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COVARIANCE FUNCTION OF ELEVATIONS ON A CRATERED PLANETARY SURFACE

PART I

CRATER BOWL CONTRIBUTION

June 21, 1968

A. H. Marcus

Work performed for Office of Manned Space Flight, National Aeronautics and Space Administration, under Contract NASW-417.

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ABSTRACT

We derive the covariance function of elevations on a plane planetary surface which has been excavated by primary meteoritic impact craters, according to various models of crater size and shape, and an assumed inverse power law meteor mass distribution. At distances less than the diameter of the smallest crater, the shape of the covariance function reflects crater geometry rather than meteor mass distribution, being linear for cylindrical craters and parabolic for paraboloidal craters. At small and moderate distances the covariance function has roughly the form suggested by Chernov. The derived functions can be used in studies requiring statistical characterization of lunar surface roughness, such as mobility of lunar roving vehicles or interpretation of radar power returns.

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June 21, 1968

Lt. General Samuel C. Phillips
Director, Apollo Program
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Washington, D. C. 20546

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Dear General Phillips:

The following report is submitted as an item of documentation of Apollo Systems Engineering Studies, under Contract NASW-417.

Covariance Function of Elevations on
a Cratered Planetary Surface - Part I -
Crater Bowl Contribution -
A. H. Marcus -
June 21, 1968 -
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Yours very truly,

A. P. Boysen, Jr.
Director

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COVARIANCE FUNCTION OF ELEVATIONS ON A CRATERED PLANETARY SURFACE

PART I

CRATER BOWL CONTRIBUTION

1.0 INTRODUCTION

In discussing the elevations of an unknown surface, it is often convenient to treat these elevations as a sample realization of a surface generated by a random process. If the random process is a gaussian process, or derived from a gaussian process, the statistical structure of the process is determined completely by its covariance function. Let the elevation of the surface at a point $\underset{\sim}{R}$ be denoted by $Z(\underset{\sim}{R})$, and let $E\{\cdot\}$ denote mathematical expectation (average over all possible realizations). The <u>covariance function</u> c, of elevations at $\underset{\sim}{R}$ and $\underset{\sim}{R}+\underset{\sim}{r}$ is then defined by the function

$$c = E\{Z(\underset{\sim}{R})Z(\underset{\sim}{R}+\underset{\sim}{r})\} - E\{Z(\underset{\sim}{R})\}E\{Z(\underset{\sim}{R}+\underset{\sim}{r})\} \qquad (1)$$

If the surface is stationary (homogeneous) and isotropic, this function is the same for any points separated by a distance r = (length of $\underset{\sim}{r}$), thus $c = c(r)$ is a function of the distance r only, not of $\underset{\sim}{R}$ or of the direction in which $\underset{\sim}{r}$ is measured. We assume the homogeneity of isotropy of the $Z(\underset{\sim}{R})$ process from now on.

Two significant uses of the covariance function have already been explored. The mean square amplitude of the power reflected by a planetary surface from a radio or radar signal can be easily related to the covariance function of the process, if the process is gaussian, or derived from a gaussian process by censoring (see, e.g., Beckmann and Spizzichino (1963) or Marcus (1967a)). In this application, it is sometimes sufficient to use only a single number derived from c(r), the

$$\underline{\text{mean}}\ \underline{\text{square}}\ \underline{\text{slope}} = q^2 = -d^2\, c(r)/dr^2\big|_{r=0} \quad , \qquad (2)$$

assuming c(r) is in fact twice differentiable at r = 0.

A second application has been mentioned by Jaeger and Schuring (1966), who also estimated c(r) directly from Ranger 7 photographs of Mare Cognitum. If a lunar vehicle responds to differences in surface elevation as a "linear filter" of the elevations, then the mean square amplitude of vibration power at given frequency (spatial wave number) ω is an easily computed function of the spectral density

$$S(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\omega_1 x_1 + i\omega_2 x_2}\, c(r)\, dx_1 dx_2 \qquad (3)$$

where, using the assumed isotropy, $\omega = (\omega_1^2 + \omega_2^2)^{1/2}$ and $r = (x_1^2 + x_2^2)^{1/2}$. The function c(r) is needed here too.

The only attempt to derive c(r) from first principles has been that of Chernov (1967). Since some of his methods figure prominently in our own treatment, we will discuss his analysis in greater detail.

2.0 CHERNOV'S ANALYSIS

An attempt to derive the spectral density $S(\omega)$ (equivalently, its inverse Fourier transform c(r)) has been made by Chernov (1967). He assumes that an initally plane surface is damaged by primary impact craters, and that the surface elevation changes resulting from the formation of craters add linearly. That is, the elevation at a point is the sum of all the elevation changes resulting from crater formation anywhere on the surface (Figure 1). This is not strictly true, as he points out, since the formation of a large crater will destroy all smaller features within its perimeter (possibly outside as well, as a consequence of the deposition of a thick layer of ejected debris near the crater rim). A correction for the loss of small features by obliteration is thus needed. The elevation changes will also depend on the elevation of the point at which the crater-forming meteorite impacted on the surface, which is in general different from the elevation at the point of interest.

Chernov also assumed some similarity principles in the formation of craters which, though possibly justifiable, are too restrictive. He assumes first that the depths of craters are proportional to their diameters, the constant of proportionality being the same number $\eta \approx 0.2$. He also assumes that the volume of the crater is proportional to the energy of the impact. These assumptions are valid for small craters, but almost certainly fail for craters larger than a few kilometers in diameter (see, e.g., Marcus (1967b) or Chabai (1965) for details).

He finally assumes that the number density of diameters of craters on the surface directly reflects the number density of meteorite masses. For the meteorite mass number density he assumes an inverse power law with index γ_1,

$$\text{(number of meteors with mass} \geq m) = (\text{const.})/m^{\gamma_1} \qquad (4)$$

(his $s = \gamma_1+1$ in our notation). The assumptions that the depth-diameter ratio of craters is constant, and that crater volume is proportional to the kinetic energy of the impacting meteorite, enable him to derive the one-dimensional spectral density

$$S(\omega) = \int_{-\infty}^{\infty} e^{i\omega r}\, c(r)dr = (\text{const.})/\omega^{5-\gamma} \ (\text{large } \omega) \qquad (5)$$

where

$$\gamma = 3\gamma_1 \qquad (6)$$

The constant is not evaluated numerically in terms of basic physical parameters.

On the other hand, <u>in the one-dimensional case</u>

$$S(\omega) = (\text{const.})\ \omega^{-\mu} \quad \begin{pmatrix} \text{for } \omega \text{ large} \\ \text{and } 1 < \mu < 3, \mu \neq 2 \end{pmatrix} \qquad (7)$$

implies

$$c(r) = c(0) - (\text{const.})\ r^{\mu-1} + o(r^{\mu-1}) \text{ for } r \text{ small} \qquad (8)$$

where $c(0) = c(r)|_{r=0}$ and $o(x)$ is any function for which $\lim_{x\to 0} o(x)/x = 0$.

For $\mu = 2$ and $\mu = 3$, and all $\varepsilon > 0$,

$$c(r) = c(0) - o(r^{\mu-1-\varepsilon}) \qquad (9)$$

If in (7) $\mu > 3$, then in the one-dimensional case

$$c(r) = c(0) - q^2r^2/2 + o(r^2) \tag{10}$$

where q^2 is the m.s. slope. These conditions are easily checked according to the value of

$$\mu = 5 - 3\gamma_1 \tag{11}$$

In practice, the special values $\mu = 2$ and $\mu = 3$ are not distinctive.

We will follow Chernov's basic idea, with the following changes: we will work in the spatial domain where the basic physical processes occur, rather than in the frequency domain. We will consider the effects of crater shape (which he ignores). We will remove some restrictions on the constancy of the crater depth-diameter and impact energy-volume ratios. We will treat the effect of the destruction of small craters by larger ones. The linear superposition principle and the inverse power law (4) we will retain.

3.0 MATHEMATICAL FOUNDATIONS

A basic mathematical formalism for such processes has been described by Matern (1960). We assume that elevation changes add linearly, and that the surface elevation $Z(\underset{\sim}{R})$ at point $\underset{\sim}{R}$ has a representation

$$Z(\underset{\sim}{R}) = \int \zeta(x,r)\ dN(x,\underset{\sim}{R}+\underset{\sim}{r}) \tag{12}$$

where $dN(x,\underset{\sim}{R}+\underset{\sim}{r})$ is a random variable, the number of craters of diameter x to $x+dx$ in a small region $d(\underset{\sim}{R}+\underset{\sim}{r})$ centered on the point $\underset{\sim}{R}+\underset{\sim}{r}$, and $\zeta(x,r)$ is the elevation change caused by a crater of diameter x which is formed at a distance r = (length of $\underset{\sim}{r}$) away from $\underset{\sim}{R}$. The integration extends over all values of x and $\underset{\sim}{r}$.

If craters are formed by primary impacts only, we can reasonably assume that $dN(x,\underset{\sim}{r})$ is a Poisson random variable with mean value

$$E\{dN(x,\underset{\sim}{r})\} = \xi(x)\ dx\ d\underset{\sim}{r} \tag{13}$$

The function $\xi(x)$ is the expected number of craters of diameter x, per unit area per unit diameter interval.

The correlation function c(r) is then readily derived (Matern, 1960),

$$c(r) = \int_0^\infty \xi(x)dx \iint \zeta(x,r_1)\ \zeta(x,r_2)\ \bar{v}_2(r_1,r_2;r)\ dr_1 dr_2 \tag{14}$$

where $\bar{v}_2(r_1,r_2;r)$ is defined by

$$\bar{v}_2(r_1,r_2;r) = \frac{\partial^2}{\partial r_1 \partial r_2} V_2(r_1,r_2;r) \tag{15}$$

$V_2(r_1,r_2;r)$ is the area in the intersection of two circles of radii r_1 and r_2 respectively, whose centers are a distance r apart. Since

$$\begin{aligned} V_2(r_1,r_2;r) = {} & r_1^2\left[\frac{\pi}{2} - \rho_1\sqrt{1-\rho_1^2} - \arcsin \rho_1\right] \\ & + r_2^2\left[\frac{\pi}{2} - \rho_2\sqrt{1-\rho_2^2} - \arcsin \rho_2\right] \end{aligned} \tag{16}$$

where

$$\begin{aligned} \rho_1 &= \left(r_1^2 + r^2 - r_2^2\right) \Big/ 2r\ r_1 \\ \rho_2 &= \left(r_2^2 + r^2 - r_1^2\right) \Big/ 2r\ r_2 \end{aligned} \tag{17}$$

we obtain

$$\bar{\nu}_2(r_1,r_2;r) = 4r_1r_2/\left[4r_1{}^2r_2{}^2 - \left(r_1{}^2+r_2{}^2-r^2\right)^2\right]^{1/2} \tag{18}$$

for

$$r_1>0,\ r_2>0,\ |r_1-r_2| < r,\ r_1+r_2 > r$$

In the remainder of the paper the following transformations will be useful:

$$\begin{aligned} u &= r_1 + r_2 \\ v &= r_1 - r_2 \end{aligned} \tag{19}$$

$$\nu_2(u,v;r) = \bar{\nu}_2(r_1,r_2;r)\ \text{Jacobian}\left(\frac{r_1,r_2}{u,v}\right)$$

$$\nu_2(u,v;r) = \frac{1}{2}\frac{u^2 - v^2}{\sqrt{u^2 - r^2}\sqrt{r^2 - v^2}} \qquad \text{for } -r<v<r,\ r<u \tag{20}$$

thus

$$c(r) = \int \xi(x)dx \iint \zeta(x,(u+v)/2)\ \zeta(x,(u-v)/2)\ \nu_2(u,v;r)du\,dv \tag{21}$$

After defining the basic physical model functions, we will work out some particular cases of (21).

In order to correctly compute the asymptotic power spectral density function $S(\omega)$, we must recognize that we are working in two dimensions. In the isotropic case,

$$S(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\omega_1x_1+i\omega_2x_2}\ c(r)dx_1dx_2$$

$$= 2\pi\int_0^{\infty} J_0(\omega r)\ c(r)rdr \tag{22}$$

where

$$r = (x_1^2+x_2^2)^{1/2}$$

$$\omega = (\omega_1^2+\omega_2^2)^{1/2}$$

and $J_o(Z)$ is the zero$\underline{\text{th}}$ order Bessel function of the first kind (Matern (1960), Eq. 2.3.8). Assuming again that for small r

$$c(r) = c(0) -(\text{const})r^{\mu-1}+o(r^{\mu-1}) \tag{8}$$

we can derive from (22), in the same way that (7) was obtained, that for large ω

$$S(\omega)\approx(\text{const})\omega^{-(\mu+1)} \tag{23}$$

Note that $S(\omega)$ should not be confused with the radial spectral density function $q_2(\omega)$, defined by

$$q_2(\omega) = 2\pi\omega S(\omega).$$

For vehicle trafficability purposes we are actually more interested in the <u>spectral density of a linear traverse</u>, say $S_1(\omega_1)$, which corresponds to a straight-line path on the surface parallel to the x_1 axis. Thus, in the (ω_1,ω_2) plane, fixing ω_1 but not ω or ω_2,

$$S_1(\omega_1) = \int_0^\infty S(\omega)d\omega_2$$

$$= \int_{\omega_1}^\infty S(\omega)\left(1-\frac{\omega_1^2}{\omega^2}\right)^{-1/2} d\omega \tag{24}$$

If ω_1 is large enough that (23) applies for $\omega \geq \omega_1$, then similarly Eq. (8) with $1<\mu<3$ is equivalent to

$$S_1(\omega_1) \approx (\text{const})\ \omega_1^{-\mu} \tag{25}$$

We see that if the behavior of c(r) for small r is given by Eq. (8), the asymptotic spectral density of a linear traverse behaves similarly in the one-and two-dimensional cases.

4.0 MODEL FUNCTIONS

We first derive the expected number density $\xi(x)$ corrected for obliteration. Suppose that a crater is a perfectly circular object whose formation destroys everything within its perimeter, but leaves everything outside intact. We assume that a crater is destroyed if, and only if, it is completely covered by a larger crater. Let the probability density function of the diameter x of a newborn crater be denoted p(x), and let the cumulative flux (number of craters formed per unit area, of any size larger than some number x_o) be denoted F. Then (Marcus, 1966a)

$$\xi(x) = \frac{p(x)}{\Lambda(x)} \{1 - \exp(-\Lambda(x)F)\} \tag{26}$$

where

$$\Lambda(x) = \frac{\pi}{4} \int_x^\infty (y - x)^2\ p(y)dy$$

In the particular case that

$$p(x) = \gamma\ x_o^{\gamma}/x^{\gamma+1} \qquad \text{if } x > x_o$$

$$p(x) = 0 \qquad \text{if } x < x_o \tag{27}$$

then

$$\Lambda(x) = \frac{\pi\ x_o^{\gamma}}{2(\gamma-1)(\gamma-2)} \frac{1}{x^{\gamma-2}} \qquad \text{if } \gamma > 2$$

so that

$$\xi(x) = \frac{2\gamma(\gamma-1)(\gamma-2)}{\pi} \frac{1}{x^3} \{1 - \exp(-\Lambda(x)F)\} \tag{28}$$

If $\Lambda F << 1$ then

$$\xi(x) \cong \left(\gamma\, x_o^{\gamma}\, F\right) / x^{\gamma+1} \qquad \text{if } x > x_o$$

If $\Lambda F >> 1$ then

$$\xi(x) \cong 2\, \frac{\gamma(\gamma-1)(\gamma-2)}{\pi} / x^3$$

For both a lightly cratered surface ($\Lambda F << 1$) and a heavily cratered surface ($\Lambda F >> 1$) we have

$$\xi(x) = sC/x^{s+1} \qquad \text{for } x > x_o \tag{29}$$

with index $s > 2$ (lightly cratered) or $s = 2$ (heavily cratered), and density coefficient $C = F\, x_o^{\gamma}$ (lightly cratered) or $C = \frac{\gamma(\gamma-1)(\gamma-2)}{\pi}$(heavily cratered). To obtain $s < 2$ we must invoke internal flooding mechanisms (Marcus, 1966b).

Unfortunately, on a heavily cratered surface, the process $dN(x,R)$ which allocates crater centers differs somewhat from a Pöisson process, since large and small craters are negatively associated (Marcus, 1966c). We will ignore this difficulty for the present.

The value of γ in the inverse power law (24) must now be established. It is usually assumed that the energy E of an explosive impact and the diameter x of the crater it forms are related by a power law

$$x = (\text{const.})\, E^{1/\gamma_2} \tag{30}$$

We may accept $\gamma_2 = 3$ for very small craters. Chernov's similarity assumptions imply this value, but for craters of a few meters diameter and larger, a higher value of γ_2 is required. Baldwin (1963) suggests that γ_2 is a slowly varying function of x with values in the range 3.2 to 3.6. As a rough average we accept

$$\gamma_2 \sim 3.4 \tag{31}$$

(see Chabai (1965) for more details on this point).

Upon combining (4) and (30), and assuming that impact velocities are not too broadly distributed, we derive an inverse power law (27) for p(x), with

$$\gamma = \gamma_1\gamma_2 \sim 3.4\gamma_1 \tag{32}$$

The value x_o corresponds to a lower cutoff on mass m_o (see (Marcus, 1967c) for discussion on this point).

It is difficult to choose a single value of γ_1 for all sizes of craters, since different lunar regions may be characterized by bombardment from essentially different meteorite populations (Marcus, 1967d). For premare craters with $x > 20$ km (which may reflect the primeval planetesimals) we may require $\gamma_1 \sim 0.62$ to 0.67, thus $\gamma \sim 2.10$ to 2.27. For postmare craters with $x > 5$ km we seem to require $\gamma_1 \sim 0.75$ to 0.84, thus $\gamma \sim 2.55$ to 2.86. Very small present meteors ($m < 1$ gm) seem to have $\gamma_1 \sim 0.88$ to 1.10 (Dohnanyi, 1967) or even $\gamma_1 \simeq 1.34$ (Hawkins and Upton, 1958) thus $\gamma \sim 2.64$ to 4.02 (using $\gamma_2 = 3$). This uncertainty cannot be resolved. It may be necessary to make p(x) time dependent, for example with $\gamma = \gamma(t)$ a function of time. We will not do so in this paper.

To include secondary crater formation in the model is possible, but only at the expense of enormous analytical complexity. We will not do so here.

It is now necessary to define the vertical relief of a crater. In the remaining sections we will assume that a crater has <u>negative</u> relief only, i.e., no crater rim or ejecta blanket. These attributes will be discussed in future papers of this series. Two assumed crater shapes will be treated, cylindrical and paraboloidal. The inital depth of the true crater bowl is assumed to be represented by a power law

$$\text{depth of crater bowl} = T_o\, x^{\delta} \tag{33}$$

Chernov assumes $\delta = 1$ explicitly; this is acceptable for $x < 1$ km but is not correct for very large craters, which seem to require $\delta \sim 0.4$ to 0.5 (Baldwin (1963), Marcus (1967b)). The value of T_o also varies slightly with diameter, perhaps $T_o \sim 0.19$ to 0.20 for $x \sim 10$ meters and $T_o \sim 0.16$ for $x \sim 1$ km. This problem also requires further study.

We will ignore internal processes which change crater shape or surface relief, although these factors are clearly of some importance.

5.0 CYLINDRICAL CRATERS

A cylindrical crater is defined by

$$\begin{aligned} \zeta(x,r) &= -\,T_o x^{\delta} && \text{if } 0 < r < x/2 \\ \zeta(x,r) &= 0 && \text{if } r > x/2 \end{aligned} \tag{34}$$

When (34) is inserted into (14) we obtain

$$c(r) = \int \xi(x)dx \cdot T_o{}^2 x^{2\delta}\, V_2\left(\frac{x}{2}, \frac{x}{2}; r\right) \tag{35}$$

In order to avoid convergence difficulties with the inverse power law (29) for $\xi(x)$, let us assume that $\xi(x)$ is truncated above at x_m:

$$\begin{aligned} \xi(x) &= \frac{sC}{1-\left(\dfrac{x_o}{x_m}\right)^s} \frac{1}{x^{s+1}} \qquad \text{for } x_o < x < x_m \\ &= 0 \qquad \text{otherwise} \end{aligned} \tag{36}$$

Combining (35), (36) and (16), we obtain

$$c(r) = \frac{sCT_o{}^2}{2\left[1-\left(\dfrac{x_o}{x_m}\right)^s\right]} r^{2+2\delta-s} \int_{\max(1,x_o/r)}^{x_m/r} \left[\frac{\pi}{2y^{s-1-2\delta}} - \frac{\sqrt{y^2-1}}{y^{s+1-2\delta}} - \frac{\arcsin(1/y)}{y^{s+1}}\right] dy \tag{37}$$

with the substitution $y = x/r$. The last two terms of the integrand are readily evaluated only when s and $s - 2\delta$ are integers. The table of indefinite integrals, Table 1, will help in the evaluation of $c(r)$. In Table 2 we give explicitly $c(r)$ for $\delta = 1$, $s = 2,3,4$, which are sketched in Figures 2-4.

Note that, in (37),

$$c(r) = 0 \quad \text{if} \quad r \geq x_m \tag{38}$$

An analysis of the case $r \to 0$ enables us to compare our results with those of Chernov. As $r \to 0$, $x_m/r \to \infty$ and $x_o/r \to \infty$. Since the integral in (37) exists on the domain $x_o/r < y < x_m/r$, we may use the fact that for sufficiently small r, the integral differs by an arbitrarily small amount from

$$\int_{x_o/r}^{x_m/r} \left[\frac{\pi}{2y^{s-1-2\delta}} - \frac{1-\left(\frac{1}{2y^2}\right)}{y^{s-2\delta}} - \frac{1-\left(\frac{1}{6y^2}\right)}{y^{s+2}} \right] dy \tag{39}$$

thus

$$c(r) \approx c(0) - c_1 r + o(r) \quad (r \to 0) \tag{40}$$

where c_1 is a constant (i.e., function of s, δ, x_o and x_m). This is of the same form as (8), with $\mu - 1 = 1$, or in the two-dimensional case

$$S_1(\omega) \approx (\text{const.})\ \omega^{-2} \quad (\omega \to \infty) \tag{41}$$

This heavy-tailed spectral density (41) implies that the surface is almost uniformly rough, in the sense that local slopes $\frac{\partial}{\partial x_i} Z(x_1,x_2)$ do not exist in mean square, (where (x_1,x_2) are the Cartesian coordinates of $\underset{\sim}{R}$, and $i = 1,2$). We will now show that this roughness is simply a consequence of the cylindrical crater geometry.

6.0 PARABOLOIDAL CRATERS

Some small fresh craters on the Moon show a roughly conical shape. But the most common geometry for a fresh crater, especially those larger than 100 meters diameter, is that of a spherical cap or a paraboloid. Some scientists prefer the spherical cap (the difference in any practical sense is slight), but for analytical simplicity we will use the paraboloid

$$\zeta(x,r) = -T_o x^\delta \left[1 - (2r/x)^2\right] \quad \text{if } 0<r<x/2$$
$$\zeta(x,r) = 0 \quad \text{if } x/2<r \tag{42}$$

Upon inserting (42) into (21), we obtain

$$c(r) = \int \xi(x)dx \cdot T_o^2 x^{2\delta} I(x,r)/2 \tag{43}$$

where

$$I(x,r) = \int_r^x \frac{du}{\sqrt{u^2-r^2}} I_1(u;x-u;r,x) \quad \text{if } r<x<2r$$

$$I(x,r) = \int_r^{x-r} \frac{du}{\sqrt{u^2-r^2}} I_1(u;r;r,x) \tag{44}$$
$$+ \int_{x-r}^x \frac{du}{\sqrt{u^2-r^2}} I_1(u;x-u;r,x) \quad \text{if } 2r\le x$$

The function I_1 is defined by

$$I_1(u;\alpha;r,x) = \int_{-\alpha}^{\alpha} \frac{(u^2-v^2)dv}{\sqrt{r^2-v^2}} \frac{\zeta\left(x,\frac{u+v}{2}\right)\zeta\left(x,\frac{u-v}{2}\right)}{T_o^2 x^{2\delta}}$$
$$= x^{-4} \int_{-\alpha}^{\alpha} \frac{(u^2-v^2)dv}{\sqrt{r^2-v^2}} \left[x^2 - (u+v)^2\right]\left[x^2 - (u-v)^2\right] \tag{45}$$

$$x^4 I_1 = \arcsin(\alpha/r) \left\{2\left(u^2-r^2\right)^3 + \left(u^2-r^2\right)^2 \left(3r^2-4x^2\right)\right.$$
$$+ \left(u^2-r^2\right)\left(2x^4-8x^2r^2 + \tfrac{9}{4}r^4\right)$$
$$\left. + r^2\left(x^4 - \tfrac{5}{2}x^2r^2 + \tfrac{5}{8}r^4\right)\right\}$$

$$+ \frac{1}{3}\,\alpha\left(r^2-\alpha^2\right)^{5/2} + \alpha\left(r^2-\alpha^2\right)^{3/2}\left[x^2 + \frac{5}{12}\,r^2 + \frac{3}{2}\left(u^2-r^2\right)\right]$$
$$+ \alpha\sqrt{r^2-\alpha^2}\left[3\left(u^2-r^2\right)^2 + \frac{9}{4}\,r^2\left(u^2-r^2\right) + x^4 - \frac{5}{2}\,x^2r^2 + \frac{5}{8}\,r^4\right] \tag{46}$$

It is evident that in general, the function I cannot be similarly reduced to elementary functions. Therefore an explicit evaluation of c(r) such as that in Table 2 is not possible. The following multiple integral formula may be useful in numerical calculations (we omit the complicated and tedious reductions):

$$c(r) = \frac{sCT_o^{\,2}}{2\left[1-\left(\frac{x_o}{x_m}\right)^s\right]}\, r^{2+2\delta-s}\left\{\int_{\max(1,x_o/r)}^{x_m/r} \frac{dy}{y^{\,s+5-2\delta}}\,\cdot \right.$$
$$\left.\int_{\max(1,y-1)}^{y}\left[\frac{\phi(w,y)}{\sqrt{1-(y-w)^2}} + \frac{\psi(w,y)}{\sqrt{w^2-1}}\right]dw\right\} \tag{47}$$

where

$$\phi(w,y) = \frac{1}{3}\,w\left(w^2-1\right)^{5/2} + w\left(w^2-1\right)^{3/2}\left(\frac{1}{3} - y^2\right)$$
$$+ w\sqrt{w^2-1}\left(y^4 - \frac{5}{2}\,y^2 + 4\right) \tag{48}$$

and

$$\psi(w,y) = \frac{1}{3}\left(y-w\right)\left[1-\left(y-w\right)^2\right]^{5/2}$$
$$+ \left(y-w\right)\left[1-\left(y-w\right)^2\right]^{3/2}\left[y^2 + \frac{3}{2}\left(y-w\right)^2 - \frac{13}{12}\right] \tag{49}$$
$$+ (y-w)\sqrt{1-(y-w)^2}\left\{y^4 + 3\left(y-w\right)^4 - \frac{5}{2}\,y^2 - \frac{15}{4}\left(y-w\right)^2 + \frac{7}{8}\right\}$$

The transformed variables are $y = x/r$ and $w = u/r$.

In Figures 5-7 we sketch c(r) for δ=1 and s=2, 3, 4.

As before, an asymptotic analysis of (47) as $r\to o$, thus $y\to\infty$, is very instructive. We omit the straightforward but lengthy calculations which lead to:

$$\int_{y-1}^{y} \frac{\phi(w,y)dw}{\sqrt{1-(y-w)^2}} = \frac{\pi}{2}\left\{\frac{1}{3}y^6 - 2y^4 + O(y^3)\right\} \tag{50}$$

and

$$\int_{y-1}^{y} \frac{\psi(w,y)dw}{\sqrt{w^2-1}} = O(y^3)$$

thus as $r\to o$, if $s \neq 2 + 2\delta$, $s \neq 2\delta$

$$c(r) = \frac{sCT_o^{\,2}}{2\left[1-\left(\frac{x_o}{x_m}\right)^s\right]}\frac{\pi}{6}\left\{\frac{x_m^{\,2+2\delta-s} - x_o^{\,2+2\delta-s}}{2+2\delta-s} - r^2\,\frac{6\left(x_o^{\,2\delta-s} - x_m^{\,2\delta-s}\right)}{s-2\delta} + O(r^3)\right\} \tag{51}$$

if $s = 2 + 2\delta$

$$c(r) = \frac{sCT_o^{\,2}}{2\left[1-\left(\frac{x_o}{x_m}\right)^s\right]}\frac{\pi}{6}\left\{\log x_m/x_o - r^2\,3(x_o^{-2} - x_m^{-2}) + O(r^3)\right\} \tag{52}$$

if $s = 2\delta$

$$c(r) = \frac{sCT_o^{\,2}}{2\left[1-\left(\frac{x_o}{x_m}\right)^s\right]}\frac{\pi}{6}\left\{\frac{x_m^{\,2} - x_o^{\,2}}{2} - r^2\,6\log\frac{x_m}{x_o} + O(r^3)\right\} \tag{53}$$

where $O(x)$ is any function for which $\lim_{x\to 0} O(x)/x$ = constant.

We thus have (8) with

$$\mu = 3 \tag{54}$$

which does not depend on the value of s. Therefore, for large ω, the spectral density $S_1(\omega)$ decreases with increasing ω at least as fast as

$$(\text{const.})\ \omega^{-3} \tag{55}$$

for all values of the parameters s, δ. This appears to disagree with Chernov's result.

7.0 DISCUSSION OF RESULTS

We have shown that for craters formed at random on a plane surface by primary meteorite impacts, the covariance function of elevations closer than the minimum crater diameter is

(a) linear for cylindrical craters

(b) parabolic for paraboloidal craters

This implies that the surface elevation is almost surely discontinuous for cylindrical craters. But for paraboloidal craters, the surface is almost surely smooth in the sense that local slopes exist in mean square. (The author conjectures that any crater shape not involving a discontinuity would yield a smooth surface in the above sense.) The shape of the short-distance correlation does not depend on the indices s and δ of the crater formation model, but does seem to reflect (to some extent, anyway) the crater geometry. This appears to disagree with Chernov's result (7) and (11).

We must now inquire whether the above asymptotic analysis has any valid physical uses. The asymptotic results (40) and (51)-(53) are meaningful only if $r < x_o$. Even though fresh impact craters smaller than one meter in diameter do not contribute significant rims or ejecta layers, their crater bowls add significantly to surface roughness. The smallest actual crater depends on the size of the smallest particles bombarding the lunar surface, which is not well determined. The terrestrial satellite data of Naumann (1966) suggests that a minimum particle mass of 10^{-13} to 10^{-12} grams may be acceptable. Other workers have suggested (Alvarez, personal communication, March 20, 1968) that the data may be compatible with a low-mass cutoff at about 10^{-9} grams. The corresponding crater diameters are, respectively, about 10^{-3} centimeters to 10^{-2} centimeters. These sizes are negligible for surface mobility and radar astronomy applications.

The theoretical power added in replacing $x_o > 0$ by $x_o = 0$ is, for the most probable values of s (i.e., γ), essentially negligible. Fixing $x_m = 100$ and $\delta = 1$, we find that c(o) with $x_o = 1$ is only 0.1% smaller than c(o) with $x_o = 0$ when $s = 2.5$, only 1.0% smaller when $s = 3.0$, and 10% smaller when $s = 3.5$. With larger x_m the discrepancy is even smaller, since the larger craters contribute most to the roughness of the surface for $s < 2+2\delta$. For practical purposes the actual value of x_o is not too important, and the asymptotic analysis leading to (40) and (51)-(53) not very relevant.

We must therefore consider the behavior of c(r) for small and moderate values of r greater than x_o. Referring now to Figures 2-7, we see that for either cylindrical or paraboloidal craters, c(r) is roughly

(1) parabolic for $s = 2$

(2) linear (exponential) for $s = 3$

(3) "logarithmic" for $s = 4$

Generalizing from these numerical cases, we conclude that the relation (8) is justified for small $r > x_o$, with

$$\mu = 3 + 2\delta - s \tag{56}$$

which reduces to Chernov's Equation (11) when $\delta=1$, $s=\gamma$, and $\gamma_2=3$.

However, we have carried out our analysis in a mathematically completely rigorous fashion, using either the same physical model as Chernov or a generalization of his model. The difference is that we have worked in the space domain using physically accurate distance interactions for cratering events; whereas, Chernov worked in the frequency domain, with no accurate frequency interpretation of a single cratering event, and with coarse approximations on the number and amplitude in the frequency domain of cratering events. We therefore believe that our analysis is more nearly correct and should be used in preference to that of Chernov.

8.0 FUTURE PROBLEMS

Some extensions of the model immediately suggest themselves. First of all, we have not considered positive relief formation such as crater rims, ejecta blankets, boulders and blocks. The latter are especially important at sizes of a few meters and smaller. Vehicle mobility studies probably demand such refinements.

Secondly, material ejected from primary craters may itself form secondary craters. This can be handled by assuming that $dN(x,\underset{\sim}{R})$ is a first-order clustering process instead of a Poisson process, but the resulting calculations are enormously more complex than those we have made in this paper.

In the same vein, we could use the known higher moment properties of the process $dN(x,\underset{\sim}{R})$ in the case that overlapping of craters is important (Marcus, 1966a, 1966c). We conjecture that the negative correlation between the numbers of large craters and small craters in a heavily cratered region introduces some (probably small) periodicities in the covariance function.

Another factor is the evolution of crater shape in time as a consequence of erosion by micrometeorites, slumping and mass movement, and filling by ejecta. Internal processes such as lava flooding, subsidence, isostatic readjustment, etc., should also be included.

Finally, the assumption (12) that crater depths are strictly additive can be improved. The extent to which the surface at $\underset{\sim}{R}$ is decreased by the formation of a crater at time t' at point $\underset{\sim}{R} + \underset{\sim}{r}$ depends on the elevation $Z(\underset{\sim}{R}+\underset{\sim}{r};t')$ at the time and place at which the crater is formed. Thus, instead of (12), we really have

$$Z(\underset{\sim}{R};t) = \int \zeta(x,r,\Delta Z(\underset{\sim}{R}+\underset{\sim}{r};t')\; dN(x,\underset{\sim}{R}+\underset{\sim}{r},t') \qquad (57)$$

where $Z(\underset{\sim}{R};t)$ is the elevation of $\underset{\sim}{R}$ at time t, $\zeta(x,r,\Delta Z)$ is the elevation change produced by the formation of a crater of diameter x a distance r away when the elevation difference at the two points at the time of formation t' is

$$\Delta Z(\underset{\sim}{R}+\underset{\sim}{r};t') = Z(\underset{\sim}{R}+\underset{\sim}{r};t') - Z(\underset{\sim}{R};t') \qquad (58)$$

and $dN(x,\underset{\sim}{R}+\underset{\sim}{r},t')$ is the number of craters of diameter x to $x + dx$ formed in $d(\underset{\sim}{R}+\underset{\sim}{r})$ during the time interval $(t',t'+dt')$ where $t' < t$. The stochastic process $Z(\underset{\sim}{R};t)$ is obviously very complicated when successive overlaps become important.

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2015-AHM-kse A. H. Marcus

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TABLE 1

	$\int \sqrt{y^2-1}\ dy/y^{s-2\delta+1}$ $s = 1$	$\int \arcsin(1/y)\ dy/y^{s+1}$
$s = 2$	$\sqrt{y^2-1} - \arccos(1/y)$	$\frac{1}{4}\left[\arcsin(1/y) - \frac{\sqrt{y^2-1}}{y^2} - \frac{2\arcsin(1/y)}{y^2}\right]$
$s = 3$	$-\frac{\sqrt{y^2-1}}{y} + \log\left(y+\sqrt{y^2-1}\right)$	$\frac{1}{9}\left[\frac{(y^2-1)^{3/2}}{y^3} - \frac{3\sqrt{y^2-1}}{y} - \frac{3\arcsin(1/y)}{y^3}\right]$
$s = 4$	$-\frac{\sqrt{y^2-1}}{2y^2} + \frac{1}{2}\arccos(1/y)$	$\frac{1}{32}\left[3\arcsin(1/y) - \frac{3\sqrt{y^2-1}}{y^2} - \frac{2\sqrt{y^2-1}}{y^4} - \frac{8\arcsin(1/y)}{y^4}\right]$

TABLE 2

CYLINDRICAL CRATERS, $\delta = 1$

$s = 2$

If $0 \leq r < x_o$,

$$c(r) = \frac{CT_o^{\,2}}{1-\left(\frac{x_o}{x_m}\right)^2}\left\{\frac{\pi}{4}\left(x_m^{\,2} - x_o^{\,2}\right)\right.$$

$$- r\left[\sqrt{x_m^{\,2} - r^2} - \sqrt{x_o^{\,2} - r^2}\right]$$

$$+ r^2\left[\arccos\left(\frac{r}{x_m}\right) - \arccos\left(\frac{r}{x_o}\right)\right]$$

$$+ \frac{r^2}{4}\left[\arcsin\left(\frac{r}{x_o}\right) - \arcsin\left(\frac{r}{x_m}\right)\right]$$

$$- \frac{r^3}{4}\left[\frac{\sqrt{x_o^{\,2}-r^2}}{x_o^{\,2}} - \frac{\sqrt{x_m^{\,2}-r^2}}{x_m^{\,2}}\right]$$

$$\left. - \frac{r^4}{2}\left[\frac{\arcsin(r/x_o)}{x_o^{\,2}} - \frac{\arcsin(r/x_m)}{x_m^{\,2}}\right]\right\}$$

If $x_o \leq r < x_m$,

$$c(r) = \frac{CT_o^{\,2}}{1-\left(\frac{x_o}{x_m}\right)^2}\left\{\frac{\pi}{4}\,x_m^{\,2} - r\sqrt{x_m^{\,2}-r^2}\right.$$

$$- r^2\left[\frac{3\pi}{8} - \arccos(r/x_m) + \frac{\arcsin(r/x_m)}{4}\right]$$

$$\left. + r^3\,\frac{\sqrt{x_m^{\,2}-r^2}}{4x_m^{\,2}} + \frac{r^4}{2x_m^{\,2}}\arcsin(r/x_m)\right\}$$

If $r \geq x_m$, $\qquad c(r) = 0$

TABLE 2

CYLINDRICAL CRATERS, $\delta = 1$

$s = 3$

If $0 \leq r < x_o$,

$$c(r) = \frac{3CT_o^2}{2\left[1-\left(\frac{x_o}{x_m}\right)^3\right]}\left\{\frac{\pi}{2}(x_m-x_o) - \frac{r}{9}\left[\left(1-\left(\frac{r}{x_m}\right)^2\right)^{3/2} - \left(1-\left(\frac{r}{x_o}\right)^2\right)^{3/2}\right.\right.$$

$$\left.- 12\left(\sqrt{1-(r/x_m)^2} - \sqrt{1-(r/x_o)^2}\right)\right] - r\left[\log\left(x_m + \sqrt{x_m^2-r^2}\right)\right.$$

$$\left.\left.- \log\left(x_o + \sqrt{x_o^2-r^2}\right)\right] - \frac{r^4}{3}\left[\frac{\arcsin(r/x_o)}{x_o^3} - \frac{\arcsin(r/x_m)}{x_m^3}\right]\right\}$$

If $x_o \leq r < x_m$,

$$c(r) = \frac{3CT_o^2}{2\left[1-\left(\frac{x_o}{x_m}\right)^3\right]}\left\{\frac{\pi}{2}x_m - \frac{r}{9}\left[6\pi + \left(1-\left(\frac{r}{x_m}\right)^2\right)^{3/2} - 12\sqrt{1-(r/x_m)^2}\right.\right.$$

$$\left.\left.+ 9\log\left(x_m + \sqrt{x_m^2-r^2}\right)\right] + r\log r + \frac{r^4}{3x_m^3}\arcsin(r/x_m)\right\}$$

If $r \geq x_m$,

$$c(r) = 0$$

TABLE 2

CYLINDRICAL CRATERS, $\delta = 1$

$s = 4$

If $0 \leq r < x_o$,

$$c(r) = \frac{2CT_o^{\,2}}{1-\left(\frac{x_o}{x_m}\right)^4}\left\{\frac{\pi}{2}\log(x_m/x_o) - \frac{1}{2}\left[\arccos(r/x_m) - \arccos(r/x_o)\right]\right.$$

$$+ \frac{3}{32}\left[\arcsin(r/x_o) - \arcsin(r/x_m)\right] - \frac{19}{32}\, r\left[\frac{\sqrt{x_o^{\,2}-r^2}}{x_o^{\,2}}\right.$$

$$\left. - \frac{\sqrt{x_m^{\,2}-r^2}}{x_m^{\,2}}\right] - \frac{r^3}{16}\left[\frac{\sqrt{x_o^{\,2}-r^2}}{x_o^{\,4}} - \frac{\sqrt{x_m^{\,2}-r^2}}{x_m^{\,4}}\right] - \frac{r^4}{4}\left[\frac{\arcsin(r/x_o)}{x_o^{\,4}}\right.$$

$$\left.\left. - \frac{\arcsin(r/x_m)}{x_m^{\,4}}\right]\right\}$$

If $x_o \leq r < x_m$,

$$c(r) = \frac{2CT_o^{\,2}}{1-\left(\frac{x_o}{x_m}\right)^4}\left\{\frac{\pi}{2}\log(x_m/r) - \frac{21}{64}\pi + \frac{13}{32}\arcsin(r/x_m)\right.$$

$$+ \frac{19}{32}\,\frac{r}{x_m^{\,2}}\sqrt{x_m^{\,2}-r^2} + \frac{1}{16}\,\frac{r}{x_m^{\,4}}\sqrt{x_m^{\,2}-r^2}$$

$$\left. + \frac{1}{4}\,\frac{r^4}{x_m^{\,4}}\arcsin(r/x_m)\right\}$$

If $r \geq x_m$,

$$c(r) = 0$$

CAPTIONS TO FIGURES

Figure 1 — Hypothetical profile of the development (top to bottom) of a cratered surface, showing superposition (S) and erasure (E) of craters by craters formed later.

Figures 2-4 — Correlation function c(r)/c(o) of elevations a distance r apart on a surface altered by cylindrical craters, as a function of crater population index S and maximum crater diameter x_m.

Figures 5-7 — Correlation function for paraboloidal craters as a function of population index S and maximum crater diameter x_m.

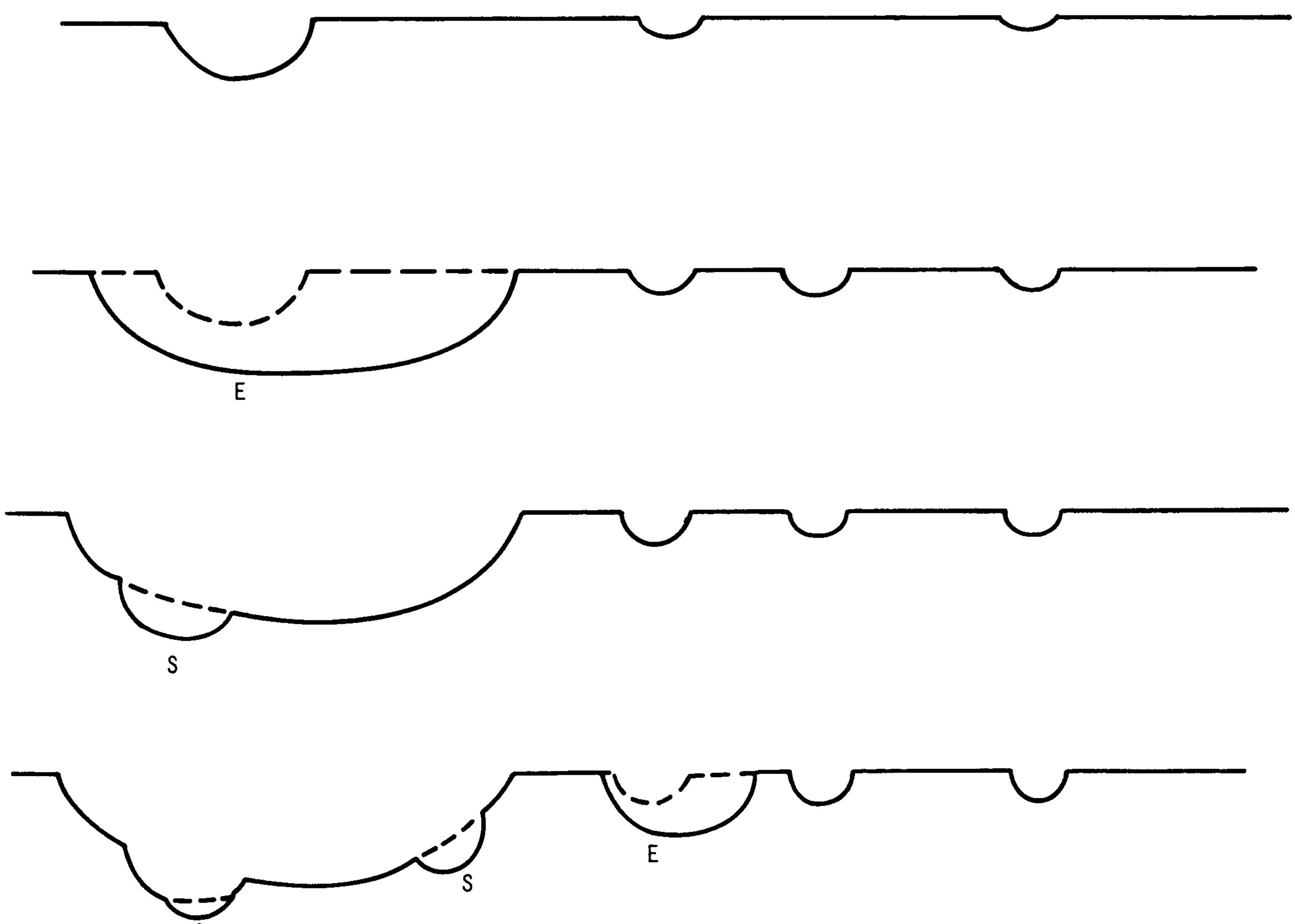

FIGURE I

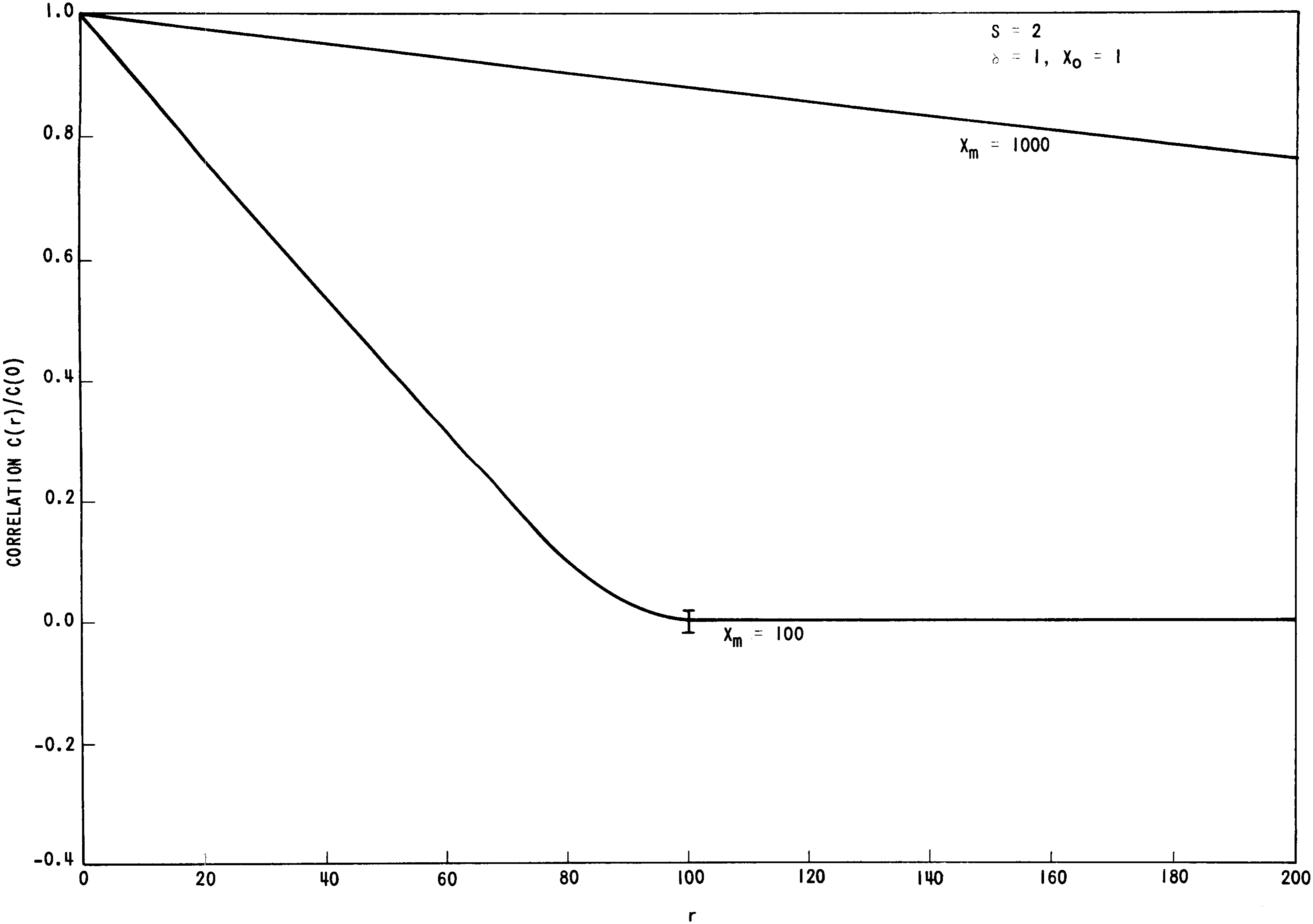

FIGURE 2 - CYLINDRICAL CRATERS

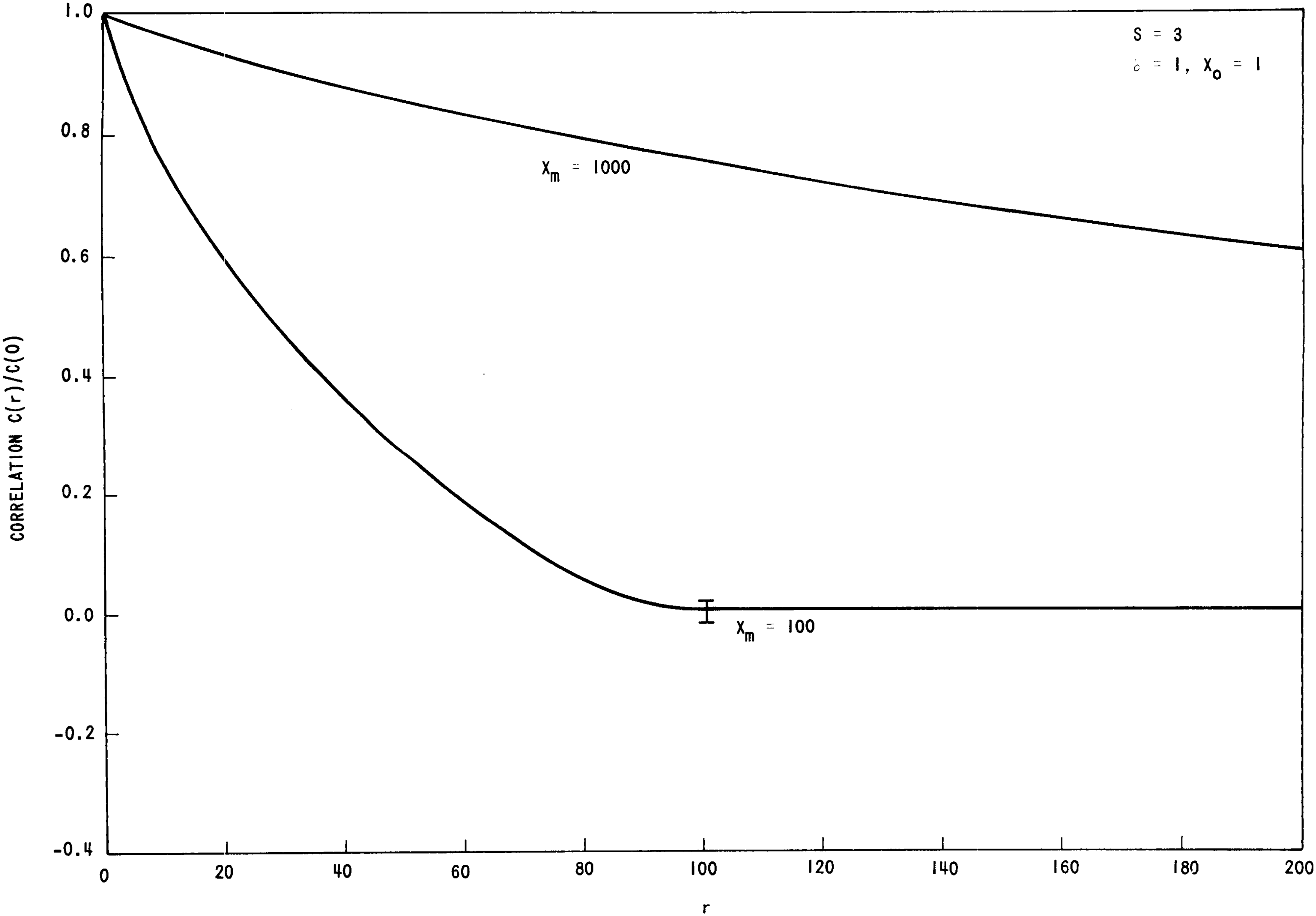

FIGURE 3 - CYLINDRICAL CRATERS

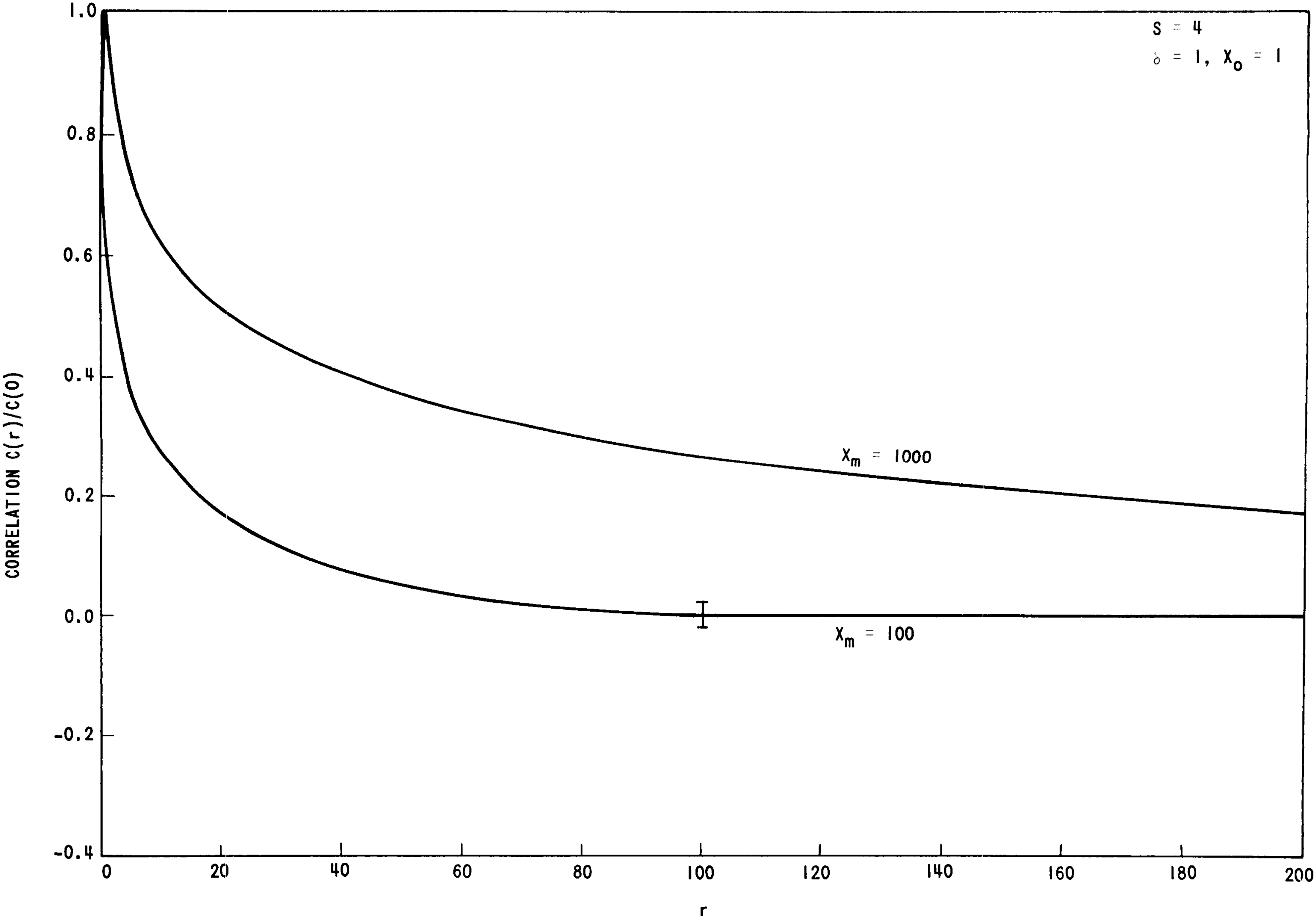

FIGURE 4 - CYLINDRICAL CRATERS

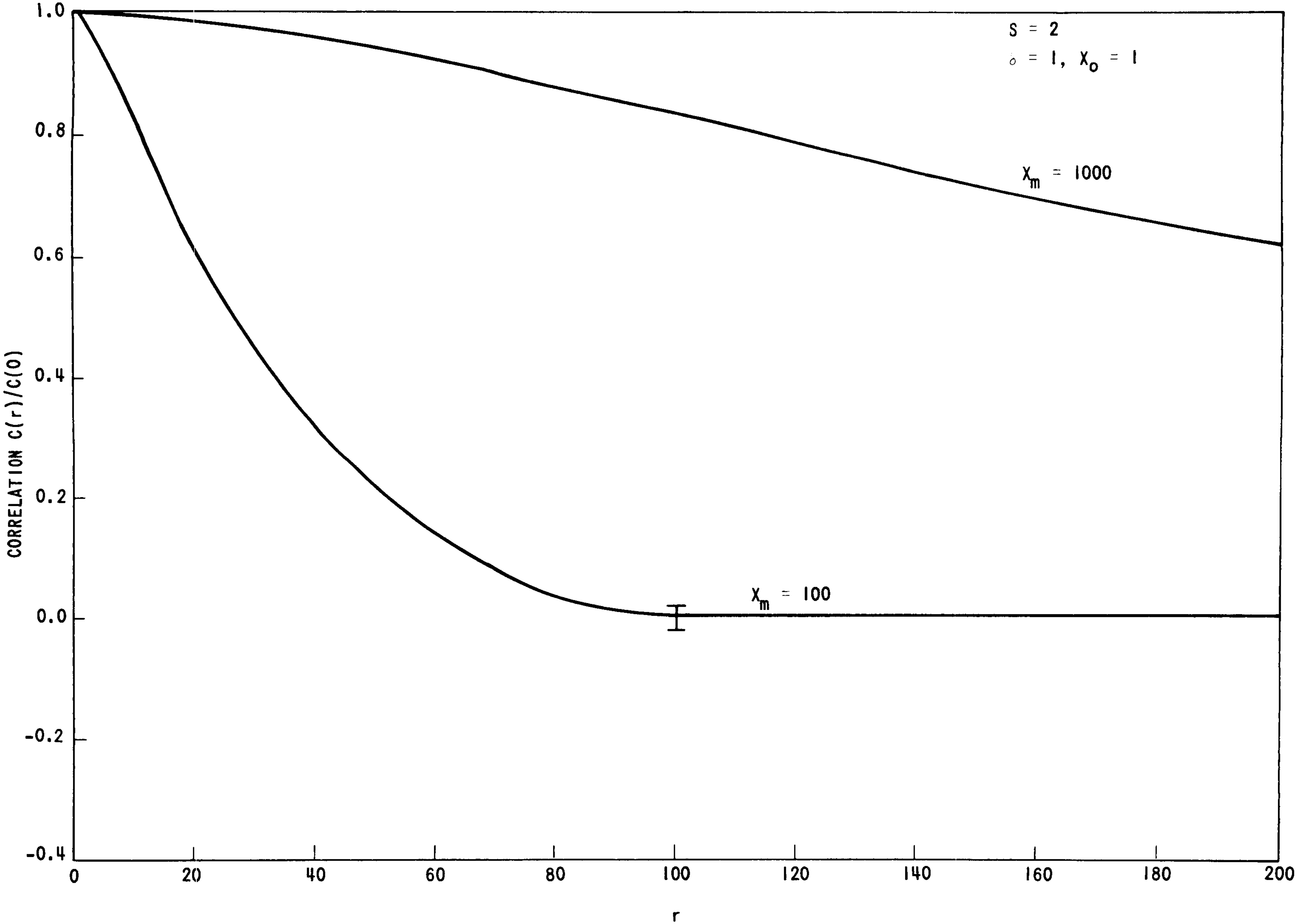

FIGURE 5 - PARABOLOIDAL CRATERS

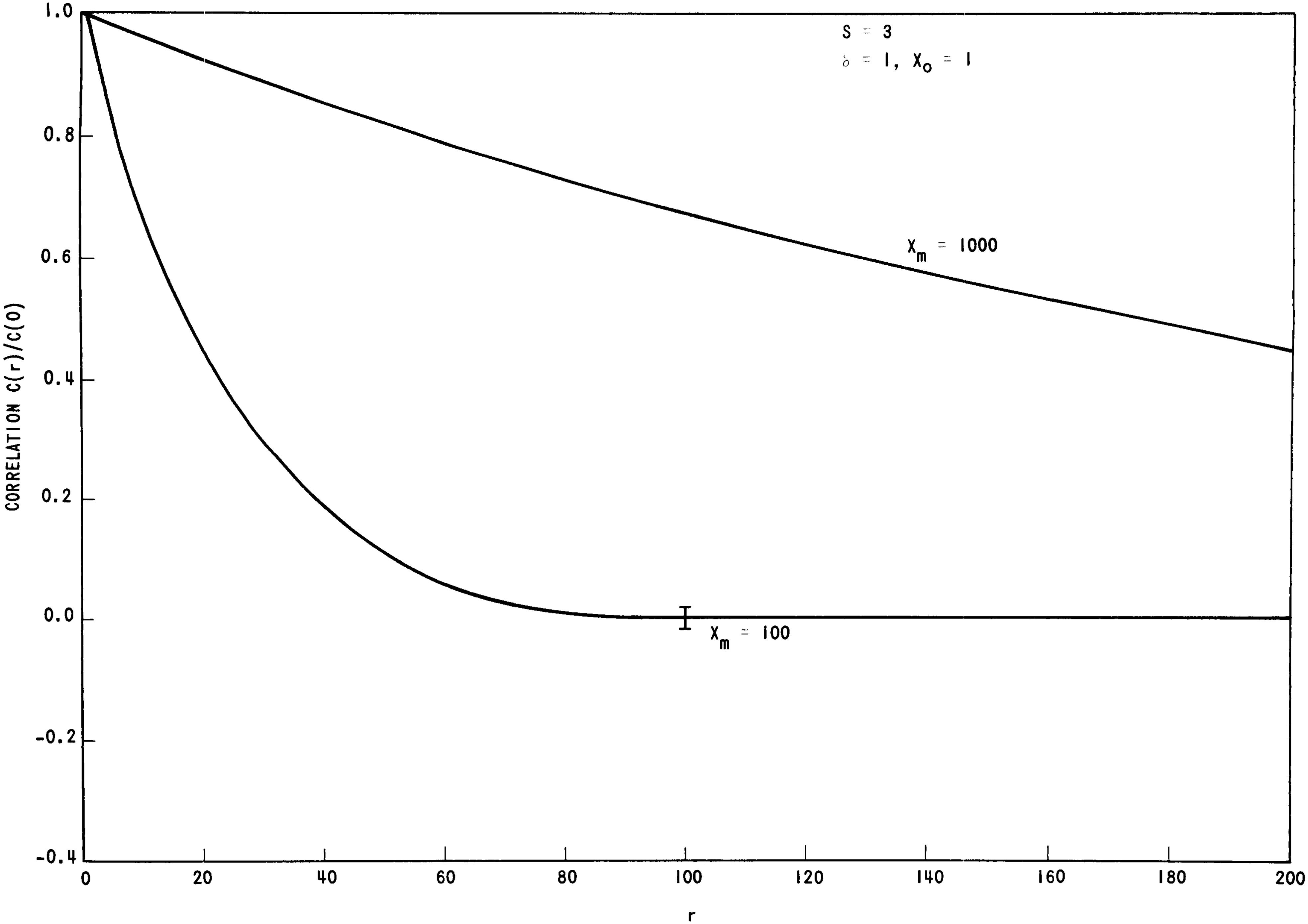

FIGURE 6 - PARABOLOIDAL CRATERS

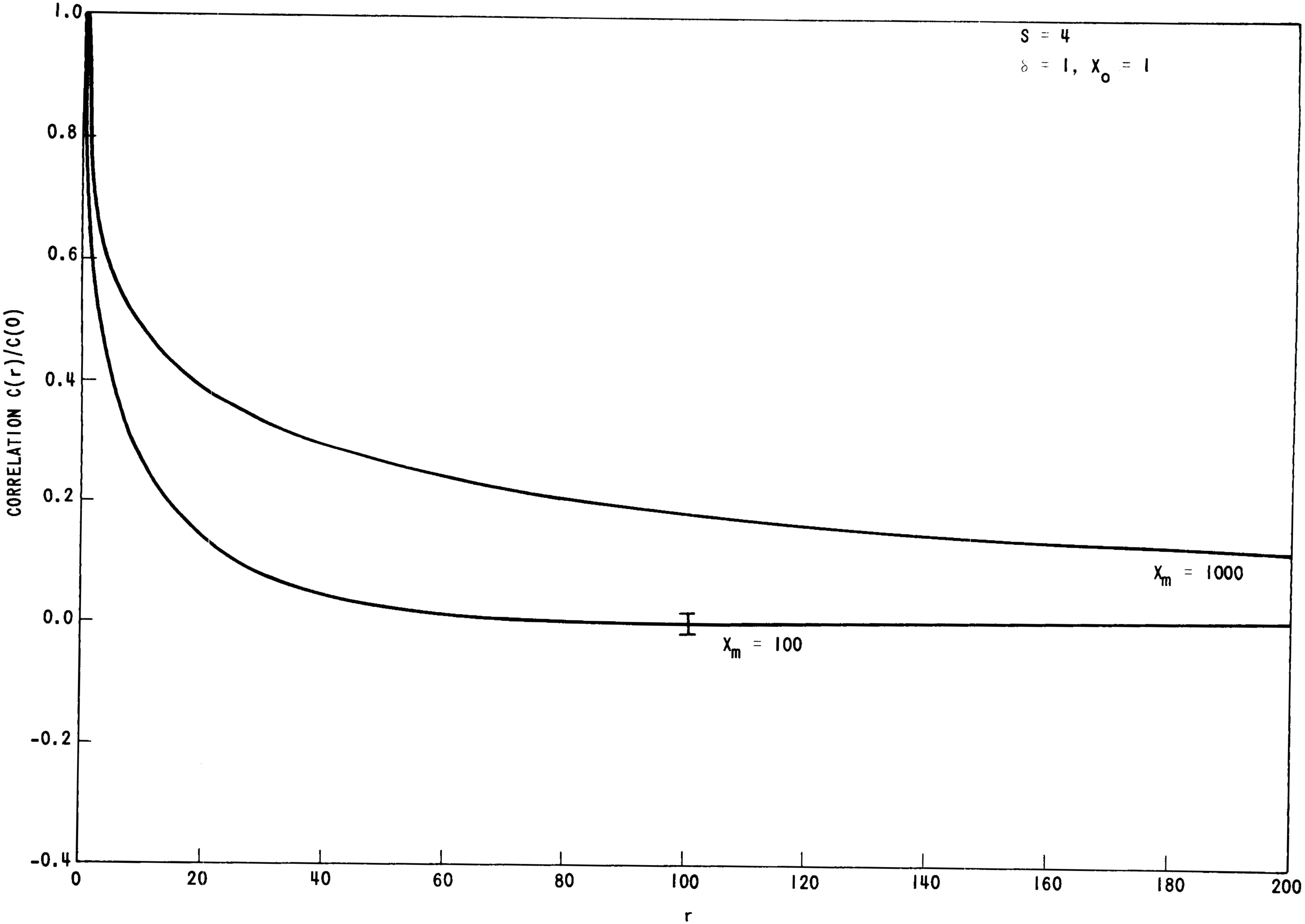

FIGURE 7 - PARABOLOIDAL CRATERS

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