NASA CONTRACTOR REPORT

NASA CR-1

NASA CR-1124

A THEORY AND MODEL OF HUMAN LEARNING BEHAVIOR IN A MANUAL CONTROL TASK

by Albert Ernest Preyss

Prepared by
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Mass.
for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1968

NASA CR-1124

A THEORY AND MODEL OF HUMAN LEARNING BEHAVIOR IN A MANUAL CONTROL TASK

By Albert Ernest Preyss

Prepared under Grant No. NsG-577 by
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Mass.

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

A THEORY AND MODEL
OF HUMAN LEARNING BEHAVIOR
IN A MANUAL CONTROL TASK

by

Albert Ernest Preyss

ABSTRACT

A theory is presented for the explanation of human learning behavior in a manual control task. In the performance of a psychomotor task, a human operator responds to sensory stimuli with limb movements. This psycho-physiological phenomenon is conceptualized herein as a single channel information processing system. A sensor, a decision center and an effector are the serially connected components of the system. Transmission and processing of information expend time, and the delay between the reception of a finite sum of component times which are assumed to be statistically independent random variables.

In the decision center responses are selected from a set of possible alternatives. Stored in memory are apriori estimates of the probability that a specific response should be inforced at the moment of decision. Response selection is determined by a rule which takes the priors into account. Learning is effected by a revision of the priors based on the weighting of certain evidence. Readily perceived events in the state history of the dynamic process being controlled are used for evidence in resolving control policy uncertainty. Bayes' theorem is the revision rule.

The model of human learning behavior is a computer program obtained from a translation of the theory into machine language. Behavior of the model depends not only on the rules of information processing postulated by the theory, but also on a set of parameters characterizing the mental and physical attributes of an individual human operator. Model behavior is compared with subject behavior measured in a motor skill experiment performed at M. I. T.'s Man Vehicle Laboratory.

As set forth, the theory explains how a human operator learns to regulate the state of a dynamic process using a relay controller. Generalization of the theory to other task contexts is discussed.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1 Background

Human learning behavior in a manual control task is the theme of this dissertation. The real impetus for intensive research on this subject came from World War II with the development of highly complex man-machine systems for employment in modern warfare. Wartime experiences with the design of sophisticated weapons and with the selection and training of personnel to man them emphasized the need for a comprehensive study of the human operator and the task variables effecting his performance. Systematic investigations of motor skills were initiated in the immediate postwar years both here and abroad. Sponsored and encouraged primarily by the military services, researchers proceeded to test various earlier hypotheses about psychomotor behavior and to revise them on the basis of fresh evidence, as well as to postulate entirely new hypotheses. Over the years, beginning around 1945, the efforts of these researchers have resulted in the accumulation of a wealth of experimental data and in a substantial amount of verbalizing about the inferences which can be drawn from these findings. Reviews of this period, such as those by Bilodeau and Bilodeau [18], Adams [2], and

Young and Stark[121] give an excellent accounting of the work which has been done by psychologists, physiologists, engineers and contributors from other disciplines. Perusal of the literature cited in these reviews reveals that although this period of investigation has produced many significant accomplishments, the list of achievements does not include a theory or model of human learning behavior in a manual control task.

1.2 Scope of Work

A theory is presented in this work for the explanation of how motor skills are learned by human operators. Based on this theory, a model of human learning behavior in a manual control task is constructed. A test of the theory is provided by a comparison of the learning behavior of the model with the learning behavior of subjects who participated in a recent experiment conducted at M.I.T.'s Man-Vehicle Laboratory.

1.3 Theoretical Approach

Motor skills learning is given a stochastic interpretation by the theory. According to this interpretation, motor skills learning is a statistical revision making process by which the human operator identifies a policy for the manual control of a dynamic process. This policy determines the limb movement he will make in response to a given sensory stimulus. Before this policy is identified, the human operator is uncertain as to which limb movement, of a possible set of alternatives, is the correct response to a given sensory stimulus. He must, nevertheless, respond to stimuli (no limb movement is also considered to be a response) while the manual control task is going on. When he does, his selection of a response alternative, the theory postulates, is based on his preferences for the alternatives in ranking them at the moment of choice, these preferences being expressed as probabilities.

A control policy is identified and therefore, a manual control task is learned when the human operator resolves his uncertainty. That is, when by this statistical revision making process he changes his preferences until all but one alternative response to a given sensory stimulus becomes improbable. Bayes' theorem is the proposed analogue of man's algorithm for revising his opinions, i.e., for changing his preferences for alternatives. Using probabilities for the orderly expression of human opinion and representing statistical revision making by Bayes' theorem are concepts which characterize an application of Bayesian statistics for the probabilistic description of human information processing. These ideas have been incorporated in the present theory to permit a complete mathematical treatment of a psychological phenomenon, the explanation of which is enchanced through quantification.

The theory postulates that the selection of response alternatives and the revision of preferences for response alternatives are functions of what is called the decision center of the human mind and that this decision center is one component of a single-channel information processing system. Also included in this information processing system are a sensor, which perceives the information upon which the decision center acts, and an effector, which executes the response decisions made by the center. An explanation of the operation of the sensor and the effector completes the description of human learning behavior provided by the theory.

1.4 Modelling Approach

A model of human learning behavior in a manual control task is readily constructed, once the theory has been developed. The model is, in fact, a digital computer program which is obtained from a straightforward translation of the theory into machine language. There are a set of read-in parameters which govern the learning behavior of the program. These parameters correspond to human psycho-physiological characteristics and can be adjusted to vary the individuality of the program. Adams(op. cit.) in a well written and succinct discussion of motor abilities makes the following statement, "Whatever the eventual approach, the need is laws for individual behavior." It is quite apparent that inter-subject intra-trial response variability is present in any experiment involving the testing of humans. To ignore the factors which cause one individual's behavior to differ from another's or to differ from one time to another is to ignore what are frequently the major sources of variance in experimental data. Recourse to a stochastic model with variable parameters represents an attempt to account for these sources of variance.

CHAPTER 2

THEORY AND MODEL

2.1 General

A theory and a model of human learning behavior in a manual control task are developed in this chapter. Regulating the state of a dynamic process is the manual control task in which the learning behavior of the human operator is explained. Although the approach taken herein may be adopted for the explanation of human operator behavior in other task contexts and some of these extensions will be discussed later, we are specifically concerned with the behavior of operators learning how to null the output of a dynamic process thru the actuation of a two-position relay controller. In addition, it is assumed that the dynamic process being controlled is time invariant and defined by a linear or nonlinear differential equation of second order and first degree. Again, it is possible to generalize our approach and consider processes not included in this restricted class. However, extensions in this direction are not discussed in the present work.

Human operators in performing manual control tasks respond to sensory stimuli with limb movements. The development of our theory begins with the conceptualization of this psycho-physiological phenomenon of response generation as a single channel information processing system. Following the presentation of this concept, we proceed to elaborate upon a description for each of the serially connected components comprising this information processing system. There are three components to be considered: a sensor, a decision center and an effector, and it is natural to treat them in this order, since it corresponds to the sequence in which we assume information is processed as it flows thru the system. Each component operates on the information transmitted to it, and with the exception of the sensor's function, these operations are interpreted stochastically. By interpreting human information processing stochastically, we are able to account for both the inter-subject and the intra-subject variability which are characteristic of human responsiveness in manual control tasks. The theory we present, therefore, is a theory which predicts the performance of individual human operators in a specific task and which explains the causes of differences in performance between individuals.

Before we begin the detailed development of the theory, we briefly outline our concept of how the human operator functions as a stochastic information processing system. In our view, information, related to the state of the dynamic process being controlled and displayed to the human operator, is perceived by the sensor, quantized and transmitted to the decision center. When the center is free to process new data, it accepts the most recently received sample of state information and decides upon a response to this stimulus. A decision is required because alternative responses to the same stimulus are possible. Stored in the memory of the decision center are the operator's preferences for the possible alternatives and we express these as probabilities. Selection of a response is governed by a rule which takes the operator's preferences into consideration. Response decisions are then passed on to the effector for execution. Time elapses between the acceptance of a sample and the completion of the selection and between this moment and the execution of the response. These intervals are treated as statistically independent random variables. During each of these cycles, the decision center may also take time out to revise the stored preferences before initiating the selection process, if it is deemed necessary. Revisions are based on the outcomes of previous response selections, a procedure which we refer to as the weighting of evidence and which we describe by an application of a set of ideas collectively called Bayesian statistics. Thus, the learning behavior of the system is characterized by a weighting procedure which revises preferences for possible response alternatives. Processing a revision adds to the delay between stimulus reception and response execution, and this increment is also treated as a random variable.

When the development of the theory is finished, a model of human learning behavior in a manual control task is constructed. This model is a computer program derived by translating the theory into machine language. The translation is accomplished by writing a source program in FORTRAN symbolism and compiling it on an IBM 7090 digital computer. The theory is tested in a later chapter by comparing the behavior of individual runs of the program with the behavior of individual human operators who performed the manual control task in a motor skill experiment. We proceed now with the theoretical development.

2.2 A Manual Control Task

A manual control or psychomotor task may be defined as a task wherein a human operator, thru a psycho-physiological process, in response to sensory stimuli, makes limb movements for the purpose of controlling a dynamic process to achieve some specified objective. In this work we are concerned with the behavior of human operators who are learning how to regulate the state of a dynamic process by pressing or releasing a key with their finger, thereby actuating a two-position relay controller. We assume that the process dynamics are second order and first degree and that the output of the process, x, is displayed to the operator. Further, the objective is to keep x nulled and operator performance is scored on the basis of the integrated absolute value of x over the duration of a trial. In this task, the complete finger movement necessary to actuate the switch once is defined as an operator's response.

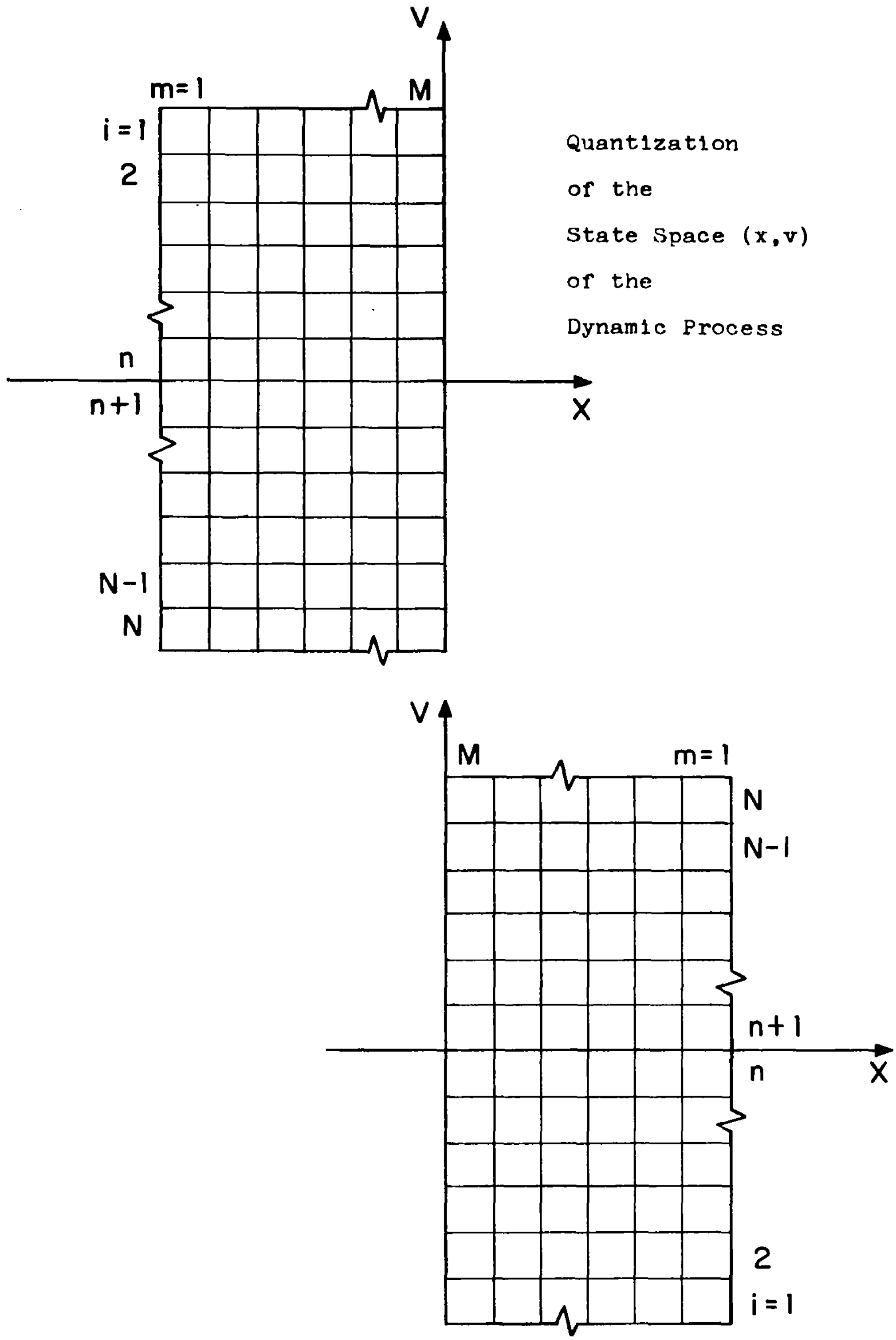
2.3 The Human Operator: A Single Channel Information Processing System

A basic postulate of this theory is that "the human operator is fundamentally a one-channel data processing system, and that a central decision mechanism must be allowed a finite time to process one S-R (stimulus-response) sequence before accepting a second." The quotation is from Adams and Creamer[4], who go on to cite what evidence is available to support this hypothesis, including their own experimental findings. Much of the evidence has been contributed by British investigators, the most prominent of whom is Welford (108). Research on this topic[106] often appears under the heading of the "psychological refractory period".

Consistent with the assumption that the human operator is a single channel information processing system we propose the following interpretation of a stimulus-response sequence. Wilde and Westcott[116] have also "sought a deeper understanding of the physiological mechanism by which a visual stimulus produces muscular movement of a human operator," and their thinking has influenced our interpretation. As we see it, the sensory apparatus, which is the first component in this serially connected system, transmits information to a decision center, the second component. This information arrives continuously, but is only sampled by the center occasionally because the processing of information requires a finite time. In the center, a decision is made on a choice of response to the stimulus. After the decision is made, the center transmits information to the effector mechanism, the third component. While the effector is executing the response, the center is preoccupied with the task of monitoring the execution, and so it can not make another response decision until the execution is complete. Part of this monitoring task involves the processing of proprioceptive signals fed back from the postural system. When the execution of a response is complete, the center accepts a new sample of sensory information and the cycle repeats itself. The time to complete one cycle is the sum of the time to make a decision and the time to execute a response. DT will be used to designate the first interval and RT the second.

2.4 The Sensor

In a manual control task man's sensory apparatus provide him with the state information necessary to effect the closed loop control of a dynamic process. His perception of the displayed output, x, and of its rate of change, v, is subject to certain limitations. We know, for example, that tones close in frequency can not be discriminated between by the ear, that angles can be resolved by the eye only to fractions of a degree and that the estimation of rates of change in stimulus dimensions is less certain than the estimation of the dimension itself. It is also known that sensory information is delayed in its transmission to the higher mental centers, so that the human operator's knowledge of the process' state is never current. Then there is the question of how this information is coded for mental processing. This question is important because man's channel capacity is limited and efficient coding is tantamount to minimizing processing times which, in turn, is essential to good performance in control tasks. Our description of the operation of man's sensory apparatus is an attempt to consider all of these factors in the simplest manner. Figure 2.1 shows a finite grid overlaying the state space of the dynamic process. It is assumed that sensory stimuli are categorized by the coordinates (m,i) of the mesh in which the process' state actually lies. We are saying, in effect,



Quantization of the State Space (x,v) of the Dynamic Process

FIGURE 2.1

that the decision center, due to measurement errors and transmission delays, is certain of the current state of the dynamic process only to within the dimensions of a mesh and furthermore, that even if the data could be resolved further, it would not be desirable since processing times would be lengthened. We are, therefore, coding by quantizing. Note that the points (x,-v) and (-x,v) have the same mesh coordinates.

2.5 The Decision Center

Subjective Probabilities - When the decision center samples the state information transmitted by the sensor, it must use this data to decide upon a response. For the task in question this means choosing between the alternatives: to switch control polarity or not to switch. During the learning phase, the human operator does not know which of these two alternatives is correct. It may be said, therefore, that a state of uncertainty exists in his mind. Thus, before he can make a response, the human operator is forced to weigh each of the alternatives and on the basis of some expression of preference, to select one. According to a definition by Good [41], "a psychological probability is a degree of belief or intensity of conviction that is used for betting purposes, for making decisions, or for any other purpose, not necessarily after mature consideration and not necessarily with any attempt at consistency with one's other opinions." When a person uses a "consistent" set of probabilities, that is, they obey the usual axioms of probability, Good says that they are called "subjective" probabilities. We accept this concept and propose the use of a probability as an expression of preference in the weighting of an alternative.

Hypothesized Control Policies - In order to determine the probability that the control polarity should be switched we go through an intermediate step which is now discussed. A control policy for the regulation of a second order dynamic process may be defined by specifying the locus of states, (x,v), dividing the region of phase space wherein the control polarity should be positive from the region wherein it should be negative. Such a locus is called a switch curve. We will assume that stored in the memory of the decision center is a set of probabilities, $p(H_i(x_m))$, for each of the MxN hypotheses,

$H_i(x_m)$: The switch curve passes thru the mesh, (x_m, v_i),

and that these probabilities are distributed so the conditions,

$$\sum_{i=1}^{N} p(H_i(x_m)) = 1, \quad m = 1,\ldots,M \qquad 2.1$$

are satisfied. By summing on i instead of on m, we avoid certain difficulties which would arise later, because a switch curve is not necessarily single-valued when expressed as a function of v. A subset of these hypotheses such as the joint hypothesis,

$$\left\{ H_r(x_1),\ H_s(x_2),\ldots,H_t(x_M) \right\},$$

may be interpreted as the definition of a control policy, since it specifies the mesh, for every x_m, wherein the control polarity should be switched. Although it is possible to base switching decisions on these joint hypotheses and their probabilities, we propose a much simpler scheme. Before we do, though, recall the labelling in figure 2.1. The reason for drawing no distinction between the states (x,-v) and (-x,v) is that we are assuming the switch curve is antisymmetric and therefore, the same decision can apply to either state.

A Selection Rule - The probabilities assigned to the hypotheses, $H_i(x_m)$, may be used in several ways to decide whether or not to switch control polarity when the sampled state is (x_m, v_j). It is not likely that a single one of these methods will characterize the decision making of all human operators. Rather, one would expect the rule for the selection of an alternative to differ among individuals. Nevertheless, we will postulate a unique representation of the selection process in order to make the development of theory and model more tractable. The selection rule we propose depends on the probability,

$$Q_j(x_m) = \sum_{i=j}^{N} p(H_i(x_m)), \qquad 2.2$$

that the switch curve at x_m passes thru a mesh whose velocity coordinate lies in the closed interval, (v_j, v_N). We refer to this as a switching probability and assume that the selection of an alternative is a Bernoulli trial with probability, $Q_j(x_m)$, of success, i.e., of switching. When we speak of switching with probability, $Q_j(x_m)$, we imply that the control polarity is opposite to the sign of x_m at the time of decision and that the switch will make the signs the same. In this case, $1-Q_j(x_m)$ is the probability that the signs are kept opposite, i.e., no switch occurs. If at the time of decision the signs are already the same, then $1-Q_j(x_m)$ is the probability that the center decides to switch control polarity to make it the opposite of $\mathrm{sgn}(x_m)$ and in this case, $Q_j(x_m)$, is the probability of not switching.

Prior Probabilities - Consider now what occurs on the first trial of a motor skill experiment. A subject has been briefed on the task he is to perform. On what does the human operator base his first response? If no clues have been provided by the briefing, any preference for a particular response must reflect a personal bias stemming from his past experience with similar or related tasks. Or, a subject might make a guess at what the dynamics of the process are and thereby be favorable disposed toward one control policy. Another subject may have very little experience with manual control tasks and may be initially inclined to treat the possible alternatives as equally likely candidates. Whatever his background, a subject's initial beliefs, those which he "brings with him," are expressed by the probabilities stored in the decision center's memory at the beginning of the experiment. These are called prior probabilities and a decision to respond for the first time is based on them.

Revising The Priors - In order to learn a psychomotor task, the human operator must resolve his uncertainty as to the location in phase space of the switch curve. He may wish, therefore, to revise his opinions and express some other preference for the possible alternatives. A revision of opinion can be treated as a change in the prior probabilities, $p(H_i(x_m))$. In the terminology of statistics the revised opinion is commonly referred to as a posterior probability. Information used for the purpose of revising an opinion shall be called evidence, E. Whatever the form of this evidence, the subject's use of it can be thought of as a weighting of the prior. Such a weighting may be represented symbolically in the following way,

$$p'(H_i) = w_i(E)\ p(H_i), \qquad 2.3$$

where the prime denotes a posterior probability and the term, $w_i(E)$, is the weighting applied by the evidence.

A Revision Rule - A trivial consequence of the product axiom of probability is a relationship known as Bayes' theorem,

$$p(H_i/E) = p(E/H_i)p(H_i)/p(E) \qquad 2.4$$

An analogy can be drawn between equations 2.3 and 2.4 if a posterior probability is taken to mean the conditional probability, $p(H_i/E)$, that the i-th hypothesis is true given the evidence, and if the weighting term is identified with the term, $p(E/H_i)/p(E)$. There have been recent investigations, see for example Phillips, Hays and Edwards [86] or Beach [10], to determine whether or not, in his estimation of posterior probabilities, man is a Bayesian (i.e., he applies a revision rule approximating Bayes' theorem which is the formally optimal rule). A dominant finding is that man is conservative: he is inefficient in resolving his uncertainty, as he is unable to make maximum use of the available evidence. Selection of an algorithm to characterize man's revision rule is complicated not only by the question of efficiency, but also by the question of uniqueness. It again seems reasonable to expect that rules for the revision of opinion differ among individuals. Thus, the analogy which has been suggested is certainly but one of many possible. However, we will, nevertheless, accept the analogy for the unique characterization of man's revision making process.

We have assumed there are N hypotheses for every m. Thus the substitution

$$p(E) = \sum_{j=1}^{N} p(E/H_j)\ p(H_j) \qquad 2.5$$

is valid. Making use of the postulated analogy and of equation 2.5 permits the following definition of the weights,

$$w_i(E) = p(E/H_i) \Big/ \sum_{j=1}^{N} p(E/H_j)\, p(H_j), \quad i = 1,\ldots,N \qquad 2.6$$

In this expression the denominator term on the right hand side can be thought of as a normalization factor which is required in order that the condition,

$$\sum_{i=1}^{N} p'(H_i) = 1 \qquad 2.7$$

be satisfied. Therefore, the formal evaluation of the weights, $w_i(E)$, can be accomplished once the priors, $p(H_i)$, are known and the N conditional probabilities $p(E/H_i)$, have been determined. When a prior is revised, the resulting posterior probability becomes the prior for the next revision and so on.

Weighting The Evidence - If a revision is made, what evidence is used and in what way? An answer to this question depends on the task itself. In the present work we are dealing with a state regulator task in which the human operator actuates a relay to null the output of a second order dynamic process. At any instant of time during the course of a trial in this task, the signs of the state variables, (x,v), and the polarity of the control can be used to distinguish which of four possible situations prevails. Each case is depicted on figure 2.2 with sketches of segments of the corresponding phase trajectory, and with the controller output called u. A decision to reverse control polarity in each of these four situations presents evidence to the operator which he can use to resolve his uncertainty as to the location of the switch curve.

In the first situation, a decision to switch might result in the outcome illustrated by the first sketch of figure 2.3. Call the position at which the trajectory crosses the x-axis, x_k. The theory postulates that the evidence,

$E_{jk}(x_m)$: Switching in the mesh (x_m, v_j), when u and x are of opposite sign and x and v are of opposite sign, results in the phase trajectory crossing the x-axis between x_k and $x_k + \Delta x$,

is used by the operator to test the hypotheses, $H_i(x_m)$, $i = 1,\ldots,N$. This implies that in order to revise his estimate of $p(H_i)$, the human operator must assign a value to each of the N conditional probabilities, $p(E_{jk}/H_i)$. Collectively, these N conditionals are part of what we call the human operator's "subjective model of the physical world." A subjective model of the physical world summarizes man's beliefs concerning the likelihood of obtaining various outcomes from an experiment

Possible Sign Combinations of the Variables x, v and u

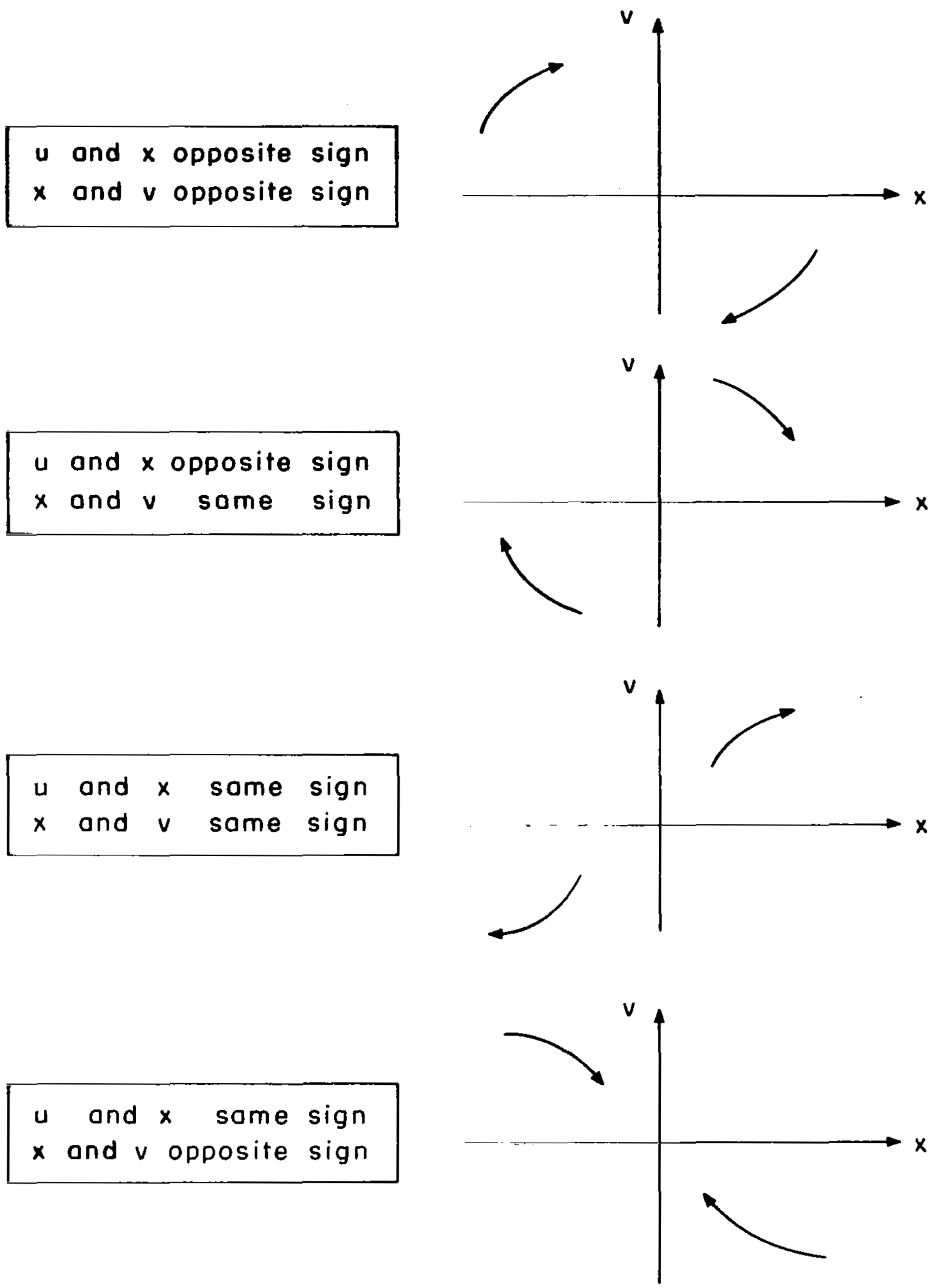
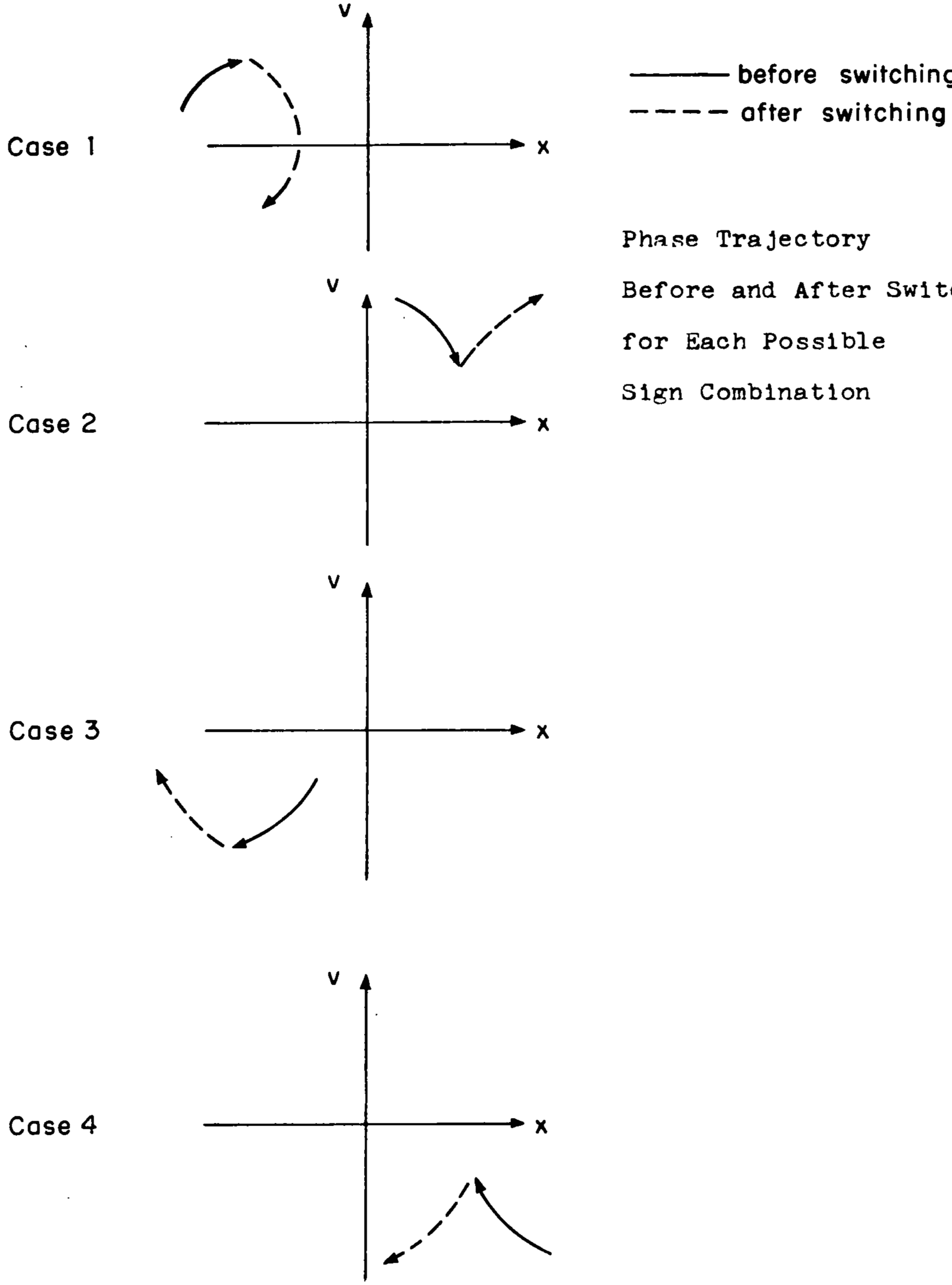


FIGURE 2.2



Phase Trajectory Before and After Switching for Each Possible Sign Combination

FIGURE 2.3

when the true state of the world is known. These beliefs develop from a lifetime of everyday experiences with the forces of nature. Like the priors, the subject brings them with him to the task. Many illustrations of man's dependence on these models to perform manual control tasks can be cited. For example, the initial limb force needed to lift an object requires an estimation of the object's weight given its size, composition, etc., and the acceleration of gravity.

Because they are subjective, models of the physical world will differ among individuals. As it was with revision and selection rules, we again find it expedient to propose a unique characterization. As part of this characterization, we now derive an expression for the conditional probability, $p(E_{jk}/H_i)$. When this is done, the evidence we assume the human operator uses in the other cases is identified and expressions for the conditionals in each of these situations are derived. This will then complete our characterization of the human operator's subjective model of the physical world.

The Conditionals, $p(E_{jk}/H_i)$ - Assume for the moment that the dynamic process is defined by the differential equation,

$$\ddot{x} = u \qquad 2.8$$

If the hypothesis, $H_i(x_m)$, that the switch curve passes thru the mesh, (x_m, v_i), is true and $1 \leq i < n$ (refer to figure 2.1), then the output of the controller, u, must have a value somewhere in the range,

$$(v_i^-)^2/2\,x^+ \leq u \leq (v_i^+)^2/\,2x^-,$$

where the plus and minus signs respectively denote the largest and smallest absolute value of the superscripted state variable in the (x_m, v_i) mesh. In a case one situation (figures 2.2 and 2.3), if the control polarity is switched when in mesh (x_m, v_j) and the above hypothesis is true, it is possible for the phase trajectory to cross the x-axis at a point, x_k, somewhere in the interval bounded by

$$d_{max} = x_m^+ - x_m^-(v_j^+/v_i^-)^2 \qquad 2.9$$

and

$$d_{min} = x_m^- - x_m^+(v_j^-/v_i^+)^2 \qquad 2.10$$

See figure 2.4, top sketch. Equations 2.9 and 2.10 are obtained from the first two integrals of equation 2.8 and the

WEIGHTING OF CASE ONE EVIDENCE

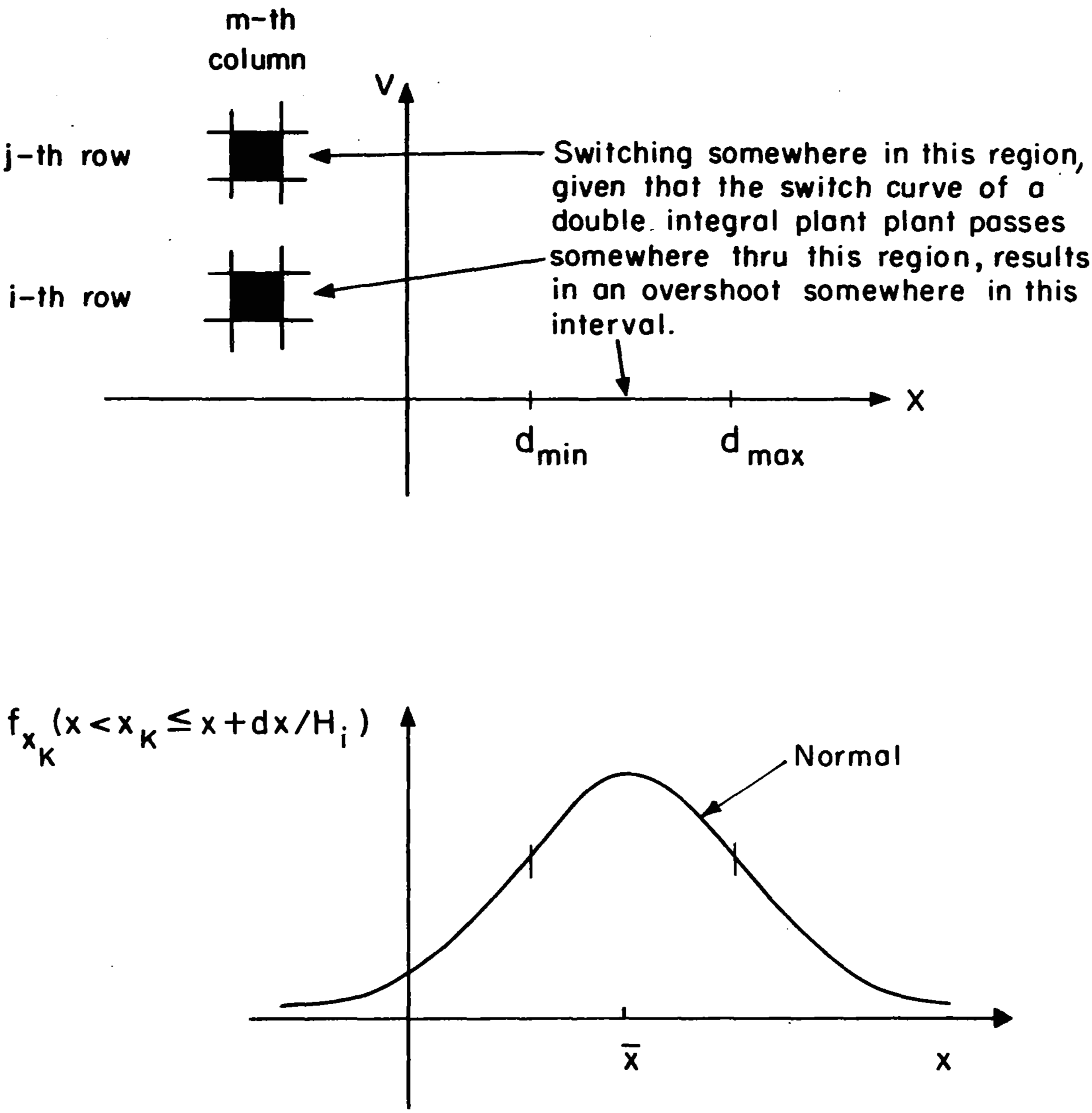


FIGURE 2.4

appropriate boundary conditions.

Say now that the dynamics of the process are not given by equation 2.8, but by some other differential equation of the second order, first degree type we are considering. If the hypothesis, $H_i(x_m)$, is true and if in a case one situation the control polarity is again switched in the mesh (x_m, v_j), what values are possible for x_k? For some dynamic processes, call them "E_+" systems, the phase trajectory will cross the x-axis somewhere between x=0 and d_{max}; whereas for others, call them "E_-" systems, the crossover will occur between d_{min} and plus infinity. These ranges apply to the example illustrated in figure 2.4. In table 2.1, we list the possible crossover ranges for the other combinations of the indices, i and j, which must be treated. Note that in most cases the ranges for E_+ and E_- systems overlap in the interval, (d_{min}, d_{max}). Because of this overlap and on the premise that a given dynamic process is equally likely to be a E_+ or E_- system, we believe it more likely for a crossover to occur within (d_{min}, d_{max}) than for it to occur outside this interval. We express our preference by assuming a normal distribution, f_{x_k}, for the probability that x_k occurs between x and x+dx given $H_i(x_m)$, and taking

$$\bar{x}(m,i,j) = (d_{max} + d_{min})/2 \qquad 2.11$$

TABLE 2.1

POSSIBLE RANGES FOR x_k

$1 \leq i,j < n$

$x_m < 0$

i,j such that	"E_+" systems	"E_-" systems
$d_{min} < d_{max} < 0$	$(d_{min},0)$	$(\bar{x}_m,d_{max})$
$d_{min} < 0 < d_{max}$	(d_{min},d_{max})	$(\bar{x}_m,d_{min}),(d_{max},\infty)$
$0 < d_{min} < d_{max}$	$(0,d_{max})$	(d_{min},∞)

for the mean of the distribution and

$$\sigma_x(m,i,j) = (d_{max} - d_{min})/2 \qquad 2.12$$

for the standard deviation. See the bottom sketch of figure 2.4. By definition of a switch curve, an undershoot, $x_k < 0$, is not possible for the example portrayed in this figure. Therefore, a truncated distribution, defined on $(0,\infty)$ for this example is actually required. Truncating the distribution is necessary, in general, as can be seen from the ranges in table 2.1. However, since the weighting of the priors is not seriously affected by ignoring this detail, the truncation is not performed and the distribution, therefore, is always defined on $(-\infty, \infty)$. The conditional probability, $p(E_{jk}/H_i)$, is obtained by integrating f_{x_k} between the appropriate limits. Whenever this integration is performed by our model, the approximation,

$$\int_{x_k}^{x_k+\Delta x} f_{x_k}(x)dx \cong f_{x_k}(x_k)\Delta x = p\ (E_{jk}/H_i) \qquad 2.13$$

is used.

It has been assumed, so far, that $1 \leq i < n$. When $i =$

n, v_i^- is zero and equation 2.9 is no longer valid. In this limiting case, $p(E_{jk}/H_n)$ approaches zero, because the magnitude of d_{max} approaches infinity. What this means is simply that we are allowing for the possibility, however unlikely, of a switch curve coincident with the x-axis. When $n < i \leq N$, the hypothesis, $H_i(x_m)$, allows for the possibility that the switch curve lies in the first or third quadrants of phase space, i.e., where x and v have the same sign. For the phase space we have defined ($v = \dot{x}$), the switch curve can not lie in these quadrants, since this would imply $v \neq \dot{x}$. It is hard to imagine what physical reasoning, if any, would lead a human operator to make such a hypothesis. But whatever their rationale, some of them behave (i.e., they switch in these quadrants) as though they temporarily held this belief. Since, by equation 2.6, we must sum over all hypotheses, we are compelled to assign values to the conditionals, $p(E_{jk}/H_i)$, $i = n+1,\ldots,N$, even though there is no physical basis for such an assignment. Our judgement of how to make this assignment in some logically consistent manner is to let

$$p(E_{jk}/H_i) = p(E_{jk}/H_n), \quad i = n+1,\ldots,N \qquad 2.14$$

At this juncture, we should point out that, as part of man's subjective model of the physical world, the conditional probabilities just introduced represent a conceptualization of momentum. In other words, when these conditionals are used to revise the prior probabilities in a case one situation, the posterior probabilities will reflect the belief that if an object moving with speed, $|v_j|$, stops in a distance $|x_k - x_m|$ after switching control polarity, it can be made to stop in a shorter (longer) distance by switching polarity when the speed is less (more) than $|v_j|$. We are assuming, therefore, that the human operator comes to the task with this belief and that the distributions of the probabilities, $p(E_{jk}/H_i)$, which we have just derived are a suitable characterization of how this belief is conceptualized in the decision center of the human mind. In passing, we note that if the human operator comes to the task with the prior probabilities set to zero in the first and third quadrants, i.e., he does not think it probable that the switch curve lies in these regions, he can learn how to control any dynamic process of the class considered simply on the evidence of where the phase trajectory crosses the x-axis after the control polarity is switched in a case one situation, provided the evidence is used as we have indicated. Even when the priors are not zero in these quadrants, it is still true that the conditionals, $p(E_{jk}/H_i)$, will enable a subject to resolve his uncertainty as to the location of the switch curve. However, situations like case two and three do provide some additional information which the human operator can use to expedite this resolution and which we now discuss.

Cases Two and Three - In a situation similar to the one illustrated by case two in figures 2.2 and 2.3, reversing the polarity of the control will, in general, cause the phase trajectory to "open-up". It is assumed that the sensor can perceive such a pattern in the state history of the dynamic process and that the decision center can correlate this change in path with the attendant increase in the integrated absolute output of the process (i.e., in the measure used for scoring performance) it produces. In other words, the human operator recognizes an improper control action. If this is true, it is reasonable to expect that he can also conclude that if the speed, $|v_j|$, at the time of switching had been greater, the trajectory would have opened even more than it did and therefore, the hypotheses, $H_i(x_m)$, for $j \leq i \leq N$ are incorrect and should be rejected. If this evidence is called E_j, it can be weighted by revising the priors with the conditional probabilities,

$$p(E_j/H_i) = 0, \quad i = j+1,\ldots,N; \quad j > n \qquad 2.15$$

As for the remaining hypotheses, we assume that the human operator does not change their relative ranking on the basis of this evidence, i.e.,

$$p(E_j/H_i) = 1, \quad i = 1,\ldots,j; \quad j > n \tag{2.16}$$

A control systems engineer may not find these conditionals to be the most appealing physical interpretation of the above evidence. However, it must be remembered that we are dealing with subjective probabilities, and the human operator, whose behavior we are explaining, is not likely to be making a sophisticated engineering analysis of his task. What we are trying to do, when proposing a set of conditional probabilities, is provide a plausible description of an unobservable mental process by inference from the outward behavior of humans.

Case three is similar to case two except that the operator now recognizes he had been using the wrong control polarity, because when he switches, the phase trajectory "closes-up". For weighting the evidence, we propose the same conditional probabilities as are given by equations 2.15 and 2.16. In our investigations of the effects of altering the weighting of the available evidence, we have found that cases two and three play only a minor role in the resolution of the human operator's uncertainty. Case one evidence dominates the learning behavior of the subject, and for this reason, we will not pursue any further justification for the conditional probabilities just postulated.

The Final Case - Switching in the fourth case does not provide the subject with evidence to use in revising the priors, $p(H_1(x_m))$. In such a situation the subject should have waited until the crossover occurs so that he could have appraised his last switching decision (i.e., case one). Subjects who have not behaved this way lose an opportunity to resolve some of their uncertainty, and they must eventually recognize this fact, if they are going to identify the correct control policy. Failure to wait long enough to perceive the consequences of a specific control action is a common mistake among beginners. Student pilots, for example, when performing certain instrument manuevers early in their training, must often be reminded to wait and see what happens after making attitude or power corrections. "Chasing the needles", as it is called, is the usual result of not heeding the reminders. Case four will be treated as though it served to remind the subject to wait. Reinforcement of this kind has been modelled by the mathematical psychologists, see Luce, Bush and Galanter[64], in some simple ways. One of their approaches is adopted here by introducing the probability, p_n, that after n reinforcements (case four occurs n times) the subject will wait for a crossover after switching in case one. The effect of the n-th reinforcement will be described in this way.

$$p_n = 1 - \alpha\ (1-p_{n-1}) \qquad 2.17$$

where α is a parameter determining the strength of each reinforcement. To finish the description α and p_0 must be specified. Like the priors, these parameters characterize the subject's past and can only be inferred from his behavior in the task.

This completes our discussion of the evidence which is available to the human operator manually controlling a dynamic process. We have postulated how the human operator weights this evidence in resolving his uncertainty as to the location of the switch curve in phase space. After the decision center chooses a response alternative, it transmits response execution commands to the effector. Before explaining the operation of this next component in our information processing system, we pause to briefly review the ideas presented in describing the decision center.

Review of Concepts - In characterizing the decision making process of a human operator engaged in a manual control task we have introduced probabilities for the orderly expression of his preferences for response alternatives. Decisions to respond, we have said, are based on these probabilities, and learning has been interpreted as a revision making process which changes them. And finally, we have postulated what evidence is needed to make these revisions. This approach represents a subtle application of Bayesian statistics for the description of human information processing.

2.6 The Effector

RT: Response Time - Executing a response, in the manual control problem we are considering, is a simple task for the human operator's motor system. All that the effector (a finger in this case) must do is depress or release a key which actuates the relay controller. This is a basic limb movement in which most adult humans are well practiced. Therefore, we need not worry about the human operator having to learn a skilled limb movement as part of his control task, and consequently no allowance need be made for adaption in the response mechanism.

In simple stimulus-response experiments, wherein the subject generally actuates some type of switch as quickly as possible after the onset of a signal, the time between stimulus and response, the reaction time, is measured. There have been attempts to account for the nonnormal distribution of the reaction time by treating the time as a sum of a fixed number of independent random variables. Each component of the sum is associated with the time taken up by some underlying process in the chain between stimulus and response. Hohle[49], in particular, obtained very satisfactory results by summing a normally distributed component with an exponentially distributed one. His conclusion was that the former component represented " the time required for organization and execution of the motor response" and the latter represented a "decision or perception" time. In the present work, it is not essential to provide a description of the time history of the limb movement, since it is only the time the switch actually occurs which matters in our explanation of human learning behavior. For this purpose the stochastic description of the response (motor) time, which we will call RT, offered by Hohle is satisfactory.

DT: Decision Time - Preceding this section on the effector, we explained the operation of man's decision center. In the center, there takes place a selection process and a revision process. On the basis of the experimental evidence provided by Hohle, Chocelle (see Hohle), Deupree and Simon(26), Restle(93), Teichner(102) and others, we have inferred that the times for revision and selection are also random variables statistically independent of each other and of the response time. We call the sum of the selection time and the revision time, the decision time, DT. If no revisions are made during a decision cycle, DT is determined by the selection time only. In order to avoid problems associated with programming a subroutine for generating random numbers of an arbitrary distribution, the probability density, for each component time of DT, is assumed uniform. For the same reason, we will, in our model, approximate the exponential distribution of the response time, RT, with a uniform distribution also. In making this approximation, the first and second moments of the uniform distribution are equated to the inferred moments of the actual distribution.

This concludes the discussion of the efector and completes the presentation of our theory for the explanation of human learning behavior in a manual control task. The next section presents a description of the model derived from this theory.

2.7 The Model

Our model of human learning behavior in a manual control task is a digital computer program (source program) which produces a machine language translation of the theory presented. As Newell, Shaw and Simon [78] have so aptly expressed it, "an explanation of an observed behavior of the organism is provided by a program of primitive information processes that generates this behavior." Herein, these primitive information processes are the selection process, the revision process, etc., which have been set forth by the theory as elements of man's technique for the identification of an unknown control policy. Since the theory has also postulated the rules for combining these processes, the computer program can be written once some final details have been considered.

For one thing, we have not yet indicated in what order revisions and selections take place. How the human operator establishes priorities in attending to several matters requiring his immediate attention is a difficult question. The order may not be fixed, and it is quite possible that the decision center can interrupt, say, the revision process, store the unfinished computations and attend to a response. Other combinations can also be conjectured. In the model, we assume that revisions come first, selections second and no interruptions of either are permitted.

For another, we have not specified how many decision cycles are required for the human operator to identify the pattern in the phase trajectory used for evidence in cases two and three. Pattern recognition capabilities vary from one individual to another, and so the number of cycles is not fixed. In the model, we assume that the human operator is capable of detecting the pattern within one decision cycle after the switch occurs.

Finally, we must provide some "numbers" for the parameters which have been left free in the theoretical development. A specification of these parameters corresponds to a specification of the psycho-physiological characteristics of some human operator. As the behavior of the model is governed by the set of numbers choses, it is possible to test the theory by attempting to match individual programs with individual human operators. What we mean by "matching" and how this has been accomplished is now discussed.

We have conducted a parametric study of the model on a digital computer. From these results we first found out how these parameters influence the behavior of the model. Then we inferred sets of parameters to provide what we believe to be a representative sample of human operator behavioral simulations. The procedure for inferring program parameters basically involved using available experimental data for the response time, RT, and fixing the selection and revision times so that the length of a decision cycle is on the order of the psychological refractory period (approximately 230 msec). Mesh dimensions, priors, etc. were educated guesses based on the results of the parametric study. Next we performed a motor skill experiment and made measurements on the response behavior of human operators. The two samples were then compared statistically to determine whether or not they came from the same parent population, i.e., whether or not they matched. In chapter three the motor skill experiment we performed is described, and in chapter four we discuss the results of the parametric study, the experiment and the statistical comparison. In table 2.2 we define the symbols used in the source program, figure 2.5 is a flow diagram of the source program and thereafter follows the source program itself.

TABLE 2.2

SYMBOLS USED IN SOURCE PROGRAM

SYMBOL	DEFINITION
ALPHA	Parameter in equation
DELTAX	x-dimension of mesh
DLXDOT	v-dimension of mesh
DT1 & DT2	Define range of uniform distribution for selection time
DT3 & DT4	Define range of uniform distribution for revision time
E(I,J)	Integrated squared error for i-th "subject" on j-th trial
EM	Output of controller ("u" in text)
I	"Subject" index
IMAX	Maximum number of "subjects" processed
IT(K)	k-th inter-response time (i.e., time between consecutive responses)
K	Response index
KTOP	Maximum number of responses
M	Column index
MMAX	Maximum number of meshes in x-direction
N	Row index
NMAX	Maximum number of meshes in v-direction

TABLE 2.2 cont.

SYMBOLS USED IN SOURCE PROGRAM

SYMBOL	DEFINITION
NSUB(I)	"Subject" designator
POW	Probability that "subject" waits for crossover in case one
P(M,N)	Probability that switch curve passes thru (M,N) mesh
RANNOF(Y)	A random variable uniformly distributed over (0.1)
RT1	Define range of uniform distribution for response time
TIME	Elapsed time from start of trial
TIMEX	Elapsed time from last switching
W(N)	Probability (evidence/hypothesis)
X(K)	Position on k-th response
XDOT(K)	Velocity on k-th response
XCROSS	Position at crossover
X1	Current position
XDOT	Current velocity
XLIM	x-boundary of grid
XDOTLM	v-boundary of grid
XLEFT(M) & XRIGHT(M)	x-values of left and right boundaries of m-th mesh, all N
XDOTHI(N) & XDOTLO(N)	v-values of top and bottom boundaries of n-th mesh, all M

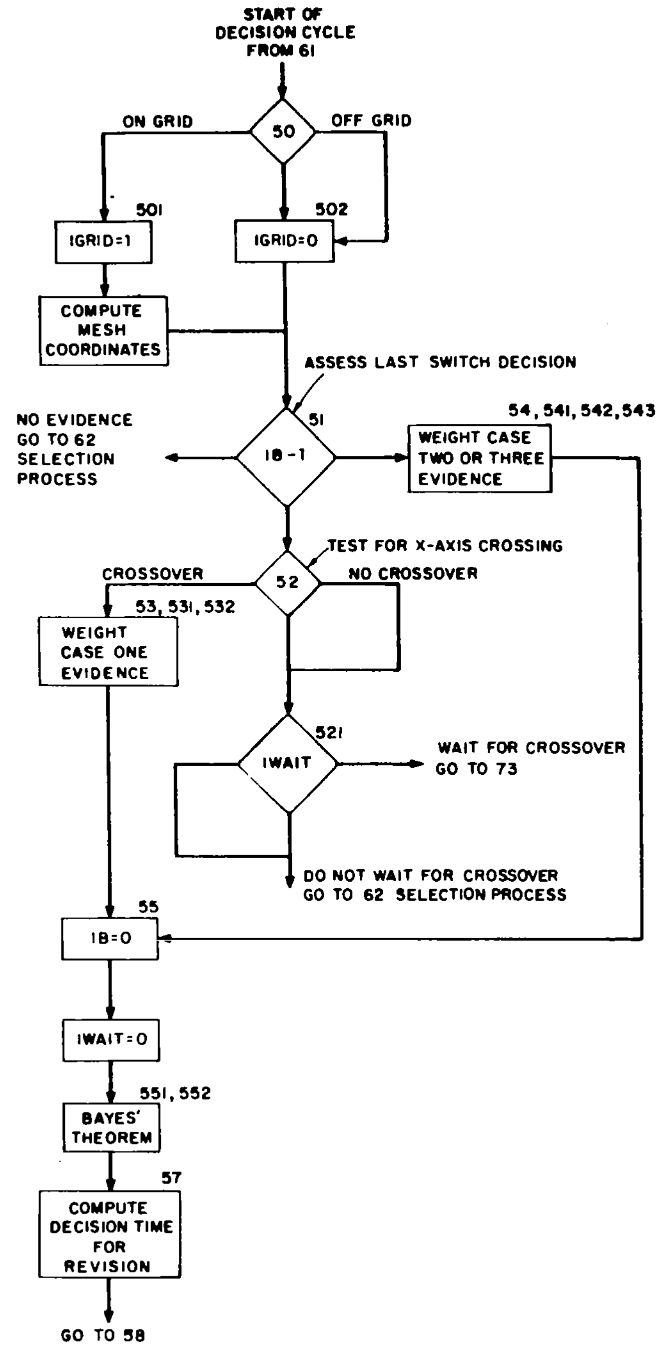


FIGURE 2.5 FLOW DIAGRAM OF SOURCE PROGRAM

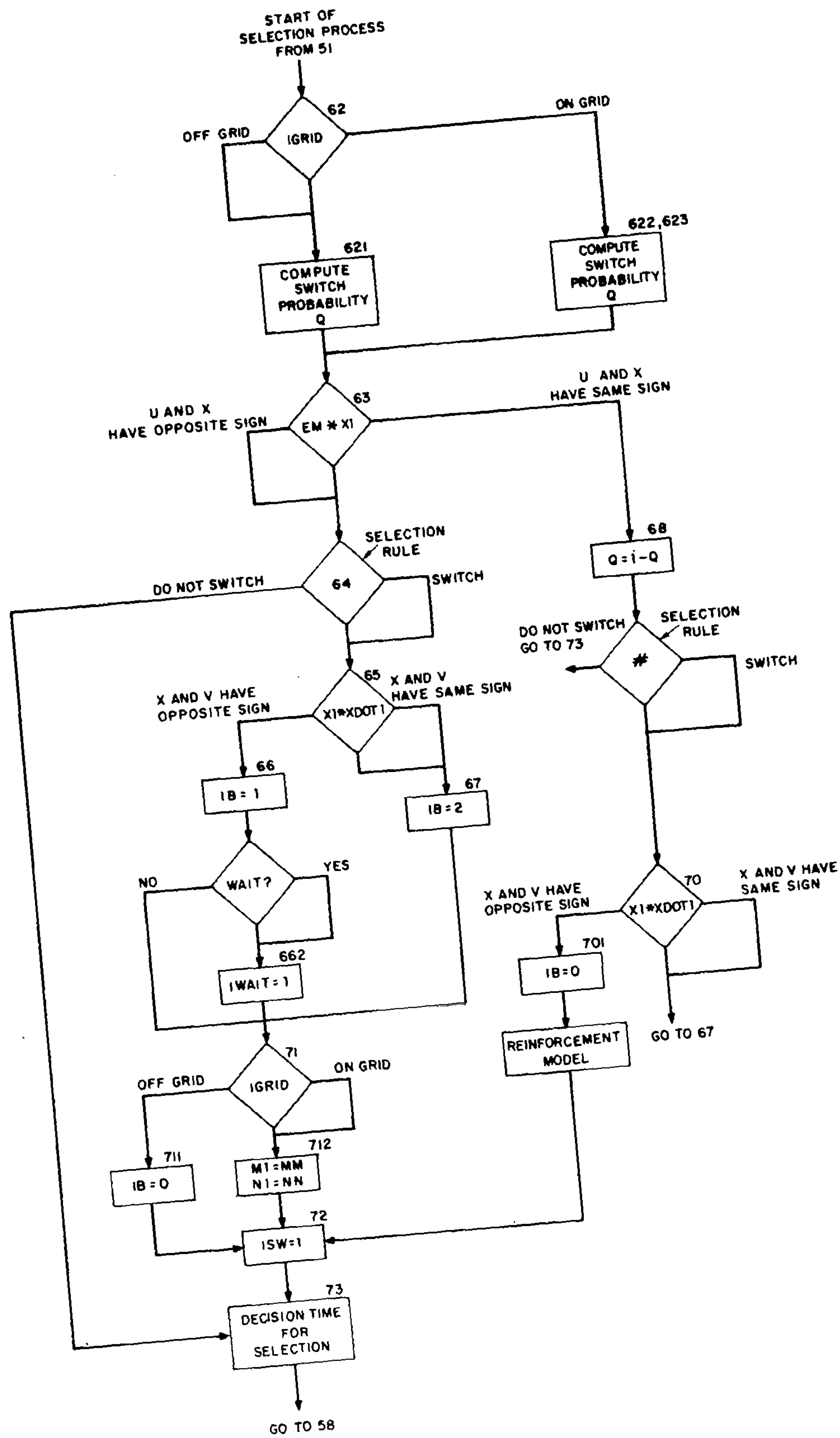


FIGURE 2.5 Flow Diagram of Source Program

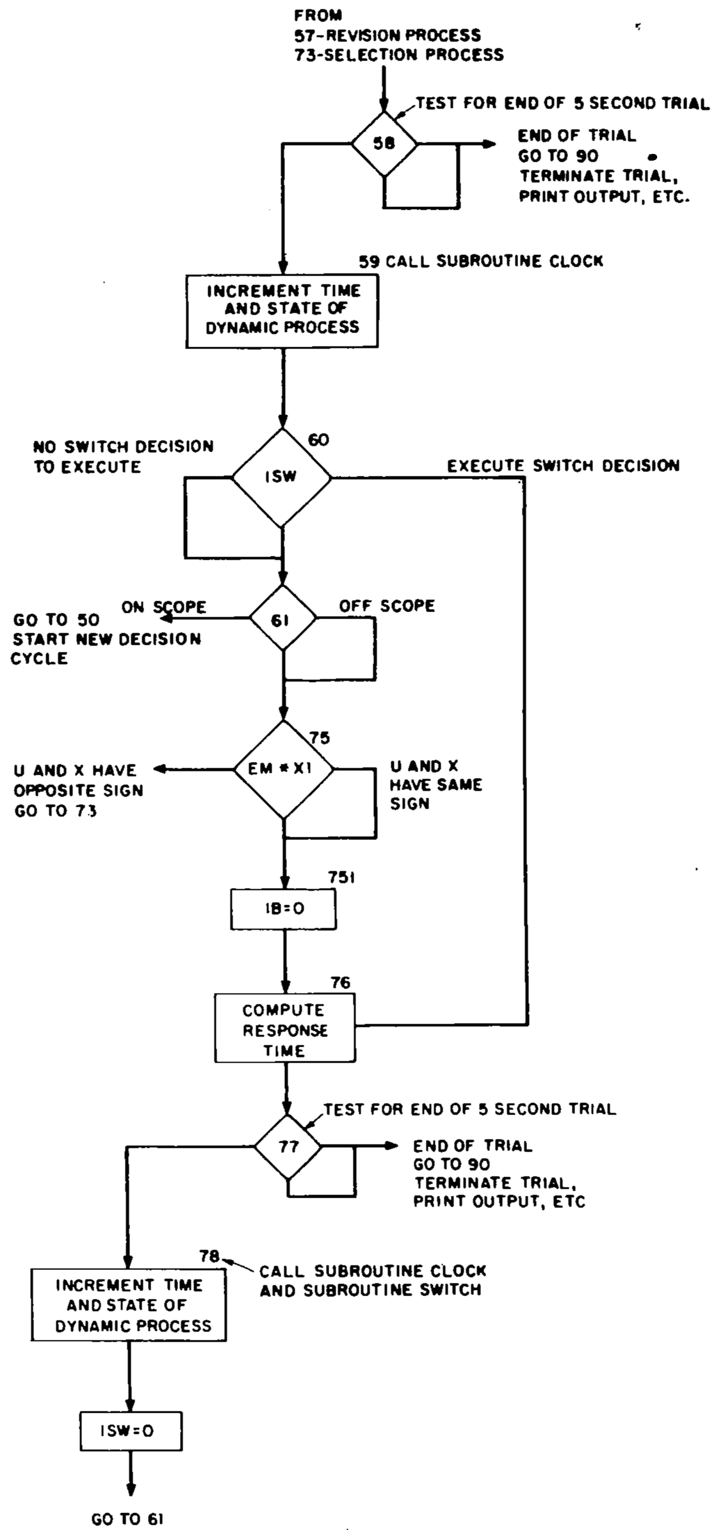


FIGURE 2.5 Flow Diagram of Source Program

SOURCE PROGRAM

```
      PREYSS' STOCHASTIC LEARNING MODEL
      SUBROUTINES CLOCK AND SWITCH ARE REQUIRED
      DIMENSION T(50,50),IT(50),X(50),XDOT(50),E(50,50),NSUB(50),
     1 P(20,40),PP(20,40),W(40),KAY(50),XLEFT(20),XRIGHT(20),XDOTHI(40)
     2 XDOTLO(40)
      COMMON EM,X1,XDOT1,TIME,TIMEX,T,X,XDOT,E
      READ 1,IMAX,NSUB1
      PROCESS EACH 'SUBJECT'
      DO 100 I = 1,IMAX
      NSUB(I) = NSUB1 - 1 + I
      READ NUMERICAL VALUES OF PSYCHOLOGICAL AND PHYSIOLOGICAL
      PARAMETERS OF MODEL
      READ 2,ID,MMAX,NMAX,MODE,DELTAX,DLXDOT,DT1,DT2,DT3,DT4,RT1,RT2,
     1 POW,ALPHA
      PRINT 3,NSUB(I),ID,MMAX,NMAX,DELTAX,DLXDOT,DT1,DT2,DT3,DT4,RT1,
     1 RT2,POW,ALPHA
      SET UP GRID IN STATE SPACE OF DYNAMIC PROCESS
      COMPUTE BOUNDARY VALUES OF X AND XDOT FOR EACH MESH
      FMMAX = MMAX
      FNMAX = NMAX
      FNMAX2 = .5 * FNMAX
      NPP = (MMAX+9) / 10
      XLIM = DELTAX * FMMAX
      XDOTLM = DLXDOT * FNMAX2
      XLEFT(1) = -XLIM
      XRIGHT(1) = -XLIM + DELTAX
      DO 22 M = 2,MMAX
      XLEFT(M) = XRIGHT(M-1)
      XRIGHT(M) = XLEFT(M) + DELTAX
   22 CONTINUE
      XDOTHI(1) = XDOTLM
      XDOTHI(NMAX) = -XDOTHI(1)
      XDOTLO(1) = XDOTLM - DLXDOT
      XDOTLO(NMAX) = -XDOTLO(1)
      NMAX02 = NMAX/2
      DO 23 N = 2,NMAX02
      NN = NMAX - N + 1
      XDOTHI(N) = XDOTLO(N-1)
      XDOTHI(NN) = -XDOTHI(N)
      XDOTLO(N) = XDOTHI(N) - DLXDOT
      XDOTLO(NN) = -XDOTLO(N)
   23 CONTINUE
      SET RANDOM NUMBER GENERATING SUBROUTINE
      B = SETUF(ID)
      DO 24 N = 1,10
      RAN = RANNOF(Y)
   24 CONTINUE
      J1 = 1
      SET AND STORE INITIAL VALUES OF P(M,N)
      MODE = 1 FOR EQUAL INITIAL PROBABILITIES IN ALL QUADRANTS
```

SOURCE PROGRAM cont.

```
C       MODE = 2 FOR EQUAL INITIAL PROBABILITIES IN QUADRANTS 2 AND 4,
C                ZERO PROBABILITIES IN QUADRANTS 1 AND 3
C       MODE = 3 FOR PROBABILITIES READ IN
C       MODE = 4 FOR PROBABILITIES PREVIOUSLY READ IN
        GO TO (25,26,27,28),MODE
 25     P1 = 1.0/FNMAX
        N2 = NMAX
 251    DO 252 N = 1,N2
        DO 252 M = 1,MMAX
        P(M,N) = P1
        PP(M,N) = P1
 252    CONTINUE
        GO TO 30
 26     P1 = 1.0/FNMAX2
        N2 = NMAX / 2
        N2P1 = N2 + 1
        DO 261 N = N2P1,NMAX
        DO 261 M = 1,MMAX
        P(M,N) = 0.0
        PP(M,N) = 0.0
 261    CONTINUE
        GO TO 251
 27     READ 4,((PP(M,N),N=1,NMAX),M=1,MMAX)
        N2 = NMAX
 28     DO 281 N = 1,NMAX
        DO 281 M = 1,MMAX
        P(M,N) = PP(M,N)
 281    CONTINUE
 30     DO 93 J = J1,51
C       PRINT CURRENT VALUES OF P(M,N)
        DO 35 NP = 1,NPP
        JM1 = J - 1
        M1 = 10*NP - 9
        M2 = XMINOF(10*NP,MMAX)
        PRINT 6,JM1,(M,M=M1,M2)
        PRINT 7
        DO 35 N = 1,N2
        PRINT 8,N,(P(M,N),M=M1,M2)
 35     CONTINUE
        IF (J-50) 45,45,93
C       START OF TRIAL
C       SET INITIAL CONDITIONS FOR EACH TRIAL
 45     EM = 7.5
        IB = 0
        ISW = 0
        IWAIT = 0
        K = 1
        KTOP = 0
        TIME = 0.0
        TIMEX = 0.0
```

```
      X(1) = -5.0
      XDOT(1) = 0.0
      GO TO 73
C     ON SCOPE
C     COMPUTE GRID POSITION
 50   IF (ABSF(XDOT1)-XDOTLM) 501,502,502
C     ON GRID
 501  IGRID = 1
      MDEL = ABSF(X1)/DELTAX .
      MM = MMAX - MDEL
      NDEL = -(X1/ABSF(X1))*XDOT1/DLXDOT
      NN = NMAX/2 - NDEL
      GO TO 51
C     OFF GRID
 502  IGRID = 0
C     ASSESS LAST SWITCH DECISION
 51   IF (IB-1) 62,52,54
C     TEST FOR AXIS CROSSING
 52   IF (XDOT1*XDOT(K)) 53,521,521
 521  IF (IWAIT) 62,62,73
C     REVISE ESTIMATES OF P(M,N)
C     B-ONE
C     WEIGHTING OF CASE ONE EVIDENCE
 53   XCROSS = X(K)-(XDOT(K)**2)/(2.0*EM)
      XCROSS = -XCROSS*EM/ABSF(EM)
      NTH = NMAX/2 - 1
      DO 531 N = 1,NTH
      DMIN = XRIGHT(M1)*(1.0-(XDOTLO(N1)/XDOTHI(N))**2) - DELTAX
      DMAX = XLEFT(M1)*(1.0-(XDOTHI(N1)/XDOTLO(N))**2) + DELTAX
      AVX = (DMAX+DMIN)/2.0
      SIGX = (DMAX-DMIN)/2.0
      XNORM = (XCROSS-AVX)/SIGX
      W(N) = EXPF(-(XNORM**2)/2.0)/SIGX
 531  CONTINUE
      NTH1 = NTH + 1
      DO 532 N = NTH1,NMAX
      W(N) = W(NTH)
 532  CONTINUE
      GO TO 55
C     B-TWO
C     WEIGHTING OF CASE TWO OR THREE EVIDENCE
 54   DO 541 N = 1,N1
      W(N) = 1.0
 541  CONTINUE
      N1P1 = N1 + 1
      IF (N1P1-NMAX) 542,542,55
 542  DO 543 N = N1P1,NMAX
      W(N) = 0.0
 543  CONTINUE
 55   IB = 0
```

```
      IWAIT = 0
C     BAYES THEOREM
      SUM = 0.0
      DO 551 N = 1,NMAX
      SUM = SUM + W(N)*P(M1,N)
 551  CONTINUE
      DO 552 N = 1,NMAX
      P(M1,N) = P(M1,N)*W(N)/SUM
 552  CONTINUE
C     DECISION TIME FOR REVISING ESTIMATES
 57   DT = DT1 + DT2*RANNOF(Y)
 58   IF (TIME+DT-5.0) 59,90,90
C     INCREMENT TIME AND STATE OF DYNAMIC PROCESS
 59   CALL CLOCK (X(K),XDOT(K),DT)
 60   IF (ISW) 61,61,76
C     CHECK WHETHER ON SCOPE
 61   IF (ABSF(X1)-XLIM) 50,75,75
C     DECIDE WHETHER OR NOT TO SWITCH
C     COMPUTE PROBABILITY OF SWITCH, Q
 62   IF (IGRID) 621,621,622
 621  Q = (1.0-XDOT1*X1/ABSF(XDOT1*X1))/2.0
      GO TO 63
 622  Q = 0.0
      DO 623 N = NN,NMAX
      Q = Q + P(MM,N)
 623  CONTINUE
 63   IF (EM*X1) 64,64,68
C     M AND X HAVE OPPOSITE SIGN
C     DECISION RULE
 64   IF (Q-RANNOF(Y)) 73,65,65
 65   IF (X1*XDOT1) 66,67,67
 66   IB = 1
      IF (POW-RANNOF(Y)) 71,662,662
 662  IWAIT = 1
      GO TO 71
 67   IB = 2
      GO TO 71
C     M AND X HAVE SAME SIGN
 68   Q = 1.0 - Q
C     DECISION RULE
      IF (Q-RANNOF(Y)) 73,70,70
 70   IF (X1*XDOT1) 701,67,67
 701  IB = 0
C     REINFORCEMENT MODEL
      POW = ALPHA*(POW-1.0) + 1.0
      GO TO 72
 71   IF (IGRID) 711,711,712
 711  IB = 0
      GO TO 72
 712  M1 = MM
```

```
      N1 = NN
 72   ISW = 1
C     DECISION TIME FOR SELECTION OF CONTROL POLICY
 73   DT = DT3 + DT4*RANNOF(Y)
      GO TO 58
C     OFF SCOPE
 75   IF (EM*X1) 73,751,751
 751  IB = 0
C     RESPONSE TIME
 76   RT = RT1 + RT2*RANNOF(Y)
 77   IF (TIME+RT-5.0) 78,90,90
C     INCREMENT TIME AND STATE OF DYNAMIC PROCESS
 78   CALL CLOCK (X(K),XDOT(K),RT)
      CALL SWITCH(I,J,K)
      ISW = 0
      GO TO 61
C     END OF TRIAL
 90   TIME = 5.0
      CALL SWITCH(I,J,K)
      KAY(J) = K
      IF (T(J,K)-.0005) 91,91,92
 91   KAY(J) = K - 1
 92   KLAST = KAY(J)
      KTOP = XMAXOF(KTOP,KLAST)
C     PRINT STATE VECTOR VALUES FOR TRIAL J
      PRINT 14,NSUB(I),J,(X(K),XDOT(K),K=2,KLAST)
 93   CONTINUE
C     OUTPUT FOR SUBJECT
      PUNCH 1,NSUB(I),KTOP
      DO 99 J = 1,50
      KLAST = KAY(J)
      DO 96 K = 1,KLAST
C     ROUND TO 4 PLACES
      IT(K) = 2000.0*T(J,K)
      FIT = IT(K)
      IF (2000.0*T(J,K)-FIT-.5) 96,94,95
 94   IF (IT(K)/2 - (IT(K)+1)/2) 95,96,96
 95   IT(K) = IT(K) + 1
 96   CONTINUE
C     FIND NUMBER OF CONTINUATION CARDS
      NC = XMINOF(KLAST/14,(KLAST+13)/14 - 1)
      IF (NC) 98,98,97
 97   PUNCH 11,NSUB(I),J,(IT(K),K=1,14),NC
      IF (NC-1) 98,98,971
 971  PUNCH 11,NSUB(I),J,(IT(K),K=15,28)
 98   K1 = NC*14 + 1
      PUNCH 11,NSUB(I),J,(IT(K),K=K1,KLAST)
 99   CONTINUE
      NPP = (IMAX+9) / 10
 100  CONTINUE
```

SOURCE PROGRAM, cont.

```
      PRINT INTEGRATED SQUARED ERRORS FOR ALL SUBJECTS
      DO 101 NP = 1,NPP
      I1 = 10*NP - 9
      I2 = XMINOF(10*NP,IMAX)
      PRINT 19
      PRINT 17,(NSUB(I),I=I1,I2)
      PRINT 15
      DO 101 J = 1,50
      PRINT 16,J,(E(I,J),I=I1,I2)
101   CONTINUE
      CALL EXIT
    1 FORMAT(40I2)
    2 FORMAT (4I2,10F4.2)
    3 FORMAT(82H1SUBJECT  ID  MMAX  NMAX  DELTAX  DLXDOT  DT1  DT2  DT3
     1 DT4  RT1  RT2  POW  ALPHA/2(4X,I2),3X,I2,4X,I2,2(4X,1F4.2),2X,
     2 1F4.2,6(1X,1F4.2),3X,1F4.2)
    4 FORMAT(20F4.3)
    6 FORMAT(1H4,I4,2X,1HM,5X,10(I2,4X))
    7 FORMAT(9X,1HN)
    8 FORMAT(7X,I3,2X,10F6.3)
   11 FORMAT(2I3,2X,14I5,I2)
   13 FORMAT(I2,E14.8,4(2X,E14.8)/(5(2X,E14.8)))
   14 FORMAT(2I3,2X,14F8.3/(8X,14F8.3))
   15 FORMAT(1H-,6X,5HTRIAL)
   16 FORMAT(5X,I5,3X,10F6.1)
   17 FORMAT(1H0,5X,7HSUBJECT,2X,10(I3,3X))
   19 FORMAT(1H1,27X,24HINTEGRATED SQUARED ERROR)
      END


      SUBROUTINE CLOCK (X,XDOT,TD)
      COMMON EM,X1,XDOT1,TIME,TIMEX
      TIME = TIME + TD
      TIMEX = TIME OF LAST SWITCH
      T = TIME - TIMEX
      COMPUTE NEW X AND XDOT
      X1 = X + XDOT*T + .5*EM*T**2
      XDOT1 = XDOT + EM*T
      RETURN
      END


      SUBROUTINE SWITCH (I,J,K)
      INCREMENTS INTEGRATED SQUARED ERROR AND SWITCHES CONTROL POLARITY
      DIMENSION T(50,50),X(50),XDOT(50),E(50,50)
      COMMON EM,X1,XDOT1,TIME,TIMEX,T,X,XDOT,E
      T(J,K) = TIME - TIMEX
      E(I,J) = E(I,J) + X(K)**2*T(J,K) + .33333333*XDOT(K)**2*T(J,K)**3
     1 + .05*EM**2*T(J,K)**5 + X(K)*XDOT(K)*T(J,K)**2
     2 + .33333333*EM*X(K)*T(J,K)**3 + .25*EM*XDOT(K)*T(J,K)**4
```

SOURCE PROGRAM cont.

```
      IF (TIME-5.0) 50,60,60
50    K = K + 1
      X(K) = X1
      XDOT(K) = XDOT1
      EM = - EM
      TIMEX = TIME
60    RETURN
      END
```

CHAPTER 3

EXPERIMENT

3.1 General

A description follows of a psychomotor experiment performed at M.I.T.'s Man Vehicle Laboratory. Over a four month period fifty paid subjects were given the opportunity to learn a manual control task. They were briefed on the task and familiarized with the apparatus, but were not allowed to practice prior to the first trial. Instructions to the subject given during the briefing are reproduced verbatim at the end of the chapter. A listing of the subject's age, sex, occupation, etc. is given in table 3.1.

3.2 Task

A subject, by actuating a two position switch, is required to null the initial misalignment between two line segments displayed on an oscilloscope in front of him (see figure 3.1). One segment, the left, remains stationary and the displacement of the other relative to it, x, satisfies the differential equation,

$$\ddot{x} = u \qquad 3.1$$

where u is the switch output and may either be +U or -U.

As the switch has no OFF position, once the segment is aligned rapid polarity changes may be used to simulate an OFF position and thereby to maintain close alignment. Each subject is given fifty, five-second trials spaced ten seconds apart. Subject performance on each trial is measured by computing the integral of the absolute value of x over the five seconds. This score is reported to the subject immediately after each trial. Every trial starts with the same initial conditions. Using the same initial conditions each time and a fixed trial length makes the scoring meaningful to the subject and useful as a measure of learning.

3.3 Task Pace

By restricting the experiment's duration to twelve and one half minutes per subject, deterioration of performance due to such effects as boredom and fatigue is effectively eliminated. It was observed that all subjects remained outwardly attentive to their task throughout this brief period. Since many of them expressed the desire to continue "the game" after their time was up, it appears that subjects were interested in the experiment and were trying hard to improve their scores. The combination of a double integral plant, a controller output of ± 7.5 cm/sec^2 and a 12.5 minute experiment results in faster learners spending the majority of their time polishing their performance, without getting bored and slower learners barely reaching an asymptotic level of performance, as judged by their scores. Limiting each trial to five seconds gives the subject enough time to respond with seven or eight switchings on the average. Fixing the intertrial length at ten seconds allows enough finger rest to keep it limber during the course of the experiment.

3.4 Controller

A micro-switch mated to a key and recessed in the subject's console (see figure 3.4) serves as the controller in the experiment. With a slight effort (operating force: 9-13 ounces) and displacement (pre-travel: 0.15 inches maximum) a subject can switch the polarity of the controller's output, u. Since finger tapping is a very basic limb movement, which people use in a variety of manual skills (i.e., typing, playing musical instruments, etc.), no training with the controller is necessary aside from a few preliminary taps to "get the feel of it". Magnitude of the controller output is set at 7.5 cm/sec^2 to avoid frequent loss of control on early trials. A loss of control situation occurs when the right segment moves off the scope. Subjects are briefed on this contingency and know the correct control polarity to use while the segment is out of sight. Trials are not terminated when this happens, but are continued for the full five seconds.

3.5 Apparatus

Figure 3.2 is a photograph of the experimenter's station, and the interconnections of the components is diagrammed in figure 3.3. Names and manufacturers of the electro-mechanical components are listed in table 3.2.

3.6 Instructions to Subject

Please be seated at the console and make yourself comfortable. You will note before you two items: a recessed key and an oscilloscope. During the course of this experiment you will actuate this key in response to a visual display presented on the oscilloscope. Depressed, the key generates a "down-command" signal; released, it generates an "up-command" signal. A "null-command" can be approximated by alternating between up and down commands rapidly (demonstrate this mode of keying).

At this time you will actuate the key using the index finger of your preferred hand. Notice the small force and minimal displacement required to switch in either direction: up for "up-command" and down for "down-command". To prevent improper keying you should keep your hand at rest on the console at all times and use only index finger motion. In addition to feeling the switching action, you should also hear a clicking sound. These tactile and audio indications of a switch will free you from having to visually monitor your responses. If the key should seem to bind, please advise me. Remember, a gentle touch is all that is necessary and any harsh keying will be brought to your attention.

Are there any questions so far?

On the face of the oscilloscope there are displayed two horizontal line segments. The left segment will remain stationary and is your reference mark. The right segment will move vertically in response to your up and down commands. It is now in the starting position, which is five centimeters below the reference mark. Before each trial, it will return to this same starting position. If it does not, let me know. The beginning of a trial is recognized by movement of the right segment away from its starting position and the end of a trial is recognized by an arresting of its motion.

Your key is not connected directly to the oscilloscope. Instead, your up and down commands are input signals to a dynamic process which is being simulated on the analog computer to your right. It is the output of this dynamic process which the displacement of the right segment represents.

Thus, your commands are being modified in some way to produce a motion of this line. This modification will not be revealed to you explicitly. However, over the course of the experiment you will learn, by observing how the right segment responds to your commands, how to regulate its motion. Before discussing your specific task, are there any questions?

Your task is simply this: align the segments and keep them aligned. You will remember that you are always commanding either up or down. Therefore, the right segment will not stay aligned unless you command "null". Even then there will be some motion. If you command "null" when the right segment is not aligned and not momentarily at rest, you can expect further motion because of the dynamic nature of the process. Is this clear?

I will not tell you what the correct keying strategy is in order to achieve alignment. Since you can only make two choices, it should be obvious that the correct strategy is some sequence of up and down commands and that the basic problem is learning when to switch from one to the other by observing how your commands influence the motion. At the start of each trial, your key should be up. This is the correct first choice in the keying sequence and it will start the right segment upward. Your second response, therefore, is a decision when to depress the key. The outcome of your second response may not be what you expect, so a third one may be necessary, and you again must decide when. You should continue in this alternating manner until the segments are aligned. Do you wish any clarification or repetition of the instructions given so far?

Should the right segment disappear from view while it is travelling upward, hold the key down until it reappears, then key as you deem necessary. The converse applies if it disappears while travelling downward. In either case it is possible that it may not reappear before the end of the trial. If this happens, it means only one thing: you did something wrong <u>before</u> it went off scope, not after.

Five seconds after the right line segment starts moving it will freeze in its position at that moment. Your keying will cause no further motion, so you may stop. This terminates the trial. Your performance on that trial will be measured and a score will be announced. This score is computed by integrating the absolute value of the misalignment over the five second interval of the trial. If you will look at this figure (show figure 3.5) you can see what this score measures. The object then is to continually improve your score.

After announcing the score, I will reset the right segment to the same initial position and the next trial will commence with the segment's first movement. Proceeding in this manner, you will be given fifty consecutive trials with no interruptions. This takes about fifteen minutes and is not tiring, so don't "save" yourself. Any final questions?

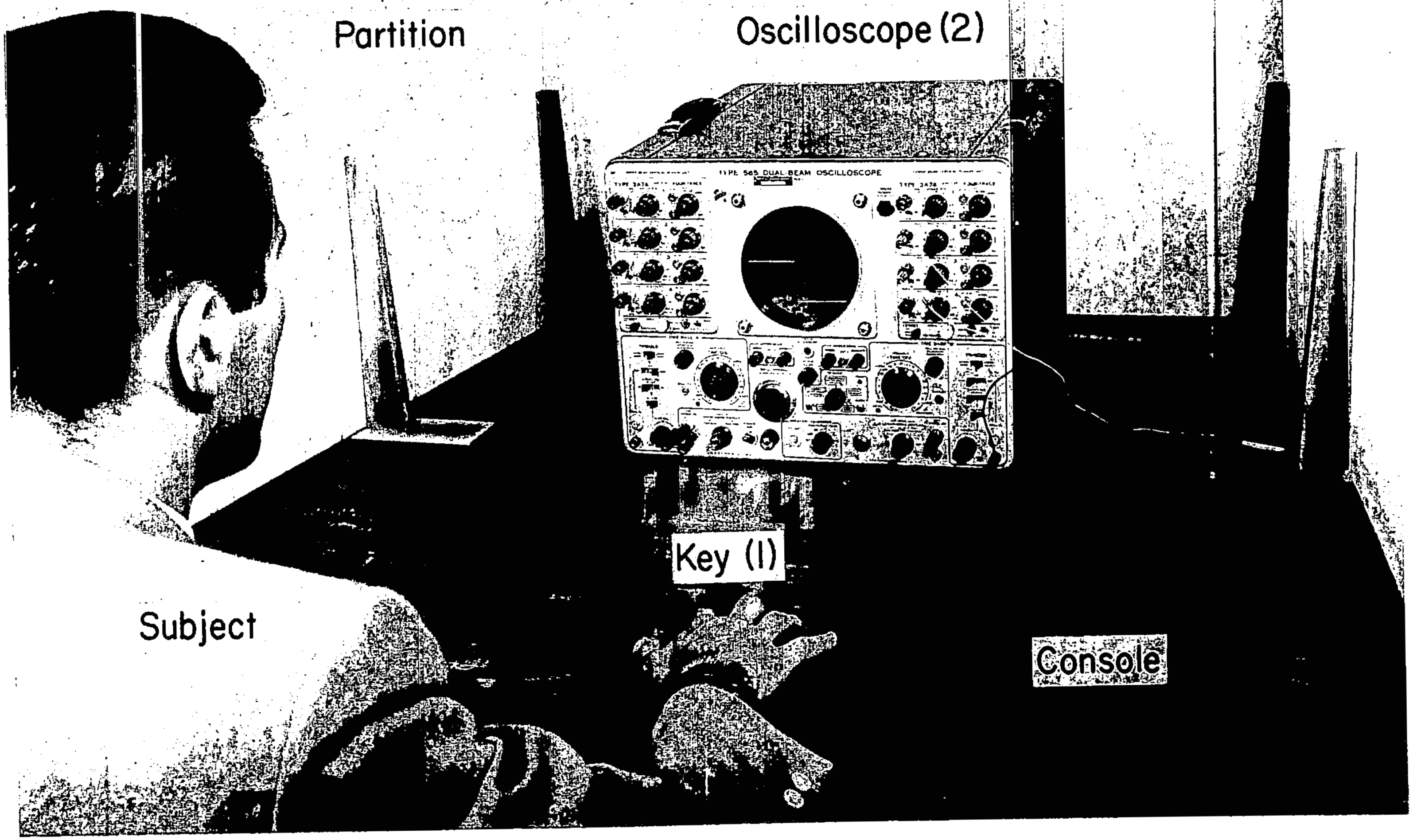

Partition
Oscilloscope (2)
Key (1)
Subject
Console
TYPE 565 DUAL-BEAM OSCILLOSCOPE

FIGURE 3.1

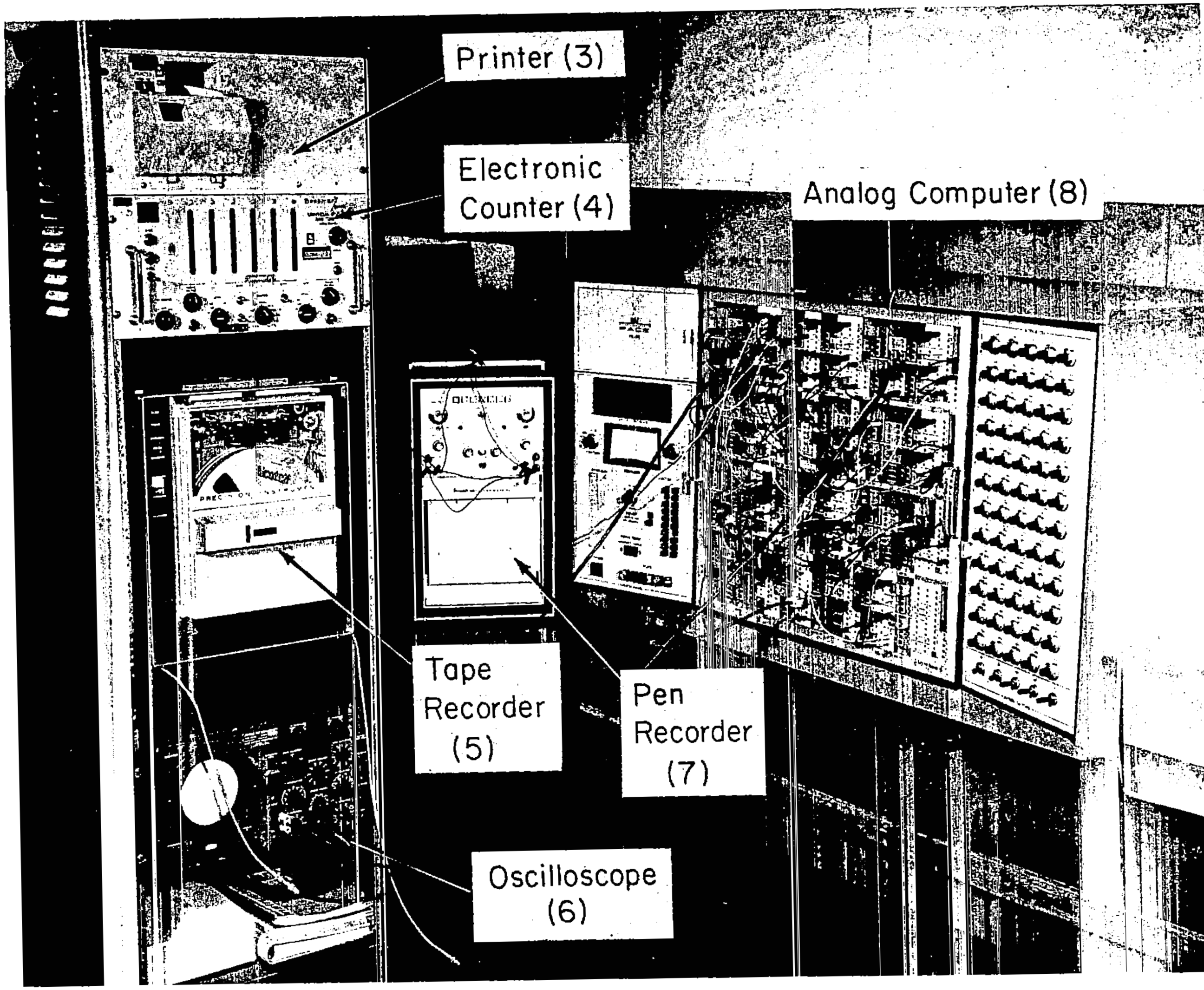

Printer (3)
Electronic Counter (4)
Analog Computer (8)
Tape Recorder (5)
Pen Recorder (7)
Oscilloscope (6)

FIGURE 3.2

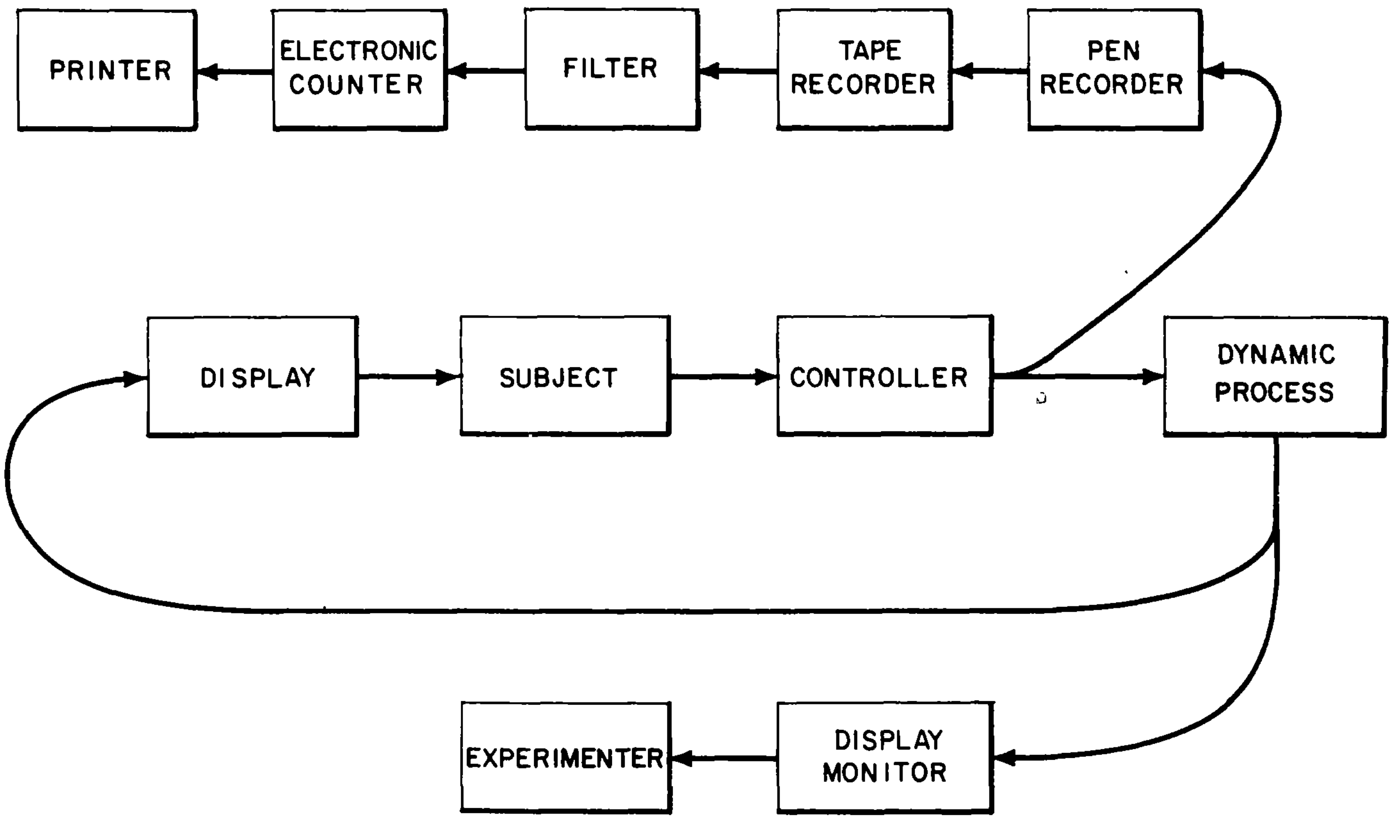


FIGURE 3.3 ARRANGEMENT OF EXPERIMENTAL APPARATUS

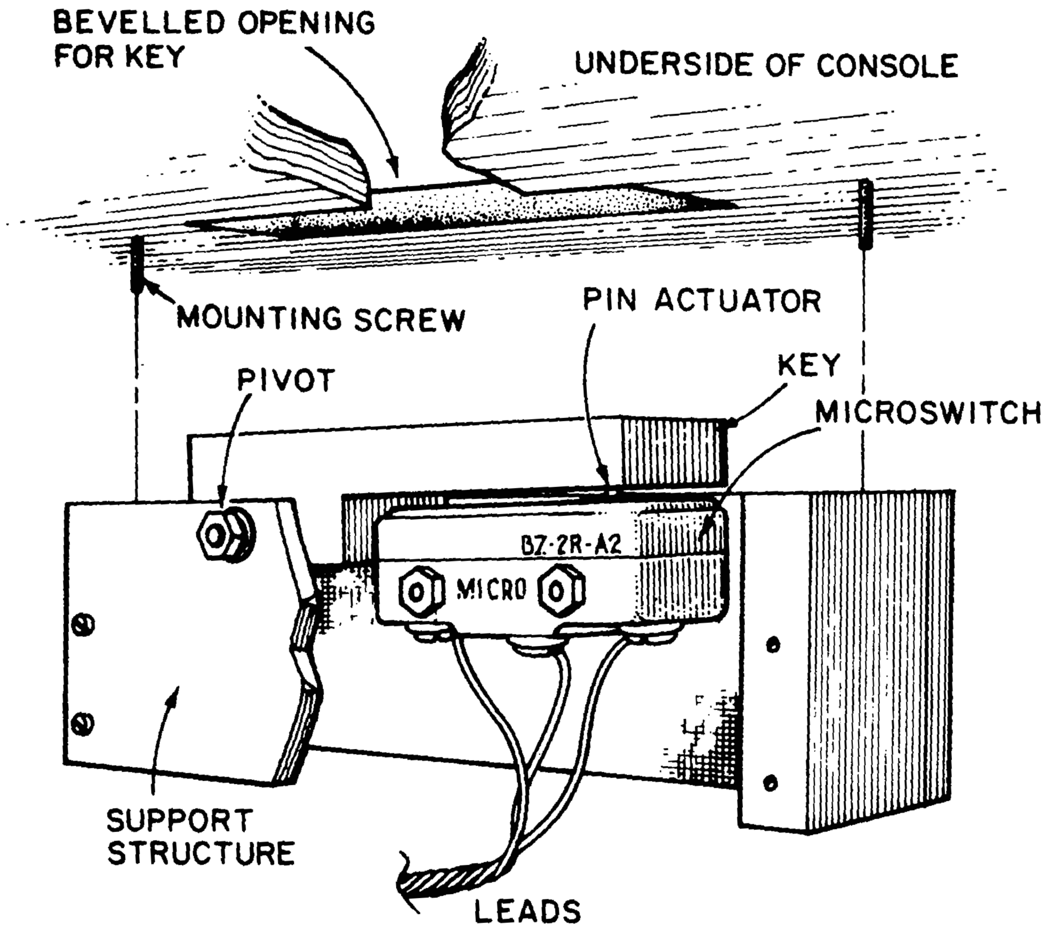
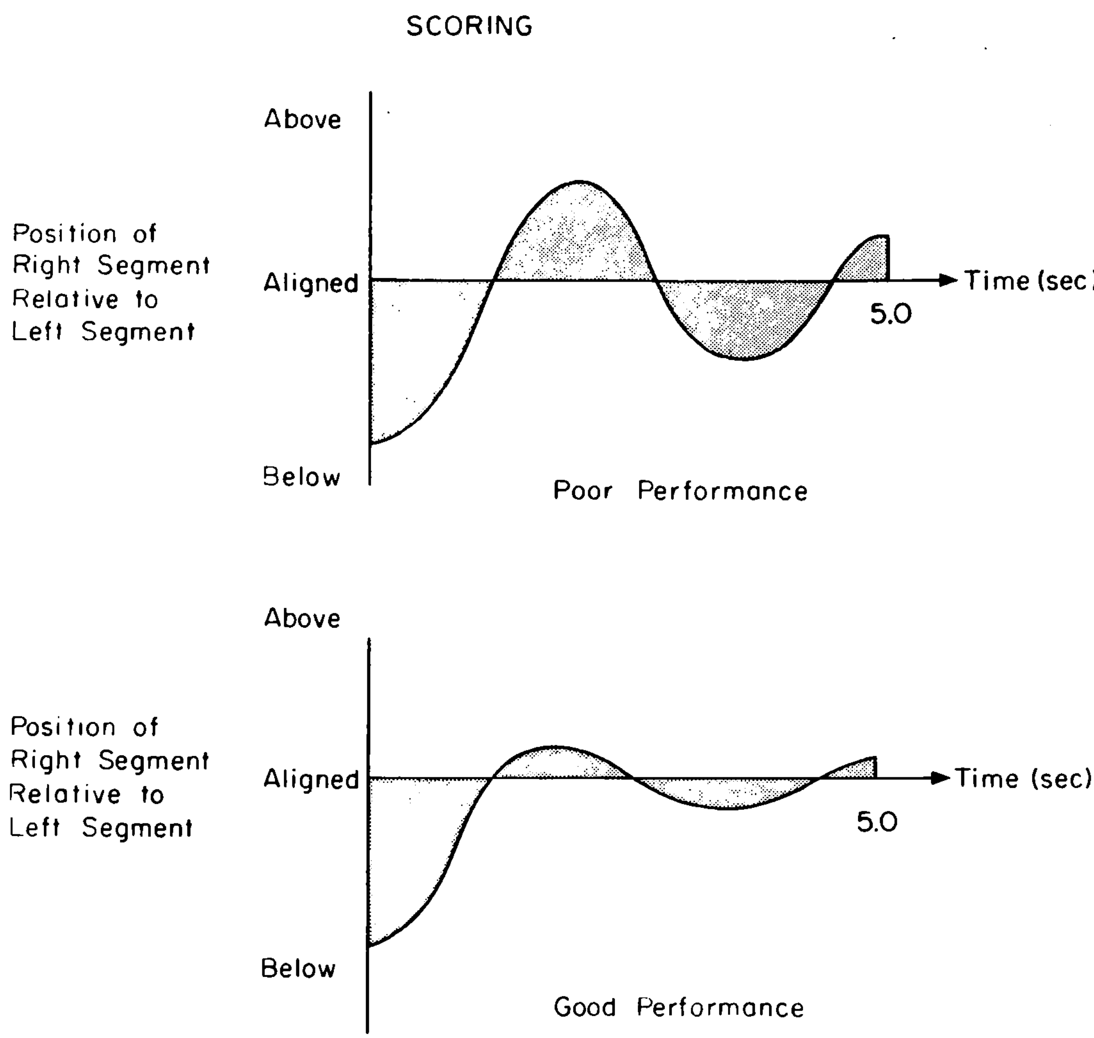


FIGURE 3.4 CUTAWAY DRAWING OF THE CONTROLLER



SCORING
Above
Position of
Right Segment
Relative to
Left Segment
Aligned
Time (sec)
5.0
Below
Poor Performance
Above
Position of
Right Segment
Relative to
Left Segment
Aligned
Time (sec)
5.0
Below
Good Performance

FIGURE 3.5

TABLE 3.1

Subjects

Number	Name	Age	Sex	Handedness	Occupation
1	A.E.H.	20	M	RH	Student
2	J.W.G.	23	F	RH	Secretary
3	M.J.M.	19	M	RH	Student
4	B.H.M.	22	M	RH	Student
5	S.M.A.	22	M	RH	Student
6	P.G.K.	24	M	RH	Officer, USAF
7	D.O.M.	24	M	RH	Officer, USAF
8	M.W.J.	24	F	RH	Secretary
9	R.W.L.	23	M	RH	Officer, USAF
10	J.C.G.	25	M	RH	Officer, USAF
11	F.H.	24	M	RH	Officer, USAF
12	J.M.Q.	21	M	RH	Student
13	B.C.M.	49	F	RH	Secretary
14	I.M.W.	23	F	RH	Secretary
15	M.C.H.	25	F	RH	Secretary
16	H.T.D.	24	M	RH	Officer, USAF
17	M.E.D.	22	M	LH	Student
18	S.M.W.	20	M	RH	Student
19	T.R.N.	20	M	RH	Student

TABLE 3.1 cont.

Subjects

Number	Name	Age	Sex	Handedness	Occupation
20	D.T.T.	19	M	RH	Student
21	A.G.	23	F	RH	Secretary
22	R.W.S.	19	M	RH	Student
23	D.W.M.	19	M	LH	Student
24	R.A.S.	19	M	RH	Student
25	M.A.R.	19	M	RH	Student
26	D.S.M.	20	M	RH	Student
27	D.F.D.	21	M	RH	Student
28	H.K.S.	25	M	RH	Officer, USAF
29	C.D.W.	22	M	RH	Student
30	R.J.R.	19	M	RH	Student
31	D.M.	20	M	RH	Student
32	D.C.M.	23	M	RH	Student
33	P.W.Y.	20	M	RH	Student
34	L.H.L.	21	M	RH	Student
35	E.G.M.	24	M	RH	Student
36	R.L.F.	24	M	LH	Student
37	J.I.S.	21	M	LH	Student
38	D.B.S.	20	M	RH	Student

TABLE 3.1 cont.

Subjects

Number	Name	Age	Sex	Handedness	Occupation
39	J.W.S.	20	M	RH	Student
40	L.P.K.	25	F	RH	Secretary
41	D.A.S.	22	F	RH	Student
42	J.A.M.	18	M	RH	Student
43	N.A.C.	26	M	RH	Student
44	S.C.R.	26	M	RH	Student
45	M.A.H.	20	M	RH	Student
46	R.E.C.	29	M	RH	Student
47	E.S.S.	21	M	RH	Student
48	D.K.M.	22	M	RH	Student
49	D.A.F.	19	M	RH	Student
50	K.A.K.	22	F	RH	Secretary

TABLE 3.2

Apparatus

Component	Specification
1 Display	Tektronix Type 565 Dual Beam Oscilloscope Type 3A74, Four Trace
2 Controller	Micro Switch Type " Z " General Purpose 15 Ampere Capacity Switch BZ - 2R - A2
3 Dynamic Process	Electronic Associates, Inc. PACE TR-48 Analog Computer
4 Pen Recorder	Brush Instruments Recorder Mark 280
5 Tape Recorder	Precision Instrument Recorder-Reproducer Series PS-200A
6 Electronic Counter	Beckman Instruments, Inc. Universal EPUT & Timer Model 7360A
7 Printer	Beckman Instruments, Inc. Digital Printer Model 1453
8 Display Monitor	DuMont Cathode Ray Oscillograph Type 304 - HR

CHAPTER 4

RESULTS

4.1 General

The results we have to present are offered in three parts. First of all, in section 4.2, there are the theoretical results which were obtained from sixty executions of the computer program written in chapter two. Model parameters, in each execution except the last ten, were varied systematically in order to study their influence on the learning behavior of the program. In the last ten executions, model parameters were selected to provide a test sample of human operator behavioral simulations. Second of all, in section 4.3, there are the experimental results which were obtained from the motor skill experiment described in chapter three. Fifty subjects performed this experiment, and the data taken on their responses is used to corroborate predictions of the theory. Third and last of all, in section 4.4, theory and experiment are compared statistically to determine whether or not the sample of operator simulations and the sample of operators are of the same parent population.

4.2 Theoretical Results

We have conducted a parametric study of the behavior of the model (i.e., the computer program) to establish how behavior is altered by changes in the psycho-physiological parameters of the model and if the alternations are consistent with our intuitive ideas of what should happen. In table 4.1, there are listed the sets of parameters which were read in to the digital computer prior to the execution of fifty runs of the program. In table 4.2 appears the scores for each program execution, and we report the integrated squared error instead of the integrated absolute error, simply because it was faster to compute. Actually, the study was larger in scope than we indicate. On the order of two hundred or more programs were executed, and so we are confident that this smaller sample provides a reliable representation of the program's behavior.

Basically, the fifty sets of parameters, as can be seen by reference to table 4.1, exhibit several variations on a theme. Programs 1-5 are used as a normative set of results upon which to make comparisons. Parameter values in these programs are not intended to characterize an "average" human operator. Programs 21-25 change the mesh dimensions of the sensory grid. Some decision center parameters are changed in programs 11-20 and 41-45 (initial distribution of the prior probabilities), 46-50 (mean and standard deviation of the revision time) and 25-35 (reinforcement strength, α). Programs

6-10 and 36-40 change the mean and standard deviation, respectively, of the response time of the effector mechanism, RT. Changes, therefore, have been made in the psycho-physiological parameters which govern the operation of all the components of our stochastic information processing system.

What do the results show? Our conclusions are based on the effect these changes have on the scores either during the initial phase of learning or during the final phase. The first five trials constitute the initial phase, and the last five trials the final. We find that, on the average, the performance of the program in controlling the dynamic process deteriorates whenever,

(a) the sensor perceives the state of the dynamic process with greater uncertainty, i.e., the mesh size is increased,

(b) the decision center is initially more uncertain of the control policy, i.e., the priors are, for example, distributed uniformly or are nonzero in the first and third quandrants,

(c) the decision center requires more time to process information, i.e., DT is increased,

(d) the decision center is slow to recognize that it must wait on the outcome of a response in order to assess whether or not it selected the correct choice, i.e., α is increased,

(e) the effector requires more time to execute a response, i.e., RT is increased.

These findings are consistent with the behavior one would expect to observe in the response performance of any information processing system, these expectations being based, in part, on the predictions of conventional control systems theory. Not to be overlooked, either, is the plain fact that the program does learn how to control a dynamic process. Also to be noted is that the learning process is convergent in all cases and the closed loop performance of the system, when it is learned, is near optimal. In this regard we should point out that the best score obtainable in this task is approximately 15.6 cm^2-sec.

A sequence of eleven sketches, presented collectively as figure 4.1, provide a most striking portrayal of learning. One can witness in this sequence the program's progress in resolving its uncertainty as to the location of the switch curve. Each sketch shows a surface, the height of which, above the reference plane at the coordinates, (x_m, v_i), represents the posterior probability, $p'(H_1(x_m))$, at the end of the indicated trial. To give a clearer visual impression, each surface, which is actually formed by a finite set of points, has been filled in and smoothed over. Program number 40, for which the priors on the first trial are all equal in the second and fourth quadrants of state space and are zero everywhere else, serves as the example. Were it practical to draw these figures for all the programs executed, one could readily distinguish slow from fast learning, partial from complete resolution of uncertainty, etc.

4.3 Experimental Results

A complete picture of human operator learning behavior in the psychomotor experiment discussed in chapter three can be developed from the measurements which were taken of the intervals between successive switches in control polarity. This interval will be referred to as an interresponse time, IRT. Interresponse times for the fifty trials performed by each subject are tabulated in appendix A. Statistical descriptors of the interresponse time for the first twenty responses of each trial are presented in the first four tables of appendix B. Tables B.1 and B.2 list the means and standard deviations of the data. A measure of skewness, alpha three, and a measure of kurtosis, alpha four, are presented in tables B.3 and B.4 respectively. All averaging has been done over the number of subjects who actually made a k-th response, and the absence of a value indicates that only one subject responded k times on the particular trial. Negative values of alpha three represent skewness to the left, positive to the right. A normal distribution has an alpha four value of 3.0. Larger values are more peaked, smaller are less. Correlation between successive responses is given by the correlation coefficient appearing in table B.5.

From the interresponse time data, the state, (x,v), of the dynamic process at each switch time has been calculated. Statistical descriptors of the state are in appendix C, where table C.1 - mean position and velocity, table C.2 - standard deviation of position and velocity, and table C.3 - covariance of state are presented. Subject performance, as measured by the integrated squared error, appears in table C.4 for each of the fifty subjects on all fifty trials. And finally, these scores have been averaged over the ensemble of subjects and the resulting mean squared error, MSE, is tabulated as a function of trial numbers in table C.5.

To provide a portrayal of learning comparable to figure 4.1, which depicts the program resolving its uncertainty, we have, for the human operator, taken the statistics on the state variables and computed the ellipsoids of concentration for the first six responses of trials 1 thru 5 and of every fifth trial thereafter. These appear as a sequence of computer drawn sketches, collectively called figure 4.2. An ellipsoid of concentration bounds a two-dimensional region over which probability is distributed uniformly such that the first and second order moments of the uniform distribution are the same as those of the actual distribution (see Cramer[23], pp. 283-285). A liberal interpretation of this definition, in our case, is to say that the n-th region shows where, in state space, "most" subjects made the n-th response. The shrinking and re-orientation of the ellipses are a vivid illustration of the ensemble's progress in identifying a control policy. In addition to this portrayal of learning, average transient responses for some of the same trials have been calculated from the state data and are presented collectively as figure 4.3

4.4 Theory and Experiment Compared

From the figures and tables presented in the previous sections, one can easily develop a qualitative appreciation for the individual differences exhibited in the learning behavior of both subjects and programs. For example, initial score, level of asymptotic performance and rate of score change are some of the readily discernible indicators provided by the integrated squared error which are useful in comparing the motor skill behavior of the programs with that of the subjects. What is important now is to answer the question of whether or not the theory developed herein is a credible explanation of human learning behavior, particularly of inter-subject, intra-subject variability. For this purpose, it is desirable to make a comparison of the subject ensemble and the test sample of programs on some quantitative basis. This has been done and is discussed next.

To establish the "similarity" between the behavior of the fifty subjects and the ten programs of the test sample, the Mann-Whitney "U" test was applied to each of the first four interresponse times of each of twelve trials. On a given trial for a given response, the sample of subject IRT'S (IRT_i : $i = 1,2,\ldots,n_1$) and the sample of model IRT's (IRT_j : $j = 1,2,\ldots,n_2$) are arranged in order; the statistic, U, counts the number of times a member of the first sample exceeds a member of the second sample, that is

$$U = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} n_{ij} \quad ,$$

where

$$n_{ij} = \begin{matrix} 1 \text{ , if } IRT_i > IRT_j \\ 0 \text{ , if otherwise} \end{matrix}$$

In the limit, as n_1 and n_2 both approach infinity in any arbitrary manner, the distribution of U is normal. In fact, for $n_1 = n_2 = 8$ the distribution differs negligibly from normal. If the random variables, IRT_i and IRT_j, have continuous cumulative distribution functions f and g respectively, the statistic U is used to test the hypothesis that f = g. Specifically, if the quantity

$$z = \left| (U - \bar{U}) / \sigma_U \right| \quad ,$$

where

$$\bar{U} = n_1 n_2 / 2$$

and

$$\sigma_U = n_1 n_2 (n_1 + n_2 + 1) / 12$$

is greater than 2.58 under the null hypothesis (f = g), the test is considered significant at the 1% level and the hypothesis of identical distributions is rejected. Table 4.3 lists the values of z calculated for the forty-eight test cases.

At the 1% level, the table shows that eighty-one percent of the cases pass the test, i.e., the hypothesis of identical distributions is acceptable. Those cases which fail the test are confined to the third and fourth responses on trials after the fifth. If one looks at the interresponse time data (appendix A) for such subjects as 11, 33, or 44 and compares it with the program data (appendix D), the reason for these failures becomes quite apparent. It can be observed that subjects develop an open-loop technique for responding when the dynamic process' state is close to the origin. This mode of behavior is an attempt by a subject to simulate an OFF position (as he was instructed) with the controller by rapidly alternating control polarity. In this mode, the subject effectively ignores state information until such a time as the error exceeds some tolerance level, and then he reverts back to a closed-loop mode of responding. Clearly the theory does not account for this, since the program makes but one response per decision cycle and does not set off pre-programmed sequences of responses. Aside from this discrepancy, the results of the "U" tests are quite favorable and offer no cause to reject the hypothesized identity of the two population distributions. Note: at the 1% level of significance, the probability of obtaining a Z-value greater than 2.58 when comparing two samples is, by definition, .01 given that the hypothesis, $f = g$, is true.

We have also applied the Mann-Whitney "U" Test to samples of the integrated squared error scores on the same trials as before. The results, which are presented in table 4.4, show that only one case is significant at either the 1% or 5% ($Z > 1.96$) level: For the human operator sample we selected the first ten subjects instead of using the entire ensemble. Performing this test on the scores is a less sensitive measure of the credibility of our theory than performing it on the IRT's, since the integration to obtain a score masks the detailed structure of the response behavior and therefore, discrepancies in this structure can be obscured from detection. Testing the IRT's on the other hand, subjects the finest grain measurement we have available on the response behavior to the scrutiny of a powerful nonparametric statistacal test.

FIGURE 4.1

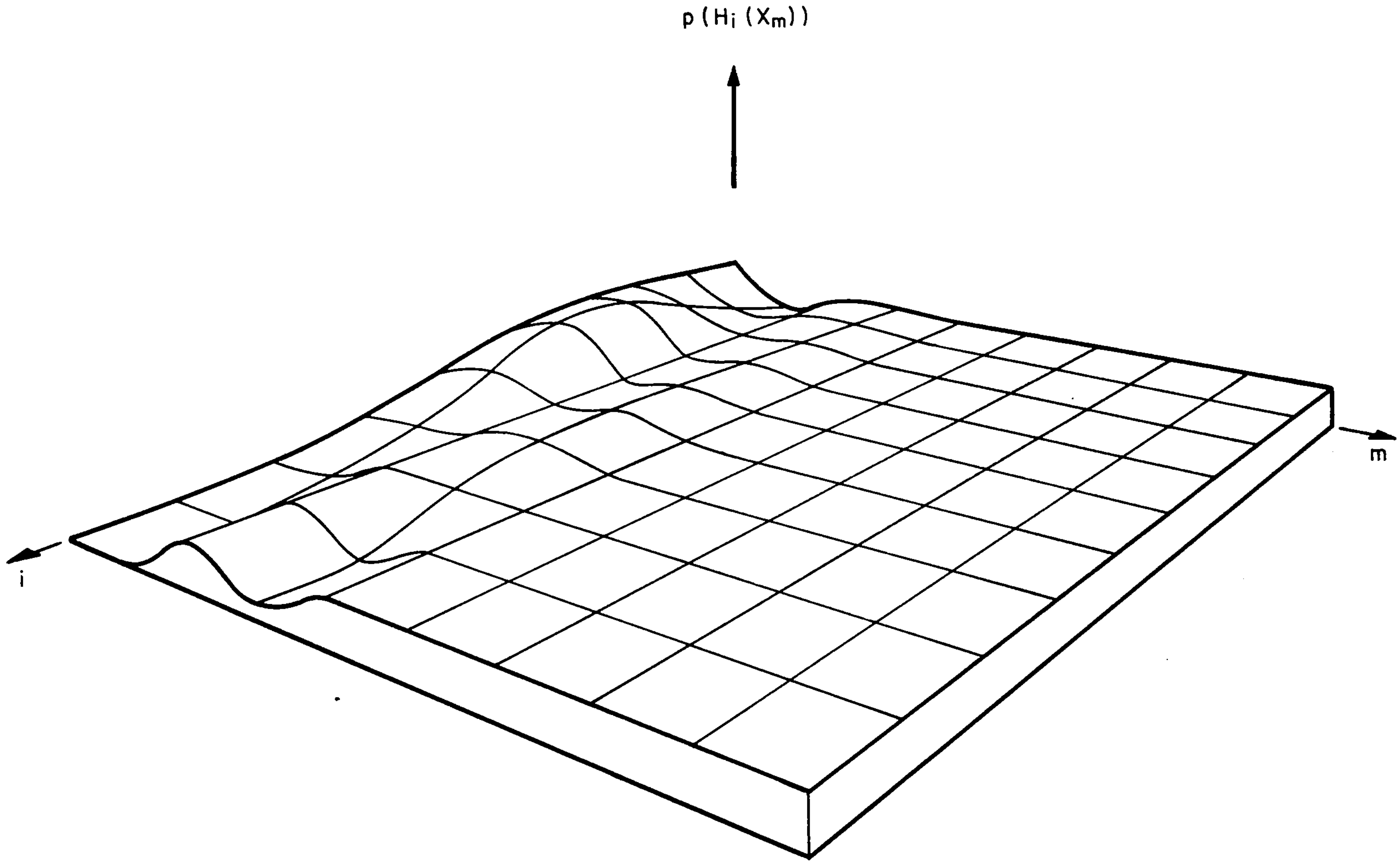
A Surface Respresentation

of the Probabilities, $p(H_1(x_m))$,

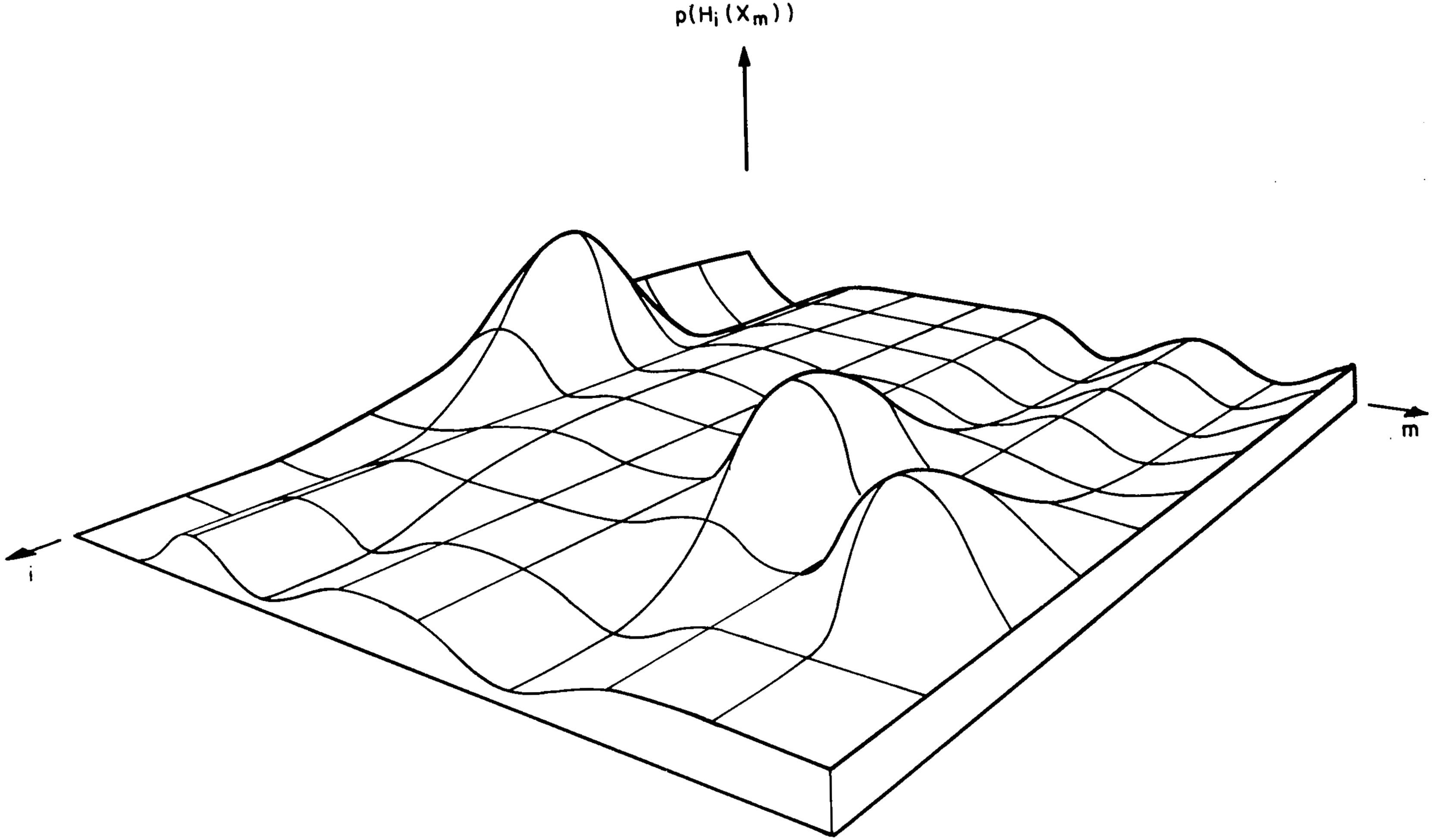
Trials 1-6, 10, 20, 30, 40 and 50

PAGES

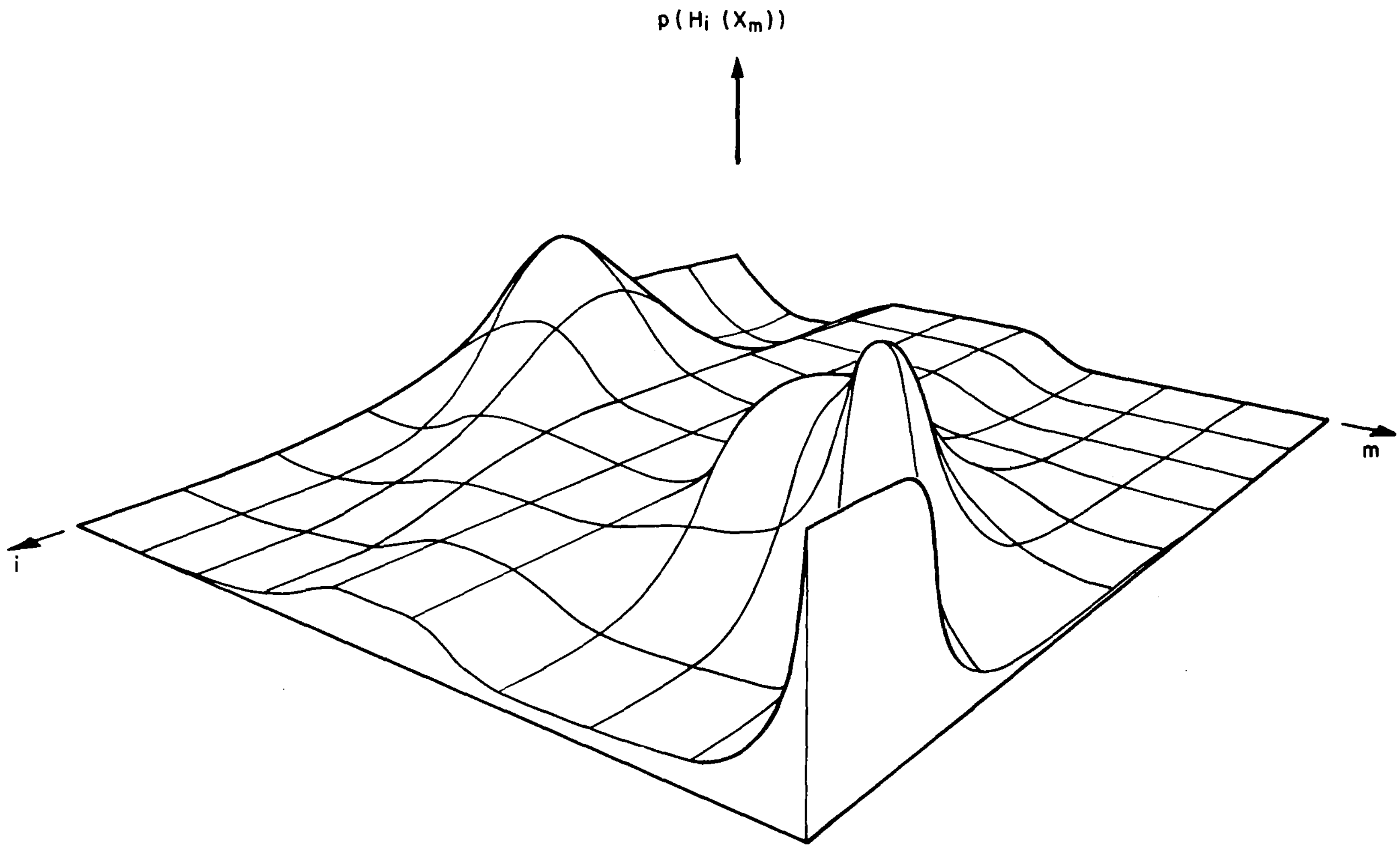
81-91



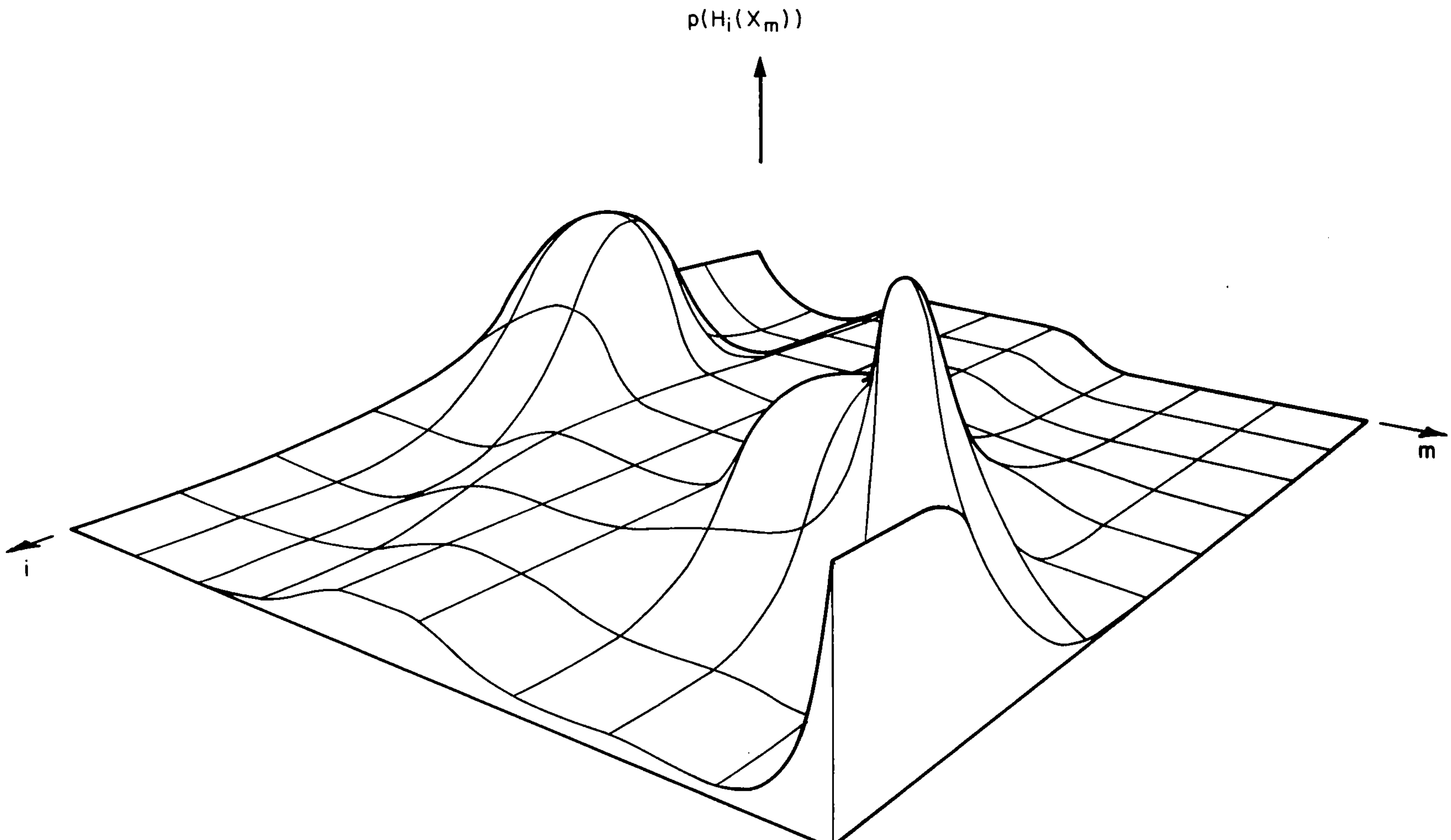
TRIAL 1



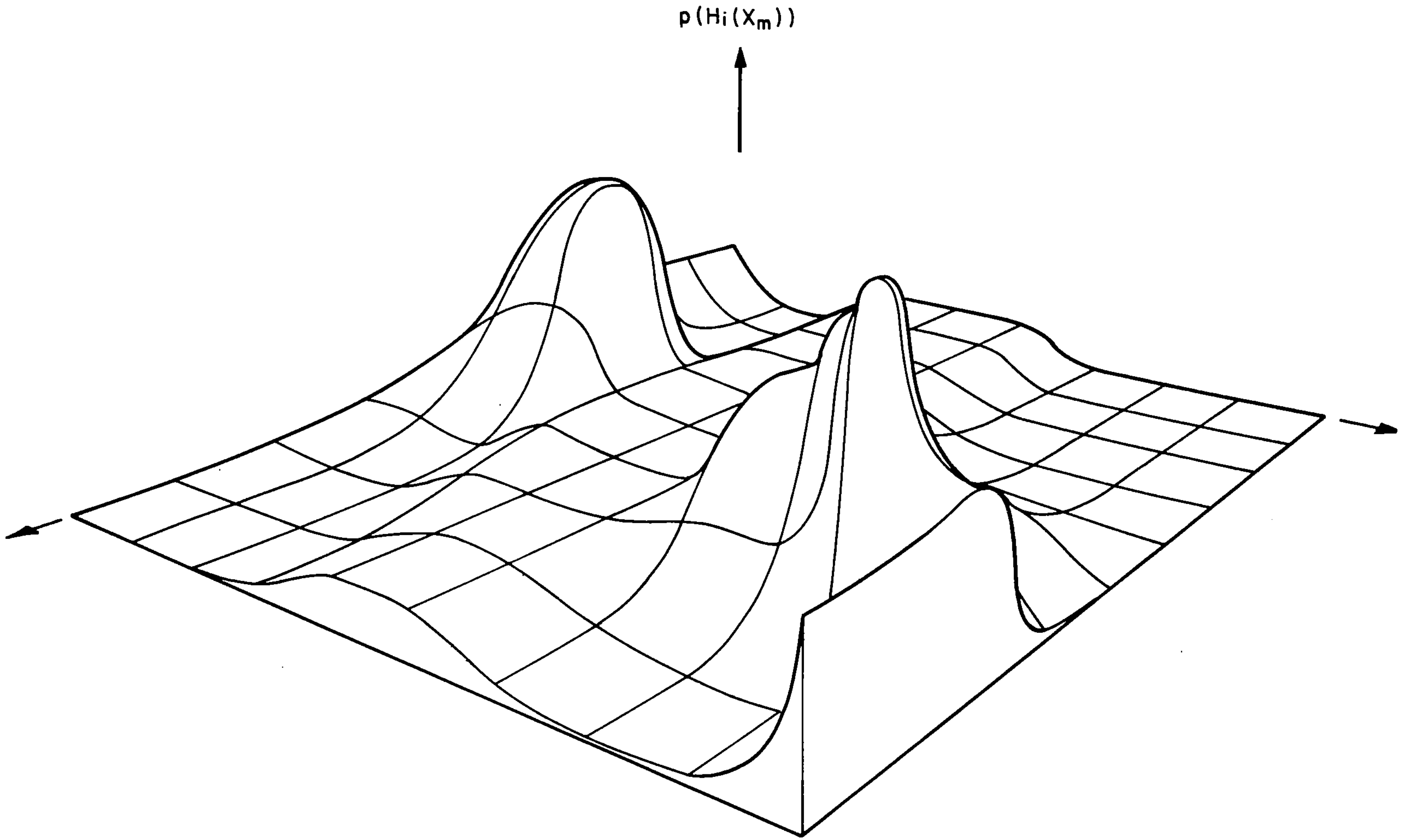
TRIAL 2



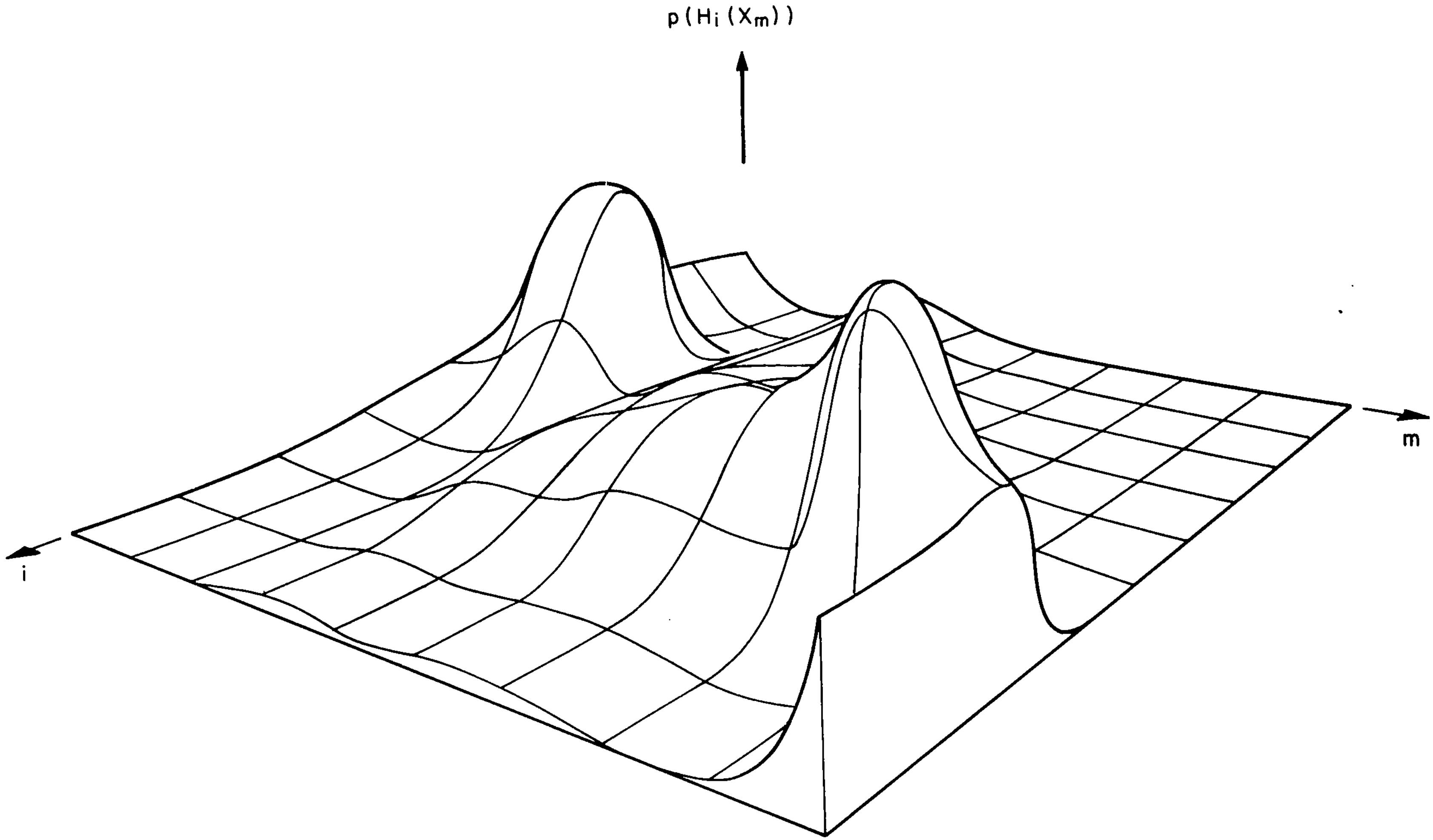
TRIAL 3



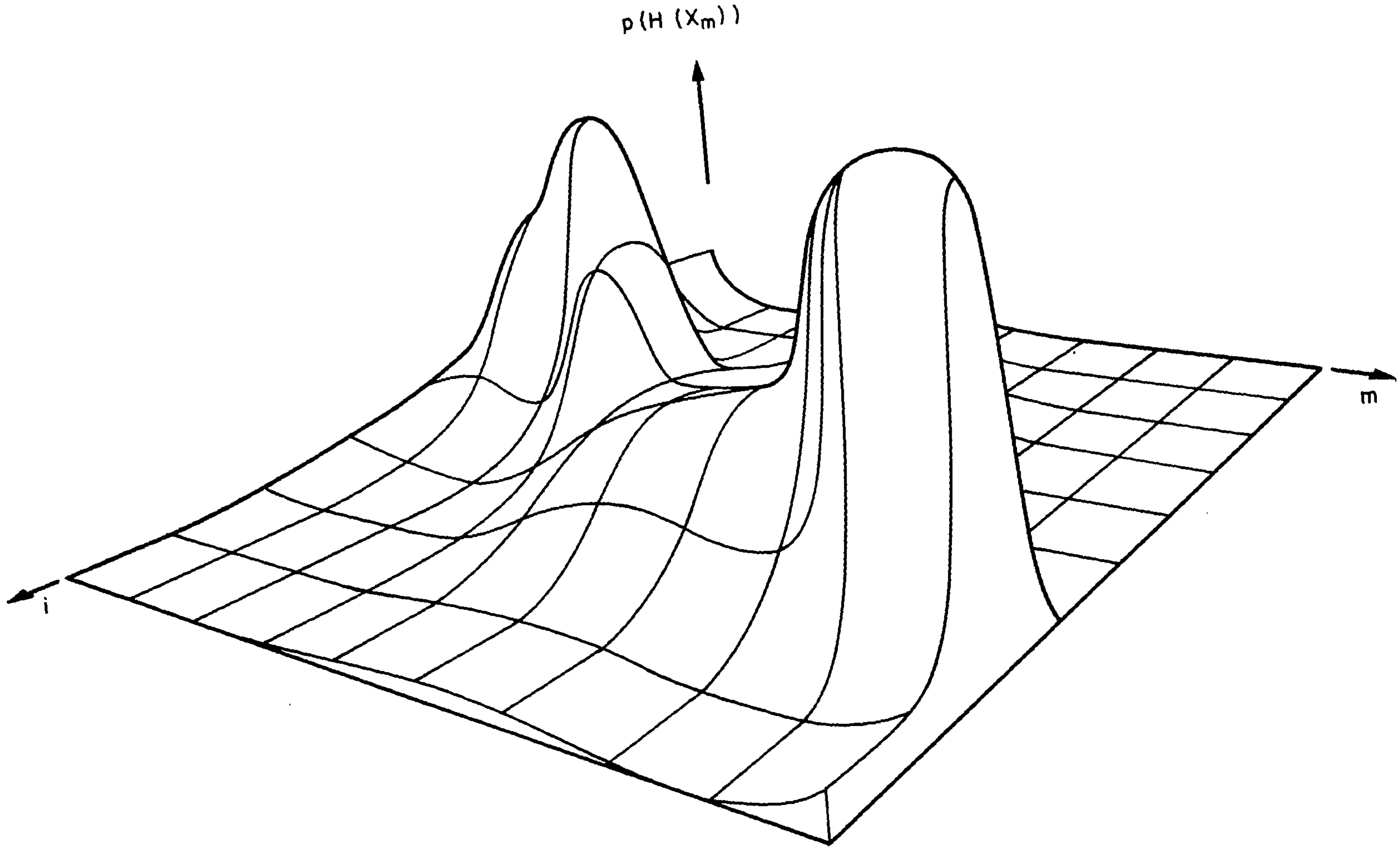
TRIAL 4



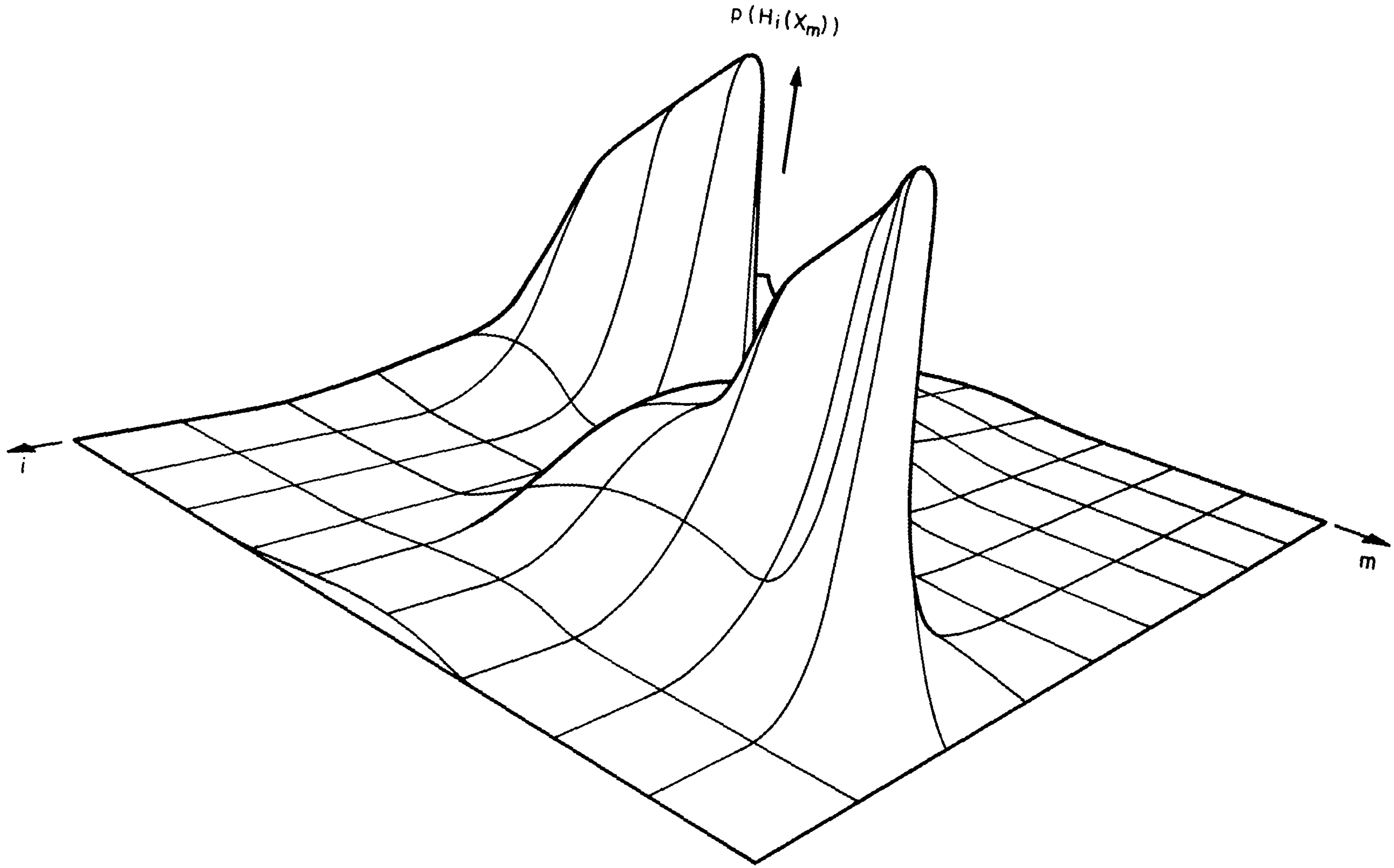
TRIAL 5



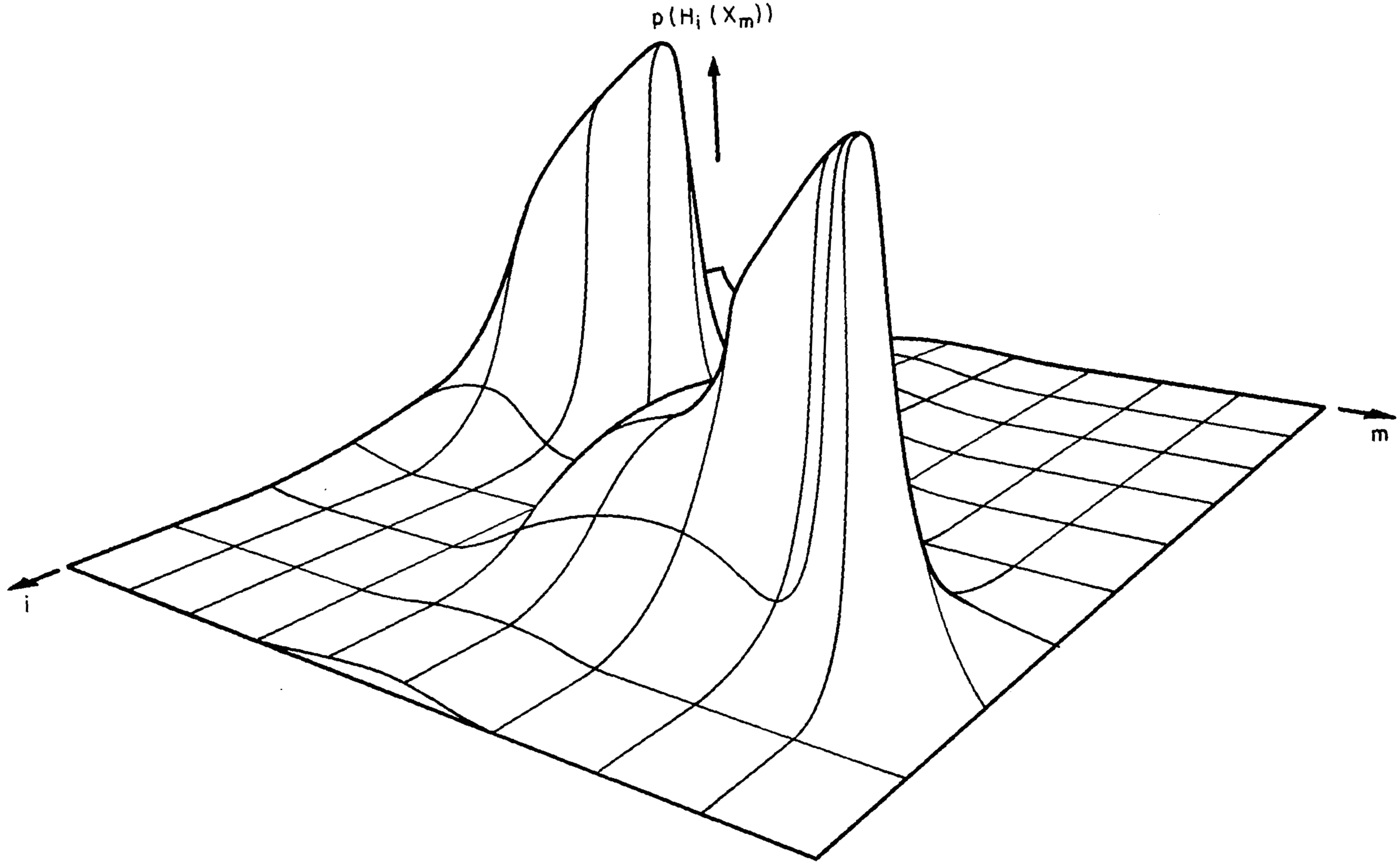
TRIAL 6



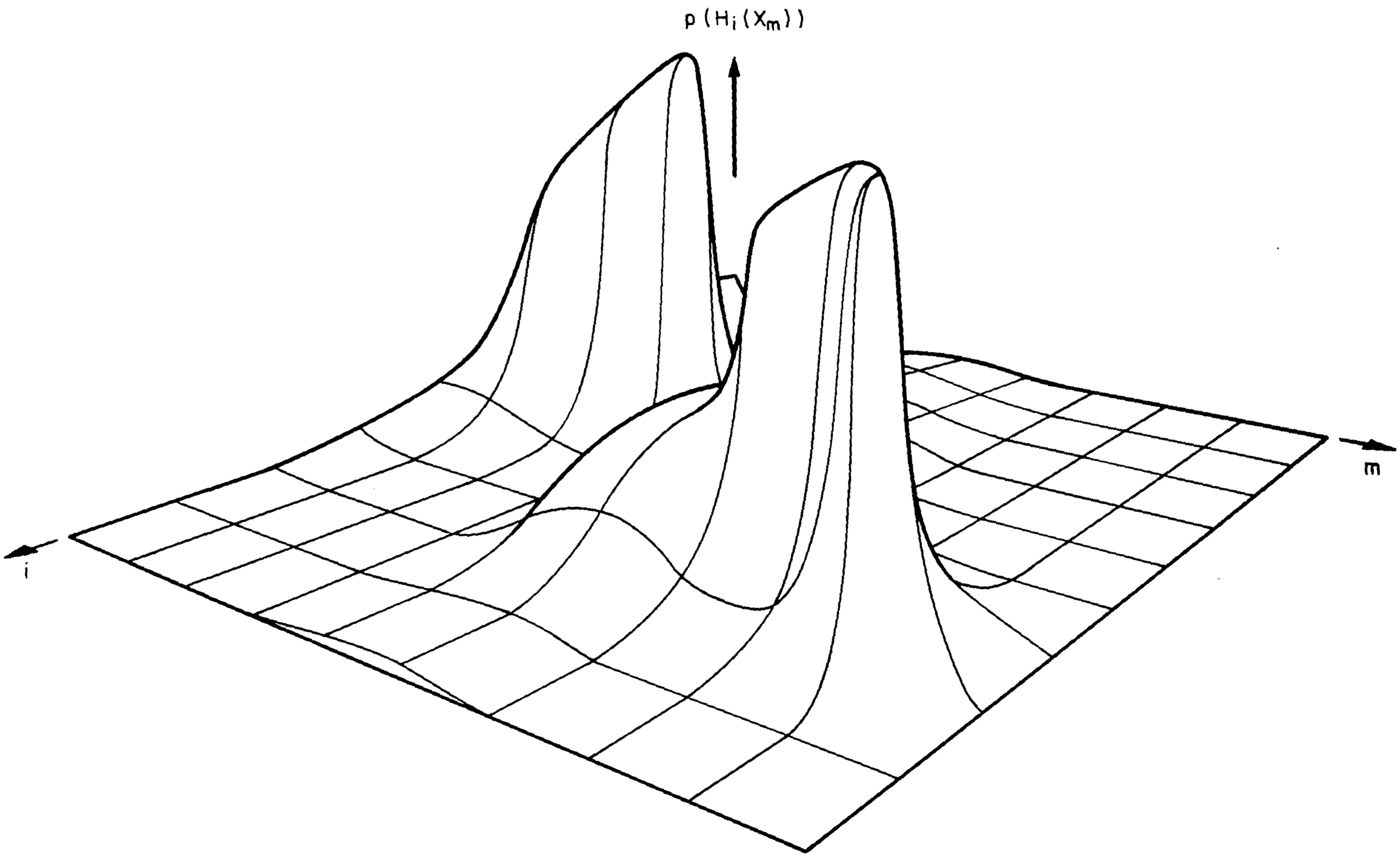
TRIAL 10



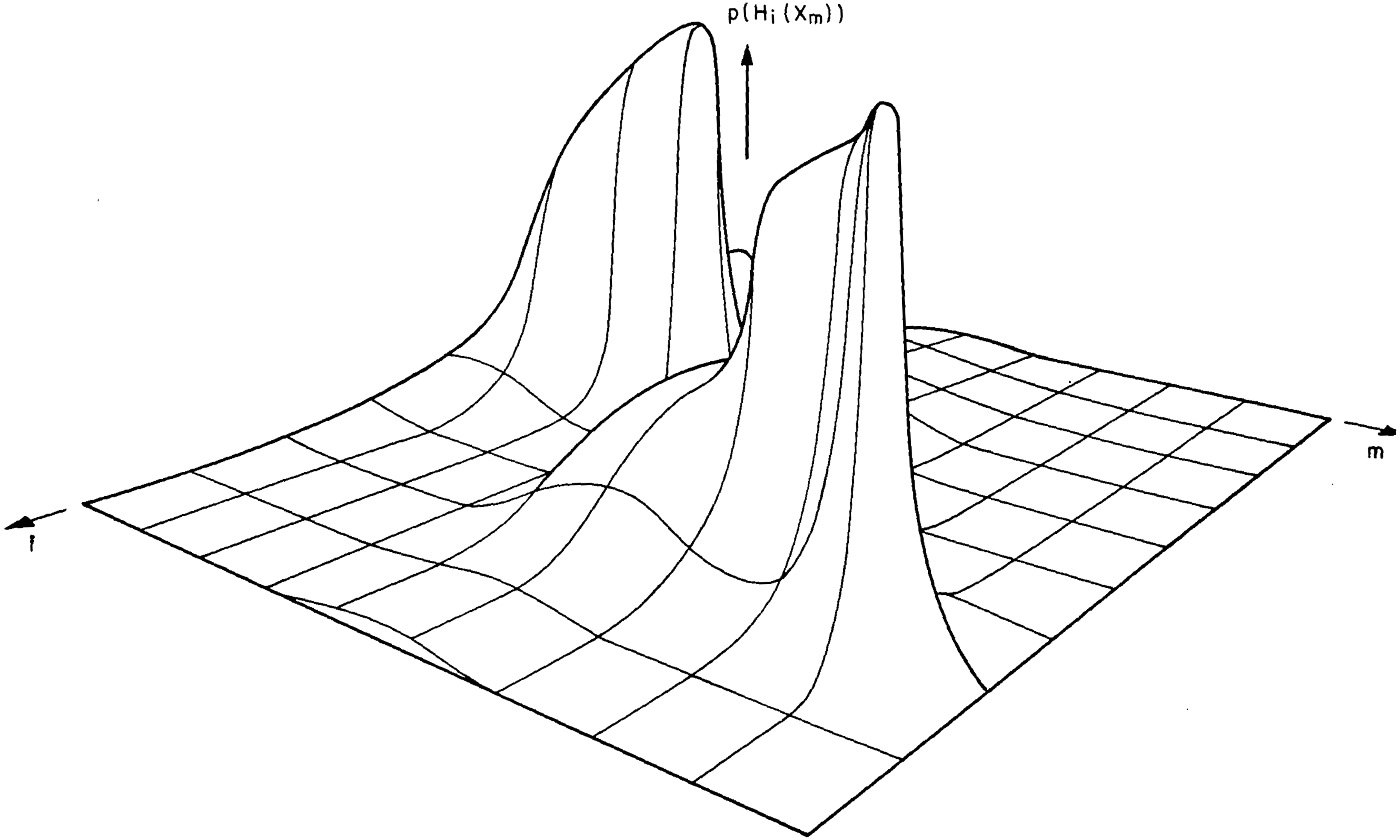
TRAIL 20



TRIAL 30



TRIAL 40



TRIAL 50

FIGURE 4.2

Ellipsoid of Concentration

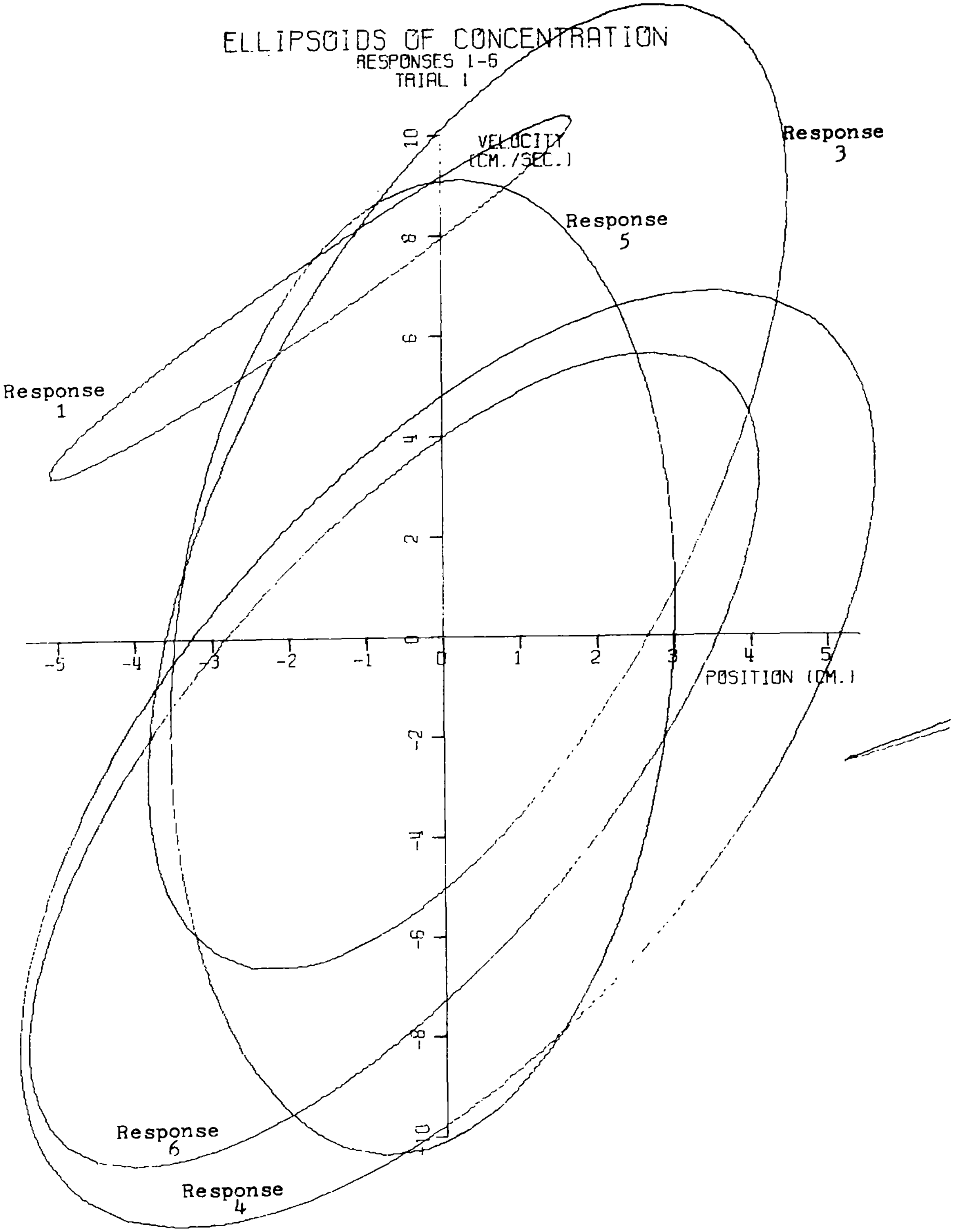
Responses 1-6

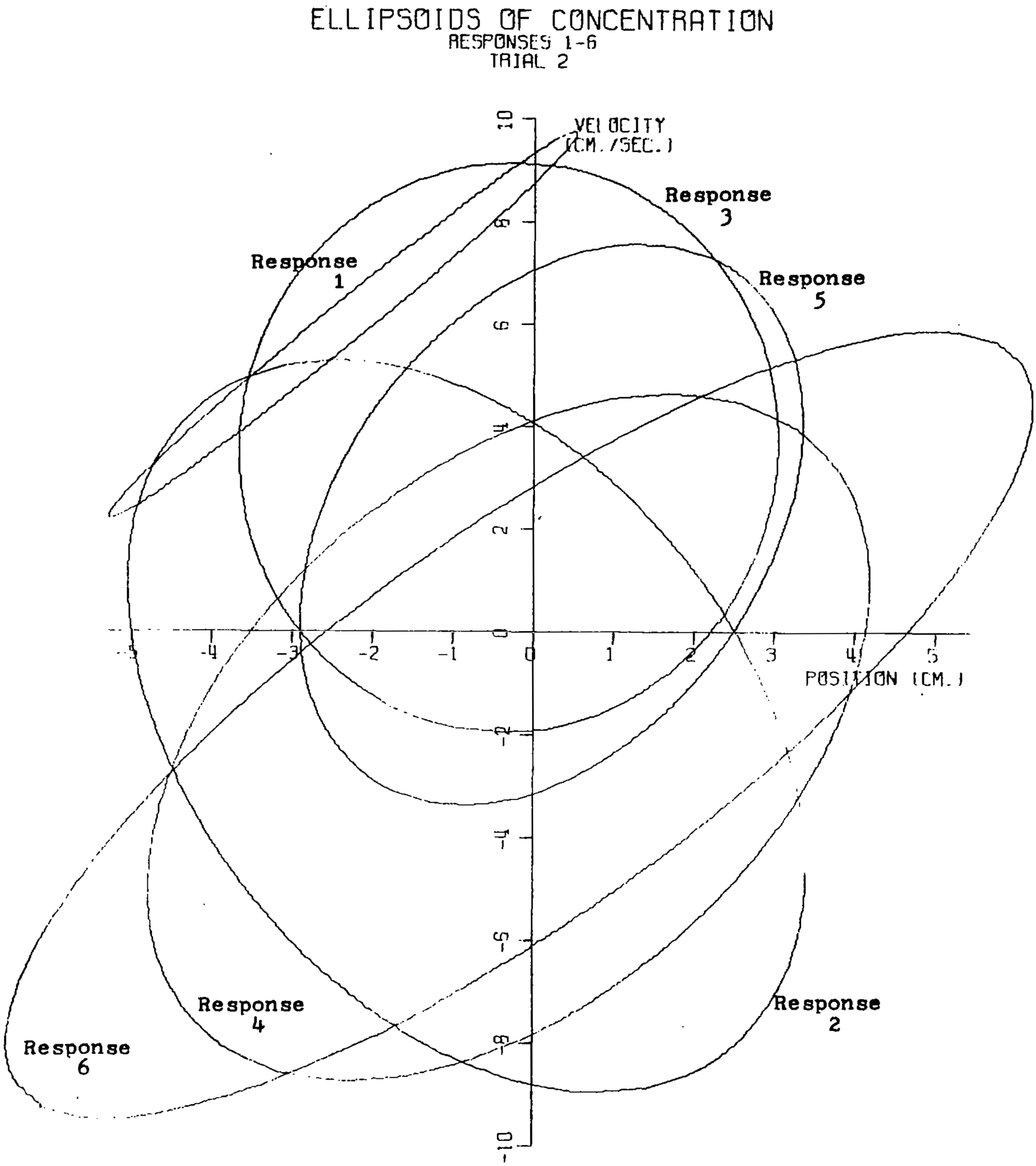
Trials 1-5, 10, 15, 20, 25, 30, 35, 40, 45 and 50

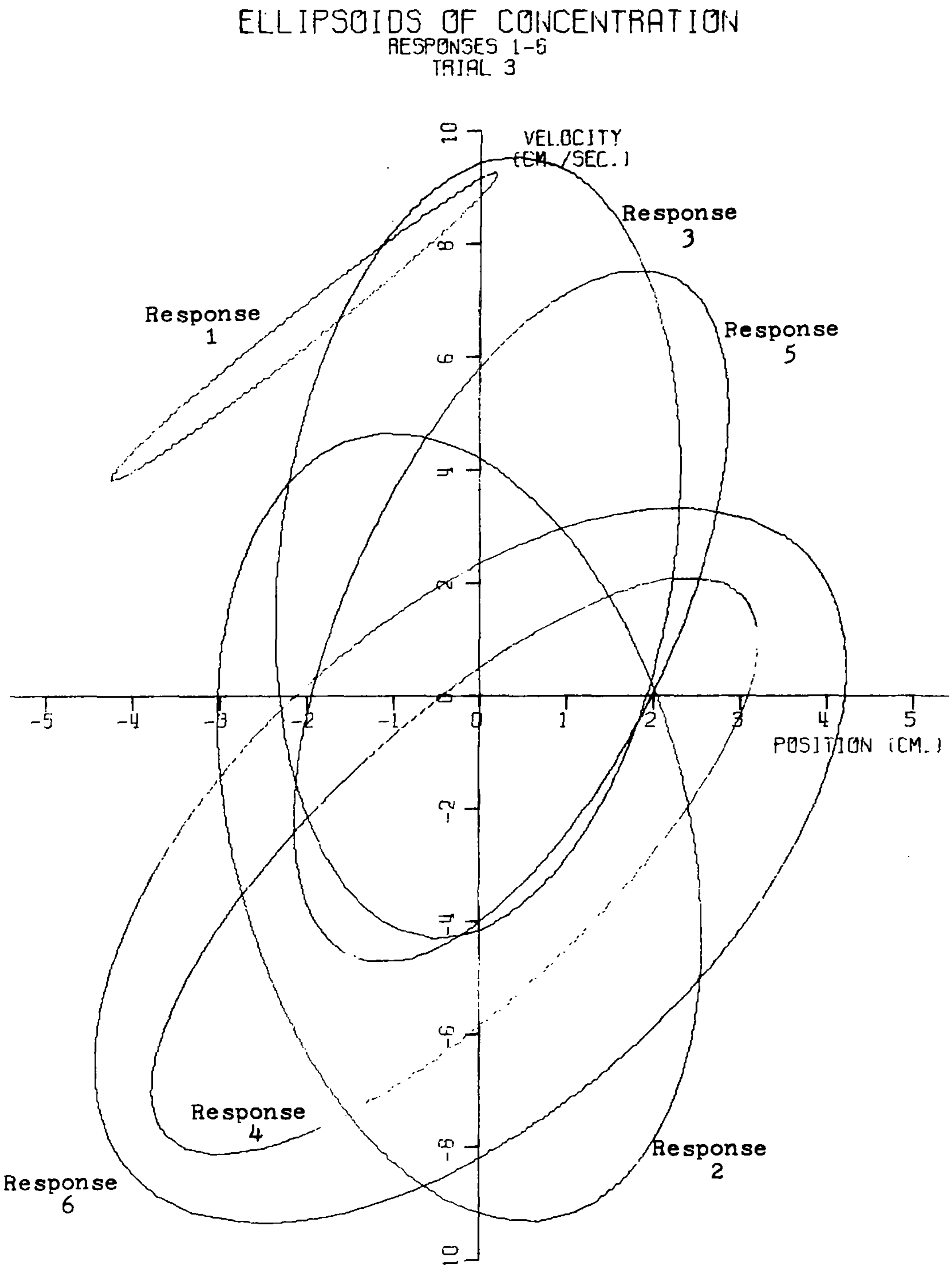
(Experimental)

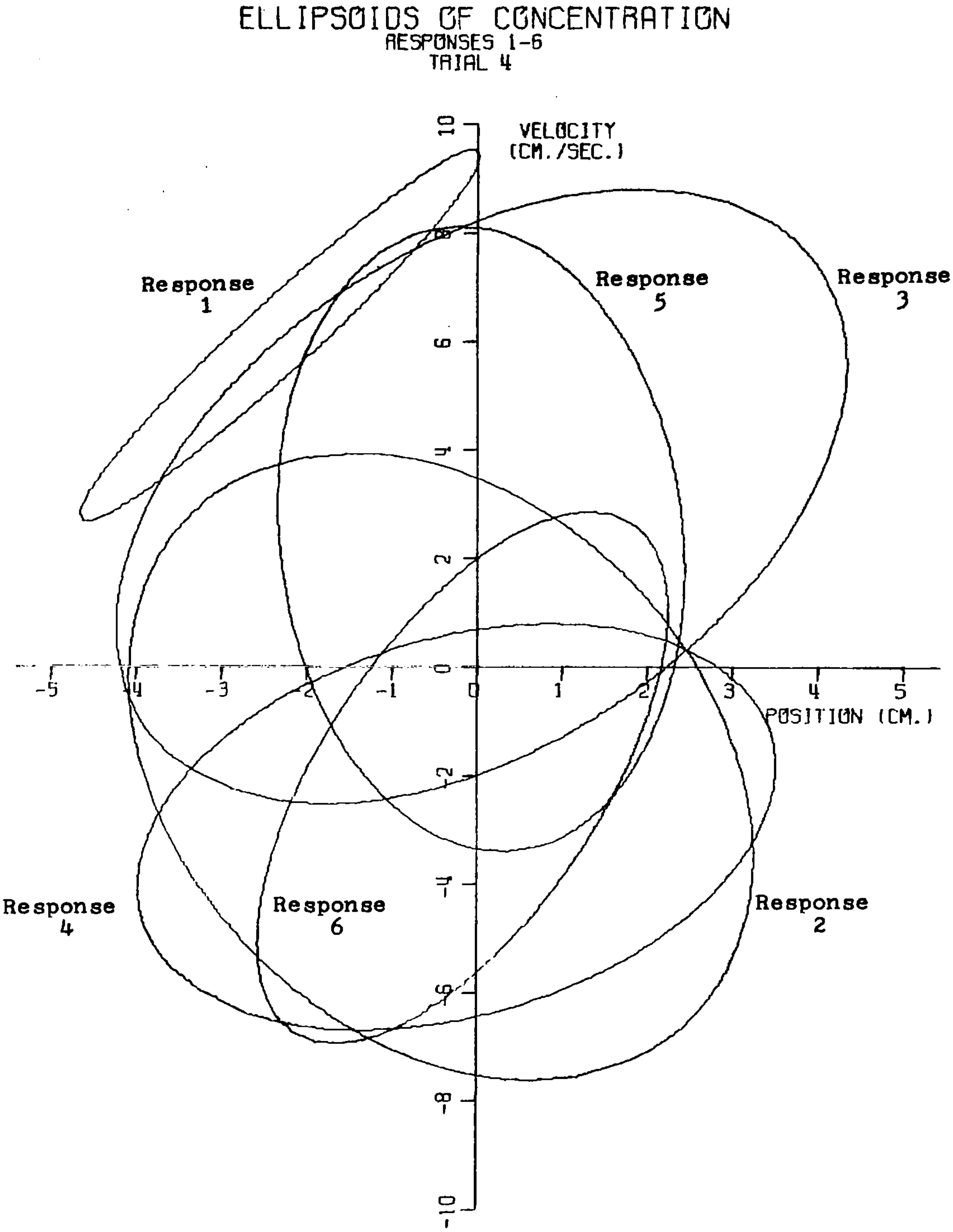
PAGES

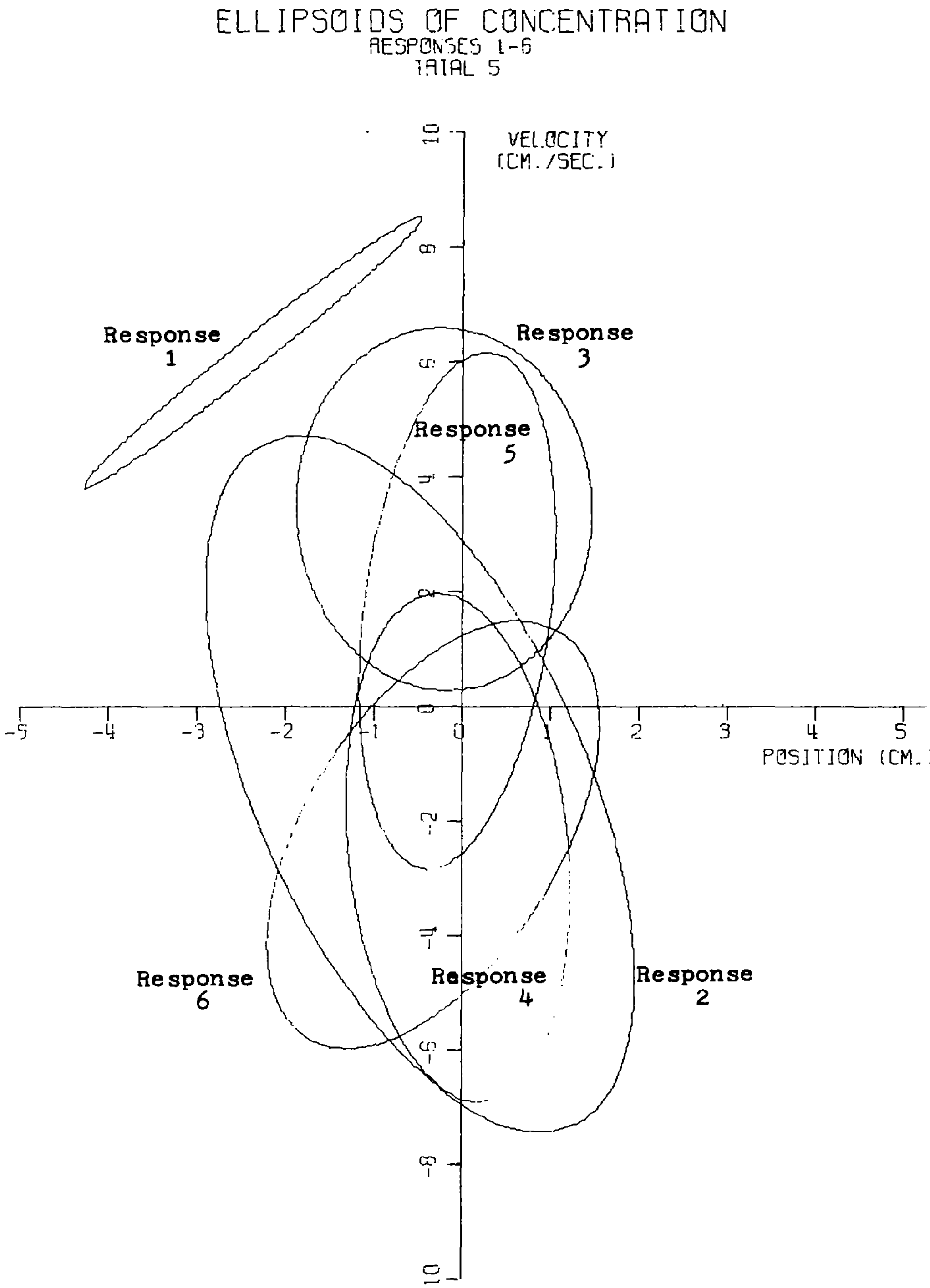
92-105


ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 1
VELOCITY
(CM./SEC.)
POSITION (CM.)
Response 1
Response 3
Response 5
Response 6
Response 4
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
0
1
2
3
4
5

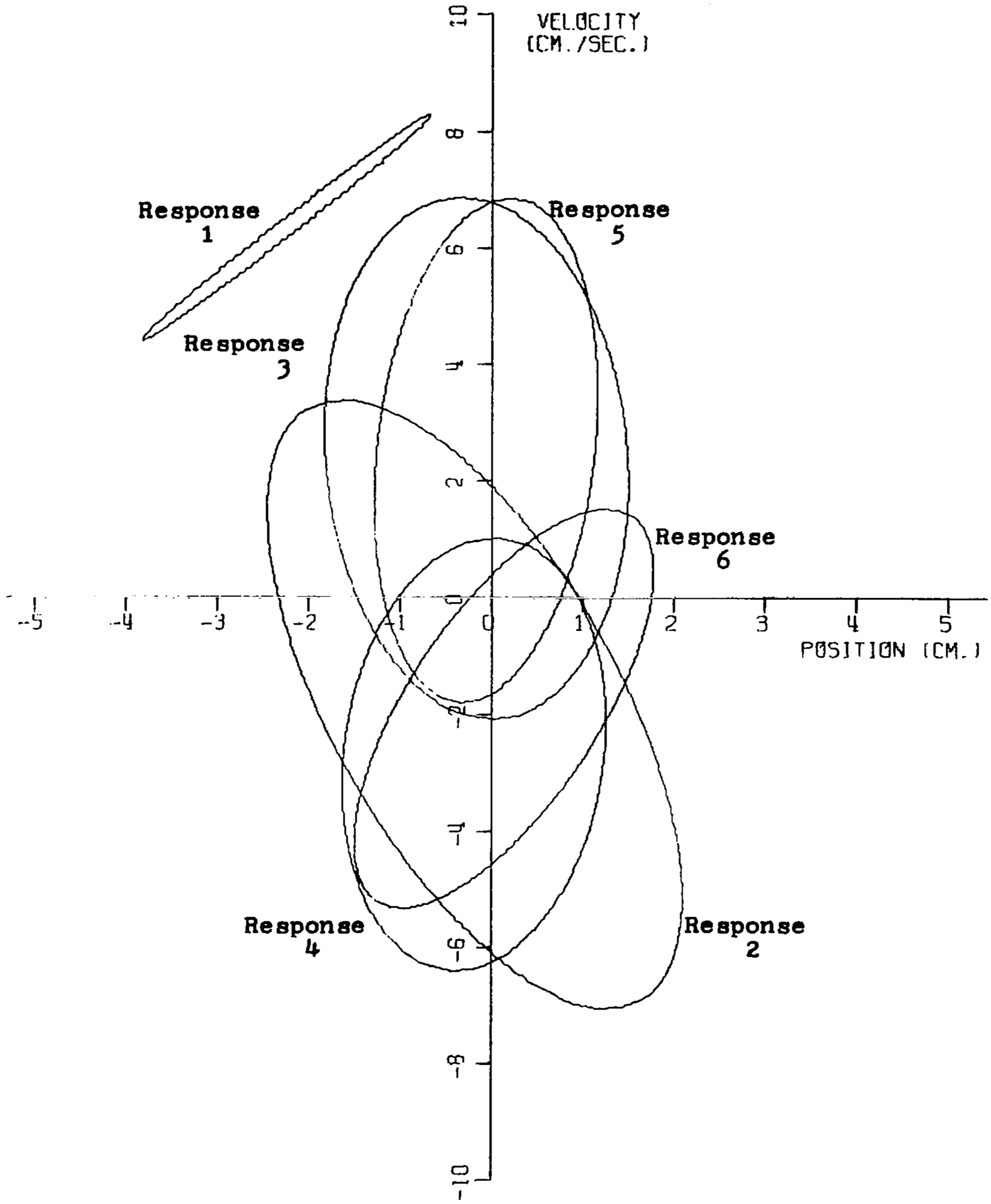

ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 2
VELOCITY (CM./SEC.)
POSITION (CM.)
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
0
1
2
3
4
5
Response 1
Response 2
Response 3
Response 4
Response 5
Response 6


ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 3
VELOCITY (CM./SEC.)
POSITION (CM.)
Response 1
Response 2
Response 3
Response 4
Response 5
Response 6
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
0
1
2
3
4
5

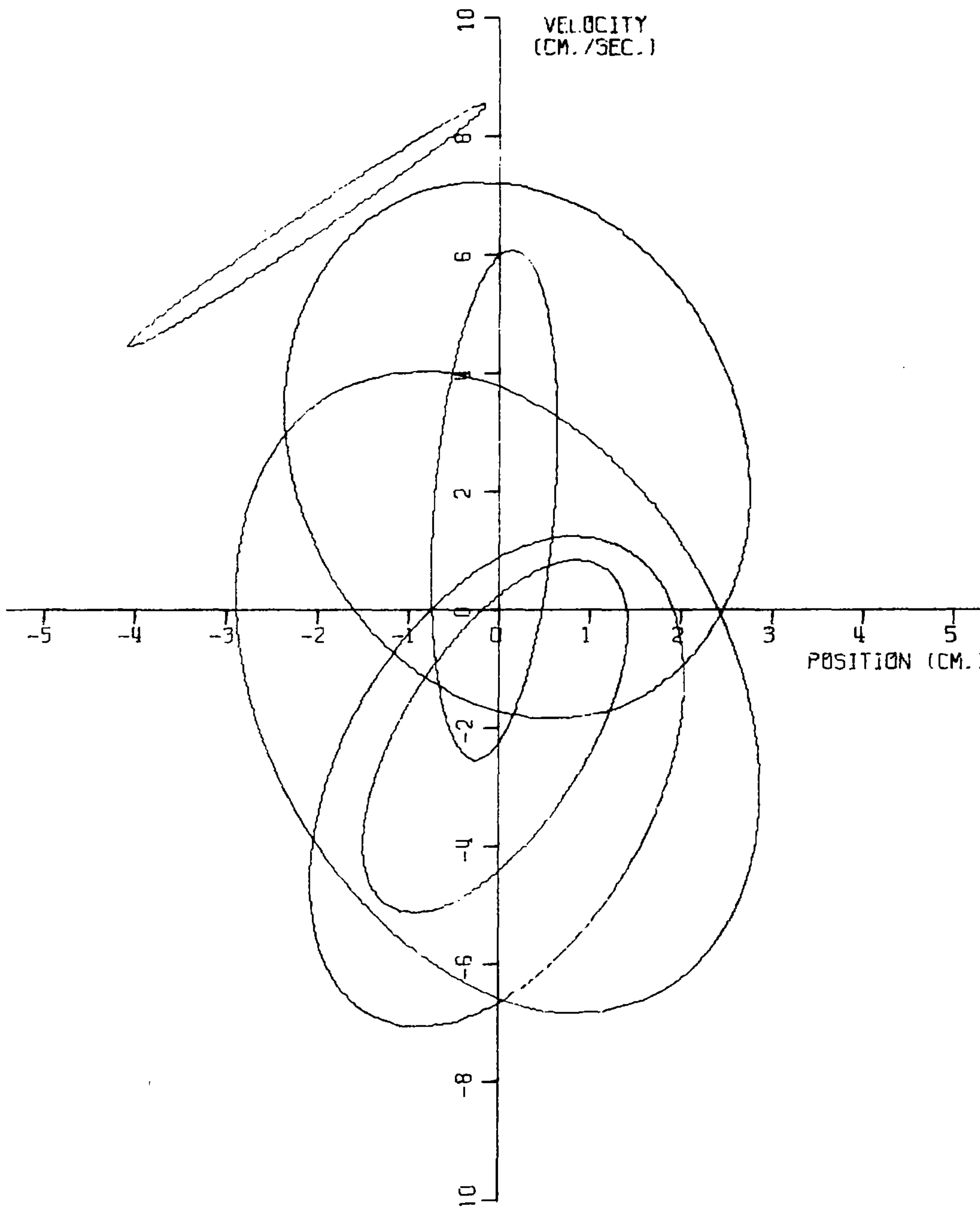

ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 4
VELOCITY (CM./SEC.)
POSITION (CM.)
Response 1
Response 2
Response 3
Response 4
Response 5
Response 6
10
6
4
2
0
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
0
1
2
3
4
5

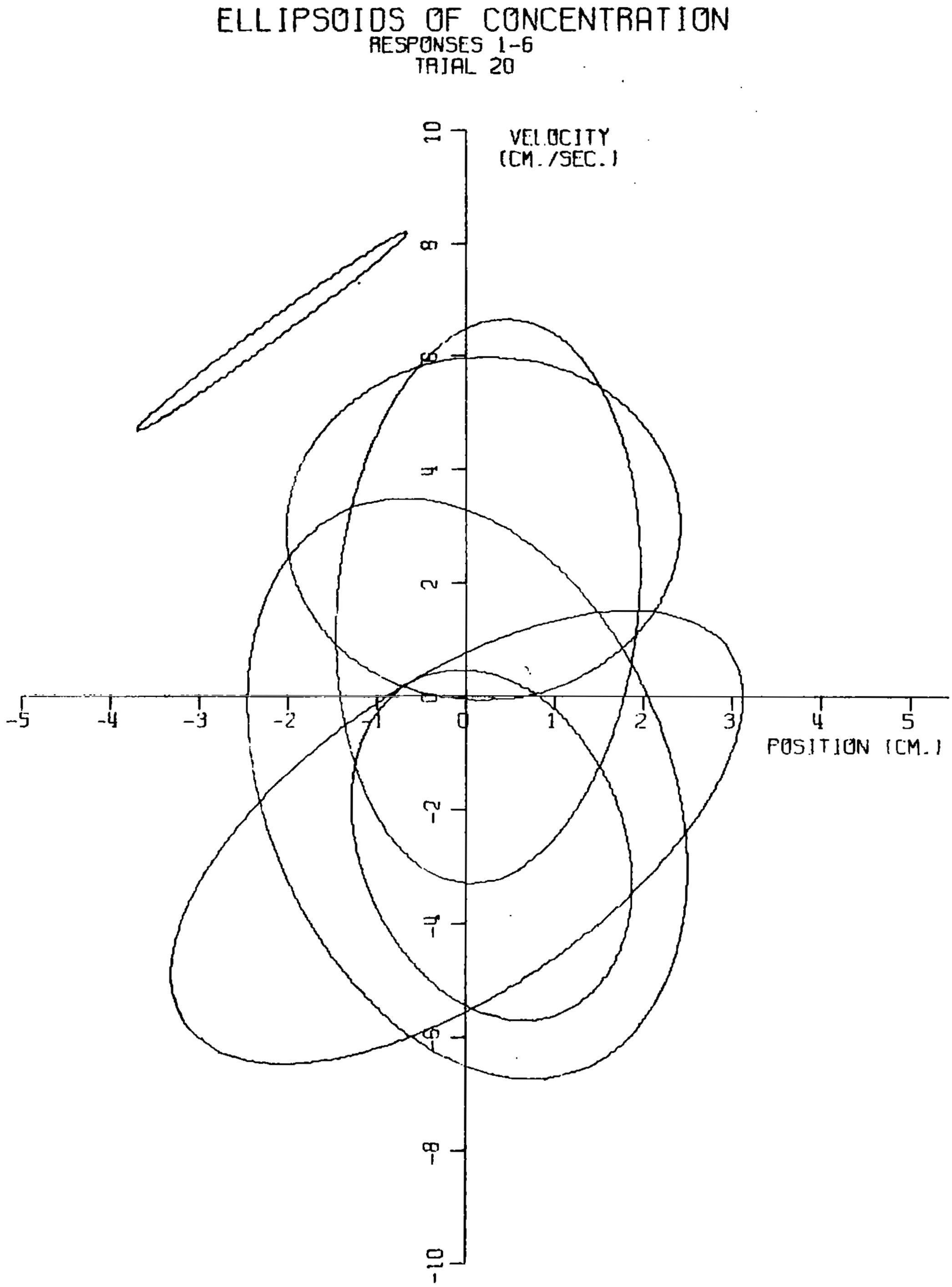

ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 5
VELOCITY
(CM./SEC.)
POSITION (CM.)
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
0
1
2
3
4
5
Response
1
Response
3
Response
5
Response
6
Response
4
Response
2

ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 10

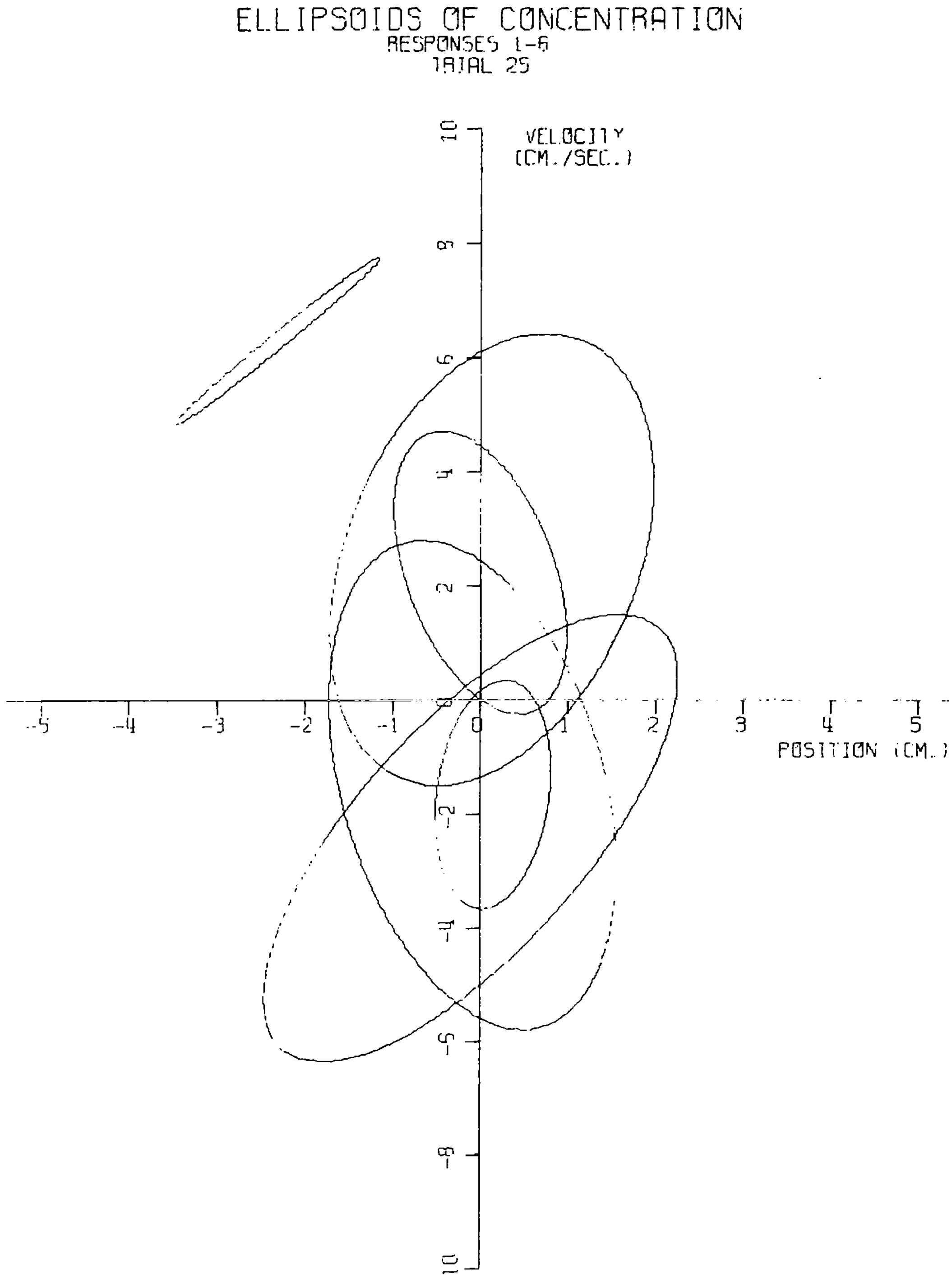


ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 15

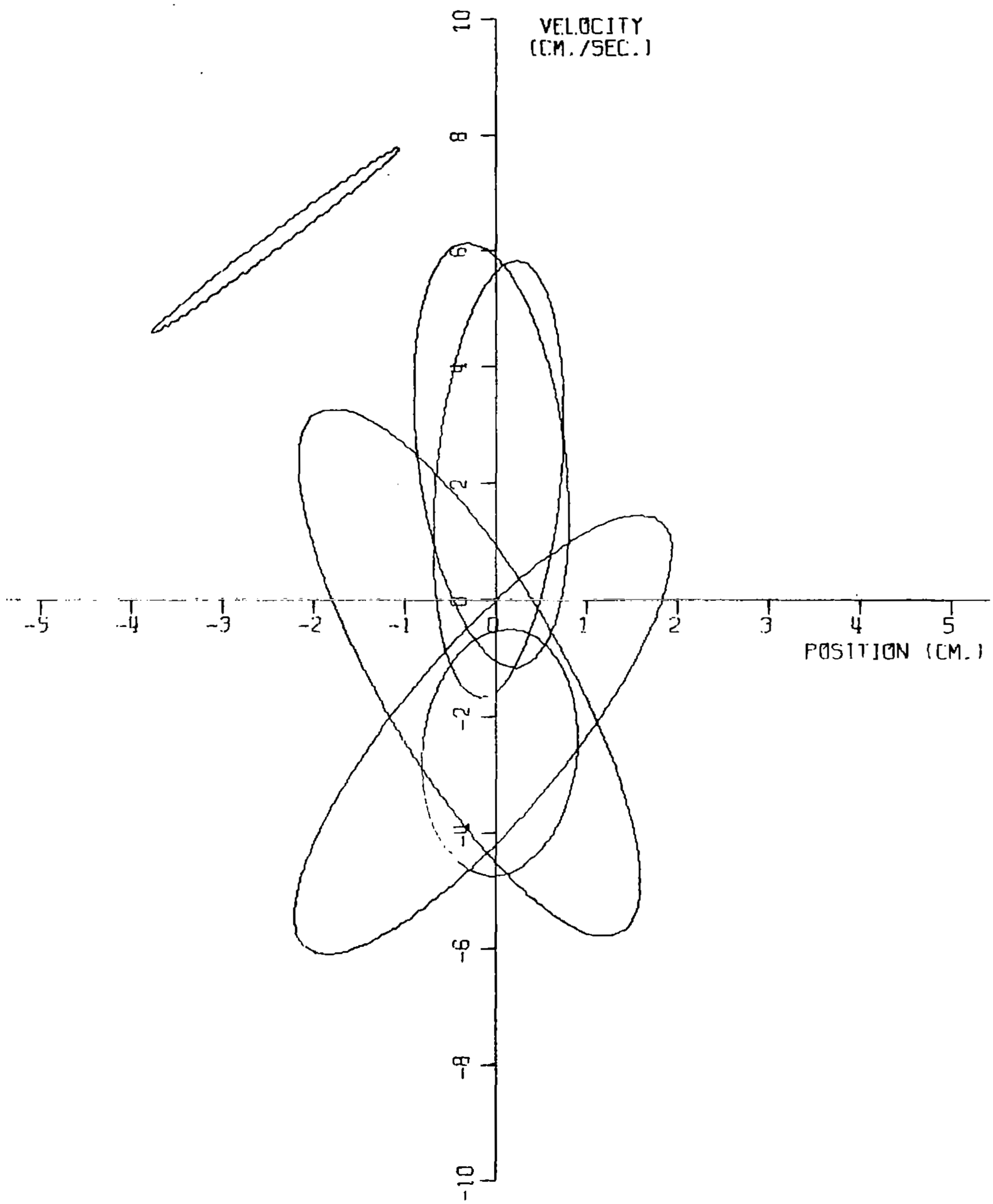


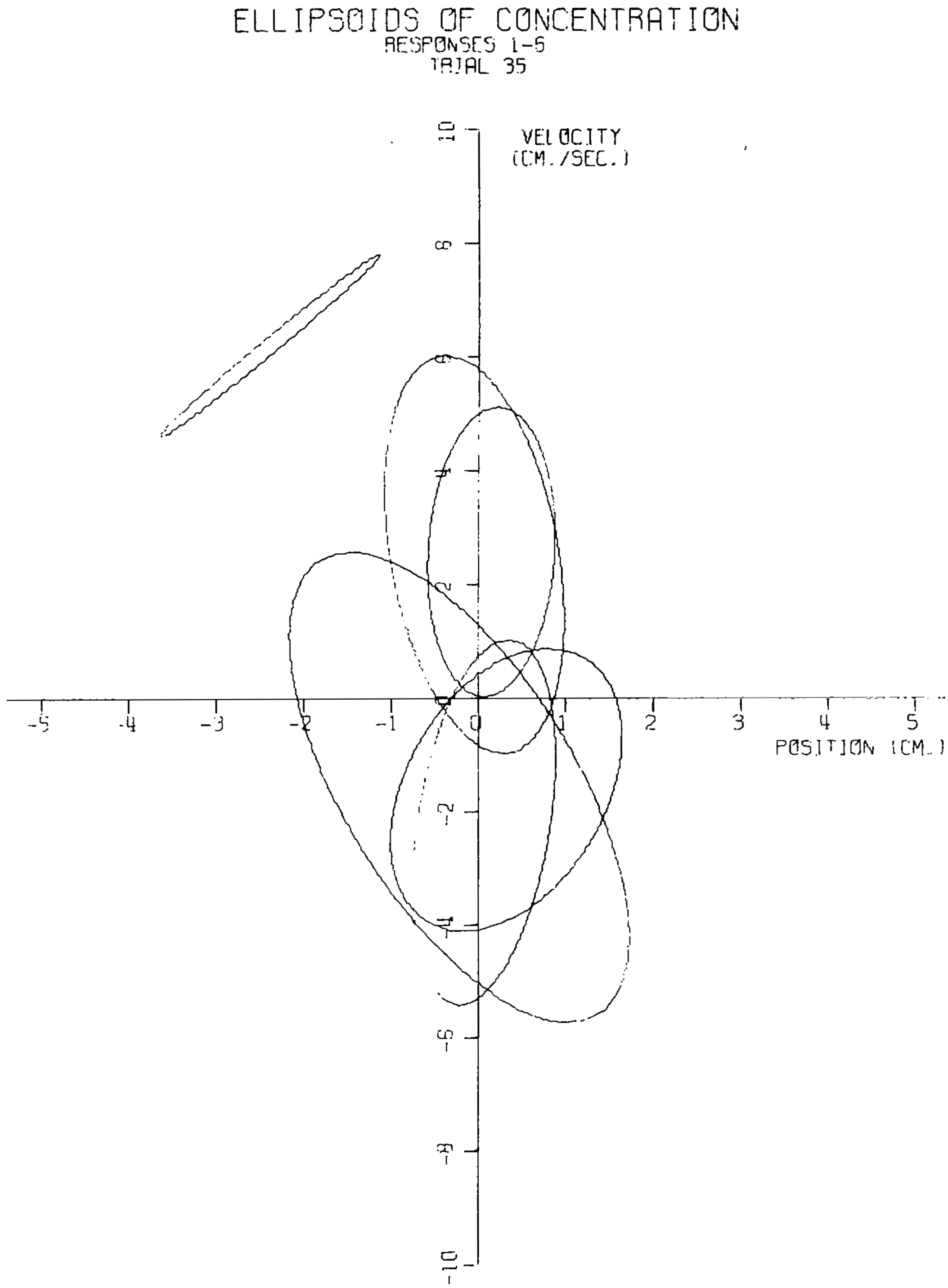


ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 20
VELOCITY
(CM./SEC.)
POSITION (CM.)
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
1
2
3
4
5

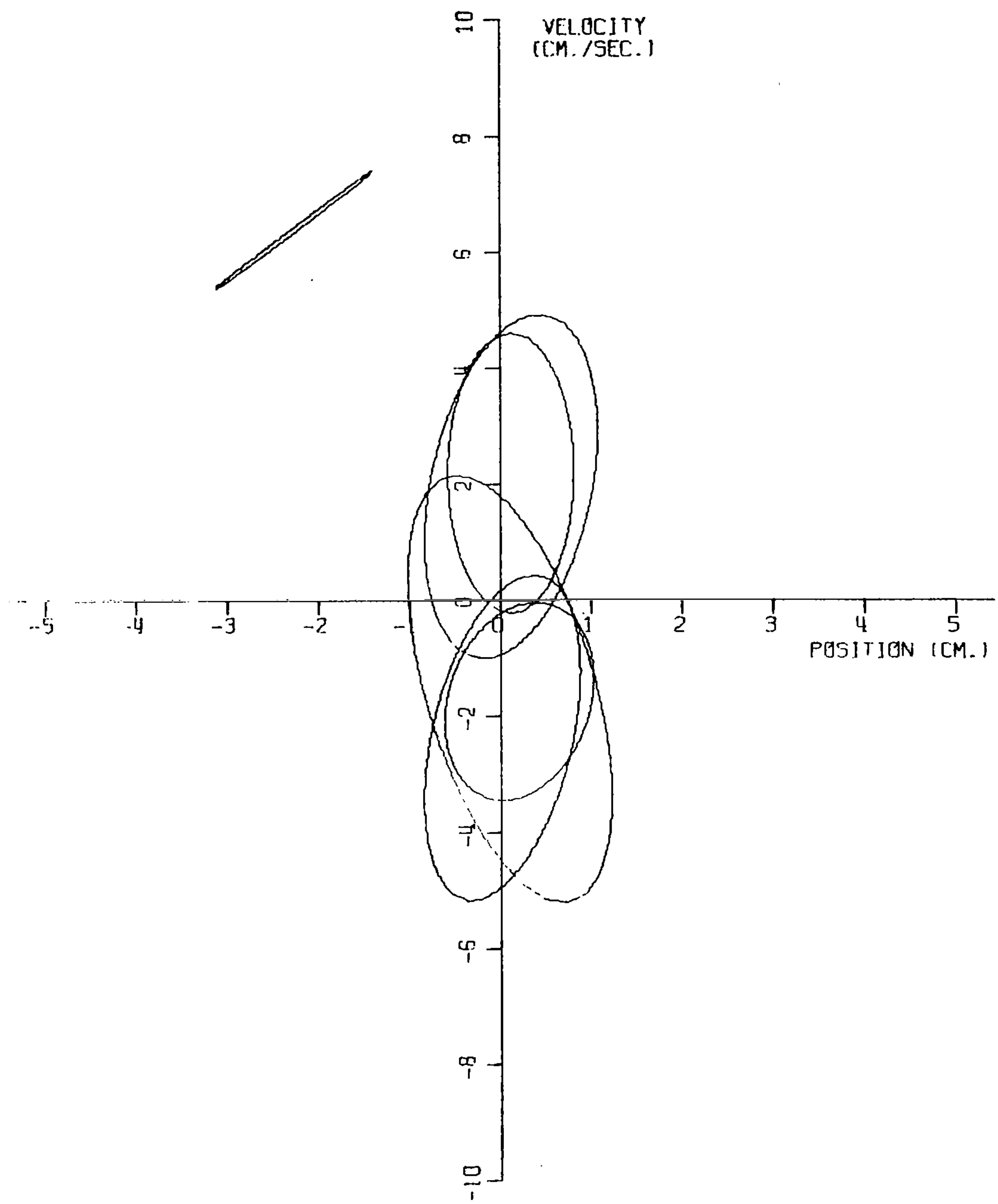

ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 25
VELOCITY
(CM./SEC.)
POSITION (CM.)
10
9
6
4
2
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
0
1
2
3
4
5

ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 30

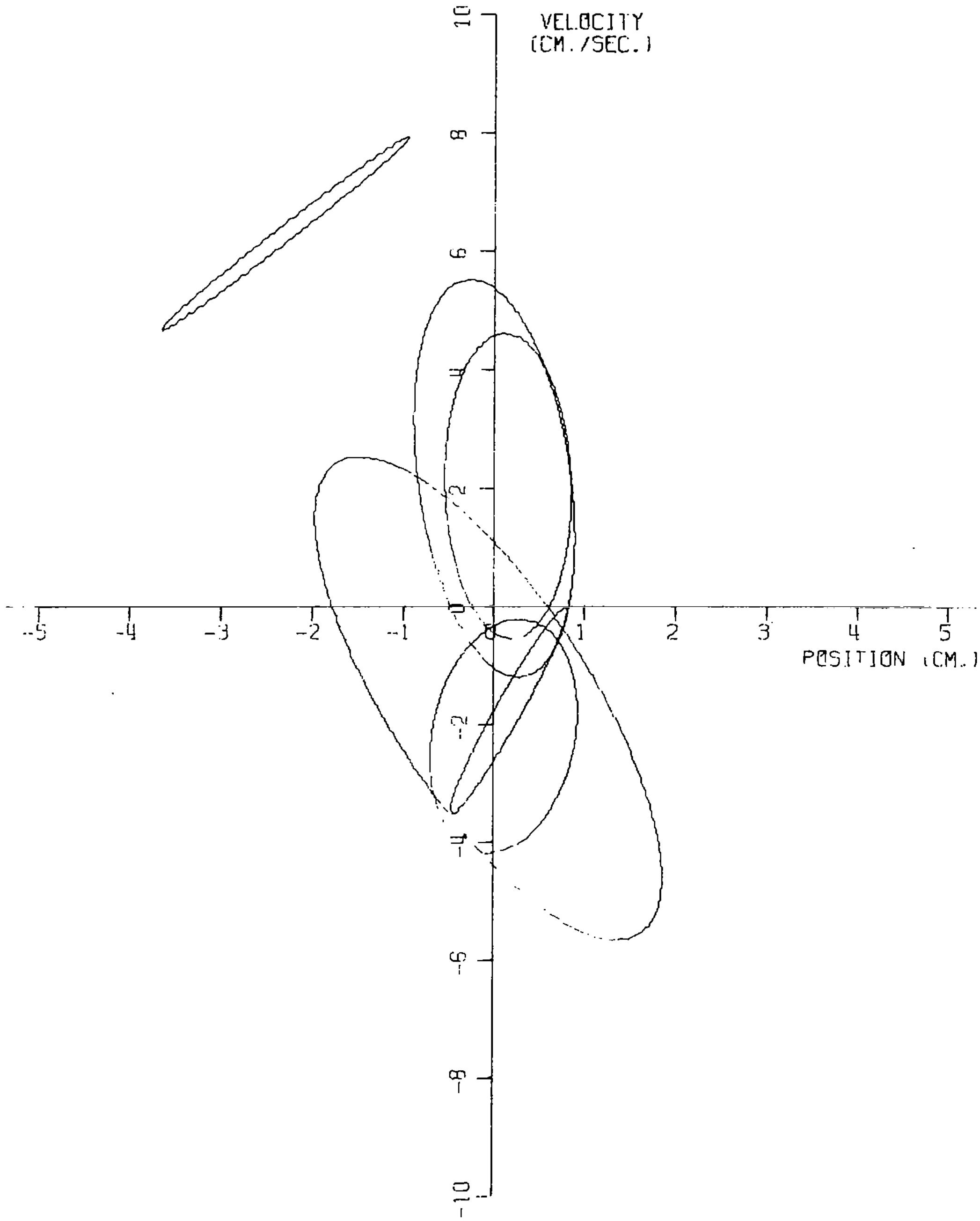



ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 35
VELOCITY
(CM./SEC.)
POSITION (CM.)
10
8
6
4
2
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
0
1
2
3
4
5

ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 40



ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 45



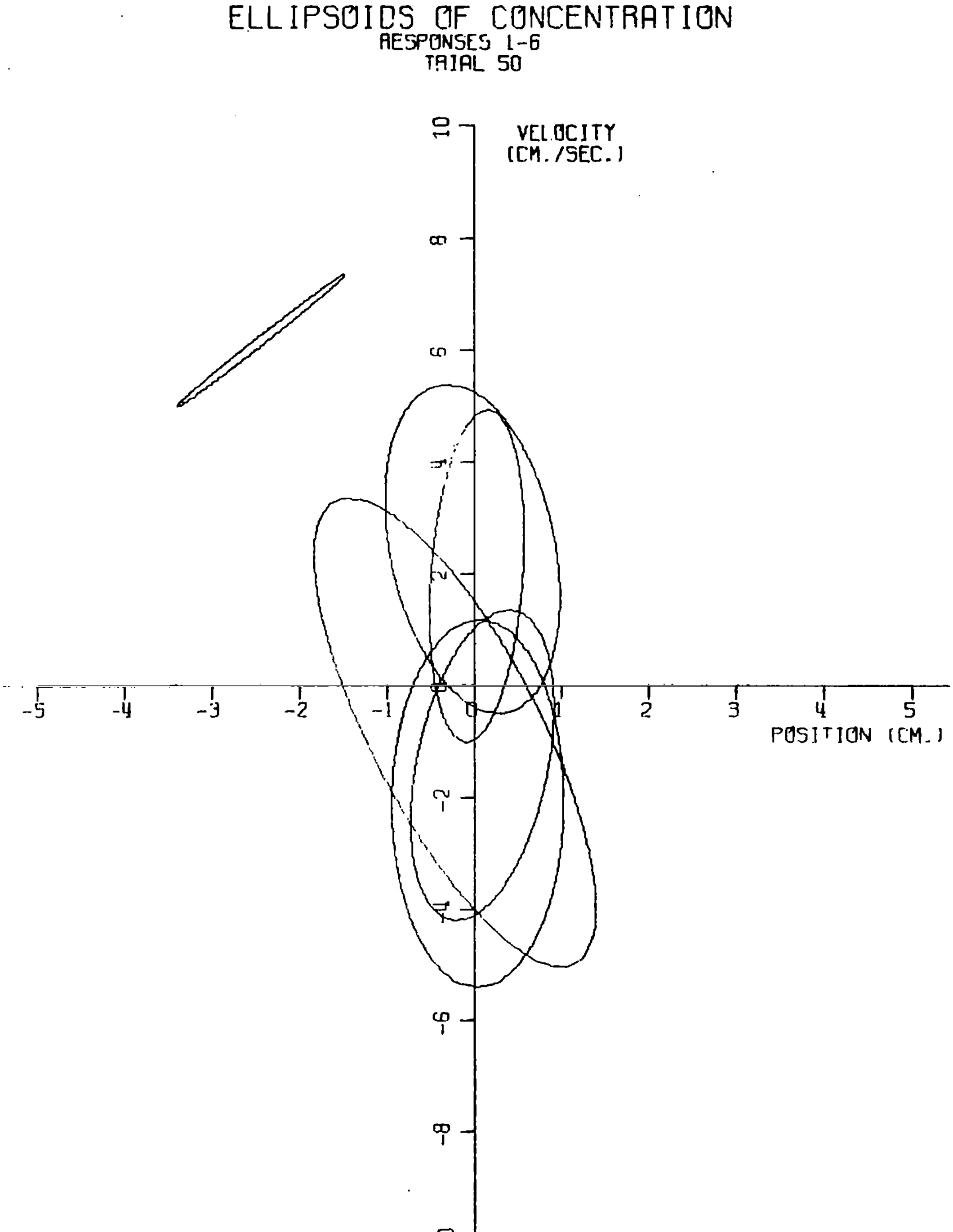

ELLIPSOIDS OF CONCENTRATION
RESPONSES 1-6
TRIAL 50
VELOCITY (CM./SEC.)
POSITION (CM.)
10
8
6
4
2
0
-2
-4
-6
-8
-10
-5
-4
-3
-2
-1
1
2
3
4
5

FIGURE 4.3

Average Transient Response

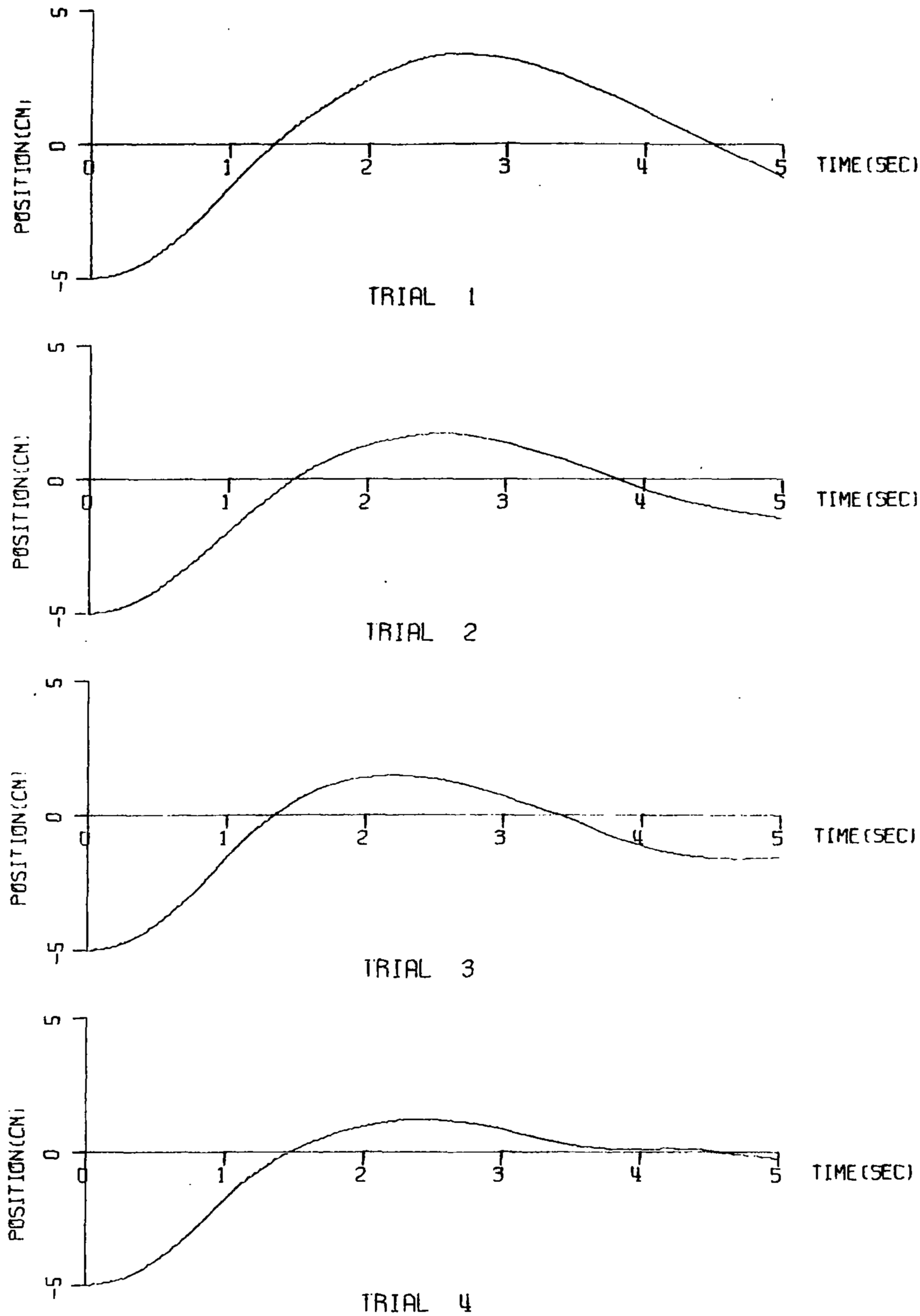
Trials 1-8, 10, 15, 20, 25, 30, 40 and 50

(Experimental)

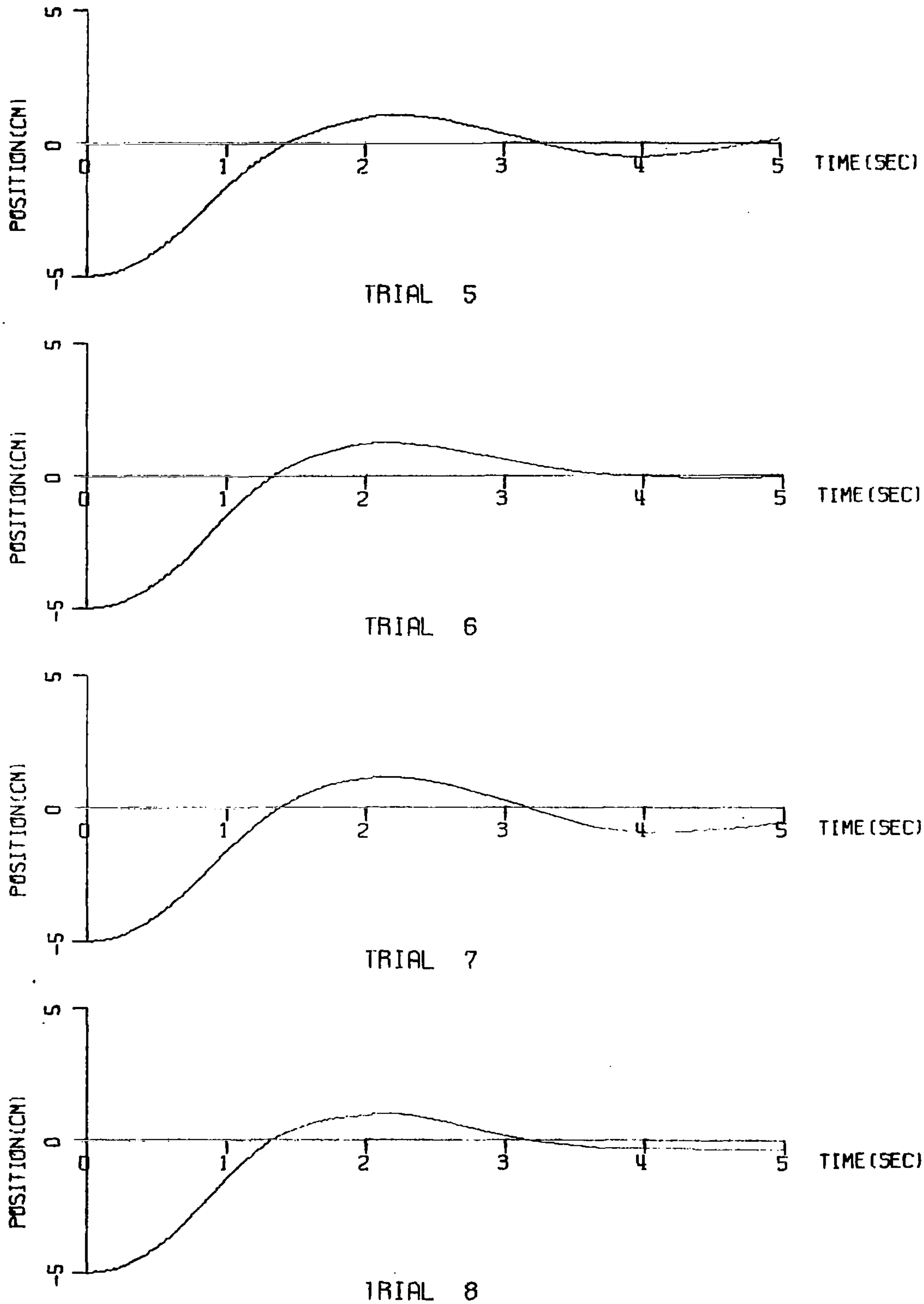
PAGES

106-109

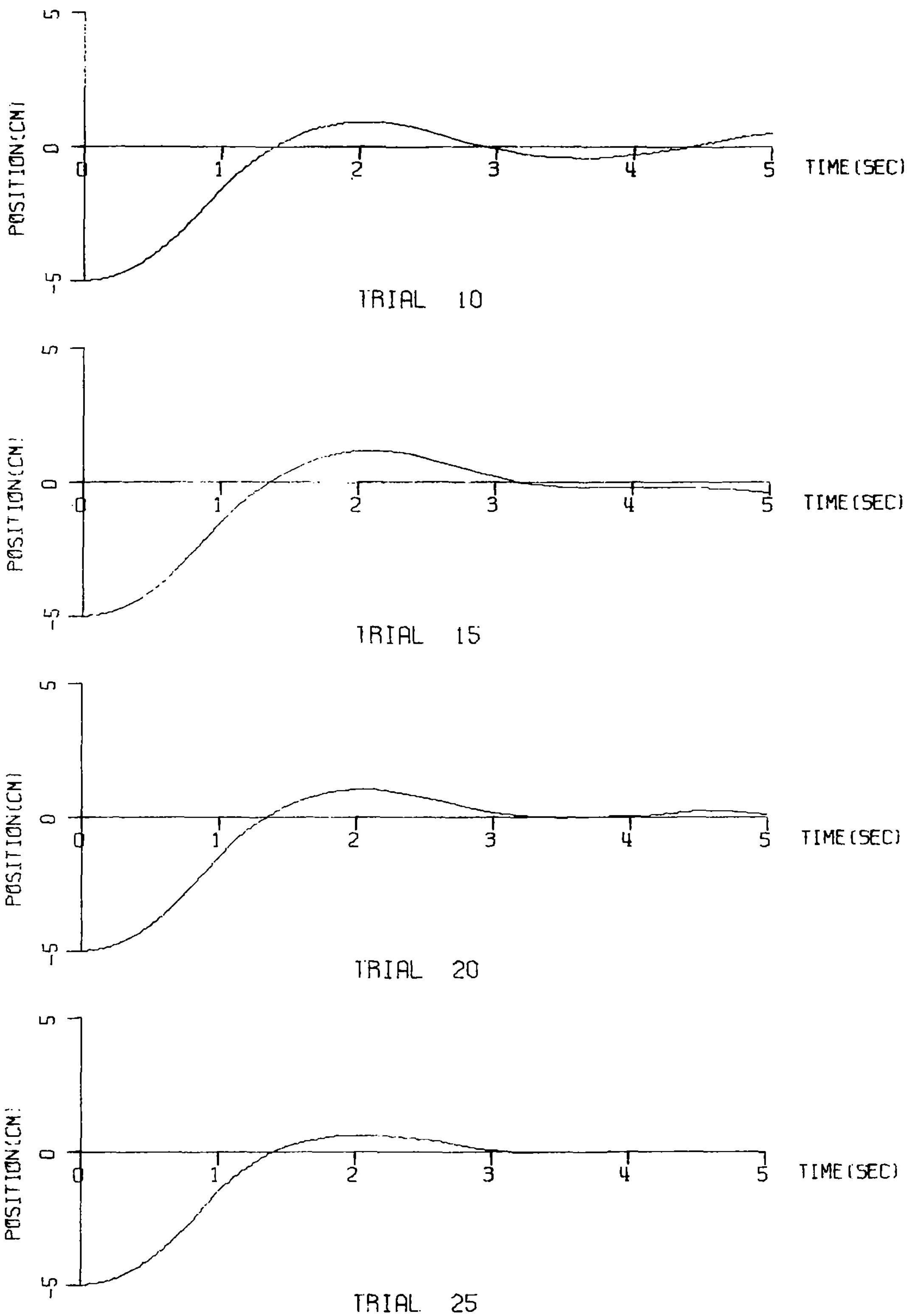
AVERAGE TRANSIENT RESPONSE



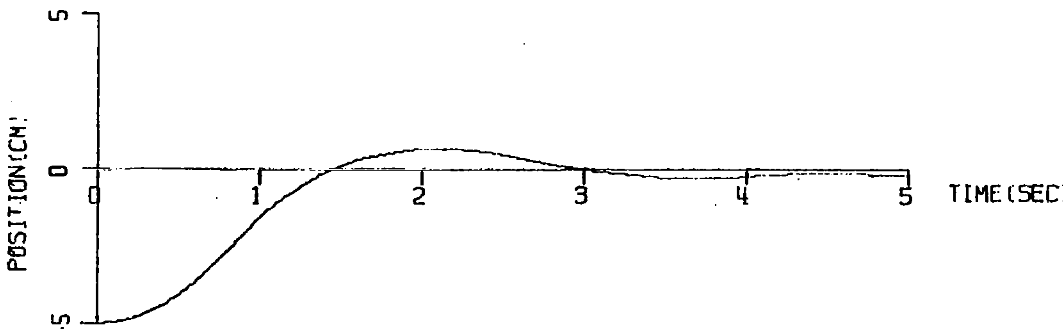
AVERAGE TRANSIENT RESPONSE



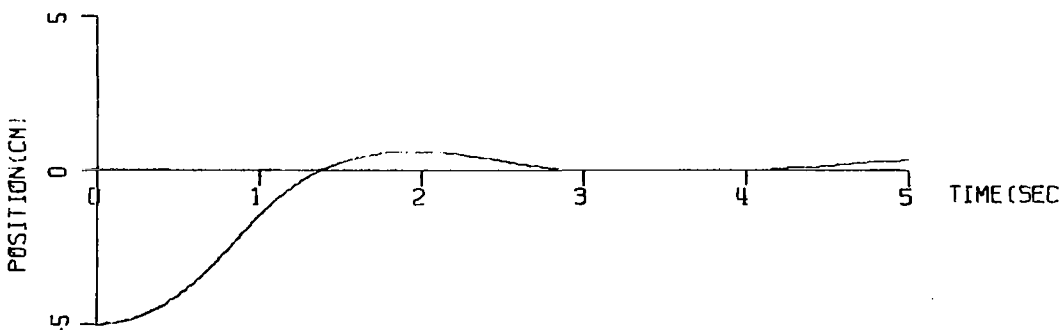
AVERAGE TRANSIENT RESPONSE



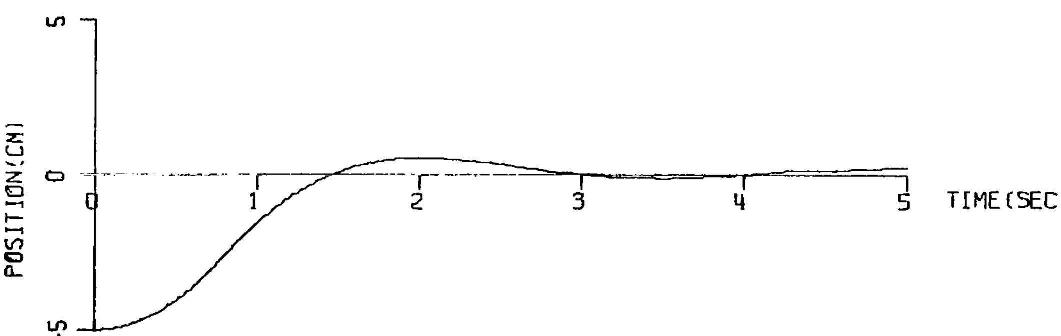
AVERAGE TRANSIENT RESPONSE



TRIAL 30



TRIAL 40



TRIAL 50

TABLE 4.1

PROGRAM PARAMETERS

PROGRAM	MMAX	NMAX	MODE	DELTAX	DLXDOT
1-5	10	20	2	.5	1.0
6	"	"	"	"	"
7	"	"	"	"	"
8	"	"	"	"	"
9	"	"	"	"	"
10	"	"	"	"	"
11-15	"	40	"	"	"
16-20	"	"	1	"	"
21	20	40	2	.25	.5
22	10	"	"	.5	"
23	5	20	"	1.0	1.0
24	"	10	"	"	2.0
25-27	10	40	"	.5	1.0
28-30	"	"	"	"	"
31-33	"	"	1	"	"
34-35	"	"	"	"	"
36-40	"	20	2	"	"
41-45	"	"	3-4	"	"
46	"	"	"	"	"
47	"	"	"	"	"
48	"	"	"	"	"
49	"	"	"	"	"
50	"	"	"	"	"
51-60	20	40	3-4	.25	.5

TABLE 4.1 cont.

PROGRAM PARAMETERS

DT1	DT2	DT#	DT4	RT1	RT2	P_o	α
.04	.02	.02	.02	.14	.02	1.0	.0
"	"	"	"	.10	"	"	"
"	"	"	"	.12	"	"	"
"	"	"	"	.16	"	"	"
"	"	"	"	.18	"	"	"
"	"	"	"	.20	"	"	"
"	"	"	"	.14	"	"	"
"	"	"	"	"	"	"	"
"	"	"	"	"	"	"	"
"	"	"	"	"	"	"	"
"	"	"	"	"	"	"	"
"	"	"	"	"	"	"	"
"	"	"	"	"	"	.0	1.0
"	"	"	"	"	"	"	.95
"	"	"	"	"	"	"	1.0
"	"	"	"	"	"	"	.95
"	"	"	"	.13	.04	1.0	.0
"	"	"	"	.14	.02	"	"
.02	.02	"	"	"	"	"	"
.03	.02	"	"	"	"	"	"
.05	.02	"	"	"	"	"	"
.06	.02	"	"	"	"	"	"
.04	.04	"	"	"	"	"	"
"	.02	"	"	"	"	"	"

TABLE 4.2

Integrated Squared Error

(Centimeters Squared Seconds)

"Subjects" 1-60

(Simulation)

PAGES

112-117

INTEGRATED SQUARED ERROR

(CENTIMETERS SQUARED SECONDS)

SUBJECT TRIAL	1	2	3	4	5	6	7	8	9	10
1	75.1	58.0	23.4	61.6	54.3	39.6	42.7	70.2	62.2	56.0
2	16.7	26.4	28.9	26.3	34.6	36.4	31.9	26.5	31.5	32.7
3	22.6	31.5	38.7	16.0	47.4	55.9	40.6	23.5	26.1	20.6
4	19.1	26.2	17.3	23.6	23.2	27.5	21.0	16.3	28.2	21.7
5	37.2	18.4	16.1	45.0	18.8	19.6	17.7	34.4	49.1	21.9
6	37.9	17.1	28.1	23.4	16.9	17.7	16.5	18.0	21.6	16.3
7	16.4	16.6	19.2	29.5	18.3	16.1	30.4	19.5	21.2	45.0
8	16.5	16.8	15.9	18.6	16.5	24.8	17.1	23.7	18.1	16.8
9	16.7	16.5	16.2	16.0	16.3	16.7	15.9	23.3	17.5	17.6
10	16.7	18.7	30.3	15.8	15.9	18.2	16.8	18.1	17.6	17.3
11	17.4	17.0	18.6	19.5	16.8	16.7	16.9	17.3	18.0	17.6
12	16.5	17.1	15.9	16.8	16.5	16.9	16.7	31.3	16.9	17.1
13	37.2	17.4	16.5	16.8	16.7	17.0	16.5	17.1	21.4	17.9
14	16.8	17.3	23.3	15.8	16.7	16.5	16.8	16.9	17.2	17.2
15	17.0	17.2	16.1	15.9	16.7	16.6	17.7	15.9	25.4	17.7
16	17.8	17.1	16.1	17.2	19.1	16.4	16.7	16.8	17.9	17.8
17	17.0	17.1	16.7	16.7	16.6	16.5	16.9	16.0	17.5	17.5
18	16.7	16.7	22.3	17.3	17.7	19.2	24.3	15.9	16.2	22.2
19	17.8	17.7	17.0	17.0	17.0	16.5	19.6	17.4	16.8	17.5
20	16.9	16.9	16.8	16.7	19.2	16.8	17.0	16.7	17.2	18.4
21	24.3	16.8	16.6	16.7	17.4	16.5	16.7	17.0	17.3	15.9
22	19.7	19.3	19.7	17.1	17.0	16.4	18.2	16.1	17.1	40.0
23	17.3	16.8	17.1	17.8	16.7	16.8	18.8	16.6	17.4	28.2
24	16.9	17.2	31.7	17.0	16.8	16.4	17.2	16.0	17.5	24.4
25	17.0	25.2	17.3	17.3	17.4	16.7	17.2	25.2	17.6	17.8
26	16.8	16.6	16.9	17.1	18.3	18.1	16.6	15.8	17.4	17.6
27	17.5	17.4	16.9	17.0	17.2	16.5	17.0	15.8	17.4	17.7
28	16.9	17.1	17.0	16.9	16.9	17.2	17.2	17.3	17.0	17.9
29	17.2	17.7	17.9	17.2	16.9	16.9	16.7	17.1	15.8	17.1
30	17.1	17.3	17.5	17.0	16.7	16.8	17.4	15.9	17.3	16.8
31	16.8	16.9	16.8	17.4	17.0	16.5	17.0	16.7	17.8	17.5
32	17.6	16.6	16.9	16.9	17.1	16.5	16.6	17.3	17.4	16.8
33	17.5	17.1	16.5	17.1	17.4	16.5	16.8	15.9	17.3	17.6
34	16.9	17.2	17.2	21.3	17.2	16.7	16.8	15.9	17.3	17.5
35	16.9	17.5	17.5	16.8	16.6	16.5	16.8	15.8	16.9	17.7
36	17.4	17.8	16.8	17.0	31.5	16.5	16.7	17.3	17.3	31.5
37	17.0	17.0	17.3	16.7	18.0	16.5	16.5	17.1	16.8	17.7
38	17.4	17.0	16.8	16.9	16.7	16.7	17.0	16.9	20.5	16.8
39	16.9	17.6	17.8	17.5	17.1	16.6	17.2	16.5	17.6	17.7
40	17.9	16.8	16.7	18.9	16.9	16.4	16.6	17.2	16.9	17.6
41	16.7	16.8	17.1	17.3	17.0	16.4	16.6	16.6	17.3	17.9
42	17.1	16.9	17.1	16.7	17.4	16.7	16.6	16.0	17.1	17.7
43	16.8	16.8	16.9	16.9	16.9	16.5	16.9	16.8	16.7	17.6
44	17.1	16.8	18.0	17.7	17.1	28.8	16.9	17.0	17.4	17.6
45	17.0	16.7	17.4	17.2	17.1	24.4	16.6	16.4	17.1	16.6
46	16.8	16.7	17.6	16.9	17.4	16.5	17.2	16.1	17.8	17.8
47	17.0	17.3	17.5	16.8	16.6	16.6	18.0	17.2	16.2	17.7
48	17.1	23.7	17.7	16.7	17.8	16.4	17.0	17.0	17.4	17.6
49	16.9	17.5	32.5	16.8	18.5	16.5	21.0	16.9	17.3	17.7
50	16.9	17.1	17.1	16.7	16.9	16.4	17.3	17.3	16.8	17.5

INTEGRATED SQUARED ERROR

(CENTIMETERS SQUARED SECONDS)

SUBJECT	11	12	13	14	15	16	17	18	19	20
TRIAL										
1	24.8	17.7	55.2	34.0	59.2	92.4	65.7	75.3	51.8	58.9
2	35.3	39.8	17.5	20.1	28.0	31.0	23.8	36.8	45.3	55.4
3	27.8	20.2	26.2	25.8	22.7	22.9	49.1	17.1	18.5	30.2
4	18.4	23.9	18.3	28.3	16.9	34.8	21.9	17.4	17.4	21.0
5	16.6	29.4	18.0	18.9	17.1	18.5	21.8	23.3	20.7	18.9
6	32.6	20.4	15.8	16.5	18.9	18.8	16.8	21.6	26.8	18.2
7	20.3	17.7	18.0	33.2	17.3	16.0	15.7	17.9	15.9	23.9
8	15.8	18.2	15.9	18.0	15.8	23.4	20.2	22.9	25.5	16.6
9	15.9	19.9	16.4	16.3	37.5	16.9	18.5	15.9	15.9	16.4
10	16.9	23.0	17.6	25.2	17.8	16.9	20.1	15.9	16.6	17.0
11	16.1	16.6	18.3	20.3	26.2	17.1	19.6	16.6	16.4	18.1
12	15.8	28.9	49.3	17.0	33.6	30.9	15.8	19.8	16.5	17.0
13	15.9	24.8	16.7	17.2	16.6	17.0	19.6	16.6	15.9	17.5
14	16.9	17.9	16.5	16.8	16.0	17.1	16.9	16.6	16.0	17.1
15	21.6	17.5	16.6	17.7	18.4	15.8	15.9	17.1	17.7	16.6
16	15.8	17.9	16.8	17.9	25.6	16.4	17.2	17.0	37.2	16.8
17	15.7	17.0	18.7	16.6	15.8	16.3	16.3	30.1	15.8	17.0
18	15.9	16.6	16.8	18.0	16.4	15.9	15.8	16.9	16.5	28.1
19	16.0	17.4	17.0	16.8	15.9	15.8	25.2	16.6	16.7	17.9
20	16.1	16.9	17.0	17.3	16.0	17.0	15.9	17.5	16.0	16.9
21	15.9	17.6	17.1	16.8	16.2	16.9	16.0	17.3	16.7	17.4
22	15.9	17.0	17.5	16.7	15.9	23.3	31.0	16.8	16.4	18.4
23	15.9	17.1	17.3	16.8	15.8	16.0	16.0	17.2	16.1	17.3
24	17.6	17.1	17.2	16.7	15.8	16.0	15.8	17.2	15.8	16.9
25	15.8	16.8	16.8	17.7	16.1	16.3	16.1	16.8	16.1	16.9
26	15.8	17.1	33.2	18.3	16.2	17.1	16.2	16.8	21.9	17.6
27	15.8	17.1	17.0	17.0	16.3	16.2	19.4	17.1	15.7	16.8
28	16.4	17.2	16.8	17.6	16.9	16.0	20.6	18.2	16.7	17.7
29	16.7	16.8	16.8	16.9	15.9	16.6	15.8	16.7	16.6	17.5
30	15.8	17.1	16.6	16.8	15.9	17.4	16.5	17.5	15.9	17.0
31	16.7	17.0	17.3	17.6	15.8	16.8	16.8	17.2	15.8	16.8
32	22.4	16.9	16.7	16.9	16.2	19.4	16.4	17.7	16.2	17.7
33	16.5	17.2	16.8	16.8	16.7	16.9	20.9	18.4	16.2	16.9
34	16.2	17.3	17.3	17.7	16.0	17.0	16.1	16.7	15.9	17.3
35	16.0	16.7	16.8	17.1	16.8	20.6	15.9	16.9	15.9	16.8
36	16.1	16.9	17.0	16.9	16.2	17.2	15.9	17.0	15.9	17.3
37	16.5	17.0	16.8	17.0	16.6	17.1	15.9	16.8	16.3	16.9
38	15.9	17.5	16.9	16.7	15.9	18.4	17.4	17.1	16.2	17.1
39	18.1	18.7	16.9	16.6	15.9	16.8	15.8	17.1	15.9	16.6
40	16.2	16.9	17.5	17.3	16.0	17.0	15.8	17.1	16.0	16.6
41	16.0	16.6	16.9	16.9	16.2	17.1	16.3	17.0	17.4	16.8
42	15.8	17.0	17.0	17.0	16.0	17.5	16.2	16.6	16.6	16.9
43	16.2	16.9	16.6	17.3	16.0	16.9	15.8	42.6	15.9	17.1
44	15.9	17.3	16.7	17.1	16.5	17.0	15.9	17.8	16.3	16.9
45	16.5	17.6	17.3	17.1	16.7	16.7	16.1	17.1	15.9	17.4
46	15.8	17.0	17.1	16.7	16.4	17.3	15.8	17.3	16.4	17.2
47	23.7	16.9	17.1	17.5	16.5	16.7	16.6	16.7	15.9	17.7
48	15.9	16.8	16.8	17.0	16.7	16.9	16.0	16.8	15.8	17.6
49	16.5	16.8	17.4	17.4	16.7	16.8	15.9	16.7	17.8	17.1
50	15.9	17.7	16.8	16.9	16.9	16.7	15.9	17.1	15.8	16.9

INTEGRATED SQUARED ERROR

(CENTIMETERS SQUARED SECONDS)

SUBJECT / TRIAL	21	22	23	24	25	26	27	28	29	30
1	40.8	50.2	68.3	70.9	29.0	25.7	51.4	38.4	25.7	22.8
2	31.4	25.5	16.1	18.4	29.2	25.3	31.2	19.7	46.0	18.4
3	40.4	19.5	22.8	22.5	37.3	50.3	37.9	32.4	37.5	29.6
4	20.1	18.3	16.6	23.7	24.4	32.1	34.1	29.3	34.7	28.3
5	21.6	40.5	17.4	19.2	23.4	32.5	27.4	39.4	31.1	42.1
6	23.8	17.1	17.6	37.0	18.4	51.2	31.9	19.9	17.8	31.1
7	18.6	32.3	17.2	16.9	22.1	21.9	25.6	18.0	25.8	18.8
8	44.1	16.3	17.2	17.7	26.4	20.9	27.9	18.4	27.0	17.3
9	28.7	16.7	17.4	20.0	20.3	17.4	34.7	43.9	18.5	17.1
10	17.6	19.8	19.4	21.4	19.7	29.8	19.8	17.3	17.6	23.7
11	17.8	16.5	16.7	22.0	23.1	19.8	21.1	28.5	17.7	34.2
12	16.1	16.3	18.2	20.1	24.0	17.0	24.6	18.1	16.8	15.9
13	16.4	16.3	24.3	24.2	19.6	19.1	20.4	17.5	18.2	16.6
14	15.8	16.4	17.7	24.3	20.8	22.3	19.6	18.3	19.8	16.2
15	16.1	16.7	17.8	23.3	26.5	18.3	19.5	17.5	22.2	18.9
16	20.3	16.5	17.7	24.6	25.2	21.4	23.6	17.6	31.9	16.0
17	16.0	16.5	17.2	24.1	18.0	22.2	20.3	17.4	17.9	15.9
18	16.2	16.2	17.5	24.5	22.5	18.0	20.6	16.9	16.8	17.7
19	16.3	16.6	17.3	23.4	28.6	21.9	17.8	17.5	17.8	17.2
20	16.2	16.7	20.5	24.3	18.6	17.3	18.9	16.9	17.5	15.9
21	15.9	17.5	18.9	24.4	23.8	17.6	21.1	18.6	18.1	17.4
22	17.9	16.7	18.0	23.5	22.8	18.1	22.4	16.7	16.8	16.0
23	16.2	16.5	16.6	23.4	19.6	18.9	23.9	17.0	23.5	16.1
24	16.3	16.4	19.4	23.3	20.4	19.1	23.0	16.8	16.7	16.6
25	15.9	16.7	17.1	24.2	19.1	16.9	20.1	17.0	16.9	16.9
26	15.8	16.4	16.8	23.0	20.3	17.2	20.6	16.6	17.2	16.5
27	15.9	34.6	18.6	23.2	17.9	16.9	20.2	26.6	16.7	16.8
28	15.9	16.2	18.1	23.7	21.0	21.3	21.9	17.0	16.8	16.9
29	16.2	16.5	18.8	23.9	18.0	18.4	23.3	16.7	17.1	17.2
30	16.4	16.3	17.4	24.3	23.6	18.8	19.9	16.8	16.9	17.0
31	15.9	16.3	17.4	32.5	18.0	19.9	19.5	17.1	16.8	16.8
32	15.9	16.6	17.9	23.6	23.2	21.3	19.2	17.7	25.8	16.9
33	15.9	16.6	17.2	23.4	19.2	19.8	20.9	17.5	17.4	17.3
34	16.0	16.4	17.1	23.7	23.7	20.1	26.3	17.1	16.7	17.1
35	15.9	16.5	17.4	26.0	21.1	22.2	24.4	25.7	17.0	16.8
36	16.4	17.1	17.4	21.8	20.0	22.3	18.5	18.1	16.9	17.4
37	34.2	16.5	18.1	22.7	19.3	21.4	21.6	17.0	17.1	16.7
38	16.0	16.3	17.4	24.3	19.4	19.7	23.9	16.9	16.7	17.5
39	15.9	16.4	18.4	24.6	17.4	16.8	21.6	16.9	16.8	17.4
40	15.9	16.6	26.2	24.8	17.3	20.1	21.5	17.4	17.7	16.9
41	15.8	16.4	18.2	23.6	18.5	17.4	18.3	17.2	17.1	16.6
42	21.5	17.1	17.9	23.3	20.1	18.4	21.3	16.6	16.7	16.9
43	16.2	16.5	17.3	23.1	17.6	19.8	20.0	17.0	16.9	17.1
44	16.0	16.3	18.4	23.5	20.5	17.4	25.3	17.0	17.8	17.1
45	16.5	16.7	18.4	23.6	20.2	23.4	21.1	17.9	16.8	17.2
46	16.0	16.6	18.2	23.9	19.1	19.7	23.5	16.9	16.8	18.6
47	16.1	16.6	17.0	24.3	17.9	17.5	19.9	16.8	16.9	16.7
48	15.9	16.5	16.9	23.4	18.1	23.0	21.4	17.0	17.1	17.5
49	16.0	16.4	17.4	23.3	17.9	17.7	16.8	17.2	17.3	17.2
50	16.0	16.2	18.4	23.9	18.3	17.2	19.9	17.4	17.6	16.7

INTEGRATED SQUARED ERROR

(CENTIMETERS SQUARED SECONDS)

SUBJECT	31	32	33	34	35	36	37	38	39	40
TRIAL										
1	34.2	54.4	54.6	65.1	48.8	77.9	44.7	49.2	49.1	82.6
2	74.2	31.7	58.3	80.0	65.1	45.0	21.9	33.5	25.5	20.4
3	50.7	82.4	39.9	27.4	22.5	17.5	16.6	21.3	28.6	19.2
4	57.6	42.1	37.8	16.5	23.2	35.2	17.5	25.8	17.4	16.7
5	28.8	35.7	41.4	27.4	19.9	22.2	31.0	22.4	38.3	19.0
6	24.7	17.7	32.7	27.2	18.1	33.9	18.7	15.8	19.1	33.0
7	48.0	21.0	36.4	16.2	23.1	16.8	15.7	21.1	16.7	18.3
8	34.0	31.0	39.9	16.8	20.6	19.6	17.9	22.1	15.8	18.7
9	28.0	20.3	21.9	17.6	17.1	18.7	16.5	16.1	17.5	17.5
10	20.3	34.7	20.1	16.3	16.7	16.9	16.4	19.0	16.1	17.1
11	25.7	24.5	22.9	23.5	19.7	17.5	16.5	15.9	16.3	17.1
12	20.9	22.6	17.6	19.6	28.2	18.2	16.4	17.1	15.8	16.7
13	19.8	18.4	21.4	19.3	17.0	17.0	17.2	30.3	26.9	18.3
14	24.4	19.8	22.3	23.4	17.2	16.8	22.5	16.0	17.7	16.7
15	28.8	17.6	21.7	16.5	45.0	16.7	32.1	15.8	15.9	16.6
16	17.9	23.2	18.8	16.8	17.1	17.7	16.7	15.8	16.4	17.3
17	17.6	19.6	20.3	22.6	17.9	17.7	16.0	16.0	16.2	19.6
18	23.5	18.2	26.3	17.8	19.0	17.3	15.9	16.5	16.1	21.4
19	18.5	21.5	22.5	17.0	17.0	17.2	21.4	16.6	16.5	17.0
20	23.6	20.2	19.9	16.9	17.9	17.0	17.0	15.9	17.2	16.8
21	23.8	20.8	18.1	17.0	17.1	17.0	16.5	17.3	15.7	17.5
22	18.6	20.2	19.9	17.4	26.6	16.5	16.7	16.5	15.8	17.1
23	17.7	17.6	20.0	17.4	17.1	17.2	18.6	16.6	15.7	18.7
24	19.6	19.4	20.2	32.9	16.7	16.6	16.5	19.5	16.0	16.7
25	18.4	21.9	20.8	16.9	17.9	17.6	17.0	15.8	24.1	16.9
26	21.0	21.7	20.6	17.1	17.7	17.8	17.1	17.8	16.3	16.9
27	21.2	23.7	20.3	17.2	16.9	16.9	17.2	16.2	16.4	17.6
28	21.1	18.4	17.9	16.7	17.0	17.1	17.4	16.8	15.8	16.8
29	24.8	24.3	18.4	17.2	16.9	17.2	17.4	16.7	15.8	17.1
30	18.9	27.9	17.4	17.1	17.1	21.8	16.9	17.0	16.1	17.4
31	18.1	20.5	17.4	17.4	18.1	17.6	17.1	19.1	21.9	17.4
32	22.8	21.5	19.9	16.8	17.5	17.0	17.5	17.2	16.0	16.9
33	21.3	20.9	19.8	16.6	17.1	17.0	16.8	17.8	15.9	17.8
34	20.7	19.3	21.4	17.8	18.3	17.2	17.0	16.9	31.5	16.6
35	21.5	21.5	20.9	17.2	17.2	16.8	17.5	17.2	15.9	17.8
36	17.8	21.7	17.4	21.7	16.9	16.9	17.0	17.5	16.9	17.7
37	21.3	20.6	19.3	17.4	16.9	17.2	17.1	16.9	15.7	17.4
38	24.5	18.2	17.4	25.2	24.2	17.1	16.8	17.2	15.9	16.8
39	17.0	23.5	23.6	17.0	16.7	17.5	17.0	17.5	16.1	16.6
40	18.9	17.8	18.6	16.8	17.0	16.8	16.5	17.1	15.9	26.0
41	23.1	18.1	18.7	16.8	18.4	16.8	17.0	17.5	18.5	17.6
42	27.3	21.9	19.1	16.7	19.5	17.7	17.4	17.7	16.2	16.8
43	19.4	20.0	19.6	16.7	16.6	17.2	18.0	16.8	16.2	17.4
44	28.4	18.6	19.1	17.3	17.3	16.6	16.8	17.2	16.4	16.7
45	17.6	19.7	20.3	22.4	16.7	16.9	16.8	17.0	16.2	16.8
46	26.3	27.4	21.3	17.2	17.0	18.3	17.7	32.9	15.9	17.8
47	18.0	22.1	19.1	17.0	17.8	17.1	17.5	17.8	16.4	16.7
48	16.8	20.1	17.7	17.5	17.1	23.4	16.7	17.3	15.9	17.0
49	20.0	21.2	20.2	19.0	17.0	17.1	17.0	17.0	16.5	17.6
50	21.7	18.8	20.6	16.8	16.8	16.9	17.5	17.5	15.7	17.9

INTEGRATED SQUARED ERROR

(CENTIMETERS SQUARED SECONDS)

SUBJECT TRIAL	41	42	43	44	45	46	47	48	49	50
1	31.5	39.1	28.1	64.1	38.0	55.9	65.9	65.3	70.0	67.5
2	37.5	28.6	16.7	39.0	28.6	16.1	29.9	21.6	25.9	42.9
3	19.3	38.5	15.9	22.2	18.3	48.2	18.6	43.1	35.8	25.0
4	20.1	21.7	38.5	24.4	16.9	19.1	37.8	16.3	18.7	17.0
5	19.2	26.9	17.7	21.3	17.0	27.7	19.6	16.1	17.8	32.3
6	28.9	24.9	17.3	25.7	20.3	16.0	18.5	31.4	38.4	25.9
7	16.9	18.5	16.5	29.2	17.3	17.3	19.6	23.1	19.7	19.7
8	15.9	17.9	17.5	16.9	25.2	16.1	15.9	15.9	16.5	16.5
9	36.9	16.6	16.6	18.3	19.2	17.2	17.0	15.8	16.3	17.1
10	16.5	15.9	16.9	18.8	17.5	15.8	20.4	16.8	16.8	17.0
11	15.8	19.8	23.1	19.2	17.8	17.2	15.7	16.1	16.7	18.2
12	17.3	22.1	17.0	18.5	18.5	17.1	17.2	16.7	17.4	17.5
13	16.3	16.7	17.3	17.4	16.8	16.0	16.5	16.0	43.1	16.7
14	17.9	16.2	16.7	16.7	18.4	15.9	15.7	18.4	16.5	17.3
15	23.3	16.4	16.9	18.6	17.4	16.4	16.0	16.5	17.0	19.6
16	16.7	15.9	17.8	17.4	16.8	17.0	16.1	16.6	17.5	16.8
17	16.4	15.9	19.5	17.3	17.2	15.8	15.9	16.7	16.8	16.8
18	16.6	18.7	17.5	16.9	17.9	19.8	15.7	16.6	17.4	31.6
19	17.5	16.0	16.8	17.7	16.6	34.6	16.1	24.4	17.0	18.6
20	17.3	15.9	17.1	17.7	16.8	16.0	16.1	17.0	18.6	16.7
21	17.3	17.5	17.7	17.0	17.1	16.5	15.9	19.3	17.4	16.8
22	17.1	17.0	17.1	16.8	16.6	16.0	27.9	17.6	17.7	16.8
23	16.7	17.3	24.2	34.4	18.4	17.6	16.0	16.7	17.6	17.0
24	21.5	16.8	16.8	17.3	16.9	16.4	16.1	16.7	31.8	17.0
25	17.0	17.8	22.2	18.6	17.3	23.4	18.8	16.9	17.1	16.8
26	16.8	17.1	17.7	17.7	25.6	18.1	16.8	16.7	16.9	17.0
27	19.4	18.7	16.9	16.9	17.0	15.9	16.6	17.5	17.3	17.4
28	17.2	17.3	17.1	17.8	17.4	17.7	16.7	17.3	17.0	16.7
29	17.0	16.9	16.6	16.9	17.6	16.6	15.8	17.3	16.6	24.7
30	17.0	16.9	16.9	17.1	17.3	15.9	16.4	17.0	17.2	18.3
31	17.2	16.8	17.6	17.0	16.8	16.5	21.4	17.1	17.4	17.1
32	16.8	16.8	17.9	16.9	16.7	15.9	15.8	17.4	17.1	17.5
33	17.2	17.5	17.3	17.3	16.8	16.1	16.1	17.2	17.1	17.0
34	17.1	16.9	17.3	16.6	16.8	15.9	15.8	30.4	23.7	16.7
35	16.7	17.2	16.9	17.0	17.4	15.8	15.7	17.6	17.4	17.6
36	17.8	16.9	17.6	17.2	17.7	15.7	16.5	17.0	16.7	17.2
37	17.0	17.5	16.8	17.0	17.1	16.2	16.7	16.9	16.8	17.1
38	17.1	16.7	17.1	16.9	16.9	16.4	15.9	19.7	17.5	17.0
39	17.8	16.7	16.8	16.7	17.3	16.9	16.4	16.7	17.1	16.6
40	16.6	24.6	37.6	16.7	16.6	16.1	15.8	16.7	16.8	17.7
41	16.8	16.9	17.4	17.0	17.3	15.9	17.0	17.6	17.2	17.0
42	17.8	17.4	16.8	16.7	20.3	15.9	15.8	17.5	17.6	17.1
43	17.3	16.7	16.8	16.8	17.2	15.8	16.5	17.0	17.0	17.8
44	16.9	16.8	16.8	17.0	16.8	15.7	16.0	17.5	18.4	16.8
45	16.8	17.3	18.5	16.8	17.5	16.4	15.9	16.7	17.6	17.6
46	17.3	16.8	17.2	16.8	17.7	16.1	15.8	17.3	16.9	16.7
47	16.8	16.9	16.9	16.7	17.4	16.0	16.3	16.9	16.8	17.5
48	17.3	17.5	16.8	17.3	17.0	16.2	15.8	17.2	16.8	16.9
49	16.7	17.1	17.3	18.6	17.4	16.5	16.1	17.4	17.1	17.4
50	17.0	16.9	17.4	17.4	18.1	15.9	15.8	16.9	17.6	17.1

INTEGRATED SQUARED ERROR

(CENTIMETERS SQUARED SECONDS)

SUBJECT TRIAL	51	52	53	54	55	56	57	58	59	60
1	30.0	37.8	30.3	43.2	33.7	53.2	65.3	72.8	76.3	63.9
2	33.6	34.8	31.8	35.4	33.2	48.4	32.2	17.2	17.8	39.5
3	18.2	26.3	30.7	27.2	40.5	32.6	17.2	35.9	28.3	25.1
4	38.4	22.5	30.8	19.8	23.5	22.1	21.7	44.6	17.4	16.8
5	20.3	20.9	31.8	18.5	26.9	22.1	29.7	24.0	18.3	19.4
6	21.3	27.4	29.5	16.3	30.0	21.4	16.5	17.3	28.3	17.5
7	32.1	28.8	32.2	24.4	18.1	19.0	20.7	24.7	19.0	17.5
8	33.3	26.2	19.8	20.1	18.2	26.9	19.7	18.7	17.6	17.0
9	38.0	25.9	26.9	20.4	18.0	20.5	33.3	17.3	20.5	20.2
10	34.1	20.6	19.2	19.3	17.9	17.0	18.2	17.1	16.7	16.4
11	36.4	19.5	18.4	17.3	18.1	16.2	17.2	17.1	24.5	27.3
12	40.6	20.2	19.0	19.1	18.5	16.3	37.2	18.5	15.9	18.3
13	17.0	24.0	19.2	16.6	20.7	16.3	17.4	16.1	16.1	16.6
14	17.8	28.7	22.4	16.3	18.4	39.8	16.6	18.4	17.6	16.5
15	16.3	18.7	22.7	20.3	19.0	31.5	16.6	17.9	16.0	19.9
16	18.3	28.7	19.9	22.2	21.2	23.3	17.4	19.0	18.3	16.6
17	16.9	19.8	28.0	19.1	18.2	16.5	17.8	18.6	15.9	16.8
18	16.3	30.5	27.2	20.2	18.4	16.3	17.3	21.0	15.9	16.8
19	18.2	18.8	19.1	18.2	18.6	16.4	16.4	27.7	15.9	27.3
20	16.2	19.1	19.3	17.5	17.9	16.2	16.2	18.0	16.3	21.9
21	18.1	20.1	23.2	19.8	17.3	16.4	16.2	16.4	16.0	18.6
22	17.8	19.5	20.6	17.3	18.3	16.3	16.0	17.4	16.3	18.2
23	18.5	19.2	25.3	19.4	17.6	16.4	16.1	16.4	17.4	17.0
24	16.3	18.6	41.9	19.0	41.1	16.3	16.1	16.2	15.9	17.1
25	17.6	20.7	21.2	21.1	19.1	16.4	16.3	16.0	15.9	17.3
26	16.8	27.7	17.9	16.8	17.2	16.7	16.0	16.0	16.4	17.5
27	16.2	20.5	19.0	18.0	19.2	16.2	16.1	16.0	15.9	16.5
28	18.1	20.2	19.3	18.3	18.0	18.1	16.2	16.6	15.9	17.3
29	16.1	18.8	22.6	18.2	18.2	33.0	15.9	22.0	15.9	17.1
30	16.2	20.1	19.2	17.0	19.2	16.2	16.0	16.0	16.1	17.0
31	39.3	19.2	20.0	17.6	22.8	16.4	18.3	16.3	17.1	16.4
32	16.2	21.6	18.6	18.6	17.1	16.6	16.3	17.7	15.8	15.9
33	16.8	26.7	16.9	17.3	18.1	16.2	16.1	16.2	15.8	16.1
34	16.3	21.5	16.8	19.3	18.6	16.2	15.9	16.3	15.9	16.1
35	16.5	24.9	17.5	17.7	18.6	16.2	16.2	16.1	15.9	16.0
36	16.5	18.9	20.7	16.5	20.4	16.5	16.1	16.4	15.9	16.0
37	16.7	19.5	18.1	16.5	19.1	16.2	16.0	16.2	16.0	15.9
38	16.5	22.4	17.9	17.0	20.5	16.9	22.4	16.2	15.9	16.0
39	16.2	24.0	17.0	16.3	20.2	17.8	15.9	16.3	16.2	16.0
40	16.6	20.2	17.6	19.7	19.3	16.6	16.0	17.5	16.0	16.1
41	16.3	20.6	18.4	18.2	17.3	16.2	16.0	25.2	15.9	16.1
42	24.8	18.5	19.0	18.0	18.3	16.3	16.0	16.4	15.9	15.9
43	16.7	26.5	23.1	16.3	19.9	16.1	16.1	16.2	16.0	16.0
44	16.3	25.0	17.3	16.2	39.9	16.2	16.0	16.5	15.9	35.7
45	17.7	19.4	17.0	17.8	18.6	16.2	16.0	16.5	16.0	16.0
46	17.2	21.3	17.3	16.5	19.9	16.3	16.0	16.3	15.9	16.2
47	16.3	25.6	16.5	16.2	18.3	16.2	16.3	16.2	15.9	16.1
48	17.0	20.6	16.3	17.5	17.3	16.2	16.0	16.2	15.8	16.1
49	16.4	21.8	16.3	16.0	17.5	16.3	16.1	16.2	18.1	15.9
50	16.5	19.2	16.2	16.3	18.0	16.3	19.9	16.3	16.0	20.0

TABLE 4.3

Z - VALUES

FOR MANN - WHITNEY U TEST

ON INTER-RESPONSE TIMES

RESPONSE TRIAL	1	2	3	4
1	1.74	0.77	2.17	0.96
2	0.63	1.11	2.16	0.07
3	1.92	0.14	1.25	0.74
4	0.95	0.52	1.70	0.40
5	1.48	1.33	0.89	0.43
6	1.86	0.50	3.34	3.06
7	0.73	1.07	3.17	2.48
8	1.11	0.89	2.34	1.88
9	2.12	0.12	2.40	2.91
10	0.40	0.85	2.87	2.80
25	0.71	0.93	3.14	3.22
50	0.75	1.62	4.10	2.50

TABLE 4.4

Z - VALUES

FOR MANN - WHITNEY U TEST

ON INTEGRATED SQUARED ERRORS

TRIAL	Z
1	.903
2	.376
3	.827
4	.526
5	.300
6	.376
7	.450
8	.266
9	2.630
10	1.352
25	1.276
50	1.051

CHAPTER 5

CONCLUSION

5.1 Discussion of Results

In the preceding chapter, we made a statistical comparison of a sample of human operator behavioral simulations obtained from computer program executions and a sample of human operator behavioral data obtained from a psychomotor experiment. Through this comparison we have sought to determine whether or not the samples came from the same parent population, i.e., are they statistical images of one another. The results of the comparison show that, with the exceptions noted and justifiably excused, there is no statistical reason for rejecting the hypothesis of identical parent population distributions. Although this is a favorable outcome and offers us a quantitative basis for having confidence in the proposed theory, we hesitate to conclude that this result, by itself, is sufficient evidence upon which to argue for the credibility of the theory. We hesitate because of the inherent limitations of any statistical test, namely, the possibility that a false hypothesis can be accepted and the possibility that other theories can pass the same test. However, if this result is weighed together with the results of the parametric study in section 4.2 and the experimental findings referenced in support of assumptions made in chapter two, the case for credibility is strengthened considerably. Therefore, supported by this collective evidence, we conclude that the theory provides a credible explanation of human learning behavior in the type of manual control task considered.

5.2 Summary

We have, in chapter two, developed a theory for the explanation of human learning behavior in a manual control task. In explaining how the human operator acquires a motor skill, we have endeavored to account for the inter-subject, intra-subject varability which is observable in psychomotor experiments. This variability has been attributed to the stochastic nature of human information processing, which we have assumed to be a sequential operation involving three subsystems: the sensor, the decision center and the effector. Each of these components has been treated as a probabilistic system, and stochastic descriptions of how they function have been provided. Our interpretation of Bayesian statistics for the characterization of the decision center's decision making has been, perhaps, our most important contribution to the understanding and conceptualization of human learning behavior. From the theory we have derived a model of human learning behavior in a manual control task. This has been accomplished by a straightforward translation of the theory into the machine language of a digital computer. A set of read-in parameters, corresponding to human psycho-physiological characteristics, gives the model an individuality. Consequently, we have been able to execute a number of computer programs, which, on the basis of a hypothesis test, have been shown to be a statistical image of an ensemble of human operators. The number of parameters required to establish the identity of the model is relatively small considering the complexity of the process being simulated and the detailed similarity it offers.

5.3 Generalizations

We now explore the possibility of generalizing the approach of the theory developed herein, for the purpose of explaining human learning behavior in other manual control task contexts.

Continuous Controller - The first extension we wish to consider is to tasks where the controller output can be varied continuously over a bounded range by the operator, but where there is otherwise no difference from the task we have already treated. In the task we have treated, we assumed that in the decision center there are stored probabilities for the MxN hypotheses,

$H_i(x_m)$: the switch curve passes thru the mesh, (x_m, v_i)

Let us reword these hypotheses so that they read,

$H_i(x_m)$: the controller output, u, in the mesh, (x_m, v_i), equals u_0,

where u_0 may either be +U or -U. Written in this form, it is clear that the control alternatives are $\pm$ U and that the probability distribution for $H_i(x_m)$ is discrete. To transition to a continuous controller we write,

$H_i(x_m)$: the controller output, u, in the mesh, (x_m, v_i), equals or is less than u_0,

where u_0 is now a continuous variable defined on the interval (-U, +U), and the distribution of u is also continuous. With this definition one can trace through the steps of the derivation in section 2.5 and see that basically the only change necessary in the development is to replace summation signs by integrals and discrete distributions by continuous ones, where appropriate.

Pursuit Tasks - In the state regulator problem we have considered, the terminal state to which the dynamic process is being forced, is fixed. In a pursuit task, the terminal state may change with time, and so, control decisions must be based on an estimate of the anticipated terminal state at the expected time of convergence. In other words, the decision center must make predictions of the future course of events. Therefore, in the information processing sequence, we must insert a prediction operation. In addition, the center's memory must store not only the probabilities of response alternatives for reaching the null state, but also the probabilities for reaching all other meshes in state space which are possible locations of the terminus.

Other Tasks - Extensions of the theory to other task contexts, including compensatory tracking problems and controlling dynamic processes not in the class to which we have restricted the present development, are also conceivable. However, in such task contexts it is doubtful that our interpretation of the evidence, E, is still applicable. Since we have not, as yet, studied these situations in any detail, we will not speclate on how the theory may be generalized to handle them.

5.4 Applications

Adaptive Control - One application of this work, which we would like to discuss, is in the field of adaptive control systems. If the computer program presented in chapter two is examined carefully, it can be seen that without the input-output statements and the superfluous subroutines for keeping score, simulating the dynamic process, etc., the logic of the program requires relatively few FORTRAN statements. In fact, if the priors are set to zero in the first and third quadrants, thereby eliminating the need for weighting case two and three evidence, if p_0 is set to one, if the sensor function is deleted, and if a few other nonessentials are removed, the program reduces to a very few statements. In such a form, it does not appear that there would be any great difficulty in constructing a special purpose digital computer to execute the control logic. If provision is made not to allow the probabilities to go to zero, the program will learn, un-learn, and re-learn control policies. A modified version of our model of human learning behavior we believe, therefore, has the potential to perform as the logic element of an adaptive control system.

Psychomotor Testing - A second application, for which the theory holds promise, is in the selection of pilot trainees. The introduction in World War II of psychomotor testing, by the military to select flight crews, marked the beginning of a continuing search for improved techniques to determine the aircrew polential of individual applicants. In the development of our theory, we have identified explicitly the determinants of human behavior in a manual control task. These included the selection rule, revision rule, prior probabilities, decision and response times, etc. Reflected in these determinants of behavior are the operator's past experience in manual control tasks, the efficiency of his information processing, his physiological limitations and the like. While it is true that these qualities alone are not sufficient to judge the aircrew potential of an individual, they are, nevertheless, important aptitude indicators. It may be possible, therefore, to devise a method, based on the theory, for statistically inferring the characteristics of an individual's information processing system from his performance in a single manual control task. We have, in fact, already done something similar to this in determining the model parameters for generating our sample of behavioral simulations.

5.5 A Final Comment

In the revision making process, as we have described it, evidence is weighted in order to revise the priors, $p(H_i)$, for all i. That is to say, in any given decision cycle probabilities in only one column of the grid may be revised. If the sum in equation 2.1 had been taken on m instead of on i, only one row at a time would have been revised. From a computer program written with all summations taken on m, we have determined that there is little difference between row and column revisions. However, if the evidence is weighted in order to revise the priors in two or more columns (or rows), or if any one of a number of alternative revision schemes is used, it is possible that the program's learning behavior will differ appreciably from its behavior in the present formulation. For example, if the evidence is used to revise all the priors in the grid simultaneously, we would expect the program to learn faster than it does now. Applying other schemes for effecting revisions is certainly one area where further investigation is recommended.

REFERENCES

1. Adams, J.A., "Human Tracking Behavior," Psychological Bulletin, Vol. 58, No. 1, 1961, pp. 55-79

2. Adams, J.A., "Motor Skills," Annual Review of Psychology Vol. 15, 2964, pp. 181-202

3. Adams, J.A. and Chambers, R.W., "Response to Simultaneous Stimulation of Two Sense Modalities," Journal of Experimental Psychology, Vol. 62, No. 2, 1962, pp. 198-206

4. Adams, J.A. and Creamer, L.R., "Data Processing Capabilities of the Human Operator," Journal of Engineering Psychology, Vol. 1, 1962, pp. 150-158

5. Ammons, R.B., "Knowledge of Performance," WADC TR 54-14, February 1954

6. Ammons, R.B. and Ammons, C.H., "Movement Analysis of the Performance of Simple Perceptual-Motor Task Under Various Conditions," WADC Technical Report 54-36, April 1954

7. Atkinson, R.C., <u>Studies in Mathematical Psychology</u>, Stanford University Press, 1964

8. Atkinson, R.C., Bower, G.H., Crothers, E.J., <u>An Introduction to Mathematical Learning Theory</u>, John Wiley and Sons, Inc., 1965

9. Bahrick, H.P., Fitts, P.M. and Briggs, G.E., "Learning Curves, Facts or Artifacts?" Psychological Bulletin, Vol. 54, No. 3, 1957, pp. 256-268

10. Beach, L.R., "Accuracy and Consistency in the Revision of Subjective Probabilities," Human Factors in Electronics, Vol. HFE-7, No. 1, March 1966

11. Bekey, G.A., "An Investigation of Sampled Data Models of the Human Operator in a Control System," ASD-TDR-62-36, May 1962

12. Bekey, G.A., Meissinger, H.F. and Rose, R.E., "A Study of Model Matching Techniques for the Determination of Parameters in Human Pilot Models," NASA CR-143, Jan. 1965

REFERENCES cont.

13. Bellman, R., Adaptive Control Processes: A Guided Tour, Princeton University Press, 1961

14. Bendat, V.S., "Interpretation and Application of Statistical Analysis for Random Physical Phenomena," IRE Transactions on Bio-Medical Engineering, Vol. BME-9, No. 1, January 1962, pp. 31-43

15. Bendat, V.S. "Mathematical Analysis of Average Response Values for Nonstationary Data," IEEE Transactions on Bio-Medical Engineering, Vol. BME-11, No. 3, July 1964, pp. 72-81

16. Bennett, E., Degan, J. and Spiegel, J., Human Factors in Technology, McGraw-Hill Book Co., Inc., 1963

17. Bertone, C.M., "A Bibliography of Russian Scientific and Technological Literature in Manual Control and Associated Areas," NASA CR-199, April 1965

18. Bilodeau, E.A., and Bilodeau, I.McD., "Motor Skills Learning," Annual Review of Psychology, Vol. 12, 1961, pp. 243-280

19. Brown, R.H., "Visual Sensitivity to Differences in Velocity," Psychological Bulletin, Vol. 58, No. 2, March 1961, pp. 89-103

20. Burke, C.V., Narasimban, R. and Benepe, O.V., "Some Problems in the Spectral Analysis of Human Behavior Records," WADC Technical Report 53-27, July 1953

21. Bush, R.R. and Mosteller, F., Stochastic Models For Learning, John Wiley and Sons, Inc., 1955

22. Cahill, H.E. and Hovland, C.I., "The Role of Memory in the Acquisition of Concepts," Journal of Experimental Psychology, Vol. 59, No. 3, March 1960, pp. 137-144

23. Cramer, H., Mathematical Methods of Statistics, Princeton University Press, 1946

24. Davenport, W.B., Jr. and Root, W.L., Random Signals and Noise, McGraw-Hill Book Co., Inc., 1958

26. Deupree, R.H. and Simon, J.R., "Reaction Time and Movement as a Function of Age, Stimulus, Duration and Task Difficulty," Ergonomics, Vol. 6, 1963, pp. 403-414

27. Elkind, J.I., "Further Studies of Multiple Regression Analysis of Human Pilot Dynamic Responses: A Comparison of Analysis Techniques and Evaluation of Time-Varying Measurements," ASD-TDR-63-618, March 1964

28. Elkind, J.I. and Darley, D. L., "The Statistical Properties of Signals and Measurements of Simple Manual Control Systems," ASD-TDR-63-85, July 1963

29. Elkind, J.I. and Green, D.M., "Measurement of Time-Varying and Nonlinear Dynamic Characterists of Human Pilots," ASD-TR-61-225, December 1961

30. Elkind, J.I., Starr, E.A., Green, D.M. and Darley, D.L., "Evaluation of a Technique for Determining Time-Variant Dynamic Characteristics of Human Pilots," NASA TN D-1897, May 1963

31. Ely, J.H., Bowen, H.M. and Orlansky, J., "Man-Machine Dynamics," Chapter VII of the Joint Services Human Engineering Guide to Equipment Design, WADC TR 57-582, November 1957

32. Ely, J.H., Thomson, R.M. and Orlansky, J., "Design of Controls," WADC Technical Report 56-172, November 1956

33. Eriksen, C.W. and Hake, H.W., "Absolute Judgements As a Function of Stimulus and Response Categories," WADC Technical Report 54-162

34. Farnsworth, P.R., McNemar, O. and McNemar, Q., "Annual Review of Psychology," Vol. 15, 1964, Annual Reviews, Inc., pp. 181-202

35. Feller, W., An Introduction to Probability Theory and its Applications, John Wiley and Sons, Inc., 1950

36. Fitts, P.M. and Peterson, J.R., "Information Capacity of Discrete Motor Responses," Journal of Experimental Psychology, Vol. 67, No. 2, February 1964, pp. 101-112

REFERENCES cont.

37. Flaherty, B.E., Psychophysiological Aspects of Space Flight, Columbia University Press, 1961

38. Fu, K.S. and Gould, E.E., "An Adaptive Pattern Recognizing Model of the Human Operator Engaged in a Time Varying Control Task," TR-EE66-8, Purdue University June 1966

39. Gersho, A., "Characterization of Time-Varying Linear Systems," Proc. of the IEEE, Vol. 51, No. 1 January 1963, p. 238

40. Gnedenko. B.V., The Theory of Probability, Chelsea Publishing Company, New York, N.Y. 1962

41. Good, I.J., The Estimation of Probabilities, Research Monograph No. 30, The M.I.T. Press, 1965

42. Graham, D. and McRuer, D., Analysis of NonLinear Control Systems, John Wiley and Sons, Inc., 1961

43. Green, B.F., Jr., Digital Computers in Research, McGraw-Hill Book Co., Inc., 1963

44. Guenther, W.C., Analysis of Variance, Prentice-Hall, Inc., 1964

45. Hagen, K.E., "The Dynamic Behavior or Parameter Adaptive Control Systems," Y368, Instrumentation Laboratory M.I.T., June 1964

46. Hall, I.A.M., "Effects of Controlled Element on the Human Pilot," WADC Technical Report 57-509, August 1958

47. Hays, W.L., Statistics for Psychologists, Holt, Rinehart and Winston, 1963

48. Held, R., "Plasticity in Sensory-Motor Systems," Scientific American, Vol. 213, No. 5, November 1965

49. Hohle, R.H., "Inferred Components of Reaction Times as Functions Foreperiod Duration," Journal of Experimental Psychology, Vol. 69, No. 4, 1965, pp. 382-386

50. Horowitz, I.M., "Plant Adaptive Systems vs Ordinary Feedback Systems," IRE Transactions on Automatic Control, Vol. AC-7, No. 1, January 1962, pp. 48-56

51. Howard, R.A., "Dynamic Inference, "Technical Report No. 10, Operations Research Center, M.I.T., December 1964

52. Hsu, J.C. and Meserve, W.E., "Decision-Making in Adaptive Control Systems," IRE Transactions on Automatic Control, Vol. AC-7, No. 1, January 1962, pp. 24-32

53. Kalman, R.E. and Bucy, R.S., "New Results in Linear Filtering and Prediction Theory," Journal of Basic Engineering, March 1961, pp. 95-108

54. Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems," Journal of Basic Engineering, March 1960, pp. 35-45

55. Kopp, R.E. and Orford, R.J., "Linear Regression Applied to System Identification for Adaptive Control Systems," AIAA Journal, Vol. 1, No. 10, October 1963

56. Karlin, L. and Mortimer, R.G., "Effect of Verbal, Visual and Auditory Augmenting Cues on Learning a Complex Motor Skill," Journal of Experimental Psychology, Vol. 65, 1963, pp. 75-79

57. Kendall, M.G. and Stuart, A., The Advanced Theory of Statistics, Vol. 1 and Vol. 2, Hafner Publishing Company, 1963

58. Kreifeldt, J.G., "A Sampled-Data Pursuit Tracking Model," IEEE Transactions on Human Factors in Electronics, Vol. HFE-6, No. 1, September 1965, pp. 65-73

59. Krendel, E.S., "A Preliminary Study of the Power-Spectrum Approach to the Analysis of Perceptual-Motor Performance," AF Technical Report No. 6723, October 1951

60. Krendel, E.S., "Spectral Density Study of Tracking Performance," WADC Technical Report 52-11, January 1952

61. Laning, J.H., Jr. and Battin, R.H., Random Processes in Automatic Control, McGraw-Hill Book Company, Inc. 1956

REFERENCES cont.

62. Lee, R.C.K., Optimal Estimation, Identification, and Control, The M.I.T. Press, Cambridge, 1964

63. Luce, R.D., Bush, R.R. and Galanter, E., Handbook of Mathematical Psychology, Vol. 1, John Wiley and Sons, Inc., 1963

64. Luce, R.D., Bush, R.R. and Galanter, E., Handbook of Mathematical Psychology, Vol. II, John Wiley and Sons, Inc., 1963

65. Luce, R. D., Bush, R.R. and Galanter, E., Handbook of Mathematical Psychology, Vol. III, John Wiley and Sons, Inc., 1965

66. MacDonald, J.S., "Experimental Studies of Handwriting Signals," Technical Report 443, Research Laboratory for Electronics, M.I.T., March 31, 1966

67. Mann, H.B., Whitney, D.R., "On a Test of Whether One of Two Random Variables Is Stochastically Larger Than The Other," Annals of Mathematical Statistics, Vol. 18, 1947, pp. 50-60

68. Margolis, M. and Leondes, C.T., "On the Philosophy of Adaptive Control For Progress Adaptive Systems," AFOSRTN 59-1199, January 1960

69. McRuer, D., Graham, D., Krendel, E. and Reisener, W., Jr., "Human Pilot Dynamics in Compensatory Systems," AFFDL-TR-65-15, May 1965

70. McRuer, D.T. and Krendel, E.S., "The Man-Machine System Concept," Proceedings of the IRE, Vol.50, No. 5, May 1962, pp. 1117-1123

71. Mermelstein, P. and Eden, M., "Experiments on Computer Recognition of Connected Handwritten Words," Information and Control, Vol. 7, 1964, pp. 250-270

72. Merriam, C.W., III, Optimization Theory and the Design of Feedback Control Systems, McGraw-Hill Book Co., Inc.,

73. Miller, D.C., "A Model for the Adaptive Response of the Human Controller to Sudden Changes in Controlled

Process Dynamics," BS/MS Thesis, M.I.T., June 1965

74. Miller, D.C., "The Effects of Performance-Scoring Criteria on Compensatory Tracking Behavior," IEEE Transactions on Human Factors in Electronics, Vol. HFE-6, No. 1, September 1965, pp. 62-65

76. Minsky, M.L., "Some Methods of Artificial Intelligence and Heuristic Programming," Mechanisation of Thought Processes, Vol. I, Her Majesty's Stationery Office, 1959, pp. 3-36

77. Mishkin, E., and Braun, L., Jr., Adaptive Control Systems, McGraw-Hill Book Co., Inc. 1961

78. Newell, A., Shaw, J.C. and Simon, H.A., "Elements of a Theory of Human Problem Solving," Psychological Review, Vol. 65, No. 3, pp. 151-166, 1958

79. North, J.D., Lomnicki, Z.A. and Zaremba, S.K., "The Design and Interpretation of Human Control Experiments," Ergonomics, Vol. I, No., 4, August 1958, pp. 314-327

80. North, J.D. and Lomnicki, Z.A., "Further Experiments on Human Operators in Compensatory Tracking Tasks," Ergonomics, Vol. 4, No. 4, October 1961, pp. 339-353

81. North, J.D., "Manual Control As a Stochastic Process," Ergonomics, Vol. VI, No. 3, July 1963, pp. 247-254

82. Osburn, P.V., "Investigation of a Method of Adaptive Control," Sc.D Thesis, M.I.T., 1961

83. Parzen, E., "An Approach to Time Series Analysis," The Annals of Methematical Statistics, Vol. 32, 1961, pp. 951-989

84. Pew R.W., "Temporal Organization in Skilled Performance," The University of Michigan Technical Report, 02814-11-T, May 1963

85. Pew, R.W., "Performance of Human Operators in a Three-State Relay Control System With and Without Explicit Velocity Information," Sixth Annual Symposium of the

Professional Group on Human Factors in Electronics, IEEE, May 1965

86. Phillips, L.D., Hays, W.L. and Edwards, W., Conservatism in Complex Probabilistic Inference," Human Factors in Electronics, Vol. HFE-7, No. 1, March 1966

87. Platzer, H.L., "The Phase-Plane as a Tool for the Study of Human Behavior in Tracking Problems," WADC Technical Report 55-444, November 1955

88. Polya, G., How to Solve It, Princeton University Press, 1945

89. Poulton, E.C., "On the Stimulus and Response in Pursuit Tracking," Journal of Experimental Psychology, Vol. 53, No. 3, 1957, pp. 189-194

90. Raiffa, H. and Schlaifer, R., Applied Statistical Decision Theory, Harvard University Press, 1961

91. Ragazzini, J.R., and Franklin, G.F., Sampled-Data Control Systems, McGraw-Hill Book Co., Inc. 1958

92. Ragazzini, J.R. and Chang, S.S.L., "Noise and Random Processes," Proceedings of the IRE, Vol. 50, No. 5, May 1962, pp. 1146-1150

93. Restle, F., Psychology of Judgement and Choice; A Theoretical Essay, John Wiley and Sons, Inc., 1961, pp. 168-172

94. Reynolds, B., "The Effect of Learning on the Predictability of Psychomotor Performance," Journal of Experimental Psychology, Vol. 44, 1952, pp. 189-198

95. Schwartz, M., Information Transmission, Modulation, and Noise, McGraw-Hill Book Co., Inc., 1959

96. Senders, J.W., "The Human Operator as a Monitor and Controller of Multidegree of Freedom Systems," IEEE Transactions on Human Factors In Electronics, Vol. HFE-5, No. 2, September 1964, pp. 2-5

97. Smith, O.J.M., "Nonlinear Computations in the Human Controller," IRE Transactions on Bio-Medical Electronics,

Vol. BME-9, No. 2, April 1962, pp. 125-128

98. Snyder, D.L., "A Nonlinear Study of Compensatory Manual Control Systems," IEEE Transactions on Human Factors in Electronics, Vol, HFE-5, No. 1, September 1964, pp. 25-28

99. Staats, A.W., Human Learning, Holt, Rinehart and Winston, Inc., 1964

100. Stevenson, S.A., "A Bibliography of Aerospace Medical Division Reports in the Fields of Engineering Psychology and Training Psychology 1945-1960," Behavioral Sciences Laboratory Report, WADC, May 1960

101. Swets, J.A., Signal Detection and Recognition by Human Observers, John Wiley and Sons, Inc., 1964

102. Teichner, W.H., "Recent Studies of Simple Reaction Time," Psychology Bulletin, Vol. 51, 1954, pp. 128-149

103. Tou, J.T., Digital and Sampled-Data Control Systems, McGraw-Hill Book Co., Inc., 1959

104. Tou, J.T., Modern Control Theory, McGraw-Hill Book Co., Inc., 1964

105. Truxal, J.G., "Review of Control Developments," Proceedings of the IRE, Vol. 50, No. 5, May 1962, pp. 781-786

106. Vines, M.A., "The Intermittency of Control Movements and the Psychological Refractory Period," British Journal of Psychology, Vol. 38, 1948, pp. 149-157

107. Wald, A. and Wolfowitz, J., "On a Test Whether Two Samples Are From the Same Population," Annuals of Mathematical Statistics, Vol. II, 1940, pp. 147-162

108. Welford, A.T., "The Psychological Refractory Period and the Timing of High-Speed Performance - A Review and A Theory," British Journal of Psychology, Vol. 43, 1952, pp. 2-19

109. Welford, A.T., "Timing and Organization of Manual Skill-

ed Performance," Acta Psychology, Vol. II, 1955, pp. 185-186

110. Welford, A.F., "The Measurement of Sensory-Motor Performances: Survey and Reappraisal of Twelve Years' Progress," Ergonomics, Vol. 4, 1960, pp. 189-230

111. Wiener, N., Cybernetics, The M.I.T. Press, 1948

112. Wiener, N., Estrapolation, Interpolation, and Smoothing of Stationary Time Series, The M.I.T. Press, 1949

113. Wiener, N., Nonlinear Problems in Random Theory, The M.I.T. Press, 1958

114. Wierwille, W.W. and Gagne, G.A., "A Theory for the Optimal Deterministic Characterization of the Time-Varying Dynamics of the Human Operator," NASA CR-170, February 1965

115. Wilcoxon, F., "Individual Comparisons By Ranking Methods," Biometrics Bulletin, Vol. I, 1945, pp. 80-83

116. Wilde, R.W. and Westcott, J.H., "The Characteristics of The Human Operator Engaged in a Tracking Task," Automatica, Vol. I, 1962, pp. 5-19

117. Woodworth, R.S. and Schlosberg, H., Experimental Psychology, Henry Holt and Company, 1954

118. Woodworth, R.S., Dynamics of Behavior, Henry Holt and Company, 1958

119. Young, L.R., Green, D.M., Elkind, J.I. and Kelly, J.A., "The Adaptive Dynamic Response Characteristics of the Human Operator in Simple Manual Control," NASA TN D-225, April 1964

120. Young, L.R. and Meiry, J.L., "Bang-Bang Aspects of Manual Control in High-Order Systems," IEEE Transactions on Automatic Control, Vol. AC-10, No. 3, July 1965

121. Young, L.R. and Stark, L., "Biological Control Systems -

A Critical Review and Evaluation," NASA CR-190, March 1965

122. Zadeh, L.A., "Frequency Analysis of Variable Networks," Proceedings of the IRE, Vol. 38, No. 3, March 1950, pp. 291-299

123. Zadeh, L.A., "From Circuit Theory to System Theory," Proceedings of the IRE, Vol. 50, No. 5, May 1962, pp. 856-865

124. Zadeh, L.A., "Optimum Nonlinear Filters," Journal of Applied Physics, Vol. 24, No. 4, April 1953, pp. 396-404

125. Zadeh, L.A., "On the Respresentation of Nonlinear Operators," IRE WESCON Convention Record, Part 2, August 1957, pp. 105-113

126. Zadeh, "Time-Varying Networks, I," Proceedings of the IRE, Vol. 49, No. 10, October 1961, pp. 1488-1503