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NATURAL FREQUENCIES OF ORTHOTROPIC CIRCULAR PLATES OF VARIABLE THICKNESS

by

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DEPARTMENT
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ABSTRACT

The present investigation is concerned with the natural frequencies of orthotropic circular plates of variable thickness. In particular, a thickness variation of the form $h = h_o (1-HR^n)$ has been selected. The derivation of the differential equation governing the motion of the plate is based on the classical formulation of the theory of plates. The solution of this equation for the axisymmetric case is obtained by an application of the method of Frobenius. Characteristic equations for the natural frequencies of clamped and simply supported plates are derived and numerical results are presented for several plates of various shapes.

LIST OF SYMBOLS

a	plate radius
a_o, b_o, c_o, d_o	undetermined constants in deflection solution
a_k, b_k, c_k, d_k	coefficients of Frobenius series for specific exponents of singularity
A_k, B_k, C_k, D_k	$= a_k/a_o$, b_k/b_o, c_k/c_o, d_k/d_o, respectively
D_o	$= E_r h_o^3/12$
D_r	$= E_r h^3/12$
E_r, E_φ, $E_{r\varphi}$	material constants
f	frequency, cycles/sec.
$\bar{f}$	$= [(a^2/h_o)\sqrt{(\rho/E_r)}]\, f$
g_k	coefficients of Frobenius series in general form
G	material constant
h	plate thickness
h_o, h_a	thickness of the plate at r=0, r=a, respectively

H	$= 1-(h_a/h_o)$, non-dimensional thickness coefficient
j	exponents of singularity in Frobenius series
k	index of Frobenius series
M_r, M_φ, $M_{r\varphi}$	radial, circumferential, and twisting moments per unit length
n	exponent of thickness expression
q	lateral load per unit area
Q_r	transverse shear force per unit length
r	radial coordinate
R	$= r/a$, non-dimensional radial coordinate
t	time
w	lateral displacement
X, Y, Z	expressions defined by Eqs. (18), (19), (20), respectively
z	coordinate normal to and measured from the median surface of the plate

α	$= E_{r\varphi}/E_r$, non-dimensional material constant
β	$= E_{\varphi}/E_r$, non-dimensional material constant
γ	$= G/E_r$, non-dimensional material constant
δ	step function notation
ϵ_r, ϵ_{φ}	radial and circumferential normal strains
$\gamma_{r\varphi}$	shearing strain
λ	$= \rho\omega^2 h_o a^4/D_o$, a frequency parameter
ρ	mass density of the plate
σ_r, σ_{φ}	radial and circumferential bending stresses
$\tau_{r\varphi}$	shearing stress
φ	circumferential coordinate
ω	circular frequency, rad./sec.
( ),	partial derivative of ( ) with respect to the subscripts following the comma
( )′	= d( )/dR

I. INTRODUCTION

The literature of recent years contains many analyses of isotropic and orthotropic circular plates of both uniform and variable thickness. The transverse vibrations of cylindrically orthotropic circular plates has been analyzed by Pandalai and Patel [1], and Minkarah and Hoppmann [2]. In these investigations clamped as well as simply supported plates of constant thickness were considered. The present analysis is an extension of this work and concerns itself with the investigation of the natural frequencies of orthotropic circular plates of variable thickness.

In the classical formulation of problems concerning plates of variable thickness the effects of both transverse shear deformation and transverse normal stress are neglected, and hence, the stress solutions do not satisfy the prescribed surface tractions at the upper and lower surfaces of the plate. However, it has been shown by Essenberg [3] that the displacements predicted by the theory are reliable provided the maximum thickness of the plate is small compared to the radius of the plate. Of course the slope of the plate surface is small compared to unity. In view of this argument, the classical theory of thin plates is assumed valid for the present investigation.

The material properties of the plates considered in past investigations ([1], [2]) and in the present analysis as well are characterized by the principal directions of orthotropy at a point; these being the radial and circumferential directions. Such orthotropy may occur naturally in some cross sections of wood, or may be manufactured, at least approximately, by reinforced plastics, by thin impregnated laminations wound around a cylindrical core, or by isotropic plates which have been stiffened by radial or circumferential ribs. It has been found by Carrier [4] that, as a consequence of this form of orthotropy, the origin of the coordinate system represents a singular point of the material properties of the plate. The effect of this singularity on the displacement solution is discussed in the last section of the thesis.

In what follows, the governing equation for the free vibration of such plates is derived in terms of the lateral deflections of their median surface. This equation is then solved, for the axisymmetric case, by the method of Frobenius. The characteristic equations for the natural frequencies are derived for a clamped and a simply supported plate, and some numerical examples are considered. Finally, a brief discussion of the results is presented.

2. DERIVATION OF THE GOVERNING EQUATION

First, it is assumed that the circular plates analyzed in the present investigation are governed by the small deflection theory of plates. That is, the following simplifying assumptions are made: (1) The maximum thickness of the plate is small in comparison with the radius of the plate. (2) The magnitude of the lateral deflection is small compared to the local thickness of the plate. (3) The rotations are very small compared to the strains and as a result the stretching of the median surface of the plate is considered negligible. (4) An element of the plate along a normal to the median surface in the undeformed plate remains straight and normal to the deformed median surface, and its extension is negligible. Hence, the transverse shear strains are taken to be zero.

The above assumptions lead to the strain displacement relations

$$\epsilon_r = -z\, w_{,rr}$$

$$\epsilon_\varphi = -z\left(\frac{w_{,r}}{r} + \frac{w_{,\varphi\varphi}}{r^2}\right) \tag{1}$$

$$\gamma_{r\varphi} = -2z\left(\frac{w_{,\varphi}}{r}\right)_{,r}$$

in polar coordinates which are the most convenient for the present problem.

In the above equation, ϵ_r and ϵ_φ are the radial and circumferential normal strains, $\gamma_{r\varphi}$ is the shearing strain, w is the lateral displacement of the median surface of the plate, r and φ are the radial and circumferential coordinates, z is the coordinate normal to and measured from the median surface of the plate (see Fig. 1), and, finally, a comma after a symbol denotes partial differentiation of the symbol with respect to the coordinates indicated by the subscripts following the comma.

Next, it is assumed that the plates are made of orthotropic material, i.e., the elastic properties of the plate in the radial and circumferential directions are different. In view of the fact that the small deflection theory of the plate is assumed to be valid, the pertinent stress-strain relations for such a plate may be written [5] as

$$\begin{aligned} \sigma_r &= E_r \, \epsilon_r + E_{r\varphi} \, \epsilon_\varphi \\ \sigma_\varphi &= E_{r\varphi} \, \epsilon_r + E_\varphi \, \epsilon_\varphi \\ \tau_{r\varphi} &= G \, \gamma_{r\varphi} \end{aligned} \tag{2}$$

where σ_r and σ_φ are normal stresses in the radial and circumferential directions, $\tau_{r\varphi}$ is the shearing stress, and E_r, E_φ, $E_{r\varphi}$ and G are the material constants. These stress-strain relations together with the strain displacement relations of Eq. (1) may now be used to derive expressions for the bending moments per unit length in terms of the deflection as

$$M_r = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_r \, zdz = - D_r \left[w,_{rr} + \alpha \left(\frac{w,_r}{r} + \frac{w,_{\varphi\varphi}}{r^2} \right) \right]$$

$$M_\varphi = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\varphi \, zdz = - D_r \left[\alpha w,_{rr} + \beta \left(\frac{w,_r}{r} + \frac{w,_{\varphi\varphi}}{r^2} \right) \right] \quad (3)$$

$$M_{r\varphi} = - \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{r\varphi} \, zdz = 2D_r \gamma \left(\frac{w,_\varphi}{r} \right),_r$$

where, M_r, M_φ and $M_{r\varphi}$ are the radial, circumferential, and twisting moments per unit length, $h = h(r,\varphi)$ is the local thickness of the plate, $D_r = E_r h^3/12$, $\alpha = E_{r\varphi}/E_r$, $\beta = E_\varphi/E_r$, and $\gamma = G/E_r$.

During bending, in addition to satisfying Eqs. (3) the plate must also satisfy the equilibrium equation

$$\frac{1}{r} (rM_r),_{rr} - \frac{2}{r^2} (rM_{r\varphi}),_{r\varphi} + \frac{1}{r^2} M_{\varphi,\varphi\varphi} - \frac{1}{r} M_{\varphi,r} = - q \quad (4)$$

where q is the intensity of the lateral load on the plate. Since the present problem is concerned with the free vibrations of the plate this intensity is

$$q = - \rho \, h \, w,_{tt} \quad (5)$$

where ρ is the mass density of the plate, t is the time and $w = w\ (r,\varphi,t)$.

Substitution of Eqs. (3) and (5) into Eq. (4) yields the governing equation for the free vibrations of the plate as

$$D_r \left[w,_{rrrr} + 2\,\frac{w,_{rrr}}{r} + 2\alpha \left(\frac{w,_{rr\varphi\varphi}}{r^2} - \frac{w,_{r\varphi\varphi}}{r^3} + \frac{w,_{\varphi\varphi}}{r^4} \right) + \right.$$

$$+ \beta \left(-\frac{w,_{rr}}{r^2} + \frac{w,_r}{r^3} + 2\,\frac{w,_{\varphi\varphi}}{r^4} + \frac{w,_{\varphi\varphi\varphi\varphi}}{r^4} \right) +$$

$$\left. + 4\gamma \left(\frac{w,_{rr\varphi\varphi}}{r^2} - \frac{w,_{r\varphi\varphi}}{r^3} + \frac{w,_{\varphi\varphi}}{r^4} \right) \right] +$$

$$+ D_{r,r} \left[2\,w,_{rrr} + 2\,\frac{w,_{rr}}{r} + \alpha \left(\frac{w,_{rr}}{r} + 2\,\frac{w,_{r\varphi\varphi}}{r^2} - 2\,\frac{w,_{\varphi\varphi}}{r^3} \right) + \right.$$

$$\left. - \beta \left(\frac{w,_r}{r^2} + \frac{w,_{\varphi\varphi}}{r^3} \right) + 4\gamma \left(\frac{w,_{r\varphi\varphi}}{r^2} - \frac{w,_{\varphi\varphi}}{r^3} \right) \right] +$$

$$+ D_{r,rr} \left[w,_{rr} + \alpha \left(\frac{w,_r}{r} + \frac{w,_{\varphi\varphi}}{r^2} \right) \right] +$$

$$+ D_{r,\varphi} \left[2\alpha\,\frac{w,_{rr\varphi}}{r^2} + 2\beta \left(\frac{w,_{r\varphi}}{r^3} + \frac{w,_{\varphi\varphi\varphi}}{r^4} \right) + \right.$$

$$\left. + 4\gamma \left(\frac{w,_{rr\varphi}}{r^2} - \frac{w,_{r\varphi}}{r^3} + \frac{w,_{\varphi}}{r^4} \right) \right] +$$

$$+ D_{r,\varphi\varphi}\left[\alpha \frac{w,_{rr}}{r^2} + \beta\left(\frac{w,_r}{r^3} + \frac{w,_{\varphi\varphi}}{r^4}\right)\right] +$$

$$+ D_{r,r\varphi}\left[4\gamma\left(\frac{w,_{r\varphi}}{r^2} - \frac{w,_\varphi}{r^3}\right)\right] = - \rho\, h\, w,_{tt} \qquad (6)$$

For the axisymmetric case this equation reduces to

$$D_r\, w,_{rrrr} + 2(D_r + r\, D_{r,r}) \frac{w,_{rrr}}{r} +$$

$$+ \left(- \beta D_r + (2+\alpha)\, r\, D_{r,r} + r^2\, D_{r,rr}\right) \frac{w,_{rr}}{r^2} +$$

$$+ \left(\beta D_r - \beta r D_{r,r} + \alpha\, r^2\, D_{r,rr}\right) \frac{w,_r}{r^3} = - \rho h\, w,_{tt} \qquad (7)$$

For the present axisymmetric problem the thickness can vary only in the radial direction and it is assumed to be given by

$$h = h_o\left[1 - \frac{h_o - h_a}{h_o}\left(\frac{r}{a}\right)^n\right] = h_o\,(1 - HR^n) \qquad (8)$$

where, h_o and h_a represent the thickness of the plate at the center $r = 0$, and at the edge of the plate $r = a$, respectively, n is any positive integer and the definitions of R and H are obvious. (For a uniform thickness plate $H = 0$). Thus, by definition, the quantity $D_r = E_r h^3/12$ becomes $D_r = E_r h_o^3 (1-HR^n)^3/12$. Since this quantity must be positive for a full plate everywhere in the interval $0 \leq r \leq a$ it is required that H be less than unity ($H < 1$). Substitution of the thickness h given by Eq. (8) into Eq. (7) leads to the equation

$$(1 - 2\,HR^n + H^2 R^{2n})\, w,_{RRRR} +$$

$$+ 2\left[1 - (2 + 3n)\,HR^n + (1 + 3n)\,H^2R^{2n}\right]\frac{w,_{RRR}}{R} +$$

$$+ \left\{-\beta + \left[2\beta - 3n(1+\alpha+n)\right]HR^n + \left[-\beta + 3n\,(1+\alpha+3n)\right]H^2R^{2n}\right\}\frac{w,_{RR}}{R^2} +$$

$$+ \left\{\beta - \left[\beta(2-3n) + 3\alpha n\,(n-1)\right]HR^n + (1-3n)(\beta-3\alpha n)H^2R^{2n}\right\}\frac{w,_R}{R^3} +$$

$$+ \frac{\rho\, h_o\, a^4}{D_o}\, w,_{tt} = 0 \qquad (9)$$

where $D_o = E_r\, h_o^3/12$. Equation (9) is the governing equation for the axisymmetric free vibration of an orthotropic plate having a thickness variation $h = h_o\,(1-HR^n)$.

The deflection solution to Eq. (9) must satisfy the boundary conditions which depend upon the manner in which the edge of the plate is supported. While for a clamped plate the displacement and the slope must vanish, the displacement and the radial moment must vanish for a simply supported plate. Thus, at $r = a$ the displacement w must satisfy for a clamped plate:

$$w = 0 \qquad \text{and } w' = 0 \qquad (10a)$$

for a simply supported plate:

$$w = 0 \qquad \text{and} \quad w'' + \alpha\, w'/R = 0 \tag{10b}$$

In writing the second of Equations (10b) use has been made of Eq. (3).

In addition to satisfying these conditions the solution must also satisfy "regularity" conditions at r = 0, the center of the plate. These require that at the center the slope be finite* and that the inertia forces be in equilibrium with the resultant internal transverse shear force. The magnitude of the intensity of shear, Q_r, at a radius r may be determined from the condition

$$2\pi\, r\, Q_r = \int_0^r \rho\, h\, 2\pi\, r\, w,_{tt}\, dr \tag{11}$$

It may also be expressed in terms of the moments [5] and hence (see Eq.(3)) in terms of the displacement w. This relation is

$$r\, Q_r = (r\, M_r),_r - M_\varphi$$

$$= -\left\{D_r\left[r\, w,_{rrr} + w,_{rr} - \beta\,\frac{w,_r}{r}\right] + r\, D_{r,r}\left[w,_{rr} + \alpha\,\frac{w,_r}{r}\right]\right\} \tag{12}$$

In view of Eqs. (11) and (12) the second of the "regularity" conditions requires that at r = 0, the expression on the last line of Equation (12) must vanish.

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*For the present axisymmetric case this condition implies that $w,_r = 0$.

3. METHOD OF SOLUTION

In attempting to solve the governing Equation (9) first the non-dimensional variable $R = r/a$ has been introduced. Next, the displacement w is assumed to have the form

$$w\ (R,t) = W(R)\ e^{i\omega t} \tag{13}$$

where W(R) is a function only of R, $i = \sqrt{-1}$ and ω is the circular frequency. When Eq. (13) is substituted into Eq. (9) the time and space variables are separated and an ordinary differential equation in terms of W is obtained as

$$\begin{aligned}
&(1 - 2HR^n + H^2R^{2n})W'''' + \\
&+ 2\left[1 - (2+3n)HR^n + (1+3n)H^2R^{2n}\right]\frac{W'''}{R} + \\
&+ \left\{-\beta + \left[2\beta-3n(1+\alpha+n)\right]HR^n + \left[-\beta+3n(1+\alpha+3n)\right]H^2R^{2n}\right\}\frac{W''}{R^2} + \\
&+ \left\{\beta - \left[\beta(2-3n) + 3\alpha n\ (n-1)\right]HR^n + (1-3n)(\beta-3\alpha n)H^2R^{2n}\right\}\frac{W'}{R^3} + \\
&- \lambda W = 0
\end{aligned} \tag{14}$$

where

$$\lambda = \frac{\rho\omega^2 h_o a^4}{D_o} \tag{15}$$

and a prime over a symbol denotes differentiation with respect to R.

For the solution of Eq. (14) the method of Frobenius is adopted. A series solution for W, about the regular singular point $R = 0$, is assumed in the form

$$W(R) = \sum_{k=0}^{\infty} g_k R^{j+k} \tag{16}$$

where k is the index of the summation, j denotes the exponents of singularity, the g_k represent the coefficients of the series, and g_o is, by assumption, the coefficient of the first term in the series. Since Eq. (14) is a fourth order differential equation, Frobenius' method will yield four exponents of singularity in the solution. Hence, the right hand side of Eq. (16) represents the sum of four series each corresponding to a particular value of j; furthermore, the g_k then represent four sets of coefficients, one for each series.

Substituting the assumed solution (16) into Eq. (14) the following equation is obtained:

$$\sum_{k=0}^{\infty} X(j+k) g_k R^{j+k-4} - \sum_{k=n}^{\infty} Y(j+k-n) H g_{k-n} R^{j+k-4} +$$

$$- \sum_{k=2n}^{\infty} Z(j+k-2n) H^2 g_{k-2n} R^{j+k-4} - \sum_{k=4}^{\infty} \lambda g_{k-4} R^{j+k-4} = 0 \tag{17}$$

where the indices of the summations have been manipulated in such a manner that R appears to the same power in each term and the functions X, Y and Z are defined as

$$X\ (j+k) = (j+k)(j+k-2)[(j+k-1)^2-\beta] \tag{18}$$

$$Y\ (j+k-n) = (j+k-n)\ \Big\{(j+k-n-1)[2(j+k+2n-1)(j+k-n-2)-2\beta + + 3n(1+\alpha+n)] + \beta(2-3n) + 3\alpha n(n-1)\Big\} \tag{19}$$

$$Z\ (j+k-2n) = (j+k-2n)\Big\{(j+k-2n-1)[-(j+k+4n-1)(j+k-2n-2) + + \beta-3n(1+\alpha+3n)] + (1-3n)(3\alpha n-\beta)\Big\} \tag{20}$$

The functional notations X (j+k), Y (j+k-n), and Z (j+k-2n) have been adopted to indicate that these expressions depend not only on the ratios of the material properties α and β, and the power of the thickness variation, n, but also on the value of the exponent of singularity, j, and the index of summation, k, for which they are to be evaluated. Upon introducing the step function notation

$$\delta(k-m) = \begin{cases} 1 & k-m \geq 0 \\ 0 & k-m < 0 \end{cases} \tag{21}$$

and collecting the coefficients of successive powers of R, Eq. (17) may be written in the form

$$X\ (j)\ g_o\ R^{j-4} +$$

$$+ \Big\{X\ (j+1)g_1 - \delta(1-n)\ Y(j+1-n)Hg_{1-n} - \delta(1-2n)\ Z(j+1-2n)H^2g_{1-2n}\Big\}\ R^{j-3} +$$

$$+ \Big\{X\ (j+2)g_2 - \delta(2-n)\ Y(j+2-n)Hg_{2-n} - \delta(2-2n)\ Z(j+2-2n)H^2g_{2-2n}\Big\}\ R^{j-2} +$$

$$+ \left\{ X\,(j+3)g_3 - \delta(3-n)\; Y(j+3-n)Hg_{3-n} - \delta(3-2n)\; Z(j+3-2n)H^2 g_{3-2n} \right\} R^{j-1} +$$

$$+ \sum_{k=4}^{\infty} \left\{ X(j+k)g_k - \delta(k-n)\; Y(j+k-n)Hg_{k-n} - \delta(k-2n)\; Z(j+k-2n)H^2 g_{k-2n} + \right.$$

$$\left. - \lambda g_{k-4} \right\} R^{j+k-4} = 0 \qquad (22)$$

Now, for W(R) given by Eq. (16) to be a solution of Eq. (14) the coefficient of each term of R in Eq. (22) must vanish identically. Thus, equating to zero the coefficient of the term R^{j-4}, the term with the lowest power of R, yields the indicial equation

$$X(j) = j(j-2)[(j-1)^2 - \beta] = 0 \qquad (23)$$

The four roots of this equation are

$$j = 0,\ 2,\ 1 + \sqrt{\beta},\ 1 - \sqrt{\beta} \qquad (24)$$

For each value of j in Eq. (24), the vanishing of the coefficients of the terms R^{j-3}, R^{j-2}, and R^{j-1} gives the following equations for determining g_1, g_2, and g_3:

$$X(j+1)g_1 = \delta(1-n)\; Y(j+1-n)Hg_{1-n} + \delta(1-2n)\; Z(j+1-2n)H^2 g_{1-2n} \qquad (25)$$

$$X(j+2)g_2 = \delta(2-n)\; Y(j+2-n)Hg_{2-n} + \delta(2-2n)\; Z(j+2-2n)H^2 g_{2-2n} \qquad (26)$$

$$X(j+3)g_3 = \delta(3-n)\; Y(j+3-n)Hg_{3-n} + \delta(3-2n)\; Z(j+3-2n)H^2 g_{3-2n} \qquad (27)$$

Finally, for the determination of each of the coefficients g_k for $k \geq 4$, the following recurrence relation is obtained when the coefficient of the term R^{j+k-4} is equated to zero:

$$X(j+k)g_k = \delta(k-n)\ Y(j+k-n)Hg_{k-n} + \delta(k-2n)\ Z(j+k-2n)H^2 g_{k-2n} + \\ + \lambda\ g_{k-4} \qquad (k \geq 4) \tag{28}$$

This relation determines each of the coefficients g_k $(k \geq 4)$ in terms of the preceding g's, and hence in terms of g_o, for each j in Eq. (24).

An inspection of Eqs. (25) thru (28) indicates that it is not possible to write a simple expression for the g_k explicitly in terms of g_o for a general value of n. However, for any particular n the corresponding solution, W(R), can be written [6] as

$$W(R) = a_o \sum_{k=0,4,5,\ldots}^{\infty} A_k R^k + b_o \sum_{k=0}^{\infty} B_k R^{k+2} + c_o \sum_{k=0}^{\infty} C_k R^{k+1+\sqrt{\beta}} + \\ + d_o \sum_{k=0}^{\infty} D_k R^{k+1-\sqrt{\beta}} \tag{29}$$

In Eq. (29) a_o, b_o, c_o, and d_o are undetermined constants, and $A_k = a_k/a_o$, $B_k = b_k/b_o$, $C_k = c_k/c_o$, and $D_k = d_k/d_o$ where the a_k, b_k, c_k, and d_k correspond to the g_k calculated with j = 0, 2, $1+\sqrt{\beta}$, and $1-\sqrt{\beta}$, respectively. The form of the solution given by Eq. (29) applies only for the case in which $\beta \neq p^2/4$, where p is any integer. For cases in which $\beta = p^2/4$

special forms for the solution are required [6]. The isotropic plate ($\beta = 1$), in particular, is one of these cases since the exponents of singularity become $j = 0, 0, 2, 2$, which represents two sets of equal roots, each set differing by an integer.

The solution for $W(R)$ of Eq. (29) must satisfy the "regularity" and boundary conditions discussed previously. In order that the slope be finite at the center of the plate ($R = 0$) the solution corresponding to the root $j = 1-\sqrt{\beta}$ is inadmissible since its derivative is always singular at the origin; hence, $d_o = 0$. It is noted that this condition also insures that the deflection is finite at the center. The second "regularity" condition to be satisfied at the center (Eq. 12) may be written in terms of the non-dimensional variable R as

$$D_r\left[RW''' + W'' - \beta \frac{W'}{R}\right] + RD_r'\left[W'' + \alpha \frac{W'}{R}\right] = 0 \tag{30}$$

Upon substituting Eq. (29) with $d_o = 0$, noting that $\beta \neq 1$ for this form of the solution, and using the definition of D_r, the condition (30) requires that $b_o = 0$. With these results the solution becomes

$$W(R) = a_o \sum_{k=0,4,5,\ldots}^{\infty} A_k R^k + c_o \sum_{k=0}^{\infty} C_k R^{k+1+\sqrt{\beta}} \tag{31}$$

where the remaining constants are to be determined from the boundary conditions at the edge of the plate.

When the boundary conditions (10a) and (10b) are enforced at the edge of the plate (R = 1), the following sets of simultaneous equations are obtained:

$$a_o \sum_{k=0,4,5,\ldots}^{\infty} A_k + c_o \sum_{k=0}^{\infty} C_k = 0$$

$$a_o \sum_{k=4,5,\ldots}^{\infty} kA_k + c_o \sum_{k=0}^{\infty} (k+1+\sqrt{\beta})\ C_k = 0 \qquad \text{(32a)}$$

$$a_o \sum_{k=0,4,5,\ldots}^{\infty} A_k + c_o \sum_{k=0}^{\infty} C_k = 0$$

$$a_o \sum_{k=4,5,\ldots}^{\infty} k(k+\alpha-1)A_k + c_o \sum_{k=0}^{\infty} (k+1+\sqrt{\beta})(k+\alpha+\sqrt{\beta})C_k = 0 \qquad \text{(32b)}$$

where Eqs. (32a) and (32b) correspond to the clamped and simply supported plate, respectively. The solution of Eqs. (32a) and (32b) for non-trivial values of a_o and c_o requires that the determinant of the coefficients of a_o and c_o in Eqs. (32a) and (32b) vanishes. These conditions yield the following characteristic equations for the determination of the natural frequencies

for a clamped plate:

$$\left[\sum_{k=0,4,5\ldots}^{\infty} A_k\right]\left[\sum_{k=0}^{\infty} (k+1+\sqrt{\beta})C_k\right] - \left[\sum_{k=4,5,\ldots}^{\infty} kA_k\right]\left[\sum_{k=0}^{\infty} C_k\right] = 0 \qquad \text{(33a)}$$

for a simply supported plate:

$$\left[\sum_{k=0,4,5,\ldots}^{\infty} A_k\right]\left[\sum_{k=0}^{\infty} (k+1+\sqrt{\beta})(k+\alpha+\sqrt{\beta})C_k\right]-\left[\sum_{k=4,5,\ldots}^{\infty} k(k+\alpha-1)A_k\right]\left[\sum_{k=0}^{\infty} C_k\right]=0 \qquad (33b)$$

The roots of Eqs. (33a) and (33b) give the values of λ which, when substituted into the equation

$$f = \frac{\omega}{2\pi} = \frac{\lambda^{\frac{1}{2}}}{2\pi}\left(\frac{D_o}{\rho h_o a^4}\right)^{\frac{1}{2}} \qquad (34)$$

determine the natural frequencies of the plate.

4. ILLUSTRATIVE EXAMPLE

The equations developed in the previous section were applied to several clamped and simply supported plates of various shapes. In particular plates of uniform thickness and those with thickness variations corresponding to n = 1 and n = 3 were considered. For all cases the material parameters α and β were taken to be 0.3 and 1.44, respectively. The numerical calculations were made restricting the series in Eqs. (33a) and (33b) to include terms up to the twelfth power in λ. The results for $\bar{f} = \left[(a^2/h_o)\sqrt{(\rho/E_r)}\right] f$ corresponding to the lowest two frequencies (see Eq. 34) were computed and are presented in the following table.

		Clamped Plate		Simply Supported Plate	
n	H	$\bar{f}_1$	$\bar{f}_2$	$\bar{f}_1$	$\bar{f}_2$
1	0.1	0.4577	1.793	0.2382	1.356
1	0.25	0.3980	1.617	0.2151	1.234
1	0.5	0.2980	1.309	0.1759	1.023
3	0.1	0.4667	1.836	0.2458	1.399
3	0.25	0.4196	1.723	0.2343	1.340
3	0.5	0.3386	1.509	0.2154	1.222
uniform thickness		0.4975	1.908	0.2536	1.436

All the computations were performed on an IBM 360 computer at the Institute's Computer Center.

5. DISCUSSION OF RESULTS

A method for determining the natural frequencies of clamped and simply supported orthotropic circular plates of radially varying thickness has been presented. From an examination of the numerical results it may be concluded that the natural frequencies for clamped and simply supported plates of a given material tend to decrease as the ratio of the edge thickness of the plate to its thickness at the center is decreased, and tend to increase as the thickness exponent n increases.

The solution for W(R) as given by Eq. (29) is valid everywhere in the plate where the material is properly orthotropic. It has been noted, however, that the form of cylindrical orthotropy considered in this investigation cannot exist at the center of the plate. In order to correct the theory for this singularity in the material properties, the neighborhood of the center of the plate could be treated as an isotropic core of radius, say, Δ. The "regularity" conditions at the center would then be applied to the solution for the isotropic core; and for the complete solution the continuity of $w, (w,_r)$, M_r and Q_r could be enforced at $r = \Delta$, the boundary of the isotropic core and the orthotropic plate. The solution which has been obtained in this investigation may be considered a limiting case of this approach, where Δ is allowed to go to zero. It is this inexactness of the solution at the origin which causes the theory to predict infinite moments at the center for cases where $\beta < 1$, and zero moments at the center for cases where $\beta > 1$. However, it is hoped that the present theory will predict frequencies in good agreement with those which may be predicted by the complete solution with an isotropic core of a very small radius.

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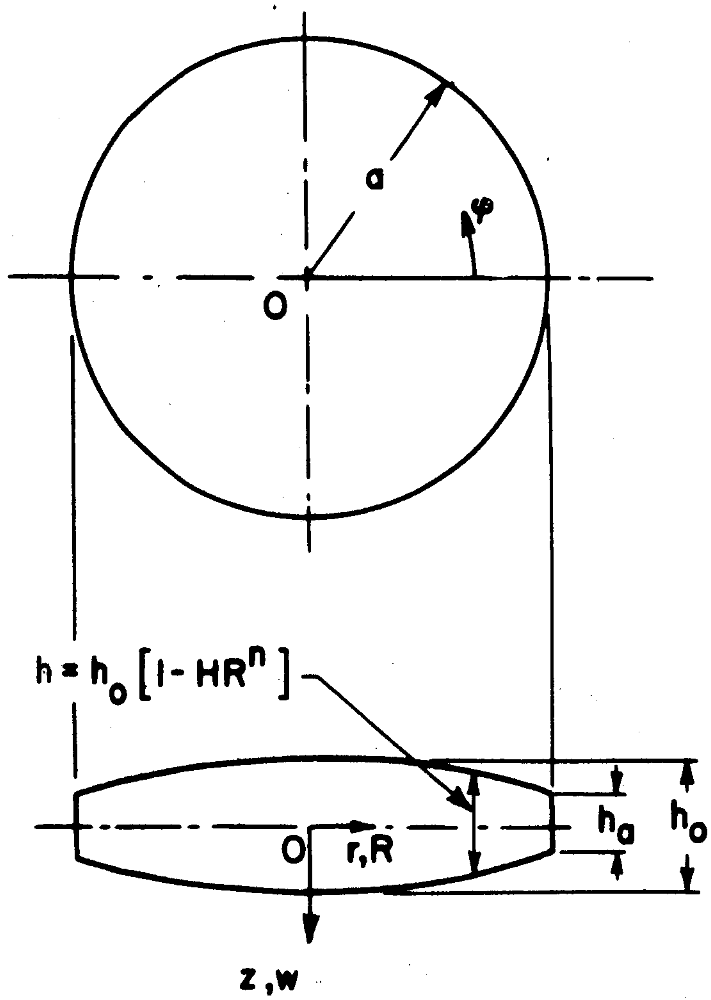

Fig.1 Plate Geometry

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13. ABSTRACT

The present investigation is concerned with the natural frequencies of orthotropic circular plates of variable thickness. In particular, a thickness variation of the form $h = h_o (1-HR^n)$ has been selected. The derivation of the differential equation governing the motion of the plate is based on the classical formulation of the theory of plates. The solution of this equation for the axisymmetric case is obtained by an application of the method of Frobenius. Characteristic equations for the natural frequencies of clamped and simply supported plates are derived and numerical results are presented for several plates of various shapes.

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