THE ROSENBLUTH-FERRARO-DUNGEY SHEATH

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Abstract

This document reviews and evaluates the Rosenbluth-Ferraro-Dungey model of the interaction region between a cold collisionless plasma and a vacuum magnetic field.

It is our belief that the microstructure of plasma-vacuum interfaces is far from well represented by this model. We believe that its use in magnetospheric models has been successful only because the sheath characteristics used have been especially insensitive to the fine structure of this model.

1. Review and Discussion of the Rosenbluth-Ferraro-Dungey Sheath

The Rosenbluth-Ferraro-Dungey sheath is a simplified model of the transition region between a cold, collisionless, streaming plasma and a vacuum magnetic field. It has been used as the microscopic model for the interaction of the solar wind and strong planetary magnetic fields (Dungey 1958, Ferraro 1952, and Beard 1960, 1964) and as a model of the sheath in a pinch collapse (Rosenbluth 1963).

In these early papers, the following sheath physics was expected (see Figure 1). The plasma streams in from the left and impinges on the magnetic field which extends off to the right. As the plasma advances into the magnetic field, the electrons are turned first by the u x B forces acting on them. The more massive ions advance further into the magnetic field until they are turned by an electric field due to the charge separation. Both species then stream away from the sheath eventually regaining their original velocities. On this basis, one can distinguish four distinct regions in the interaction. Far to the left of the interface is the region where only the smallest deviations from free streaming occur. In this region the electric and magnetic fields and the induced currents and charges are very small and asymptotically approach zero at minus infinity. We will call this the "far upstream" region. Moving toward the interface, the next region is the "magnetic sheath". In this region the electric field is very small but the magnetic field grows rapidly and almost reaches its vacuum value. Next, is the charge separation region. At its left end the electrons are turned around and at its right end the ions are turned around. Electric fields are large and the region contains only ions. Finally, the entire right half space is the vacuum region and contains no particles from the plasma stream. It does contain, however, a magnetic field and at plus infinity the current sheet which supports it.

Historically, sheath analysis began with estimates of the length scale associated with the magnetic sheath λ_m. (See for example Dungey 1958, Ferraro 1952 and Beard 1960.) These estimates were based on approximations valid in the far upstream and magnetic sheath regions but invalid in the charge separation region. These approximations and calculations are reviewed in Section 3. Analysis gave for the magnetic sheath scale:

$$\lambda_m = \left[\frac{1}{2\mu_0 e^2 N} \frac{mM}{m+M}\right]^{\frac{1}{2}}$$

$$= 3.75 \text{ km} \frac{1}{\sqrt{N \text{ cm}^3}}$$

Since the number density, N, in the solar wind is of the order of ten particles per cubic centimeter, the magnetic sheath is very thin compared to the scale of the magnetosphere (∿60,000 km) so it has been idealized in magnetospheric models as vanishingly thin and the electric field effects have been consistantly neglected.

An important result of these early studies was the formulation of the "pressure balance" relation based on conservation of momentum considerations. These lead to the following jump condition between quantities at plus and minus infinity (see Section 2).

$$2N(m+M)U^2 = \frac{B^2}{2\mu_0}$$

Because this result is exact and does not depend on a detailed knowledge of the interaction physics, it plays the same important role here that the Rankine-Hugoniot relations play in collision dominated shock theory.

Recently however, critical examination of this sheath model has indicated some of its limitations. It was not until Sestero (1965) and Davies (1967) performed a numerical integration of the sheath equations that the fine structure of the charge separation region was obtained. Based on their calculations, Sestero and Davies concluded that the previous physical picture of the sheath was incorrect. In the solutions they obtained, either electrons or ions could turn first and the entire sheath could either be neutral or contain excess charge depending on the values of upstream particle density and velocity (details are presented in Section 3). Davies calculated that for the parameters appropriate to the solar wind, electrons turned first but the sheath was not neutral.

The excess charge introduces an undesirable electric field in the vacuum region whose magnitude is -10 volts/m for the solar wind parameters. Parker (1967), unhappy even with the electric field which exists in the charge separation region when the sheath is neutral, suggested that particles from the ionosphere could easily travel along the magnetic field lines and discharge the layer. If this is the case, the more extensive field caused by the excess charge will also be discharged by drawing particles either from the interior of the magnetosphere or from the ionosphere. In this case, electric field effects will be negligible and a study of neutral sheaths would be required. Parker and Lerche (1967) have suggested however, that even neutral sheaths may be physically unacceptable since the presence of streaming velocities parallel to the interface causes secondary vacuum magnetic fields which require either another source current or high particle pressures to assure confinement.

In addition to these detail difficulties with the simple stationary sheath model, no demonstration has been made to show that the sheath is stable. In fact, it is more reasonable to believe that it is not. Counterstreaming of the cold plasmas occurs throughout the left half space of Figure 1 and electrostatic instabilities seem unavoidable. What one expects is that initially electron streaming instabilities dominate and rapid electron thermalization occurs. Subsequently the ions are thermalized as they stream through the warm electrons. The details of these processes and the description of the final state are beyond the scope of linear theories.

All in all, it is the author's belief that the microstructure of plasma-vacuum interfaces is far from well represented by the Rosenbluth-Ferraro-Dungey sheath. We believe that the use of the model in magnetospheric models has been successful only because the sheath characteristics used have been especially insensitive to the fine structure.

2. Formulation of the Problem

The self consistant system of equations governing the cold, collisionless sheath has been formulated with varying degrees of completeness in previous papers (Ferraro 1952, Dungey 1958, Rosenbluth 1963, Sestero 1964, 1965, Bertotti 1963, Bernstein and Rabinowitz 1959, Beard 1960, 1964, and Davies 1967). We will present only the barest outline here so that the equations will be at hand.

We wish to consider the confinement of a one dimensional plasma, that is, a plasma in which all the parameters depend on only a single space variable, x. The Vlasov equation for a hydrogenic plasma and Maxwell's equations for the fields are:

$$u\frac{\partial f_\pm}{\partial x} \pm \frac{e}{m_\pm}\left[\left\{E_x + v B_z - w B_y\right\}\frac{\partial f_\pm}{\partial u} - u B_z \frac{\partial f_\pm}{\partial v} + u B_y \frac{\partial f_\pm}{\partial w}\right] = 0 \tag{1}$$

and

$$\frac{\partial B_x}{\partial x} = 0 = \frac{\partial E_z}{\partial x} = \frac{\partial E_y}{\partial x} \tag{2}$$

$$\frac{\partial E_x}{\partial x} = \frac{q(x)}{\varepsilon_0} \tag{3}$$

$$J_x = 0 \tag{4}$$

$$J_y = -\frac{1}{\mu_0}\frac{\partial B_z}{\partial x} \tag{5}$$

$$J_z = \frac{1}{\mu_0}\frac{\partial B_y}{\partial x} \tag{6}$$

The charge distribution and the currents may be written in terms of the distribution function as:

$$q(x) = \sum_{\text{species}} \pm e \int_{-\infty}^{+\infty} d^3v \, f_\pm \tag{7}$$

and

$$J(x) = \sum_{\text{species}} \pm e \int_{-\infty}^{+\infty} d^3v \, v \, f_\pm \tag{8}$$

Potentials are introduced for B and E.

$$\begin{aligned} B &= \nabla \times A \\ &= \left[0, -\frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x}\right] \end{aligned}$$

and (9)

$$\begin{aligned} E &= -\nabla \phi \\ &= \left[-\frac{\partial \phi}{\partial x}, 0, 0\right] \end{aligned}$$

Because collisions have been neglected, one can obtain constants of the motion for each species by integrating the subsidiary Lagrange equations (Chandrasekhar 1962). The Vlasov equations will then be satisfied by any distribution functions which depend only on these constants. They are:

$$C_1 = \frac{m_\pm}{2}\left(u^2+v^2+w^2\right) \pm e\phi$$

$$C_2 = v \pm \frac{eA_y}{m_\pm} \tag{10}$$

$$C_3 = w \pm \frac{eA_z}{m_\pm}$$

A change of variables from [u, v, w] to [C_1, C_2, C_3] permits the evaluation of the charge and currents when a distribution function is chosen. In specifying the distribution functions we will require at minus infinity that: a) the plasma be cold and have a center of mass velocity [U_x, U_y, U_z] and that b) the electric and magnetic fields be zero and that c) the values of the field potentials ϕ and A go to zero. Since these potentials can only be found to within an additive constant we are free to make this choice.

The distribution function chosen is:

$$f_{\pm}(C_1, C_2, C_3) = m_{\pm}^3 |U_x| N \; \delta\left(C_1 - \frac{m_{\pm}}{2}\{U_x^2 + U_y^2 + U_z^2\}\right) \delta(C_2 - U_y)\; \delta(C_3 - U_z) \tag{11}$$

and when this is introduced into (7) and (8) one obtains:

$$q_{\pm} = \frac{\pm 2eN \left|\frac{U_x}{U}\right|}{\left[1 \mp \frac{2e\phi}{m_{\pm}U^2} - \left(\frac{U_y}{U} \mp \frac{eA_y}{m_{\pm}U}\right)^2 - \left(\frac{U_z}{U} \mp \frac{eA_z}{m_{\pm}U}\right)^2\right]^{\frac{1}{2}}} \tag{12}$$

$$J_{x\pm} = 0 \tag{13}$$

$$J_{y\pm} = \frac{\pm 2eN \left|\frac{U_x}{U}\right| \left(U_y \mp \frac{eA_y}{m_{\pm}}\right)}{\left[1 \mp \frac{2e\phi}{m_{\pm}U^2} - \left(\frac{U_y}{U} \mp \frac{eA_y}{m_{\pm}U}\right)^2 - \left(\frac{U_z}{U} \mp \frac{eA_z}{m_{\pm}U}\right)^2\right]^{\frac{1}{2}}} \tag{14}$$

$$J_{z\pm} = \frac{\pm 2eN \left|\frac{U_x}{U}\right| \left(U_z \mp \frac{eA_z}{m_{\pm}}\right)}{\left[1 \mp \frac{2e\phi}{m_{\pm}U^2} - \left(\frac{U_y}{U} \mp \frac{eA_y}{m_{\pm}U}\right)^2 - \left(\frac{U_z}{U} \mp \frac{eA_z}{m_{\pm}U}\right)^2\right]^{\frac{1}{2}}} \tag{15}$$

The self consistant system of equations describing the sheath is then:

$$-\frac{\partial^2\phi}{\partial x^2} = \sum_{\text{SPECIES}} \frac{q_\pm(\phi, A_y, A_z)}{\varepsilon_o} \tag{16}$$

$$-\frac{\partial^2 A_y}{\partial x^2} = \sum_{\text{SPECIES}} \mu_o J_{y\pm}(\phi, A_y, A_z) \tag{17}$$

$$-\frac{\partial^2 A_z}{\partial x^2} = \sum_{\text{SPECIES}} \mu_o J_{z\pm}(\phi, A_y, A_z) \tag{18}$$

and the boundary conditions are

$$\left.\begin{aligned} \phi &= 0 = \frac{\partial\phi}{\partial x} && (19)\\ A_y &= 0 = \frac{\partial A_y}{\partial x} && (20)\\ A_z &= 0 = \frac{\partial A_z}{\partial x} && (21) \end{aligned}\right\} \text{ at } x = -\infty$$

The general three dimensional collisionless system has an exact integral which can be obtained by multiplying the Vlasov equations through by the velocity vector (u, v, w), adding the two Vlasov equations and integrating over the entire velocity space.

Currents and charge densities which appear can be replaced using Maxwell's equations. The following result is obtained.

$$\nabla \cdot \left[\sum \int_{-\infty}^{+\infty} d^3v\, m_\pm \mathbf{v}\mathbf{v} f_\pm - \epsilon_o \mathbf{E}\mathbf{E} - \frac{1}{\mu_o}\mathbf{B}\mathbf{B} + \left(\frac{\epsilon_o E^2}{2} + \frac{B^2}{2\mu_o} \right) \mathrm{I} \right] = 0 \tag{22}$$

For the system considered here this reduces to:

$$2N(m+M)U_x^2 = \frac{B_y^2 + B_z^2}{2\mu_o} - \frac{\epsilon_o E_x^2}{2}$$

$$+ 2N|U_x| U \sum m_\pm \left[1 \mp \frac{2e\phi}{m_\pm U^2} - \left(\frac{U_y}{U} \mp \frac{eA_y}{m_\pm U} \right)^2 - \left(\frac{U_z}{U} \mp \frac{eA_z}{m_\pm U} \right)^2 \right]^{\frac{1}{2}} \tag{23}$$

This integral is frequently called the "pressure balance" relation and expresses the conservation of momentum normal to the interface. It is of considerable importance because it is an exact integral of the system of equations and does not depend on the detail structure of the sheath. It provides a jump condition at the interface which plays the same macroscopic role as the Rankine-Hugoniot relations in collision dominated shock theory.

In Equations (12) through (15) and again in (23) square roots of the form

$$\left[1 \mp \frac{2e\phi}{m_\pm U^2} - \left(\frac{U_y}{U} \mp \frac{eA_y}{m_\pm U} \right)^2 - \left(\frac{U_z}{U} \mp \frac{eA_z}{m_\pm U} \right)^2 \right]^{\frac{1}{2}}$$

are to be taken. These forms are particularly important in the analysis of the sheath because they specify the relations between the variables at the particle turning points. Using the constants of the motion given in (11) it can easily be shown that

$$1 \mp \frac{2e\phi}{m_\pm U^2} - \left(\frac{U_y}{U} \mp \frac{eA_y}{m_\pm U} \right)^2 - \left(\frac{U_z}{U} \mp \frac{eA_z}{m_\pm U} \right)^2 = \frac{u^2}{U^2} \tag{24}$$

Since this form is always positive the square root is always real. It is zero at the turning points where u is zero. The locus of turning point coordinates in the (A_y, A_z, ϕ) space can be obtained therefore by setting (24) equal to zero. For example, in Figure 2 the upper curve labeled ITC (ion turning curve) is obtained by setting (24) equal to zero for the ions while the lower curve labeled ETC is obtained by setting (24) equal to zero for the electrons. A typical solution crosses both the ITC and the ETC with the electrons being turned back at the ETC and the ions at the ITC.

Finally, because of the separation of the ion and electron turning points, the expressions for $q_\pm$ and $J_\pm$ given in (13) to (16) properly apply only before either species is turned back. Once a species is turned back, its contribution to charge and current is zero and the appropriate term in (13) to (16) is deleted.

3. The "Straight In" Case

The simplest configuration occurs when U_y and U_z are taken to be zero. In this case, particles far from the sheath have no velocity components parallel to the interface and stream "straight in" to the sheath.

A consistant solution of (18) then has A_z equal to zero everywhere which in turn means that w is zero and the particles move in the x,y plane as shown in Figure 1. The remaining equations still cannot be solved in closed form in general but one exact solution does exist. For the special cases in which the electron and ion masses are equal the electric field is zero everywhere and the magnetic field can be obtained analytically. Consider the asymptotic form of (16) in the far upstream region where A_y and ϕ are small.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{2e^2 N(m+M)}{\epsilon_0 U^2 mM}\phi = \frac{e^3 N}{\epsilon_0 U^2}\left(\frac{1}{m^2} - \frac{1}{M^2}\right) A_y^2 + O\left(A_y^3, \phi^2\right) \qquad (25)$$

Now if m=M, (25) has no solution which goes to zero at minus infinity except the trivial one $\phi=0$. Then (17) becomes

$$\frac{\partial^2 A_y}{\partial x^2} = \frac{4e^2 N \mu_0}{M} \frac{A_y}{\left[1 - \frac{e^2 A_y^2}{M^2 U^2}\right]^{\frac{1}{2}}} \qquad (26)$$

and this is integrable (see for example Dungey 1958). The length scale associated with the size of the magnetic sheath is found to be:

$$\lambda_m = \left[\frac{M}{4e^2 N \mu_0} \right]^{\frac{1}{2}} \tag{27}$$

While this solution is exact it is of relatively little interest because it shows none of the phenomena associated with the electric field and is based on an unphysical mass ratio.

An historically important but approximate result can also be obtained using the charge neutral assumption which attempts to include small electric field effects by replacing Poisson's Equation (16) with the algebraic condition that $\Sigma q_{\pm}$ be zero. This replacement was justified on the physical grounds that the plasma cannot support significant charge separation because of its high conductivity. The condition that charge separation be negligible leads to the relation:

$$\phi = \frac{e(M-m)}{2Mm} A_y^2 \tag{28}$$

Then (17) becomes

$$\frac{\partial^2 A_y}{\partial x^2} = 2e^2 \mu_0 N \frac{m+M}{mM} \frac{A_y}{\left[1 - \frac{e^2 A_y^2}{mMU^2} \right]^{\frac{1}{2}}} \tag{29}$$

which is identical in form to (26) and therefore has the same integral. The length scale associated with the magnetic sheath now shows the unequal masses and is:

$$\lambda_m = \left[\frac{mM}{2e^2 \mu_o N(m+M)} \right]^{\frac{1}{2}} \tag{30}$$

The same results can be obtained in a much more satisfying way, however, by considering the asymptotic forms of ϕ and A_y at minus infinity (Sestero 1965). Where ϕ and A_y are small, (25) shows the asymptotic behavior of ϕ and (31) shows the asymptotic behavior of A_y:

$$\frac{\partial^2 A_y}{\partial x^2} = 2e^2 N \mu_o \frac{m+M}{mM} A_y + O(A_y^2, \phi^2) \tag{31}$$

The asymptotic forms can be obtained immediately by integration.

$$A_y = A_y(-\infty) \operatorname{Exp}\left[\left(\frac{m+M}{mM} 2e^2 N \mu_o \right)^{\frac{1}{2}} x \right] \tag{32}$$

$$\phi = \frac{e(M-m)}{2Mm} \frac{1}{1 + \frac{4u^2}{c^2}} A_y^2 \tag{33}$$

and

$$\lambda_m = \left[\frac{mM}{(M+M) 2\mu_o N e^2} \right]^{\frac{1}{2}} \tag{34}$$

Note that (33) contains only the non-homogeneous part of ϕ because the homogeneous part does not go to zero at minus infinity.

All of the previous analysis deals only with the regions far from the interface. The fine structure of the sheath due to electric field effects has not proved amenable to analysis. An effort has been made however to integrate the equations numerically and thereby obtain a complete solution.

In 1965, Sestero presented the results of his numerical integration of the equations governing the sheath. Then in 1967, Davies performed a similar integration extending the results to include relativistic effects. Both investigators found anomalous behavior of the solutions. Prior to their calculations it was generally believed that ions, because of their greater momentum, would always be turned back after the electrons and that the sheath would be neutral in the sense that the excess positive charge in the charge separation region would just cancel the excess negative charge in the magnetic sheath. Both Sestero and Davies produced solutions in which this was not the case. In some cases, ions were turned back first and in general the sheaths were not neutral but had excess charge which could be either positive or negative.

The integrations of Sestero and Davies are performed by using the asymptotic forms for ϕ and A_y to obtain starting values for the functions. Equations (16) and (17) are then numerically integrated and the momentum integral (23) used to provide a check on the integration. Sestero's solutions are shown in Figure 2 in the A_y, ϕ space. The trajectory of a solution is traced out in this space as an implicit function of x. Consider the solution for U/c = .45. Starting at small values of A_y and ϕ in the lower left corner, the solution moves to the right passing through the electron turning curve, then through the ion turning curve and finally becomes linear in the region beyond the last turning point. The slope in this linear region gives the ratio of the electric and magnetic fields so the fact that the slope is non-zero shows the existence of an electric field in the vacuum region.

The existence of this electric field is an unexpected result of the calculations. In order for the solution to be consistant with the boundary conditions at minus infinity it is necessary to have not only a current source at plus infinity but also a charge source. Such a charge source was not part of the original conceptualization of the sheath which was to be a transition region between a field free plasma and a vacuum magnetic field. In geophysical applications, for example, while the magnetic field is easily identified as due to currents inside the earth the existence of a large scale electric field is difficult to imagine. This is because there are so many charged particles present in the magnetosphere that neutralization seems highly likely. The addition of sheath curvature effects will somewhat reduce the extent of the field by causing a $1/R^2$ fall off in the field but this will not solve the conceptual problems associated with the electric field.

That such a charge is actually necessary can be seen by considering the electric field due to the plasma charge and charge $Q\delta(x-\infty)$. Since the charge depends only on the single space variable x and if the last turning point is arbitrarily taken to be at x=0 then:

$$E_x(x) = \frac{1}{2\varepsilon_0}\int_{-\infty}^{+\infty} dx'\, q(x')\,\frac{x-x'}{|x-x'|} - \frac{Q}{2\varepsilon_0} \tag{35}$$

If x is less than zero, the field in the plasma is obtained.

$$E_x(x<0) = \frac{1}{2\varepsilon_0}\int_{-\infty}^{x} dx'\, q(x') - \frac{1}{2\varepsilon_0}\int_{x}^{0} dx'\, q(x') - \frac{Q}{2\varepsilon_0} \tag{36}$$

Introducing the asymptotic form for q in the first integral and taking the limit as x goes to minus infinity we obtain:

$$E_x(-\infty) = \lim_{x \to -\infty} \frac{4U^2/c^2}{1 + 4U^2/c^2} \frac{e^3 N}{\varepsilon_o U^2} \left(\frac{1}{M^2} - \frac{1}{m^2} \right) \frac{\lambda_m}{2} A_y^2(-\infty)\, e^{2x/\lambda_m}$$

$$- \frac{1}{2\varepsilon_o} \int_{-\infty}^{0} dx'\, q(x') - \frac{Q}{2\varepsilon_o}$$

$$= - \frac{1}{2\varepsilon_o} \int_{-\infty}^{0} dx'\, q(x') - \frac{Q}{2\varepsilon_o} \qquad (37)$$

But the boundary condition at minus infinity is that E_x be zero; therefore the additional charge is equal to the excess sheath charge but opposite in sign. The additional charge need not be all at plus infinity but can be spread throughout the positive half space. The only requirement is that its sum be the value specified above.

4. Neutral Sheaths

Analysis shows the existence of electric fields in both the charge separation region and the vacuum field region. Parker (1967), discussing the electric field in the charge separation region, suggested that such a field would be rapidly discharged in the geophysical case by charged particles drawn from the ionosphere since all the geomagnetic field lines pass through the ionosphere at high latitude and particle diffusion along the field lines is relatively unhindered. If these arguments are valid for the charge separation field, they will equally apply to the vacuum electric field caused by the excess sheath charge.

Since it seems likely that electric fields cannot exist for long at the boundary of the magnetosphere, we wish to consider the possible configurations of neutral sheaths. In a neutral sheath, we distinguish an active streaming plasma of solar origin and an immobile passive plasma of magnetospheric origin. The passive plasma does not produce currents at the sheath but only acts to neutralize charge imbalance in the active plasma causing ϕ to be zero everywhere. Now the problem is to solve for A_y and A_z using (17), (18), (20), and (21). The passive charge distribution required is given by:

$$q_{PASSIVE} = -\sum q_{\pm}(0, A_y, A_z) \qquad (38)$$

In the "straight in" case, where U_y and U_z are both zero, A_z is zero everywhere and A_y is obtained by solving (39).

$$\frac{\partial^2 A_y}{\partial x^2} = 2\mu_o N e^2 A_y \left[\frac{1/M}{\left[1 - \frac{e^2 A_y^2}{M^2 U^2}\right]^{\frac{1}{2}}} + \frac{1/m}{\left[1 - \frac{e^2 A_y^2}{m^2 U^2}\right]^{\frac{1}{2}}} \right] \tag{39}$$

Again A_y falls off exponentially at minus infinity with the same length scale given in (34). Further, Figure 2 shows that the electrons will always turn back first. The physics of the "straight in" interaction holds no surprises.

If, however, one permits non-zero values of U_y and U_z, thereby introducing streaming components parallel to the interface, new physical effects occur. We note first that there is no translational invariance in either the y or z direction. The solutions therefore are not simply those of the "straight in" case with parallel components superimposed. Rather, since these parallel streaming components give rise to additional currents, the physics is more complicated. Consider, for example, the addition of the streaming velocity U_z. There is now a negative current in the z direction in the magnetic sheath due to the upward streaming electrons and a positive current in the charge separation region due to the upward streaming ions. These currents give rise to the magnetic field potential A_z. This extra component is undesirable, however, since it necessitates the addition of an unphysical current at plus infinity or the addition of electron and ion thermal effects (Lerche 1967).

5. Conclusions and Research Directions

The Rosenbluth-Ferraro-Dungey sheath is probably a poor microscopic model of the transition region between a streaming plasma and a magnetic field. The extraneous electric and magnetic fields required by consistant solutions of the general sheath equations introduce conceptual difficulties which are unavoidable for arbitrary directions of the incident plasma. We are presently integrating (17) and (18) for the general neutral sheath to estimate the magnitude of the induced fields but we do not expect the results to change the general conclusions.

We attribute the success of the model not to its detailed correctness but rather to the fact that the sheath characteristics modeled have been independent of the detail. For example, a particularly important result, the "pressure balance", Equation (23) is correct for <u>any</u> sheath which exists even if it is warm as long as the variations in field variables normal to the sheath are larger than the variations parallel to the sheath.

The warm sheath, a better model, is much more difficult to quantify however because of the existence of trapped orbits and the non-stationary character of the flow. While some effort has been made in this direction (Lerche 1967) the results lack generality. We hope to continue our research with warm sheaths and study numerical simulations of these transition regions.

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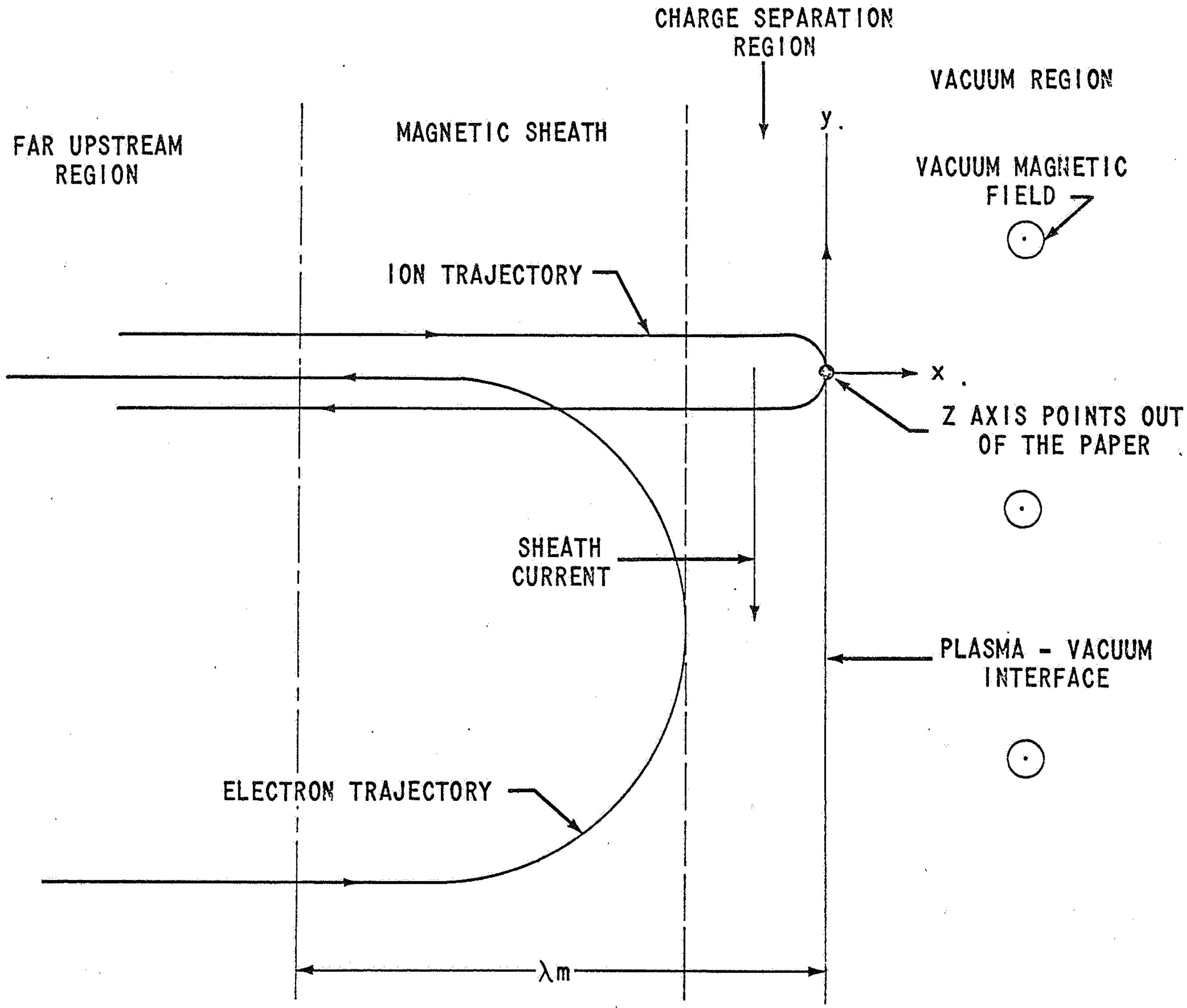

FIGURE I - SCHEMATIC OF THE ROSENBLUTH-FERRARO-DUNGEY SHEATH

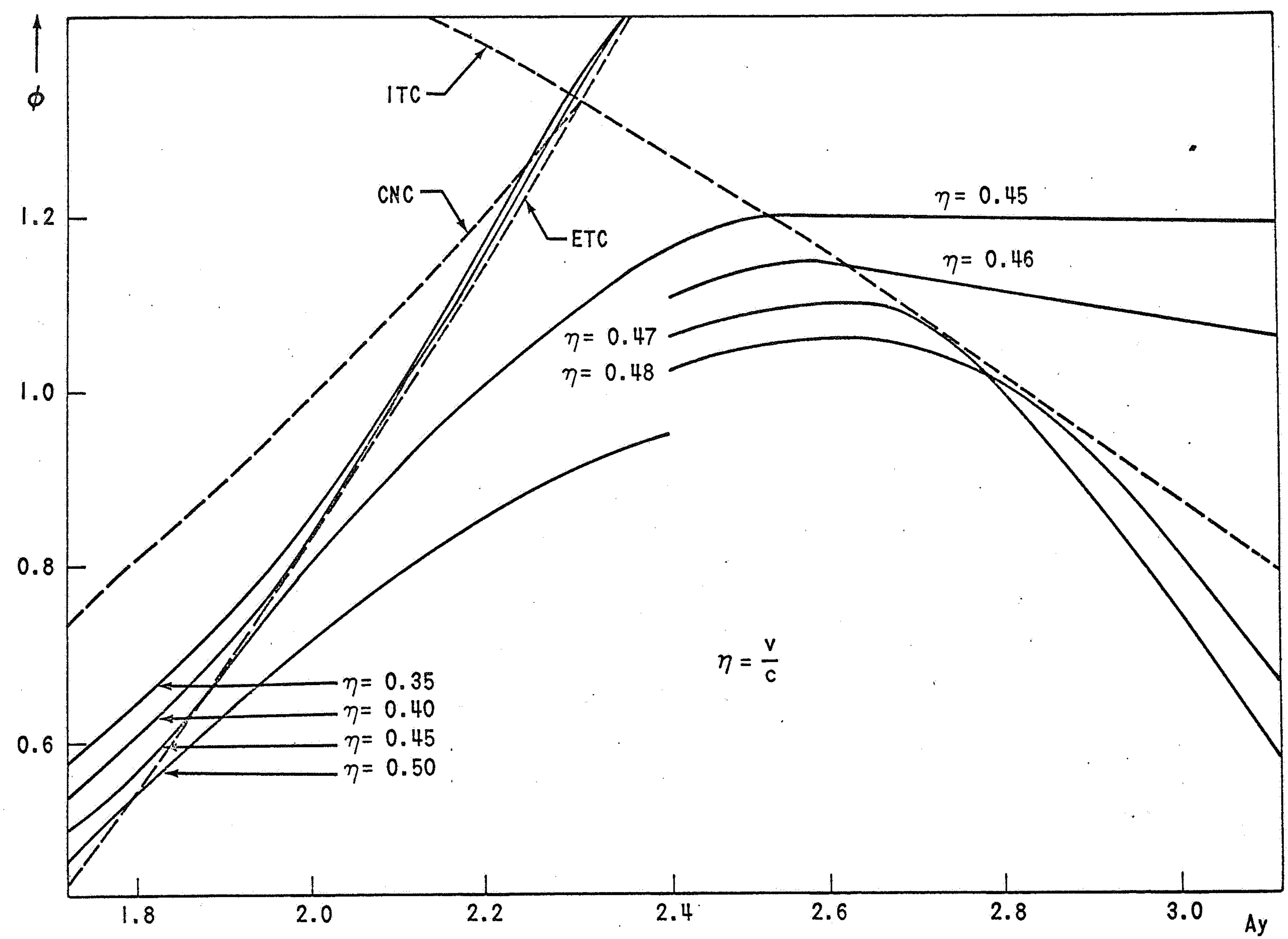

FIGURE 2 - BEHAVIOR OF THE SOLUTIONS NEAR THE ELECTRON AND ION TURNING CURVES (SESTERO 1965)