NASA TECHNICAL NOTE **NASA TN D-4704**

c.1

NASA TN D-4704

DESIGN OF BODIES TO PRODUCE SPECIFIED SONIC-BOOM SIGNATURES

by Raymond L. Barger

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1968

DESIGN OF BODIES TO PRODUCE SPECIFIED SONIC-BOOM SIGNATURES

By Raymond L. Barger

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

DESIGN OF BODIES TO PRODUCE SPECIFIED SONIC-BOOM SIGNATURES

By Raymond L. Barger
Langley Research Center

SUMMARY

A mathematical procedure is described for calculating the shape of a body of revolution that will generate a specified signature at a given Mach number and lateral spacing from the body. The applicability of the method is contingent on the requirement that the signature be physically obtainable. Results of a wind-tunnel study with three sample bodies support qualitatively the validity of the theory.

INTRODUCTION

The Whitham theory (ref. 1) has been shown to predict accurately both near-field and far-field signatures produced by a body of revolution (which may be an "equivalent" body for volume alone or for volume and lift (ref. 2)) of given design. Although variations in the shape of the generating body do not normally prevent its signature from developing into the well-known N-shaped wave at large distances, there are some signature shapes that do not attain the classical N-wave form in the far field. One such example is described in reference 1 and another is given in the present paper.

Moreover, even if the signature does approach an N-wave, it may develop so gradually that it does not attain its final asymptotic form within the distance corresponding to objectionable overpressures (ref. 3). If it is determined that certain such wave forms are less annoying or less destructive than others, then it would be advantageous to be able to design bodies that will generate them. Finally, near-field signatures are of interest in connection with certain sonic-boom studies such as wind-tunnel investigations and signature measurements using a probe aircraft.

The purpose of this paper is to describe a procedure for designing bodies corresponding to prescribed pressure signatures.

SYMBOLS

$c = \left(\gamma M^2\right) \Big/ (2\beta)^{1/2}$

F function defined by equation (2)

$k = 2^{-1/2}(\gamma + 1)M^4\beta^{-3/2}$

l body length

M Mach number

p overpressure, difference between local pressure and undisturbed pressure

p_0 undisturbed pressure

$R = (s/\pi)^{1/2}$

s distribution of sources associated with generating body

x,r cylindrical coordinates with origin at nose of generating body and positive x-axis directed along body axis

$X = x - \beta r$

y characteristic parameter, related to x and r by equation (3); the value of y on a characteristic is determined as distance from body nose at which extrapolated characteristic intersects x-axis

$\beta = \sqrt{M^2 - 1}$

γ ratio of specific heats

ξ,η dummy integration variables

ϕ transformed F-function (eq. (4))

Subscripts:

1 first intersection point

2 last intersection point

BASIC RELATIONS

According to reference 1, the relative overpressure p/p_0 on a characteristic line y = Constant, at a distance r from the axis of the generating body is given by (see eq. (69) of ref. 1)

$$\frac{p(y)}{p_0} = \frac{c}{r^{1/2}} F(y) \tag{1}$$

where $F(y)$ is determined from the source distribution by (see eq. (14) of ref. 1)

$$F(y) = \begin{cases} \dfrac{1}{2\pi} \displaystyle\int_0^y (y - \xi)^{-1/2} s''(\xi) d\xi & (\text{for} \quad y > 0) \\ 0 & (\text{for} \quad y \leqq 0) \end{cases} \tag{2}$$

Denote by X the shifted coordinate $x - \beta r$ related to y by (see eq. (12) of ref. 1)

$$X = y - kr^{1/2} F(y) \tag{3}$$

A function of fundamental importance in determining the signature is the function $\phi(X)$ defined by

$$\phi(X) = \frac{c}{r^{1/2}} F\big(y(X)\big) \tag{4}$$

This function is in general multivalued, but the $\phi(X)$ curve is well defined because it is the result of transforming the graph of $F(y)$ (considering this graph as a geometrical figure in the yF-plane) by the transformation of coordinates:

$$X = y - kr^{1/2} F \tag{3a}$$

$$\phi = \frac{c}{r^{1/2}} F \tag{4a}$$

Since the Jacobian of this transformation is $\frac{c}{r^{1/2}}$, the areas will be multiplied by that factor as a result of the transformation. It is to be noted that the Jacobian is constant for fixed Mach number and radial distance r.

Suppose a straight line having slope $\left(kr^{1/2}\right)^{-1}$ is drawn through the graph of the function $F(y)$ in such a way that the areas of the lobes cut off by the line are equal, as in the Whitham procedure for locating a shock (pp. 319-327 of ref. 1). Denote the abscissas of the first and last intersection points by y_1 and y_2, respectively. Then, under the transformation defined by equations (3a) and (4a), this straight line is transformed into a vertical line representing a shock in the signature. All characteristics y for $y_1 < y < y_2$ have been "absorbed" in the shock. The fact that the transformed areas of the lobes cut off by a shock line are still equal was utilized to provide an automatic calculation system for locating the shocks in the procedure described in reference 4.

The signature is identical to the $\phi(X)$ curve except that the multivalued portions of the $\phi(X)$ curve are replaced by a kind of average discontinuity, representing a shock, in the signature. Thus, the shape of the signature between shocks is the same as that of the $\phi(X)$ curve.

It is apparent that the form of the signature is independent of the actual size and shape of the two lobe areas cut out of the $F(y)$ curve by a shock line. The only requirement is that the areas of the lobes be equal.

DERIVATION OF BODY SHAPE

Basic Procedure

If a specified pressure signature is physically obtainable, it is possible to derive the shape of a generating body which will produce that signature by a step-by-step inversion of the mathematical relations of the preceding section. As will be apparent from the construction, the generating body is not unique when the specified signature contains shocks, as will normally be the case for a reasonable standoff distance.

The first step in the procedure is to construct from the signature a function $\phi(X)$ that can be uniquely related to a generating body. Of course, the signature itself, without modification, could be taken to be $\phi(X)$. However, the bodies obtained in this manner often have features which may be undesirable, such as a highly cusped nose section or too large a base area. Alternatively, one can construct equal-area lobes (dashed lines in fig. 1) intersected by the shock.

In order to be physically obtainable, the signature and the function $\phi(X)$ obtained from it must satisfy certain constraints. Inasmuch as $\int_0^\infty F(y)dy = 0$ for any body (eq. (29), ref. 1), then the net area of the signature itself must be zero. Limitations are imposed on $\phi(X)$ by the fact that the corresponding function $F(y)$ cannot be multivalued but can possess discontinuities. Therefore, the slope of $\phi(X)$ is limited by the slope $\frac{d\phi}{dX} = -\frac{c}{kr}$ of a line in the X,ϕ system corresponding to a vertical line in the y,F

system. (See fig. 1.) For a body with a conical nose section,

$$\left.\frac{d\phi}{dX}\right|_{\substack{\phi = 0 \\ X = 0}} = -\frac{c}{kr}$$

Once $\phi(X)$ has been constructed, the function $F(y)$ is obtained by inverting the transformation defined by equations (3) and (4):

$$y = X + \frac{kr}{c}\phi \tag{5}$$

$$F = \frac{r^{1/2}}{c}\phi \tag{6}$$

Then, with the $F(y)$ resulting from this transformation, the source distribution $s(y)$ is determined from equation (2) by the usual rule for inverting the Abel integral equation together with an integration by parts:

$$s(y) = 4\int_0^y F(\eta)\sqrt{y - \eta}\, d\eta$$

where the terms containing a factor of $s(0)$ or $s'(0)$ have been omitted because the theory is only applicable to pointed bodies (for which $s(0) = s'(0) = 0$). Finally, the distribution of radii of the equivalent body is defined by $R(x) = \sqrt{\frac{s(x)}{\pi}}$.

The F-function and corresponding body shape obtained from the function $\phi(X)$ of figure 1 are shown in figure 2 for $M = 1.2$ and $r = l$.

Alternate Procedure for Constructing F(y)

The problem of satisfying the inequality constraints on the slope of the function $\phi(X)$ constructed from the signature is eliminated by an alternate procedure for obtaining $F(y)$ from the given signature. This procedure is as follows:

(1) Apply the inverse transformation (eqs. (5) and (6)) directly to the given desired pressure signature.

(2) In the transformed function, replace each straight-line section representing a transformed shock discontinuity by a curve that forms equal area lobes on either side of the line in such a way that the resulting function is not multivalued. Then, $F(y)$ can be defined to be this function.

COMPARISON WITH EXPERIMENT

Generating Bodies

The procedure described in the preceding section was used to design, from specified wave shapes, the three bodies shown in figure 3, together with their respective F-functions. The first two wave forms were selected because of their intrinsic interest and the third was of use in pressure-probe calibration tests. For each body, the base diameter shown indicates the diameter at which the body was faired into the sting mount. Bodies with smaller base diameters would correspond to F-functions having increased negative area, which would be compensated by an equal increase in the subsequent positive section.

Results of Measurement

The design signatures for $M = 1.2$ and $r = 0.665$ meter (corresponding to r/l values of 4.2, 4.2, and 1.25 for bodies 1, 2, and 3, respectively) are shown in figure 4 together with the measured signatures. (Only part of the signature is shown for body 3 because only part of the signal could be measured, since the entire signature length exceeded the longitudinal traversal capability of the pressure-probe mount.) The measurements were obtained in tests in the Langley 8-foot transonic pressure tunnel using pressure measurement techniques similar to methods which have already been described elsewhere (ref. 5).

The experimentally measured wave shapes approximate the theoretical shapes with some rounding off of the corners. This failure to measure sharp vertices of a signature is a problem which appears to be, to some extent, unavoidable in wind-tunnel sonic-boom studies (ref. 6). However, in this case, at least part of this error, as well as the variations in the nearly constant signature of body 3, can be attributed to relatively loose machining tolerances. The near-field signature shape is sensitive to slight deviations in the shape of the generating body, as is apparent from the form of equation (2).

According to the Whitham approximation, the shock system of body 2 begins to form at a lateral distance of about 2 meters. Within the available spacing of 0.665 meter, no shocks were recorded by either the pressure sensors or the wind-tunnel schlieren apparatus.

It may be observed that the signature produced by body 1 would not asymptotically approach an N-wave but rather a form having the positive and negative parts of an N-wave separated by a section of zero overpressure. This section of zero overpressure corresponds to that portion of the F-function that vanishes identically. (See fig. 3.)

CONCLUDING REMARKS

A mathematical procedure for designing a body of revolution that will generate a given signature has been described. Results of wind-tunnel studies with three sample bodies tend to support the theory.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 25, 1968,
126-61-03-13-23.

REFERENCES

1. Whitham, G. B.: The Flow Pattern of a Supersonic Projectile. Commun. Pure App. Math., vol. V, no. 3, Aug. 1952, pp. 301-348.

2. Walkden, F.: The Shock Pattern of a Wing-Body Combination, Far From the Flight Path. Aeron. Quart., vol. IX, part 2, May 1958, pp. 164-194.

3. McLean, F. Edward: Some Nonasymptotic Effects on the Sonic Boom of Large Airplanes. NASA TN D-2877, 1965.

4. Middleton, Wilbur D.; and Carlson, Harry W.: A Numerical Method for Calculating Near-Field Sonic-Boom Pressure Signatures. NASA TN D-3082, 1965.

5. Maglieri, Domenic J.; Ritchie, Virgil S.; and Bryant, John F., Jr. (with appendix B by Virgil S. Ritchie): In-Flight Shock-Wave Pressure Measurements Above and Below a Bomber Airplane at Mach Numbers From 1.42 to 1.69. NASA TN D-1968, 1963.

6. Carlson, H. W.; and Morris, O. A.: Wind-Tunnel Sonic-Boom Testing Techniques. J. Aircraft, vol. 4, no. 3, May-June 1967, pp. 245-249.

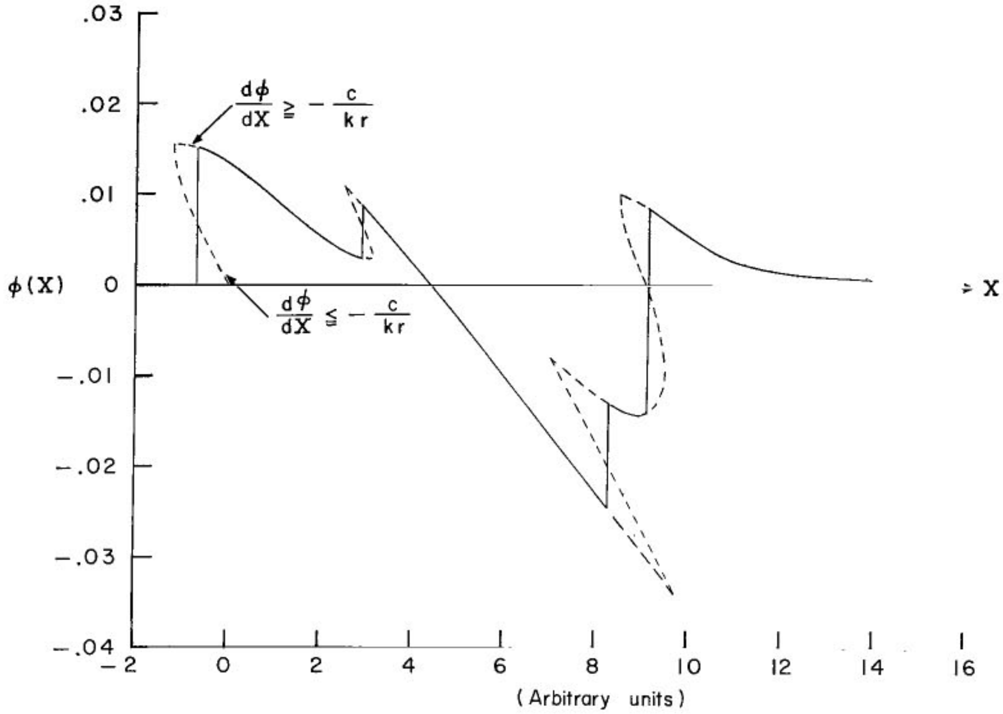


Figure 1.- Construction of Φ(X) from given pressure signature.

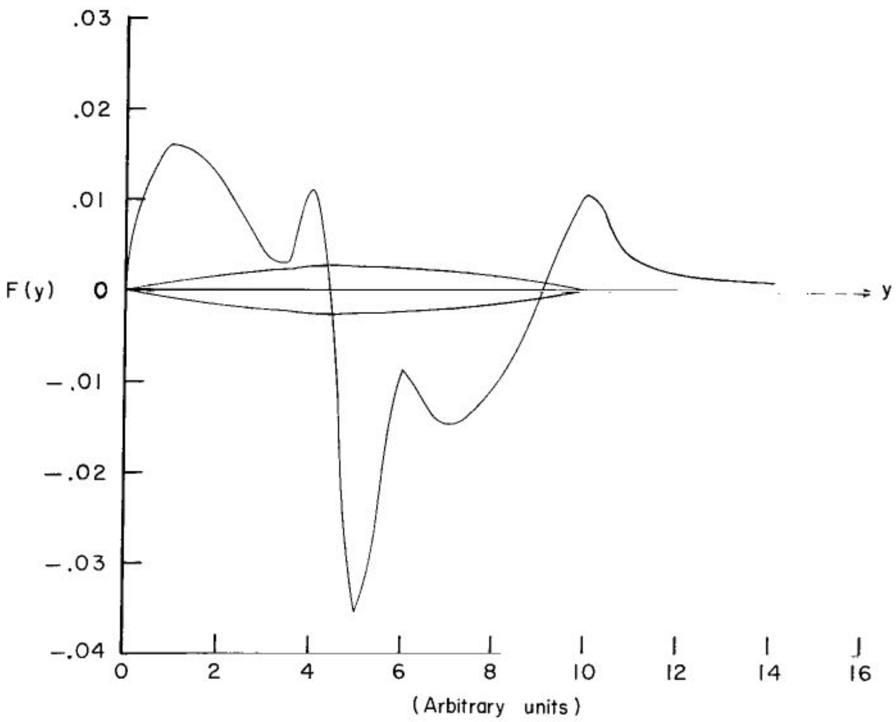


Figure 2.- F(y) and resulting body shape obtained from function Φ(X) in figure 1.

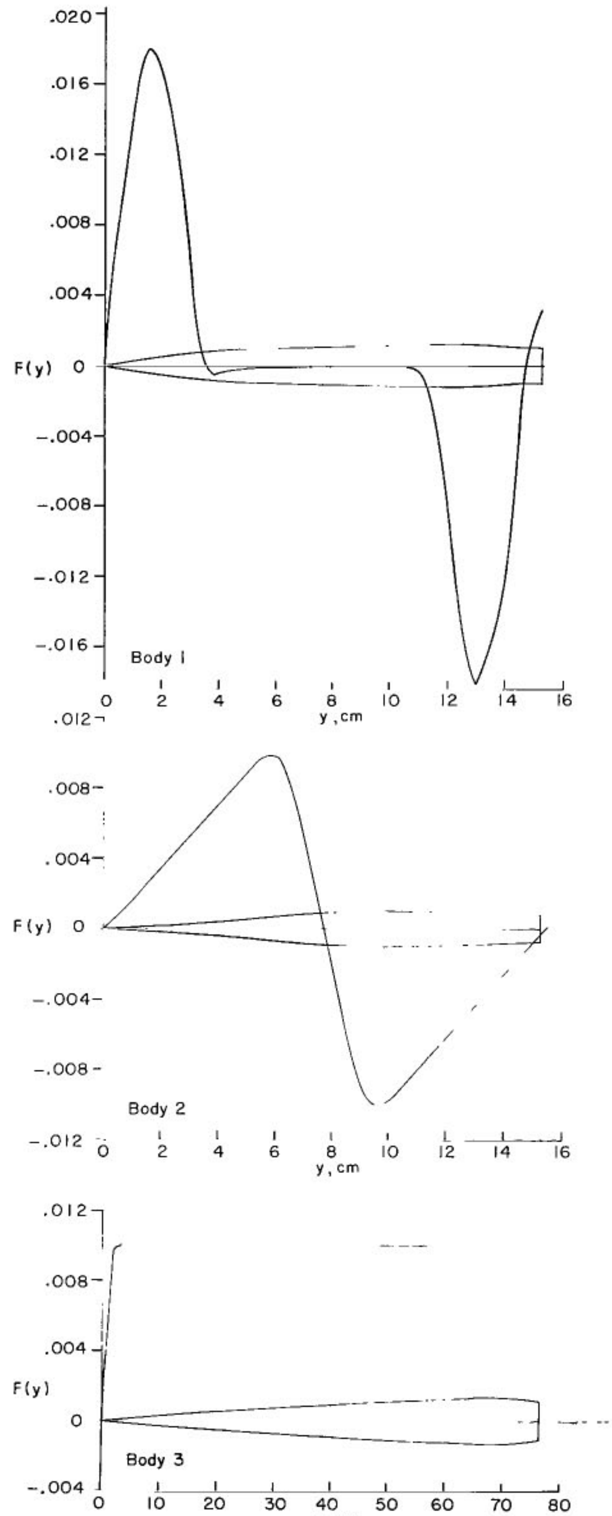
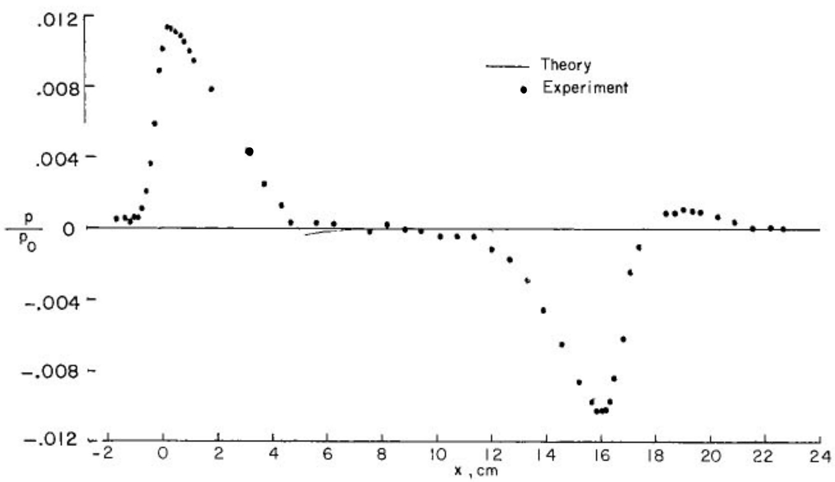
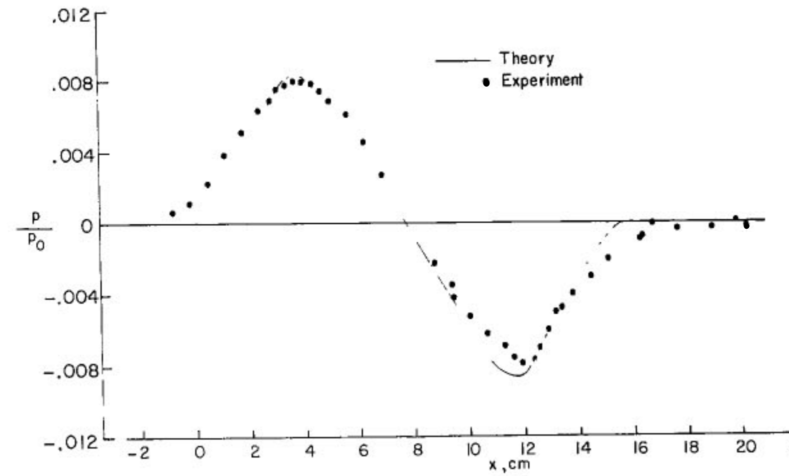


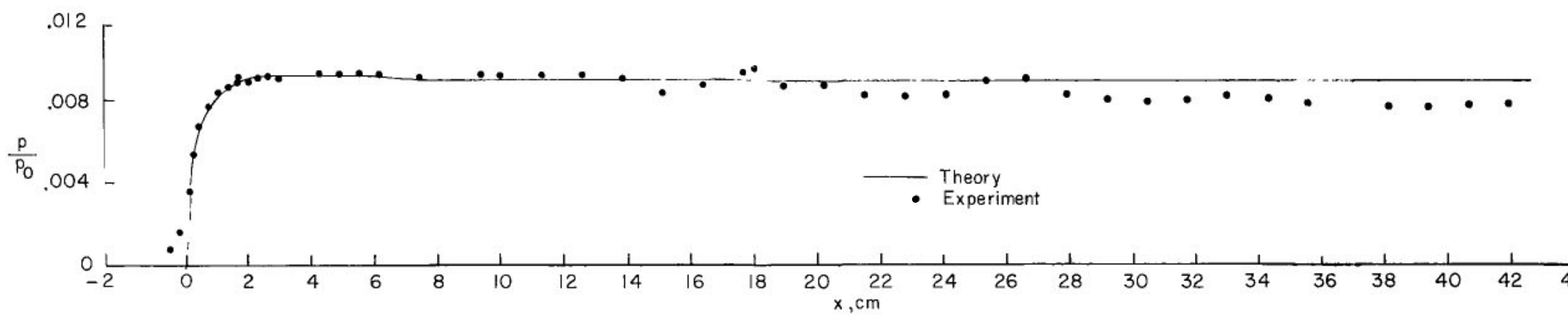
Figure 3.- Calculated F-functions and corresponding bodies for experimental investigation.



(a) Body 1.



(b) Body 2.



(c) Fore portion of body 3.

Figure 4.- Specified and measured pressure signatures. ($M = 1.2$ and $r = 0.665$ m.)

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546

OFFICIAL BUSINESS

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

040 001 26 51 3DS 68216 00903
AIR FORCE WEAPONS LABORATORY/AFWL/
KIRTLAND AIR FORCE BASE, NEW MEXICO 8711

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

— NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546