THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
DEPARTMENT OF AEROSPACE ENGINEERING

Final Report

Fluid Mechanics of Magnetically Balanced Arcs in Cross Flows

S. W. BOWEN

Supported by:

National Aeronautics and Space Administration
Grant No. NGR 23-005-128
Washington, D.C.

Administered through:

July 1968

OFFICE OF RESEARCH ADMINISTRATION • ANN ARBOR

Final Report

FLUID MECHANICS OF MAGNETICALLY BALANCED ARCS IN CROSS FLOWS

S. W. Bowen

ORA Project 07912

Supported by

National Aeronautics and Space Administration
Grant No. NGR 23-005-128
Washington, D. C.

Administered through

OFFICE OF RESEARCH ADMINISTRATION
The University of Michigan
Ann Arbor

July 1968

SUMMARY OF EFFORT

NASA Grant NGR 24-005-128 has been concerned with the properties of magnetically balanced arcs in a supersonic flow of air.

In the following summary, several findings of interest are briefly discussed. For a further discussion of these points, the references listed in Appendix A should be consulted.

The magnetically balanced arc behaves somewhat like a solid body, impervious with respect to the external flow. This behavior was experimentally verified by the fact that small magnesium tracer particles introduced into the arc near the upstream anode root were carried along within the arc column for a substantial fraction of the arc length, at velocities on the order of 10 m/sec. The free stream velocity was about 600 m/sec. The significant similarity parameters for the magnetically balanced arc are:

1. The Lorentz convection parameter

$$L_c = \frac{\rho_1 \, BI \, d_1}{u_1^2}$$

where B and I are the magnetic field and arc current, ρ_1, d_1 and u_1 are a reference density arc, dimension and velocity given by $u_1 = \sqrt{\sigma_1 \, EB \, d_s / \rho_1}$, σ_1 is a reference electrical conductivity and E the electrical field strength.

2. The power loading

$$P_L = EI_1/\phi_1$$

where ϕ_1 is a reference heat flux potential

$$\phi = \int kdT$$

and k the thermal conductivity.

3. The Prandtl number

$$Pr_1 = \mu_1 h_1/\phi_1$$

and

4. The Eckert number

$$K_1 = u_1^2/h_1$$

where h_1 is a reference enthalpy.

A combination of the above parameters, the joule heating parameter, can be formed

$$K_{JH} = P_L/Pr_1 \sqrt{L_c}$$

K_{JH} is the ratio of joule heating to energy transfer by convection and is useful in characterizing when L_c is "large" or "small".

L_c plays a dominant role in determining the arc behavior. One find that P_L correlates with L_c for both subsonic and supersonic flows.

For large L_c

$$EI/\phi \sim \sqrt{L_c}$$

while for lower L_c one must correlate $(E - E_o)I/Pr_1 \phi_1$ vs L_c.

Numerical studies performed on a digital computer indicate a sharp ridge in the interior temperature distribution is developed together with an interior doublet flow pattern. The temperature ridge is pushed forward in the Lorentz direction with increasing values of the magnetic field. The doublet flow pattern does not appear to strongly depend on external boundary conditions.

Some qualitative experimental confirmation for this internal arc structure was obtained by comparing the side on transverse luminosity profile, obtained by numerically integrating the visible air radiation volume emission coefficient along the line of sight through the arc, with the experimentally measured profiles. Although the relative intensity distributions are similar, they are not exactly the same and the experimental radiances are 3 orders of magnitude greater than theoretically calculated. The reason for this difference is not understood.

The reason for the arc slant is not understood. It does not appear to be related to the Hall effect. Although usually the anode root is upstream, this behavior can be reversed using blunt rather than cone-cylinder electrodes. It seems to be related to the flow field and shock wave pattern between the electrodes. The slant angle is usually quite close to the Mach angle. It is interesting that even subsonic balanced arcs on rail electrodes show a slant*. One may conjecture that the arc slant may be related to the stability of the arc. Such an analysis is far beyond that presently possible however.

Appendix A contains a list of all publications generated on this NASA Grant.

Appendix B is an abstract of the Ph.D. thesis of L.M. Nicolai which was satisfactorily accepted by the Graduate School and has been submitted to NASA for publication as a NASA Contractor report.

---

*Winograd, Y.Y. and Klein, J.F., "Electric Arc Stabilization in Crossed Convective and Magnetic Fields," AIAA Paper 68-709, AIAA Fluid and Plasma Dynamics Conference, June 1968.

APPENDIX A

List of Publications from NASA Grant NGR 23-005-128

Kuethe, A.M., Harvey, R.L., Nicolai, L.M., "Model of an Electric Arc Balanced Magnetically in a Gas Flow," AIAA Paper 67-96 presented at Fifth AIAA Aerospace Science Meeting, Jan. 23-26, 1967.

Kuethe, A.M., Harvey, R.L., Nicolai, L.M., "Model of an Electric Arc Balanced Magnetically in a Gas Flow," First Annual Report, ORA Project 07912, Apr. 1967, 07912-1-P.

Kuethe, A.M., "Fluid Mechanics of Magnetically Balanced Arcs in Cross Flows," Progress Report, ORA Project 07912, Sept. 1967, 07912-3-P.

Bowen, S.W., "Fluid Mechanics of Magnetically Balanced Arcs in Cross Flows," Progress Report, ORA Project 07912, Mar. 1968, 07912-4-P.

Kuethe, A.M., "Fluid Mechanical Structure of Balanced Arcs," Invited Lecture, Proceedings Midwest Conference on Mechanics, Colorado State University (extended abstract, in Press).

Harvey, R.L., "An Experimental and Theoretical Investigation of Magnetically Balanced Arcs," Ph.D. Thesis, University of Michigan, 1968.

Nicolai, L.M., "An Experimental and Theoretical Analysis of the Convected Balanced Arc," Ph.D. Thesis, University of Michigan, 1968.

APPENDIX B

An Experimental and Theoretical Analysis of the Convected Balanced Arc*

The purpose of this investigation was to examine the behavior of a D.C. electric arc in a cross-flow and transverse magnetic field. The experimental part of the investigation was carried out on high current arcs (175 to 500 amperes) balanced by a magnetic field B in a supersonic airstream at Mach numbers M_∞ of 2.5, 3.0, and 3.5.

The arc slanted in the airstream with the anode spot upstream of the cathode. The slant angle was close to the free stream Mach angle and increased slightly with increasing current. The velocity normal to the arc varied approximately as ${u_{\infty_n}}^4 \sim I$. The measured electric field E was nearly constant for a given M_∞.

A reference station 1/3 the distance from the anode to the cathode was selected for all the measurements. The streamwise d_S and transverse d_T dimensions were measured from photographs with d_T about 50% larger than d_S. Both dimensions increased with current according to $d \sim \sqrt{I}$. The value of B at the reference station was nearly constant for a given M_∞. The normal force coefficient C_D based upon d_T was nearly constant at 1.75 for all three Mach numbers.

---

*Ph. D. Abstract by Leland Malcolm Nicolai.

Microdensitometer scans of the arc photographs showed the peak temperature well forward of the arc axis. Results from the numerical solution of the governing equations showed the same feature. The measured radiance distribution was three orders of magnitude larger than the calculated distribution. The measured and calculated radiance distributions had similar shapes, however, suggesting similar kinetic temperature distributions.

The theoretical analysis was based upon a continuum fluid and the experimental observation that the convected balanced arc is impervious to the free stream. The physical model chosen for the arc column consists of 1) an arc core, 2) an external flow and 3) an interaction region. The model assumes that energy transfer by radiation and mass transfer is small compared with EI.

A characteristic velocity for the arc core is $u_1 = \sqrt{\sigma_1 \, E \, B \, d_S / \rho_1}$. Based upon u_1, a reference temperature T_1 and dimension d_1 the significant similarity parameters for the arc core are $L_c = \rho_1 \, B I \, d_1 / u_1^2$, $P_L = EI_1/\phi_1$, $Pr_1 = \mu_1 \, h_1/\phi_1$ and $K_1 = u_1^2/h_1$. Here the quantities μ_1, h_1, ϕ_1, ρ_1 and σ_1 are the viscosity, static enthalpy, heat flux potential, density and electrical conductivity respectively, evaluated at T_1. Since the arc is balanced with C_D nearly constant, $\sqrt{L_c} \sim \rho_\infty u_{\infty_n} d_1/\mu_\infty$.

As L_c becomes large (i.e. large u_{∞_n}) the energy equation simplifies to the joule heating and convection terms. At large L_c the coefficient of the joule heating term $K_{JH} = EI/\phi_1 \, Pr_1 \sqrt{L_c}$ becomes constant and we must have $EI/\phi_1 \, Pr_1 \sim \sqrt{L_c}$. The characteristic relation for large L_c and constant pressure is $E \sim u_{\infty_n}^{2/3} / I^{1/3}$.

The experimental results from Roman (ARL Report 66-0191) and UM (the present investigation) were used to test the theoretical model. For large L_c the theory simplies the energy equation considerably and predicts $EI/\phi_1 \sim \sqrt{L_c}$ which is in close agreement with UM data. Roman's data, however, displays an $EI/\phi_1 \sim L_c$ behavior and is in the low L_c region (i.e. low u_{∞_n}) where the simplified energy equation is not valid.

If the data from Roman and UM is expressed as log $\left\{(E - E_0)I/Pr_1 \phi_1\right\}$ versus log L_c, Roman's data collapses about a curve having a slope of 1.0 and the UM data collapses about a curve having a slope near 1/2. A continuous curve drawn through the two sets of data appears to describe the macroscopic behavior of the convected balanced arc from low L_c to large L_c.