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FINAL REPORT

DEVELOPMENT OF NON RESONANCE PIEZOELECTRIC ACTUATED FATIGUE MACHINE

by

F. Everett Reed
William T. Hogan
Robert W. Reid
John M. Rush
David L. Hartford
Arun K. Trikha

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95 Russell Street, Littleton, Massachusetts 01460 • Telephone 486-3526 area code 617

SUMMARY

A piezoelectric actuated fatigue machine has been designed, built and tested. The machine is capable of subjecting a specimen to a cyclic load up to ± 2000 lbs., and deflection amplitudes up to ± .005 in. The first natural frequency of the machine and specimen system is approximately 770 cycles per second. The operating frequency range of the machine could be increased substantially beyond the first natural frequency by proper electronic modification of the input signal.

The load transducer consists of piezoelectric crystal discs bonded together so as to form a cylinder. The design incorporates precompression of the crystal stack by means of tie rods in order that the epoxy bond between the discs will not be subjected to a tensile load when the stack is energized.

The mechanical design of the machine is such as to maximize the first natural frequency of the structure. Such design was dictated by the requirements of a nearly constant magnification factor over the desired frequency operating range to insure accurate input-output response.

A number of specimens of varying axial stiffnesses were used to investigate the influence of the sample characteristics on the performance of the machine.

The response of the machine to a known input signal was measured by means of a portion of the stack acting as a load cell.

SYMBOLS

		Units
a	cross sectional area	meter^2
c'	capacitance of one piezoelectric disc	farads
C_{33}^{E}	Young's modulus for piezoelectric material along thickness at constant electric field	newtons/meter^2
D	electric displacement	coulomb/meter^2
d_{33}	piezoelectric constant that relates strain with applied field	meter/volt
E	electric field strength	volt/meter
E_p	peak value of electric field strength	volt/meter
e_{33}	piezoelectric constant equal to $d_{33}C_{33}^{E}$	newton/volt meter
F	axial force	newton
f	frequency in cycles per second	cycles/second
I	current	ampere
n	total number of piezoelectric discs in the fatigue machine	
P	electrical power	watts
q	Young's modulus of elasticity for non-piezoelectric material	newton/meter^2
T	axial thickness of piezoelectric disc	meter
t	time	second
u	axial displacement	meter
V	voltage difference across piezoelectric disc	volts
x	coordinate in the axial direction	meter
ϵ^{T}	piezoelectric dielectric constant measured at constant stress	farad/meter
μ	a material damping constant	newton second/meter^2
ρ	mass density	kilograms
σ	axial stress	newton/meter^2
ω	angular frequency equal to $2\pi f$	radian/second

INTRODUCTION

Up to the present time the machines designed to test fatigue specimens under random loading have been large, expensive and generally low cyclic frequency. The purpose of this investigation was to design, build and test a more effective machine, using piezoelectric crystals as the actuating element.

Since the deflection required to stress structural materials is small, the most suitable machine for fatigue testing should be one which transmits its power as large forces moving over small distances. In order to translate the input signal into a load on the specimen without distortions, the machine should operate at frequencies below its resonant frequency and in order to have reasonably short testing times, the operating frequencies and hence the resonant frequency should be high.

The program to be followed in a random or a programmed loading is almost always represented in terms of electrical signals. These signals may be generated by punched cards (read electrically), by a random noise generator whose output is modified and shaped by filters, or by the magnetic tape or other record of an actual stress environment. This signal must be amplified and fed through one or more transducers to appear as a stress loading on the sample. Customarily the signal has been used to modulate an hydraulic servo valve controlling the flow of hydraulic fluid. By this process the natural frequency of the test system is rapidly lowered, first because of the low frequency response of the electrodynamic servo valve, and secondly because of the standing waves generated in the hydraulic fluid. It would surely be desirable to transmit the forces to the sample through an electrical to mechanical force transducer rather than through the steps from electrical to hydraulic to mechanical.

When one explores the properties of the different types of electrical current or voltage to mechanical force transducers, they are found to fall into the classification of electromagnetic transducers, electrostatic transducers, and transducers which are characterized by the change in length of materials under the action of an electromagnetic or electrostatic field, i.e. magnetostrictive or piezoelectric respectively.

The most common type of electrical to mechanical transducer is based upon electromagnetic induction. This is the basis of electric motors, the electromagnetic shakers, solenoids, etc. However, this type of machine is characterized by large displacements and small forces. Because of saturation, the force per unit area which can be developed by magnetic fields is quite limited. Since the areas must be large to generate large forces, it is difficult to build an electromagnetic machine which has a high natural frequency, So, likewise, the force per unit area generated by an electrostatic machine is relatively small. Also it may be difficult to build an electromagnetic or an electrostatic machine in which the force is linearly proportional to the electrical input.

When the transducers that operate by change in internal structures of materials induced by magnetic or electrostatic fields are explored, it is found that the change in length resulting from magnetostriction is too small to be useful for any transducer other than one operating at resonance. When operating off resonance, the cross sectional area of the transducer would have to be so large in order to bring about a favorable trade-off between the magnetostrictive strain and the loading strain that it would make an impractical transducer. Until recently this same characteristic would be found with electrostatic transducers, such as those based on quartz or rochelle salt. However, there has been a rapid development in transducers based upon barium-, zirconium-, and lead-titanate and these, when properly manufactured and oriented, change their lengths considerably more than other piezoelectric materials under the effects of electrostatic fields.

A structural fatigue machine based upon the use of these crystals is quite feasible. The strain (deformation per unit length) in these crystals obtained through the electrostatic fields is proportional to the potential gradient in the crystals. The force which the crystals generate is proportional to the area. Thus, to increase the capacity of a machine which uses piezoelectric crystals as actuators, it is only necessary to increase the cross-sectional area of the crystals. Although this increase in area will result in some additional deflections in the structure which transmits the crystal motion to the test specimen, the height of the crystal stack will be changed very little and hence the natural frequency of the test machine can be kept high even for large capacity machines. Furthermore, the crystals are moderately efficient transducers (although they have a large capacitance and therefore generate impedance problems) and so a crystal fatigue machine should be reasonable in its power demand and should have minimum problems of heating and noise.

Since all of these theoretical generalizations had not been established by the construction of an actual piezoelectric actuated fatigue machine, it was desirable to build a small prototype to test the principle.

The present investigation has demonstrated the feasibility of piezoelectric actuated fatigue machines. The prototype machine operated at reasonable reactive power levels, presented no heat dissipation problems and displayed fidelity of input-output response over a substantial frequency range.

As a result of the present investigation, a rational basis now exists for the design of larger piezoelectric actuated fatigue machines.

DESIGN AND CONSTRUCTION

The fatigue machine (see Figs. 1 and 2) consists of four major items, namely, the actuating piezoelectric crystal stacks, the static load eliminator, the machine frame, and specimen clamps. In addition to the above major items the machine has a cooling oil flow system, legs and suitable work-top surface.

The top and bottom piezoelectric crystal stacks each consist of an inner and outer hollow crystal cylinder. The inner cylinder is 11.4 inches long with a 2.00" O. D. and a .312" I. D. The outer cylinder is 10.9 inches long with a 3.125" O. D. and a 2.375" I. D. The inner crystal cylinder fits concentrically into the outer crystal cylinder and the annular space between the two cylinders together with the annular space between the outer crystal cylinders and the cylindrical aluminum jacket form the cooling oil passage. The crystal cylinders were fabricated from 1/4 inch thick discs of lead zirconate-lead titanate piezoelectric ceramic. Each inner stack contains 45 piezoelectric discs and each outer stack contains 43 piezoelectric discs.

Each crystal cylinder consists of ceramic discs and thin (.002") perforated nickel electrodes bonded together in alternate layers by means of epoxy. The polarity of every other disc is reversed (see electrical schematic, Fig. 12) so that one electrode serves to conduct the current to the top of one ceramic disc and the bottom of the adjacent ceramic disc. The "high" points of the perforated electrode surface penetrates the epoxy and make contact with the silvered face of the ceramic disc. Immediately after the stack is assembled, the stack is put under compression so as to squeeze the epoxy to a thin uniform layer. After the epoxy has partially set, it is cured in an oven. The curing process consists of approximately 12 hours at 165°F. When the curing process has been completed, the electrical connections are made by soldering leads to protruding tabs on the nickel electrodes. All leads are duplicated in order to enhance the reliability of the machine. The leads and tabs are electrically insulated by thin sheets of fiberglas in regions where the geometry allows such application and by insulating varnish in the remaining sensitive areas. Both the inner and outer stacks are equipped with steel tie rods, the purpose of which is to provide precompression of the stack so the epoxy bonds will always be under a compressive load. The inner stack tie rod is0.190" I. D. x 14 1/2" long whereas each of the four tie rods used in the outer stack is .095" I. D. x 12 3/4" long.

The purpose of the second major item of the fatigue machine, the static load eliminator is to prevent the build-up of a static compressive or tensile stress in the crystal stacks after the machine is completely assembled, including the securing of the specimen in the clamps. The static load eliminator consists of a very close fitting double faced piston and cylinder.

The cylinder is filled with a viscous and incompressible fluid (Glycerine), care being taken to remove all air bubbles, and the cylinder is then sealed by means of "O" rings. One end of the outer crystal stack is attached to the piston, the cylinder is attached to one of the aluminum plates that form part of the machine frame. The piston is free to move in the cylinder but because of the high viscous forces between the piston and the cylinder, the piston is restricted to a very slow rate of motion even under high forces. As a result, the piston will, in sufficient time, move relative to the cylinder (and therefore relative to the machine frame) so as to completely relieve any steady (static) stress that may have been built up in the crystal stack as a result of the method of assembling the machine or as a result of differential thermal expansion which might occur after the machine is completely assembled.

Before the cylinder is sealed, the fluid pressure is brought up to 400 psig by means of a hand pump to prevent the entry of air into the system. A minute quantity of air would reduce the stiffness of the static load eliminator to an unacceptably low value. This pressure level is subsequently maintained by means of compressed gas and is monitored by means of a pressure gage so that any leakage in the system can be readily detected.

The final step in preparing the machine for testing a specimen is to secure the specimen in the clamps. Immediately before this is done, the position of the cylinder of the static load eliminator in the frame of the machine is adjusted with the aid of a jig so that the piston is in its mid-position and therefore free to relieve subsequent static loads in either direction.

The third major machine item, the machine frame, consists of two (12" x 12" x 3" thick) aluminum plates to which the outer stacks are attached, one directly and one through the static load eliminator, and four (2.0" dia., 10.5" long) steel rods which connect the aluminum plates.

The fourth and last major item of the fatigue machine is the specimen and clamps. The clamps are made of steel and are attached to the inner stacks by an epoxy bond through an insulating material (fiberglas).

The general shape of the fatigue machine is a result of maximizing the first natural frequency. Two major considerations enter into this operation, namely, the total length of crystals must be kept as short as possible and the two thick plates that form part of the machine frame must be kept as close together and therefore the stiffness to mass ratio of the frame as high as possible. The first consideration is established by selecting the most suitable crystal material and cross sectional area. The second is best satisfied by splitting the stack and folding it back upon itself. It should be pointed out that it is quite important to make the machine parts as stiff as possible, not only for the purpose of achieving a high natural frequency for the machine, but also to achieve the required force-elongation load requirement set by a particular fatigue sample without demanding an excessively high applied voltage. When a low frequency voltage is applied to the crystal stack, the stack can experience maximum axial elongation with zero axial force, or maximum axial force with zero elongation or any intermediate load-elongation combination; the particular combination is of course set by the combined stiffness of the specimen and the machine. The solid lines in Figure 3 show the relationship between applied electric field and load-elongation combination experienced by the stack. The two broken lines shown in Figure 3 are the lines along which the machine would operate if the specimen stiffness were 521,000 lbs/in. The lower broken line is based on a machine stiffness (exclusive of the specimen) of 400,000 lbs/in, which is the calculated value for the present machine. It is interesting to note from Figure 3 that a change in machine stiffness from infinity to the calculated value at a fixed value of the applied voltage results in a reduction of the axial load to 60% of its former value.

In addition to the four major items described above, the machine is equipped with a cooling oil flow system. This oil flow system provides cooling for the inner and outer crystal stacks and consists of a pump, filter and reservoir with sufficient exposed area to effect heat dissipation to the laboratory. Transformer oil is used in the cooling system so as to enhance the electrical insulation of the stacks.

The evaluation of the machine requires the measurement of the voltage, current and frequency of the electrical power supplied to the stacks as well as the strain or force applied to the specimen by the piezoelectric stack.

The magnitude of the voltage and current delivered to the stack was displayed on the oscilloscope simultaneously by use of a preamplifier which incorporated a high frequency electronic chopper. The voltage applied across each piezoelectric disc was displayed on the oscilloscope through a calibrated attenuator probe. The current supply to the two complete crystal stacks was measured by a calibrated current probe and displayed on the oscilloscope.

The axial force generated by the piezoelectric stack can be measured by one piezoelectric disc in each stack. This sensing crystal was located adjacent to the specimen clamp. The specimen and sensing crystal were located as close as possible in order that the sensing crystal output would remain a faithful indication of the force experienced by the specimen under dynamic conditions. This sensing element consists of a single 1/4" thick disc (2.00" O. D. x 0.312" I. D.) of piezoelectric ceramic bonded by epoxy to the remainder of the inner stack.

The power supply obtained to conduct the experimental evaluation of the machine was a rotary power supply originally designed to drive an electromagnetic shaker. This power supply was available from the Raytheon Co. as excess government equipment. The use of a second-hand power supply of this nature was indicated in our proposal in view of the substantial savings when contrasted to a modern electronic power supply. However, the use of a rotary power supply excludes a random capability and restricts the fatigue machine to operation at constant frequencies or to a smooth frequency sweep.

The power supply is capable of operating at frequencies up to the required 2000 cps. However, it was necessary to incorporate new transformers between the rotary power supply and the crystal stack so that the final output would be at the high voltage required by the crystal stack rather than the low voltage required by the electromagnetic shaker.

In anticipation of possible interaction between the power supply and the fatigue machine, it was determined that the high inductance of the alternators combined with high capacitance of the crystal stack would result in an electrical frequency low enough, about 70 cps, so that it should not result in difficulties. Likewise, it was determined that the torsional frequency of the motors and alternators against the alternator field would be low enough, about 25 cps, so that it should not create difficulties. However, these determinations are approximate because of the limited data on the alternator.

RESULTS AND DISCUSSION

Fidelity of input-output

The fidelity with which the machine translates an input voltage signal into a mechanical load on the test specimen is of prime importance in the design of the piezoelectric fatigue machine. In order for the machine to faithfully translate an input voltage into a load output, it is necessary to operate the machine well below the first natural frequency of the machine where the magnification factor will be essentially constant. In order to predict the first natural frequency, a mathematical model of the machine and piezoelectric stacks was developed and a computer program was written in order to perform the required calculations. The model was developed by representing the fatigue machine as an eleven degree-of-freedom system. The analysis as well as the computed frequency response curve for the model is presented in Appendix A. This mathematical model predicts a first natural frequency at approximately 900 cycles per second.

The measured frequency response of the machine is shown in Fig. 4. The ordinate in Figure 4 is the voltage output of one of the sensing crystals. This voltage is directly related to the force on the sensing crystal and hence on the specimen. The two curves shown in Figure 4 represent operation at two positions of the static load eliminator, namely, at the top of its travel and at mid position. From the measured response curve it appears that the frequency response of the machine is sensitive to the operating condition of the static load eliminator. Under the conditions investigated the first natural frequency was found to be approximately 770 cps.

Applied electric field vs load output

Although the complete mathematical model and associated computer program could readily be used to predict the performance of the machine over the full range of load and frequency, for operation well below the first natural frequency the potential which must be applied across each crystal to produce a given load on the sample as well as the power can readily be computed if it is assumed that all crystals experience the same loading at any instant. On the basis of this assumption the stress as a function of the strain and electric field strength in the crystal stack are given by the expression

$$\sigma = C_{33}^{E} \left(\frac{\partial u}{\partial x}\right) - e_{33}E \, * \qquad (1)$$

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*This equation together with the equation for the electric displacement $D = d_{33}\sigma + \epsilon^{T}E$ suffice to determine the static and low frequency behavior of piezoelectric crystals [Ref. 1].

where E = electric field strength (volts/meter)

σ = stress in the longitudinal direction of the stack (newton/meter^2)

$\frac{\partial u}{\partial x}$ = longitudinal strain in the crystal stack (meter/meter)

e_{33} = piezoelectric constant (newton/volt meter)

C_{33}^E = Young's modulus of elasticity measured along the thickness at constant electric field (newton/meter^2)

The above equation can be easily solved for the electric field strength that must be applied in order to produce a given stress and strain in the crystal stack.

$$E = \frac{C_{33}^E}{e_{33}} \left(\frac{\partial u}{\partial x}\right) - \frac{1}{e_{33}} \sigma \qquad (2)$$

But since $d_{33} = e_{33}/C_{33}^E$ = piezoelectric constant that relates strain with applied field and σ = axial force/cross sectional area = F/a, the applied electric field strength as a function of the strain and stress in the stack is given by the expression

$$E = \frac{1}{d_{33}} \left(\frac{\partial u}{\partial x}\right) - \frac{1}{d_{33} C_{33}^E} \left(\frac{-F}{a}\right) \qquad (3)$$

where for this application

$d_{33} = 590 \times 10^{-12}$ (meters/volt)

$C_{33}^E = .0484 \times 10^{+12}$ (newton/meter^2)

$a = 20.3 \times 10^{-4}$ (meter^2)(average)

Three specimens were used in the experimental evaluation phase of the program. Specimens A and B were made of .050" sheet steel and had a computed stiffness of 521,000 $\#_f$/in and 461,000 $\#_f$/in respectively. Specimen C was made of .050" sheet aluminum alloy and had a computed stiffness of 153,000 $\#_f$/in. Figures 5, 6 and 7 show the calculated voltage which must be applied across each crystal to produce a given force on specimen A, B and C respectively. The force experienced by the specimens that failed as shown in Figures 5, 6 and 7 was estimated from S-N curves available in the literature.

Power requirements

The apparent power (the product of applied voltage and current without considering phase) delivered by the power supply to a single piezoelectric disc is

$$= (E\,T)\,I$$

where T = axial thickness of one piezoelectric disc (meter)

I = current supplied to one disc (amperes)

Since the current supplied to the disc electrode is equal to the product of the electrode area (a) and the current density (which in turn can be set equal to the time derivative of the electric displacement, D), the apparent power supplied to one disc is given by the expression

$$= (E\,T)\,\{\frac{\partial}{\partial t}[D]\,a\} \qquad (4)$$

$D = \epsilon^T E + d_{33}\sigma$ = electric displacement (coulomb/meter^2)

ϵ^T = the dielectric constant measured at constant stress (farad/meter)

$E = E_p e^{j\omega t}$ where E_p = peak electric field strength

Thus the peak apparent power supplied to the entire crystal stack is given by the expression

$$P = n T E_p 2\pi f [\epsilon^T E_p + d_{33}(\frac{-F}{a})]\,a \qquad (5)$$

where for this particular machine

P = peak apparent power (watts) delivered to the entire stack
n = total number of crystal discs in the machine, 176.
T = 0.00635 meters
E_p = peak applied electric field strength (volts/meter)
f = frequency of applied signal (cycles per second)
$a = 20.3 \times 10^{-4}$ (meters^2) (average)
$\epsilon^T = 3.01 \times 10^{-8}$ (farad/meter)
$d_{33} = 590 \times 10^{-12}$ (meter/volt)

Equations 5 and 3 can now be combined to yield an expression for the apparent power as a function of the voltage applied across each crystal. This function is shown in graphical form in Figures 8, 9 and 10 as well as measured values for specimens A, B and C respectively.

The dielectric losses in ceramic piezoelectric crystals are usually expressed as a dissipation factor which is equal to the ratio of effective series resistance to effective series reactance. For the piezoelectric material used in the fatigue machine the value of this dissipation factor was estimated at approximately 0.02 at 1000 cps by the supplier. Since the series resistance is small compared to the series reactance, the dissipation factor in this case will be essentially equal to the power factor which in turn is equal to the ratio of power actually dissipated in the stack to apparent power (KVA) supplied to the stack. The power factor was measured when the stack was loaded with a specimen of stiffness equal to 521,000 $\#_f$/in and driven at a frequency well below the first natural frequency, namely, 440 cps. The value of the power factor so measured was found to be less than .02.

Reliability of the stack

The piezoelectric fatigue machine was operated for a total time of approximately 50 hours over a frequency range of 50 - 2000 cps and an applied voltage range of 0 - 2000 volts (peak) while loaded with the three specimens of various stiffness. In this time no structural failure or epoxy bond separation occurred in the piezoelectric crystals. However, a failure did occur after approximately the first 20 hours of operation between the fiberglas insulating disc and the steel terminator on one of the inner stacks. The reason for the failure is believed to be lack of proper surface preparation before the epoxy was applied to the fiberglas.

Sensing Crystal Voltage Output

The open circuit output voltage of the sensing crystal is proportional to the force applied to the crystal and therefore can be used as a measure of the axial force applied to the specimen by the actuating crystals. If the inner steel tie rod did not pass through the sensing crystal, the ratio of the force applied to the sensing crystal and the force applied to the specimen would be unity. However, when the tie rod passes through the sensing crystal, the force in the crystals is greater than the force in the specimen due to the load carried by the tie rod. With the tie rod in place the ratio of the force in the sensing crystal to the force in the specimen is given by the expression

$$1 + \frac{1}{2}\,\frac{[\text{tie rod stiffness}]}{[\text{specimen stiffness}]}$$

The value of the above ratio is 1.058 for specimen A (steel, Stiffness = 521,000 #/in) and 1.191 for specimen C (aluminum, stiffness = 153,000 #/in).

CONCLUSIONS

The feasibility of a fatigue machine actuated by a stack of bonded piezoelectric crystals has been demonstrated.

The fatigue machine has been operated for approximately 50 hours over a frequency range of 50 to 2000 cps while applying loads up to approximately ± 1300 lbs. The piezoelectric stack did not show signs of deterioration.

The measured first natural frequency of 770 cps indicates a substantial frequency range over which the machine can be operated at nearly constant amplification of the input signal.

The measured relation between the applied field strength and the mechanical load on the specimen as well as the apparent power and real power delivered to the stack are in reasonable agreement with the predicted values. Calibration of the machine is required, including zero frequency operation, before a more detailed comparison between measured and predicted values of force and displacement can be achieved.

RECOMMENDATIONS

In view of the fact that the feasibility of the concept of a bonded piezoelectric stack has now been demonstrated, it is recommended that the performance of the present machine be accurately calibrated while operating in the DC as well as AC mode. This calibration should involve measurements of the force, strain and total elongation experienced by specimens of various stiffness and an experimental study of the static load eliminator.

The capability of the present design should be extended by incorporation of a device which would apply a preload to the fatigue specimen. Such a preload could be applied to the specimen through the center tie rod of the inner stack located on that half of the machine that contains the static load eliminator. A sketch of a possible arrangement is shown in Figure 11.

Upon completion of calibration and incorporation of a preload device, the operating capability of the fatigue machine should be extended to the original ultimate intent of random excitation by replacing the present rotary power supply with an electronic power supply.

REFERENCES

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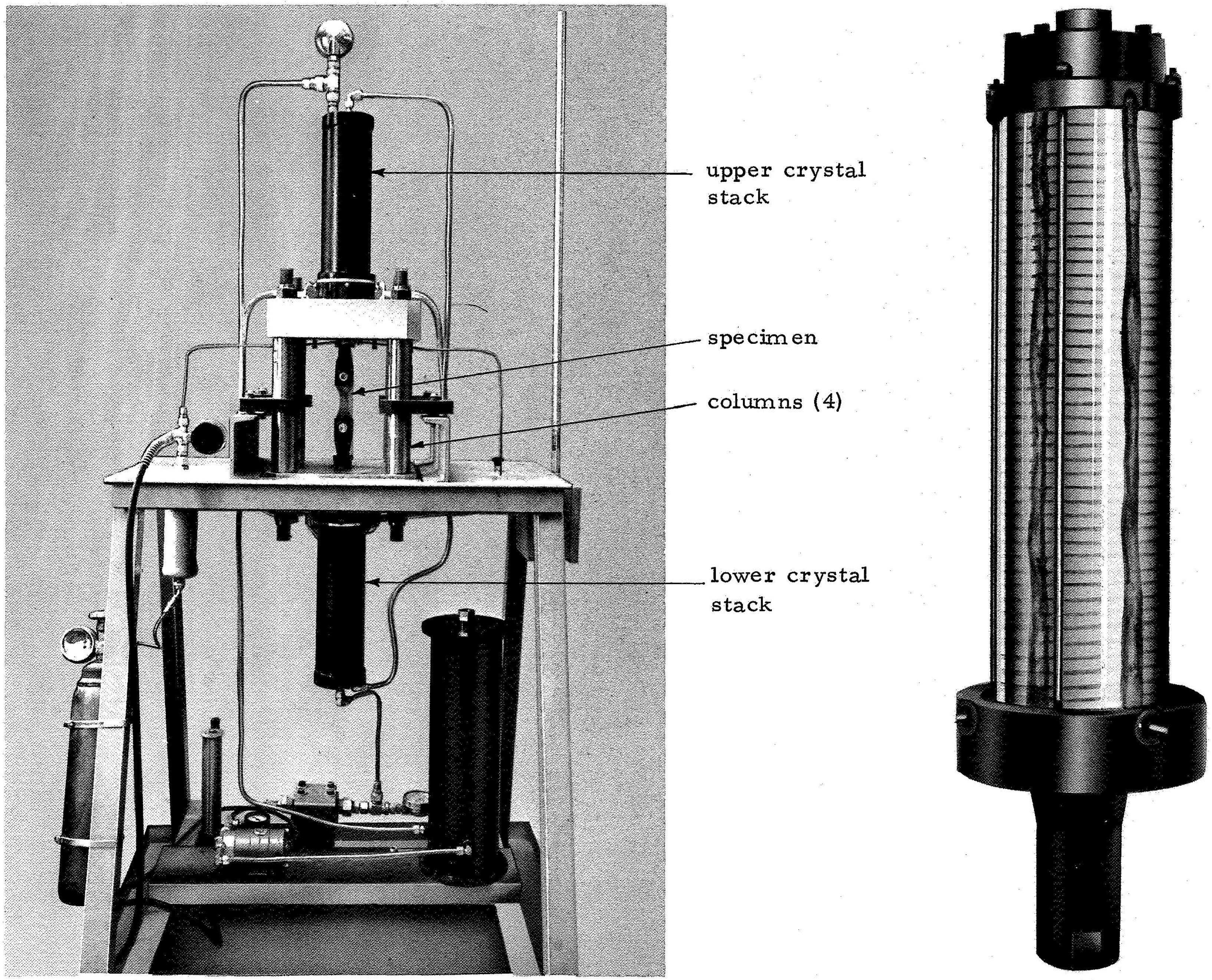

Random Fatigue Testing Machine

Piezoelectric Crystal Assembly

Fig. 1

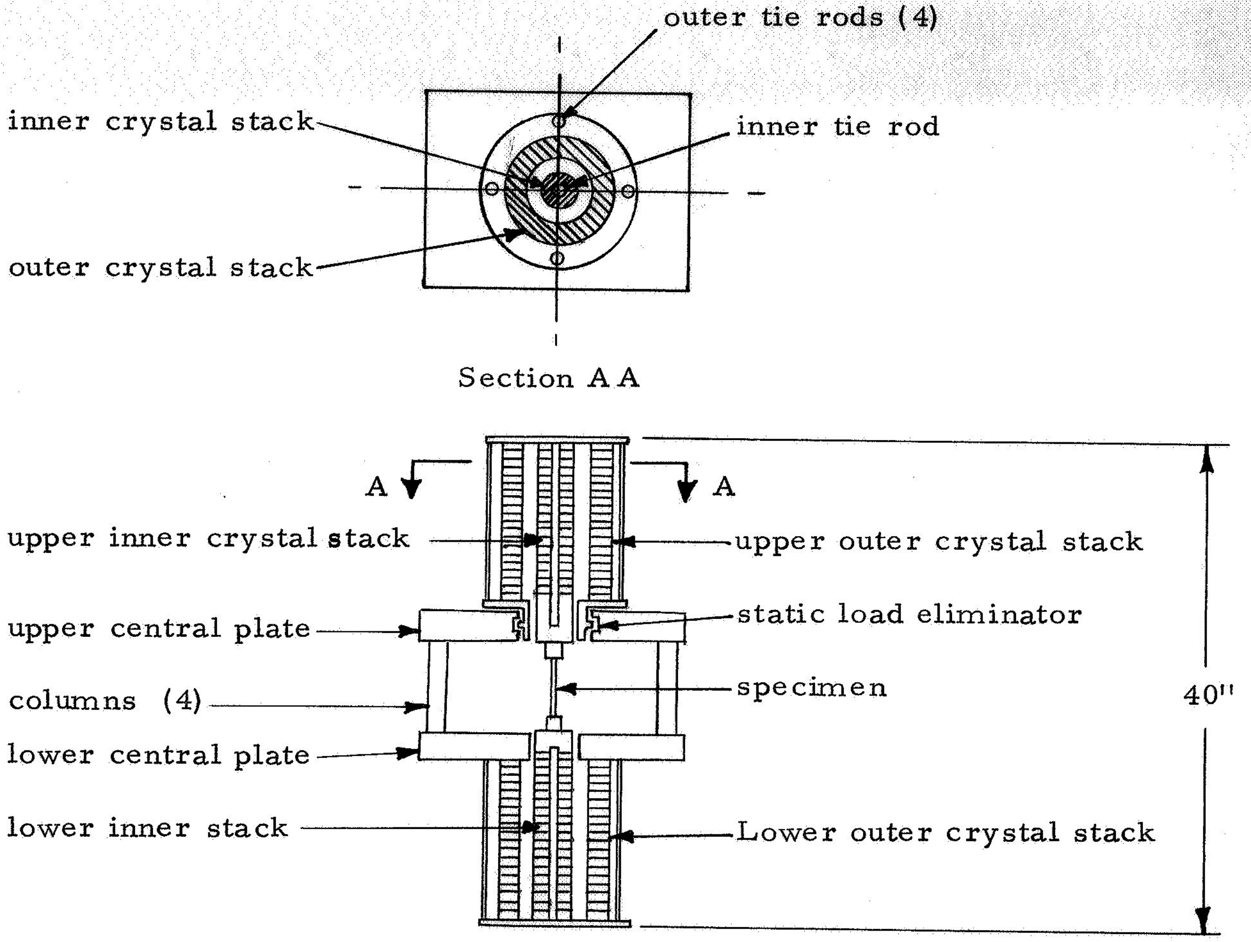

Fig. 2 - Schematic of fatigue machine elements

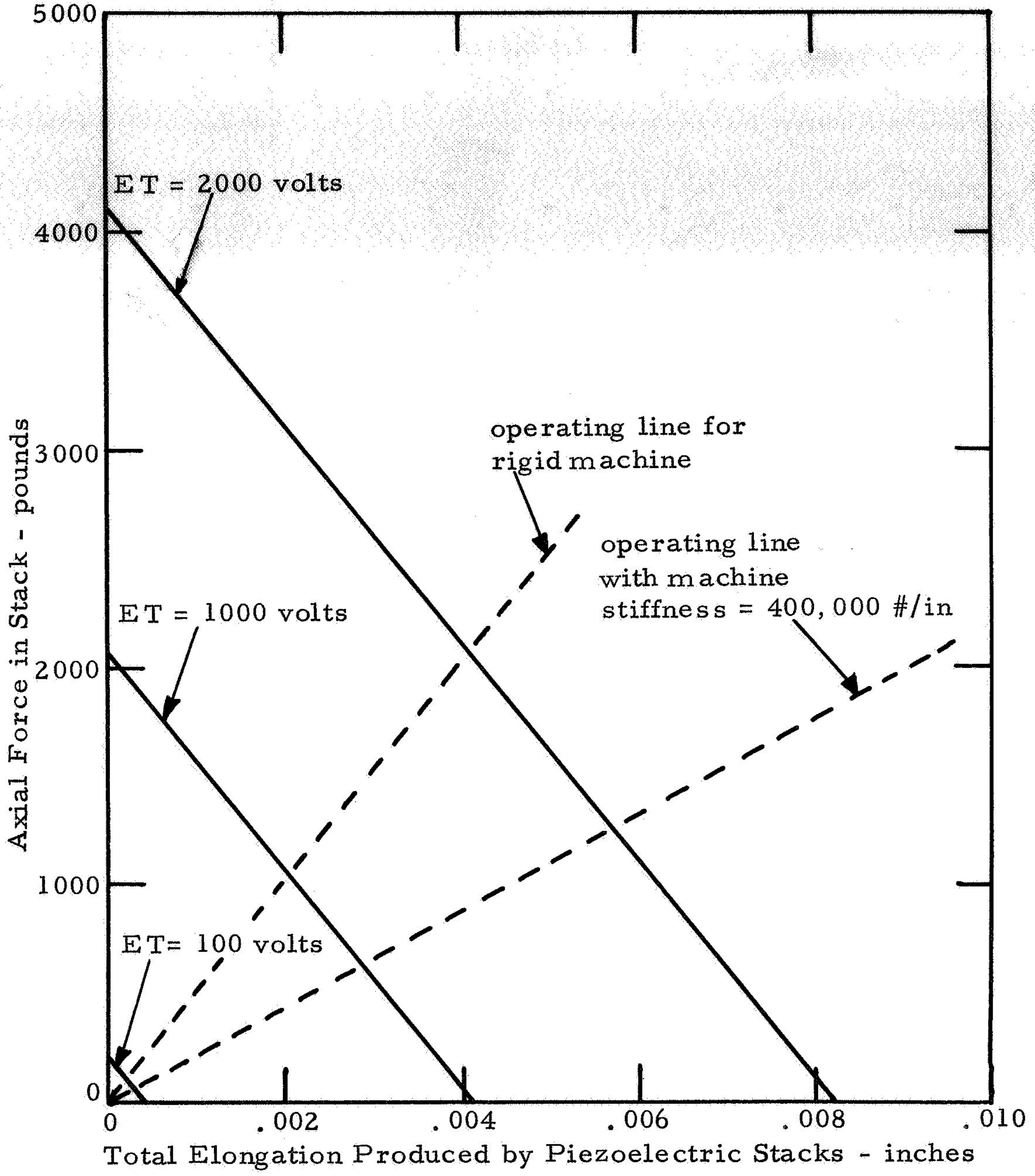

Fig. 3 - The three solid lines show the possible range of axial force and total elongation produced by the piezoelectric stack for an applied voltage of 100, 1000 and 2000 volts.

The two broken lines represent the locus of points along which the machine will operate when loaded with a specimen of stiffness equal to 521,000 lbs/in.

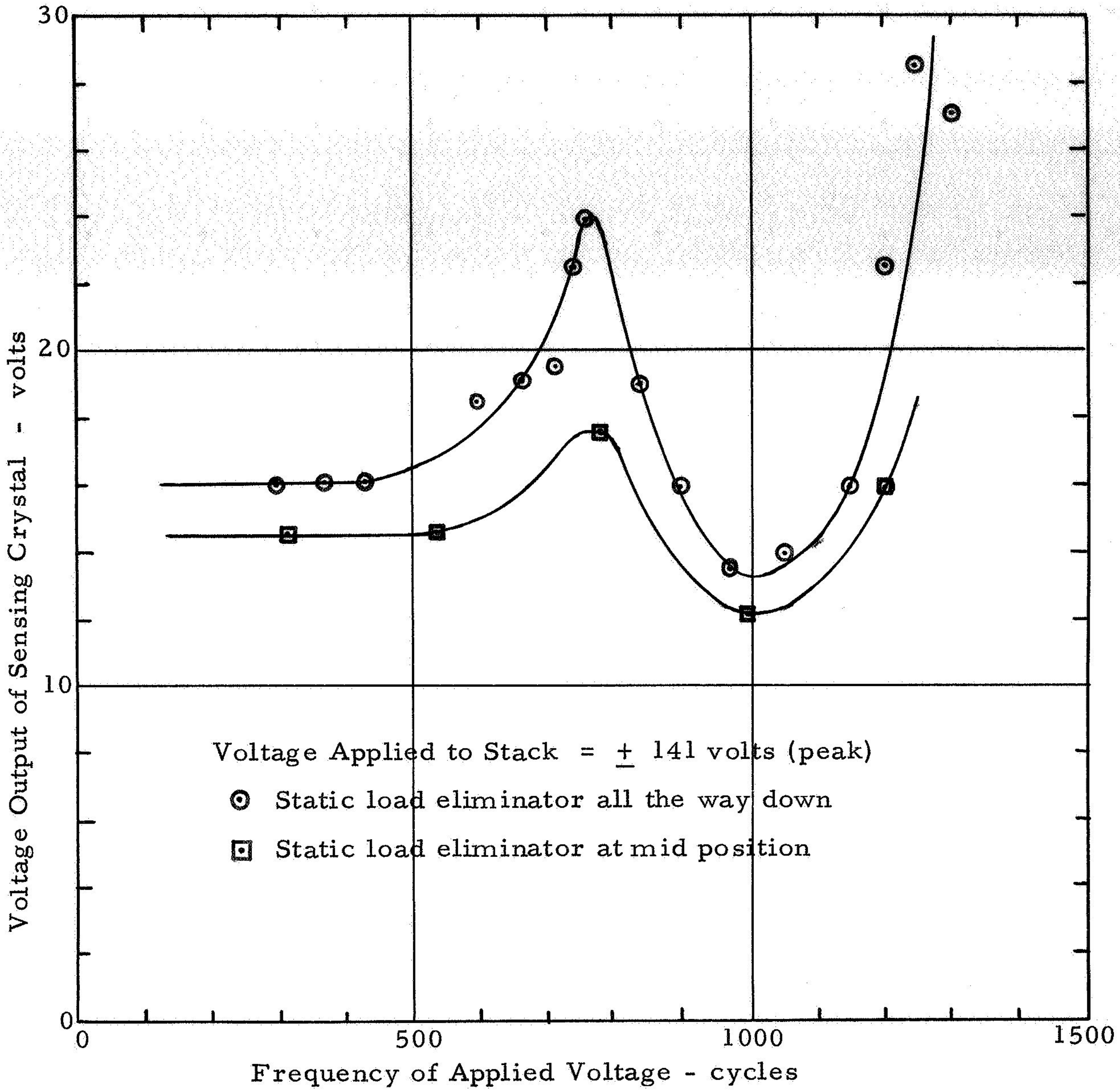

Fig. 4 - Frequency response curve of the fatigue machine when the machine is driven by alternating voltage and loaded by a specimen with a stiffness equal to 521,000 lbs/in.

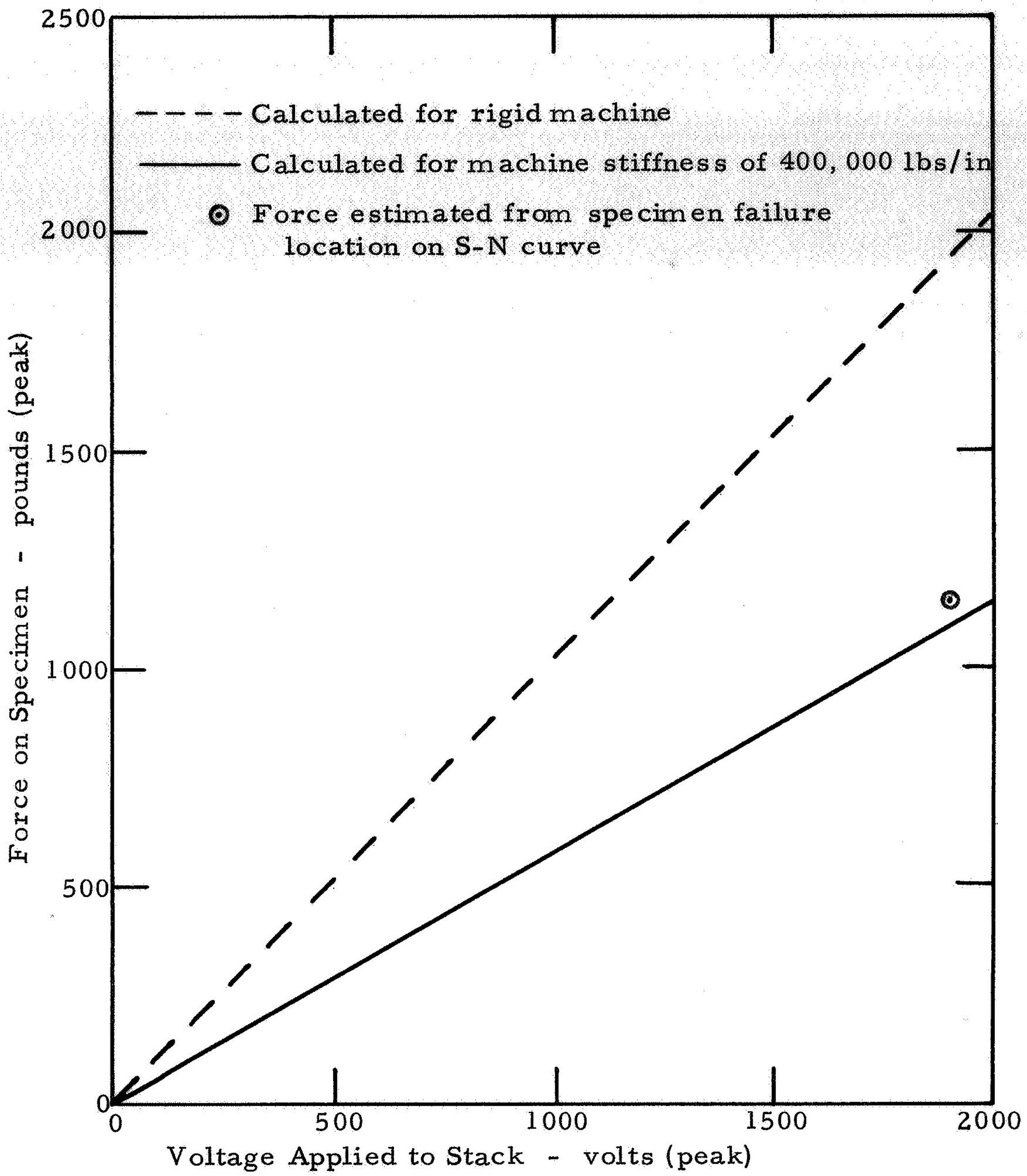

Fig. 5 - Voltage applied to the piezoelectric stack vs the developed axial force on specimen A.

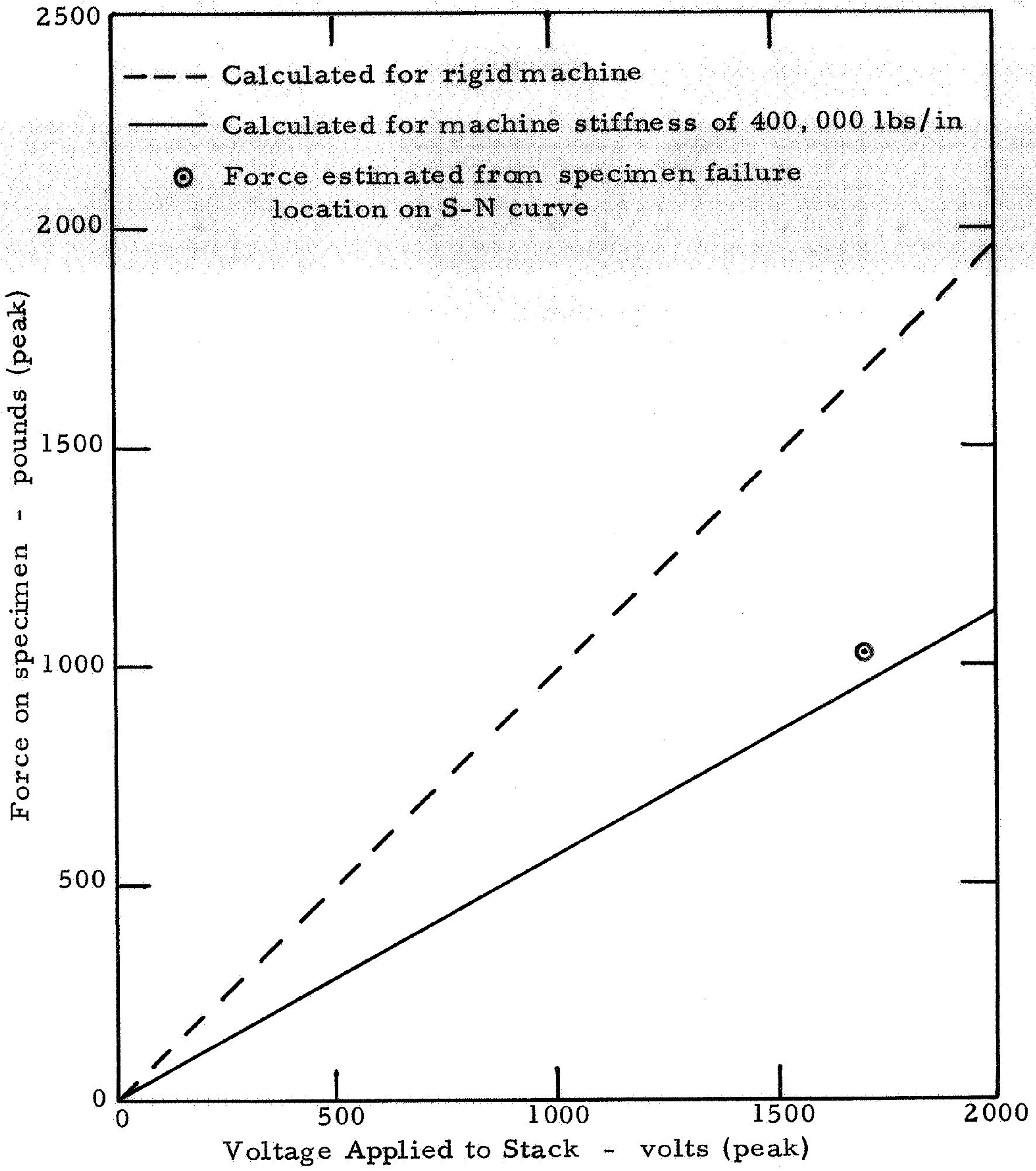

Fig. 6 - Voltage applied to the piezoelectric stack vs the developed axial force on specimen B.

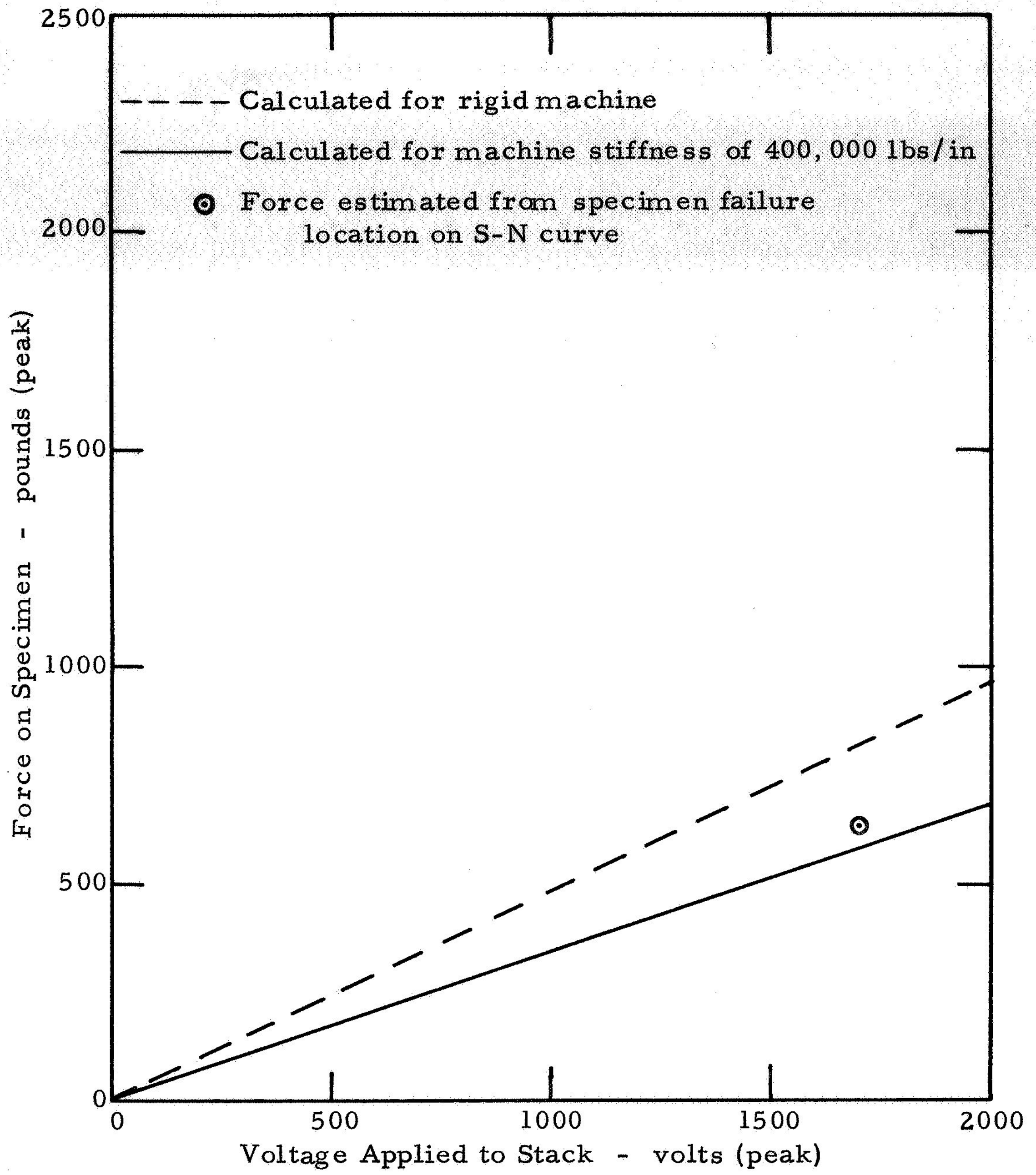

Fig. 7 - Voltage applied to the piezoelectric stack vs the developed axial force on specimen C.

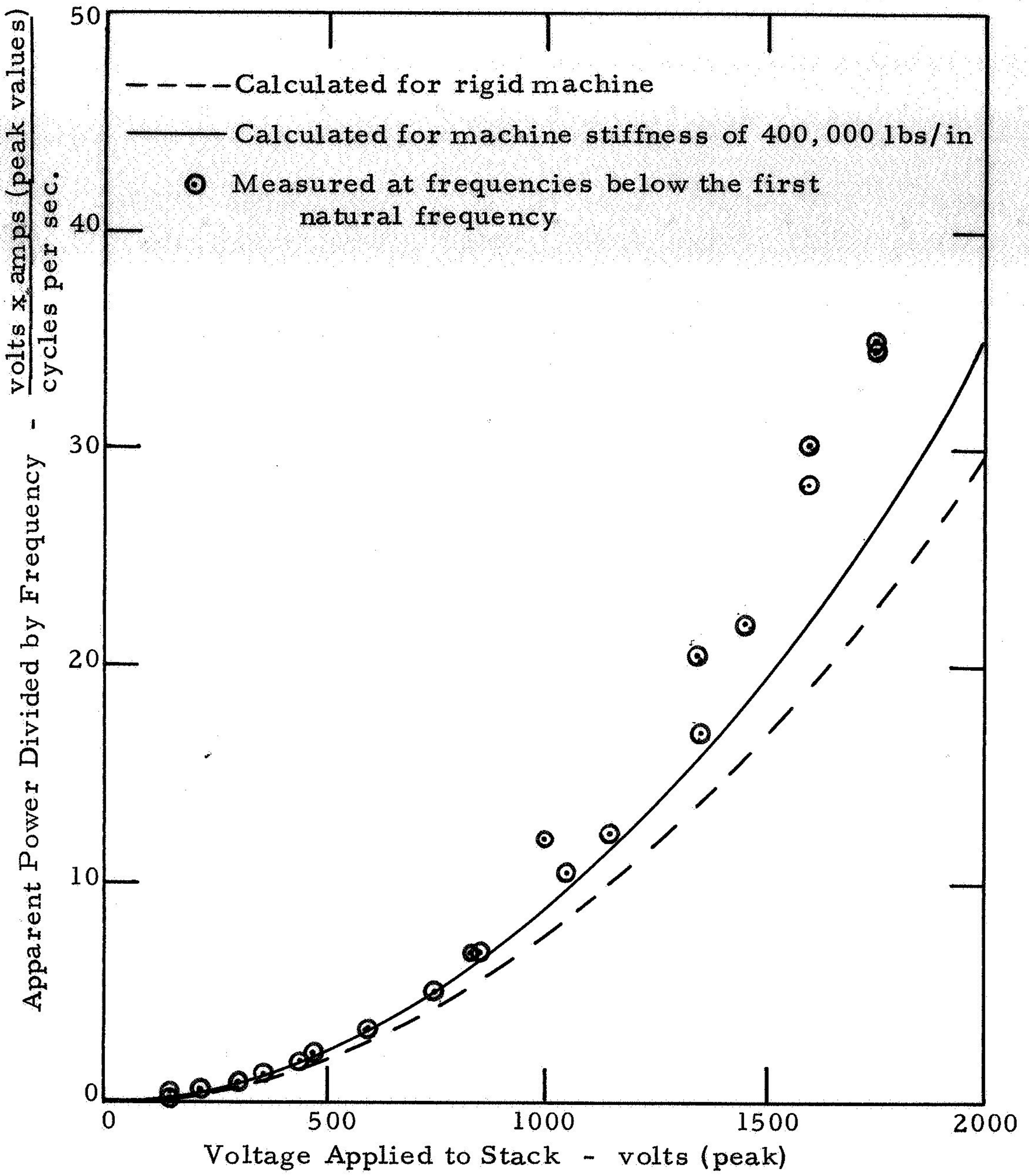

Fig. 8 - Calculated and measured apparent power supplied to the piezoelectric stack (divided by the frequency) vs the applied voltage when the machine is loaded by specimen A.

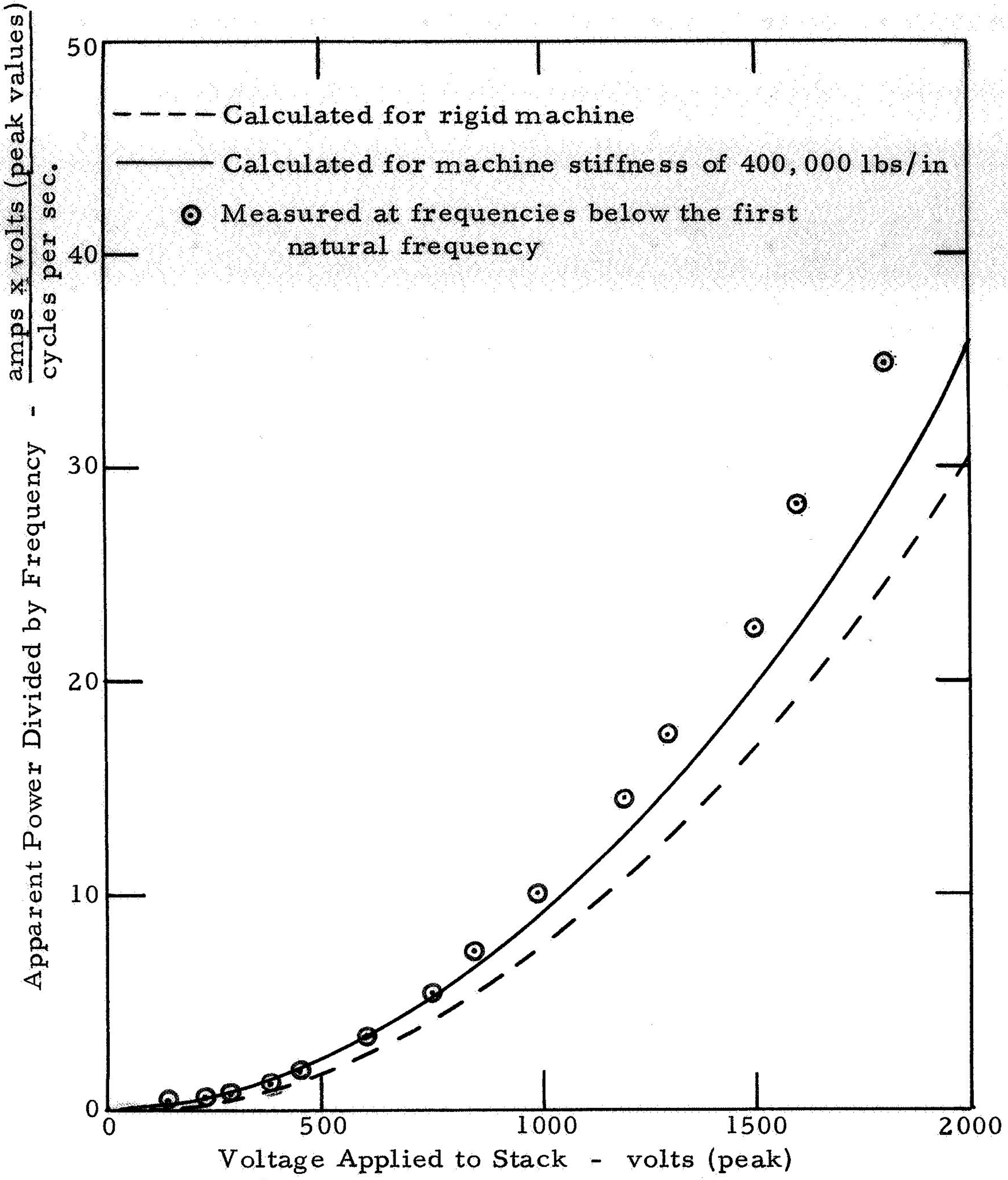

Fig. 9 - Calculated and measured apparent power supplied to the piezoelectric stack (divided by the frequency) vs the applied voltage when the machine is loaded by specimen B.

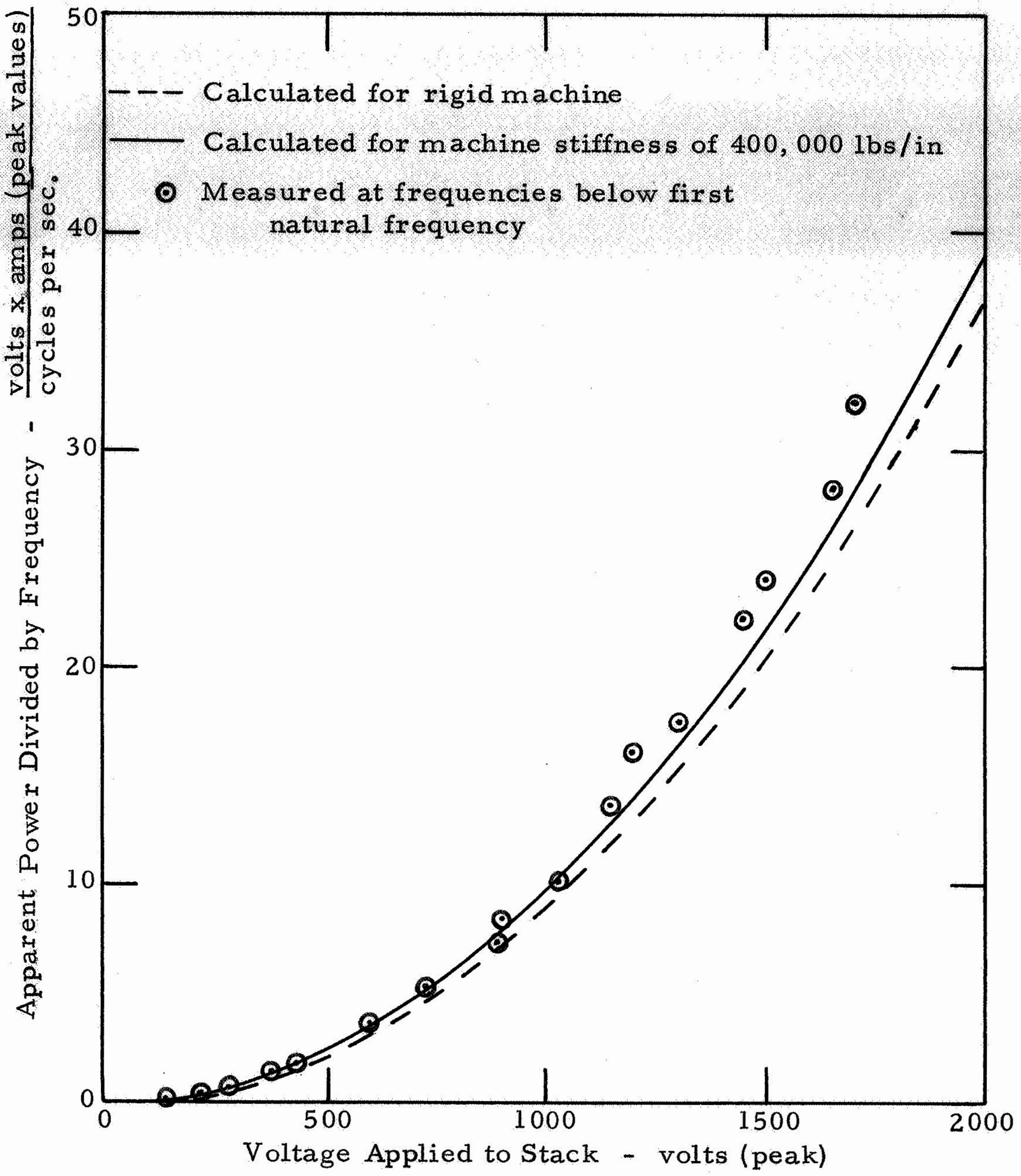

Fig. 10 - Calculated and measured apparent power supplied to the piezoelectric stack (divided by the frequency) vs the applied voltage when the machine is loaded by specimen C.

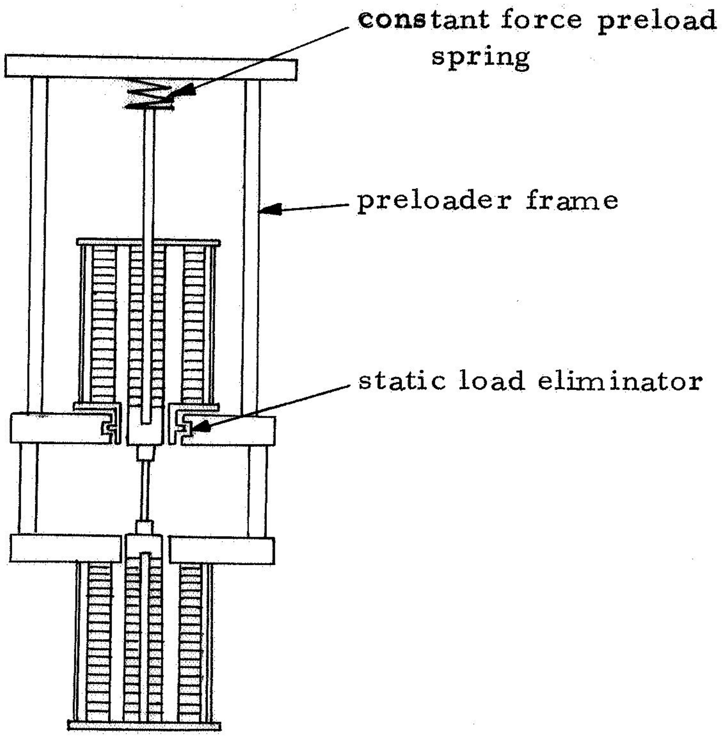

Fig. 11 - Schematic of Fatigue Machine Equipped with a Preloading Device

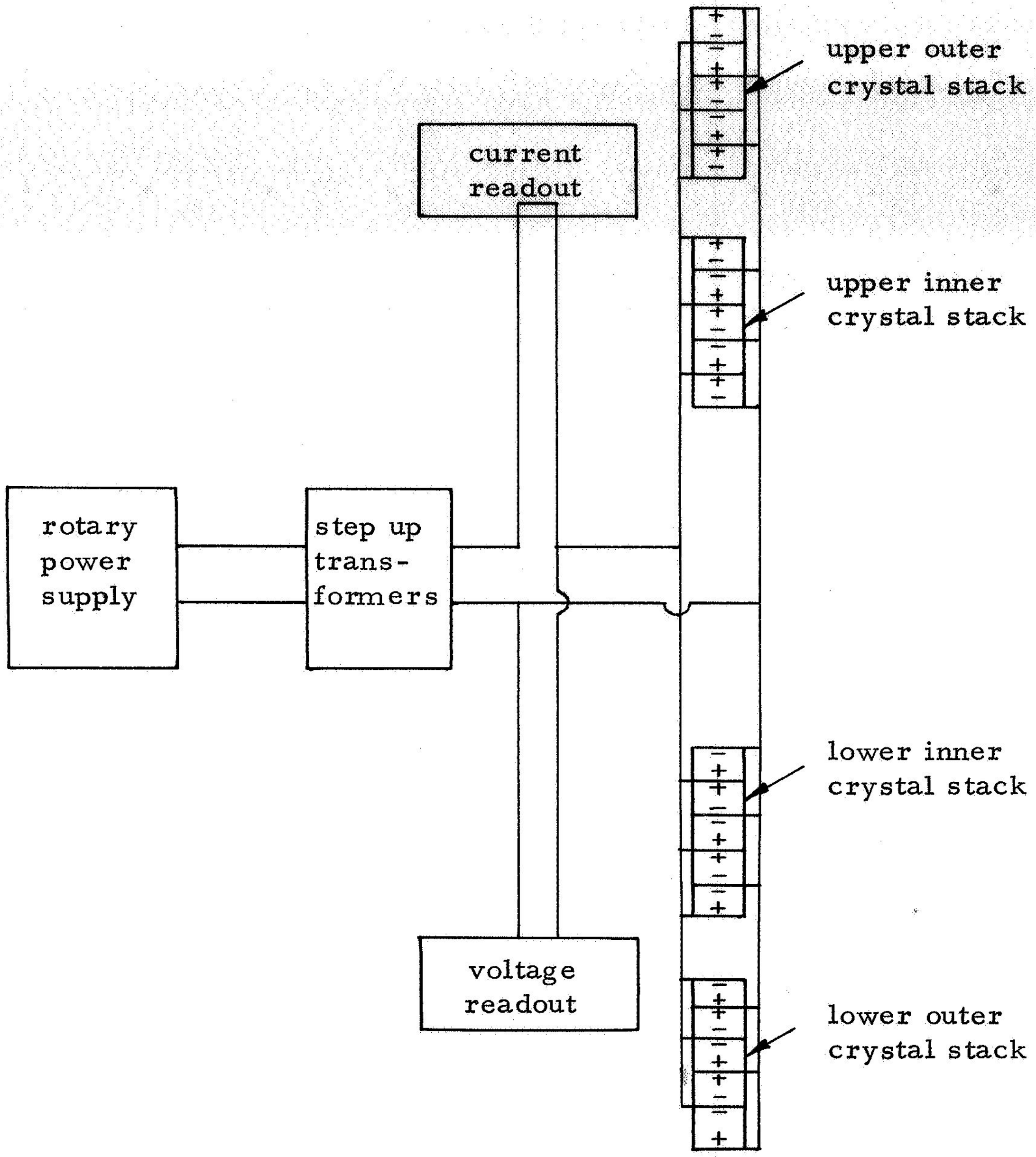

Fig. 12 - Electrical Schematic of the Piezoelectric Fatigue Machine

APPENDIX A

ANALYSIS OF PIEZOELECTRIC FATIGUE-TESTING MACHINE

ANALYSIS OF PIEZOELECTRIC FATIGUE-TESTING MACHINE

Mathematical Model

For an analysis of the natural frequency, and the deflection at different frequencies, the piezoelectric fatigue-testing machine can be represented adequately by an eleven degree-of-freedom system as shown in Fig. A-1.

The two outer, and the two inner crystal stacks are represented as distributed mass systems, because of their length being comparatively large, and the speed of sound in crystal material being comparatively small. The rest of the structure is represented by a system of masses interconnected with springs as follows:

m_a = 1/3 mass of upper plate plus static load eliminator cylinder
m_b = 2/3 mass of upper plate plus 1/3 mass of columns
m_c = 1/2 mass of lower plate plus 1/3 mass of columns
m_d = 1/2 mass of lower plate plus mass of stack shield assembly and lower flange
m_e = 1/2 mass of folded stack connector plate plus mass of outer stack end plate
m_f = 1/2 mass of folded stack connector plate
m_g = mass of specimen clamp plus mass of termination plus 1/3 mass of specimen
m_h = mass of specimen clamp plus mass of termination plus 1/3 mass of specimen
m_i = 1/2 mass of folded stack connector plate
m_j = 1/2 mass of folded stack connector plate plus mass of outer stack end plate
m_k = mass of static load eliminator piston, stack shield assembly and upper flange

k_1 = stiffness of static load eliminator
k_2 = stiffness of upper plate
k_3 = stiffness of columns
k_4 = stiffness of lower plate
k_5 = stiffness of outer crystal stack

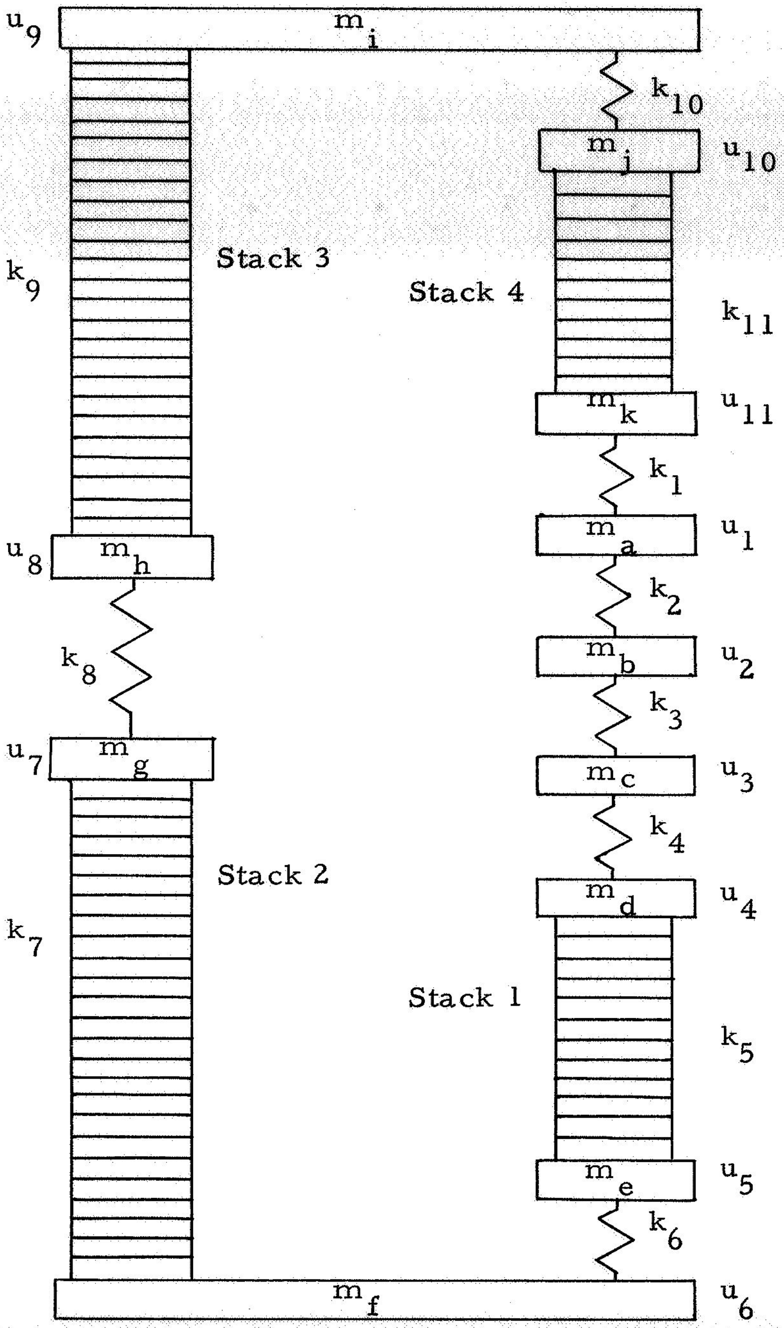
u_9
m_i
k_{10}
m_j
u_{10}
Stack 3
k_9
Stack 4
k_{11}
m_k
u_{11}
k_1
m_a
u_1
u_8
m_h
k_2
m_b
u_2
k_8
k_3
u_7
m_g
m_c
u_3
k_4
Stack 2
m_d
u_4
k_7
Stack 1
k_5
m_e
u_5
k_6
m_f
u_6

Figure A-1

k_6 = stiffness of folded stack connector plate
k_7 = stiffness of inner crystal stack
k_8 = stiffness of specimen clamps, terminations and specimen combined
k_9 = stiffness of inner crystal stack
k_{10} = stiffness of folded stack connector plate
k_{11} = stiffness of outer crystal stack

Consider an element of length, dx, enclosed between the sections AA and BB (Fig. A-2)

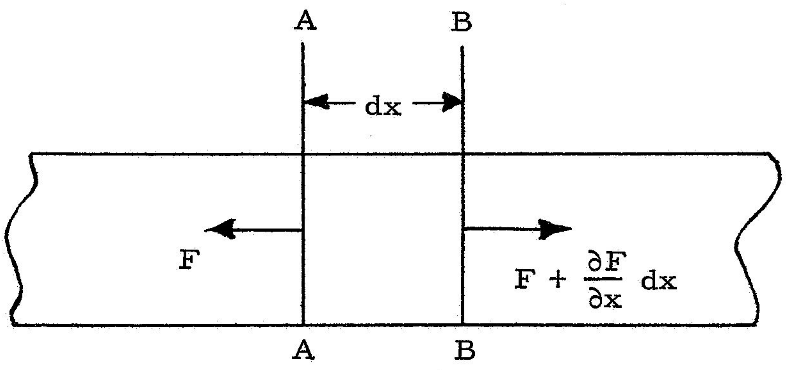

Fig. A-2

For an ordinary material, the effects of damping on the vibrations are included by assuming the stress in any element to be the sum of a deformation stress and a stress related to the velocity of strain, i.e.,

$$\sigma = q \frac{\partial u}{\partial x} + \mu \frac{\partial^2 u}{\partial x . \partial t} \qquad \text{(A-1)}$$

where
σ = stress at any element
q = Young's modulus of elasticity
u = the longitudinal displacement; $\frac{\partial u}{\partial x}$ is the longitudinal strain
x = distance along x-axis
t = time
μ = a material damping constant

For a piezoelectric crystal, if an electric field E acts along the crystal length, then equation (A-1) is modified to

$$\sigma = C_{33}^{E} \left(\frac{\partial u}{\partial x}\right) + \mu \frac{\partial^2 u}{\partial x \, \partial t} - e_{33} E \qquad \text{(A-2)}$$

where C_{33}^{E} = Young's modulus of elasticity measured along the thickness at constant electric field

e_{33} = piezoelectric constant

Thus for the element AABB, the force acting on the face AA is

$$F = a \left[C_{33}^{E} \left(\frac{\partial u}{\partial x}\right) + \mu \frac{\partial^2 u}{\partial x \, \partial t} - e_{33} E\right]$$

where a is the cross sectional area

The force acting on the face BB = F + $\frac{\partial F}{\partial x}$ dx in the direction of positive x.

The mass of the element AABB = $\rho . a$ dx where ρ is the mass density

Therefore, by Newton's law of motion

$$\frac{\partial F}{\partial x} dx = \rho \, a \, dx \frac{\partial^2 u}{\partial t^2}$$

or

$$a \left[C_{33}^{E} \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^3 u}{\partial x^2 \partial t}\right] dx = \rho \, a \, dx \frac{\partial^2 u}{\partial t^2}$$

or

$$C_{33}^{E} \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^3 u}{\partial x^2 \partial t} = \rho \frac{\partial^2 u}{\partial t^2} \qquad \text{(A-3)}$$

the solution of which is given by

$$u = \left[A_1 e^{j\beta x} + A_2 e^{-j\beta x}\right] e^{j\omega t} \qquad \text{(A-4)}$$

where

$$\beta = \left[\frac{\omega^2 \rho}{q + j\omega\mu}\right]^{1/2}$$

By determining the deflection across the specimen at different frequencies (at a constant value of applied electric field strength) and plotting a graph, an estimate of the natural frequency of the system can be made. A computer program has been written to determine the deflection across the specimen as a function of driving frequency. Some results of this program are shown in Figure A-3.

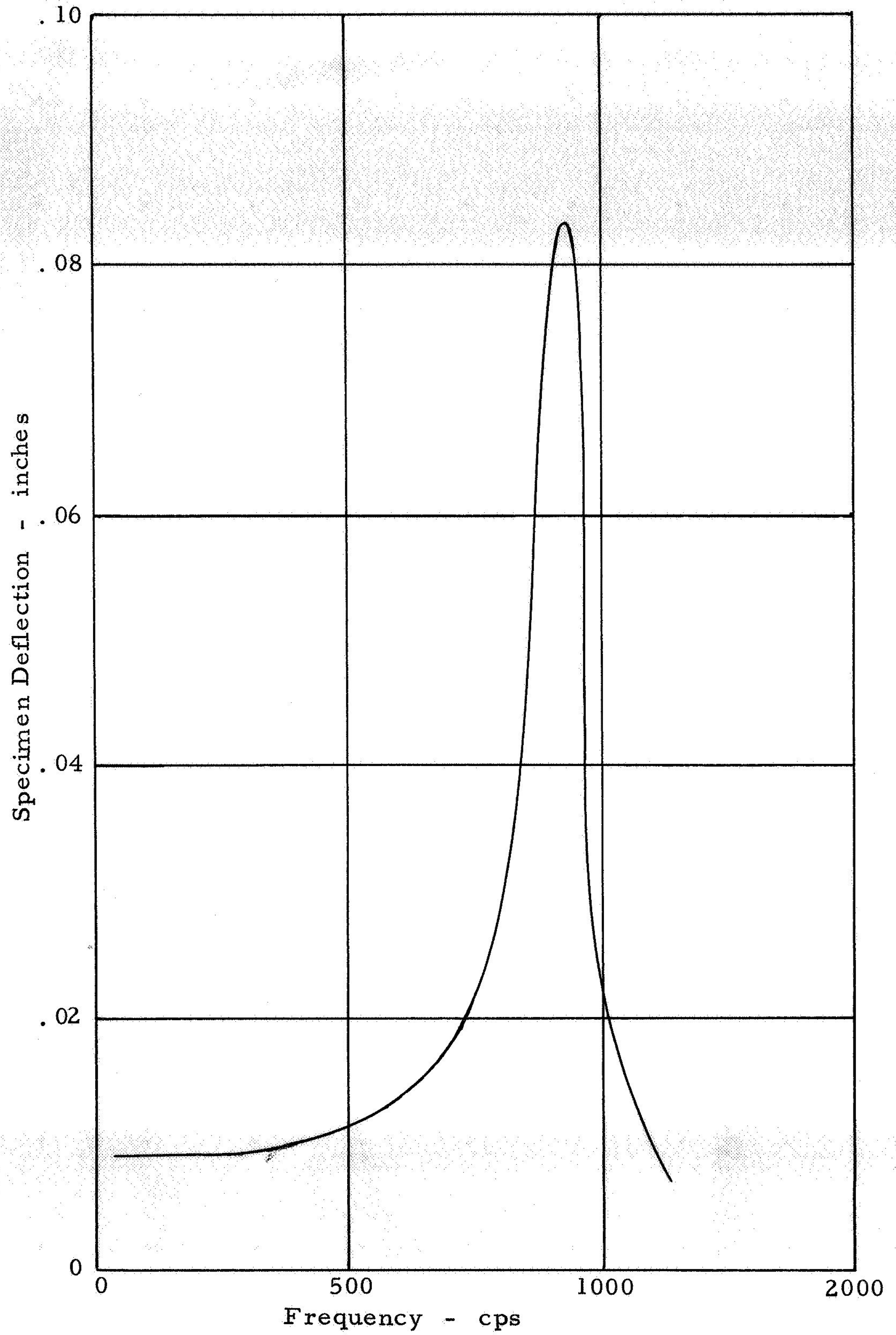

Fig. A-3 - Calculated specimen deflection vs frequency when the applied voltage is 2000 volts for specimen B.