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THEORETICAL INTENSITIES
of
RECOMBINATION LINES

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Leo Goldberg

Harvard College Observatory

February 1968

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ABSTRACT

A review is given of the effects of departures from thermal equilibrium on the intensities of radio recombination lines under conditions found in H II regions. The equilibrium intensities are modified in two respects. First, the transition rate is proportional to the departure coefficient b_n of the initial level. Second, the magnitude of the stimulated emission is proportional to $d \ln b_n/dn$ and may be larger or smaller than the equilibrium value $\left(1 - e^{-h\nu/kT_e}\right)$ depending on whether the sign of the derivative is positive or negative, respectively. In hydrogen, the b_n values lie in the range 0.75-1.0 for all n greater than 50 and hence only the second effect is of major importance. In multi-electron atoms, however, the b_n values may exceed unity by one or more orders of magnitude and $d \ln b_n/dn$ may be positive, negative, or zero, depending on values of the principal quantum number and the electron density. Some applications of the theory are presented, based on recent calculations of the b_n factors for hydrogen and carbon.

THEORETICAL INTENSITIES OF RECOMBINATION LINES

Leo Goldberg

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I. INTRODUCTION

Early attempts to interpret the intensities of radio frequency recombination lines were based on the theory put forward by Kardashev (1959) to support his suggestion that such lines should be observable. Kardashev argued that since the populations of discrete hydrogen levels of high quantum number differed at most by only a few per cent from those in thermodynamic equilibrium, the deviations from equilibrium could be neglected. It has since become apparent, however, (Goldberg, 1966) that, while the absolute values of the level populations may be very close to equilibrium, only very slight differential departures in the equilibrium populations of the lower and upper levels of a transition may lead to population inversions and appreciable amplifications of the line intensity by stimulated emission. Furthermore, the populations of the high levels of multi-electron atoms may exceed their equilibrium values by as much as one or more orders of magnitude (Goldberg and Dupree, 1967).

II. THE TRANSFER EQUATION

In deriving expressions for the line intensities, we shall assume as a starting point that an H II region is a homogeneous medium emitting and absorbing radiation of frequency ν, both in the continuum by free-free transitions and in a discrete line by transitions between upper level n and lower level m. We recognize that H II regions are far from homogeneous and that they display large fluctuations in density, temperature and velocity. But in the absence of realistic models for the variations, we prefer to neglect them completely in the first approximation in the hope that applications of the theory will lead to the derivation of approximate models and that these in turn will provide the basis for a more realistic theory. We write the equation of transfer for a central ray as follows:

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \ , \tag{1}$$

where I_ν is the specific intensity, $d\tau_\nu$ is the increment of optical depth defined in the usual way by

$$d\tau_\nu = -k_\nu \; dx \quad , \tag{2}$$

and S_ν, the source function, is given by

$$S_\nu = j_\nu/k_\nu \quad ; \tag{3}$$

dx is the increment of path length in cm, k_ν is the absorption coefficient in cm^{-1}, and j_ν is the volume emission in ergs cm^{-3} sec^{-1} $ster^{-1}$. The specific intensity and the absorption and emission coefficients are written as the sum of contributions from the line and the continuum, respectively:

$$I_\nu = I_c + I_L \quad ; \tag{4}$$

$$k_\nu = k_c + k_L \quad ; \tag{5}$$

$$j_\nu = j_c + j_L \quad . \tag{6}$$

The solution of equation (1) for an isothermal cloud of constant density is

$$I_\nu = S_\nu(1 - e^{-\tau_\nu}) \quad . \tag{7}$$

Let r be the ratio of the emergent intensity in the line to that in the continuum. Then

$$r = \frac{I_\nu - I_c}{I_c} = \eta_\nu \frac{(1 - e^{-\tau_\nu})}{(1 - e^{-\tau_c})} - 1 \, , \tag{8}$$

where

$$\eta_\nu = S_\nu / B_\nu(T_e) \, ,$$

since the source function for the continuous radiation is the Planck function $B_\nu(T_e)$. Under conditions of thermodynamic equilibrium, the level populations are given by the Saha-Boltzmann equation for temperature T_e and hence $\eta_\nu = 1$. We use asterisks to denote the equilibrium values of the various quantities. Then the equilibrium value of r is

$$r^* = \frac{(1 - e^{-\tau_\nu{}^*})}{(1 - e^{-\tau_c})} - 1 \, . \tag{9}$$

In the optically-thin case, when $\tau_L << \tau_c << 1$,

$$r^* = \frac{\tau_L{}^*}{\tau_c} = \frac{k_L{}^*}{k_c} \, . \tag{10}$$

It is understood that the expressions for both $k_L{}^*$ and k_c include the factor $1 - \exp(-h_\nu/kT_e)$, which corrects for stimulated emission in thermodynamic equilibrium.

We now derive expressions for r in the more general case when the populations of atomic levels depart from their equilibrium values. The non-equilibrium populations affect the observed line intensities in two important respects. First, the transition rates are altered in direct proportion to the populations. Second, the factor by which the absorption coefficient must be multiplied to allow for stimulated emission becomes

$$1 - (b_n/b_m) \exp (-h\nu/kT_e), \tag{11}$$

where b is the ratio of the actual level population to the equilibrium value.

Hence when $b_n > b_m$, the absorption coefficient may be reduced below its equilibrium value on account of increased stimulated emission and may even assume negative values. When $b_n < b_m$, the absorption coefficient is greater than its equilibrium value as a consequence of reduced stimulated emission.

We first obtain a non-equilibrium expression for η_ν in terms of the b-values.

Let

$$S_\nu = \frac{j_c + j_L}{k_c + k_L} \quad , \tag{12}$$

with

$$j_c = k_c \, B(T_e) \quad , \tag{13}$$

and

$$j_L = b_n j_L^* = b_n k_L^* \, B(T_e) \quad , \tag{14}$$

where b_n is the ratio of the actual population N_n to that given by the Saha-Boltzmann equation, N_n^*. The line absorption coefficient may also be expressed in terms of its equilibrium value k_L^* by

$$k_L = b_m k_L^* \left[\frac{1 - (b_n/b_m) \exp(-h\nu/kT_e)}{1 - \exp(-h\nu/kT_e)} \right] \quad . \tag{15}$$

We evaluate $\eta_\nu = S_\nu/B_\nu(T_e)$ by substituting for S_ν its value in equation (12) and using equations (13), (14), and (15). We obtain

$$\eta_\nu = \frac{1 + b_n(k_L^*/k_c)}{1 + b_m(k_L^*/k_c)\beta} \tag{16}$$

where

$$\beta = \left[\frac{1 - (b_n/b_m)\exp(-h\nu/kT_e)}{1 - \exp(-h\nu/kT_e)}\right] . \qquad (17)$$

The increment of optical depth is

$$d\tau_\nu = -(k_c + k_L)\,dx \qquad (18)$$

and for a homogeneous cloud,

$$\tau_\nu = \tau_c\left(1 + b_m \frac{k_L^*}{k_c}\beta\right) . \qquad (19)$$

We note that when $T_e = 10^4$ °K and $\nu = 5000$ MHz $h\nu/kT_e = 2.40 \times 10^{-5}$. Hence equation (16) may be simplified by letting

$$\exp(-h\nu/kT_e) \approx 1 - \frac{h\nu}{kT_e} . \qquad (20)$$

We find

$$\beta \approx (b_n/b_m)\left(1 - \frac{kT_e}{h\nu}\frac{\Delta b}{b_n}\right) , \qquad (21)$$

where $\Delta b = b_n - b_m$. Since Δb is usually very small, we may put

$$\beta = (b_n/b_m)\left(1 - \frac{kT_e}{h\nu}\frac{d\,\ell n\, b_n}{dn}\Delta n\right) . \qquad (22)$$

When the gas is optically thin, we may approximate the ratio r by expanding the exponentials in equation (8), retaining the quadratic terms in τ_ν and τ_c. We obtain

$$r = \eta_\nu \frac{\tau_\nu(1 - \tau_\nu/2)}{\tau_c(1 - \tau_c/2)} - 1 \ . \qquad (23)$$

Substituting for η_ν and τ_ν from equations (16) and (19), we find

$$r/r^* = b_n - \frac{1}{2}\tau_c b_n \left[1 + \frac{b_m}{b_n}\beta(1 + r^* b_n)\right] . \qquad (24)$$

In most cases of interest, $|\beta| >> 1$ and $r^* b_n << 1$. Hence

$$r/r^* = b_n \left(1 - \frac{1}{2}\tau_c \frac{b_m}{b_n}\beta\right) . \qquad (25)$$

We put

$$r/r^* = b_n(1 + \frac{1}{2}\tau_c\gamma) , \qquad (26)$$

where

$$\gamma = \frac{d\,\ell n\, b_n}{dn}\Delta n \ .$$

Thus, both the absolute value of b_n and its rate of change with n cause the line intensity to deviate from its value in thermodynamic equilibrium. We note that the line should appear either in emission or absorption according to the sign and value of $d\,\ell n\, b_n/dn$ and the value of the optical depth in the continuum. When $(d\,\ell n\, b_n/dn) > 0$, the line will be in emission and its transition rate will be proportional to b_n. Furthermore, when $(kT_e/h\nu)(d\,\ell n\ b_n/dn)\Delta n$ exceeds unity, β is negative and the line intensity ratio r is increased by stimulated emission by an amount equal to $\tau_c r^* b_n \gamma/2$. Note that, according to equation (19), negative values of β may reduce the optical depth relative to its equilibrium value. Very large amplification by stimulated emission may occur when β assumes large negative values, such that $b_m(k_L^*/k_c)\beta < -1$ and the total optical depth becomes negative.

When $(d\,\ell n\ b_n/dn)$ is negative, $\beta > 1$ and approaches $kT_e/h\nu$ in the limit when $b_m - b_n \rightarrow b_m$. An optically thin recombination line will be observed in absorption when $d\,\ell n\ b_n/dn$ is negative and

$$\frac{1}{2}\ \tau_c\ b_m\beta\ >\ b_n \tag{27}$$

or

$$\frac{1}{2}\ \tau_c \left(1 - \frac{kT_e}{h\nu}\ \frac{d\,\ell n\ b_n}{dn}\ \Delta n\right) > 1\ ,$$

$$\tau_c \gtrsim - \frac{2h\nu}{kT_e}\ \frac{1}{(d\,\ell n\ b_n/dn)\ \Delta n}\ . \tag{28}$$

III. POPULATIONS OF ENERGY LEVELS

The populations of atomic energy levels must be calculated by equating the rates at which atoms enter and leave a given level by all relevant processes. The most important processes for the population of levels in H II regions are 1) electronic capture and cascade from higher levels, 2) three-body recombination by electron impact, and 3) collisional transitions from nearby levels by electron impact. The levels are depopulated principally by 1) spontaneous downward transitions, 2) ionization by electron impact, and 3) collisional transitions to nearby levels by electron impact. The bare nuclei of hydrogenic ions recombine only by direct radiative capture but ions with one or more electrons may also capture electrons by dielectronic recombination, which often occurs at a much higher rate than radiative capture and has an entirely different dependence upon n and ℓ. Hence we shall discuss the problem of level populations separately for the two kinds of ions.

A. Hydrogenic Atoms

Early calculations for hydrogenic atoms in H II regions were based on the assumption that the populations are determined purely by radiative processes, i.e., that the levels are occupied by radiative capture and cascade from higher levels and that they are depopulated by spontaneous transitions to lower levels. The capture-cascade equations for hydrogenic atoms with an infinite number of energy levels were first solved by Baker and Menzel (1938), by the so-called n-method, which assumes that atoms in a level with principal quantum number n will be distributed among the sub-levels $n\ell$ in proportion to their statistical weights $(2\ell + 1)$. The same problem was solved more exactly and elegantly by Seaton (1959). The capture-cascade equations for the populations of the sub-levels $n\ell$ were solved by Burgess (1958) for 12 levels, by Searle (1958) for 10 levels, and by Pengelly (1964) for an infinite number of levels.

The n-method implies that collisional transitions between sub-levels $n\ell$ and $n\ell'$ occur at a faster rate than radiative transitions between levels n and n', whereas the $n\ell$-method is valid only in the absence of collisions. Pengelly and Seaton (1964) showed that for $T_e = 10^4$ °K and $N_e = 10^4 \text{ cm}^{-3}$, the populations of the sub-levels are proportional to their statistical weights for values of $n \gtrsim 15$ in hydrogen and for values of $n \gtrsim 22$ in ionized helium. At lower densities the collisional $n\ell \rightarrow n\ell'$ transitions may be neglected up to correspondingly larger values of n, but no matter how low the electron density there will always be a value of n above which the collisional transitions must be taken into account. Similarly, Seaton (1964) showed that, for the same values of the temperature and electron density, the rate of collisional transitions $n \rightarrow n \pm 1$ exceeds that of spontaneous radiative transitions for $n \gtrsim 38$ and that collisional ionization is the predominant process for the depopulation of levels with $n \gtrsim 82$.

Seaton calculated the level populations of hydrogen including radiative processes, collisional ionization and three-body recombination and collisional $n \rightarrow n \pm 1$ transitions, assuming $T_e = 10^4$ °K and $N_e = 10^4 \text{ cm}^{-3}$. The dashed curve in Fig. 1 shows the resulting variation of b_n with n. Seaton's curve for the pure radiative case

($N_e = 0$) is also shown for comparison. The radiative Gaunt factors were taken as unity in the calculation of both curves. The level populations have also been calculated for values of N_e smaller than 10^4 cm^{-3} by Binh-Dy and McCarroll (1967), by Dyson (1967) and by Hayler (1967). Hayler's results for $N_e = 10, 10^2, 10^3, 10^4$, and 10^5 are shown in Figs. 1, 2, and 3. Accurate Gaunt factors and collisional transitions with $\Delta n = \pm 2$ were included in the rate equations. As the electron density increases, the transition from radiatively controlled to collisionally controlled populations occurs at progressively lower values of n.

Seaton (1964) showed that the intensity per unit frequency of the overlapped high series members of hydrogen in gaseous nebulae is proportional to b_n; he suggested that spectral measurements might give important information on physical processes in nebulae. Such measurements have recently been made on spectra of NGC 7027, NGC 7662 and the Orion nebula taken with the 100-inch and 200-inch telescopes of the Mt. Wilson and Palomar Observatories (Greenstein and Hayler, 1968). The results are in good agreement with Hayler's calculations for values of the electron density appropriate to each object.

In Fig. 2, the quantity log $(d\,\ell n\, b_n/dn)\,(kT_e/h\nu)$ or log γ is plotted against the principal quantum number n, with $T_e = 10^4$ °K and with $N_e = 10^4, 10^3, 10^2$, and 10 cm^{-3}. As N_e decreases, the maximum value of log γ increases and occurs at increasingly larger values of n. The effect of departures from thermal equilibrium on the line intensity is determined chiefly by the quantity $\tau_c\gamma/2$ in equation (25). The optical depth τ_c in the free-free continuum may be related to the frequency and emission measure by an approximate expression due to Altenhoff et al. (1960), which, for a temperature of 10^4 °K, does not deviate from the exact formula of Oster (1961) by more than five per cent in the range of frequencies 100-30,000 MHz. The approximate expression is

$$\tau_c = 8.235 \times 10^{-2} T_e^{-1.35} \nu^{-2.1} E \,, \tag{29}$$

where ν is in GHz and E, the emission measure, is in pc cm^{-6}. In Fig. 3, we have plotted log $\tau_c\gamma/2$ as a function of ν for four values of the electron density, with $T_e = 10^4$ °K and $E = 10^5$. The adopted value of E is well above the average for H II regions (Westerhout, 1958), and since high values of E tend to be associated with high values of N_e, it is obvious from the figure that appreciable amplification by stimulated emission is chiefly to be expected at frequencies below 5-10 GHz. The percentage amplification for an optically-thin layer is proportional to E and hence may easily be estimated from Fig. 3 for any value of E and values of N_e up to 10^4 cm^{-3}.

B. Comparison Between Theory and Observations

The brightness temperature of the free-free continuum in an optically thin gas is $T_c = T_e \cdot \tau_c$, with τ_c given by equation (29). Similarly, the brightness temperature of the line center is $T_L = T_e \cdot \tau_L$ where τ_L is the optical depth in the line center, the line absorption coefficient being determined by Doppler broadening. The resulting expression for r^* (Mezger and Höglund, 1967) is

$$\Delta\nu_L r^* = 2.036 \times 10^4 \, \nu_L^{2.1} \, T_e^{-1.15} , \tag{30}$$

where ν_L is the line frequency in GHz and $\Delta\nu_L$ is the half-power width in KHz, equal to 1.66 times the Doppler width $\Delta\nu_D$. Several authors have pointed out that the right-hand side of equation (30) should be divided by $1 + \left(N_{He^+} / N_{H^+}\right)$ to allow for the contribution of ionized He to the free-free continuum. The ratio N_{He^+} / N_{H^+} may be derived from the relative intensities of the adjacent α lines of H and He.

Equation (30) has been widely used for the determination of T_e from observations of $\Delta\nu_L \cdot r$ in a number of H II regions, on the assumption that $r = r^*$. From measurements of the 109α line at 5009 MHz in twenty galactic radio sources, Mezger and Höglund (1967) derived a mean value of 5800 °K, with individual values between 4000 °K and 9500 °K. Similarly, Gardner and McGee (1967), using the Parkes telescope, obtained an average temperature of about 5000 °K from observations of 126α at 3249 MHz in sixteen sources. Other investigators have obtained similiar results. Optical determinations of the electron temperature have tended to be substantially larger (Peimbert, 1967; Aller and Liller, 1959; O'Dell, 1966). For example, recent photoelectric measurements of forbidden-line intensities in the spectra of O^+, O^{+2} and N^+ give average temperatures of 10,700 °K, 8700 °K and 8400 °K for Orion, M8 and M17, respectively, whereas the corresponding values in °K from the 109α line are 6400^{+680}_{-1500}, 5790^{+880}_{-1410}, and 4910^{+520}_{-1150}, respectively.

If the discrepancy is caused by small departures from thermodynamic equilibrium (Goldberg, 1966), equation (26) rather than equation (30) should be used for the determination of electron temperatures. Peimbert (1967) believes that at least part of the discrepancy can be accounted for by temperature fluctuations in the ionized regions. Unfortunately, it is not a simple matter to calculate the effect of departures from equilibrium on the line intensities, because radio astronomical determinations of the emission measure and hence of τ_c usually refer only to the brightest central regions of a nebula, whereas the line observations are averaged over the entire nebula in most cases. As is evident from equation (9), $r^* \to 0$ in optically thick regions and, since τ_c varies roughly as ν^{-2}, as the frequency decreases the observed line intensity tends to refer more and more to those parts of the nebula where the emission measure and τ_c are low. Thus, because of saturation effects, it is difficult to observe the maser effect at the low frequencies where it should be most pronounced.

Since, according to equation (26), the departures from equilibrium (LTE) depend on τ_c and hence on the emission measure, Mezger and Höglund (1967) attempted to find a correlation between E and T_e as derived from the 109α line through equation (30). The result was inconclusive. Dieter (1967) attempted to find a like correlation for the 158α line, also with negative results. It should be noted that the maser effect is proportional to $(d \ln b_n/dn)$ as well as to τ_c and that high values of E are associated with large N_e and therefore with small $d \ln b_n/dn$.

Gardner and McGee (1967) showed that in thermodynamic equilibrium the intensity ratio of the lines 158β and 126α, which have nearly the same frequency, is 0.224, whereas their observed values in M17 and Orion are 0.22 and 0.13, respectively. A similar comparison of β/α intensities by Palmer et al. (1967), shows that the relative intensities of the 137β and 109α lines in Orion M17 and IC 434 are less than the LTE value of 0.28 by varying amounts, the observed ratios being 0.19, 0.24, and 0.22, respectively. Williams (1967) has measured the ratios 197β/156α and 225γ/156α, and finds frequent large departures from the theoretical ratios. Similarly, the ratio of the 148δ and 94α lines in Orion is about 20 per cent higher than the calculated LTE value (Gordon and Meeks, 1967).

A more decisive observational test of departures from equilibrium can be made from a comparison of intensities of β and α transitions from the same upper level, as proposed by Gardner and McGee (1967), by Palmer (1967), and by Dyson (1967). It has been shown that such observations can lead to unambiguous determinations of both T_e and of $d\,\ell n\, b_n/dn$, by the solution of two simultaneous equations. The relevant measurements have not yet been carried out.

C. Non-hydrogenic Atoms

Burgess (1964) has shown that under appropriate conditions, notably those of high temperature and low density, the total rate of dielectronic recombination may exceed that of radiative recombination, by as much as two to three orders of magnitude. Application of this discovery to the calculation of ionization equilibria in the solar corona seems to have removed the long-standing discrepancy between the ionization temperature of the corona and values derived by other methods.

Dielectronic recombination must also play an important role in planetary nebulae and in H II regions. Since the largest contributions to the total rate of dielectronic recombination are made by electrons with energies slightly less than E_{ij}, the energy of the resonance transition in the recombining ion, the rate is approximately proportional to $\exp(-E_{ij}/kT_e)$. Table 1 gives values of E_{ij} for a number of ions known to be abundant in gaseous nebulae, together with values of α_d and α_r calculated by Dupree (1967) for temperatures of

10,000 °K and 20,000 °K, using Burgess' (1965a) approximate general formula for α_d and a hydrogenic expression due to Seaton (1959) for α_r. Note that there is very little if any correlation between E_{ij} and the ionization potential, because most of the resonance transitions involve no change in the principal quantum number. For most ions, the total rate of radiative recombination is substantially greater than the dielectronic rate, even when the temperature is as high as 20,000 °K. Si, C, and low stages of S, Ca and Mg are notable exceptions and require appropriate handling in calculations of ionization equilibrium.

Even more important than its influence on the total rate of recombination is the effect of dielectronic recombination on the populations of highly-excited levels (Goldberg and Dupree, 1967). The rate coefficient for radiative capture is approximately proportional to n^{-3} (Seaton, 1959) and therefore favors the occupation of the lowest levels. Conversely, the rate coefficient for dielectronic recombination has a totally different dependence on n and ℓ, which strongly favors the population of very high levels. Even when the total rate of radiative capture is much larger than the dielectronic rate, the latter process may be more important for the high levels. Thus, the distribution of atoms among the excited states in a radiatively controlled environment can be totally different in the case of a complex atom than for hydrogen.

The effect is illustrated by approximate calculations of the b_n factors for Ca I (Goldberg and Dupree, 1967), which were performed for $T_e = 10^4$ °K and $N_e = 10^2$ and 10^4 cm^{-3}. The levels were assumed to be populated by dielectronic recombination, by three-body electronic recombination, by cascades from higher levels and depopulated by collisional ionization and by spontaneous downward transitions. The rate coefficients for the dielectronic recombination of Ca II were taken from Burgess (1965b), and hydrogenic approximations were used for the other cross-sections. The results are given in Table 2. The b_n's increase with increasing n until they reach a maximum at some value of the principal quantum number n_{max}, which is determined by the electron density. They then decrease rapidly toward the series limit as collisional ionization assumes control of the level populations.

We may estimate the numerical relationship between n_{max} and N_e by assuming that the maximum value of b_n occurs when the rate C_{ni} of collisional ionization out of level n becomes equal to the total rate A_n of spontaneous transitions downward. We write (Seaton 1959, 1964)

$$\frac{C_{ni}}{A_n} = \frac{4.38 \times 10^{-15}\, n_{max}^{7}\, T_e^{-1/2}\, N_e}{3\, \ell n\, n_{max} - 3.13} = 1 \quad , \qquad (31)$$

where C_{ni} is derived from classical theory and A_n is the hydrogenic rate summed over all lower levels from $n'' = 4$ to $n'' = n - 1$. With $T_e = 10^4$, $N_e = 10^4$ we obtain $n_{max} = 80$, and with $T_e = 10^4$, $N_e = 10^2$, $n_{max} = 155$, in good agreement with the values in Table 2.

At the time the above calculations were made, the rate coefficients for dielectronic recombination in carbon were unknown, but a rough extrapolation of the results for calcium indicated that a pair of radio recombination lines observed by Palmer et al. (1967) in the objects NGC 2024 and IC 1795 could reasonably be attributed to the 109α and 110α lines of C I. Quite recently, the relevant rate coefficients for the recombination of singly-ionized carbon have been computed by Shore (1967) and employed in the solution of the rate equations by Neal Baker (1967). We re-examine the carbon identification in the light of these improved calculations.

If we assume that the lines observed by Palmer et al. (1967) are indeed the 109α and 110α lines of C I, their intensities require that the upper levels of both transitions be overpopulated by a factor of about 60, provided that both the carbon lines and the neighboring hydrogen lines are emitted in the same volume and that the carbon-hydrogen abundance ratio has the solar value 5×10^{-4}. The results of Baker's calculations are given in Figs. 4, 5, and 6. Figure 4 shows the variation of b_n with n for $T_e = 10^4$ °K and $N_e = 10, 10^2, 10^3$, and 10^4 cm^{-3}. The values of b_n shown are too small to account for the observed intensities. As shown in Fig. 5, the b_n's may be greatly increased by raising the temperature, but a value much higher than 10,000 °K seems unreasonable. Another possibility is that of amplification by stimulated emission but this seems to be ruled out because db_n/dn is zero at $n = 100$ when $N_e = 10^2\ cm^{-3}$ and is negative at larger values of the electron density.

Finally, Fig. 6 shows values of r and r^* calculated as a function of n for both H and C with $T_e = 10,000$ °K, $N_e = 100\ cm^{-3}$, and an emission measure $E = 1.8 \times 10^5\ cm^{-6}$ pc. The observed ratio r_H/r_C is on the order of 10, whereas the calculated values of r in Fig. 6 imply a ratio of about 1000 at $n = 100$. Note that, with increasing n, the intensity r_C decreases rapidly toward zero as τ_C increases and, in accordance with equation (28), the line goes over into absorption. The value of r_C at $n = 110$ is increased by an order of magnitude when the temperature is raised to 14,000 °K, but the steep rate of fall of r_C with n is in conflict with the observation that the 109α and 110α lines in NGC 2024 are equal in intensity.

IV. DISCUSSION

Thus, barring unexpected errors in the cross-sections, the process of overpopulating high levels by dielectronic recombination does not seem to account quantitatively for the anomalous intensities of the supposed carbon lines, unless the region of line formation is considerably greater in volume and/or density than that producing the hydrogen lines. It is also possible that the lines are formed by radiative recombination in an H I region, where all of the carbon should be singly-ionized (Seaton, 1951). This possibility was also examined by Goldberg and Dupree (1967) and ruled out because the generally accepted value of 10^{-3} cm^{-3} for the electron density was too low to produce recombination lines in a volume of reasonable size. Recently, however, Pikel'ner (1967) and Spitzer and Tomasko (1967) have shown that the hydrogen in H I regions may be partially ionized by energetic particles, and that the electron density in clouds may be on the order of 10^{-2} cm^{-3} or greater. A second development of significance is the revelation that the interstellar densities of O, C^+, Si^+, and Al^+ inferred from rocket ultraviolet spectra of δ and π Scorpii are about 100 times greater than would be expected from observations of the 21-cm line of neutral hydrogen (Stone and Morton, 1967). The assumption by Stone and Morton that the ultraviolet lines are formed in the H I regions is supported by the presence of the O I line, since oxygen should be ionized in H II regions.

It would be interesting to try to observe radio recombination lines in the direction of δ and π Scorpii. The expected brightness temperature may be estimated from the equation (Kardashev, 1959)

$$T_B = \frac{10^6 \cdot E}{T_e^{3/2} \cdot \Delta\nu_D} , \qquad (32)$$

which is valid in thermal equilibrium. From the observed intensity of the C^+ resonance line $\lambda1334.5$, Stone and Morton found 9×10^{18} cm^{-2} C^+ atoms along the line of sight to the two stars.

The distance is 170 pc, and therefore the mean density of C^+ atoms is 1.5×10^{-2} cm^{-3}, which is also a lower limit to the mean density of free electrons. Since much of the gas in the line of sight will be concentrated in clouds, the actual electron density will be greater. Hence, it seems reasonable to adopt $N_e = 5.0 \times 10^{-2}$ cm^{-3} and $T_e = 60$ °K, values which are consistent with Pikel'ner's density-temperature model. Finally, we take $\Delta\nu_D = 50$ kHz, the value observed in NGC 2024 (Palmer _et al_. 1967). The emission measure is found to be 0.15 and $T_B \sim 7 \times 10^{-4}$ °K. The brightness temperature seems rather too small to be detected but its value might be increased considerably by departures from thermal equilibrium and by integration over a greater path length. Considering how little is known about the composition, structure, and physical state of the interstellar medium, any calculated value of the expected intensity of the radio carbon lines is hardly more than a guess. It is not even certain whether such lines formed in H I regions will appear in emission or in absorption, since the appearance of the lines will depend on whether the brightness of the background emission is larger or smaller than the source function of the spectral line, and the required b_n factors have not yet been calculated for temperatures on the order of 100 °K.

To conclude, it is not yet clear whether the lines attributed to the 109α and 110α lines of C I are formed in H II or in H I regions. Additional observations are urgently needed at both higher and lower frequencies in order to provide clues to the correct recombination mechanism and hence to the physical conditions under which the lines are being formed.

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TABLE 1

Total rate coefficients for recombination, α_d, and α_r, for $T_e = 10^4$ and 2×10^4 °K.

			$T_e = 10^4$ °K		$T_e = 2.0 \times 10^4$ °K	
Ion	I.P.(eV)	E_{ij}(eV)	α_d	α_r	α_d	α_r
He I	24.6	40.8	6.42-30*	4.12-13	4.30-20	2.48-13
C I	11.3	9.3	1.47-14	2.06-13	1.14-12	1.17-13
II	24.4	12.6	2.88-15	1.34-12	1.60-12	7.70-13
III	47.9	8.0	3.61-13	3.73-12	1.33-11	2.22-12
IV	64.5	308.0	negl.	7.45-12	negl.	4.54-12
N I	14.5	11.4	8.58-16	2.06-13	2.30-13	1.17-13
II	29.6	12.5	7.82-16	1.34-12	3.93-13	7.70-13
III	47.4	16.2	8.34-17	3.73-12	3.58-13	2.22-12
IV	77.5	10.0	4.98-14	7.45-12	5.83-12	4.54-12
V	97.9	431.0	negl.	1.28-11	negl.	7.82-12
O I	13.6	14.9	4.58-17	2.06-13	9.02-14	1.17-13
II	35.1	14.9	4.67-17	1.34-12	9.15-14	7.70-13
III	54.9	15.7	3.41-17	3.73-12	1.10-13	2.22-12
IV	77.4	19.7	2.11-18	7.45-12	6.82-14	4.54-12
V	113.9	12.0	6.63-15	1.28-11	2.46-12	7.82-12
Ne I	21.6	26.9	4.18-23	2.06-13	8.75-17	1.17-13
II	41.1	25.3	5.75-22	1.34-12	4.83-16	7.70-13
III	63.5	22.8	2.40-20	3.73-12	4.82-15	2.22-12
IV	97.1	21.7	4.38-20	7.45-12	4.59-15	4.54-12
V	126.2	22.1	5.06-20	1.28-11	6.63-15	7.82-12
VI	157.9	26.6	1.49-21	1.99-11	2.76-15	1.22-11
Mg I	7.6	4.4	9.45-12	1.19-13	4.38-11	6.45-14
II	15.0	53.5	negl.	9.29-13	negl.	5.00-13
III	80.1	38.6	negl.	3.73-12	1.55-19	2.22-12
Si I	8.2	6.9	9.43-13	1.19-13	1.78-11	6.45-14
II	16.3	10.3	1.00-13	9.29-13	1.38-11	5.00-13
III	33.5	8.9	3.72-13	2.78-12	2.30-11	1.55-12
IV	45.1	105.0	negl.	5.83-12	negl.	3.37-12
V	166.8	50.4	negl.	1.28-11	negl.	7.82-12

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* The entry 6.42-30 stands for 6.42×10^{-30}, etc.

Table 1 - continued

			$T_e = 10^4$ °K		$T_e = 2.0 \times 10^4$ °K	
Ion	I.P.(eV)	E_{ij}(eV)	α_d	α_r	α_d	α_r
S I	10.4	9.8	3.25-14*	1.19-13	3.47-12	6.45-14
II	23.4	10.4	1.35-14	9.29-13	2.01-12	5.00-13
III	35.0	11.7	9.05-15	2.78-12	2.78-12	1.55-12
IV	47.3	15.8	5.52-16	5.83-12	1.83-12	3.37-12
V	72.7	13.3	5.80-15	1.03-11	4.58-12	5.98-12
Ca I	6.1	3.1	2.75-11	7.42-14	6.05-11	3.72-14
II	11.9	25.3	8.93-21	6.05-13	7.37-15	3.37-13
III	51.2	18.9	7.42-19	2.78-12	1.52-14	1.55-12
IV	67.3	19.4	1.66-18	5.83-12	3.96-14	3.37-12
V	84.5	19.6	1.37-18	1.03-11	4.10-14	5.98-12

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* The entry 6.42-30 stands for 6.42 × 10^{-30}, etc.

TABLE II

Approximate b_n-factors for Ca I with $T_e = 10^4$ °K

n	b_n	
	$N_e = 10^2$	$N_e = 10^4$
10	77	66
20	234	209
50	1230	1068
100	4270	645
200	2000	22

Fig. 1. Solid curves: b_n factors for hydrogen, $T_e = 10^4$ °K, after Hayler (1967). Dashed curve: calculations by Seaton (1964) for $N_e = 10^4 \mathrm{cm}^{-3}$, $T_e = 10^4$ °K.

Fig. 2. Variation of log γ with principal quantum number n for different values of the electron density N_e (Hayler, 1967).

Fig. 3. Variation of the quantity log $\frac{1}{2}\gamma\tau_c$ with the frequency of recombination lines in GHz, for different values of the electron density. (Hayler, 1967).

Fig. 4. b_n factors for C I calculated for $T_e = 10^4$ °K and different values of the electron density. (Baker, 1967).

Fig. 5. b_n factors for C I calculated for $N_e = 10^2 \mathrm{cm}^{-3}$ and three different values of the electron temperature. (Baker, 1967).

Fig. 6. Variation of r_H^*, r_c and r_c^* with n, calculated for $T_e = 10^4$ °K and $N_e = 10^2 \mathrm{cm}^{-3}$. (Baker, 1967).

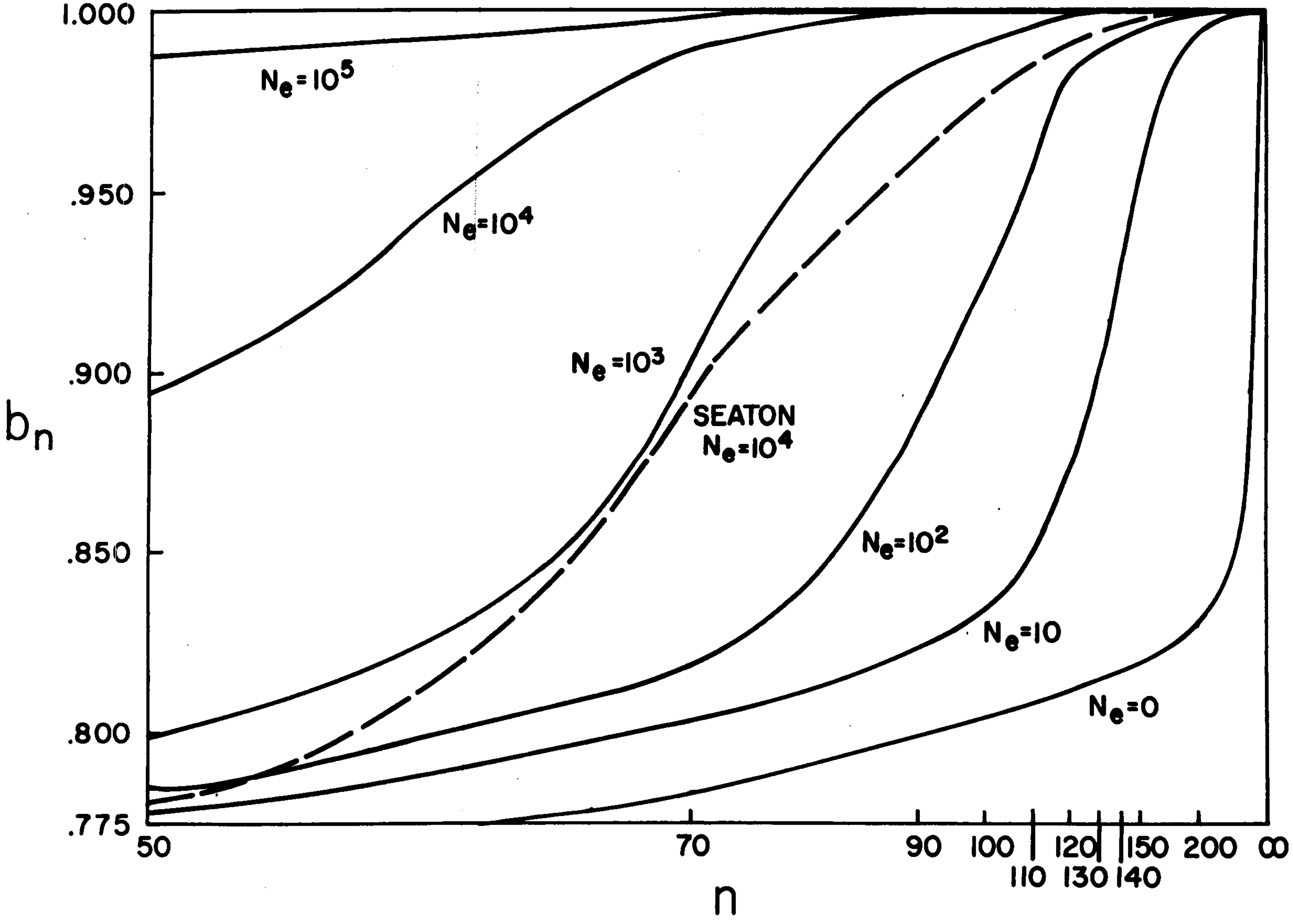

Fig. 1. Solid curves: b_n factors for hydrogen, $T_e = 10^4$ °K, after Hayler (1967). Dashed Curve: calculations by Seaton (1964) for $N_e = 10^4 cm^{-3}$, $T_e = 10^4$ °K.

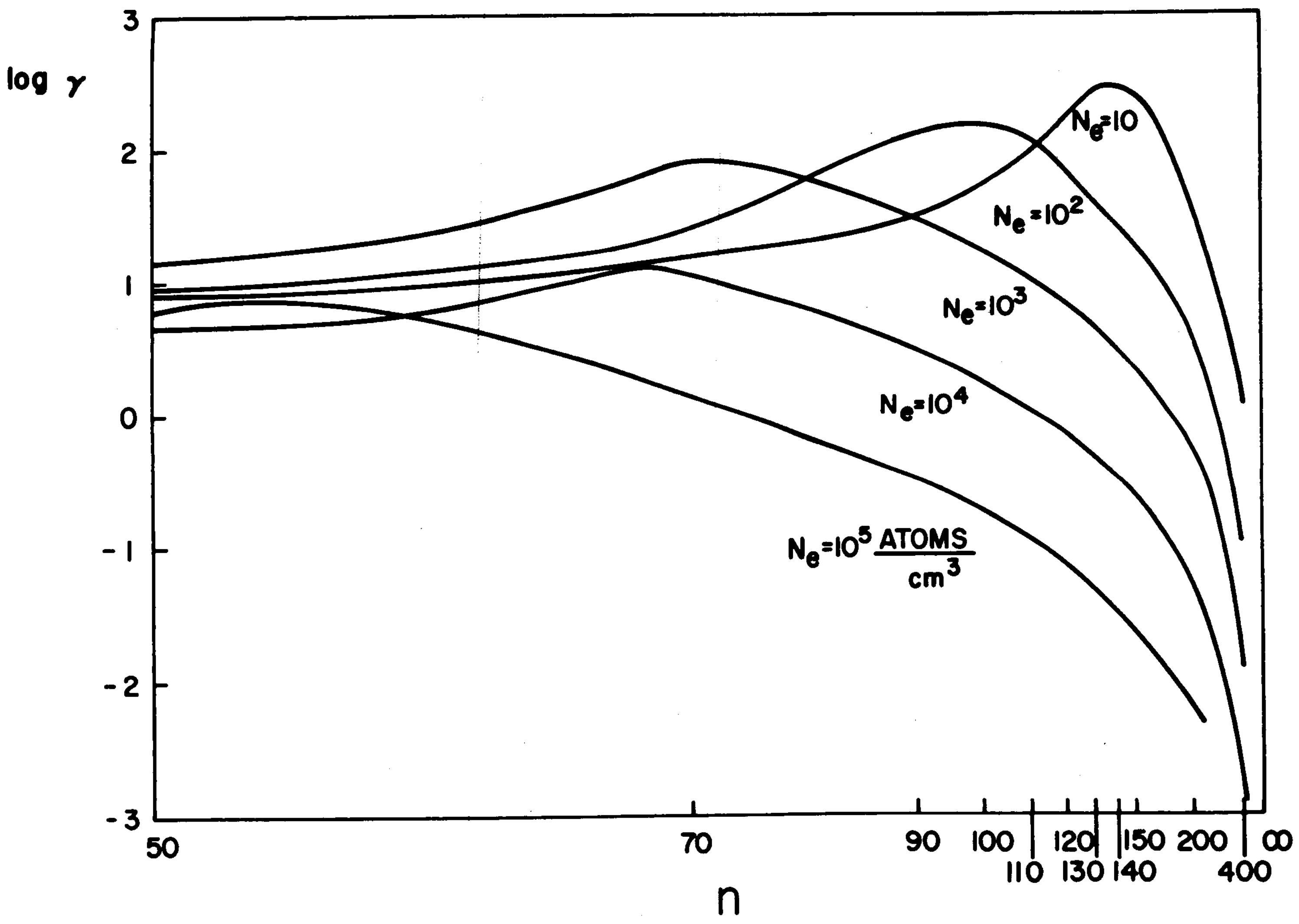

Fig. 2. Variation of log γ with principal quantum number n for different values of the electron density N_e (Hayler, 1967).

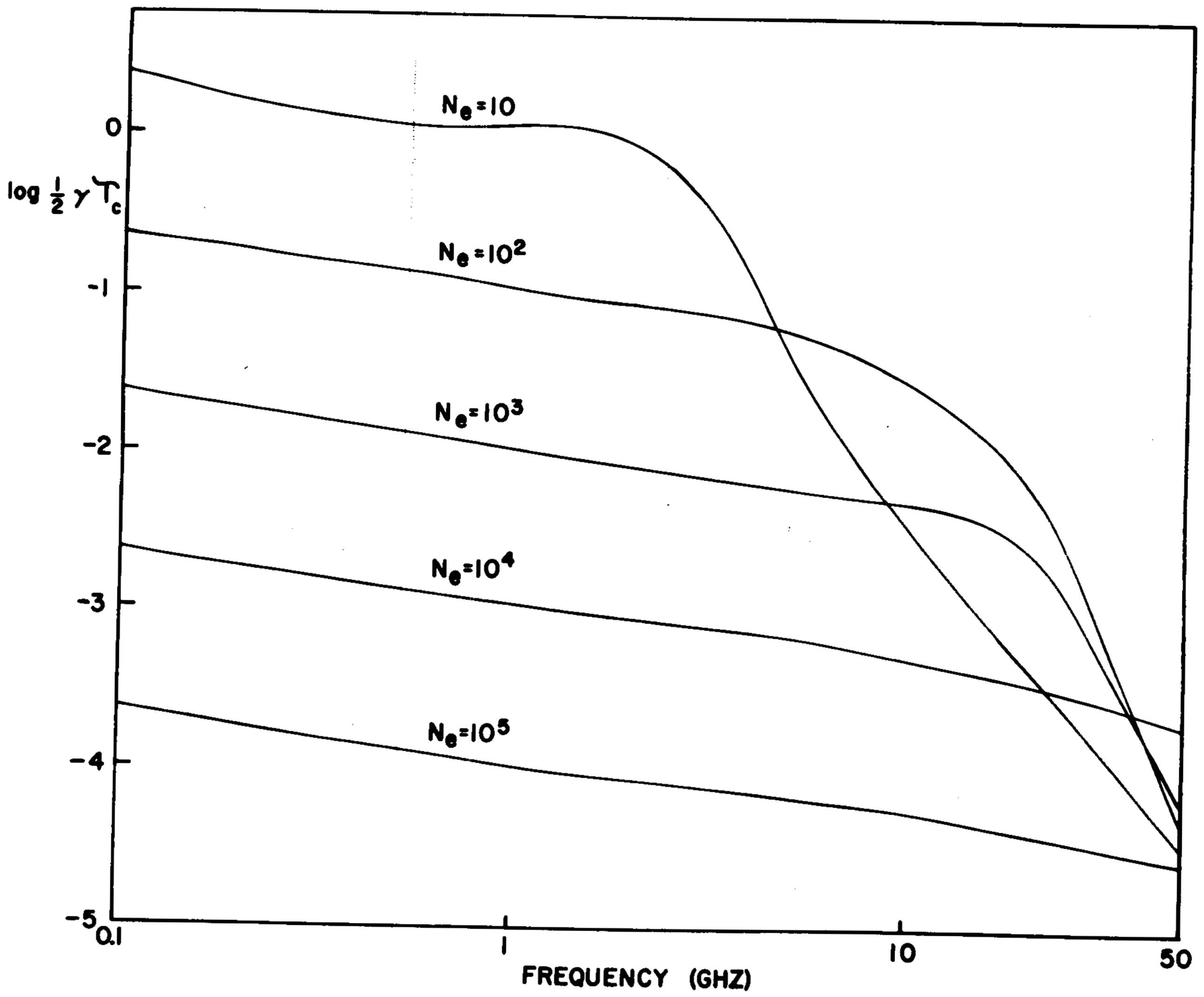

Fig. 3. Variation of the quantity $\log \frac{1}{2} \gamma \tau_c$ with the frequency of recombination lines in GHz, for different values of the electron density. (Hayler, 1967).

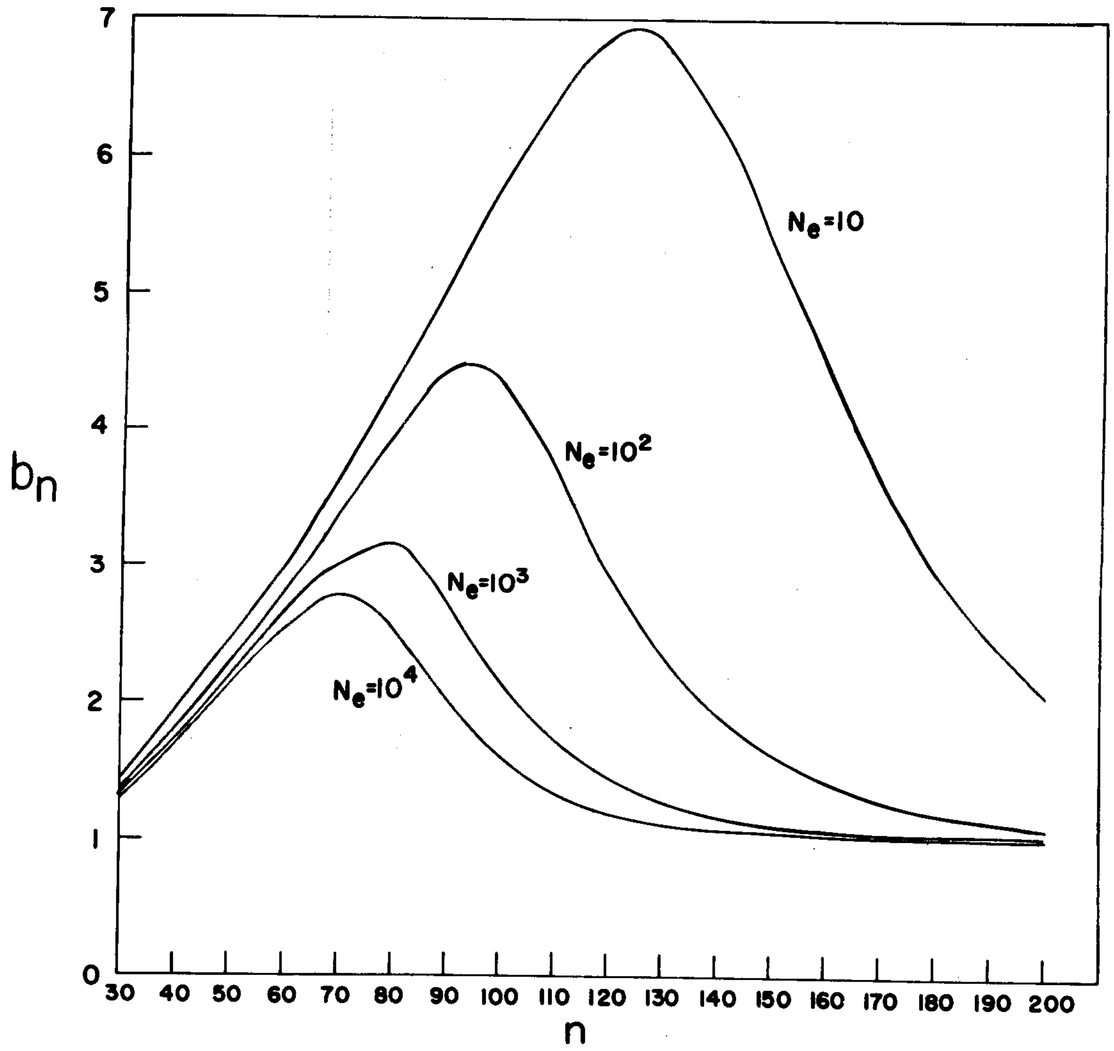

Fig. 4. b_n factors for C I calculated for $T_e = 10^4$ °K and different values of the electron density, (Baker, 1967).

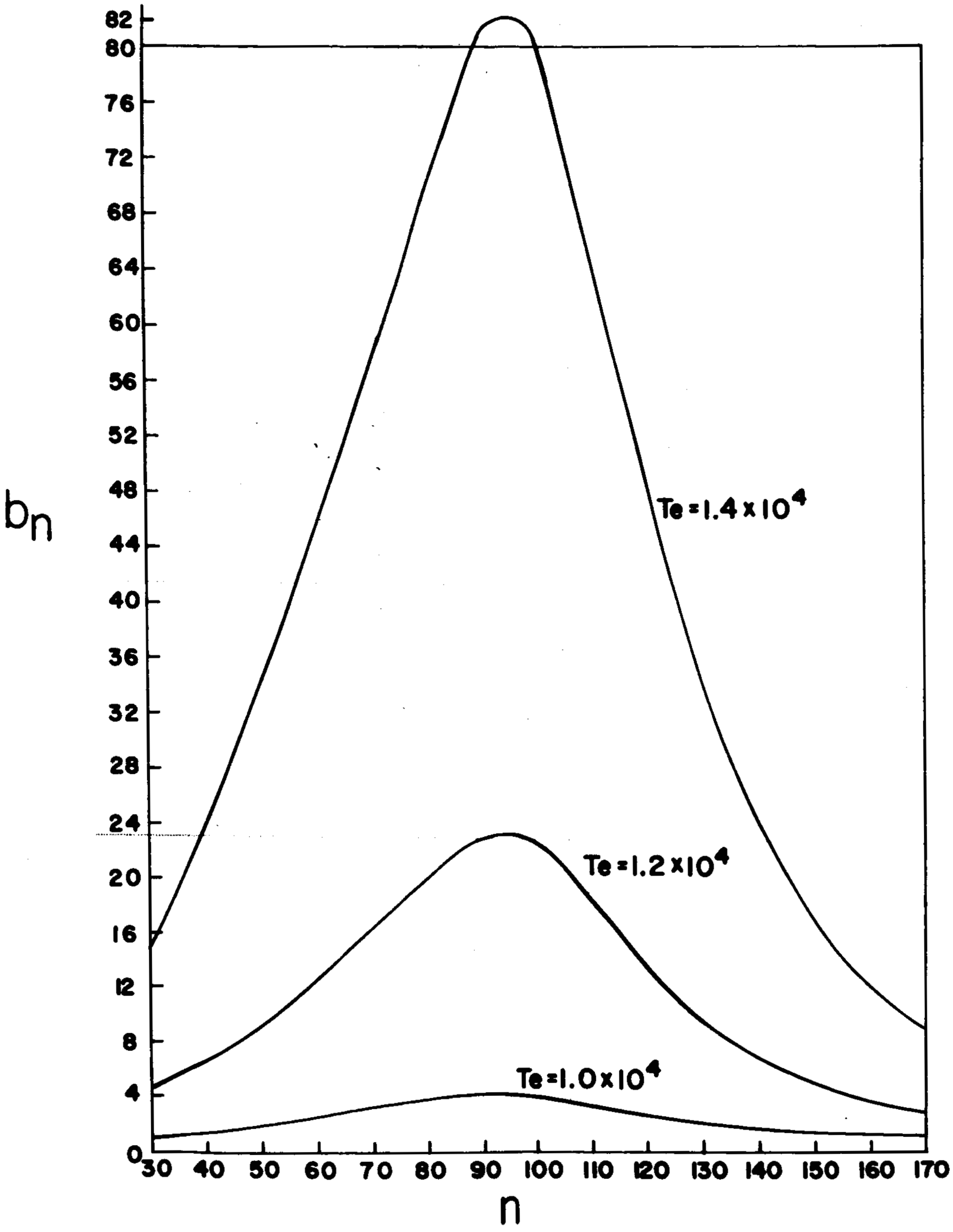

Fig. 5. b_n factors for C I calculated for $N_e = 10^2 cm^{-3}$ and three different values of the electron temperature. (Baker, 1967).

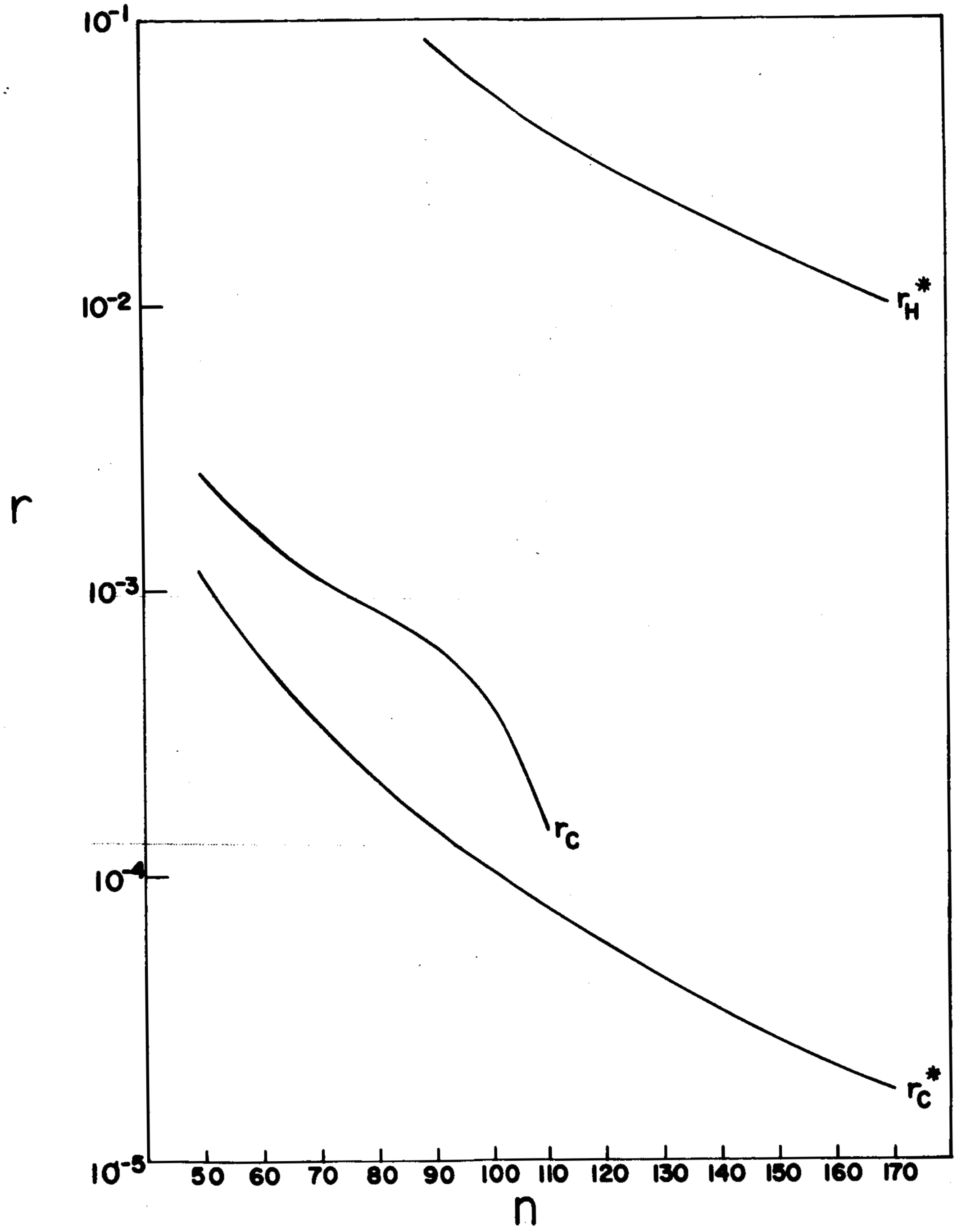

Fig. 6. Variation of r_H^*, r_c and r_c^* with n, calculated for $T_e = 10^4$ °K and $N_e = 10^2 \text{cm}^{-3}$. (Baker, 1967).