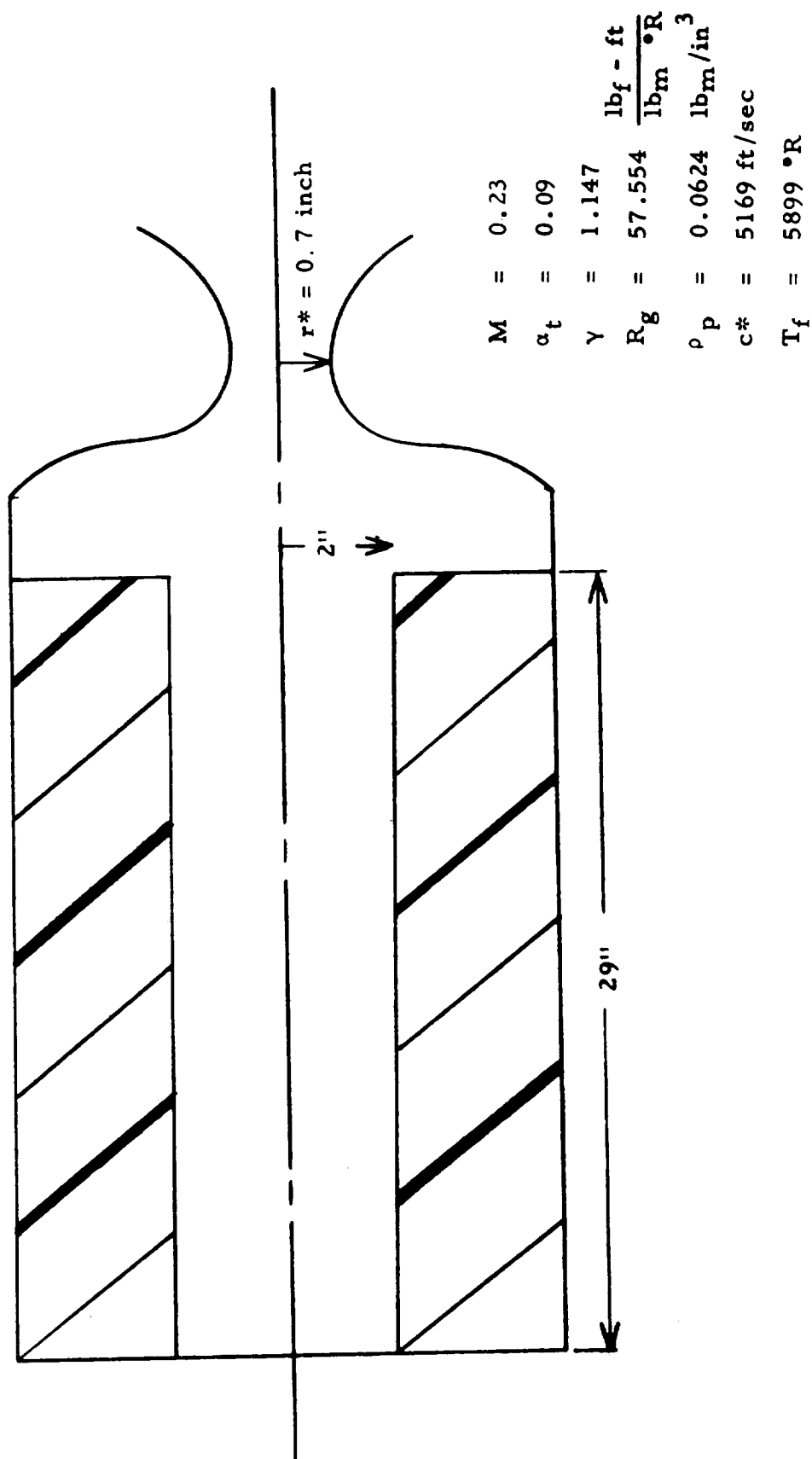


Figure A3. Configuration Used in Parametric Study

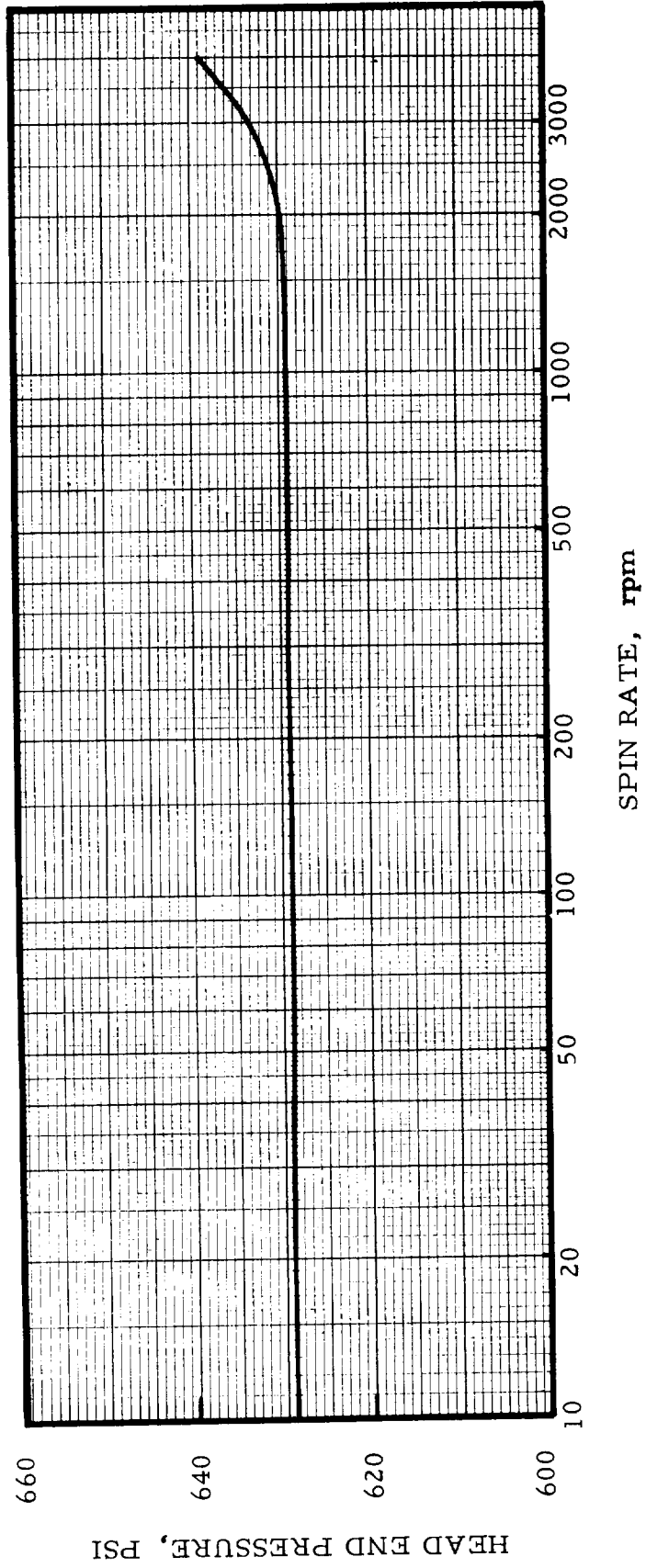


Figure A4. Head End Pressure Versus Spin Rate

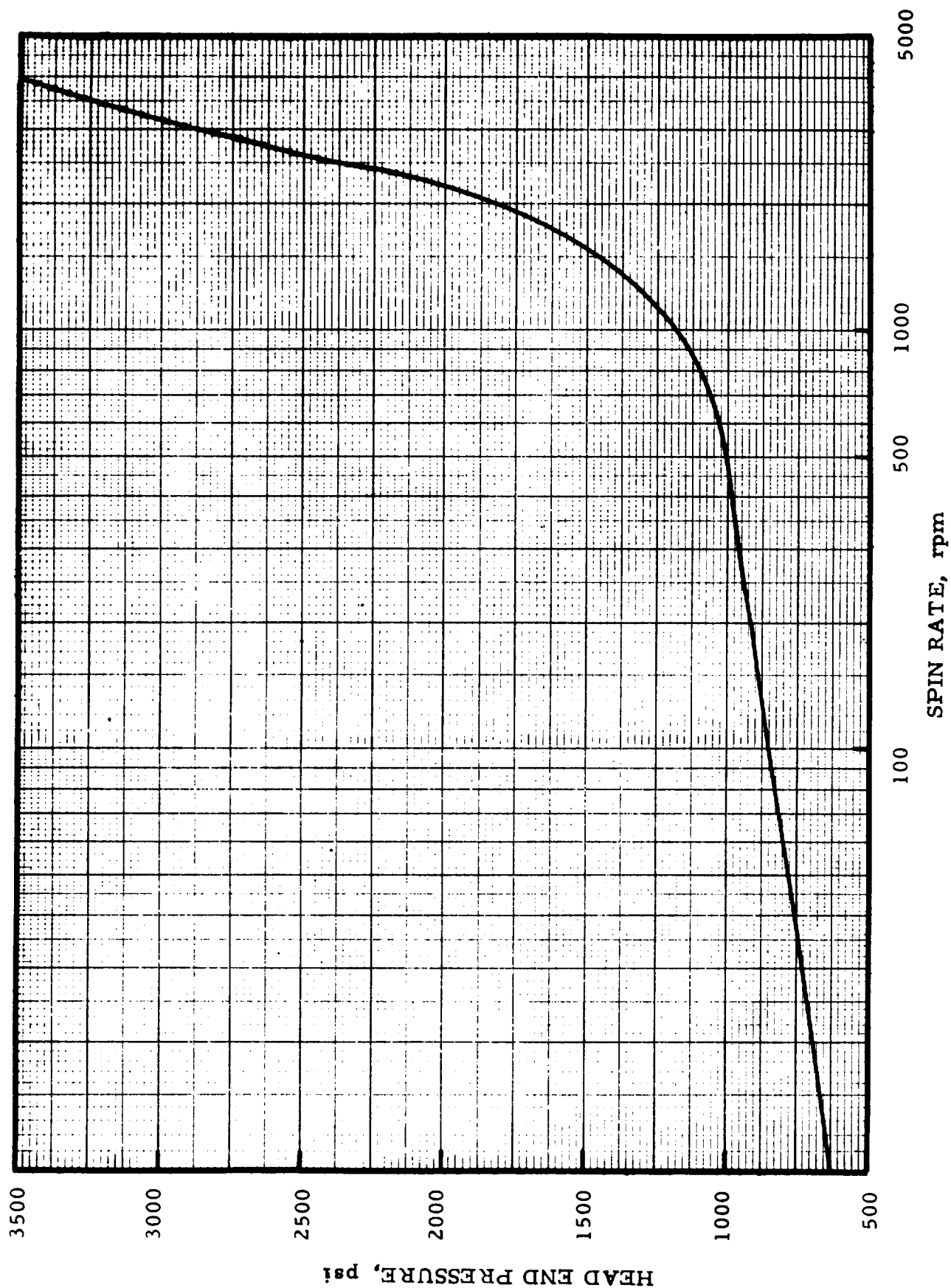


Figure A5. Head End Pressure Versus Spin Rate for Highly Metallized Propellant

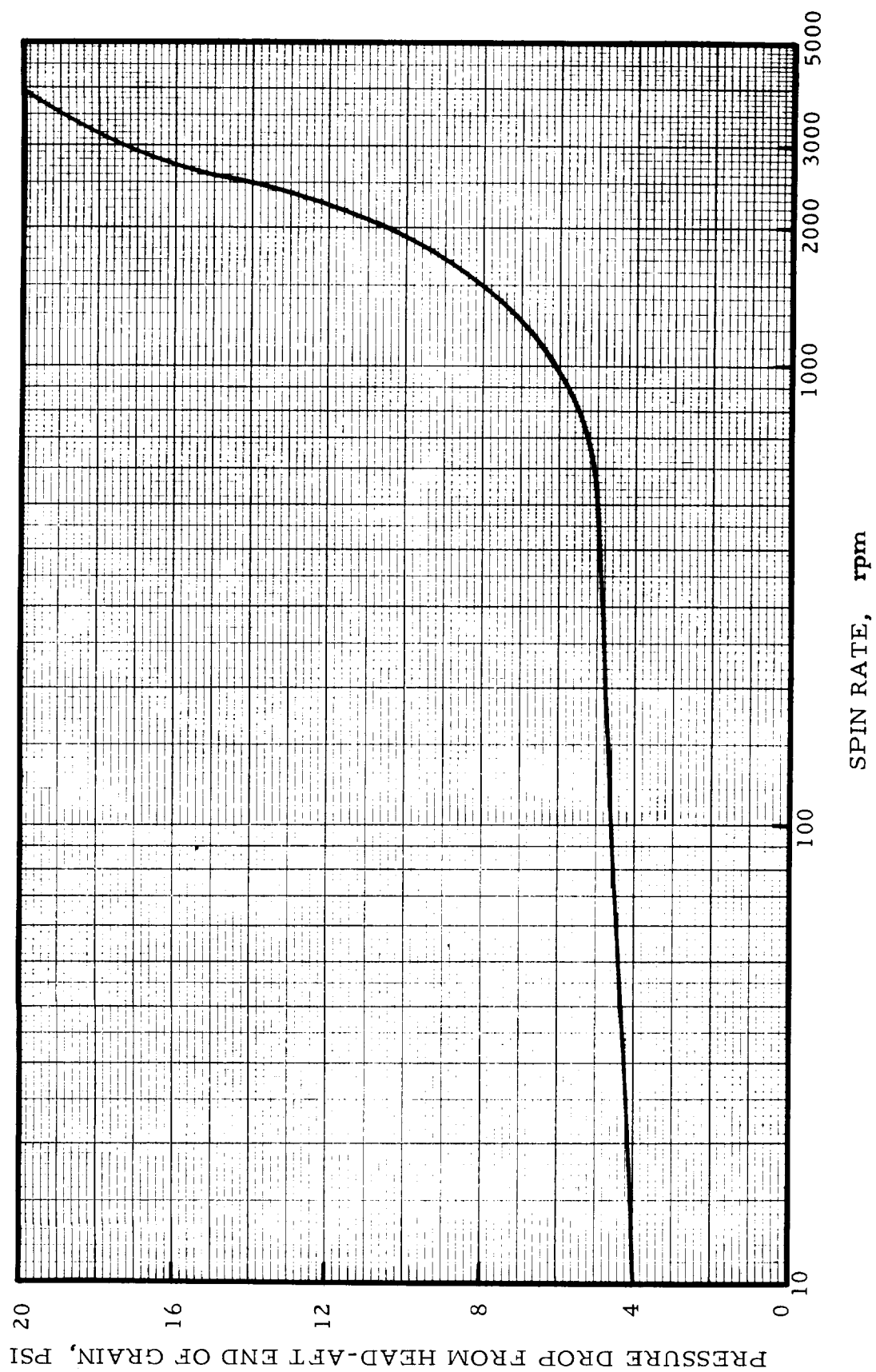


Figure A6. Pressure Drop Versus Spin Rate for Metallized Propellant

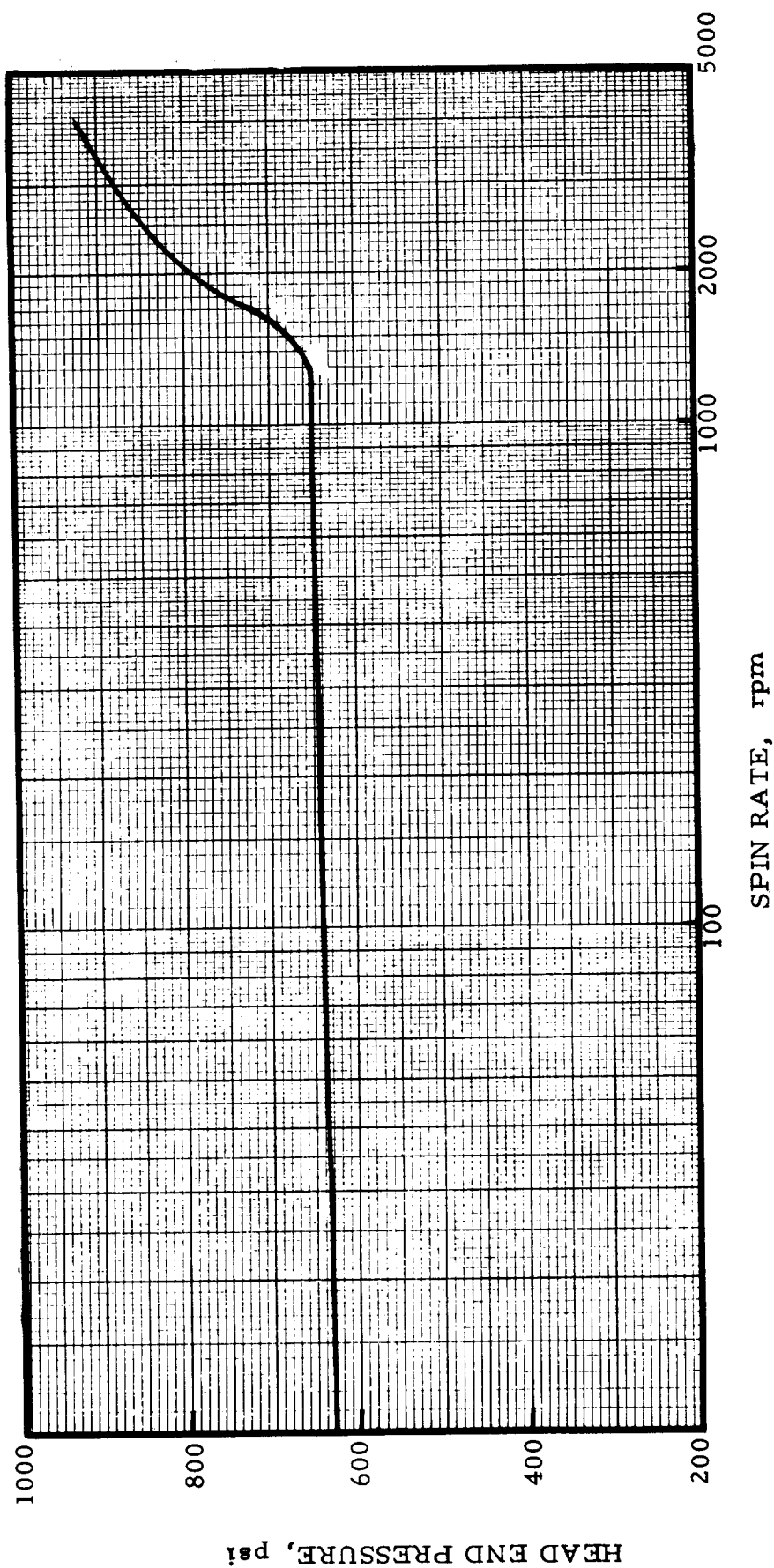


Figure A7. Head End Pressure Versus Spin Rate for Non-Metallized Propellant

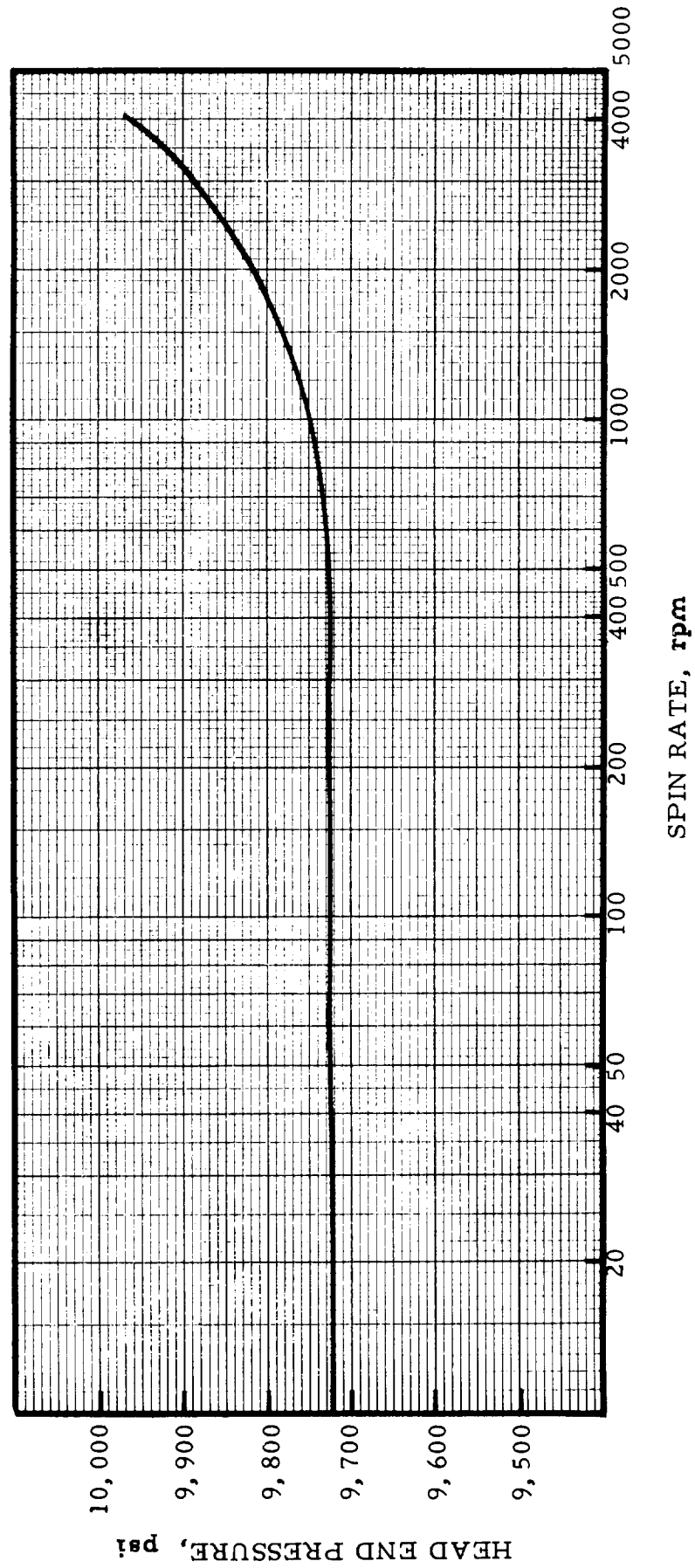


Figure A8. Effect of Increase in Pressure Exponent

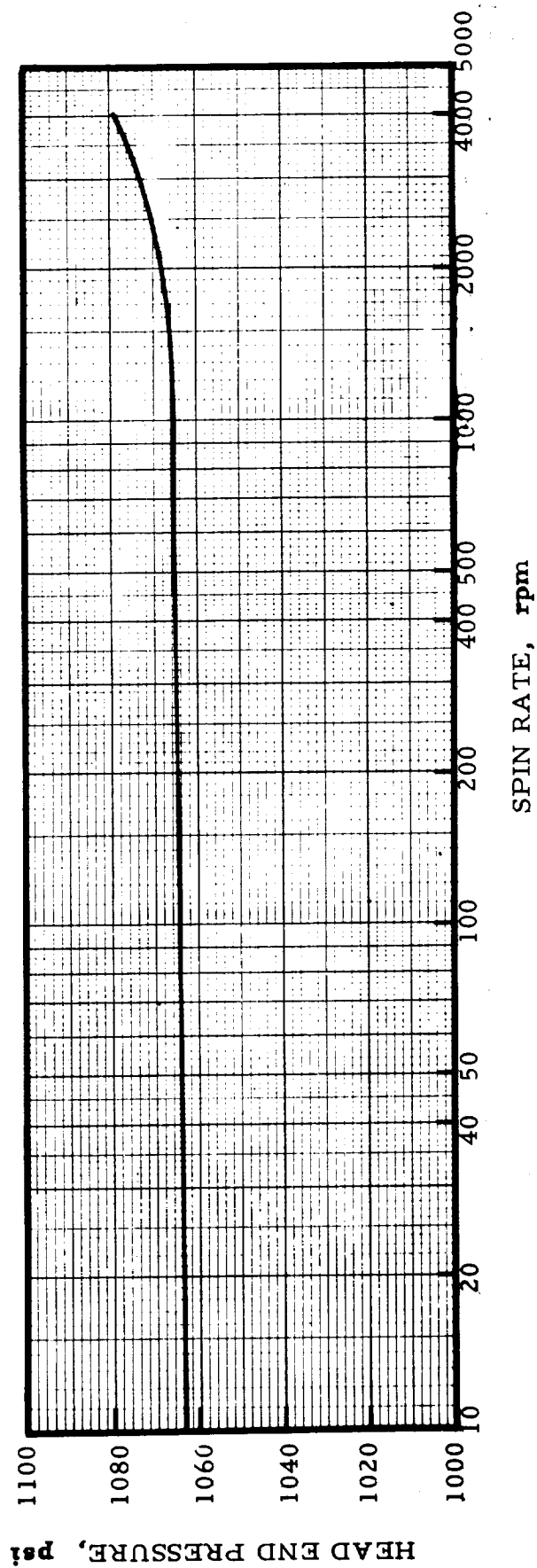


Figure A9. Effect of Burning Rate Coefficient

APPENDIX B

NOZZLE WALL BOUNDARY LAYER THEORY

Several investigations have presented analyses for the prediction of the boundary layer development on a conical nozzle wall in which the free stream velocity distribution was assumed to be a potential vortex. Each of these analyses was investigated to determine its suitability for application in predicting the boundary layer characteristics in the nozzle of a spinning rocket motor. It was anticipated that the extension of an existing boundary layer analysis by employing a form of Reynold's analogy would yield a method whereby the heat transfer coefficients could be calculated along the nozzle wall.

Taylor (ref. 34) presented the first analysis of the boundary layer development in a nozzle with a swirling flow free stream defined by a potential vortex velocity distribution. The following usual boundary layer assumptions were made.

1. The flow is axi-symmetric and steady.
2. The boundary layer thickness is small compared to the local nozzle radius.
3. The pressure gradient normal to the wall is small.
4. The pressure gradient of the free stream is imposed on the boundary layer.

In addition to the above, the analysis was restricted to an incompressible, laminar flow with negligible axial velocity in the free stream. The viscous boundary layer equations were reduced in accordance with these assumptions and integrated through the boundary layer to yield the momentum integral equations. Assumed velocity distributions, which satisfied the boundary conditions, were substituted into the momentum equations which could then be integrated, employing an arbitrary function, to yield two differential equations. These equations were then solved using numerical techniques for local boundary layer thickness and velocity. Taylor's analysis was directed toward swirl atomizers used in liquid spray systems and thus the assumptions are not compatible with the compressible high velocity flows associated with rocket nozzles. The boundary layer in a rocket nozzle must be considered to be compressible and turbulent; axial velocity in the free stream cannot be neglected near the throat.

Binnie and Harris (ref. 35) also analyzed the boundary layer flow in a swirl atomizer in a manner similar to Taylor except that axial velocity in the core was not assumed negligible. They derived the boundary layer equations for laminar incompressible flow when the free stream possesses both angular and axial velocity. This was an improvement upon Taylor's work; however, the assumption of an incompressible laminar boundary layer is not acceptable for use in rocket nozzle applications.

Cook (ref. 36) asserted that the practice of assuming that the component of velocity in the boundary layer in the angular or theta direction reached the free stream velocity over the same boundary layer thickness as the velocity component in the axial direction was incorrect. He postulated that in a laminar three-dimensional boundary layer there is a "thickness" related to the free stream angular velocity and a "thickness" related to the axial free stream velocity. The problem of Taylor was used to demonstrate his method in order that the results could be compared with Taylor's results.

These three analyses for the least complicated conditions of a laminar incompressible boundary layer in a nozzle with swirling flow are not in agreement. There are considerable differences between the boundary layer parameters as computed by these analyses. Therefore, it is apparent that boundary layer theory for the most simple conditions of swirling flow in a nozzle is not well defined.

An analysis of the boundary layer in the conical nozzle of a cyclone dust separator has been presented by Weber (ref. 37). Whereas the previous analyses have employed boundary layer equations derived from the Navier-Stokes equations, Weber derived the momentum integrals directly from applying the momentum equation in a direction along the nozzle wall and in the angular direction around the nozzle axis. He thus was able to obtain solutions for both laminar and turbulent boundary layers. However, the analysis assumes incompressible flow and the axial velocity in the free stream is neglected.

In order to use the two momentum integral equations derived by Weber to obtain a solution for the boundary layer development in a rocket motor nozzle, the velocity profiles and boundary conditions would have to be adjusted as follows. The velocity profile in the angular, θ , direction or in planes normal to the cone axis was assumed to be a one-seventh power distribution.

$$v = v_{\delta} (y/\delta)^{1/7} \quad (1)$$

where $v_{\delta} = K/r_w$ (potential vortex in freestream)

y = normal distance from wall

and δ = boundary layer thickness

The velocity profile in the direction parallel to and along the wall was assumed to be

$$u = v_{\delta} E \left[(y/\delta)^{1/7} - (y/\delta) \right] \quad (2)$$

where $E = f(x)$. These velocity distributions satisfy the boundary conditions

$$(1) u = 0, \quad v = 0 \quad \text{at } y = 0$$

$$\text{and } (2) u = 0, \quad v = v_{\delta} \quad \text{at } y = \delta.$$

For flow in a rocket nozzle boundary condition (2) must become

$$(2) u = u_{\delta}, \quad v = v_{\delta} \quad \text{at } y = \delta.$$

where $u_{\delta} = f(x)$. The velocity distribution for v could remain the same and the velocity distribution for u could be assumed to be

$$u = u_{\delta} (y/\delta)^{1/7} \quad (3)$$

The problem, of course, would be the determination of the freestream axial velocity as a function of x when swirl effects are present.

Expressions for the shear stress in the θ direction and the x direction are required in addition to the velocity profiles. Weber used an empirical expression for the shear stress

$$\tau_w = 0.0225 \rho V^{7/4} (\nu / y)^{1/4} \quad (4)$$

where τ_w = shear stress

$$V = u^2 + v^2 \text{ (total velocity),}$$

ν = kinematic fluid viscosity,

and ρ = fluid density.

The x and θ components of shear stress are then determined from the above equation and substituted into the two momentum equations. Weber then applied numerical techniques to solve for the boundary layer thickness as a function of the cone radius.

In order to calculate heat transfer coefficients for a turbulent boundary layer, von Karman's form of Reynolds analogy (ref. 38), which relates the Stanton number to the friction coefficient, is usually employed.

$$St = \frac{f/2}{1 + \sqrt{f/2} \left[5 (P_r - 1) + 5 \log_e \left[(5 P_r + 1) / 6 \right] \right]} \quad (5)$$

where St = Stanton number

P_r = Prandtl number

and f = friction factor (from experimental correlation)

For a flat plate, the friction factor may be related to shear stress by the equation

$$\tau_w = (f/2) \rho U^2 \quad (6)$$

where

U = Freestream velocity

Thus, equations (5) and (6) cannot be applied directly with Weber's equation for shear stress, (4) to predict heat transfer coefficients since these equations are for one-dimensional flow over a flat plate.

Elliott and Bartz (ref. 19) assumed that equation (5) for a flat plate may be used in a rocket nozzle with one-dimensional flow when the free-stream conditions and the energy and momentum boundary layer thicknesses are the same as those on the flat plate. A correlation of friction layer as a function of Mach number and momentum thickness is employed with equation (5) to calculate heat transfer coefficients.

A similar approach could be used to calculate heat transfer coefficients in a nozzle with swirl flow if a correlation for the friction factor in a three-dimensional boundary layer could be developed. However, the accuracy of the coefficients is sensitive to the friction factor correlation and any approximation or errors involved in determining the friction factor would be reflected in calculated values for the heat transfer coefficient.

In summary, the boundary layer approach would yield only a rough approximation to the solution due to the previously discussed assumptions necessary for a solution and the lack of a correlation for friction factor which could be applied with the flow field found in the nozzle of a spinning rocket motor.

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