A Program to Calculate Particle Spectra

Produced in High Energy Proton-Proton Collisions by

The Two-Temperature Statistical Model

by

J. R. Wayland

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UNIVERSITY OF MARYLAND
DEPARTMENT OF PHYSICS AND ASTRONOMY
COLLEGE PARK, MARYLAND

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I. INTRODUCTION

In the past few years, the lack of success of older statistical model led to new models that rely more on phenomenological observations. One of the most successful models is the two-temperature model of Wayland and Bowen [1]. The analytic form of this model allows easy extension of the predictions to cosmic ray energies.

We will give a brief outline of the two-temperature model in Section II. In Section III, we consider the application of the model. The parameters for the $\pi^{\pm}$, $K^{\pm}$, p and $\bar{p}$ spectra are given. Copies of the programs can be obtained from the author.

II. THE TWO-TEMPERATURE MODEL

The main assumption of the two-temperature model is that there are two characteristic temperatures associated with each type of particle produced; one is associated with the transverse momentum distribution (T_o), the other with the longitudinal momentum distribution (T). The temperature T_o (a constant) describes the natural "thermal" motion of a hot interaction region. Thus, the transverse temperature will give a transverse momentum distribution that is independent of the incident energy. This will be reflected in a constant value of $\langle p_t \rangle$, the average transverse momentum. However, T_o will be different for different mass particles. The temperature, T, can be considered as the quasi-equilibrium condition at which the various particles "boil off" as the interaction volume cools. T will be greater than T_o because it will include the effect of the collective motion of the interacting gas along the direction of the incoming nucleon. We start with a quantum- statistical mechanical average occupation number density

$$\nu = \frac{1}{\exp[\sqrt{p^2 + m^2}/T] \pm 1}\,, \quad \begin{array}{l} + \rightarrow \text{fermions} \\ - \rightarrow \text{bosons} \end{array} \tag{1}$$

where p and m are the momentum and mass of the particles. The normalized momentum distributions are:

$$\omega_\ell^N(p_\ell^*)dp_\ell^* = \frac{T}{m^2c^3}\,\frac{\sum\limits_{k=1}^{\infty} (\pm)^{k+1}[\exp(-\frac{k\mu_1}{T})/k^{3/2}][1 + \frac{k\mu_1}{T}]}{\sum\limits_{k=1}^{\infty} (\pm)^{k+1}K_2(kmc^2/T)/k}\, dp_\ell^* \tag{2}$$

$$\omega_t^N(p_t^*)dp_t^* = \frac{p_t^*\mu_2}{T_o m^2 c^2}\,\frac{\sum\limits_{k=1}^{\infty} (\pm)^{k+1}K_1(k\mu_2/T_o)}{\sum\limits_{k=1}^{\infty} (\pm)^{k+1}K_2(kmc^2/T_o)/k}\, dp_t^* \tag{3}$$

where

$$\mu_1^2 = p_\ell^{*2} + m^2,$$

$$\mu_2^2 = p_t^{*2} + m^2,$$

$$p_\ell^* = \text{longitudinal momentum},$$

$$p_t^* = \text{transverse momentum},$$

and the K's are modified Bessel functions. Starred quantities are in C.M. system. The flux of particles is given by

$$\frac{d^2N}{dp^*d\Omega^*} = \frac{2V_oT^2}{T_o m^2 c^4 h^3} p^{*2}\mu_2 \frac{\sum_{k=1}^{\infty} (\pm)^{k+1} K_1(\frac{k\mu_2}{T_o}) \sum_{k=1}^{\infty} (\pm)^{k+1} \exp(\frac{-k\mu_2}{T})(1 + \frac{k\mu_1}{T})/k^{3/2}}{\sum_{k=1}^{\infty} (\pm)^{k+1} \frac{K_2(kmc^2/T_o)}{k}} \quad (4)$$

We can write

$$T = \text{constant } (E_o^*)^{1/4} \quad (5)$$

which allows us to make calculations as a function of the incident energy, E_o. For pions the interaction volume varies as

$$V_o = V_o' \left\{1 + \frac{k'}{\sigma\sqrt{2\pi}} \exp\{ \frac{-\langle p^*\rangle^2}{2\sigma^2} [1 - \cos (2 \langle\theta^*\rangle - \theta^*)]^2\}\right\} \quad (6)$$

where

$$\tan \langle\theta^*\rangle = \frac{\langle p_t^*\rangle}{\langle p_\ell^*\rangle} = \frac{\pi}{2}\sqrt{\frac{T_o}{T}\frac{K_2(m/T)K_{5/2}(m/T_o)}{K_2(m/T_o)K_{5/2}(m/T)}}, \quad (7)$$

and

$$\langle p^*\rangle^2 = \langle p_t^*\rangle^2 + \langle p_\ell^*\rangle^2 . \quad (8)$$

III. USE OF PROGRAM

One must supply the following input parameters for the program:

W1	=	the rest mass of the proton in GeV/c^2,
EØ	=	the incident energy of the proton in GeV,
W	=	the rest mass of the produced particle in GeV/c^2,
AL	=	the angle in the lab system at which the particles are produced (in radians),
TØ	=	the transverse temperature in GeV,
A5	=	the conversion factor for the longitudinal temperature,
C	=	the normalizat on constant,
A	=	index for passes through program,
SG	=	the first constant for meson volume,
AK	=	the second constant for meson volume,
AM	=	+1 for fermions, -1 for bosons,
M1	=	the number of terms kept in the sums.

The values of the fixed parameters in the correct format are shown in Table I.

For energies $E_o < 50$ GeV, one should take M1 = 3 to insure accurate results for low momentum secondaries. However, above 50 GeV, M1 = 1 is normally acceptable. The A term should be zero (blank) except on the cards for the last spectra wanted. Then one should use any positive floating number greater than zero.

The required Bessel functions are generated by function subprograms. These are generated by a series approximation method [2]. Of course, one can easily use existing programs if it is more expedient.

If the program is to be used for incident energies greater than 1000 GeV the version given in appendix C should be used.

If one does not want the volume variation for the pions set AK = SG = 0.

Table I

Particle	To	A5	C	SG	AK
π^+	1.4E-01	2.89E-01	1.6E+03	5.0E-02	1.7E-01
π^-	1.4E-01	2.41E-01	1.5E+03	5.0E-02	1.7E-01
K^+	1.15E-01	1.92E-01	1.4E+04	-	-
K^-	1.15E-01	1.28E-01	3.3E+04	-	-
P	1.2E-01	3.01E+00	1.8E+03	-	-
$\bar{P}$	1.4E-01	1.02E-01	6.0E+03	-	-

Appendix A

A Listing of the Fortran II Program

```
C
C     THE FOLLOWING PROGRAM COMPUTES THE SPECTRA OF PARTICLES
C     PRODUCED IN PROTON-PROTON COLLISIONS AT HIGH ENERGIES BY THE
C     TWO TEMPERATURE STATISTICAL MODEL.  THE INPUT PARAMETERS REQUIRED
C     ARE AS FOLLOWS
C        W1 = THE REST MASS OF THE PROTON IN GEV/C*C
C        EO = THE INCIDENT ENERGY OF THE PROTON IN GEV
C        W  = THE REST MASS OF THE PRODUCED PARTICLE IN GEV/C*C
C        AL = THE ANGLE IN THE LAB SYSTEM AT WHICH THE PARTICLES ARE
C                PRODUCED (IN RADIANS)
C        TO = THE  TRANVERSE TEMPERATURE IN GEV
C        A5 = THE CONVERSION FACTOR FOR THE LONGITUDAL TEMPERATURE
C        C  = THE NORMALIZATION CONSTANT
C        A  = INDEX FOR PASSES THROUGH PROGRAM
C        SG = THE FIRST CONSTANT FOR MESON VOLUME
C        AK = THE SECOND CONSTANT FOR THE MESON VOLUME
C        AM = +1 FOR FERMIONS, -1 FOR BOSONS
C        M1 = THE NUMBER OF TERMS KEPT IN THE SUMS
C
   10 READ INPUT TAPE 5, 900, W1, EO, W, AL, TO, A5, C, A
  900 FORMAT(8E10.6)
      READ INPUT TAPE 5, 903, SG, AK, AM, M1
  903 FORMAT(3E10.6, I10)
C
C     COMPUTE THE KINEMATIC FACTORS TO CONVERT FROM LAB TO CMS (TO LAB)
C
      T1= EO - W1
      ETCM = SQRTF((W1 + W1)**2 + 2.0*T1*W1)
      GAMMA = (T1+ W1 + W1)/ETCM
      ETA = SQRTF(T1*(T1 + 2.0*W1))/ETCM
      BETA = ETA/GAMMA
C
C     COMPUTE THE  PARAMETERS FOR COMPUTATION OF CROSS-SECTION
C
      T = A5*ETCM**0.25
    9 B2 = (AL*180.0)/3.1415926
      WRITE OUTPUT TAPE 6, 901, B2, TO, T, C, EO, W
  901 FORMAT(///20X,14HANGLE IN LAB =, F7.3,5X, 4HTO =, F7.3,5X,3HT =,
     $ F7.3, 5X, 3HC =, F8.3 , 5X, 4HEO =, F7.3, 5X, 3HW =, F7.4//
     $          20X, 12HANGLE IN CM , 5X, 15HMOMENTUM IN LAB, 5X,
     $ 14HMOMENTUM IN CM, 5X, 16HCM CROSS SECTION, 5X,
     $ 17HLAB CROSS SECTION/)
      PL = 1.00
   57 EL = SQRTF(PL*PL + W*W)
      B1 = SINF(AL)/(GAMMA*(COSF(AL) - (BETA*EL/PL)))
      A2 = ATANF(ABSF(B1))
      IF(B1) 14, 15, 15
   14 ACM = 3.1415926 -A2
      GO TO 16
   15 ACM = A2
   16 B3 = ACM*(180.0/3.1415926)
      P = SQRTF((PL*SINF(AL))**2 + (GAMMA**2)*(PL*COSF(AL) -
     1 BETA*EL)**2)
      IF(AK) 21, 21, 22
   21 C1 = C
      GO TO 56
   22 AP1 = (SQRTF(W*T/1.5707963)*BK52(W/T))/BK2(W/T)
      AP2 = (SQRTF(1.5707963*W*TO)*BK52(W/TO))/BK2(W/TO)
      AP = SQRTF(AP1*AP1 + AP2*AP2)
      A1 = SG*SQRTF(6.2831852)
      ATH = ATANF(AP2/AP1)
```

```
      C1 = C*(1. + AK*EXPF(-(AP*AP*(1. - COSF(2.*ATH - ACM))**2)
     1 /(2.*SG*SG))/A1)
   56 P1 = P*COSF(ACM)
      P2 = P*SINF(ACM)
      U1 = SQRTF(P1*P1 + W*W)
      U2 = SQRTF(P2*P2 + W*W)
C
C     COMPUTE THE CROSS SECTION IN THE CMS
C
      X = U2/TO
      Y = U1/T
      B4 = 0.0
      B5 = 0.0
      DO  67 I=1, M1
      AI = I
      X1 = AI*X
      B4 = B4 +  BK1(X1)*AM**(I+1)
      Y1 = AI*Y
   67 B5 = B5 +  (EXPF(-Y1)*(1.0 + Y1)*AM**(I+1))/(AI**1.5)
      DCS = C1*P*P*T*T*U2*B4*B5
C
C     COMPUTE THE CROSS SECTION IN THE LAB
C
      ECM = SQRTF(P*P + W*W)
      CONV = (ECM*PL*PL)/(EL*P*P)
      DCSL = CONV*DCS
      WRITE OUTPUT TAPE 6, 902, B3, PL, P, DCS, DCSL
  902 FORMAT(20X, F7.3, 9X, F10.6, 10X, F10.6, 10X, F10.5, 10X, F10.5)
      PL = PL + 1.00
      IF(PL -  FO ) 57, 57, 5
    5 IF(A) 10, 10, 4
    4 CALL EXIT
      END
      FUNCTION BK1(X)
      IF(X - 2.0) 20, 21, 21
   20 S= X/2.
      T = X/3.75
      BK1 =(1.0/X)*(X* LOGF(X/2.0)*X*(0.5 + .87890594*T*T +
     1 .51498869*T**4 + .15084934*T**6 + .02658733*T**8 +
     2 .00301532*T**10 + .00032411*T**12) + 1. +
     3 .15443144*S*S - .67278579*S**4 - .18156879*S**6 -
     4 .01919402*S**8 - .00110404*S**10 - .00004686*S**12)
      RETURN
   21 S = 2./X
      BK1 = EXPF(-X)*(1.0/SQRTF(X))*(1.25331414 + .23498619*S -
     1 .03655620*S*S + .01504268*S**3 - .00780353*S**4 +
     2 .00325614*S**5 - .00068245*S**6)
      RETURN
      END
      FUNCTION BIO(X)
      T = X/3.75
      BIO = 1.0 + 3.5156229*T*T + 3.0899424*T**4 +
     1  1.2067492*T**6 + .2659732*T**8 + .0360768*T**10 +
     2 .0045813*T**12
      RETURN
      END
      FUNCTION BKO(X)
      IF(X - 2.0) 100, 101, 101
  100 R = X/2.0
      BKO =  -LOGF(R)*BIO(X) - 0.57721566 + 0.42278420*R*R +
     1 .23069756*R**4 + .0348859*R**6 + .00262698*R**8 +
```

```
    2 .0001075*R**10 + .0000074*R**12
      RETURN
  101 S = 2.0/X
      BKO = (EXPF(-X)/SQRTF(X))*(1.25331414 - 0.07832358*S +
    1 .02189568*S**2- .01062446*S**3 + .0058782*S**4 -
    2 .0025154*S**5 + .00053208*S**6)
      RETURN
      END
      FUNCTION BK2(X)
      BK2 = (2.0/X)*BK1(X) + BKO(X)
      RETURN
      END
      FUNCTION BK52(X)
      BK52 = SQRTF(1.707963/X)*EXPF(-X)*(1.+3./X+3./(X*X))
      RETURN
      END
*     DATA
```

Appendix B

A Listing of the Fortran IV Program

```
$IBFTC MAIN
C
C     THE FOLLOWING PROGRAM COMPUTES THE SPECTRA OF PARTICLES
C     PRODUCED IN PROTON-PROTON COLLISIONS AT HIGH ENERGIES BY THE
C     TWO TEMPERATURE STATISTICAL MODEL.  THE INPUT PARAMETERS REQUIRED
C     ARE AS FOLLOWS
C        W1 = THE REST MASS OF THE PROTON IN GEV/C*C
C        EO = THE INCIDENT ENERGY OF THE PROTON IN GEV
C        W  = THE REST MASS OF THE PRODUCED PARTICLE IN GEV/C*C
C        AL = THE ANGLE IN THE LAB SYSTEM AT WHICH THE PARTICLES ARE
C                PRODUCED (IN RADIANS)
C        TO = THE  TRANVERSE TEMPERATURE IN GEV
C        A5 = THE CONVERSION FACTOR FOR THE LONGITUDAL TEMPERATURE
C        C  = THE NORMALIZATION CONSTANT
C        A  = INDEX FOR PASSES THROUGH PROGRAM
C        SG = THE FIRST CONSTANT FOR MESON VOLUME
C        AK = THE SECOND CONSTANT FOR THE MESON VOLUME
C        AM = +1 FOR FERMIONS, -1 FOR BOSONS
C        M1 = THE NUMBER OF TERMS KEPT IN THE SUMS
C
   10 READ(5,900) W1, EO, W, AL, TO, A5, C, A
  900 FORMAT(8E10.6)
      READ(5,903) SG, AK, AM, M1
  903 FORMAT(3E10.6, I10)
C
C     COMPUTE THE KINEMATIC FACTORS TO CONVERT FROM LAB TO CMS (TO LAB)
C
      T1= EO - W1
      ETCM = SQRT((2.*W1)**2 + 2.*T1*W1)
      GAMMA = (T1+ W1 + W1)/ETCM
      ETA = SQRT(T1*(T1 + 2.*W1))/ETCM
      BETA = ETA/GAMMA
C
C     COMPUTE THE  PARAMETERS FOR COMPUTATION OF CROSS-SECTION
C
      T = A5*ETCM**0.25
    9 B2 = (AL*180.0)/3.1415926
      WRITE(6,901) B2, TO, T, C, EO, W
  901 FORMAT(///20X,14HANGLE IN LAB =, F7.3,5X, 4HTO =, F7.3,5X,3HT =,
     $ F7.3, 5X, 3HC =, F8.3 , 5X, 4HEO =, F7.3, 5X, 3HW =, F7.4//
     $         20X, 12HANGLE IN CM , 5X, 15HMOMENTUM IN LAB, 5X,
     $ 14HMOMENTUM IN CM , 5X, 16HCM CROSS SECTION, 5X,
     $ 17HLAB CROSS SECTION/)
      PL = 1.00
   57 EL = SQRT(PL*PL + W*W)
      B1 = SIN(AL)/(GAMMA*(COS(AL) - (BETA*EL/PL)))
      A2 = ATAN(ABS(B1))
      IF(B1) 14, 15, 15
   14 ACM = 3.1415926 -A2
      GO TO 16
   15 ACM = A2
   16 B3 = ACM*(180.0/3.1415926)
      P = SQRT((PL*SIN(AL))**2 + (GAMMA**2)*(PL*COS(AL) - BETA*EL)**2)
      IF(AK) 21, 21, 22
   21 C1 = C
      GO TO 56
   22 AP1 = (SQRT(W*T/1.5707963)*BK52(W/T))/BK2(W/T)
      AP2 = (SQRT(1.5707963*W*TO)*BK52(W/TO))/BK2(W/TO)
      AP = SQRT(AP1*AP1 + AP2*AP2)
      ATH = ATAN(AP2/AP1)
      C1 = C*(1. + AK*EXP(-(AP*AP*(1. - COS(2.*ATH - ACM))**2)
```

```
     1 /(2.*SG*SG))/(SG*.3993899))
  56 P1 = P*COS(ACM)
     P2 = P*SIN(ACM)
     U1 = SQRT(P1*P1 + W*W)
     U2 = SQRT(P2*P2 + W*W)
C
C     COMPUTE THE CROSS SECTION IN THE CMS
C
     X = U2/T0
     Y = U1/T
     B4 = 0.0
     B5 = 0.0
     DO  67 I=1, M1
     AI = I
     X1 = AI*X
     B4 = B4 +  BK1(X1)*AM**(I+1)
     Y1 = AI*Y
  67 B5 = B5 + (EXP(-Y1)*(1. + Y1)*AM**(I+1))/(AI**1.5)
     DCS = C1*P*P*T*T*U2*B4*B5
C
C     COMPUTE THE CROSS SECTION IN THE LAB
C
     ECM = SQRT(P*P + W*W)
     CONV = (ECM*PL*PL)/(EL*P*P)
     DCSL = CONV*DCS
     WRITE(6,902) B3, PL, P, DCS, DCSL
 902 FORMAT(20X, F7.3, 9X, F10.6, 10X, F10.6, 10X, F10.5, 10X, F12.8)
     PL = PL + 1.00
     IF(PL -  EO ) 57, 57, 5
   5 IF(A) 10, 10, 4
   4 RETURN
     END
$IBFTC I0
     FUNCTION BI0(X)
     IF(X- 3.75) 3, 3, 4
   3 T = X/3.75
     BI0 = 1.0 + 3.5156229*T*T + 3.0899424*T**4 +
    1  1.2067492*T**6 + .2659732*T**8 + .0360768*T**10 +
    2 .0045813*T**12
     RETURN
   4 T = 3.75/X
     BI0 = (EXP(X)/SQRT(X))*(0.39894228 + 0.01328592*T +
    $ 0.00225319*T*T - 0.00157565*T**3 + 0.00916281*T**4
    $ - 0.02057706*T**5 + 0.02635537*T**6
    $ -0.01647633*T**7 + 0.00392377*T**8)
     RETURN
     END
$IBFTC K0
     FUNCTION BK0(X)
     IF(X - 2.0) 100, 101, 101
 100 R = X/2.0
     BK0 =  -ALOG(R)*BI0(X) - 0.57721566 + 0.42278420*R*R +
    1 .23069756*R**4 + .0348859*R**6 + .00262698*R**8 +
    2 .0001075*R**10 + .0000074*R**12
     RETURN
 101 S = 1.0/X
     BK0 = EXP(-X)*SQRT(S)*(1.2533141373 - 0.1566641816*S
    $ +0.0881112782*S*S -0.0913909546*S**3
    $ +0.1344596228*S**4 -0.2299850328*S**5
    $+0.3792409730*S**6 - 0.5247277331*S**7)
     RETURN
```

```
      END
$IBFTC K1
      FUNCTION BK1(X)
      IF(X - 2.0) 20, 21, 21
   20 S= X/2.
      T = X/3.75
      BK1 = (1./X)*(X*ALOG(S)*X*(.5 + .87890594*T*T +
     1 .51498869*T**4 + .15084934*T**6 + .02658733*T**8 +
     2 .00301532*T**10 + .00032411*T**12) + 1. +
     3 .15443144*S*S - .67278579*S**4 - .18156879*S**6 -
     4 .01919402*S**8 - .00110404*S**10 - .00004686*S**12)
      RETURN
   21 S = 2./X
      BK1 = EXP(-X)*(1./SQRT(X))*(1.25331414 + .23498619*S -
     1 .03655620*S*S + .01504268*S**3 - .00780353*S**4 +
     2 .00325614*S**5 - .00068245*S**6)
      RETURN
      END
$IBFTC K2
      FUNCTION BK2(X)
      BK2 = (2.0/X)*BK1(X) + BK0(X)
      RETURN
      END
$IBFTC K52
      FUNCTION BK52(X)
      BK52 = (1.2533141*EXP(-X)*(1.+3./X+3./(X*X)))/SQRT(X)
      RETURN
      END
$DATA
```

Appendix C

A Listing of the Program for Energies Greater than 1000 GeV

```
$IBFTC MAIN
C
C     THE FOLLOWING PROGRAM COMPUTES THE SPECTRA OF PARTICLES
C     PRODUCED IN PROTON-PROTON COLLISIONS AT HIGH ENERGIES BY THE
C     TWO TEMPERATURE STATISTICAL MODEL.  THE INPUT PARAMETERS REQUIRED
C     ARE AS FOLLOWS
C        W1 = THE REST MASS OF THE PROTON IN GEV/C*C
C        EO = THE INCIDENT ENERGY OF THE PROTON IN GEV
C        W  = THE REST MASS OF THE PRODUCED PARTICLE IN GEV/C*C
C        AL = THE ANGLE IN THE LAB SYSTEM AT WHICH THE PARTICLES ARE
C               PRODUCED (IN RADIANS)
C        TO = THE  TRANVERSE TEMPERATURE IN GEV
C        A5 = THE CONVERSION FACTOR FOR THE LONGITUDAL TEMPERATURE
C        C  = THE NORMALIZATION CONSTANT
C        A  = INDEX FOR PASSES THROUGH PROGRAM
C        SG = THE FIRST CONSTANT FOR MESON VOLUME
C        AK = THE SECOND CONSTANT FOR THE MESON VOLUME
C        AM = +1 FOR FERMIONS, -1 FOR BOSONS
C        PC = THE LOWEST MOMENTUM OF PRODUCED PARTICLE IN GEV/C
C        M1 = THE NUMBER OF TERMS KEPT IN THE SUMS
C
C     THIS VERSION IS FOR INCIDENT ENERGIES GREATER THAN 1000 GEV
C
   10 READ(5,900) W1, EO, W, AL, TO, A5, C, A
  900 FORMAT(8E10.6)
      READ(5,903) SG, AK, AM, PC, M1
  903 FORMAT(4F10.6, I10)
C
C     COMPUTE THE KINEMATIC FACTORS TO CONVERT FROM LAB TO CMS (TO LAB)
C
      T1= EO - W1
      ETCM = SQRT((2.*W1)**2 + 2.*T1*W1)
      GAMMA = (T1+ W1 + W1)/ETCM
      ETA = SQRT(T1*(T1 + 2.*W1))/ETCM
      BETA = ETA/GAMMA
C     COMPUTE THE  PARAMETERS FOR COMPUTATION OF CROSS-SECTION
C
C
      T = A5*ETCM**0.25
    9 B2 = (AL*180.0)/3.1415926
      WRITE(6,901) TO, T, C, EO, W
  901 FORMAT(///20X, 4HTO =, F10.4, 5X, 3HT =, F10.4, 5X, 3HC =, F10.4,
     $ 5X, 4HEO =, E14.7, 5X, 3HW =, F10.4//
     $ 20X, 15HMOMENTUM IN LAB, 10X,
     $ 14HMOMENTUM IN CM, 10X, 16HCM CROSS SECTION, 10X,
     $ 17HLAB CROSS SECTION/)
      PL = PC
   57 EL = SQRT(PL*PL + W*W)
      IF(AK) 21, 21, 22
   21 C1 = C
      GO TO 56
   22 AP1 = (SQRT(W*T/1.5707963)*BK52(W/T))/BK2(W/T)
      AP2 = (SQRT(1.5707963*W*TO)*BK52(W/TO))/BK2(W/TO)
      AP = SQRT(AP1*AP1 + AP2*AP2)
      ATH = ATAN(AP2/AP1)
      C1 = C*(1. + AK*EXP(-(AP*AP*(1. - COS(2.*ATH - ACM))**2)
     1 /(2.*SG*SG))/(SG*.3993899))
   56 P1 = (PL*COS(AL))/(2.*GAMMA)
      P2 = PL*SIN(AL)
      P = SQRT(P1*P1 + P2*P2)
      U1 = SQRT(P1*P1 + W*W)
```

```
      U2 = SQRT(P2*P2 + W*W)
C
C     COMPUTE THE CROSS SECTION IN THE CMS
C
      X = U2/TO
      Y = U1/T
      B4 = 0.0
      B5 = 0.0
      DO  67 I=1, M1
      AI = I
      X1 = AI*X
      B4 = B4 +  BK1(X1)*AM**(I+1)
      Y1 = AI*Y
   67 B5 = B5 + (EXP(-Y1)*(1. + Y1)*AM**(I+1))/(AI**1.5)
      DCS = C1*P*P*T*T*U2*B4*B5
C
C     COMPUTE THE CROSS SECTION IN THE LAB
C
      ECM = SQRT(P*P + W*W)
      CONV = (ECM*PL*PL)/(EL*P*P)
      DCSL = CONV*DCS
      WRITE(6,902) PL, P, DCS, DCSL
  902 FORMAT(20X, 1PE14.7, 10X, 1PE14.7, 10X, 1PE14.7, 10X, 1PE14.7)
      PL = PL + PC
      IF(PL - 10.*PC) 57, 30, 30
   30 PC = 10.*PC
      PL = PC
      IF(PC - EO) 57, 5, 5
    5 IF(A) 10, 10, 4
    4 RETURN
      END
$IBFTC IO
      FUNCTION BIO(X)
      IF(X- 3.75) 3, 3, 4
    3 T = X/3.75
      BIO = 1.0 + 3.5156229*T*T + 3.0899424*T**4 +
     1  1.2067492*T**6 + .2659732*T**8 + .0360768*T**10 +
     2 .0045813*T**12
      RETURN
    4 T = 3.75/X
      BIO = (EXP(X)/SQRT(X))*(0.39894228 + 0.01328592*T +
     $ 0.00225319*T*T - 0.00157565*T**3 + 0.00916281*T**4
     $ - 0.02057706*T**5 + 0.02635537*T**6
     $ -0.01647633*T**7 + 0.00392377*T**8)
      RETURN
      END
$IBFTC KO
      FUNCTION BKO(X)
      IF(X - 2.0) 100, 101, 101
  100 R = X/2.0
      BKO =  -ALOG(R)*BIO(X) - 0.57721566 + 0.42278420*R*R +
     1 .23069756*R**4 + .0348859*R**6 + .00262698*R**8 +
     2 .0001075*R**10 + .0000074*R**12
      RETURN
  101 S = 1.0/X
      BKO = EXP(-X)*SQRT(S)*(1.2533141373 - 0.1566641816*S
     $ +0.0881112782*S**2 -0.0913909546*S**3
     $ +0.1344596228*S**4 -0.2299850328*S**5
     $+0.3792409730*S**6 - 0.5247277331*S**7)
      RETURN
      END
```

```
$IBFTC K1
      FUNCTION BK1(X)
      IF(X - 2.0) 20, 21, 21
   20 S= X/2.
      T = X/3.75
      BK1 = (1./X)*(X*ALOG(S)*X*(.5 + .87890594*T*T +
     1 .51498869*T**4 + .15084934*T**6 + .02658733*T**8 +
     2 .00301532*T**10 + .00032411*T**12) + 1. +
     3 .15443144*S*S - .67278579*S**4 - .18156879*S**6 -
     4 .01919402*S**8 - .00110404*S**10 - .00004686*S**12)
      RETURN
   21 S = 2./X
      BK1 = EXP(-X)*(1./SQRT(X))*(1.25331414 + .23498619*S -
     1 .03655620*S*S + .01504268*S**3 - .00780353*S**4 +
     2 .00325614*S**5 - .00068245*S**6)
      RETURN
      END
$IBFTC K2
      FUNCTION BK2(X)
      BK2 = (2.0/X)*BK1(X) + BK0(X)
      RETURN
      END
$IBFTC K52
      FUNCTION BK52(X)
      BK52 = (1.2533141*EXP(-X)*(1.+3./X+3./(X*X)))/SQRT(X)
      RETURN
      END
$DATA
```